Chapter 7

Problems on Quadratic Equations

POINTS TO REMEMBER

1. Method of solving problems on quadratic equations :

   (i) Denote the unknown quantity in the given problem by x.

   (ii) Form the quadratic equation with the help of the given condition or statement.

   (iii) Then solve for x.

   Then x will be the required answer.

   Note. Lengths, areas, volume, money etc. is always taken as positive, neglecting the negative values.

EXERCISE 7

Q. 1. Find two numbers whose sum is 40 and product 375.

Sol. Sum of two numbers = 40

Let first number = x

Then, second number = 40 − x

According to the condition,

$$x(40 - x) = 375$$
$$\Rightarrow 40x - x^2 = 375$$
$$\Rightarrow 40x - x^2 - 375 = 0$$
$$\Rightarrow -x^2 + 40x - 375 = 0$$
$$\Rightarrow x^2 - 40x + 375 = 0$$
$$\Rightarrow x^2 - 15x - 25x + 375 = 0$$
$$\begin{cases} \because -40 = -15 - 25 \\ 375 = (-15)(-25) \end{cases}$$
$$\Rightarrow x(x - 15) - 25(x - 15) = 0$$
$$\Rightarrow (x - 15)(x - 25) = 0$$

[Zero Product Rule]

Either x − 15 = 0, then x = 15

or   x − 25 = 0, then x = 25

(i) If first number = 15, then second number = 40 − 15 = 25

(ii) If first number = 25, then second number = 40 − 25 = 15

Hence, two required numbers are 15 and 25 **Ans.**

Q. 2. The difference between two integers is 4. Their product is 221. Find the numbers.

Sol. Let the required natural numbers be x and (4 − x).

Then,   x(4 − x) = 221

$$4x - x^2 = 221$$
$$\Leftrightarrow x^2 - 4x + 221 = 0$$
$$\Leftrightarrow x^2 - 17x + 13x + 221 = 0$$
$$\Leftrightarrow x(x - 17) - 13(x - 17) = 0$$
$$\Leftrightarrow (x - 17)(x - 13) = 0$$
$$\Leftrightarrow x = 17 \text{ or } x = 13$$

Hence, the required numbers are 17 and 13.

Q. 3. The sum of a natural number and its reciprocal is $\dfrac{65}{8}$. Find the natural number.

Arundeep's Foundation Math-X

Sol. Let the natural number $= x$

Then, its reciprocal $= \dfrac{1}{x}$

According to the condition,

$x + \dfrac{1}{x} = \dfrac{65}{8}$ $\Rightarrow 8x^2 + 8 = 65x$

[Multiplying by $8x$]

$\Rightarrow 8x^2 - 65x + 8 = 0$

$\begin{cases} \because 8 \times 8 = 64 \\ \therefore -65 = -64 - 1 \\ 64 = (-64) \times (-1) \end{cases}$

$\Rightarrow \quad 8x^2 - 64x - x + 8 = 0$

$\Rightarrow 8x(x-8) - 1(x-8) = 0$

$\Rightarrow \quad (x-8)(8x-1) = 0$

[Zero Product Rule]

Either $x - 8 = 0$, then $x = 8$

or $8x - 1 = 0$, then $8x = 1 \Rightarrow x = \dfrac{1}{8}$

But, it is not possible as it is not a natural number

$\therefore$ Required number = **8 Ans.**

Q. 4. Divide 27 into two parts such that the sum of their reciprocals is $\dfrac{3}{20}$.

Sol. Let the two parts be x and $(27 - x)$.

Then, $\dfrac{1}{x} + \dfrac{1}{27-x} = \dfrac{3}{20}$

$\Leftrightarrow \dfrac{27 - x + x}{x(27-x)} = \dfrac{3}{20}$

$3x(27 - x) = 27 \times 20$

$\Rightarrow \quad x(27 - x) = 9 \times 20$

$\Rightarrow \quad x^2 - 27x + 180 = 0$

$\Rightarrow x^2 - 15x - 12x + 180 = 0$

$\Rightarrow x(x - 15) - 12(x - 15) = 0$

$\Rightarrow \quad (x - 15)(x - 12) = 0$

$\Rightarrow \quad x = 15 \text{ or } x = 12$

Hence, the required two parts are 15 and 12.

