Chapter 10

Factor Theorem

POINTS TO REMEMBER

1. **Polynomial :** An expression of the forms $p(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots\ldots + a_{n-1} x + a_n$ where $a_0, a_1, a_2, \ldots\ldots\ldots a_{n-1}, a_n$ are the real numbers and $a_0 \neq 0$, is called a polynomial in x of degree n.

2. **Value of a polynomial $p(x)$ at $x = \alpha$:** The value of a polynomial $p(x)$ at $x = \alpha$ is obtained by substituting $x = \alpha$ in the given polynomial and it is rested by $p(\alpha)$.

3. **Division Algorithm for polynomials :** On dividing a polynomial $p(x)$ by a polynomial $d(x)$, let the quotient be $q(x)$ and the remainder be $r(x)$, then

   $p(x) = d(x) \times q(x) + r(x)$ where either $r(x) = 0$ or deg. $r(x) <$ deg. $d(x)$.

   Here $p(x)$ is called dividend, $d(x)$ is divisor, $q(x)$ is quotient and $r(x)$ is the remainder.

   Note : When a polynomial $p(x)$ is divided by $(x - \alpha)$, then the remainder is a constant, which can be zero or non-zero.

4. **Remainder Theorem :**

   If a polynomial $p(x)$ is divided by $(x - \alpha)$, then the remainder is $p(\alpha)$.

   Proof : When a polynomial $p(x)$ is divided by $(x - \alpha)$, then by division algorithm, we obtain quotient $q(x)$, and a constant remainder c such that.

   $p(x) = (x - \alpha) \cdot q(x) + c$ ...(*i*)

   On substituting $x = \alpha$ in (*i*), we get :

   $p(\alpha) = (\alpha - \alpha) \cdot q(\alpha) + c = 0 \cdot q(a) + c = 0 + c = c$

   Hence, remainder $= p(\alpha)$.

   Results : (*i*) When $p(x)$ is divided by $(x + \alpha)$, then the remainder $= p(-\alpha)$.

   (*ii*) When $p(x)$ is divided by $(ax + b)$, then remainder $= p\left(\frac{-b}{a}\right)$.

5. **Factor Theorem :**

   Let $p(x)$ be a polynomial and α be the real number. Then $(x - \alpha)$ is a factor of $p(x)$ if $p(x) = 0$

   Proof : When know by remainder theorem that when $p(x)$ is divided by $(x - \alpha)$, then remainder $= p(\alpha)$.

   Now, if $(x - \alpha)$ is a factor $p(x)$, then remainder $= 0 \Rightarrow p(\alpha) = 0$.

   Hence, $(x - \alpha)$ is a factor of $p(x)$ if $p(\alpha) = 0$.

   Results : 1. $(x + 2)$ is a factor of $p(x)$ if $p(-\alpha) = 0$

   2. $(ax + b)$ is a factor of $p(x)$ if $p(x)$ of $p\left(\frac{-b}{a}\right) = 0$.

EXERCISE 10 (A)

Without actual division, find the remainder when :

Q. 1. $p(x) = 3x^2 - 5x + 7$ is divided by $(x - 2)$.

Sol. $p(x) = 3x^2 - 5x + 7$ ...(*i*)

Let $x - 2 = 0$, then $x = 2$

Now, substituting the value of x in (*i*), we get

$$p(2) = 3(2)^2 - 5(2) + 7$$
$$= 3 \times 4 - 5 \times 2 + 7$$
$$= 12 - 10 + 7 = 19 - 10 = 9$$

$\therefore$ Hence remainder = **9 Ans.**

Q. 2. $p(x) = 2x^3 - 5x^2 + 3x - 10$ is divided by $(x - 3)$

Sol. $p(x) = 2x^3 - 5x^2 + 3x - 10$ ...(*i*)

Let $x - 3 = 0$, then $x = 3$

Now, substituting the value of x in (*i*), we get

$$p(3) = 2(3)^3 - 5(3)^2 + 3 \times 3 - 10$$
$$= 2 \times 27 - 5 \times 9 + 9 - 10$$
$$= 54 - 45 + 9 - 10 = 63 - 55 = 8$$

Hence remainder = 8 **Ans.**

Q. 3. $p(x) = 5x^3 - 12x^2 + 17x - 6$ is divided by $(x - 1)$.

Sol. $p(x) = 5x^3 - 12x^2 + 17x - 6$ ...(*i*)

Let $x - 1 = 0$, then $x = 1$

Substituting the value of x in (*i*), we get

$$p(1) = 5(1)^3 - 12(1)^2 + 17(1) - 6$$
$$= 5 \times 1 - 12 \times 1 + 17 \times 1 - 6$$
$$= 5 - 12 + 17 - 6 = 22 - 18 = 4$$

Hence remainder = **4 Ans.**

Q. 4. $p(x) = 8x^3 - 16x^2 + 14x - 5$ is divided by $(2x - 1)$.

Sol. $p(x) = 8x^3 - 16x^2 + 14x - 5$ ...(*i*)

Let $2x - 1 = 0$ then $2x = 1$

$$\Rightarrow x = \frac{1}{2}$$

Substituting the value of x in (*i*), we get

$$p\left(\frac{1}{2}\right) = 8\left(\frac{1}{2}\right)^3 - 16\left(\frac{1}{2}\right)^2 + 14 \times \frac{1}{2} - 5$$
$$= 8 \times \frac{1}{8} - 16 \times \frac{1}{4} + 14 \times \frac{1}{2} - 5$$
$$= 1 - 4 + 7 - 5 = 8 - 9 = -1$$

