Chapter 16

Similarity (As a Size-Transformation)

POINTS TO REMEMBER

1 Similarity of Figures

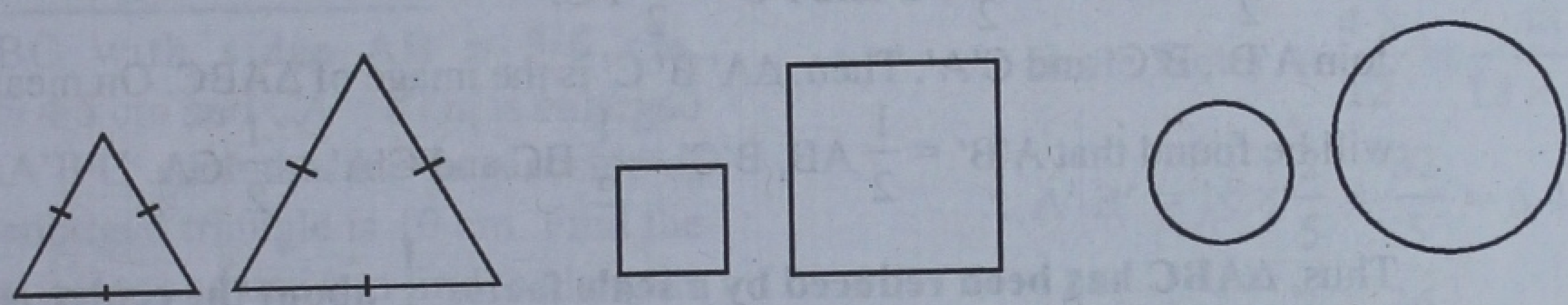
Any two figures are said to be similar, if they have exactly the same shape but not necessarily the same size.

Example :

(*i*) Two equilateral Δ*s* are always similar.

(*ii*) Two squares are always similar.

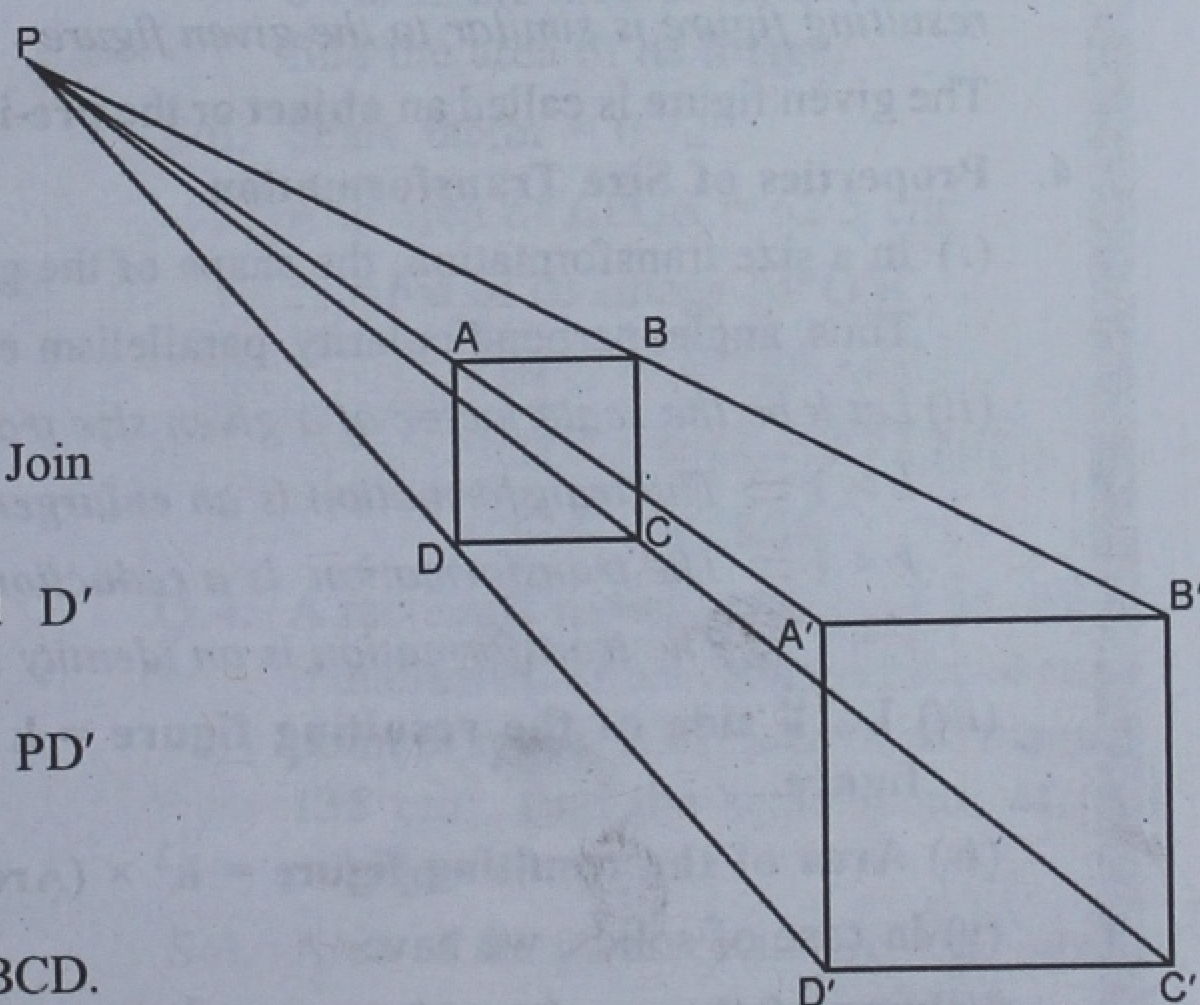
(*iii*) Two circles are always similar.



