Chapter 17

Similarity of Triangles

POINTS TO REMEMBER

1. **Similar Triangles :**

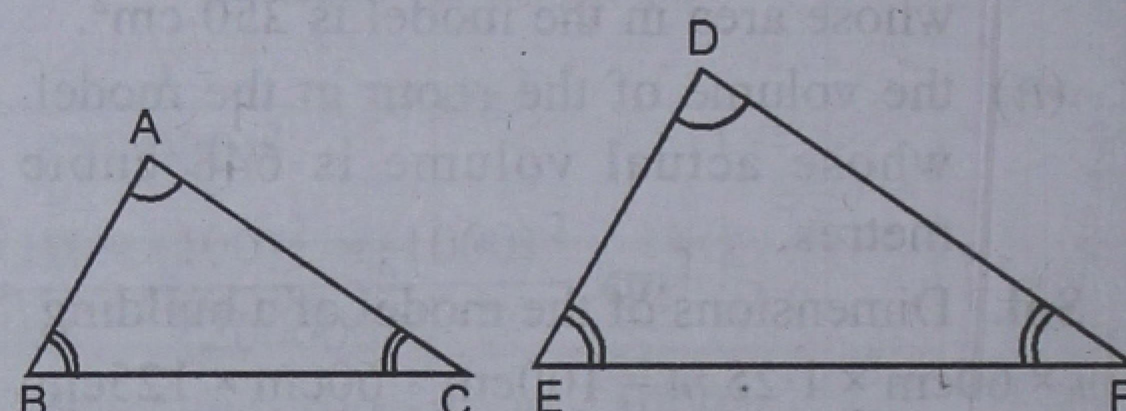
   Δ ABC and Δ DEF are said to be similar, if their corresponding angles are equal and their corresponding sides are proportional.

   i.e., when ∠A = ∠D, ∠B = ∠E, ∠C = ∠F

   and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.

   Then we write, **ΔABC ~ ΔDEF.**

   The sign '~' is read as **'is similar to'.**



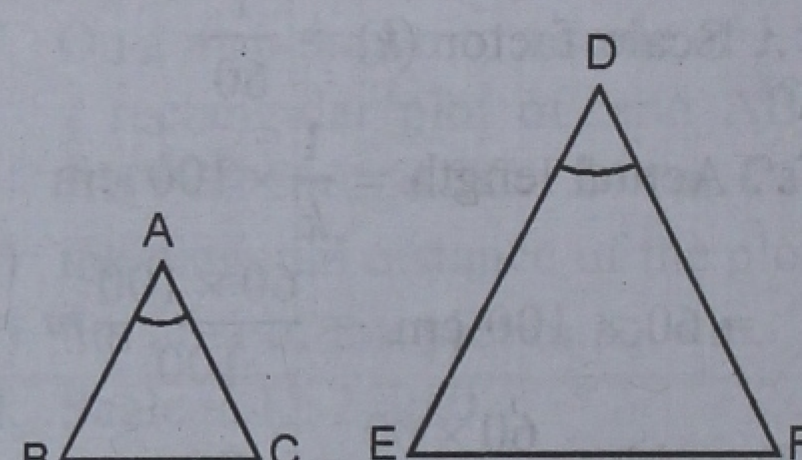
2. **Three Similarity Axioms For Triangles :**

   (*i*) **SAS-Axiom :** *If two triangles have a pair of corresponding angles equal and the sides including them proportional, then the triangles are similar.*

   If in ΔABC and ΔDEF, we have

   ∠A = ∠D and $\frac{AB}{DE} = \frac{AC}{DF}$ then,

   ΔABC ~ ΔDEF.

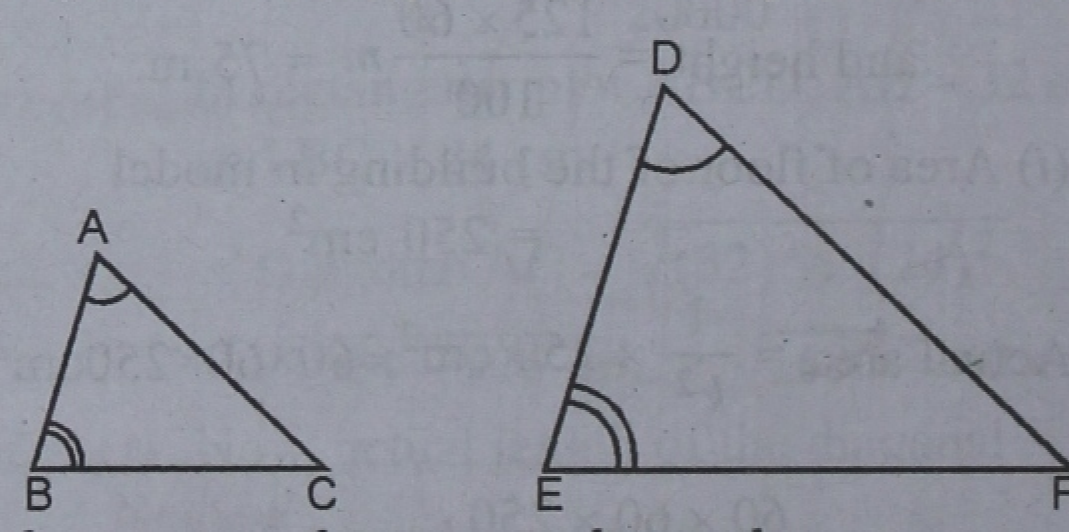


   (*ii*) **AA–Axiom or AAA–Axiom :** *If two triangles have two pairs of corresponding angles equal, the triangles are similar.*

   If in ΔABC and ΔDEF, we have

   ∠A = ∠D and ∠B = ∠E, then

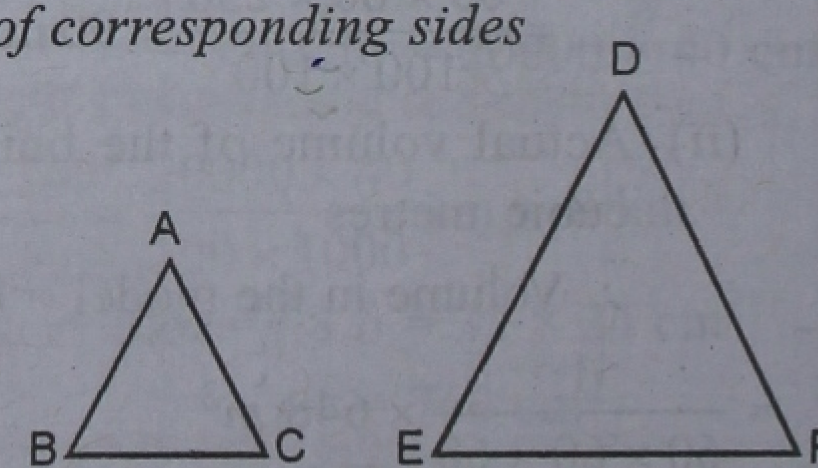
   ΔABC ~ ΔDEF.



   (*iii*) **SSS–Axiom :** *If two triangles have their three pairs of corresponding sides proportional, then the triangles are similar.*

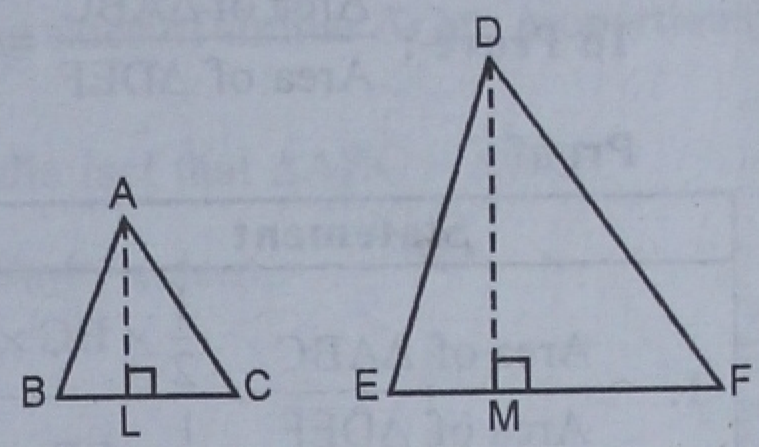
   If in ΔABC and ΔDEF, we have

   $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$, then ΔABC ~ ΔDEF.



