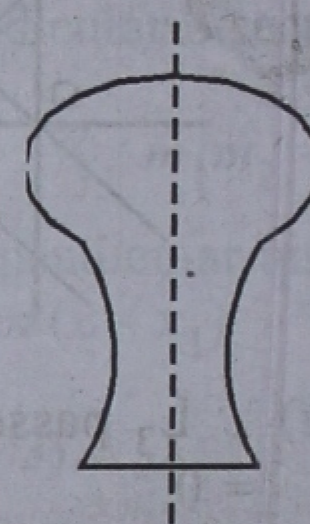
Unit 4
Geometry

Chapter 15
Symmetry

POINTS TO REMEMBER

I. Line of Symmetry :



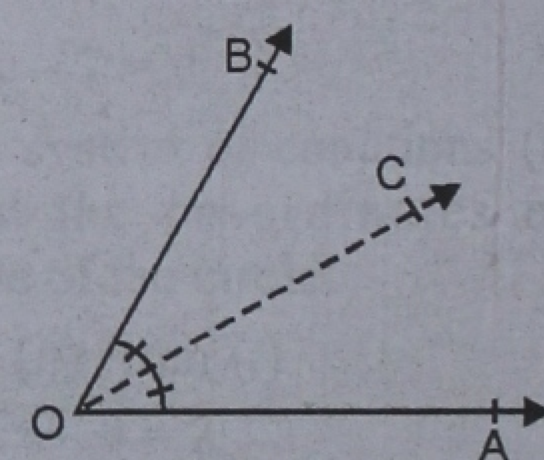
Trace the given figure and the dotted line on a piece of paper and fold it along the dotted line. You will find that the two parts of the figure on both sides of the line coincide with each other.

Thus, the dotted line divides the given figure into two identical figures. We say that the given figure is **symmetrical about the dotted line.**

Line of symmetry. *Whenever a line divides a given figure into two congruent figures, i.e., two identical halves, we say that the given figure is symmetrical about that line.* And, in this case the line is called the **axis of symmetry or line of symmetry.**

Some Examples :

1. *An angle with equal arms is symmetrical about its bisector.*

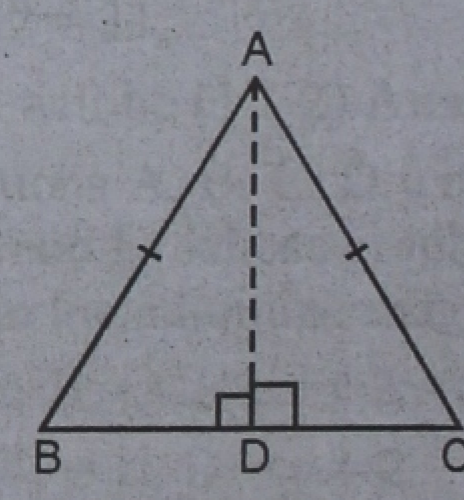


Let ∠AOB be a given angle with equal arms OA and OB and let OC be the bisector of ∠AOB.

Clearly, OC divides ∠AOB into two identical halves.

∴ OC is the line of symmetry of ∠AOB.

2. *An isosceles triangle has one line of symmetry, namely the bisector of the vertical angle.*



Let ΔABC be an isosceles triangle with AB = AC.

Let AD be the bisector of ∠A.

Then, AD ⊥ BC and D is the mid-point of BC.

Now , in Δs ABD and ACD, we have

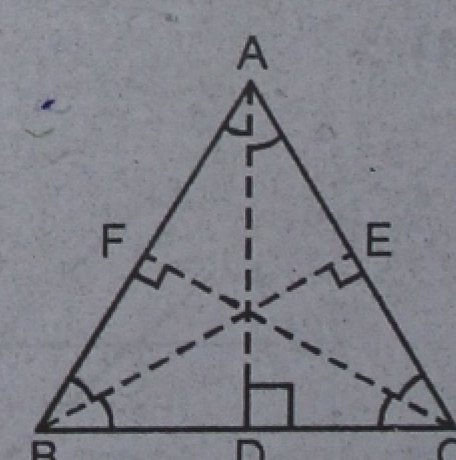
AB = AC, BD = DC and AD = AD.

∴ Δ ABD ≅ Δ ACD

Thus, AD divides Δ ABC into two congruent triangles.

∴ AD is the line of symmetry of ΔABC.

3. *An equilateral triangle has three lines of symmetry, namely the bisector of each of its angles.*



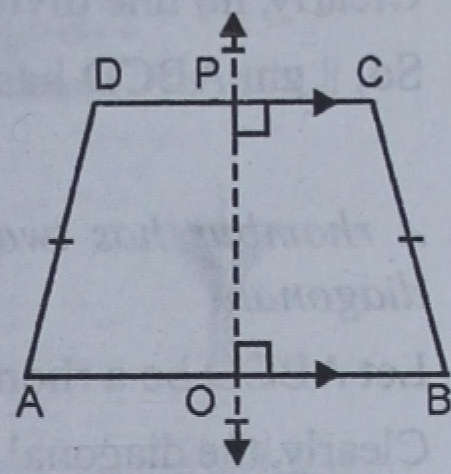
Let ΔABC be an equilateral triangle and let AD, BE and CF be the bisectors of ∠A, ∠B and ∠C respectively.

Clearly, AD divides ΔABC into two congruent triangles.

∴ AD is the line of symmetry of ΔABC.

Similarly, BE as well as CF is the line of symmetry of ΔABC.

4. *An isosceles trapezium has one line of symmetry, namely the perpendicular bisector of its parallel sides.*



Let ABCD be an isosceles trapezium in which

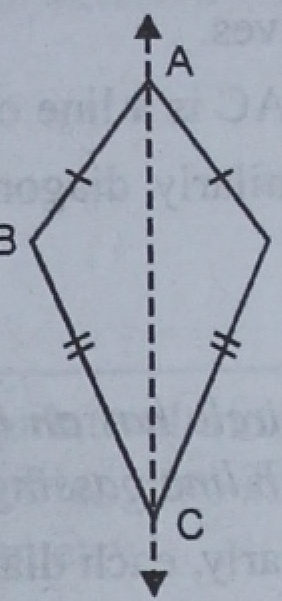
AB || DC and AD = BC.

Let PQ be a line which is the perpendicular bisector of AB as well as DC.

Clearly, PQ divides ABCD into two identical halves.

∴ PQ is the line of symmetry of trap. ABCD.

5. *A kite has the vertical diagonal as the line of symmetry.*

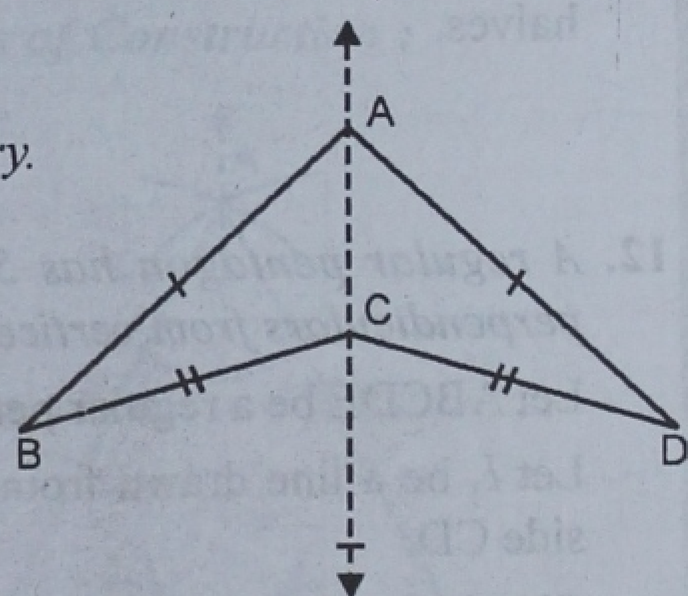


Let ABCD be the kite in which AB = AD and BC = DC.

Clearly, the vertical diagonal AC divides the kite ABCD into two identical halves.

∴ AC is the line of symmetry of kite ABCD.

6. *The vertical diagonal of an arrowhead is the line of symmetry.*

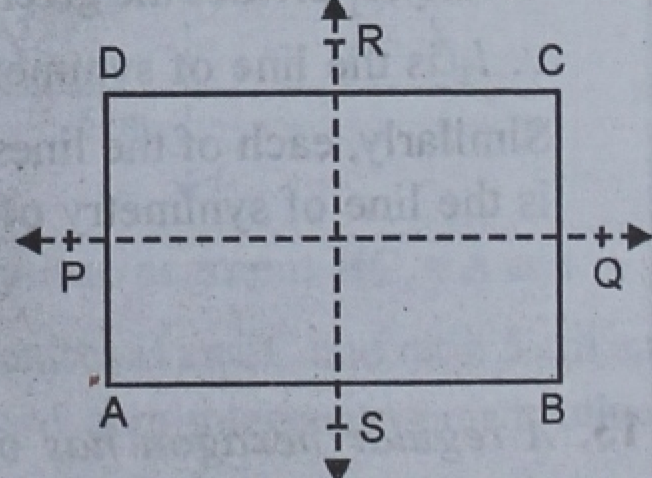


Let ABCD be an arrowhead in which AB = AD and BC = DC.

Clearly, the line AC divides the figure into two identical halves.

Hence, AC is the line of symmetry of the arrowhead ABCD

7. *A rectangle has two lines of symmetry, namely each of the lines joining the mid-points of its opposite sides.*



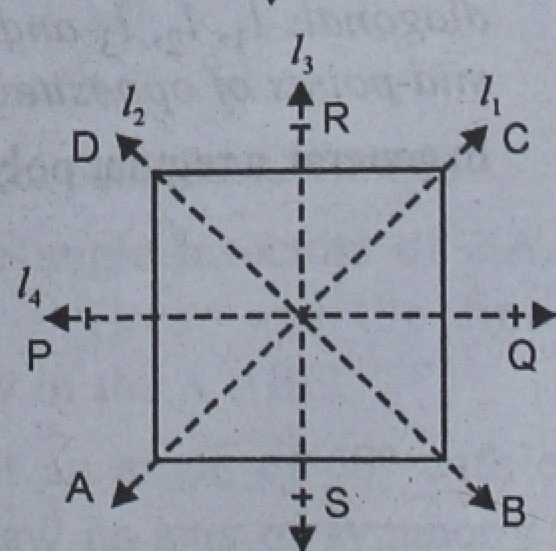
Let ABCD be a rectangle and let PQ be the line joining the mid-points of one pair of opposite sides AD and BC.

Clearly, PQ divides rect. ABCD into two identical halves.

∴ PQ is a line of symmetry of rect. ABCD.

Similarly, if R and S be the mid-points of DC and AB respectively, then RS is a line of symmetry of rect. ABCD.

8. *A square has 4 lines of symmetry, namely the two diagonals and the lines joining the mid-points of its opposites sides.*



Let ABCD be a square in which l_1 and l_2 be its diagonals ; l_3 be the line joining the mid-points of AB and DC and l_4 be the line joining the mid-points of AD and BC.

Each one of l_1, l_2, l_3 and l_4 divides the sq. ABCD into two identical halves.

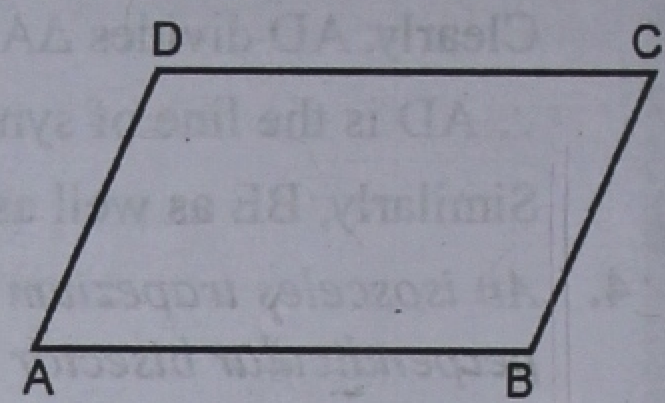
∴ Each one of these lines is the line of symmetry of sq. ABCD.