Hence remainder = **–1 Ans.**

Q. 5. $p(x) = 9x^2 - 6x + 2$ is divided by $(3x - 2)$.

Sol. $p(x) = 9x^2 - 6x + 2$ ...(*i*)

Let $3x - 2 = 0$, then $3x = 2 \Rightarrow x = \frac{2}{3}$

Substituting the value of x in (*i*), we get

$$p\left(\frac{2}{3}\right) = 9\left(\frac{2}{3}\right)^2 - 6 \times \frac{2}{3} + 2$$
$$= 9 \times \frac{4}{9} - 6 \times \frac{2}{3} + 2$$
$$= 4 - 4 + 2 = 6 - 4 = 2$$

Hence remainder = **2 Ans.**

Q. 6. $p(x) = x^3 - 2x^2 - 5x + 6$ is divided by $x + 2$.

Sol. $p(x) = x^3 - 2x^2 - 5x + 6$ ...(*i*)

Let $x + 2 = 0$, then $x = -2$

Substituting the value of x in (*i*), we get

$$p(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$$
$$= -8 - 2 \times 4 + 10 + 6$$
$$= -8 - 8 + 10 + 6 = 16 - 16 = 0$$

Hence remainder = **0 Ans.**

Q. 7. $p(x) = 8x^2 - 2x - 15$ is divided by $(2x + 3)$.

Sol. $p(x) = 8x^2 - 2x - 15$ ...(*i*)

Let $2x + 3 = 0$, then $2x = -3 \Rightarrow x = \frac{-3}{2}$

Substituting the value of x in (*i*), we get

$$p\left(\frac{-3}{2}\right) = 8\left(\frac{-3}{2}\right)^2 - 2\left(\frac{-3}{2}\right) - 15$$

$= 8\left(\frac{9}{4}\right) + 3 - 15$

$= 18 + 3 - 15 = 21 - 15 = 6$

Hence remainder = **6 Ans.**

Q. 8. On dividing $(ax^3 + 9x^2 + 4x - 10)$ by $(x + 3)$, we get 5 as remainder. Find the value of a.

Sol. $p(x) = ax^3 + 9x^2 + 4x - 10$

Let $x + 3 = 0$, then $x = -3$

$\therefore$ Remainder $= p(-3)$

$= a(-3)^3 + 9(-3)^2 + 4(-3) - 10$

$= -27a + 81 - 12 - 10$

$= -27a + 59$

$\because$ Remainder = 5 (given)

$\therefore -27a + 59 = 5 \Rightarrow -27a = 5 - 59$

$\Rightarrow -27a = -54 \Rightarrow a = \frac{-54}{-27} = 2$

$\therefore$ $a = 2$ **Ans.**

Q. 9. When $(x^3 + 3x^2 - kx + 4)$ is divided by $(x - 2)$, the remainder is k. Find the value of k.

Sol. Let $f(x) = x^3 + 3x^2 - kx + 4$.

By remainder theorem, when $f(x)$ is divided by $(x - 2)$,

The remainder $= f(2)$

$= 2^3 + 3.2^2 - k.2 + 4$

$= 8 + 12 - 2k + 4$

$= 24 - 2k.$

According to given, $24 - 2k = k$

$\Rightarrow 3k = 24 \Rightarrow k = 8.$

Q. 10. The polynomials $p(x) = (ax^3 + 3x^2 - 3)$ and $q(x) = (2x^3 - 5x + a)$, when divided by $(x - 4)$ leave the same remainder in each case. Find the value of a.

Sol. $p(x) = ax^3 + 3x^2 - 3$

$q(x) = 2x^3 - 5x + a$

Let $x - 4 = 0$, then $x = 4$

$\therefore p(4) = a(4)^3 + 3(4)^2 - 3$

$= 64a + 48 - 3 = 64a + 45$ ...(*i*)

$q(4) = 2(4)^3 - 5 \times 4 + a$

$= 2 \times 64 - 20 + a$

$= 128 - 20 + a = 108 + a$ ...(*ii*)

$\because$ In each case the remainder is same

$\therefore 64a + 45 = 108 + a$

$\Rightarrow 64a - a = 108 - 45$

$\Rightarrow 63a = 63 \Rightarrow a = \frac{63}{63} = 1$

Hence, $a = 1$ **Ans.**

Q. 11. The polynomial, $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ when divided by $(x - 1)$ and $(x + 1)$ leaves the remainders 5 and 19 respectively. Find the values of a and b. Hence, find the remainder when $f(x)$ is divided by $(x - 2)$.