Q. 5. The sum of two numbers is 12 and the sum of their squares is 74. Find the numbers.

Sol. Sum of two numbers = 12

Let first number $= x$

Then, second number $= 12 - x$

According to the condition,

$(x)^2 + (12 - x)^2 = 74$

$\Rightarrow x^2 + 144 - 24x + x^2 = 74$

$\Rightarrow 2x^2 - 24x + 144 - 74 = 0$

$\Rightarrow \quad 2x^2 - 24x + 70 = 0$

$\Rightarrow \quad x^2 - 12x + 35 = 0$ [Dividing by 2]

$\Rightarrow x^2 - 7x - 5x + 35 = 0$

$\begin{cases} \because -12 = -7 - 5 \\ 35 = (-7) \times (-5) \end{cases}$

$\Rightarrow x(x - 7) - 5(x - 7) = 0$

$\Rightarrow \quad (x - 7)(x - 5) = 0$

[Zero Product Rule]

Either $x - 7 = 0$, then $x = 7$

or $\quad x - 5 = 0$, then $x = 5$

(i) If $x = 7$, then first number = 7

and, second number = 12 − 7 = 5

(ii) If $x = 5$, then first number = 5

and, second number = 12 − 5 = 7

Hence, required numbers are 5, 7 **Ans.**

Q. 6. Find two consecutive natural numbers, the sum of whose squares is 145.

Sol. Let first natural number $= x$

then, second number $= x + 1$

According to the condition,

$(x)^2 + (x + 1)^2 = 145$

$\Rightarrow x^2 + x^2 + 2x + 1 = 145$

$\Rightarrow 2x^2 + 2x + 1 - 145 = 0$

$\Rightarrow \quad 2x^2 + 2x - 144 = 0$

$\Rightarrow \quad x^2 + x - 72 = 0$

[Dividing by 2]

$\Rightarrow \quad x^2 + 9x - 8x - 72 = 0$

$$\left\{ \begin{array}{l} \because \ 1 = 9 - 8 \\ -72 = 9 \times (-8) \end{array} \right\}$$

$\Rightarrow x (x + 9) - 8 (x + 9) = 0$

$\Rightarrow \qquad (x + 9) (x - 8) = 0$

[Zero Product Rule]

Either, $x + 9 = 0$, then $x = -9$

Which is not possible, as it is not a natural number.

or $\ x - 8 = 0$, then $x = 8$

$\therefore$ First natural number $= 8$

and second number $= 8 + 1 = 9$ **Ans.**

Q. 7. Find two consecutive positive even integers whose product is 224.

Sol. Let first positive even integer $= 2x$

then, second even integer $= 2x + 2$

According to the condition,

$\qquad 2x (2x + 2) = 224$

$\Rightarrow \qquad 4x^2 + 4x = 224$

$\Rightarrow \quad 4x^2 + 4x - 224 = 0$

$\Rightarrow \qquad x^2 + x - 56 = 0$ [Dividing by 4]

$\Rightarrow x^2 + 8x - 7x - 56 = 0$

$$\left\{ \begin{array}{l} \because \ 1 = 8 - 7 \\ -56 = 8 \times (-7) \end{array} \right\}$$

$\Rightarrow x (x + 8) - 7 (x + 8) = 0$

$\Rightarrow \qquad (x + 8) (x - 7) = 0$

[Zero Product Rule]

Either, $x + 8 = 0$, then $x = -8$,

which is not possible, as it is not a positive integer.

or $x - 7 = 0$, then $x = 7$

$\therefore$ First positive even integer

$\qquad = 2x = 2 \times 7 = 14$

and second even integer

$\qquad = 14 + 2 = 16$ **Ans.**

Q. 8. The sum of the squares of three consecutive odd numbers is 2531. Find the numbers.

Sol. Let the smallest odd number be x, then the other two consecutive odd numbers are $x + 2$ and $x + 4$.