3. Results on Area of Similar Triangles (Theorems)

Theorem 1. *The areas of two similar triangles are proportional to have squares on their corresponding sides.*



Given : Δ ABC ~ Δ DEF.

To Prove : $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{AB^2}{DE^2} = \dfrac{BC^2}{EF^2} = \dfrac{AC^2}{DF^2}$

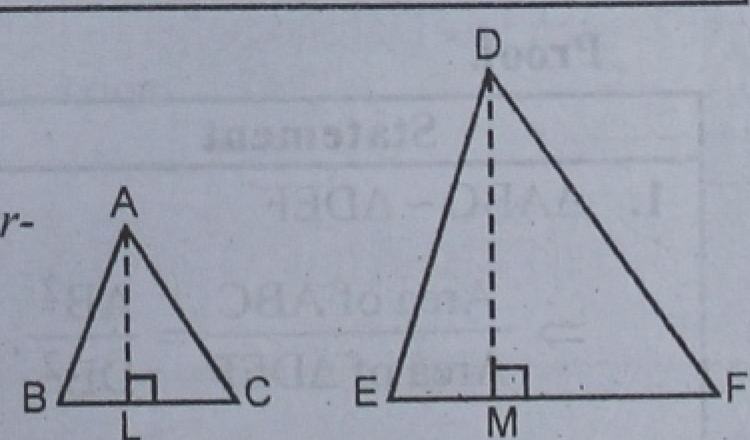
Construction : Draw AL $\perp$ BC and DM $\perp$ EF.

Proof.

Statement	Reason
1. $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times EF \times DM}$	Area of $\Delta = \frac{1}{2} \times$ Base $\times$ Height
$\Rightarrow \dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{BC}{EF} \times \dfrac{AL}{DM}$. ...I	
2. In Δ ALB and Δ DME,	
(*i*) $\angle ALB = \angle DME$	Each equal to 90°.
(*ii*) $\angle ABL = \angle DEM$	$\Delta ABC \sim \Delta DEF \Rightarrow \angle B = \angle E$.
$\therefore \Delta ALB \sim \Delta DME$	AA–axiom for similarity of Δs.
$\Rightarrow \dfrac{AL}{DM} = \dfrac{AB}{DE}$. ...II	Corresponding sides of similarity Δs are proportional.
3. $\Delta ABC \sim \Delta DEF$	Given.
$\Rightarrow \dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$. ...III	Corresponding sides of similar Δs are proportional.
4. $\dfrac{AL}{DM} = \dfrac{BC}{EF}$.	From II and III.
5. Substituting $\dfrac{AL}{DM} = \dfrac{BC}{EF}$ in I, we get :	
$\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{BC^2}{EF^2}$. ...IV	
6. Combining III and IV, we get :	
$\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{AB^2}{DE^2} = \dfrac{BC^2}{EF^2} = \dfrac{AC^2}{DF^2}$.	

Hence, $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{AB^2}{DE^2} = \dfrac{BC^2}{EF^2} = \dfrac{AC^2}{DF^2}$.

Theorem 2. *The areas of two similar triangles are proportional to the squares on their corresponding altitudes.*



Given : $\Delta ABC \sim \Delta DEF$, $AL \perp BC$ and $DM \perp EF$.

To Prove : $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{AL^2}{DM^2}$.

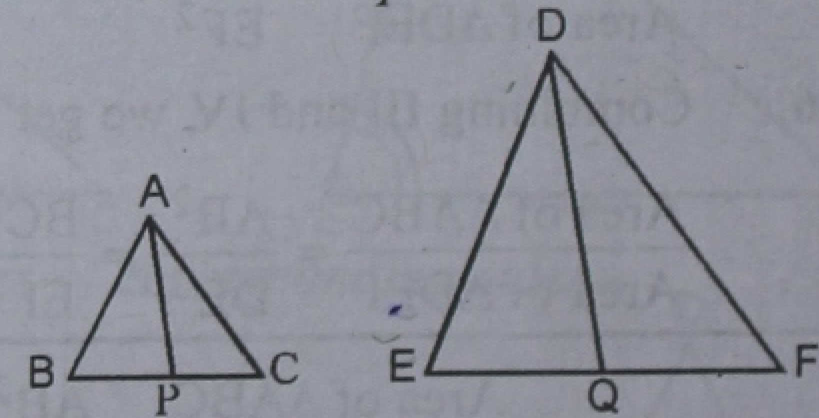
Proof :

Statement	Reason
1. $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times EF \times DM}$	Area of $\Delta = \frac{1}{2} \times$ Base $\times$ Height.
$\Rightarrow \dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{BC}{EF} \times \dfrac{AL}{DM}$. ...I	
2. In ΔALB and ΔDME, we have	
(*i*) $\angle ALB = \angle DME$	Each equal to 90°.
(*ii*) $\angle ABL = \angle DEM$	$\Delta ABC \sim \Delta DEF \Rightarrow \angle B = \angle E$.
$\therefore \Delta ALB \sim \Delta DME$	AA–Axiom for similarity of Δs.
$\Rightarrow \dfrac{AB}{DE} = \dfrac{AL}{DM}$. ...II	Corresponding sides of similar Δs are proportional.
3. $\Delta ABC \sim \Delta DEF$	Given.
$\Rightarrow \dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$ ...III	Corresponding sides of similar Δs are proportional.
4. $\dfrac{BC}{EF} = \dfrac{AL}{DM}$	From II and III
5. Substituting $\dfrac{BC}{EF} = \dfrac{AL}{DM}$ in I, we get : $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{AL^2}{DM^2}$.	

Hence, $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{AL^2}{DM^2}$.

Theorem 3. *The areas of two similar triangles are proportional to the squares on their corresponding medians.*

Given : $\Delta ABC \sim \Delta DEF$ and AP, DQ are their medians.



To Prove : $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \dfrac{AP^2}{DQ^2}$

Proof.

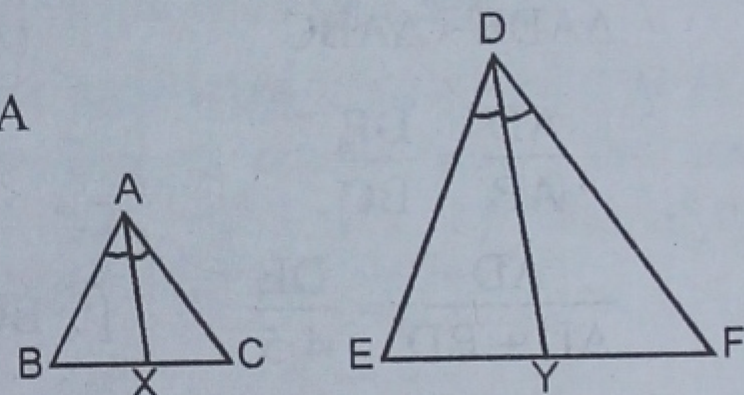
Statement	Reason
1. $\Delta ABC \sim \Delta DEF$	Given.
$\Rightarrow \dfrac{\text{Area of ABC}}{\text{Area of } \Delta DEF} = \dfrac{AB^2}{DE^2}$. ...I	Areas of two similar Δs are proportional to the squares on corresponding sides.

2. $\Delta ABC \sim \Delta DEF$ $\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{2BP}{2EQ} = \frac{BP}{EQ}$. ...II	Corresponding sides of similar Δs are proportional
3. $\frac{AB}{DE} = \frac{BP}{EQ}$ and $\angle A = \angle D$ $\Rightarrow \Delta APB \sim \Delta DQE$	From II and the fact that $\Delta ABC \sim \Delta DEF$. By SAS–similarity axiom.
$\Rightarrow \frac{BP}{EQ} = \frac{AP}{DQ}$ ...III $\Rightarrow \frac{AB}{DE} = \frac{AP}{DQ}$ $\Rightarrow \frac{AB^2}{DE^2} = \frac{AP^2}{DQ^2}$. ...IV	From II and III.
4. $\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AP^2}{DQ^2}$.	From I and IV.

Hence, $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AP^2}{DQ^2}$.

Theorem 4. *The areas of two similar triangles are proportional to the squares on their corresponding angle bisector segments.*

Given : $\Delta ABC \sim \Delta DEF$ and AX, DY are the bisectors of $\angle A$ and $\angle D$ respectively.



To Prove : $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AX^2}{DY^2}$.

Proof.