9. *A parallelogram has no line of symmetry.*

Let ABCD be a parallelogram.

Clearly, no line divides it into two identical halves.

So, || gm ABCD has no line of symmetry.



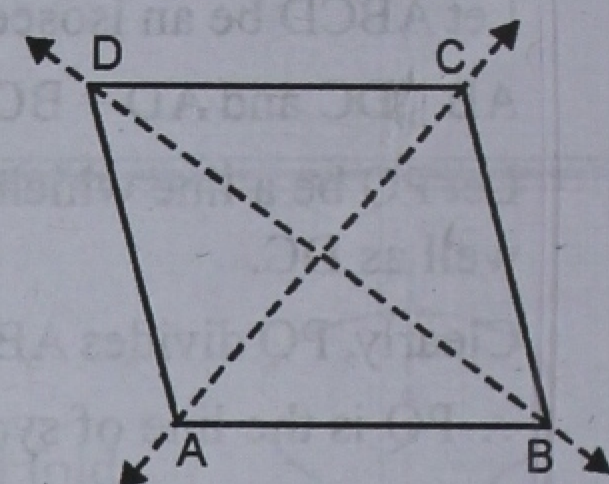
10. *A rhombus has two lines of symmetry, namely each of its diagonals.*

Let ABCD be a rhombus.

Clearly, the diagonal AC divides the rhombus into two identical halves.

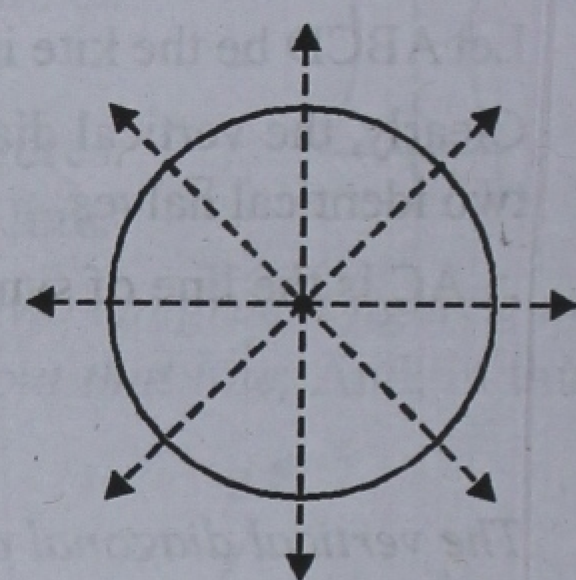
∴ AC is a line of symmetry.

Similarly, diagonal BD is a line of symmetry.



11. *A circle has an infinite number of lines of symmetry, namely each line passing through the centre of the circle.*

Clearly, each diameter of divides the circle into two identical halves.



12. *A regular pentagon has 5 lines of symmetry, namely the perpendiculars from vertices to the opposite sides.*

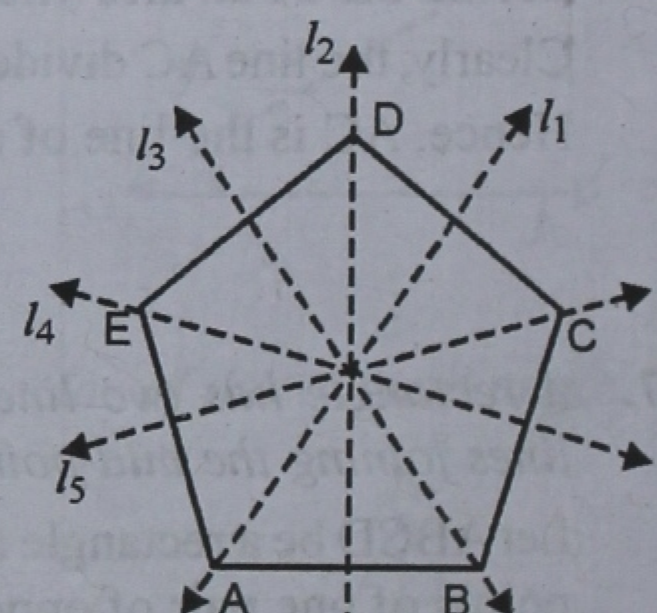
Let ABCDE be a regular pentagon.

Let l_1 be a line drawn from A, perpendicular to the opposite side CD.

Clearly, l_1 divides the given pentagon into two identical figures.

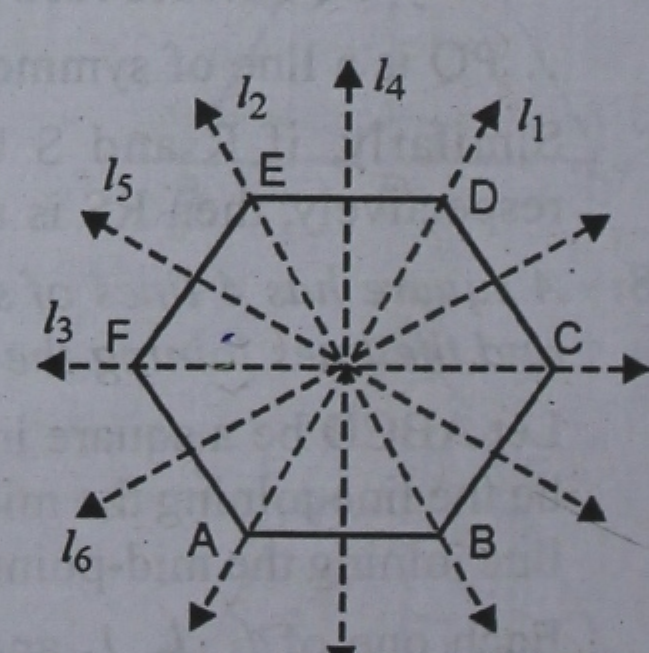
∴ l_1 is the line of symmetry of the given pentagon.

Similarly, each of the lines l_2, l_3, l_4 and l_5 shown in the figure is the line of symmetry of the given pentagon.

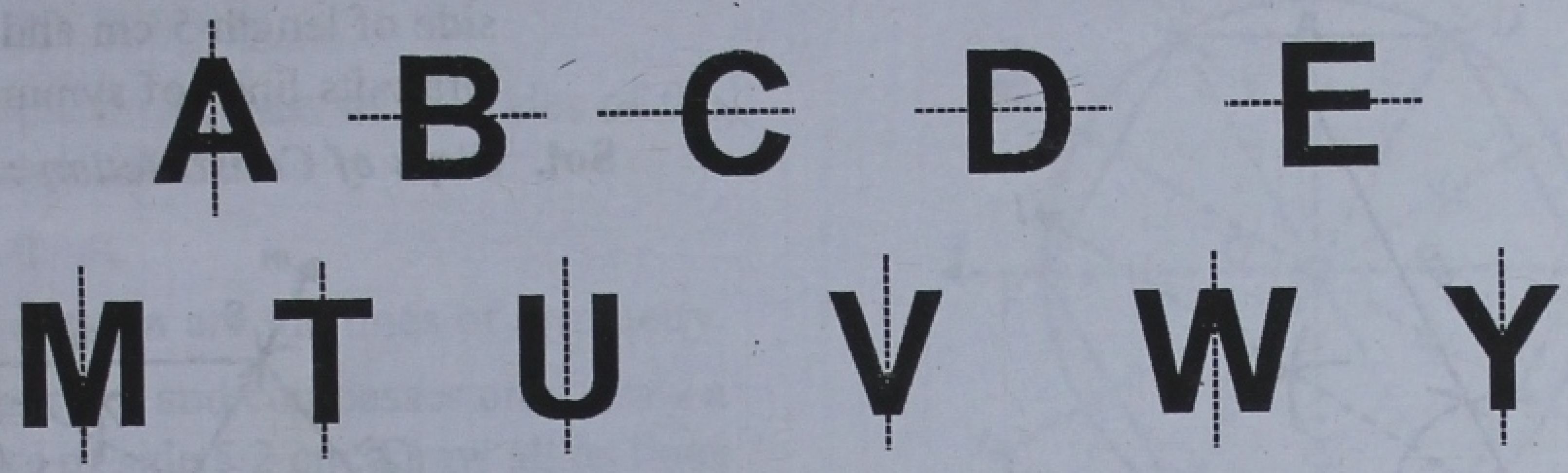


13. *A regular hexagon has 6 lines of symmetry, namely three diagonals l_1, l_2, l_3 and three lines l_4, l_5, l_6 each joining the mid-points of opposite sides of the hexagon.*

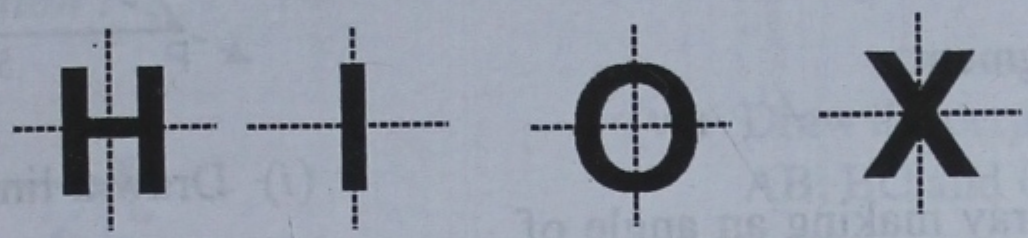
In general, a regular polygon of *n* sides has *n* lines of symmetry.



14. (*i*) *Each of the letters given below has one line of symmetry, shown by the dotted line.*



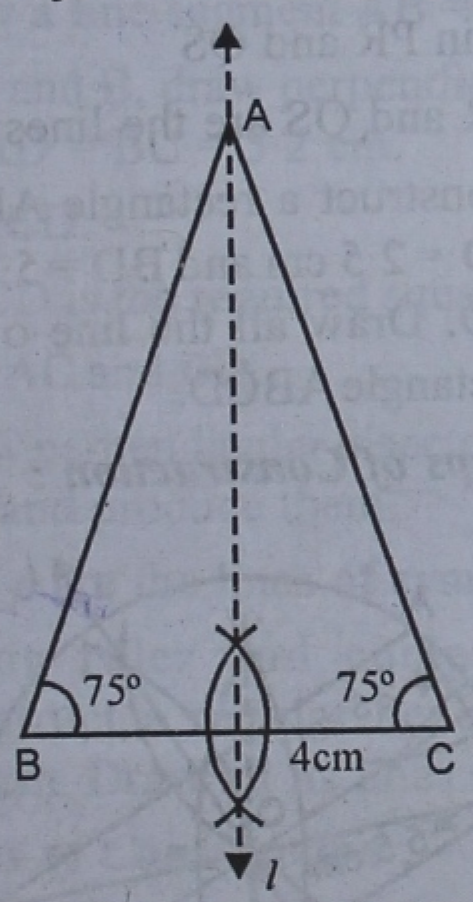
(*ii*) *Each of the letters given below has two lines of symmetry, shown by dotted lines.*



EXERCISE 15

Q.1. Construct an isosceles triangle with base = 4 cm and each base angle measuring 75°. Draw its line of symmetry.

Sol. ***Steps of Construction :***



(*i*) Draw a line segment BC = 4 cm.

(*ii*) At B and C draw rays making an angle of 75° each which meet each other at A.