According to given statement,

$\qquad x^2 + (x + 2)^2 + (x + 4)^2 = 2531$

$\Rightarrow x^2 + (x^2 + 4x + 4) + (x^2 + 8x + 16) = 2531$

$\Rightarrow x^2 + x^2 + 4x + 4 + x^2 + 8x + 16 = 2531$

$\Rightarrow \ 3x^2 + 12x + 20 - 2531 = 0$

$\Rightarrow \ 3x^2 + 12x - 2511 = 0$

$\Rightarrow \ 3x^2 + 93x - 81x - 2511 = 0$

$\Rightarrow \ 3x (x + 31) - 81 (x + 31) = 0$

$\Rightarrow \ (3x - 81) (x + 31) = 0$

$\Rightarrow \ x = \dfrac{81}{3} = 27$ or $x = -31$

but x cannot be negative $\therefore x = 27$

Then, $x = 27$, $x + 2 = 29$ and $x + 4 = 31$

Hence, the required numbers are 27, 29, 31 **Ans.**

Q. 9. Find two consecutive multiples of 3 whose product is 270.

Sol. Let first multiple of $3 = 3x$

Then, second multiple of $3 = 3x + 3$

Now, according to the condition,

$\qquad 3x (3x + 3) = 270$

$\Rightarrow \qquad 9x^2 + 9x = 270$

$\Rightarrow 9x^2 + 9x - 270 = 0$

$\Rightarrow \qquad x^2 + x - 30 = 0$ [Dividing by 9]

$\Rightarrow x^2 + 6x - 5x - 30 = 0$

$$\left\{ \begin{array}{l} \because \ 1 = 6 - 5 \\ -30 = 6 \times (-5) \end{array} \right\}$$

$\Rightarrow x (x + 6) - 5 (x + 6) = 0$

$\Rightarrow \qquad (x + 6) (x - 5) = 0$

[Zero Product Rule]

Either $x + 6 = 0$, then $x = -6$, which is not possible

or $x - 5 = 0$, then $x = 5$

$\therefore$ First multiple of $3 = 3 \times 5 = 15$

and second multiple $= 15 + 3 = 18$ **Ans.**

Q. 10. A number consists of two digits whose product is 18. If 27 is added to number, the digits interchange their places. Find the number.

Sol. Let the units digit = x

∵ Product of two digits = 18

Then, tens digit = $\dfrac{18}{x}$.

∴ Number $= x + 10\left(\dfrac{18}{x}\right) = x + \dfrac{180}{x}$

When the digit of the number are interchanged

then units digit $= \dfrac{18}{x}$

and tens digit $= x$

∴ Number $= \dfrac{18}{x} + 10x$.

Now according to the condition,

$$x + \dfrac{180}{x} + 27 = \dfrac{18}{x} + 10x$$

$$\Rightarrow \quad 10x + \dfrac{18}{x} - x - \dfrac{180}{x} - 27 = 0$$

$$\Rightarrow 10x^2 + 18 - x^2 - 180 - 27x = 0$$

[Multiplying by x]

$$\Rightarrow 9x^2 - 27x - 162 = 0$$

$$\Rightarrow \quad x^2 - 3x - 18 = 0 \ [\text{Dividing by } 9]$$

$$\Rightarrow x^2 - 6x + 3x - 18 = 0$$

$$\left\{ \begin{array}{l} \because -3 = -6 + 3 \\ -18 = -6 \times 3 \end{array} \right\}$$

$$\Rightarrow x(x - 6) + 3(x - 6) = 0$$

$$\Rightarrow \quad (x - 6)(x + 3) = 0$$

[Zero Product Rule]

Either $x - 6 = 0$, then $x = 6$

or $x + 3 = 0$,

then $x = -3$, which is not possible.

∴ Number $= x + \dfrac{180}{x} = 6 + \dfrac{180}{6}$

$= 6 + 30 = 36$ **Ans.**

Q. 11. A two-digit number contains the smaller of the two digits in the unit place. The product of the digits is 40 and the difference between the digits is 3. Find the number.

Sol. Let the units digit = x

Then, tens digit = $x + 3$.

∴ Number $= x + 10(x + 3)$

$= x + 10x + 30 = 11x + 30$

Product of two digits = 40

According to the condition,

$$x(x + 3) = 40$$

$$\Rightarrow \qquad x^2 + 3x = 40$$

$$\Rightarrow \qquad x^2 + 3x - 40 = 0$$

$$\Rightarrow x^2 + 8x - 5x - 40 = 0$$

$$\left\{ \begin{array}{l} \because 3 = 8 - 5 \\ -40 = 8 \times (-5) \end{array} \right\}$$

$$\Rightarrow x(x + 8) - 5(x + 8) = 0$$

$$\Rightarrow \qquad (x + 8)(x - 5) = 0$$

[Zero Product Rule]

Either $x + 8 = 0$, then $x = -8$, which is not possible

or $x - 5 = 0$, then $x = 5$

∴ Number $= 11x + 30 = 11 \times 5 + 30$

$= 55 + 30 = 85$ **Ans.**

Q. 12. The sum of the numerator and denominator of a certain fraction is 10. If 1 is subtracted from both the numerator and denominator, the fraction is decrease by $\dfrac{2}{21}$. Find the fraction.