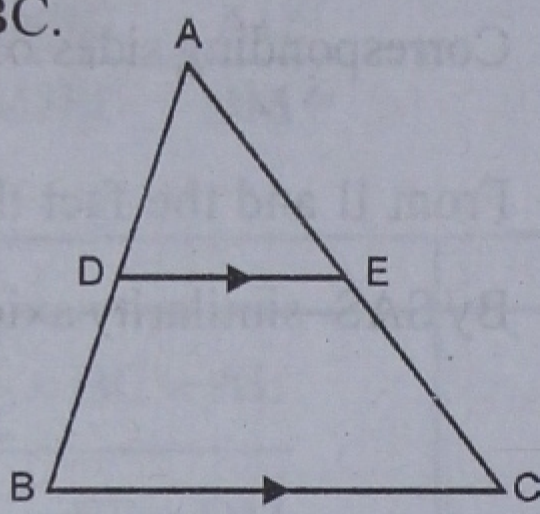
Statement	**Reason**
1. $\frac{\text{Area of } \Delta\ ABC}{\text{Area of } \Delta\ DEF} = \frac{AB^2}{DE^2}$. ...I	Areas of similar Δs are proportional to the squares of the corresponding sides.
2. $\Delta ABC \sim \Delta DEF$ $\Rightarrow \angle A = \angle D \Rightarrow \frac{1}{2} \angle A = \frac{1}{2} \angle D$ $\Rightarrow \angle BAX = \angle EDY$. ...II	Given. $\angle BAX = \frac{1}{2} \angle A$ and $\angle EDY = \frac{1}{2} \angle D$.
3. In ΔABX and ΔDEY, we have $\angle BAX = \angle EDY$ $\angle B = \angle E$ $\therefore \Delta ABX \sim \Delta DEY$ $\Rightarrow \frac{AB}{DE} = \frac{AX}{DY} \Rightarrow \frac{AB^2}{DE^2} = \frac{AX^2}{DY^2}$ ...III	 From II. $\Delta ABC \sim \Delta DEF$. By AA-similarity axiom.
4. From I and III, we get : $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AX^2}{DY^2}$.	

EXERCISE 17

Q. 1. In the figure, DE || BC.

(*i*) Prove that ΔADE and ΔABC are similar.

(*ii*) Given that AD = $\frac{1}{2}$ BD, calculate DE. If BC = 4·5 cm. **(2004)**



Sol. In ΔABC, DE || BC

BC = 4·5 cm

In ΔADE and ΔABC

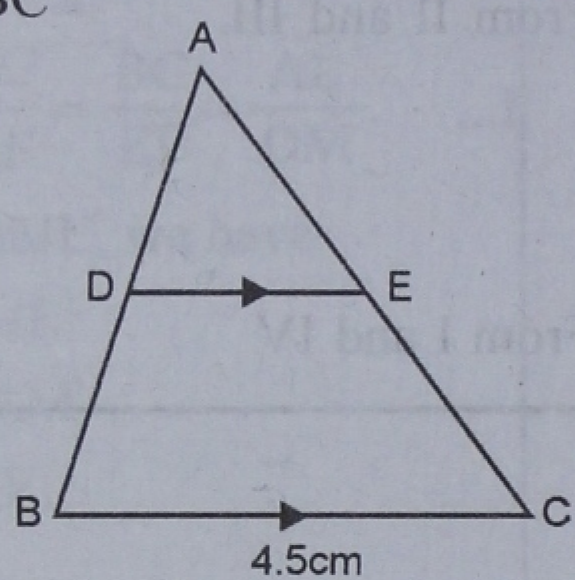
∠ADE = ∠ABC

{Corresponding angles}

∠AED = ∠ACB

and ∠A = ∠A

(Common)



∴ ΔADE ~ ΔABC (AAA axiom)

$$\therefore \quad \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \quad \frac{AD}{AD + BD} = \frac{DE}{4\cdot5} \quad [\because BC = 4\cdot5 \text{ cm}]$$

$$\Rightarrow \quad \frac{\frac{1}{2}BD}{\frac{1}{2}BD + BD} = \frac{DE}{4\cdot5} \left[\because AD = \frac{1}{2}BD\right]$$

$$\Rightarrow \quad \frac{\frac{1}{2}BD}{\frac{3}{2}BD} = \frac{DE}{4\cdot5} \Rightarrow \frac{1}{2} \times \frac{2}{3} = \frac{DE}{4\cdot5}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{DE}{4\cdot5} \Rightarrow DE = \frac{1}{3} \times 4\cdot5 = 1\cdot5$$

Hence, DE = 1·5 cm **Ans.**

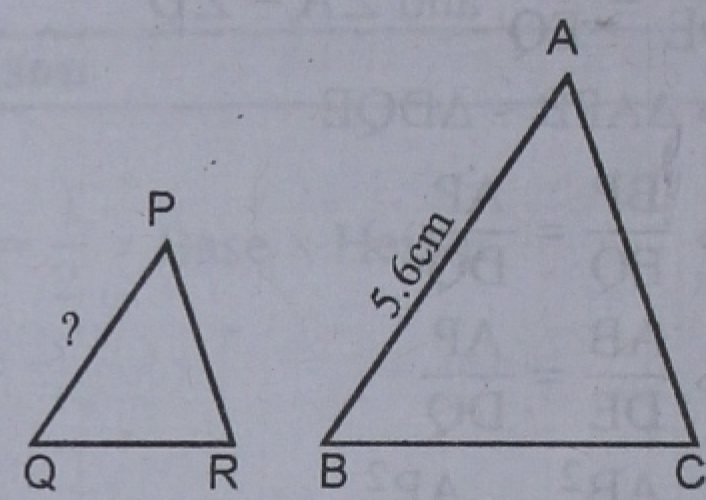
Q.2. (*i*) ΔABC and ΔPQR are similar triangles such that area (ΔABC) = 49 cm^2 and area (ΔPQR) = 25 cm^2.

If AB = 5·6 cm, find the length of PQ.

(*ii*) ΔABC and ΔPQR are similar triangles such that area (ΔABC) = 28 cm^2 and area (ΔPQR) = 63 cm^2. If PR = 8·4 cm, find the length of AC.

(*iii*) ΔABC ~ ΔDEF. If BC = 4cm, EF = 5cm and area (ΔABC) = 32cm^2, determine the area of ΔDEF.

Sol. (*i*) ∵ ΔABC ~ ΔPQR (given)



$$\therefore \frac{\text{area }(\Delta ABC)}{\text{area }(\Delta PQR)} = \frac{AB^2}{PQ^2}$$

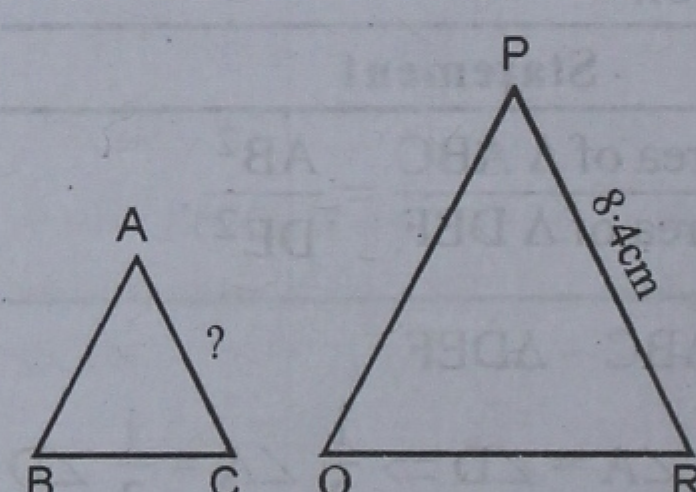
(∵ Areas of similar triangles are proportional to the squares on their corresponding sides)

$$\Rightarrow \frac{49}{25} = \frac{(5\cdot6)^2}{PQ^2} \Rightarrow \frac{(7)^2}{(5)^2} = \frac{(5\cdot6)^2}{PQ^2}$$

$$\Rightarrow \frac{5\cdot6}{PQ} = \frac{7}{5} \Rightarrow PQ = \frac{5\cdot6 \times 5}{7}$$

⇒ PQ = 4·0 cm **Ans.**

(*ii*) ∵ ΔABC ~ ΔPQR (Given)



$$\therefore \frac{\text{area }(\Delta ABC)}{\text{area }(\Delta PQR)} = \frac{AC^2}{PR^2}$$