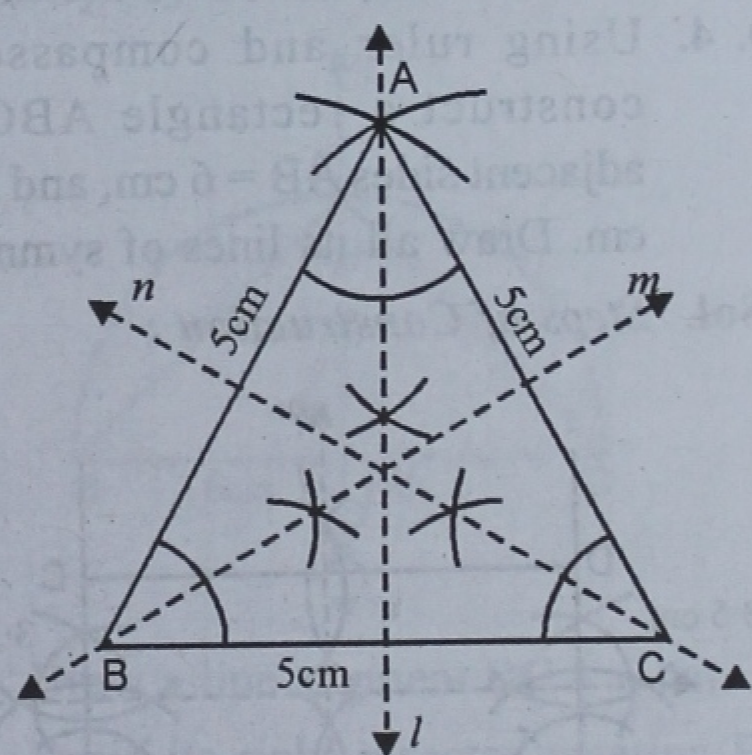
Δ ABC is an isosceles triangle.

(*iii*) Draw the right bisector of BC which meets at A.

This right bisector of side BC is the required line of symmetry.

Q.2. Construct an equilateral triangle each of whose sides is of length 5 cm. Draw all its lines of symmetry.

Sol. ***Steps of Construction :***



(*i*) Draw a line segment BC = 5 cm.

(*ii*) With centres B and C and radii 5 cm each, draw two arcs intersecting each other at A.

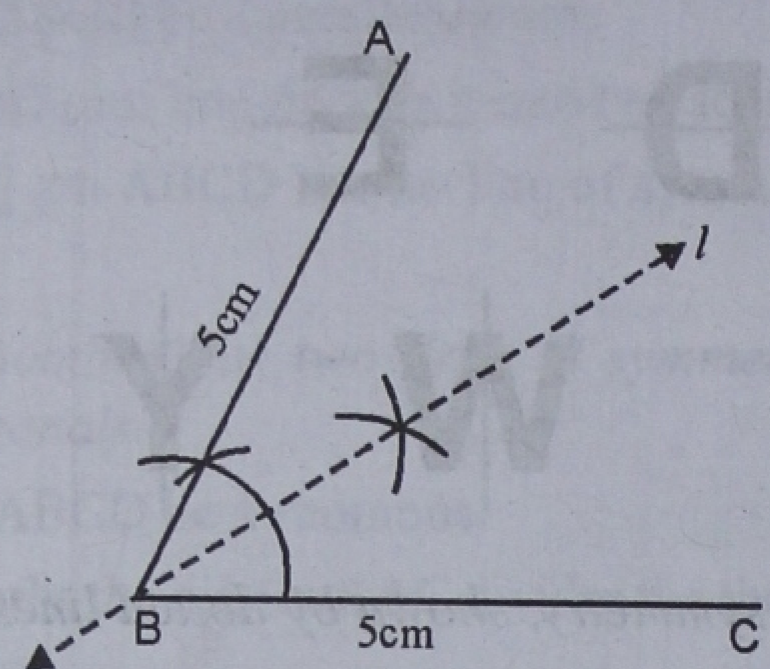
(*iii*) Join AB and AC.

ΔABC is an equilateral triangle.

(*iv*) Draw the angle bisectors of ∠A, ∠B and ∠C. These are the lines of symmetry of the Δ ABC.

Q.3. Construct an angle of 60° with equal arms. Draw its axis of symmetry.

Sol. ***Steps of Construction :***



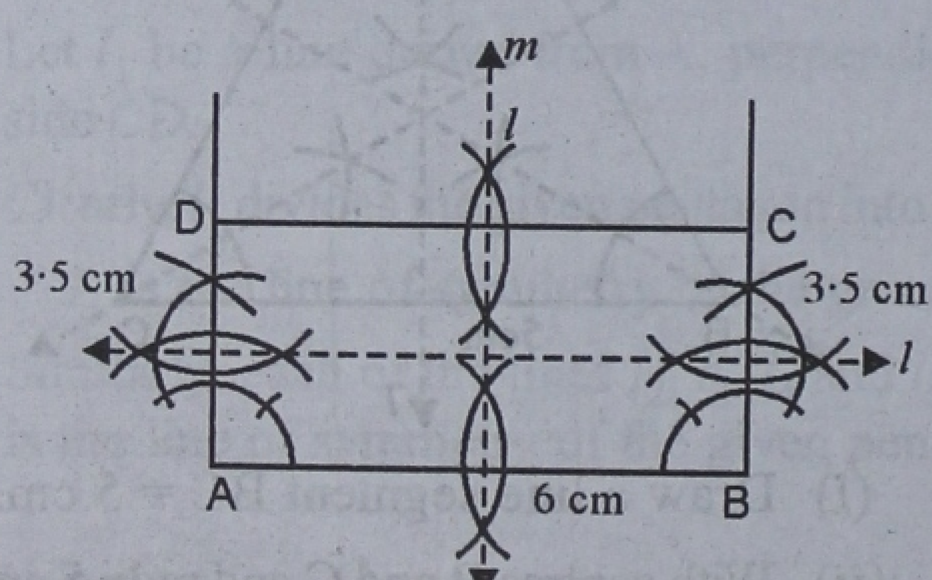
(*i*) Draw a line segment.
BC = 5 cm.

(*ii*) At B, draw an ray making an angle of 60° with the help of ruler and compasses and cut off BA = 5 cm.
∠ABC is the angle of 60°.

(*iii*) Draw the bisector of ∠ABC.
This is the required axis of symmetry.

Q. 4. Using ruler and compasses only, construct a rectangle ABCD with adjacent sides AB = 6 cm. and BC = 3·5 cm. Draw all its lines of symmetry.

Sol. ***Steps of Construction :***



(*i*) Draw a line segment AB = 6 cm.

(*ii*) At A and B, draw rays making are angle of 90° each and cut off AD = BC = 3·5 cm.

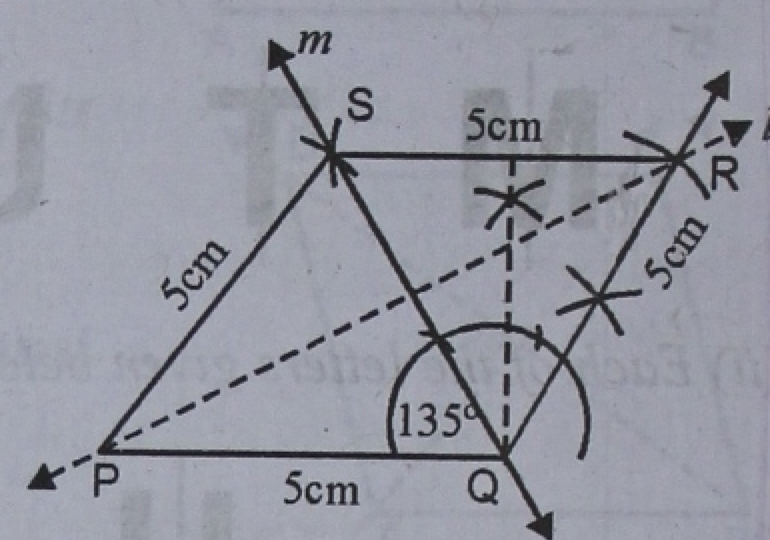
(*iii*) Join DC.
ABCD is the rectangle

(*iv*) Draw the mid-poinjts of each side.

(*v*) Join the mid-points of opposite sides these are the lines of symmetry.

Q. 5. Construct a rhombus PQRS with each side of length 5 cm and ∠PQR = 135°. Draw its lines of symmetry.

Sol. ***Steps of Construction :***



(*i*) Draw a line segment PQ = 5 cm.

(*ii*) At Q draw a ray making an angle of 135° and cut off QR = 5cm.

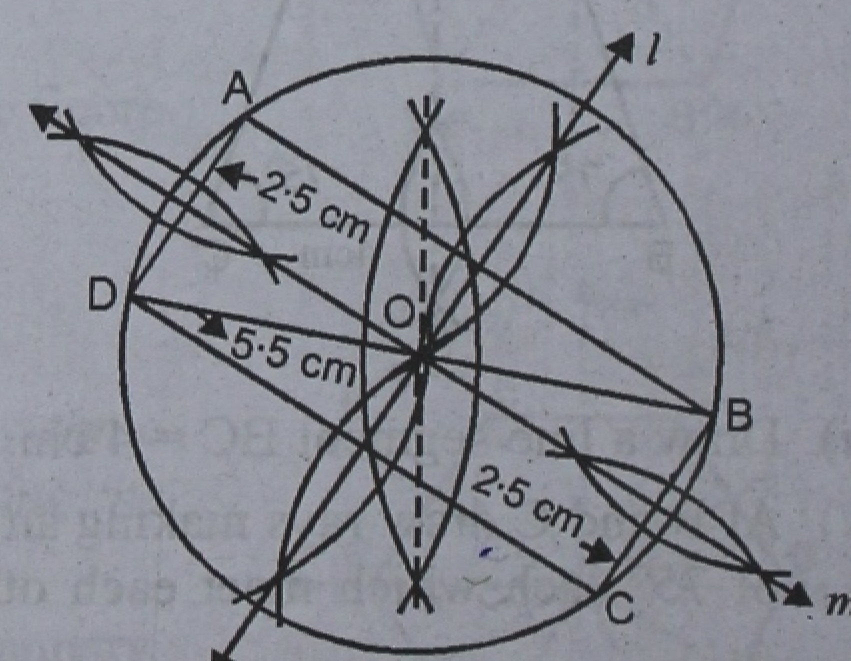
(*iii*) With centre P and R draw two arcs of 5 cm radius each intersecting each other at S.

(*iv*) Join PS and RS.
PQRS is the rhombus.

(*v*) Join PR and QS
PR and QS are the lines of symmetry.

Q. 6. Construct a rectangle ABCD in which AD = 2·5 cm and BD = 5·5 cm. Measure CD. Draw all the line of symmetry of rectangle ABCD.

Sol. ***Steps of Construction :***



(*i*) Draw a line segment BD = 5·5 cm.

(*ii*) With BD as diameter, draw a circle.

(*iii*) With centre D and B and radius 2·5 cm., draw arcs intersecting the circle at A and C respectively.

(*iv*) Join AB, AD, CB and CD. ABCD is the required rectangle on measurment the side CD, it is 4·9 cm.

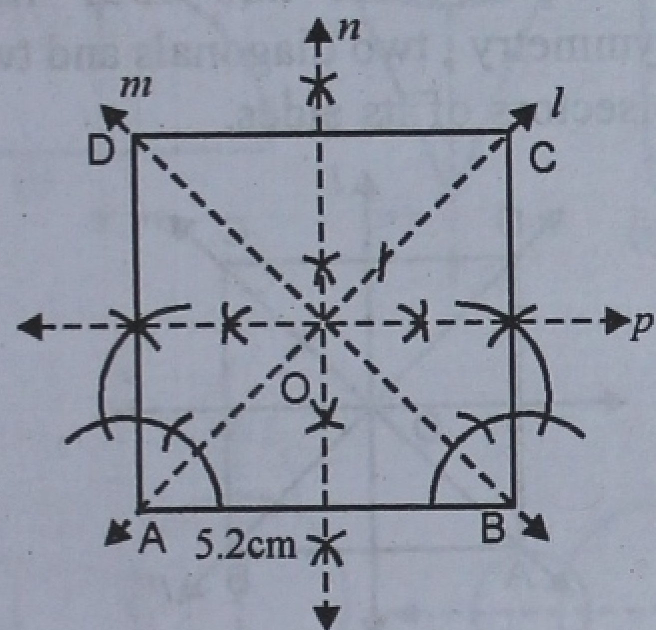
(*v*) Find the mid-points of the sides of the rectangle ABCD.