Similar Figures

2. Similarity As a Size Transformation

(*i*) **Enlargement :** Let us be given a figure, say a quadrilateral ABCD. With the help of this quadrilateral, we shall construct a similar quadrilateral, each of whose sides is twice the corresponding sides of ABCD.



Method : Mark a point P outside ABCD. Join PA, PB, PC and PD.

Produce them to A′, B′, C′ and D′ respectively such that :

PA′ = 2PA, PB′ = 2PB, PC′ = 2PC and PD′ = 2PD.

Join A′ B′, B′C′, C′D′ and D′A′.

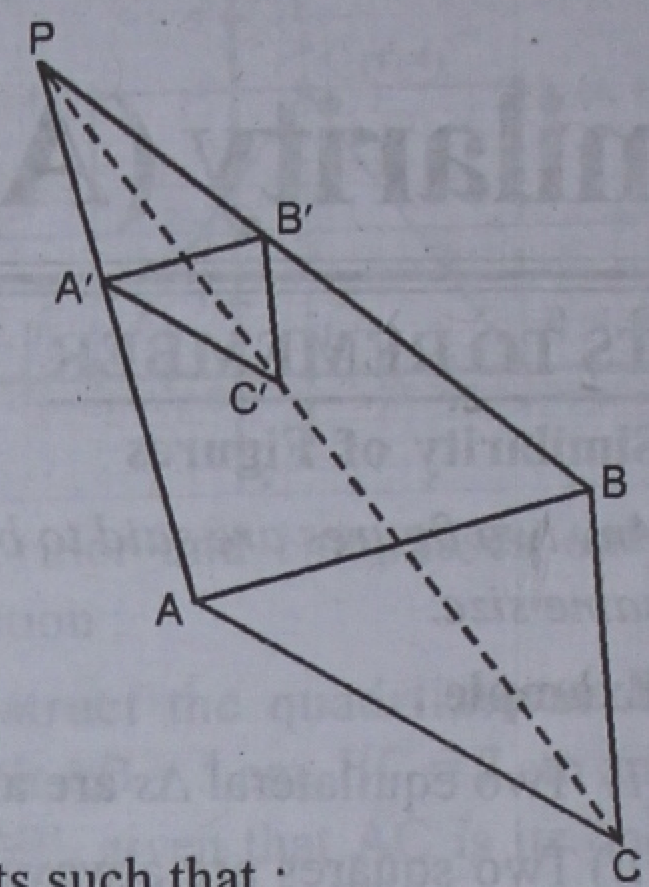
Then, A′B′C′D′ is called the **image** of ABCD.

On measurement, it will be found that :

A′B′ = 2AB, B′C′ = 2BC, C′D′ = 2CD and D′A′ = 2DA.

We say that the object ABCD has been enlarged by a scale factor 2 about the centre of enlargement P to give the image A′B′C′D′.

(*ii*) **Reduction :** Let us be given a figure, say a ΔABC. With the help of this triangle, we shall construct a similar triangle, each of whose sides is equal to half the corresponding sides of ΔABC.



Method : Take a point P outside ΔABC.

Join PA, PB and PC.

Mark points A′, B′ and C′ on these line segments such that :

$PA' = \frac{1}{2} PA$, $PB' = \frac{1}{2} PB$ and $PC' = \frac{1}{2} PC$.