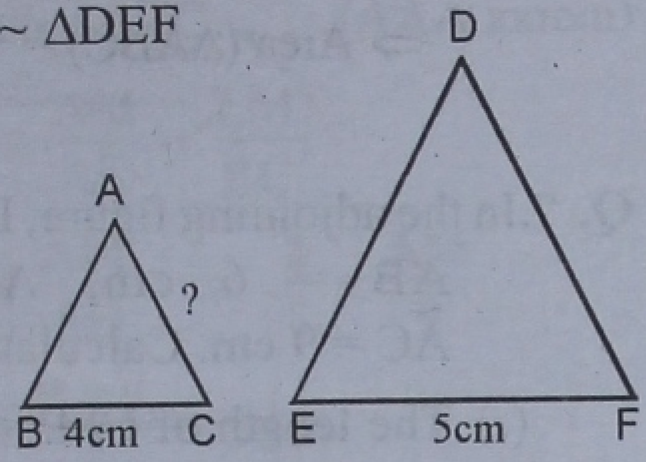
(∵ Areas of similar triangles are proportional to the squares of their corresponding sides)

$$\Rightarrow \frac{28}{63} = \frac{AC^2}{(8\cdot4)^2} \Rightarrow \frac{4}{9} = \frac{AC^2}{(8\cdot4)^2}$$

$$\Rightarrow \frac{(2)^2}{(3)^2} = \frac{AC^2}{(8\cdot4)^2} \Rightarrow \frac{2}{3} = \frac{AC}{8\cdot4}$$

$$\Rightarrow AC = \frac{2 \times 8\cdot4}{3} = 2 \times 2\cdot8 = 5\cdot6 \text{ cm}$$ **Ans.**

(*iii*) $\because \Delta ABC \sim \Delta DEF$



$$\therefore \frac{\text{area }(\Delta ABC)}{\text{area }(\Delta DEF)} = \frac{BC^2}{EF^2}$$

($\because$ Areas of similar triangles are proportional to the squares on their corresponding sides)

$$\Rightarrow \frac{32cm^2}{\text{area }(\Delta DEF)} = \frac{(4)^2}{(5)^2} = \frac{16}{25}$$

$\Rightarrow$ 16 × area (Δ DEF) = 32 × 25

$$\Rightarrow \text{area }(\Delta DEF) = \frac{32 \times 25}{16} = 50 \text{ cm}^2$$ **Ans.**

Q. 3. The area of two similar triangles are 48 cm^2 and 75 cm^2 respectively. If the altitude of the first triangle be 3·6 cm, find the corresponding altitude of the other.

Sol. Area of first triangle = 48 cm^2

And, area of second triangle = 75 cm^2

Altitude of first triangle = 3·6 cm

Let, altitude of second triangle = x cm

$\because \Delta s$ are similar

$\therefore$ Their areas are proportional to the squares of their corresponding altitudes.

$$\therefore \frac{\text{Area of first triangle}}{\text{Area of second triangle}} = \frac{(3 \cdot 6)^2}{x^2}$$

$$\Rightarrow \frac{48}{75} = \frac{(3 \cdot 6)^2}{x^2} \Rightarrow 48x^2 = (3 \cdot 6)^2 \times 75$$

$$\Rightarrow x^2 = \frac{(3 \cdot 6)^2 \times 75}{48} = \frac{3 \cdot 6 \times 3 \cdot 6 \times 75}{48}$$

$$= \frac{3 \cdot 6 \times 3 \cdot 6 \times 25}{16} = \frac{3 \cdot 6 \times 3 \cdot 6 \times 5 \times 5}{4 \times 4}$$

$$= \frac{(3 \cdot 6 \times 5)^2}{(4)^2}$$

$$\therefore x = \frac{3 \cdot 6 \times 5}{4} = 0 \cdot 9 \times 5 = 4 \cdot 5 \text{ cm}$$ **Ans.**

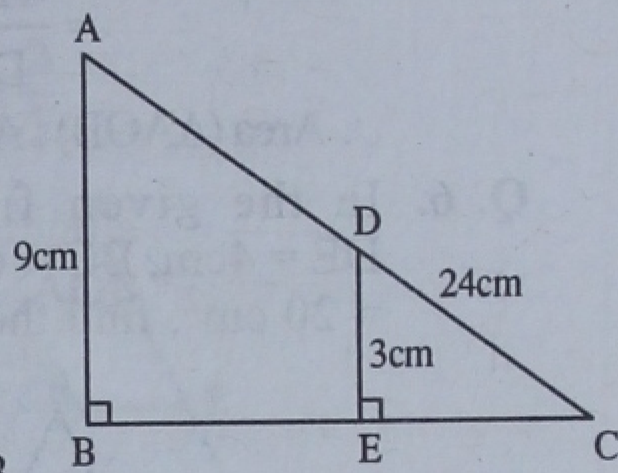
Q. 4. In the given figure, AB and DE are perpendicular to BC.

If AB = 9 cm, DE = 3 cm and AC = 24 cm, calculate AD. **(2005)**

Sol. In Δ ABC and Δ DEC,

$\because \angle B = \angle E = 90°$ and $\angle C = \angle C$

$\therefore \Delta ABC \sim \Delta DEC$ (By A. A. similarity)



$$\therefore \frac{AB}{DE} = \frac{AC}{DC}$$

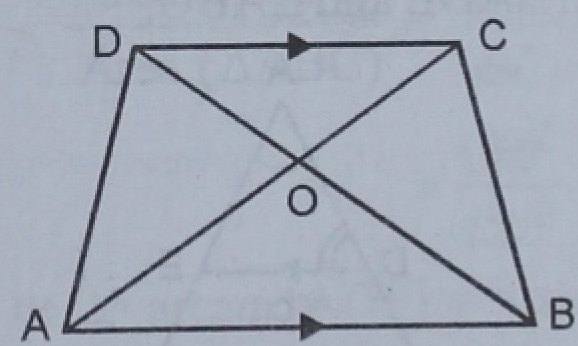
$$\therefore \frac{9}{3} = \frac{24}{DC}$$

$$\therefore DC = \frac{24 \times 3}{9} = 8 \text{ cm}$$

AD = AC – DC

$\therefore$ AD = 24 – 8 = 16 cm **Ans.**

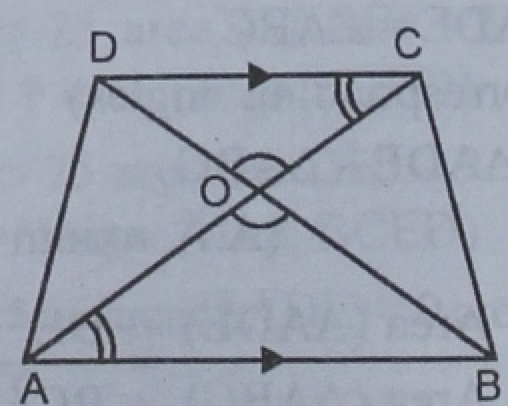
Q. 5. In the adjoining figure, ABCD is a trapezium in which AB || DC and AB = 2DC. Determine the ratio of the areas of ΔAOB and ΔCOD.



Sol. In trapezium ABCD,

AB = 2DC, DC || AB,

In ΔAOB and ΔCOD,



$\angle AOB = \angle COD$

(Vertically opposite angles)

$\angle OAB = \angle OCD$ (Alternate angles)

$\therefore \Delta AOB \sim \Delta COD$ (A.A. axiom of similarity)

$$\therefore \frac{\text{Area }(\Delta AOB)}{\text{Area }(\Delta COD)} = \frac{AB^2}{DC^2}$$