(*vi*) Join them,

then *l* and *m* are the lines of symmetry.

Q. 7. Using ruler and compasses only, draw a square of side 5·2 cm. Draw all its lines of symmetry.

Sol. ***Steps of Construction :***



(*i*) Draw a line segment AB = 5·2 cm.

(*ii*) At A and B, draw perpendicular and cut off AD = BC = 5·2 cm.

(*iii*) Join CD.

ABCD is the required square.

(*iv*) Join AC and BD.

(*v*) Draw perpendicular bisectors of AB and AD and produce them.

∴ *l*, *m*, *n* and *p* are the lines of symmetry.

Q. 8. Using ruler and compasses only, construct a regular hexagon of side 3·6 cm. Draw all its lines of symmetry.

Sol. ***Steps of Construction :***

(*i*) Draw a circle of radius 3·6 cm.

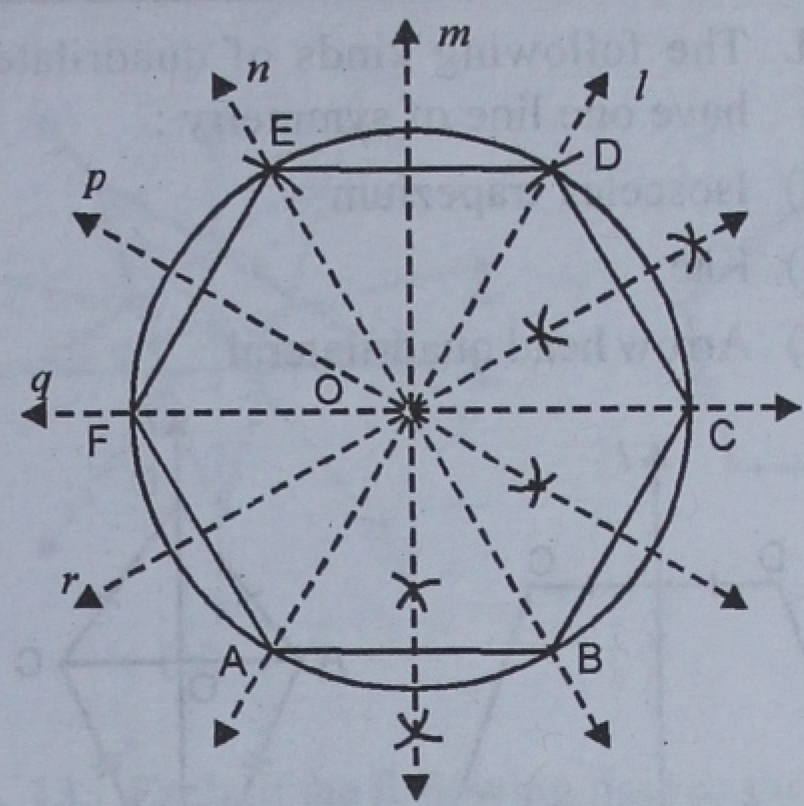
(*ii*) Take a point A on it.

(*iii*) With centre A and radius 3·6 cm., cut off the circle at B, C, D, E and F.

(*iv*) Join AB, BC, CD, DE, EF and FA.

ABCDEF is the required hexagon.

(*v*) Join AD, BE and CF and produce them both sides.

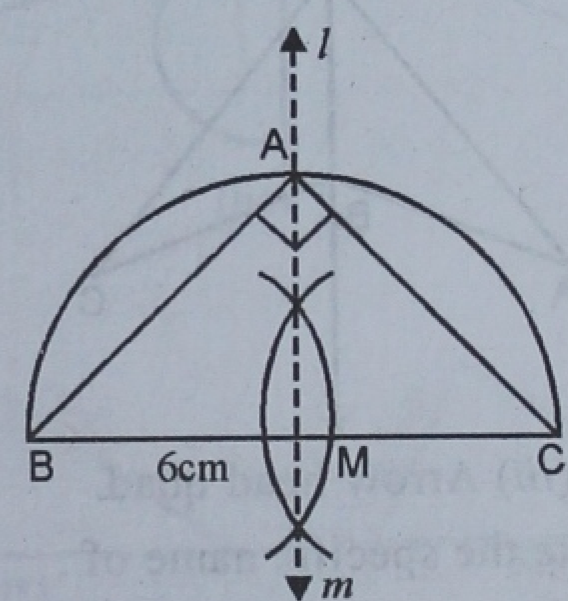


(*vi*) Draw the perpendicular bisectors of side AB, BC and CD and produced them.

∴ *l*, *m*, *n*, *p*, *q*, *r* are the six lines of symmetry.

Q. 9. Draw an isosceles right-angled triangle having hypotenuse equal to 6 cm. Draw its line of symmetry.

Sol. ***Steps of Construction :***



(*i*) Draw a line segment BC = 6 cm.

(*ii*) Draw its right bisector meeting BC at M.

(*iii*) With centre M, and BC as diameter, draw a semi-circle.

(*iv*) Produce the right bisector of BC meeting the semi-circle at A.

(*v*) Join AB and AC.

ΔABC is an isosceles right angled triangle. AM, the right bisector of side BC is the required line of symmetry.

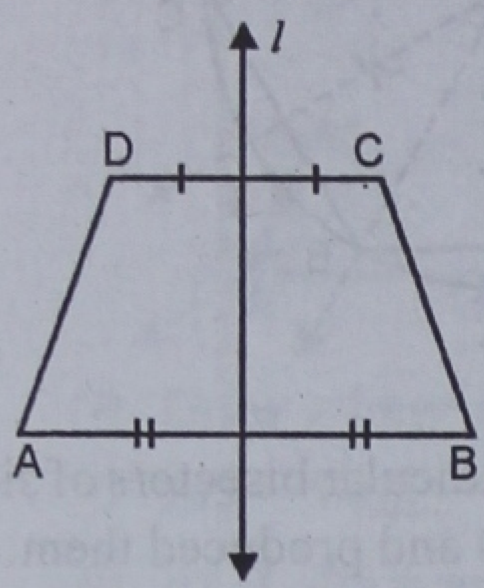
Q. 10. Write the specific names of all these quadrilaterals which have only one line of symmetry.

Sol. The following kinds of quadrilaterals have one line of symmetry :

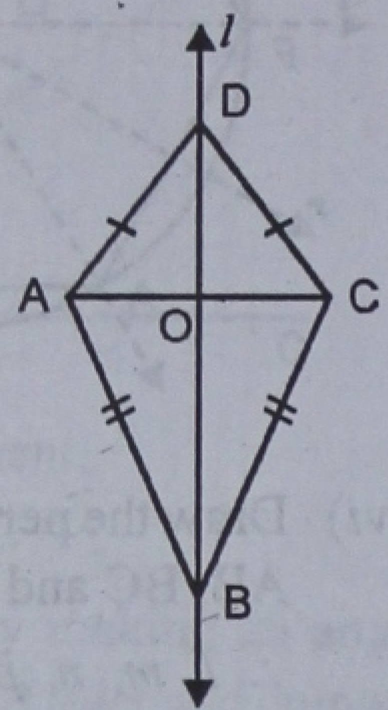
(*i*) Isosceles trapezium

(*ii*) Kite

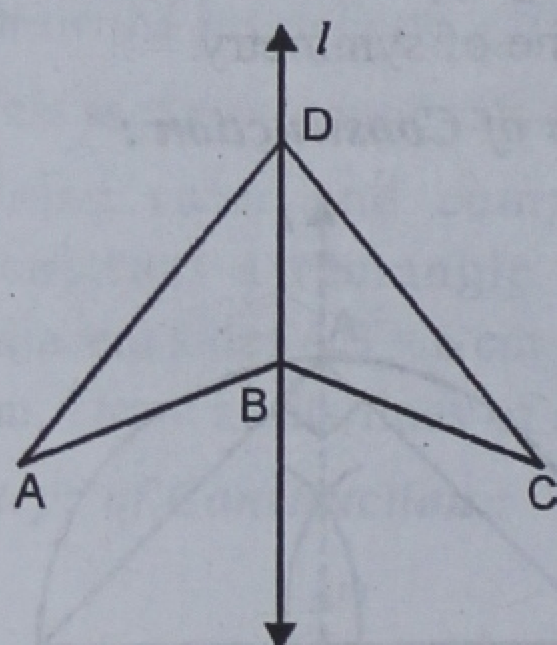
(*iii*) Arrow head quadrilateral



(*i*) Isosceles Trapezium



(*ii*) Kite



(*iii*) Arrow head quad.

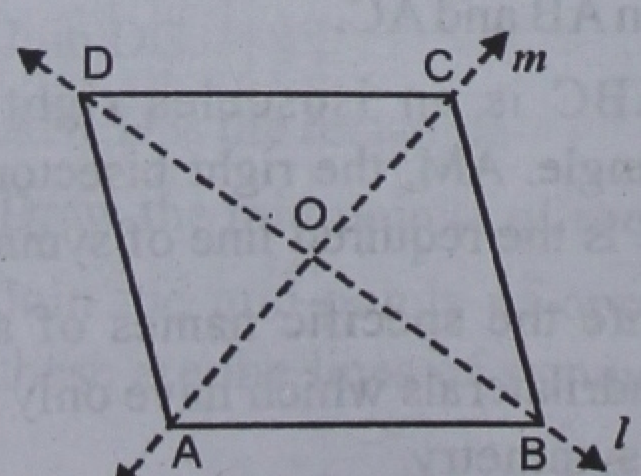
Q. 11. Write the specific name of :

(*a*) the quadrilateral having its diagonals as the only lines of symmetry ;

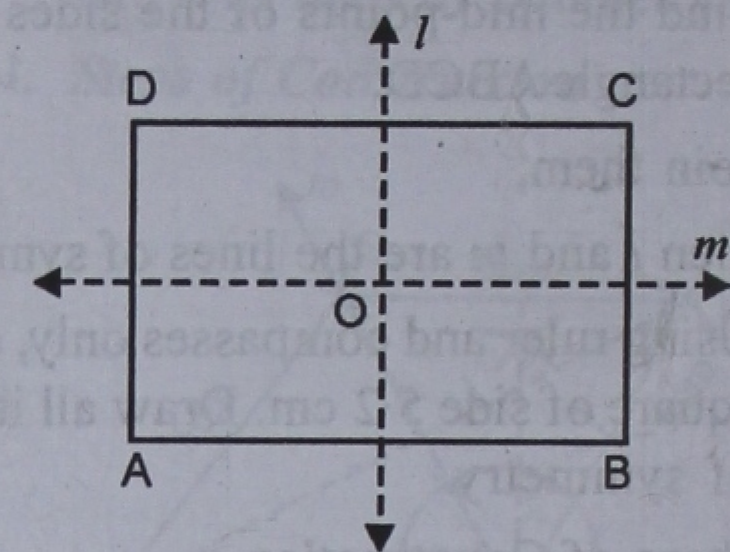
(*b*) the quadrilateral having two, but not the diagonals, as the lines of symmetry ;

(*c*) the quadrilateral having more than two lines of symmetry.

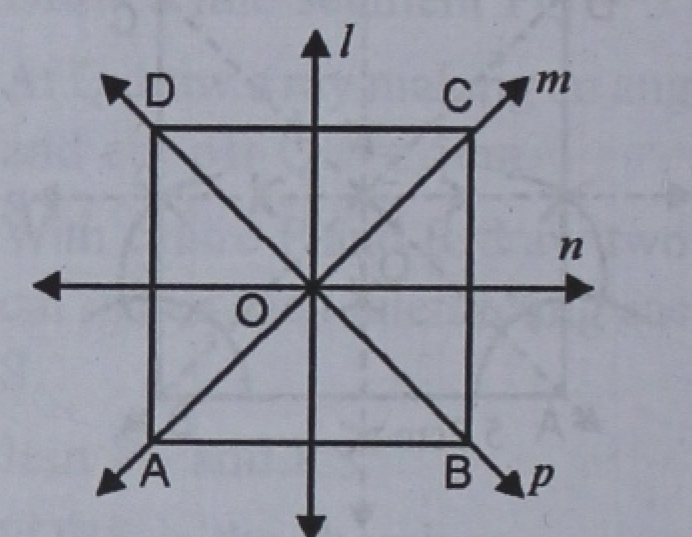
Sol. (*a*) **A Rhombus.** Its two diagonals are the lines of symmetry.