Sol. Sum of numerator and denominator of a fraction = 10

Let numerator = x

then, denominator = $10 - x$

∴ Fraction $= \dfrac{x}{10 - x}$

Subtracting 1 from both the numerator and denominator, then

Numerator = $x - 1$

and denominator = $10 - x - 1 = 9 - x$

and fraction = $\dfrac{x-1}{9-x}$

According to the condition,

$$\dfrac{x}{10-x} - \dfrac{x-1}{9-x} = \dfrac{2}{21}$$

$$\Rightarrow \dfrac{x(9-x) - (x-1)(10-x)}{(10-x)(9-x)} = \dfrac{2}{21}$$

$$\Rightarrow \dfrac{9x - x^2 - (10x - x^2 - 10 + x)}{90 - 10x - 9x + x^2} = \dfrac{2}{21}$$

$$\Rightarrow \dfrac{9x - x^2 - 10x + x^2 + 10 - x}{x^2 - 19x + 90} = \dfrac{2}{21}$$

$$\Rightarrow \dfrac{-2x + 10}{x^2 - 19x + 90} = \dfrac{2}{21}$$

$$\Rightarrow 2(x^2 - 19x + 90) = 21 \times (-2x + 10)$$

[By cross multiplication]

$$\Rightarrow 2x^2 - 38x + 180 = -42x + 210$$

$$\Rightarrow 2x^2 - 38x + 42x + 180 - 210 = 0$$

$$\Rightarrow 2x^2 + 4x - 30 = 0$$

$$\Rightarrow x^2 + 2x - 15 = 0 \quad \text{[Dividing by 2]}$$

$$\Rightarrow x^2 + 5x - 3x - 15 = 0 \quad \left\{ \begin{array}{l} \because 2 = 5 - 3 \\ -15 = 5 \times (-3) \end{array} \right\}$$

$$\Rightarrow x(x+5) - 3(x+5) = 0$$

$$\Rightarrow (x+5)(x-3) = 0$$

[Zero Product Rule]

Either $x + 5 = 0$, then $x = -5$, which is not possible

or $x - 3 = 0$, then $x = 3$

$\therefore$ Fraction = $\dfrac{x}{10-x} = \dfrac{3}{10-3} = \dfrac{3}{7}$ **Ans.**

Q. 13. Two years ago, a man's age was three times the square of his son's age. In three years' time, his age will be four times his son's age. Find their present ages.

Sol. Let the present age of the man be x years, and that of his son be y years.

Two years ago,

The age of the man = $(x - 2)$ years

and the age of his son = $(y - 2)$ years.

According to the given statement,

$$x - 2 = 3(y - 2)^2 \qquad \qquad ...(i)$$

After 3 years from now,

The age of the man = $(x + 3)$ years

and the age of his son = $(y + 3)$ years

According to the given statement,

$$x + 3 = 4(y + 3)$$

$$\Rightarrow x + 3 = 4y + 12$$

$$\Rightarrow x = 4y + 9 \qquad \qquad ...(ii)$$

Substituting, the value of x from (ii) in (i), we get :

$$3(y - 2)^2 = 4y + 9 - 2$$

$$\Rightarrow 3(y^2 - 4y + 4) = 4y + 7$$

$$\Rightarrow 3y^2 - 12y + 12 - 4y - 7 = 0$$

$$\Rightarrow 3y^2 - 16y + 5 = 0$$

$$\Rightarrow 3y^2 - 15y - y + 5 = 0$$

$$\Rightarrow 3y(y - 5) - 1(y - 5) = 0$$

$$\Rightarrow (y - 5)(3y - 1) = 0$$

$$\Rightarrow y - 5 = 0 \text{ or } 3y - 1 = 0$$

$$\Rightarrow y = 5 \text{ or } y = \dfrac{1}{3}$$

But, $y = \dfrac{1}{3}$ is not possible, $\therefore y = 5$.

From (ii), $x = 4 \times 5 + 9 = 29$.

$\therefore$ The present age of the man = 29 years

and the present age of his son = 5 years.

Q. 14. A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. Find

the time taken by the first pipe alone to fill the tank.