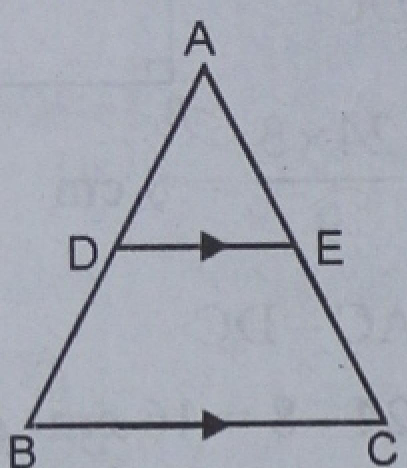
($\because$ Areas of similar triangles are proportional to the squares of their corresponding sides)

$$\Rightarrow \frac{\text{Area }(\Delta AOB)}{\text{Area }(\Delta COD)} = \frac{[2\,DC]^2}{DC^2} \quad (\because AB = 2\,DC)$$

$$= \frac{4DC^2}{DC^2} = \frac{4}{1}$$

$\therefore$ Area (ΔAOB) : Area (ΔCOD) = 4 : 1 **Ans.**

Q. 6. In the given figure, DE || BC. If DE = 4cm, BC = 6cm and area (ΔADE) = 20 cm^2, find the area of ΔABC.

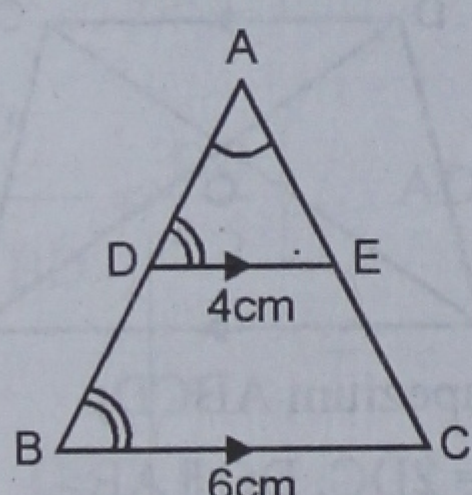


Sol. In ΔABC, DE || BC

DE = 4cm, BC = 6cm.

and area (ΔADE) = 20 cm^2

In ΔADE and ΔABC,



$\angle A = \angle A$ (common)

$\angle ADE = \angle ABC$

(corresponding angles)

$\therefore \Delta ADE \sim \Delta ABC$

(A.A. axiom of similarity)

$$\therefore \frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta ABC)} = \frac{DE^2}{BC^2}$$

$$\Rightarrow \frac{20}{\text{Area }(\Delta ABC)} = \frac{(4)^2}{(6)^2}$$

$$\Rightarrow \frac{20}{\text{Area }(\Delta ABC)} = \frac{16}{36}$$

$$\Rightarrow \text{Area }(\Delta ABC) = \frac{20 \times 36}{16} = 45\,\text{cm}^2$$

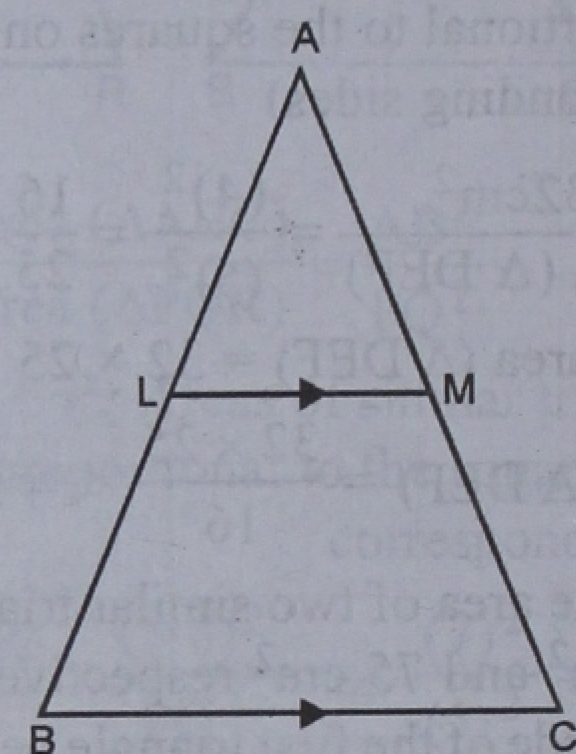
Ans.

Q. 7. In the adjoining figure, LM is parallel to BC. AB = 6 cm, AL = 2 cm and AC = 9 cm. Calculate :

(*i*) The length of CM.

(*ii*) The value of $\frac{\text{area }(\Delta ALM)}{\text{area (trap LBCM)}}$

(1996)



Sol. Given in the fig. LM || BC, AB = 6 cm

AL = 2 cm, AC = 9 cm

(*i*) To find length of CM

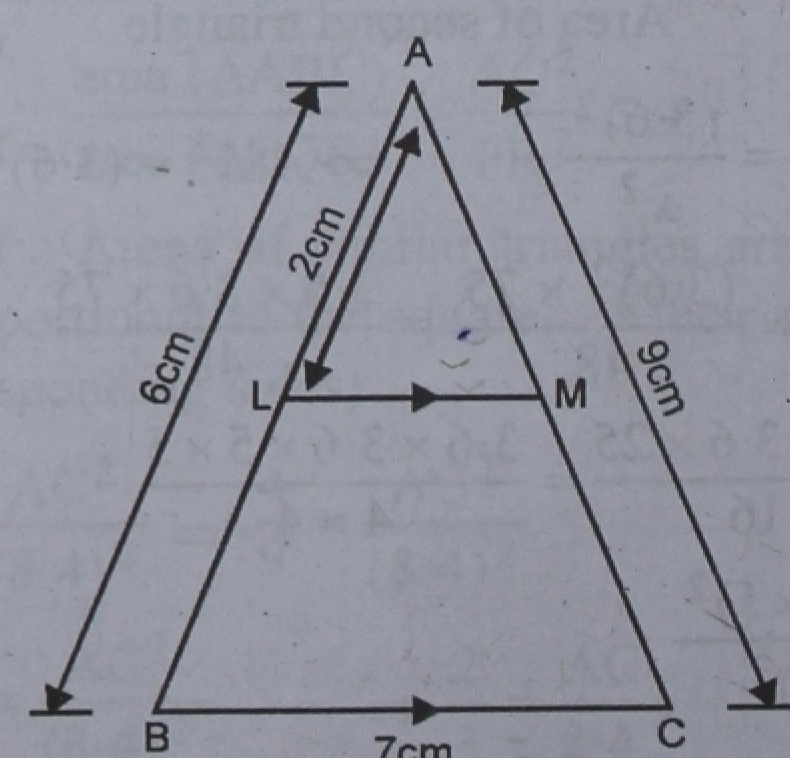
(*ii*) Ratio of $\frac{\text{Area of }\Delta ALM}{\text{Area of Trap. LBCM}}$

In ΔALM and ΔABC,

$\angle ALM = \angle ABC$ {Corresponding angle}

$\angle AML = \angle ACB$

$\angle A = \angle A$ (Common)



$\therefore \Delta ALM \sim \Delta ABC$ (AAA axiom)

$\therefore \dfrac{AL}{AB} = \dfrac{AM}{AC} = \dfrac{LM}{BC}$

$\therefore \dfrac{AL}{AB} = \dfrac{AM}{AC} \Rightarrow \dfrac{2}{6} = \dfrac{AM}{9}$

$\Rightarrow AM = \dfrac{2 \times 9}{6} = 3$ cm.

CM = AC – AM

= 9 cm – 3 cm = 6 cm

$\therefore$ Length of CM = 6 cm **Ans.**

(*i*) $\because \Delta ALM \sim \Delta ABC$ (Proved)

$\therefore \dfrac{\text{Area } \Delta ALM}{\text{area } \Delta ABC} = \dfrac{AL^2}{AB^2} = \dfrac{(2)^2}{(6)^2}$

$= \dfrac{4}{36} = \dfrac{1}{9}$

Let area of $\Delta ALM = x$ cm^2

$\therefore$ area of $\Delta ABC = 9x$ cm^2

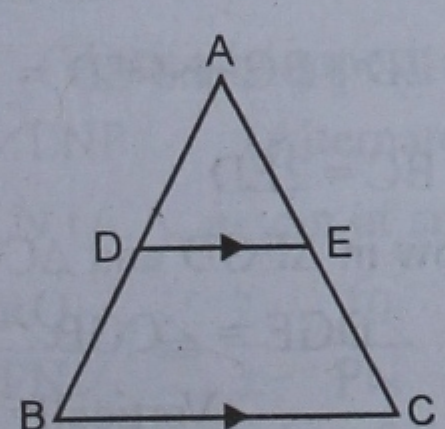
Now $\dfrac{\text{area of } \Delta ALM}{\text{area of Trap. LBCM}}$

$= \dfrac{\text{Area of } \Delta ALM}{\text{area of } \Delta ABC - \text{area of } \Delta ALM}$

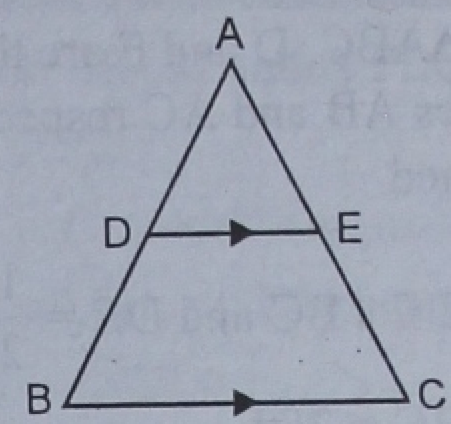
$= \dfrac{x}{9x - x} = \dfrac{x}{8x} = \dfrac{1}{8}$

$\therefore \dfrac{\text{Area of } \Delta ALM}{\text{area of Trap. LBCM}} = \dfrac{1}{8}$ **Ans.**

Q. 8. (*i*) In the given figure, DE || BC and DE : BC = 3 : 5. Calculate the ratio of the areas of Δ ADE and the trapezium BCED.