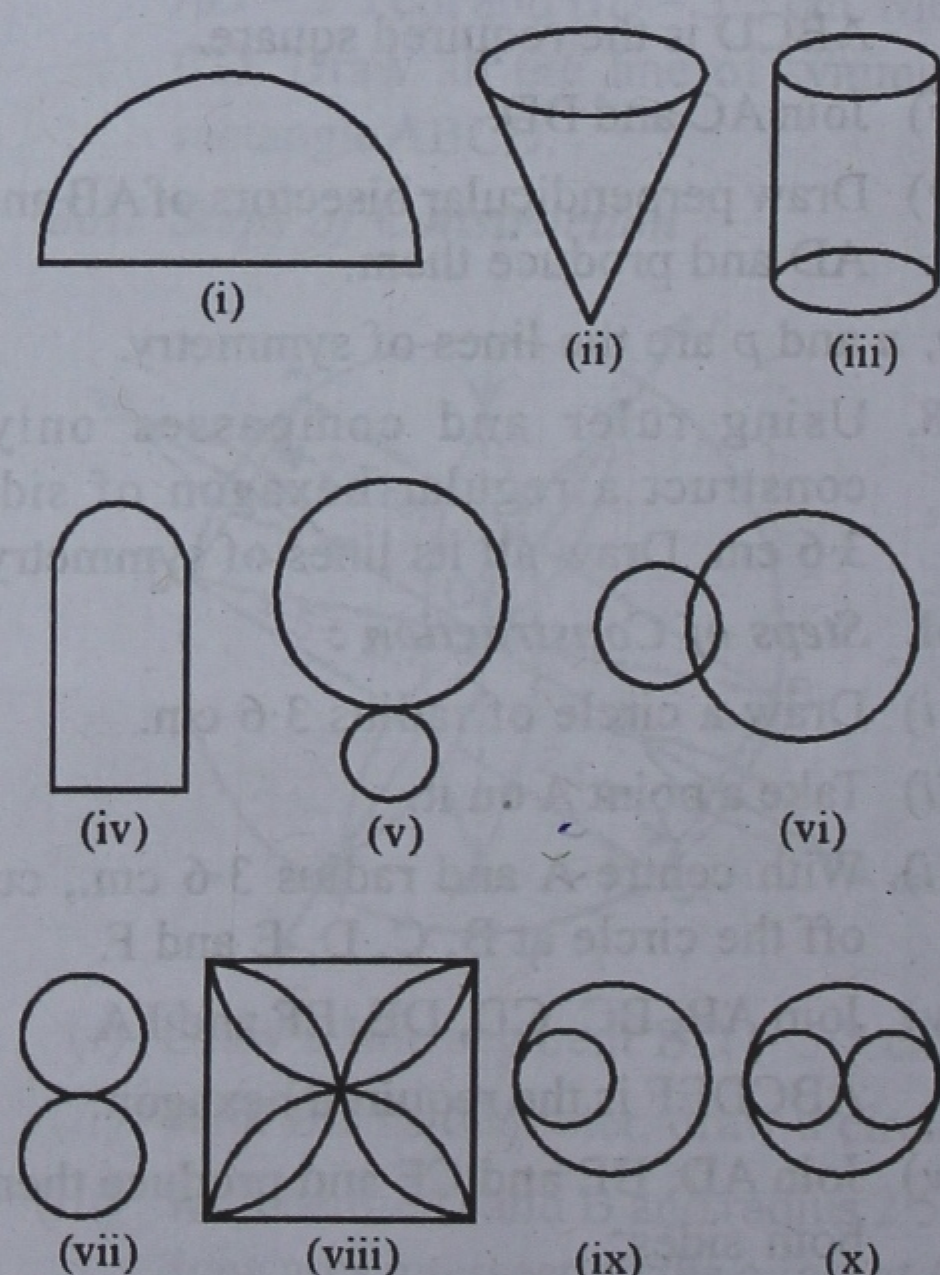
(*b*) **A Rectangle.** It has two lines of symmetry which are the perpendicular bisectors of its sides.

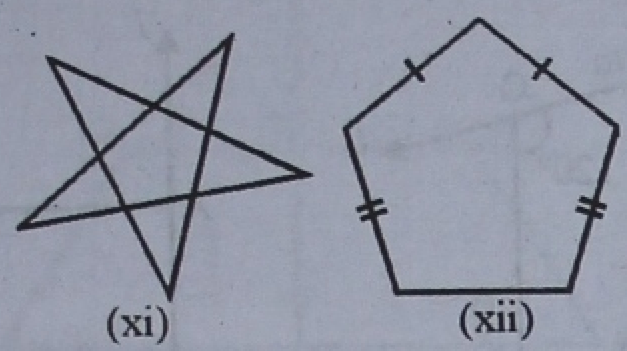


(*c*) **A Square.** It has four lines of symmetry ; two diagonals and two right bisectors of its sides.

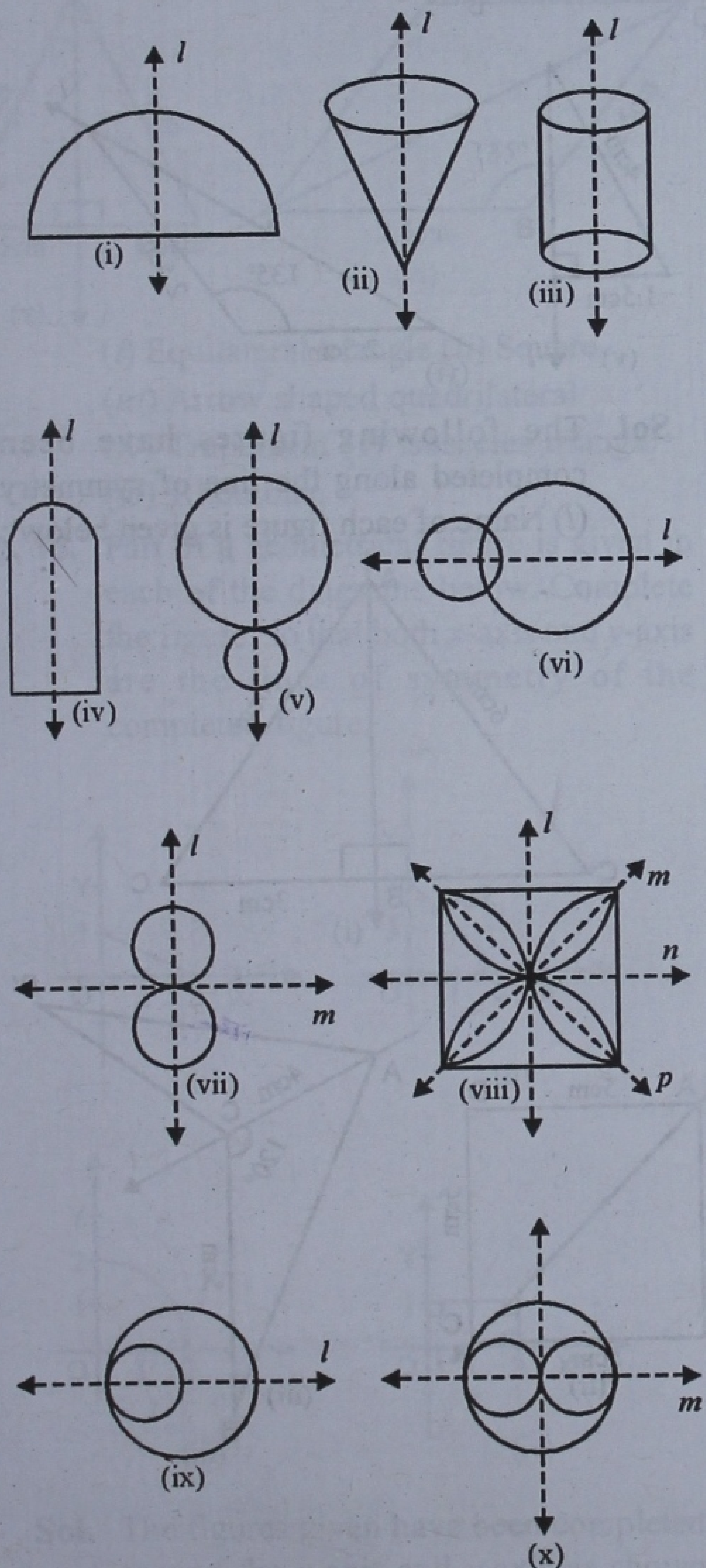


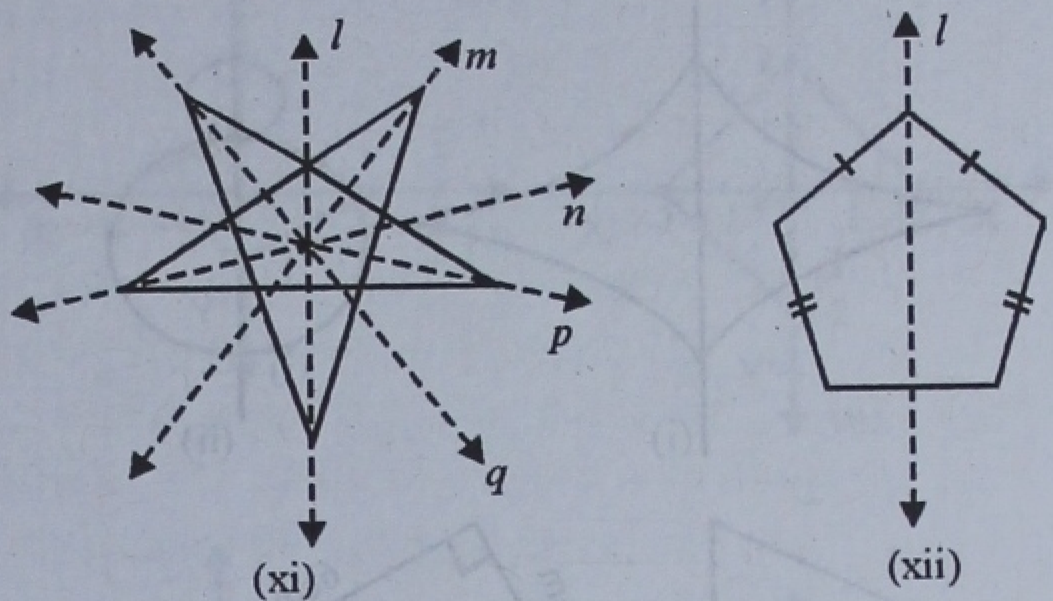
Q. 12. Copy each of the following figures in your note-book and draw in each case, the line (or lines) of symmetry. Indicate them by dotted lines.