Join A′B′, B′C′ and C′A′. Then, $\Delta A'B'C'$ is the image of ΔABC. On measurement, it will be found that $A'B' = \frac{1}{2} AB$, $B'C' = \frac{1}{2} BC$ and $C'A' = \frac{1}{2} CA$.

Thus, **ΔABC has been reduced by a scale factor $\frac{1}{2}$ about the centre of reduction P to give the image A′ B′ C′.**

3. Size Transformation

It is the process in which a given figure is enlarged or reduced by a scale factor k, such that the resulting figure is similar to the given figure.

The given figure is called an **object** or the **pre-image** and the resulting figure is called its **image.**

4. Properties of Size Transformation

(*i*) In a size transformation, the shape of the given figure is preserved.

Thus, angle, perpendicularity, parallelism etc. are preserved.

(*ii*) *Let k be the scale factor of a given size transformation. Then,*

$k > 1 \Rightarrow$ *The transformation is an* ***enlargement.***

$k < 1 \Rightarrow$ *The transformation is a* ***reduction.***

$k = 1 \Rightarrow$ *The transformation is an* ***identity transformation.***

(*iii*) **Each side of the resulting figure = k times the corresponding side of the given figure.**

(*iv*) **Area of the resulting figure = k^2 × (Area of the given figure).**

(*v*) In case of solids, we have

Volume of the resulting figure = k^2 × (Volume of the given figure).

5. Model :

The model of a plane figure and the actual figure are similar to one another. Let the model of a plane figure be drawn to the scale $1 : p$.

Then, **Scale Factor,** $k = \frac{1}{p}$.

(*i*) *Length of the model* = k × (*Length of the actual figure*).

(*ii*) *Area of the model* = k^2 × (*Area of the actual figure*).

(*iii*) *Volume of the model* = k^3 × (*Volume of the actual figure*).

6. Map :

Let the map of a plane figure be drawn to the scale 1 : p.

Then, Scale Factor, $\mathbf{k} = \dfrac{\mathbf{1}}{\mathbf{p}}$.

(i) Length in the map = k × (Actual length).

(ii) Area in the map = k^2 × (Actual area).

EXERCISE 16

Q.1. ΔABC with sides AB = 3·6 cm, BC = 4·5 cm and CA = 6 cm is enlarged to ΔA′B′C′ such that the largest side of the enlarged triangle is 10 cm. Find the scale factor and use it to find the lengths of the other sides of ΔA′B′C′.

Sol. In Δ ABC, AB = 3·6 cm, BC = 4·5 cm, and CA = 6 cm and it has been enlarged to ΔA′B′C′. In the resulting ΔA′B′C′, largest side = 10 cm.

But largest side in the given ΔABC = CA = 6 cm.

$\therefore$ Scale factor $= \dfrac{10}{6} = \dfrac{5}{3}$.

$\therefore$ A′B′ $= 3\cdot6 \times \dfrac{5}{3} = 6\cdot0$ cm.

B′C′ $= 4\cdot5 \times \dfrac{5}{3} = 7\cdot5$ cm. **Ans.**

Q.2. A ΔABC with sides AB = 16 cm, BC = 12 cm and CA = 18 cm is reduced to ΔA′B′C′ such that the smallest side of the image triangle is 4·8 cm. Find the scale factor and use it to find the lengths of the other sides of ΔA′B′C′.

Sol. In ΔABC, AB = 16 cm, BC = 12 cm and CA = 18 cm and it has been reduced to ΔA′B′C′

In the resulting ΔA′B′C′, smallest side = 4·8 cm.

But, the smallest side in Δ ABC, BC = 12 cm.

$\therefore$ Scale factor $= \dfrac{4\cdot8}{12} = \dfrac{48}{12\times10} = \dfrac{2}{5}$

$\therefore$ A′B′ $= 16 \times \dfrac{2}{5} = \dfrac{32}{5} = 6\cdot4$ cm.

C′A′ $= 18 \times \dfrac{2}{5} = \dfrac{36}{5} = 7\cdot2$ cm. **Ans.**

Q.3. A ΔPQR is reduced by a scale factor 0·72. If the area of ΔPQR is 62·5 cm^2, find the area of its image.