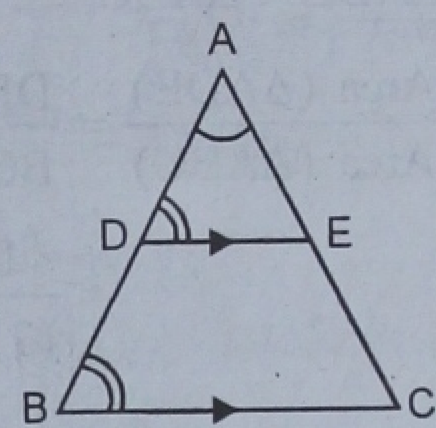
(*ii*) In ΔABC, D and E are mid-points of AB and AC respectively. Find the ratio of the areas of ΔADE and ΔABC.



Sol. (*i*) In ΔABC, DE || BC

DE : BC = 3 : 5

$\Rightarrow \dfrac{DE}{BC} = \dfrac{3}{5}$



In ΔADE and ΔABC,

$\angle A = \angle A$ (common)

$\angle ADE = \angle ABC$ (corresponding angles)

$\therefore \Delta ADE \sim \Delta ABC$

(A.A. axiom of similarity)

$\therefore \dfrac{\text{Area } (\Delta ADE)}{\text{Area } (\Delta ABC)} = \dfrac{(DE)^2}{(BC)^2}$

$= \dfrac{(3)^2}{(5)^2} = \dfrac{9}{25}$

$\Rightarrow$ 25 area (ΔADE) = 9 area (ΔABC)

(By cross multiplication)

$\Rightarrow$ 25 area (ΔADE) =

9 [area (ΔADE) + area (Trap. BCED)]

$\Rightarrow$ 25 area (ΔADE) = 9 area (ΔADE) + 9 area (Trap. BCED)

$\Rightarrow$ 25 area (ΔADE) – 9 area (ΔADE) = 9 area (Trap. BCED)

$\Rightarrow$ 16 area (ΔADE) = 9 area (Trap. BCED)

$\Rightarrow \dfrac{\text{Area } (\Delta ADE)}{\text{Area (Trap. BCED)}} = \dfrac{9}{16}$

$\therefore$ Area (Δ ADE) : area (Trap. BCED)

= 9 : 16 **Ans.**

(ii) In ΔABC, D and E are the mid-points of sides AB and AC respectively. D, E are joined.

$\therefore$ DE ∥ BC and DE = $\frac{1}{2}$ BC

or BC = 2DE

Now in Δ ADE and Δ ABC,

∠A = ∠A (common)

∠ADE = ∠ABC (corresponding angles)

$\therefore$ ΔADE ~ ΔABC

$$\therefore \frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta ABC)} = \frac{DE^2}{BC^2}$$

$$= \frac{DE^2}{(2DE)^2} = \frac{DE^2}{4DE^2}$$

$$= \frac{1}{4}$$

$\therefore$ Area (ΔADE) : Area (ΔABC)

= 1 : 4 **Ans.**

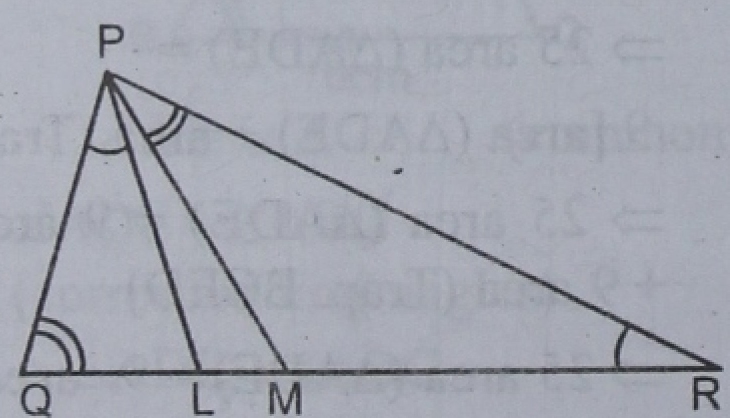
Q. 9. In a ΔPQR, L and M are two points on the base QR, such that ∠LPQ = ∠QRP and ∠RPM = ∠RQP. Prove that :

(i) ΔPQL ~ ΔRPM

(ii) QL.RM = PL.PM

(iii) PQ^2 = QL.QR

Sol. In ΔPQR, L and M are two points on the base OR. Such that ∠LPQ = ∠QRP



Proof :

(i) Now in ΔPQL and Δ RPM,

∠LPQ = ∠QRP (given)

And, ∠RQP = ∠RPM (given)

$\therefore$ ΔPQL ~ ΔRPM

(A.A. axiom of similarity)

(ii) $\therefore \frac{QL}{PM} = \frac{PL}{RM}$ ⇒ QL·RM = PL·PM.

(corresponding sides of similar triangles are proportional)

(iii) Similarly in ΔPQL and ΔPQR,

∠PQL = ∠PQR (common)

∠LPQ = ∠QRP (given)

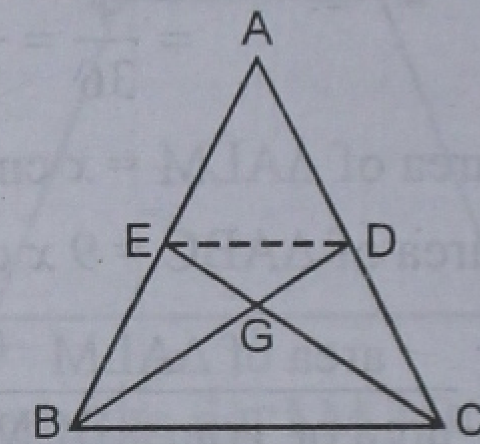
$\therefore$ ΔPQL ~ ΔPQR

$\because \frac{PQ}{QR} = \frac{QL}{PQ}$

⇒ PQ·PQ = QL·QR

⇒ PQ^2 = QL·QR. Hence Proved.

Q. 10. In the adjoining figure, the medians BD and CE of a ΔABC meet at G.



Prove that :

(i) ΔEGD ~ ΔCGB

(ii) BG = 2GD from (i) above. **(2002)**

Sol. Given : In ΔABC, BD and CE are the medians intersecting each other at G. E, D are joined

To Prove : (i) ΔEGD ~ ΔCEB

(ii) BG = 2GD

Proof : $\because$ E and D are the mid-points of the sides AB and AC of ΔABC.

$\therefore$ ED ∥ BC and ED = $\frac{1}{2}$ BC.

or BC = 2ED

(i) Now in ΔEGD and ΔCGB

∠DGE = ∠CGB

(Vertically opposite angles)

∠EDG = ∠GBC (alternate angles)

$\therefore$ ΔEGD ~ ΔCGB

(A.A. axiom of similarity)

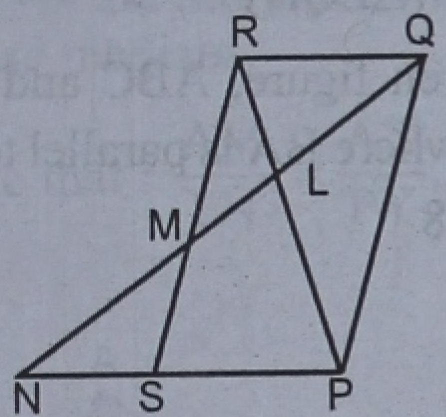
(*ii*) $\therefore \frac{GD}{BG} = \frac{ED}{BC} = \frac{ED}{2ED}$ ($\because$ BC = 2ED)

$\Rightarrow \frac{GD}{BG} = \frac{1}{2}$ $\Rightarrow$ BG = 2GD.