Sol. Let first pipe can fill the tank in = x hrs

Then, second and third pipes will take $(x-5)$ and $(x-9)$ hours respectively to fill the tank.

According to the given condition,

$$\frac{1}{x} + \frac{1}{(x-5)} = \frac{1}{x-9}$$

$$\Rightarrow \frac{x-5+x}{x(x-5)} = \frac{1}{x-9}$$

$$\Rightarrow \frac{2x-5}{x^2-5x} = \frac{1}{x-9}$$

$$\Rightarrow x^2 - 5x = (2x-5)(x-9)$$
$$\Rightarrow x^2 - 5x = 2x^2 - 18x - 5x + 45$$
$$\Rightarrow x^2 - 5x = 2x^2 - 23x + 45$$
$$\Rightarrow 2x^2 - 23x + 45 - x^2 + 5x = 0$$
$$\Rightarrow \qquad x^2 - 18x + 45 = 0$$
$$\Rightarrow \qquad x^2 - 15x - 3x + 45 = 0$$
$$\Rightarrow \quad x(x-15) - 3(x-15) = 0$$
$$\Rightarrow \qquad (x-15)(x-3) = 0$$

Either $x - 15 = 0$, Then $x = 15$

or $\qquad x - 3 = 0$, Then $x = 3$

But it is not possible

$\therefore \qquad x = 15$

$\therefore$ First pipe can fill the tank in 15 hrs.

Q. 15. A booster pump can be used for filling as well as for emptying a tank. The capacity of the tank is 2400 m³. The emptying capacity of the tank is 10 m³ per minute higher than its filling capacity and the pump needs 8 minutes lesser to empty the tank than it needs to fill it. What is the filling capacity of the pump ?

Sol. Let the filling capacity of the pump be x m³/min.

Then, emptying capacity of the pump = $(x + 10)$ m³/min.

According to the given statement,

$$\therefore \quad \frac{2400}{x} - \frac{2400}{(x+10)} = 8$$

$$\Rightarrow \frac{2400(x+10) - x(2400)}{x(x+10)} = 8$$

$$\Rightarrow \frac{2400x + 24000 - 2400x}{x^2 + 10x} = 8$$

$$\Rightarrow \qquad 24000 = 8x^2 + 80x$$
$$\Rightarrow \quad 8x^2 + 80 = 24000$$
$$\Rightarrow \quad 8x^2 + 80x - 24000 = 0$$
$$\Rightarrow \qquad x^2 + 10x - 3000 = 0$$
$$\Rightarrow \quad x^2 + 60x - 50x - 3000 = 0$$
$$\Rightarrow \quad x(x+60) - 50(x+60) = 0$$
$$\Rightarrow \qquad (x+60)(x-50) = 0$$
$$\Rightarrow \quad x = -60 \text{ or } x = 50$$

So, rejecting the negative value, we see that filling capacity of the pump is 50 m³/min. **Ans.**

Q. 16. Albert goes to his friend's house which is 12 km away from his house. He covers half of the distance at a speed of x km/hr and the remaining at $(x + 2)$ km/hr. If he takes 2 hrs 30 min. to cover the whole distance, find x.

Sol. Distance of Albert's friends house = 12 km.

Speed for half distance

i.e. for 6 km = x km/hr.

and speed for the second half distance = $(x + 2)$ km/hr.

Total time taken = 2 hrs. 30 min.

$$= \frac{5}{2} \text{ hours.}$$

According to the condition,

$$\frac{6}{x} + \frac{6}{x+2} = \frac{5}{2}$$

$$\Rightarrow \frac{6(x+2) + 6 \times x}{x(x+2)} = \frac{5}{2}$$

$$\Rightarrow \quad \frac{6x+12+6x}{x^2+2x} = \frac{5}{2}$$

$$\Rightarrow \quad \frac{12x+12}{x^2+2x} = \frac{5}{2}$$

$$\Rightarrow \quad 5(x^2+2x) = 2(12x+12)$$

[By cross multiplication]

$$\Rightarrow \quad 5x^2+10x = 24x+24$$

$$\Rightarrow 5x^2+10x-24x-24 = 0$$

$$\Rightarrow \quad 5x^2-14x-24 = 0$$

$$\begin{bmatrix} \because 5\times(-24) = -120 \\ -14 = -20+6 \\ -120 = -20\times 6 \end{bmatrix}$$

$$\Rightarrow \quad 5x^2-20x+6x-24 = 0$$

$$\Rightarrow 5x(x-4)+6(x-4) = 0$$

$$\Rightarrow \quad (x-4)(5x+6) = 0$$

[Zero Product Rule]

Either $x-4 = 0$, then $x = 4$

or $\quad 5x+6 = 0$, then $5x = -6$

$$\Rightarrow \quad x = -\frac{6}{5}$$

Which is not possible

$\therefore \quad x = 4$ Ans.