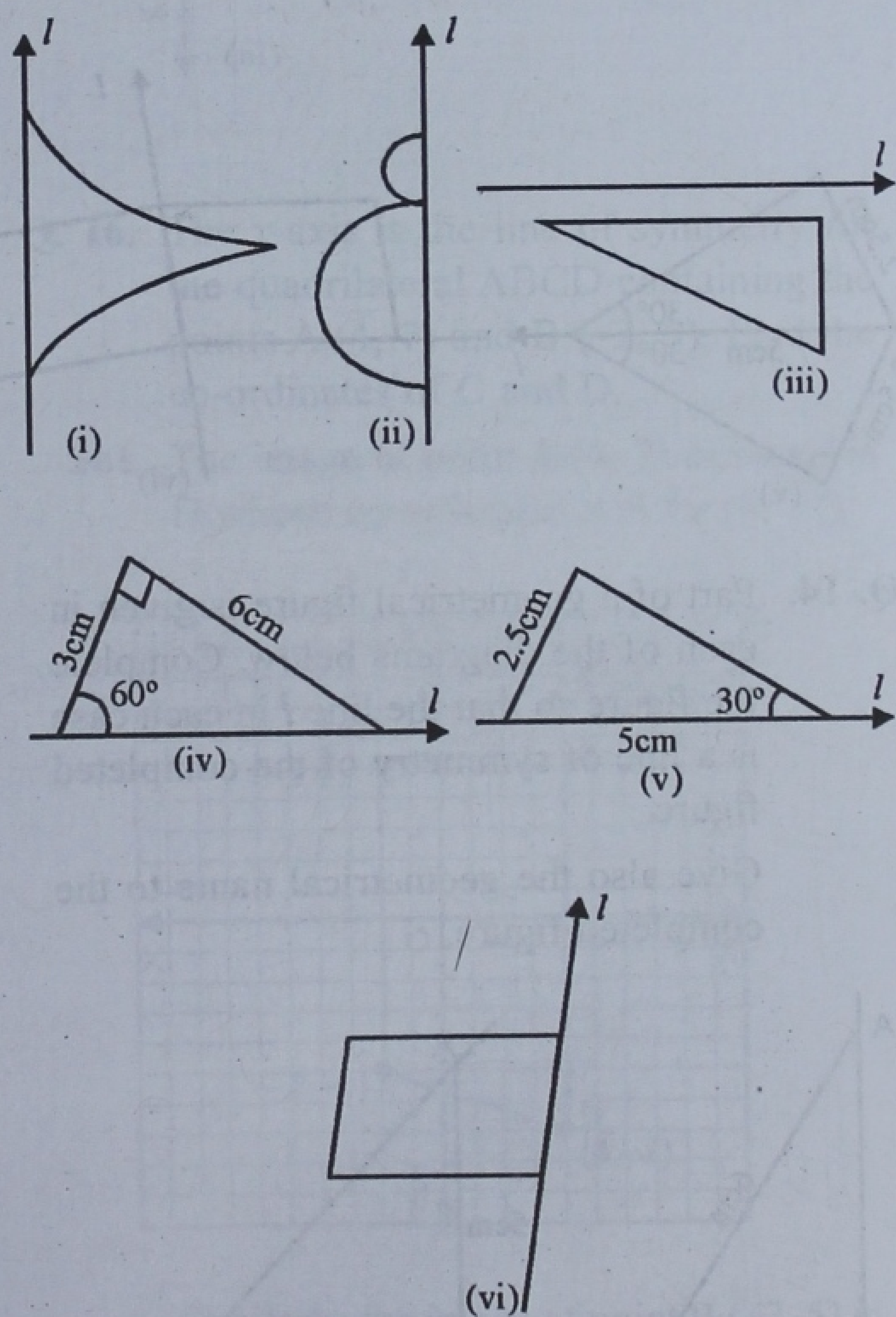


Sol. The figures given, have line or lines of symmetry as given below :

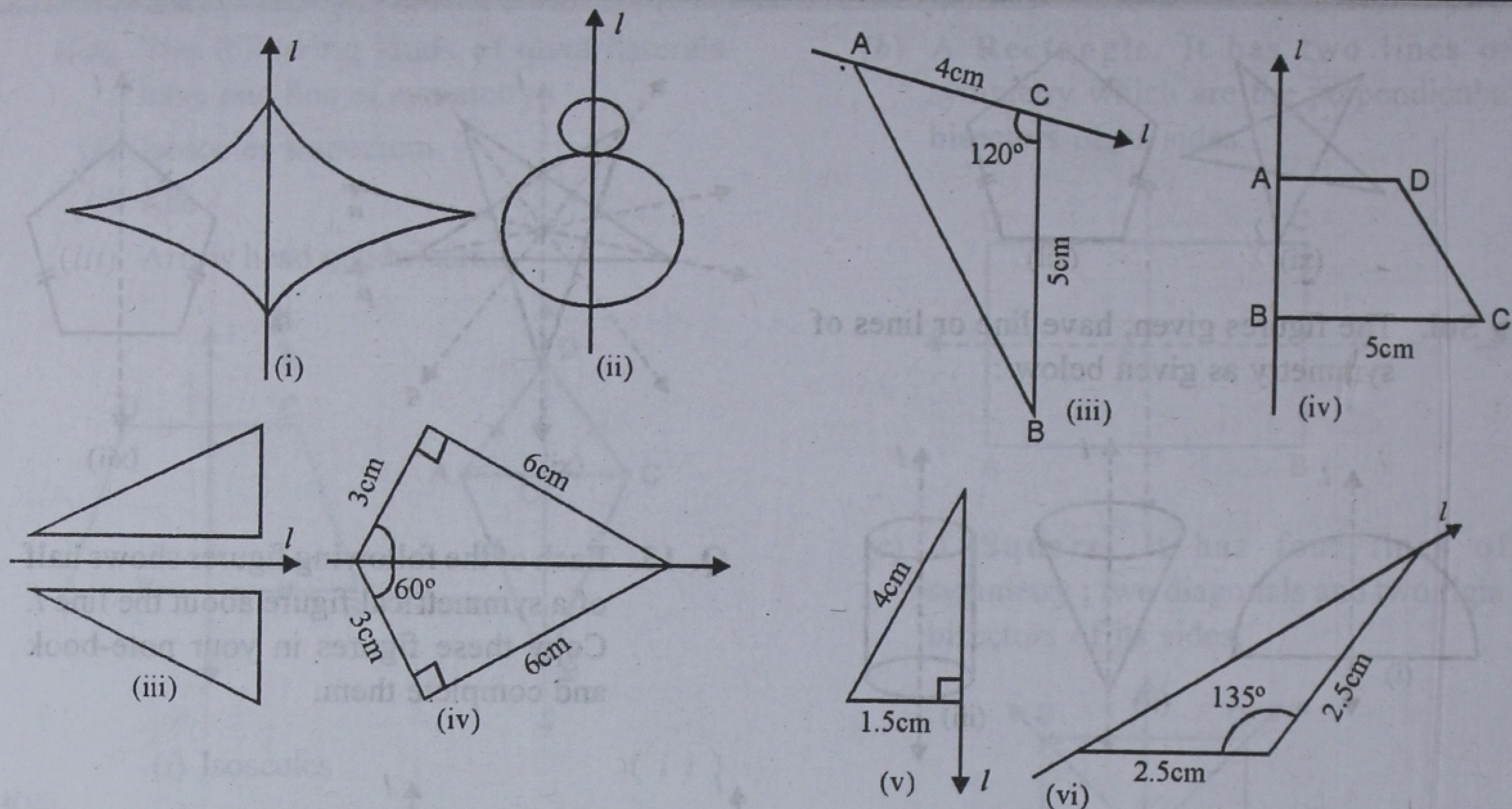


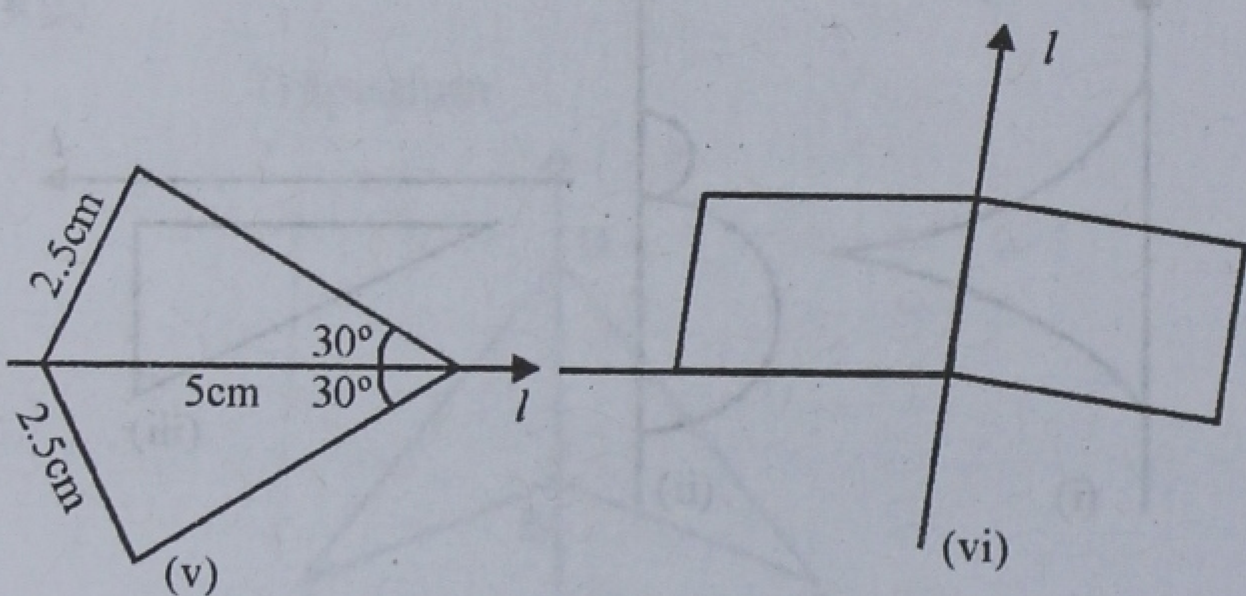


Q. 13. Each of the following figures shows half of a symmetrical figure about the line l. Copy these figures in your note-book and complete them.