Hence Proved.

11. In the adjoining figure, PQRS is a parallelogram with PQ = 15cm and RQ = 10cm. L is a point on RP such that RL : LP = 2 : 3. QL produced meets RS at M and PS produced at N.

Find the lengths of PN and RM. **(1997)**



Sol. In || gm PQRS,

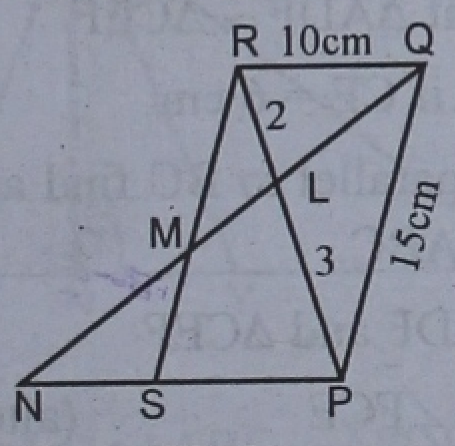
PQ = 15cm, RQ = 10cm

L is a point on RP such that

RL : LP = 2 : 3.

QL is produced to meet RS at M and PS is produced to meet at N.

(*i*) In ΔRLQ and ΔPLN,



$\angle$RLQ = $\angle$PLN

(Vertically opposite angles)

$\angle$RQL = $\angle$LNP (Alternate angles)

$\therefore \Delta RLQ \sim \Delta PLN$ (A.A. axiom of similarity)

$\therefore \frac{RL}{LP} = \frac{RQ}{PN}$ $\Rightarrow \frac{2}{3} = \frac{10}{PN}$

(Corresponding sides of similar triangles are proportional)

$\Rightarrow PN = \frac{3 \times 10}{2} = 15$ cm.

(*ii*) Similarly in ΔRLM and Δ PLQ,

$\angle$RLM = $\angle$PLQ

(Vertically opposite angles)

$\angle$LRM = $\angle$LPQ (Alternate angles)

$\therefore \Delta RLM \sim \Delta PLQ$

(A.A. axiom of similarity)

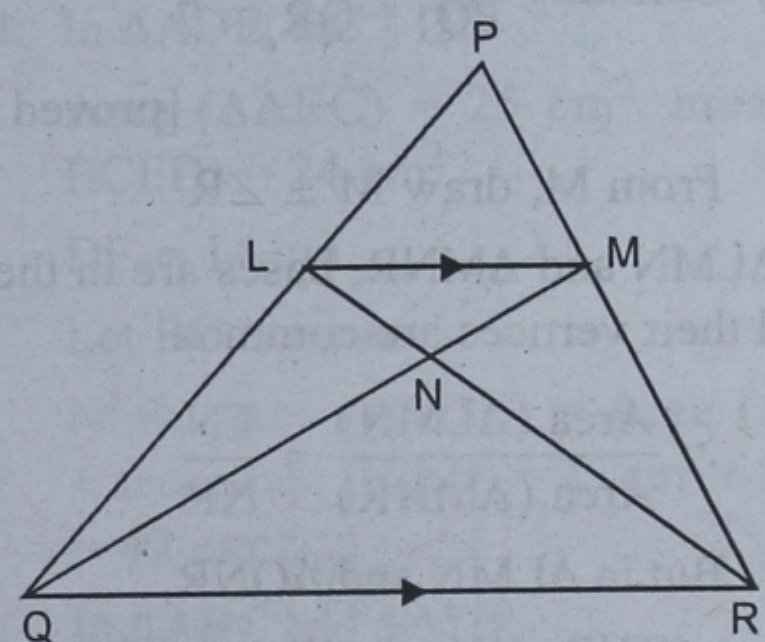
$\therefore \frac{RL}{LP} = \frac{RM}{PQ}$

(Corresponding sides of similar triangles are proportional)

$\Rightarrow \frac{2}{3} = \frac{RM}{15}$ $\Rightarrow RM = \frac{2 \times 15}{3}$

$\therefore$ RM = 10 cm **Ans.**

Q. 12. In Δ PQR, LM || QR and PM : MR = 3:4. Calculate :



(*i*) $\frac{PL}{PQ}$ and then $\frac{LM}{QR}$;

(*ii*) $\frac{\text{Area } (\Delta LMN)}{\text{Area } (\Delta MNR)}$;

(*iii*) $\frac{\text{Area } (\Delta LQM)}{\text{Area } (\Delta LQN)}$.

Sol. In ΔPQR, LM || QR and PM : MR = 3 : 4

LR and MQ are joined intersecting each other at N.

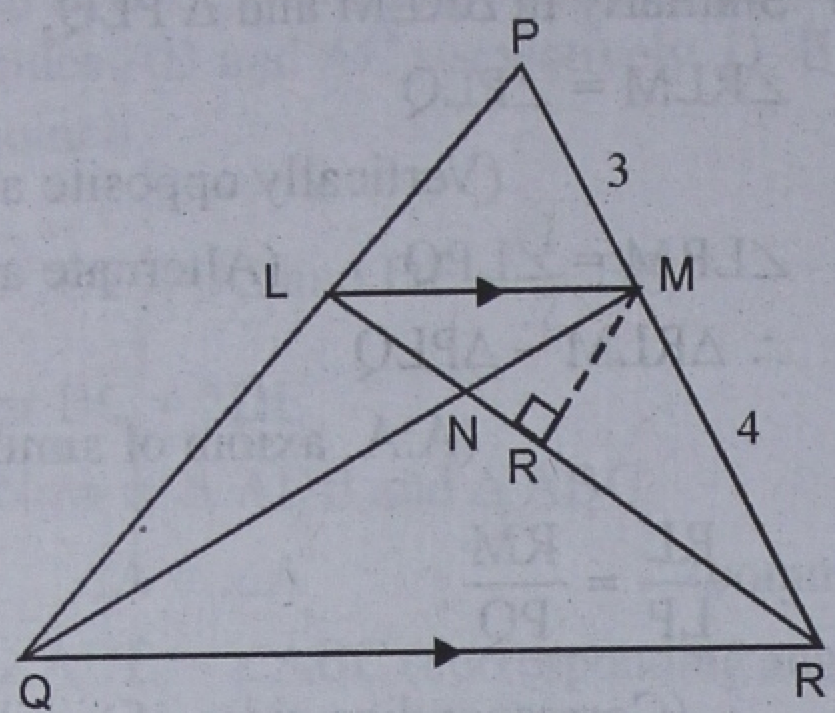
(*i*) Now in ΔPLM and ΔPQR,

$\angle P = \angle P$ (common)

$\angle$PLM = $\angle$PQR (Corresponding angles)

$\because \Delta PLM \sim \Delta PQR$

(A.A axiom of similarity)



$$\therefore \frac{PL}{PQ} = \frac{PM}{PR} = \frac{PM}{PM + MR}$$

$$= \frac{3}{3+4} = \frac{3}{7}$$

(*ii*) Similarly $\frac{PL}{PQ} = \frac{LM}{QR} = \frac{3}{7}$

[proved in (*i*)]

From M, draw M ⊥ ∠R

Now, (ΔLMN and ΔMNR, bases are in the same line and their vertices are common.

$$\therefore \frac{\text{Area }(\Delta LMN)}{\text{Area }(\Delta MNR)} = \frac{LN}{NR}$$

But in ΔLMN and ΔQNR,

∠LNM = ∠QNR (Vertically opposite angles)

∠LMN = ∠NQR (Alternate angles)

∴ ΔLMN ~ ΔQNR (A.A. axiom of similarity)

$$\therefore \frac{LN}{NR} = \frac{LM}{QR} = \frac{3}{7} \quad \text{(Proved)}$$

Hence, $\frac{\text{Area }(\Delta LMN)}{\text{Area }(\Delta MNR)} = \frac{LN}{NR} = \frac{3}{7}$