Sol. Scale factor = 0·72

And area of ΔPQR = 62·5 cm^2

$\therefore$ Area of its image ΔP′Q′R′

$= 62\cdot5 \times (0\cdot72)^2$ cm^2

$= 62\cdot5 \times 0\cdot72 \times 0\cdot72$ cm^2

$= 32\cdot4$ cm^2 **Ans.**

Q.4. A rectangle having an area of 60 cm^2 is transformed under enlargement about a point in space. If the area of its image is 135 cm^2, find the scale factor of the enlargement.

Sol. Area of the given rectangle = 60 cm^2

And area of its enlargement = 135 cm^2

Let, scale factor = k

$\therefore 135 = 60 \times k^2$ ($\because$ In area)

$\Rightarrow k^2 = \dfrac{135}{60} = \dfrac{9}{4} \Rightarrow k = \sqrt{\dfrac{9}{4}} = \dfrac{3}{2}$

$\therefore k = 1\cdot5$

Hence, scale factor of enlargement = 1·5 **Ans.**

Q. 5. On a map, drawn to a scale of 1 : 25000, a triangular plot LMN of land has the following measurements : LM = 6 cm, MN = 8 cm and $\angle LMN = 90°$. Calculate:

(*i*) the actual lengths of MN and LN in kilometres,

(*ii*) the actual area of the plot in sq. km.

Sol. Scale = 1 : 25000 $\Rightarrow$ Scale factor $(k) = \dfrac{1}{25000}$

In triangular plot LMN, LM = 6 cm, MN = 8 cm and $\angle LMN = 90°$

But, $LN^2 = LM^2 + MN^2$ (By Pythagoras Theorem)

$= (6)^2 + (8)^2 = 36 + 64 = 100 = (10)^2$

$\therefore$ LN = 10 cm.

(*i*) Now, actual length of MN $= \dfrac{1}{k}$ (8 cm)

$= 8 \times 25000$ cm. $= \dfrac{8 \times 25000}{100 \times 1000} = 2$ km.

Similarly, actual length of LN

$= 10 \times 25000$ cm. $= \dfrac{10 \times 25000}{100 \times 1000} = 2·5$ km

And actual length of LM $= \dfrac{6 \times 25000}{100 \times 1000} = 1·5$ km.

(*ii*) Actual area of the plot $= \dfrac{1}{2} \times LM \times MN$

$= \dfrac{1}{2} \times 1·5 \times 2 \text{ km}^2 = 1·5 \text{ km}^2$ **Ans.**

Q. 6. The scale of a map is 1 : 200000. A plot of land of area 10 km² is to be represented on the map. Find :

(*i*) the length in km on the ground, represented by 1 cm on the map ;

(*ii*) the area in km² that can be represented by 3 cm² on the map ;

(*iii*) the area on the map, representing the plot of land.

Sol. Scale = 1 : 200000

$\therefore$ Scale factor $(k) = \dfrac{1}{200000}$

Area of a plot = 10 km²

(*i*) Length on the map = 1 cm.

$\therefore$ Actual length $= \dfrac{1}{k} \times 1 \text{cm} = 200000 \times 1$ cm.

$= \dfrac{200000}{100 \times 1000} = 2$ km.

(*ii*) Area on the map = 3 cm²

$\therefore$ Actual area $= 3 \times \left(\dfrac{1}{k}\right)^2$

$= 3 \times 200000 \times 200000 \text{cm}^2$

$= \dfrac{3 \times 200000 \times 200000}{100 \times 100 \times 1000 \times 1000} = 12 \text{ km}^2$

(*iii*) Actual area of plot = 10 km²

$\therefore$ Area on the map = k^2 (10 km²)

$= \dfrac{1}{(200000)^2} \times 10 \text{ km}^2$

$= \dfrac{10 \times (100)^2 \times (1000)^2}{(200000)^2} \text{km}^2$

$= \dfrac{10 \times 100 \times 100 \times 1000 \times 1000}{200000 \times 200000} \text{cm}^2$

$= \dfrac{10}{4} = 2·5 \text{ cm}^2$ **Ans.**

Q. 7. On a map drawn to a scale of 1 : 20000, a rectangular plot of land ABCD has AB = 32 cm and BC = 24 cm. Calculate :

(*i*) the diagonal distance of the plot in km,

(*ii*) the area of the plot in sq. km.

Sol. Scale = 1 : 20000

$\therefore$ Scale factor $= \dfrac{1}{20000}$

In rectangular plot ABCD, AB = 32 cm and BC = 24 cm

$\therefore$ Diagonal AC $= \sqrt{(32)^2 \times (24)^2}$

$= \sqrt{1024 + 576}$ cm. $= \sqrt{1600} = 40$ cm.