Sol. $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ ...(*i*)

(*i*) When divided by $(x - 1)$,

$x - 1 = 0$ then $x = 1$

Substituting the value of x in (*i*)

$f(1) = (1)^4 - 2(1)^3 + 3(1)^2 - a \times 1 + b$

$= 1 - 2 + 3 - a + b$

$= 2 - a + b$

$\because$ Remainder = 5

$\therefore 2 - a + b = 5$

$\therefore a - b = 2 - 5 = -3$

$a - b = -3$ ...(*ii*)

(*ii*) When divided by $x + 1$, then

$x + 1 = 0 \Rightarrow x = -1$

Substituting the value of x in (*i*)

$f(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b$

$= 1 + 2 + 3 + a + b \Rightarrow 6 + a + b.$

$\because$ Remainder = 19

$\therefore 6 + a + b = 19$

$\Rightarrow a + b = 19 - 6 = 13$

$a + b = 13$ ...(*iii*)

Adding (*ii*) and (*iii*),

$2a = 10 \Rightarrow a = 5$

$\therefore \quad 5 + b = 13 \Rightarrow b = 13 - 5 = 8$

Hence, $a = 5, b = 8$

(*iii*) When $f(x)$ is divided by $x - 2$, then

$x - 2 = 0 \Rightarrow x = 2$

$\therefore f(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$

$f(2) = (2)^4 - 2(2)^3 + 3(2)^2 - 5(2) + 8$

$= 16 - 16 + 12 - 10 + 8 = 10$

Hence, remainder = 10 **Ans.**

Q.12. If $(2x^3 + ax^2 + bx - 2)$ leaves remainders 7 and -20 when divided by $(2x - 3)$ and $(x + 3)$ respectively, find the value of a and b.

Sol. $p(x) = 2x^3 + ax^2 + bx - 2$

Let $2x - 3 = 0$, then $2x = 3$

$\Rightarrow \quad x = \frac{3}{2}$

Now, $p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 + a\left(\frac{3}{2}\right)^2 + b\left(\frac{3}{2}\right) - 2$

$= 2 \times \frac{27}{8} + \frac{9}{4}a + \frac{3}{2}b - 2$

$= \frac{27}{4} + \frac{9}{4}a + \frac{3}{2}b - 2$

$= \frac{9}{4}a + \frac{3}{2}b + \frac{27}{4} - 2$

$= \frac{9}{4}a + \frac{3}{2}b + \frac{27 - 8}{4} = \frac{9}{4}a + \frac{3}{2}b + \frac{19}{4}$

$\because$ In this case, remainder = 7, then

$\frac{9}{4}a + \frac{3}{2}b + \frac{19}{4} = 7$

$\Rightarrow \frac{9}{4}a + \frac{3}{2}b = 7 - \frac{19}{4}$

$\Rightarrow 9a + 6b = 28 - 19$ [Multiplying by 4]

$\Rightarrow 9a + 6b = 9$

$\Rightarrow 3a + 2b = 3$ ...(*i*) [Dividing by 3]

Again, let $x + 3 = 0$, then $x = -3$

$\therefore p(-3) = 2(-3)^3 + a(-3)^2 + b(-3) - 2$

$= 2 \times (-27) + 9a - 3b - 2$

$= -54 + 9a - 3b - 2$

$= 9a - 3b - 56$

$\because$ In this case, remainder = -20

$\therefore 9a - 3b - 56 = -20$

$\Rightarrow 9a - 3b = -20 + 56$

$\Rightarrow 9a - 3b = 36$

$\Rightarrow 3a - b = 12$ ...(*ii*) [Dividing by 3]

Subtracting (*ii*) from (*i*), we get

$3b = -9 \quad \Rightarrow b = \frac{-9}{3} = -3$

Substituting the value of b in (1), we get

$3a + 2(-3) = 3$

$\Rightarrow 3a - 6 = 3 \Rightarrow 3a = 3 + 6 = 9$

$\Rightarrow a = \frac{9}{3} = 3$

Hence $a = 3, b = -3$ **Ans.**

EXERCISE 10 (B)

Q. 1. Using factor theorem, show that :

(*i*) $(x - 3)$ is a factor of $(x^3 + x^2 - 17x + 15)$.

(*ii*) $(x + 1)$ is a factor of $(x^3 + 4x^2 + 5x + 2)$.

(*iii*) $(3x - 2)$ is a factor of $(3x^3 + x^2 - 20x + 12)$.

(*iv*) $(3 - 2x)$ is a factor of $(2x^3 - 9x^2 + x + 12)$.

Sol. (*i*) Let $p(x) = x^3 + x^2 - 17x + 15$.

By factor theorem, $(x - 3)$ will be a factor of $p(x)$, if $f(3) = 0$

Now, $p(3) = [3^3 + 3^2 - 17 \times 3 + 15]$

$= (27 + 9 - 51 + 15) = 0$

$\therefore (x - 3)$ is a factor of $p(x)$.

$$\begin{array}{r} x^2+4x-5 \\ x-3\,\overline{)\,x^3+x^2-17x+15} \\ x^3-3x^2 \\ \hline 4x^2-17x+15 \\ 4x^2-12x \\ \hline -5x+15 \\ -5x+15 \\ \hline \times \end{array}$$

On dividing $p(x) = x^3 + x^2 - 17x + 15$ by $(x-3)$

We get, Quotient $= (x^2 + 4x - 5)$

$\therefore p(x) = x^3 + x^2 - 17x + 15$

$= (x-3)(x^2 + 4x - 5)$

$= (x-3)(x^2 + 5x - x - 5)$

$= (x-3)[x(x+5) - 1(x+5)]$

$= (x-3)(x+5)(x-1)$

Hence, $x^3 + x^2 - 17x + 5$

$= (x-3)(x+5)(x-1)$ **Ans.**

(*ii*) Let $p(x) = x^3 + 4x^2 + 5x + 2$.

By factor theorem, $(x+1)$ will be a factor of $p(x)$, if $p(-1) = 0$.

Now,

$p(-1) = [(-1)^3 + 4(-1)^2 + 5(-1) + 2]$

$= -1 + 4 - 5 + 2 = 0$

$\therefore (x+1)$ is a factor of $p(x)$.