(*iii*) Similarity we can prove that

$$\frac{\text{Area }(\Delta LQM)}{\text{Area }(\Delta LQN)} = \frac{QM}{QN}$$

But ΔQNR ~ ΔMNL [proved]

$$\therefore \frac{QN}{NM} = \frac{QR}{LM} = \frac{7}{3} \quad \text{[from (ii)]}$$

$$\therefore \frac{QN}{NM} = \frac{7}{3} \quad \therefore \frac{NM}{QN} = \frac{3}{7}$$

$$\Rightarrow \frac{NM}{QN} + 1 = \frac{3}{7} + 1$$

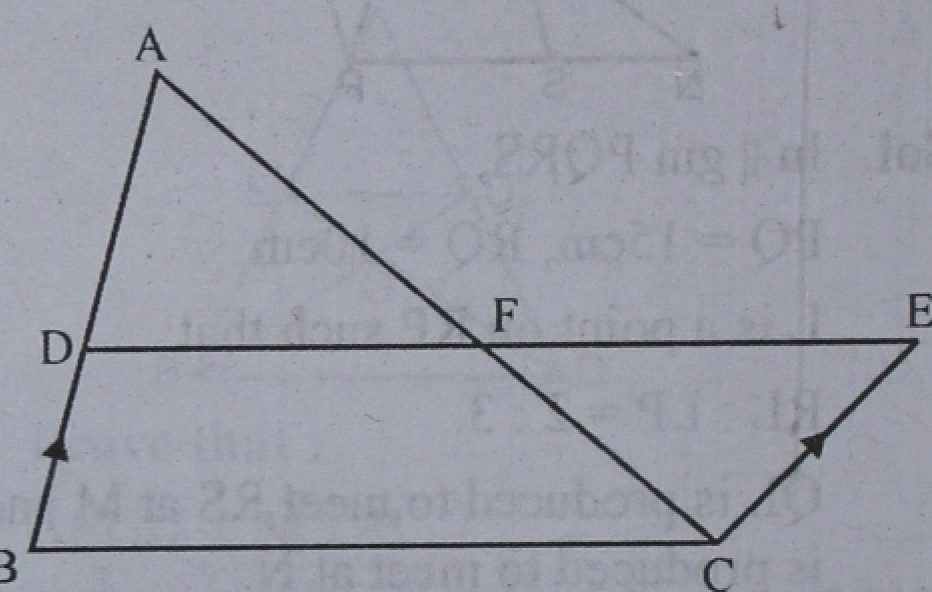
(Adding 1 to both sides)

$$\frac{NM + QN}{QN} = \frac{3+7}{7}$$

$$\Rightarrow \frac{QM}{QN} = \frac{10}{7}$$

$$\therefore \frac{\text{Area }(\Delta LQM)}{\text{Area }(\Delta LQN)} = \frac{10}{7} \quad \textbf{Ans.}$$

13. In the given figure, ABC and CEF are two triangles where BA is parallel to CE and AF : AC = 5 : 8



(*i*) Prove that ΔADF ~ ΔCEF

(*ii*) Find AD if CE = 6 cm

(*iii*) If DF is parallel to BC find area of ΔADF : area of ΔABC.

Sol. (*i*) In ΔADF and ΔCEF

∠DAF = ∠FCE (alternate angles)

∠AFD = ∠CFE (Vertically opp. angles)

∴ ΔADF ~ ΔCEF (by A.A.)

Hence proved

(*ii*) ΔADF ~ ΔCEF

$$\therefore \frac{AD}{CE} = \frac{DF}{EF} = \frac{AF}{CF}$$

$$\therefore \frac{AD}{CE} = \frac{AF}{FC}$$

$$\frac{AD}{16} = \frac{5}{8}$$

AD = 10 cm

DF || BC

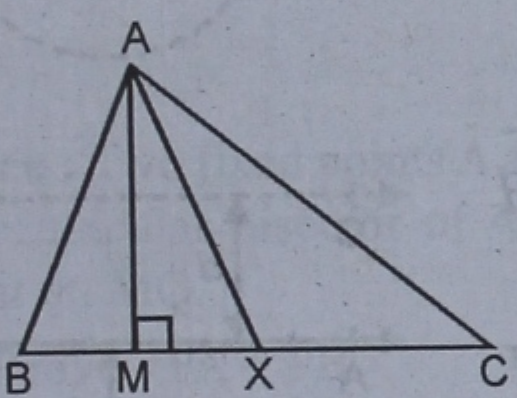
$\Delta ADF \sim \Delta ABC$

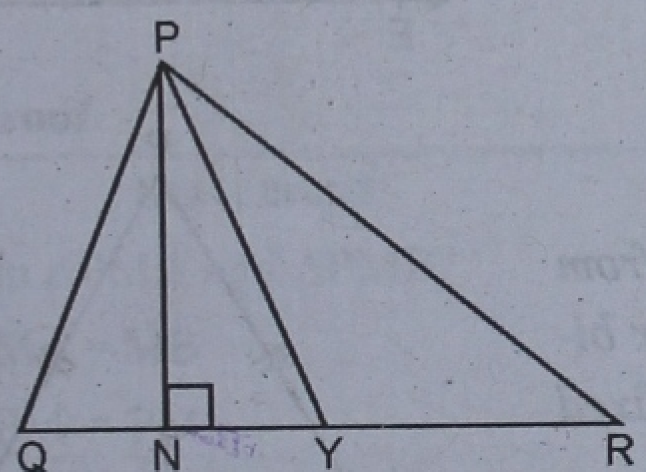
$\angle D = \angle B$ and $\angle F = \angle C$

$$\frac{\text{Ar. of } \Delta ADF}{\text{Ar. of } \Delta ABC} = \frac{AF^2}{AC^2} = \left(\frac{5}{8}\right)^2 = \frac{25}{64}$$

14. In the given figure, $\Delta ABC \sim \Delta PQR$ and AM, PN are altitudes, whereas AX and PY are medians.

Prove that : $\frac{AM}{PN} = \frac{AX}{PY}$.





Sol. Proof : $\because \Delta ABC \sim \Delta PQR$ (given)

$$\therefore \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta PQR)} = \frac{AM^2}{PN^2} \quad ...(i)$$

(Areas of similar triangles are proportional to the squares of their corresponding altitudes)

Again, $$\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta PQR)} = \frac{AX^2}{PY^2} \quad ...(ii)$$

(Areas of similar triangles are proportional to the squares of their corresponding medians)

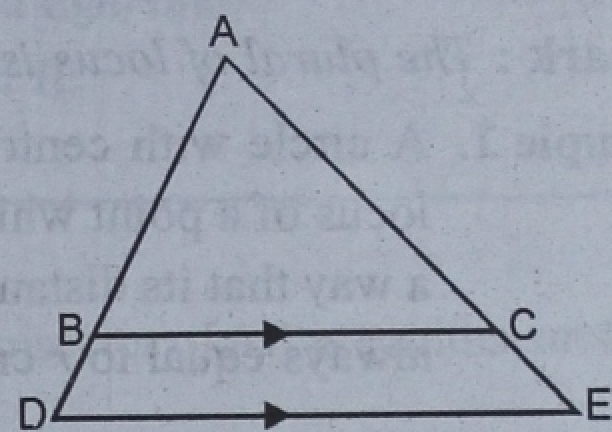
From (*i*) and (*ii*),

$$\frac{AM^2}{PN^2} = \frac{AX^2}{PY^2}$$

$$\Rightarrow \quad \frac{AM}{PN} = \frac{AX}{PY}$$

Hence Proved.

Q. 15. In the given figure BC || DE, area (ΔABC) = 25 cm², area (trap. BCED) = 24 cm² and DE = 14 cm. Calculate the length of BC.



Sol. In ΔADE, BC || DE

Area (ΔABC) = 25 cm², area (trap. BCED) = 24 cm²

DE = 14 cm.

Let BC = x cm.

Now, Area (ΔADE) = Area (ΔABC) + area (trap. BCED) = 25 cm² + 24 cm² = 49 cm²

In ΔABC and ΔADE,

$\angle A = \angle A$ (Common)

$\angle ABC = \angle ADE$ (Corresponding angles)

$\because \Delta ABC \sim \Delta ADE$ (A.A. axiom of similarity)

$$\therefore \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta ADE)} = \frac{BC^2}{DE^2}$$

(Areas of similar triangles are proportional to the square of their corresponding sides)

$$\Rightarrow \frac{25}{49} = \frac{x^2}{(14)^2}$$

$$\Rightarrow \frac{(5)^2}{(7)^2} = \frac{(x)^2}{(14)^2} \Rightarrow \frac{x}{14} = \frac{5}{7}$$

$$\Rightarrow 7x = 5 \times 14 \quad \Rightarrow x = \frac{5 \times 14}{7} = 10$$

$\therefore$ BC = 10 cm. Ans.