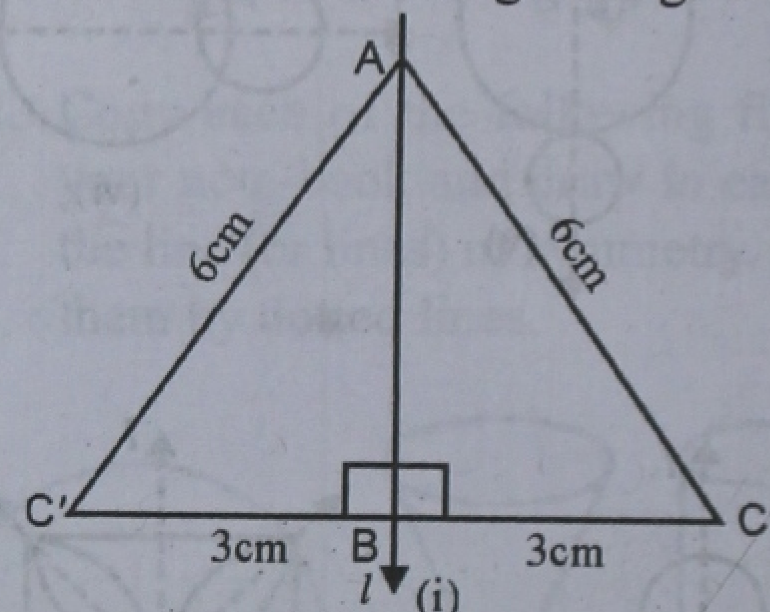


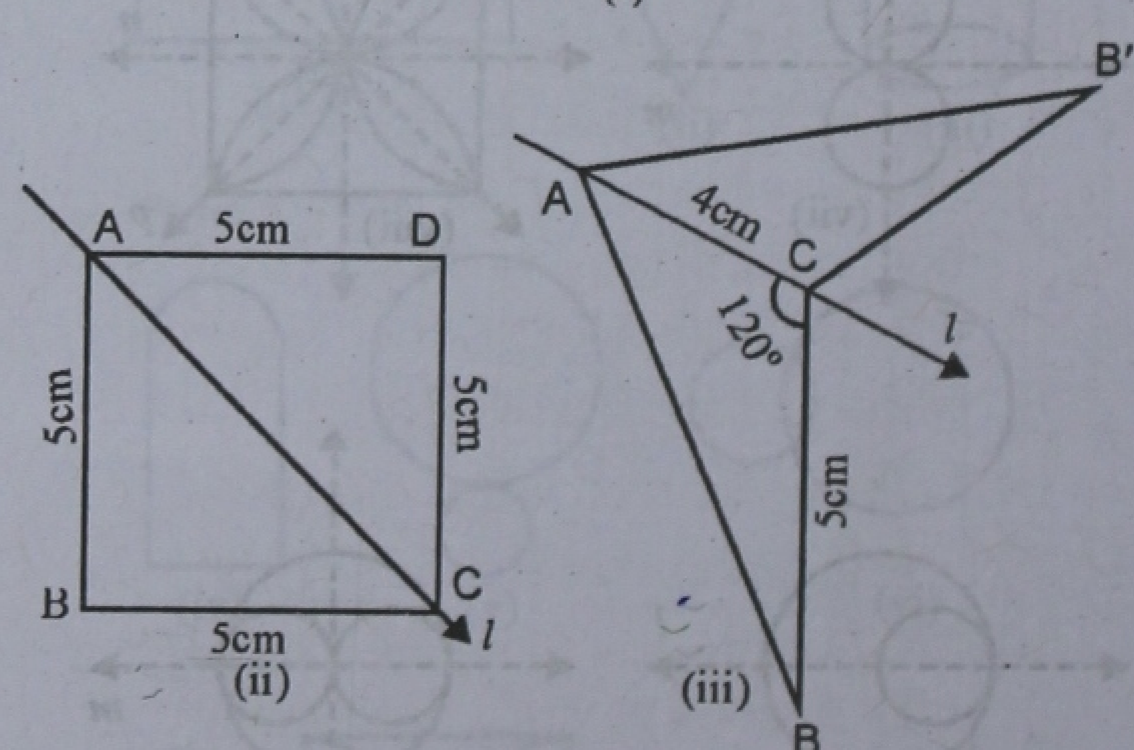
Sol. Following is given the complete figures which are symmetrical along the line (l) given in each figure :





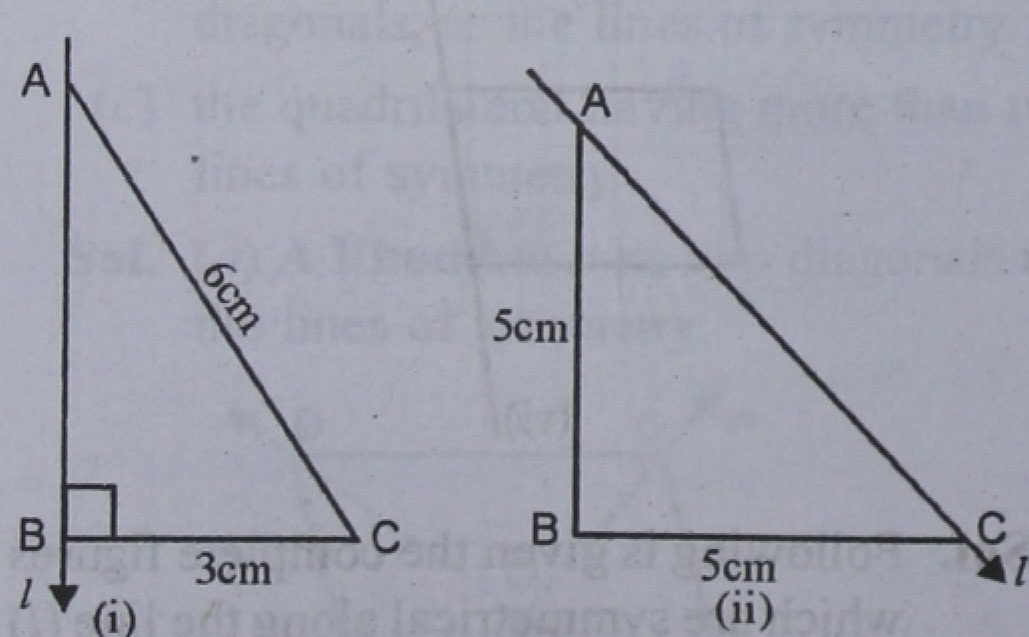
Sol. The following figures have been completed along the line of symmetry (l) Name of each figure is given below :

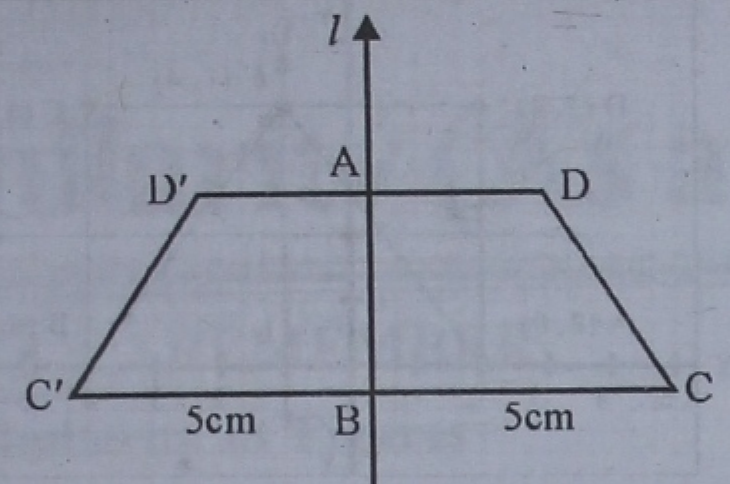




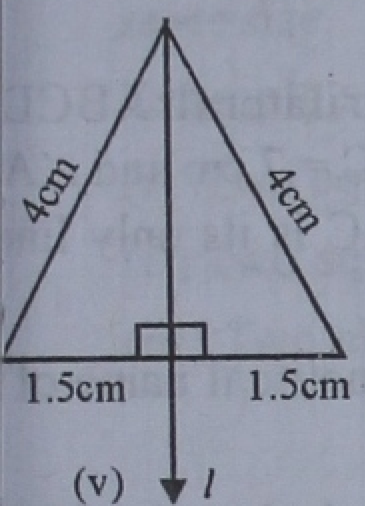
Q. 14. Part of a geometrical figure is given in each of the diagrams below. Complete the figure so that the line l in each case is a line of symmetry of the completed figure.

Give also the geometrical name to the completed figure.

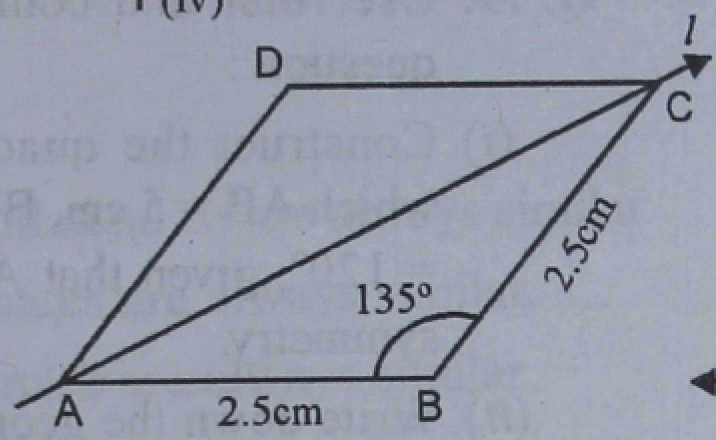




(iv)

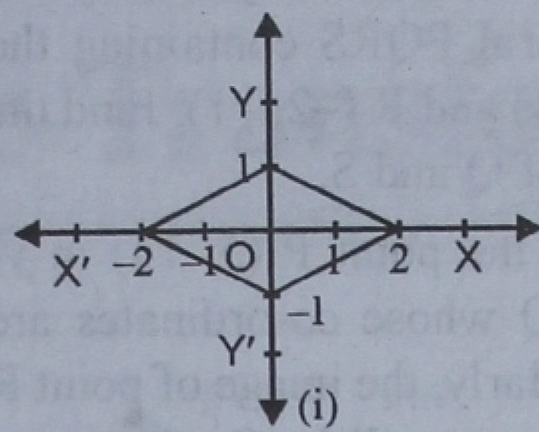


(v)

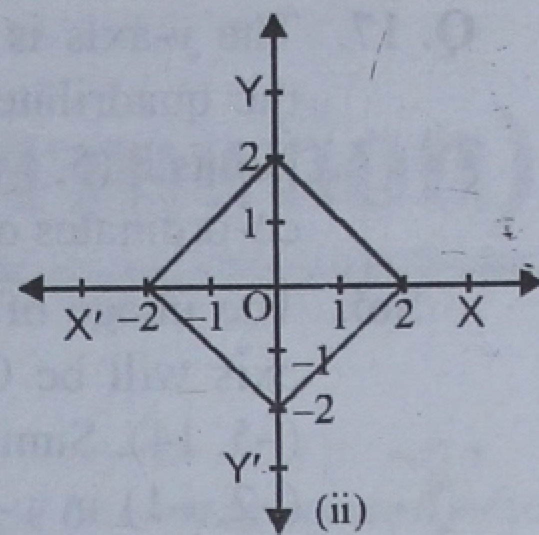


(vi)

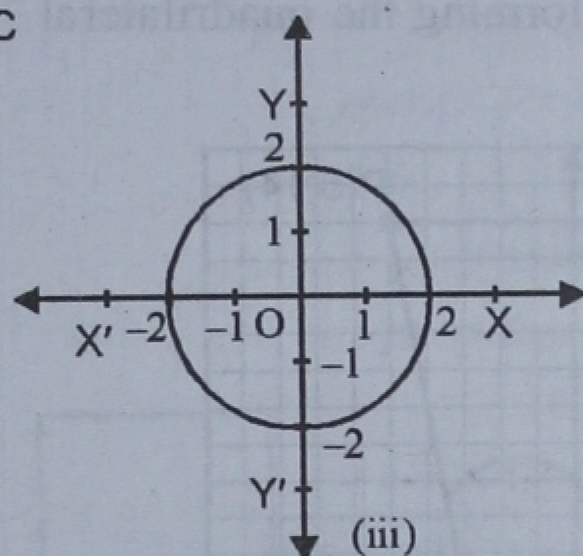
(*i*) Equilateral triangle (*ii*) Square
(*iii*) Arrow shaped quadrilateral
(*iv*) Trapezium (*v*) Isosceles triangle
(*vi*) Rhombus.