On dividing $p(x) = x^3 + 4x^2 + 5x + 2$ by $(x+1)$

$$\begin{array}{r} x^2+3x+2 \\ x+1\,\overline{)\,x^3+4x^2+5x+2} \\ x^3+x^2 \\ \hline 3x^2+5x+2 \\ 3x^2+3x \\ \hline +2x+2 \\ +2x+2 \\ \hline \times \end{array}$$

We get, Quotient $= (x^2 + 3x + 2)$

$\therefore p(x) = x^3 + 4x^2 + 5x + 2$

$= (x+1)(x^2 + 3x + 2)$

$= (x+1)[x^2 + 2x + x + 2)$

$= (x+1)[x(x+2) + 1(x+2)]$

$= (x+1)(x+2)(x+1)$

$= (x+1)^2(x+2)$

Hence, $x^3 + 4x^2 + 5x + 2$

$= (x+1)^2(x+2)$ **Ans.**

(*iii*) Let $p(x) = 3x^3 + x^2 - 20x + 12$

Also, $(3x-2) = 3\left(x - \frac{2}{3}\right) = 3\left[x - \left(+\frac{2}{3}\right)\right]$

Now,

$$p\left(\frac{2}{3}\right) = \left[3 \times \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 - 20 \times \left(+\frac{2}{3}\right) + 12\right]$$

$$= \left[3 \times \frac{(8)}{27} + \frac{4}{9} - \frac{40}{3} + 12\right]$$

$$= \left[\frac{8}{9} + \frac{4}{9} - \frac{40}{3} + 12\right]$$

$$= \left[\frac{8 + 4 - 120 + 108}{9}\right] = 0$$

$\therefore (3x-2)$ is a factor of $p(x)$

On dividing $p(x)$ by $(3x-2)$, we get :

Quotient $= (x^2 + x - 6)$

$$\begin{array}{r} x^2+x-6 \\ 3x-2\,\overline{)\,3x^3+x^2-20x+12} \\ 3x^3-2x^2 \\ \hline 3x^2-20x \\ 3x^2-2x \\ \hline -18x+12 \\ -18x+12 \\ \hline \times \end{array}$$

$\therefore p(x) = 3x^3 + x^2 - 20x + 12$
$= (3x - 2)(x^2 + x - 6)$
$= (3x - 2)[x^2 + 3x - 2x - 6]$
$= (3x - 2)[x(x + 3) - 2(x + 3)]$
$= (3x - 2)(x + 3)(x - 2)$

$\therefore 3x^3 + x^2 - 20x + 12$
$= (3x - 2)(x + 3)(x - 2)$ **Ans.**

(*iv*) $(3 - 2x)$ is a factor of $(2x^3 - 9x^2 + x + 12)$.

Sol. $p(x) = 2x^3 - 9x^2 + x + 12$ ...(*i*)

Dividing by $(3 - 2x)$, then

$3 - 2x = 0 \Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2}$

Substituting, the values of x in (*i*),

$$p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + \frac{3}{2} + 12$$

$$= 2 \times \frac{27}{8} - 9 \times \frac{9}{4} + \frac{3}{2} + 12$$

$$= \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + \frac{12}{1}$$

$$= \frac{27 - 81 + 6 + 48}{4}$$

$$= \frac{81 - 81}{4} = 0$$

$\therefore$ Remainder = 0

$\therefore (3 - 2x)$ is a factor of $p(x)$ **Ans.**

Q. 2. Use factor theorem to show that $(x + 2)$ and $(2x - 3)$ are factors of $(2x^2 + x - 6)$.

Sol. $p(x) = 2x^2 + x - 6$

Let $x + 2 = 0$, then $x = -2$

$x + 2$ is a factor of $p(x)$ if

$p(-2) = 0$ [Factor Theorem]

Now, $p(-2) = 2(-2)^2 + (-2) - 6$
$= 2 \times 4 - 2 - 6 = 8 - 8 = 0$

Hence, $x + 2$ is a factor of $p(x)$

Again, let $2x - 3 = 0 \Rightarrow 2x = 3$

then $x = \frac{3}{2}$

Now, $2x - 3$ is a factor of $p(x)$ if

$$p\left(\frac{3}{2}\right) = 0$$

$$\text{Now, } p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^2 + \frac{3}{2} - 6$$

$$= 2 \times \frac{9}{4} + \frac{3}{2} - 6$$

$$= \frac{9}{2} + \frac{3}{2} - 6$$

$$= \frac{12}{2} - 6 = 6 - 6 = 0$$

Hence, $2x - 3$ is also a factor of $p(x)$

Hence proved.

Q. 3. Find the value of a so that $(x + 6)$ is a factor of the polynomial $(x^3 + 5x^2 - 4x + a)$.

Sol. Let $p(x) = x^3 - 5x^2 - 4x + a$.