17. **(i)** A train covers a distance of 780 km at x km/hr. Had the speed been $(x-5)$ km/hr, the time taken to cover the same distance would have been increased by 1 hour. Write down an equation in x and solve it to evaluate x.

Sol. Total distance = 780 km

Speed of train = x km/hr

$\therefore$ Time taken = $\dfrac{780}{x}$ hrs.

In second case,

Speed of the train = $(x-5)$ km/hr.

Time taken = $\dfrac{780}{x-5}$ hr.

According to the condition,

$$\frac{780}{x-5} - \frac{780}{x} = 1$$

$$\Rightarrow \quad 780\left(\frac{1}{x-5}-\frac{1}{x}\right) = 1$$

$$\Rightarrow \quad 780\left(\frac{x-x+5}{x(x-5)}\right) = 1$$

$$\Rightarrow \quad \frac{780\times(5)}{x^2-5x} = 1$$

$$\Rightarrow \quad 3900 = x^2-5x$$

[By cross multiplication]

$$\Rightarrow x^2-5x-3900 = 0$$

$$\begin{bmatrix} \because \quad -5 = -65+60 \\ -3900 = -65\times 60 \end{bmatrix}$$

$$\Rightarrow \quad x^2-65x+60x-3900 = 0$$

$$\Rightarrow \quad x(x-65)+60(x-65) = 0$$

$$\Rightarrow \quad (x-65)(x+60) = 0$$

[Zero Product Rule]

Either $x-65 = 0$, then $x = 65$

or $x+60 = 0$, then $x = -60$, But it does not satisfy it.

$\therefore \quad x = 65$ Ans.

(ii) A train covers a distance of 600 km at x km/hr. Had the speed been $(x+20)$ km/hr, the time taken to cover the same distance would have been reduced by 5 hours, write down an equation in x and solve it to evaluate x. (1996)

Sol. Total distance = 600 km

Speed of the train = x km/hr.

$\therefore \quad$ Time taken = $\dfrac{600}{x}$ hrs.

In second case,

Speed of the train = $(x+20)$ km/hr.

Time taken = $\dfrac{600}{x+20}$ hrs.

According to the condition,

$$\frac{600}{x} - \frac{600}{x+20} = 5$$

$$\Rightarrow \quad 600\left[\frac{1}{x} - \frac{1}{x+20}\right] = 5$$

$$\Rightarrow \quad 600\left[\frac{x+20-x}{x(x+20)}\right] = 5$$

$$\Rightarrow \quad \frac{600 \times 20}{x(x+20)} = 5$$

$$\Rightarrow \quad 5 x(x+20) = 600 \times 20$$

(By cross multiplication)

$$\Rightarrow \quad x(x+20) = 600 \times 4$$

(Dividing by 5)

$$\Rightarrow \quad x^2 + 20x = 2400$$

$$x^2 + 20x - 2400 = 0 \textbf{ Ans.}$$

$$\Rightarrow x^2 + 60x - 40x - 2400 = 0$$

$$\begin{cases} \because\ 60 - 40 = 20 \\ 60 \times (-40) = -2400 \end{cases}$$

$$\Rightarrow \quad x(x+60) - 40(x+60) = 0$$

$$\Rightarrow \quad (x+60)(x-40) = 0$$

(Zero Product Rule)

Either $x + 60 = 0$, then $x = -60$

which is not possible being in negative

or $x - 40 = 0$, then $x = 40$

$\therefore$ Speed of the train = 40 km/hr **Ans.**

Q. 18. By increasing the speed of a car by 10 km/hr, the time of journey for a distance of 72 km. is reduced by 36 minutes. Find the original speed of the car. **(2005)**

Sol. Let original speed of car be x km/h.

$$\therefore \text{ Time} = \frac{72}{x}\ \text{hr.}$$

new speed $= x + 10$ km/h.

$$\therefore \text{ new time} = \frac{72}{x+10}\ \text{hr.}$$

difference in time $= 36$ mins.

$$\therefore \quad \frac{72}{x} - \frac{72}{x+10} = \frac{36}{60}$$

$$\frac{72x + 720 - 72x}{x(x+10)} = \frac{3}{5}$$

$$5 \times 720 = 3(x^2 + 10x)$$

$$5 \times 240 = x^2 + 10x$$

$$1200 = x^2 + 10x$$

$$x^2 + 10x - 1200 = 0$$

$$x^2 + 40x - 30x - 1200 = 0$$

$$x(x+40) - 30(x+40) = 0$$

$$(x-30)(x+40) = 0$$

$$\therefore\ x = 30$$

as $x = -40$ is not possible

$\therefore$ original speed of the car = 30 km/h.