(*i*) Now, actual length of the diagonal

$= \dfrac{1}{k} \times 40 \text{cm} = 20000 \times 40$ cm.

$= \dfrac{20000 \times 40}{100 \times 1000} \text{km} = 8$ km.

(*ii*) Area of plot $= l \times b = 32 \times 24 \text{ cm}^2$

$= 768 \text{ cm}^2$

$\therefore$ Actual area of the plot

$= 768 \times \dfrac{1}{k^2} \text{ cm}^2 \Rightarrow 768 \times (20000)^2 \text{ cm}^2$

$= \frac{768 \times 20000 \times 20000}{100 \times 1000 \times 100 \times 1000} km^2$

$= \frac{768 \times 4}{100} = \frac{3072}{100} km^2 = 30 \cdot 72\ km^2$ **Ans.**

Q. 8. The dimensions of the model of a multistorey building are 1 *m* × 60 cm × 1·25 *m*. If the model is drawn to a scale 1 : 60, find the actual dimensions of the building in metres. Find :

(*i*) the floor area of a room of the building, whose area in the model is 250 cm^2.

(*ii*) the volume of the room in the model, whose actual volume is 648 cubic metres.

Sol. Dimensions of the model of a building =1*m* × 60cm × 1·25 *m* = 100cm × 60cm × 125cm.

i.e. length = 100 cm, breadth = 60 cm. and height = 125 cm.

Scale = 1 : 60

$\therefore$ Scale factor $(k) = \frac{1}{60}$

(*a*) $\therefore$ Actual length $= \frac{1}{k} \times 100$ cm

$= 60 \times 100$ cm. $= \frac{60 \times 100}{100} m = 60 m$

Breadth $= \frac{60 \times 60}{100} m = 36m$

and height $= \frac{125 \times 60}{100} m = 75$ m.

(*b*)(*i*) Area of floor of the building in model = 250 cm^2

$\therefore$ Actual area $= \frac{1}{k^2} \times 250\ cm^2 = 60 \times 60 \times 250 cm^2$

$= \frac{60 \times 60 \times 250}{100 \times 100} = 90 m^2$

(*ii*) Actual volume of the building = 648 cubic metres

$\therefore$ Volume in the model $= k^3 \times 648\ m^3$

$= \frac{1}{60 \times 60 \times 60} \times 648\ m^3$

$= \frac{100 \times 100 \times 100 \times 648}{60 \times 60 \times 60} cm^3 = 3000\ cm^3$ **Ans.**

Q. 9. A model of a ship is made to a scale of 1 : 250. Find :

(*i*) the length of the ship, if the length of its model is 1·2 *m*.

(*ii*) the area of the deck of the ship, if the area of the deck of its model is 1·6 m^2.

(*iii*) the volume of its model, when the volume of the ship is 1 cubic kilometre.

Sol. Scale = 1 : 250

$\therefore$ Scale factor $(k) = \frac{1}{250}$

(*i*) Length of ship in the model = 1·2 *m*

$\therefore$ Actual length of the ship $= \frac{1}{k}(1.2) m$

$= 250\ (1 \cdot 2)\ m = 300\ m$

(*ii*) Area of deck of ship in model = 1·6 m^2

$\therefore$ Actual area of the deck

$= \frac{1}{k^2} \times 1 \cdot 6\ m^2 = (250)^2 \times 1 \cdot 6\ m^2$

$= 250 \times 250 \times 1 \cdot 6\ m^2 = 100000\ m^2$

(*iii*) Volume of the ship = 1 m^3

$\therefore$ Volume in the model $= k^3\ (1)\ m^3$

$= \frac{1}{(250)^3} \times 1\ km^3 = \frac{1 \times 1000 \times 1000 \times 1000}{250 \times 250 \times 250} m^3$

$= 64\ m^3$ **Ans.**

10. The model of a buidling is constructed with scale factor 1 : 30.

(*i*) If the height of the model is 80 cm, find the actual height of the building in metres.

(*ii*) If the actual volume of a tank at the top of the building is 27 m^3, find the volume of the tank on the top of the model. (2009)

Sol.(*i*) $\frac{\text{Height of model}}{\text{Height of actual building}} = \frac{1}{30}$

$\frac{80}{H} = \frac{1}{30} \Rightarrow H = 2400\ cm = 24\ m$

(*ii*) $\frac{\text{Volume of model}}{\text{Volume of tank}} = \left(\frac{1}{30}\right)^3$

$\frac{V}{27} = \frac{1}{27000} \Rightarrow V = \frac{1}{1000} m^3 = 1000\ cm^3$