Now, $(x + 6)$ is a factor of $p(x)$

$\Rightarrow p(-6) = 0$
$\Rightarrow (-6)^3 + 5(-6)^2 - 4(-6) + a = 0$
$\Rightarrow -216 + 180 + 24 + a = 0$
$\Rightarrow -36 + 24 + a = 0$
$\Rightarrow -12 + a = 0$
$\Rightarrow a = 12$ **Ans.**

Q. 4. For what value of a is the polynomial $(2x^3 + ax^2 + 11x + a + 3)$ exactly divisible by $(2x - 1)$?

Sol. Let $p(x) = 2x^3 + ax^2 + 11x + a + 3$

Now, $(2x - 1)$ is a factor of $p(x)$

$$\Rightarrow p\left(\frac{1}{2}\right) = 0$$

$$\Rightarrow 2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + 11\left(\frac{1}{2}\right) + a + 3 = 0$$

$\Rightarrow 2\left(\frac{1}{8}\right)+a\left(\frac{1}{4}\right)+\frac{11}{2}+a+3=0$

$\Rightarrow \frac{1}{4}+\frac{a}{4}+\frac{11}{2}+a+3=0$

$\Rightarrow \frac{1+a+22+4a+12}{4}=0$

$\Rightarrow \frac{5a+35}{4}=0 \Rightarrow 5a+35=0$

$\Rightarrow 5a=-35 \Rightarrow a=-\frac{35}{5}$

$\Rightarrow a=-7$

Hence, $a=-7$ **Ans.**

5. (*i*) Using Factor theorem, show that $(x-3)$ is a factor of $x^3-7x^2+15x-9$. Hence, factorise the given expression completely.

(*ii*) Using factor theorem, show that $(x-4)$ is a factor of $(2x^3+x^2-26x-40)$ and hence factorise $2x^3+x^2-26x-40$.

Sol. (*i*) Since $x-3$ is a factor of $P(x)=x^3-7x^2+15x-9$, so we must have $P(3)=0$

Now,

$P(3)=(3)^3-7(3)^2+15(3)-9$

$=27-63+45-9=72-72=0$

So, $x-3$ is the factor of given expression.

Now, factorise it by division

$$\begin{array}{r|l} & x^2-4x+3 \\ \hline x-3 & x^3-7x^2+15x-9 \\ & x^3-3x^2 \\ & -\quad + \\ \hline & -4x^2+15x \\ & -4x^2+12x \\ & +\quad - \\ \hline & +3x-9 \\ & 3x-9 \\ & -\quad + \\ \hline & \times \end{array}$$

So factors, $(x-3)(x^2-4x+3)$

$=(x-3)[x^2-3x-x+3]$

$=(x-3)[x(x-3)-1(x-3)]$

$=(x-3)(x-3)(x-1)$

$=(x-1)(x-3)^2$ **Ans.**

(*ii*) $p(x)=2x^3+x^2-26x-40$

Let $x-4=0$, then $x=4$

By Factor Theorem,

$p(4)=2(4)^3+(4)^2-26\times4-40$

$=2\times64+16-104-40$

$=128+16-104-40$

$=144-144=0$

$\therefore p(4)=0$,

Hence $x-4$ is a factor of $p(x)$

Now, dividing $p(x)$ by $x-4$,

$$\begin{array}{r|l} & 2x^2+9x+10 \\ \hline x-4 & 2x^3+x^2-26x-40 \\ & 2x^3-8x^2 \\ & -\quad + \\ \hline & 9x^2-26x \\ & 9x^2-36x \\ & -\quad + \\ \hline & 10x-40 \\ & 10x-40 \\ & -\quad + \\ \hline & \times \end{array}$$

$\therefore 2x^3+x^2-26x-40$

$=(x-4)(2x^2+9x+10)$

$=(x-4)\{2x^2+4x+5x+10)$

$\left\{\begin{array}{l}\because 2\times10=20 \\ \therefore 9=4+5 \\ 20=4\times5\end{array}\right\}$

$=(x-4)\{2x(x+2)+5(x+2)\}$

$=(x-4)(x+2)(2x+5)$ **Ans.**

Q.6. Show that $(x - 3)$ is a factor of $(2x^3 - 3x^2 - 11x + 6)$ and hence factorise $(2x^3 - 3x^2 - 11x + 6)$

Sol. $p(x) = 2x^3 - 3x^2 - 11x + 6$

Let $x - 3 = 0$, then $x = 3$

If $x - 3$ is a factor, then $p(3) = 0$

Now, $p(3) = 2(3)^3 - 3(3)^2 - 11 \times 3 + 6$

$= 2 \times 27 - 3 \times 9 - 33 + 6$

$= 54 - 27 - 33 + 6$

$= 60 - 60 = 0$

Hence, $x - 3$ is a factor of $p(x)$

Now, dividing $p(x)$ by $x - 3$

$$
\begin{array}{r}
2x^2+3x-2 \\
x-3\,\big)\,2x^3-3x^2-11x+6\,\big(\\
2x^3-6x^2 \\
-\quad + \\
\hline
3x^2-11x \\
3x^2-9x \\
-\quad + \\
\hline
-2x+6 \\
-2x+6 \\
+\quad - \\
\hline
\times
\end{array}
$$

$p(x) = (x - 3)(2x^2 + 3x - 2)$

$= (x - 3)(2x^2 + 4x - x - 2)$ $\left\{\begin{array}{l}\because 2\times(-2) = -4 \\ \therefore 3 = 4 - 1 \\ -4 = 4\times(-1)\end{array}\right\}$

$= (x - 3)\{2x(x + 2) - 1(x + 2)\}$

$= (x - 3)(x + 2)(2x - 1)$ **Ans.**

Q.7. Show that $(2x - 3)$ is a factor of $(2x^3 + 3x^2 - 5x - 6)$ and hence factorise $(2x^3 + 3x^2 - 5x - 6)$.