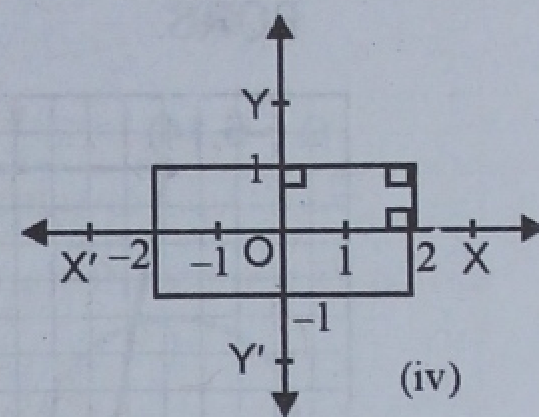
(i)



(ii)

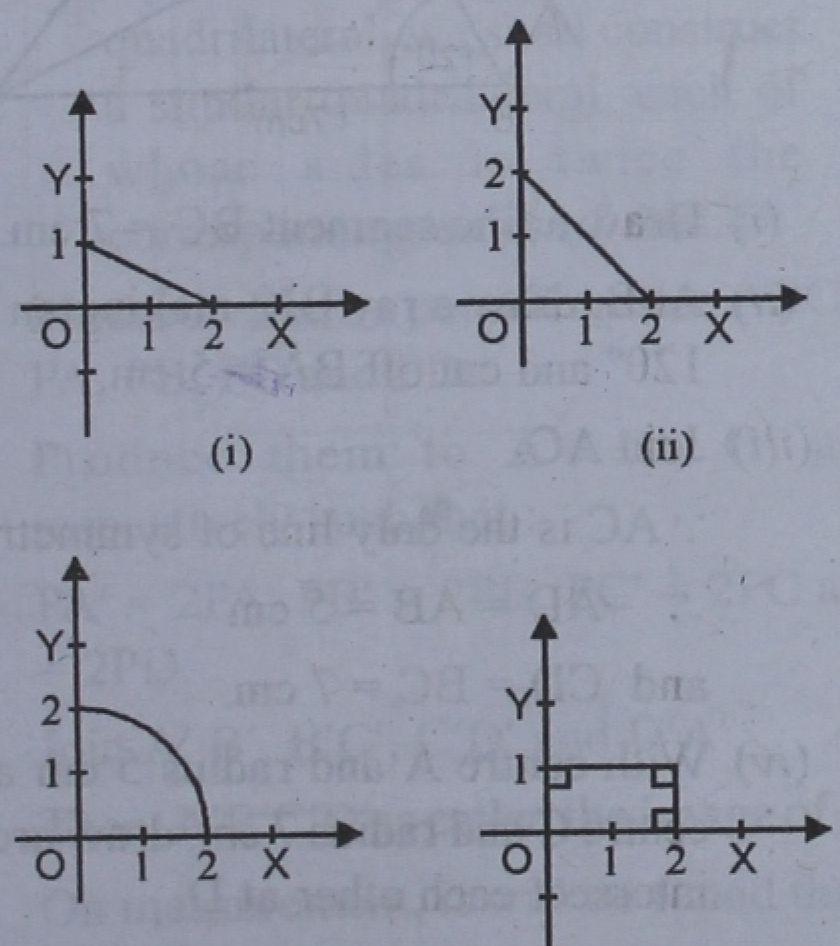


(iii)



(iv)

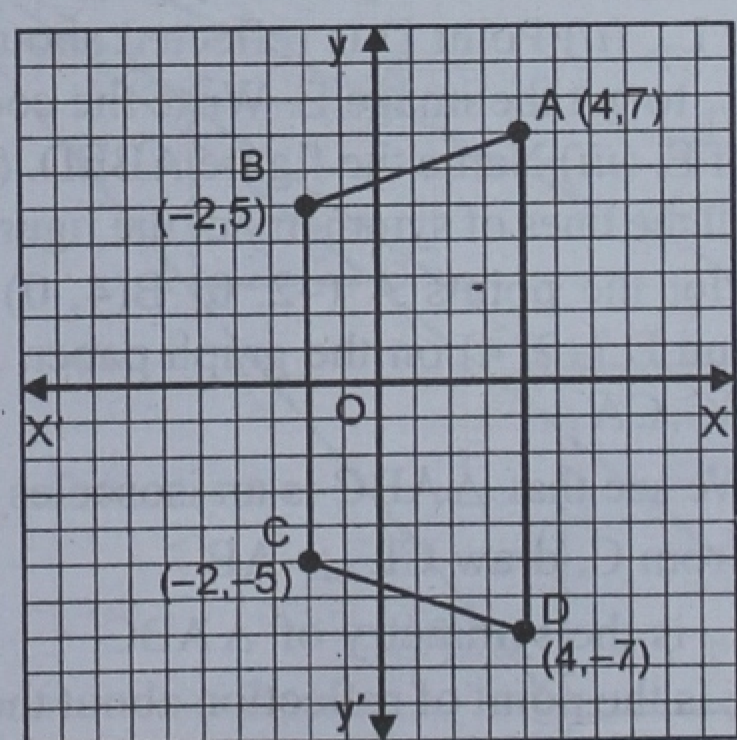
Q. 15. Part of a geometrical figure is given in each of the diagrams below. Complete the figure, so that both x-axis and y-axis are the lines of symmetry of the completed figure.



Sol. The figures given have been completed around the x-axis and y-axis as shown below :

Q. 16. The x-axis is the line of symmetry for the quadrilateral ABCD containing the points A (4, 7) and B (–2, 5). Find the co-ordinates of C and D.

Sol. The image of point A (4, 7) in x-axis is D whose co-ordinates will be (4, –7)

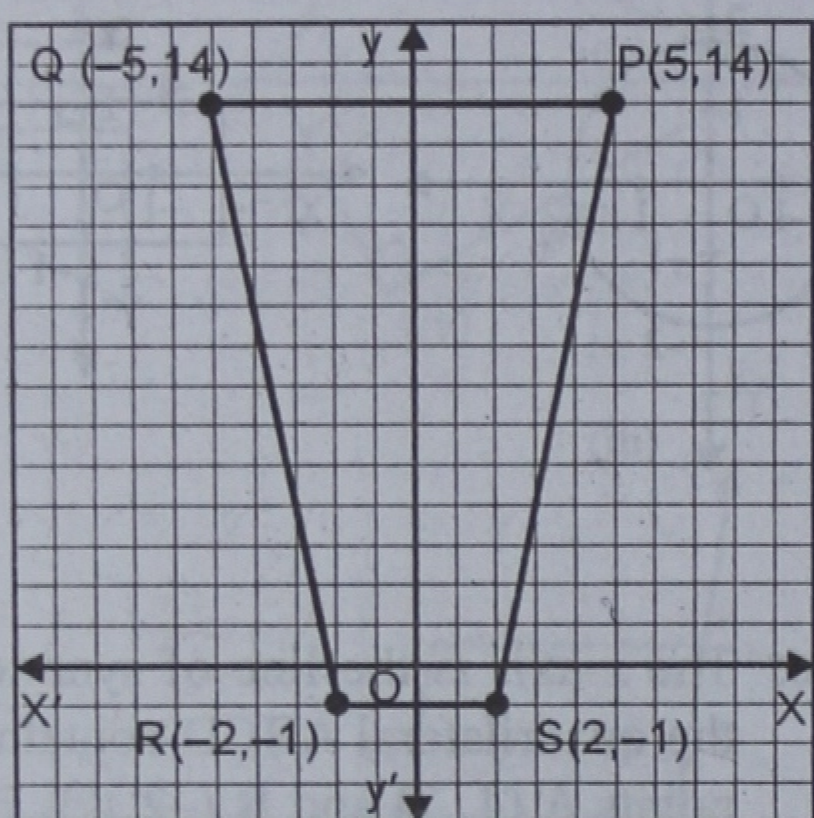


Similarly, the image of point B (–2, 5) in x-axis is C whose co-ordinates are (–2, –5).

AB, BC, CD and DA are joined. ABCD is a quadrilateral.

Q. 17. The *y*-axis is the line of symmetry for the quadrilateral PQRS containing the points P (5, 14) and R (–2, –1). Find the co-ordinates of Q and S.

Sol. The image of the point P (5, 14) in *y*-axis will be Q whose co-ordinates are (–5, 14). Similarly, the image of point R (–2, –1) in *y*-axis will be S whose co-ordinates are (2, –1). PQ, QR, RS and SP are joined forming the quadrilateral PQRS.



18. Use a graph paper for this question (Take 1 cm = 1 unit on both the axes). Plot the points A(–2, 0), B (4, 0), C(1, 4) and D(–2, 4).

(*i*) Draw the line of symmetry of Δ ABC. Name it L_1. (*ii*) Point D is reflected about the line L_1 to get the image E. Write the coordinates of E. (*iii*) Name the figure ABED. (*iv*) Draw all the lines of symmetry of the figure ABED.

Sol. Plot the points A (–2, 0) B(4, 0), C(1, 4) and D (–2, 4) on the graph paper. Join AB, BC, CA

We see that Δ ABC is an isosceles triangle.

(i) From C, draw CL ⊥ AB

∴ L_1 is the symmetry of Δ ABC.

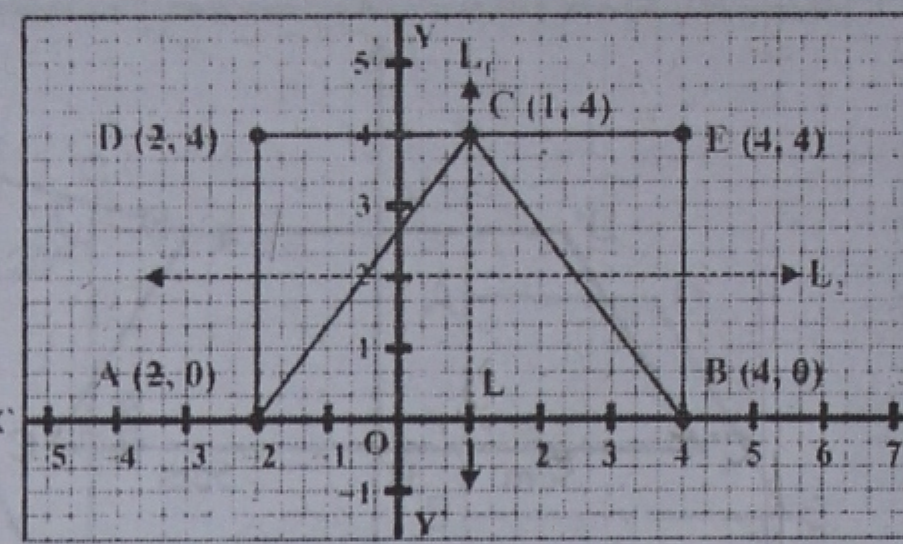
(ii) E is the point of reflection about the line L_1.

∴ E is the image of D and its co-ordinates are (4, 4)

(iii) The figure ABED is a rectangle in shape

Draw L_2, the perpendicular bisector of AD and BE

∴ L_1 and L_2 are the lines of symmetry of the figure ABED.



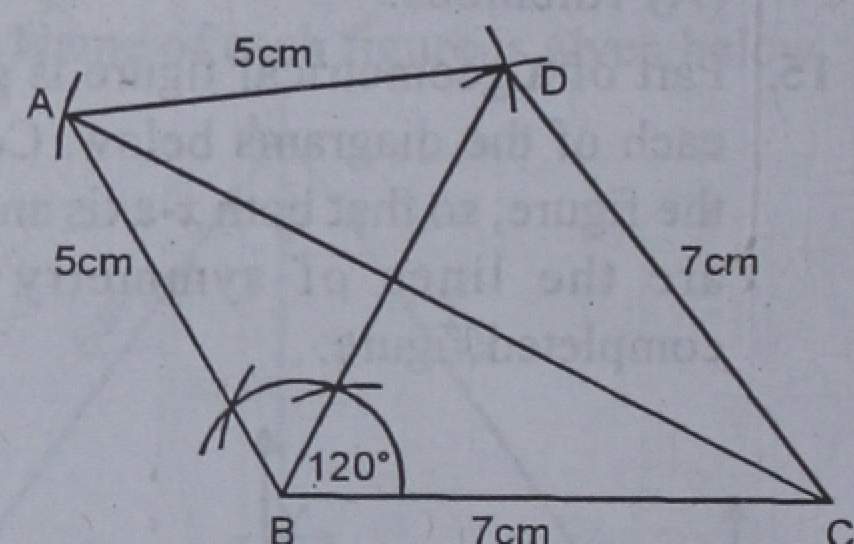
Q. 19. Use ruler and compasses only in th question :

(*i*) Construct the quadrilateral ABCD in which AB = 5 cm, BC = 7 cm and ∠AB = 120°, given that AC is its only line of symmetry.

(*ii*) Write down the geometrical name of t quadrilateral.

(*ii*) Measure and record the length of BD in cm. **(199**

Sol. Steps of Constructions :



(*i*) Draw a line segment BC = 7 cm.

(*ii*) At B, draw a ray BX, making an angle 120° and cut off BA = 5 cm.

(*iii*) Join AC.

∵ AC is the only line of symmetry.

∴ AD = AB = 5 cm

and CD = BC = 7 cm.

(*iv*) With centre A and radius 5 cm and wi centre C and radius 7 cm, draw arcs whic intersect each other at D.

Join AD and CD.

ABCD is the required quadrilateral whic is a kite in shape.

Measure the length of BD, it is 5·7 c **Ans.**