Sol. $p(x) = 2x^3 + 3x^2 - 5x - 6$

Let $2x - 3 = 0$, then $2x = 3$

$\Rightarrow \quad x = \frac{3}{2}$

If $2x - 3$ is a factor of $p(x)$, then $P\left(\frac{3}{2}\right) = 0$

Now, $p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 + 3\left(\frac{3}{2}\right)^2 - 5 \times \frac{3}{2} - 6$

$= 2 \times \frac{27}{8} + 3 \times \frac{9}{4} - \frac{15}{2} - 6$

$= \frac{27}{4} + \frac{27}{4} - \frac{15}{2} - 6$

$= \frac{54}{4} - \frac{27}{2} = \frac{27}{2} - \frac{27}{2} = 0$

Hence, $2x - 3$ is a factor of $p(x)$

Now, dividing $p(x)$ by $2x - 3$,

$$
\begin{array}{r}
x^2+3x+2 \\
2x-3\,\big)\,2x^3+3x^2-5x-6\,\big(\\
2x^3-3x^2 \\
-\quad + \\
\hline
6x^2-5x \\
6x^2-9x \\
-\quad + \\
\hline
+4x-6 \\
-4x-6 \\
-\quad + \\
\hline
\times
\end{array}
$$

$p(x) = (2x - 3)(x^2 + 3x + 2)$

$= (2x - 3)(x^2 + x + 2x + 2)$

$= (2x - 3)\{x(x + 1) + 2(x + 1)\}$ $\left\{\begin{array}{l}\because 3 = 1 + 2 \\ 2 = 1 \times 2\end{array}\right\}$

$= (2x - 3)(x + 1)(x + 2)$ **Ans.**

Q.8. Show that $(3x + 2)$ is a factor of $(6x^3 + 13x^2 - 4)$ and hence factorise $(6x^3 + 13x^2 - 4)$.

Sol. $p(x) = 6x^3 + 13x^2 - 4$

Let $3x + 2 = 0$, then $3x = -2$

$\Rightarrow \quad x = -\frac{2}{3}$

If $3x + 2$ is a factor of $p(x)$, then $P\left(\frac{-2}{3}\right) = 0$

Now, $P\left(\frac{-2}{3}\right) = 6\left(\frac{-2}{3}\right)^3 + 13\left(\frac{-2}{3}\right)^2 - 4$

$= 6 \times \left(-\frac{8}{27}\right) + 13 \times \frac{4}{9} - 4$

$= -\frac{16}{9} + \frac{52}{9} - 4 = \frac{52}{9} - 4 - \frac{16}{9} = \frac{52}{9} - \frac{52}{9} = 0$

Hence, $3x + 2$ is a factor of $p(x)$

Now, dividing $p(x)$ by $3x + 2$

$$3x+2\overline{)6x^3+13x^2-4}(2x^2+3x-2$$
$$\underline{\underset{-}{6x^3}\underset{-}{+4x^2}}$$
$$9x^2$$
$$\underline{\underset{-}{9x^2}\underset{-}{+6x}}$$
$$-6x-4$$
$$\underline{\underset{+}{-6x}\underset{+}{-4}}$$
$$\times$$

$p(x) = (3x + 2)(2x^2 + 3x - 2)$

$= (3x + 2)\{2x^2 + 4x - x - 2\}$

$= (3x + 2)\{2x(x + 2) - 1(x + 2)\}$

$= (3x + 2)(x + 2)(2x - 1)$ **Ans.**

Q.9. Show that $(2x + 1)$ is a factor of $(2x^3 + 5x^2 + 4x + 1)$ and hence factorise $(2x^3 + 5x^2 + 4x + 1)$.

Sol. $p(x) = 2x^3 + 5x^2 + 4x + 1$

Let $2x + 1 = 0$, then $2x = -1 \Rightarrow x = \frac{-1}{2}$

If $2x + 1$ is a factor of $p(x)$, then $p\left(\frac{-1}{2}\right) = 0.$

Now, $p\left(\frac{-1}{2}\right) = 2\left(\frac{-1}{2}\right)^3 + 5\left(\frac{-1}{2}\right)^2 + 4\left(\frac{-1}{2}\right) + 1$

$= 2\left(\frac{-1}{8}\right) + 5 \times \frac{1}{4} - 4 \times \frac{1}{2} + 1 = \frac{-1}{4} + \frac{5}{4} - 2 + 1$

$= \frac{-1+5}{4} - 1 = \frac{4}{4} - 1 = 1 - 1 = 0.$

Hence, $2x + 1$ is a factor of $p(x)$

Now, dividing $p(x)$ by $2x + 1$

$$2x+1\overline{)2x^3+5x^2+4x+1}(x^2+2x+1$$
$$\underline{\underset{-}{2x^3}\underset{-}{+x^2}}$$
$$4x^2+4x$$
$$\underline{\underset{-}{4x^2}\underset{-}{+2x}}$$
$$2x+1$$
$$\underline{\underset{-}{2x}\underset{-}{+1}}$$
$$\times$$

$p(x) = (2x + 1)(x^2 + 2x + 1)$

$= (2x + 1)(x + 1)^2$ **Ans.**

Q.10. If $(x - 2)$ is a factor of $2x^3 - x^2 - px - 2$

(*i*) find the value of p.

(*ii*) with the value of p, factorize the above expression completely. **(2008)**

Sol. (*i*) $\because (x - 2)$ is a factor of $2x^3 - x^2 - px - 2$

By remainder theorem

Let $x - 2 = 0 \quad \Rightarrow x = 2$

and $f(x) = 2x^3 - x^2 - px - 2$

$\therefore f(2) = 2(2)^3 - (2)^2 - p \times 2 - 2$

$= 16 - 4 - 2p - 2 = 10 - 2p$

$\because x - 2$ is a factor of $f(x)$

$\therefore$ Remainder = 0

$\therefore 10 - 2p = 0$

$\Rightarrow 2p = 10 \Rightarrow p = \frac{10}{2} = 5$

$\therefore p = 5$

(*ii*) $\therefore f(x) = 2x^3 - x^2 - 5x - 2$

Dividing $f(x)$ by $x - 2$, we get,

$f(x) = (x - 2)(2x^2 + 3x + 1)$

$= (x - 2)\{2x^2 + 2x + x + 1\}$

$= (x - 2)[2x(x + 1) + 1(x + 1)]$

$= (x - 2)(x + 1)(2x + 1)$

$$x-2\overline{)2x^3-x^2-5x-2}(2x^2+3x+1$$
$$\underline{\underset{-}{2x^3}\underset{+}{-4x^2}}$$
$$3x^2-5x$$
$$\underline{\underset{-}{3x^2}\underset{+}{-6x}}$$
$$x-2$$
$$\underline{\underset{-}{x}\underset{+}{-2}}$$
$$\times$$

Q. 11. (*i*) Find the values of a and b, if $(x - 1)$ and $(x + 2)$ are both factors of $x^3 + ax^2 + bx - 6$).

(*ii*) Given that $(x + 2)$ and $(x + 3)$ are factors of $2x^3 + ax^2 + 7x - b$. Determine the values of a and b. **(2009)**

Sol. (*i*) $p(x) = x^3 + ax^2 + bx - 6$

Let $x - 1 = 0$, then $x = 1$

If $x - 1$ is a factor of $p(x)$; then $p(1) = 0$

$p(1) = (1)^3 + a(1)^2 + b(1) - 6$

$= 1 + a + b - 6 = a + b - 5$

$\therefore a + b - 5 = 0 \Rightarrow a + b = 5$ ...(*i*)

Again $x + 2 = 0$, then $x = -2$

$\because x + 2$ is also a factor of $p(x)$

$\therefore p(-2) = 0$

Now, $p(-2) = (-2)^3 + a(-2)^2 + b(-2) - 6$

$= -8 + 4a - 2b - 6 = 4a - 2b - 14$

$\therefore 4a - 2b - 14 = 0 \Rightarrow 4a - 2b = 14$

$\Rightarrow 2a - b = 7$ [Dividing by 2] ...(*ii*)

From (*i*), $a = 5 - b$

Substituting the value of a in (*ii*), we get

$2(5 - b) - b = 7 \Rightarrow 10 - 2b - b = 7$

$\Rightarrow 10 - 3b = 7 \Rightarrow -3b = 7 - 10 = -3$

$\therefore b = \frac{-3}{-3} = 1$

$\therefore a = 5 - b = 5 - 1 = 4$

Hence, $a = 4$, $b = 1$ **Ans.**

(*ii*) $(x + 2)$ and $(x + 3)$ are the factors of $f(x) = 2x^3 + ax^2 + 7x - b$

When $x + 2$ is the factors

Then $x + 2 = 0 \Rightarrow x = -2$

$\therefore f(-2) = 0$

$\therefore f(-2) = 2(-2)^3 + a(-2)^2 + 7(-2) - b = 0$

$-16 + 4a - 14 - b = 0$

$\Rightarrow 4a - b = 30$ ....(*i*)

$\because x + 3$ is factors of $f(x)$

$\therefore x + 3 = 0 \Rightarrow x = -3$

$\therefore f(-3) = 0$

$\Rightarrow 2(-3)^3 + a(-3)^2 + 7(-3) - b = 0$

$\Rightarrow -54 + 9a - 21 - b = 0 \Rightarrow -75 + 9a - b = 0$

$\Rightarrow 9a - b = 75$ ....(*ii*)

Subtracting (*i*) from (*ii*),

$5a = 45 \Rightarrow a = \frac{45}{5} \Rightarrow a = 9$

From (*i*) $4 \times 9 - b = 30$

$\Rightarrow 36 - b = 30, -b = 30 - 36$

$\Rightarrow -b = -6 \Rightarrow b = 6$, Hence $a = 9$, $b = 6$

Q.12. If $(x^3 + ax^2 + bx + 6)$ has $(x - 2)$ as a factor and leaves a remainder 3 when divided by $(x - 3)$, find the values of a and b.

Sol. $f(x) = x^3 + ax^2 + bx + 6$

Let $x - 2 = 0$, then $x = 2$

$\because x - 2$ is a factor of $f(x)$

$\therefore f(2) = 0$

Now, $f(2) = (2)^3 + a(2)^2 + b(2) + 6$

$= 8 + 4a + 2b + 6 = 4a + 2b + 14$

$\therefore 4a + 2b + 14 = 0$

$\Rightarrow 4a + 2b = -14$ [Dividing by 2]

$\Rightarrow 2a + b = -7$ ...(*i*)

Again, let $x - 3 = 0$, then $x = 3$.

$\therefore f(3) = (3)^3 + a(3)^2 + b(3) + 6$

$= 27 + 9a + 3b + 6 = 9a + 3b + 33$

$\because$ In this case remainder = 3

$\therefore 9a + 3b + 33 = 3$

$\Rightarrow 9a + 3b = 3 - 33$

$\Rightarrow 9a + 3b = -30$

$\Rightarrow 3a + b = -10$ (Dividing by 3) ...(*ii*)

Substracting (*ii*) from (*i*)

$-a = 3 \Rightarrow a = -3$

Substituting the value of a in (*i*), we get

$2(-3) + b = -7 = -6 + b = -7$

$\Rightarrow b = -7 + 6 = -1$

Hence, $a = -3$, $b = -1$ **Ans.**

Using factor theorem, factorise each of the following :

Q. 13. $x^3 + 7x^2 + 7x - 15$

Sol. $f(x) = x^3 + 7x^2 + 7x - 15$

Let $x = 1$, then

$f(1) = (1)^3 + 7(1)^2 + 7(1) - 15$

$= 1 + 7 + 7 - 15 = 0$

$\because f(1) = 0$

$\therefore (x - 1)$ is a actor $f(x)$

$$
\begin{array}{r|l}
 & x^2 + 8x + 15 \\
x - 1 & x^3 + 7x^2 + 7x - 15 \\
 & x^3 - x^2 \\
 & -\quad + \\
 & 8x^2 + 7x \\
 & 8x^2 - 8x \\
 & -\quad + \\
 & 15x - 15 \\
 & 15x - 15 \\
 & -\quad + \\
 & \times
\end{array}
$$

Now, dividing $f(x)$ by $(x - 1)$, we get :

$f(x) = (x - 1)(x^2 + 8x + 15)$

$= (x - 1)\{x^2 + 5x + 3x + 15\}$

$= (x - 1)\{x(x + 5) + 3(x + 5)\}$

$= (x-1)(x+3)(x+5)$ **Ans.**

Q.14. $6x^3 - 7x^2 - 11x + 12$

Sol. $f(x) = 6x^3 - 7x^2 - 11x + 12$

Let $x = 1$, then

$f(1) = 6(1)^3 - 7(1)^2 - 11 \times 1 + 12$

$= 6 - 7 - 11 + 12 = 0$

$\because f(1) = 0$

$\therefore (x-1)$ is a factor of $f(x)$

$$
\begin{array}{r|l}
 & 6x^2 - x - 12 \\
\hline
x-1 & 6x^3 - 7x^2 - 11x + 12 \\
 & 6x^3 - 6x^2 \\
 & -\quad + \\
\hline
 & -x^2 - 11x \\
 & -x^2 + x \\
 & +\quad - \\
\hline
 & -12x + 12 \\
 & -12x + 12 \\
 & +\quad - \\
\hline
 & \times
\end{array}
$$

Now, dividing $f(x)$ by $(x-1)$, we get :

$f(x) = (x-1)(6x^2 - x - 12)$

$= (x+1)\{6x^2 - 9x + 8x - 12\}$

$= (x-1)\{3x(2x-3) + 4(2x-3)\}$

$= (x-1)(2x-3)(3x+4)$ **Ans.**

Q.15. $2x^3 + 7x^2 - 9$

Sol. $f(x) = 2x^3 + 7x^2 - 9$

Let $x = 1$, then

$f(1) = 2(1)^3 + 7(1)^2 - 9 = 2 + 7 - 9 = 0$

$\because f(1) = 0$

$\therefore (x-1)$ is a factor of $f(x)$

$$
\begin{array}{r|l}
 & 2x^2 + 9x + 9 \\
\hline
x-1 & 2x^3 + 7x^2 - 9 \\
 & 2x^3 - 2x^2 \\
 & -\quad + \\
\hline
 & 9x^2 - 9 \\
 & 9x^2 - 9x \\
 & -\quad + \\
\hline
 & 9x - 9 \\
 & 9x - 9 \\
 & -\quad + \\
\hline
 & \times
\end{array}
$$

Now, dividing $f(x)$ by $(x-1)$, we get :

$f(x) = (x-1)(2x^2 + 9x + 9)$

$= (x-1)\{2x^2 + 6x + 3x + 9\}$

$= (x-1)\{2x(x+3) + 3(x+3)\}$

$= (x-1)(x+3)(2x+3)$ **Ans.**

Q.16. $2x^3 + 19x^2 + 38x + 21$

Sol. $f(x) = 2x^3 + 19x^2 + 38x + 21$

Let $x = -1$, then

$f(-1) = 2(-1)^3 + 19(-1)^2 + 38(-1) + 21$

$= -2 + 19 - 38 + 21 = 0$

$\because f(-1) = 0$

$\therefore (x+1)$ is a factor of $f(x)$

$$
\begin{array}{r|l}
 & 2x^2 + 17x + 21 \\
\hline
x+1 & 2x^3 + 19x^2 + 38x + 21 \\
 & 2x^3 + 2x^2 \\
 & +\quad - \\
\hline
 & 17x^2 + 38x \\
 & 17x^2 + 17x \\
 & -\quad - \\
\hline
 & 21x + 21 \\
 & 21x + 21 \\
 & -\quad - \\
\hline
 & \times
\end{array}
$$

Now, dividing $f(x)$ by $(x+1)$, we get :

$f(x) = (x+1)(2x^2 + 17x + 21)$

$= (x+1)\{2x^2 + 14x + 3x + 21\}$

$= (x+1)\{2x(x+7) + 3(x+7)\}$

$= (x+1)(x+7)(2x+3)$ **Ans.**