Q. 19. The distance by road between two towns A and B, is 216 km, and by rail is 208 km. A car travels at a speed of km/hr and the train travels at a spee which is 16 km/hr faster than the car

(i) Write down the time taken by the car t reach town B from A, in terms of x.

(ii) Write down the time taken by the trai to reach town B from A, in terms of

(iii) If the train takes 2 hours less than th car to reach town B, obtain an equatio in x and solve it.

(iv) Hence, find the speed of the train.

Sol. Distance by road between two towns and B = 216 km and distance by ra between A and B = 208 km.

Speed of car $= x$ km/hr.

and speed of train $= (x + 16)$ km/hr.

(i) Time taken by car $= \dfrac{216}{x}$ hours

(ii) Time taken by train $= \dfrac{208}{x+16}$ hrs.

(iii) According to the condition,

$$\frac{216}{x} - \frac{208}{x+16} = 2$$

$$\Rightarrow \frac{216(x+16) - 208 \times x}{x(x+16)} = \frac{2}{1}$$

$$\Rightarrow \frac{216x + 3456 - 208x}{x^2 + 16x} = \frac{2}{1}$$

$$\Rightarrow \frac{8x + 3456}{x^2 + 16x} = \frac{2}{1}$$

$$\Rightarrow \quad 2(x^2 + 16x) = 8x + 3456$$

$$\Rightarrow 2x^2 + 32x - 8x - 3456 = 0$$

$$\Rightarrow \quad 2x^2 + 24x - 3456 = 0$$

$$\Rightarrow \quad x^2 + 12x - 1728 = 0$$

[Dividing by 2]

$$x^2 + 48x - 36x - 1728 = 0$$

$$\begin{cases} \because 12 = 48 - 36 \\ -1728 = 48 \times (-36) \end{cases}$$

$$\Rightarrow x(x + 48) - 36(x + 48) = 0$$

$$\Rightarrow \quad (x + 48)(x - 36) = 0$$

[Zero Product Rule]

Either $x + 48 = 0$, then $x = -48$ which is not possible.

or $x - 36 = 0$, then $x = 36$.

Hence, speed of the car = 36 km/hr.

(iv) ∴ Speed of train = $x + 16 = 36 + 16$

= 52 km/hr.**Ans.**

Q.20. Car A travels x km for every litre of petrol, while car B travels $(x + 5)$ km for every litre of petrol.

(i) Write down the number of litres used by car A and car B in covering a distance of 400 km.

(ii) If car A used 4 litres of petrol more than car B in covering 400 km, write an equation in x and solve it to determine the number of litres of petrol used by car B for the journey. (1997)

Sol. Car A travels per litre = x km

And car B travels per litre = $(x + 5)$ km

(i) No. of litres used to travel 400 km by

car A = $\dfrac{400}{x}$ litres

and no. of litres used to travel 400 km .

by car B = $\dfrac{400}{x+5}$ litres

(ii) According to the condition,

$$\frac{400}{x} - \frac{400}{x+5} = 4$$

$$\Rightarrow 400\left(\frac{1}{x} - \frac{1}{x+5}\right) = 4$$

$$\Rightarrow 400\left(\frac{x+5-x}{x(x+5)}\right) = 4$$

$$\Rightarrow \frac{400 \times 5}{x^2 + 5x} = \frac{4}{1}$$

$$\Rightarrow \quad 4(x^2 + 5x) = 2000$$

$$\Rightarrow 4x + 20x - 2000 = 0$$

$$\Rightarrow \quad x^2 + 5x - 500 = 0$$

(Dividing by 4)

∴ Equation will be $x^2 + 5x - 500 = 0$

$$\Rightarrow x^2 + 25x - 20x - 500 = 0$$

$$\begin{cases} 5 = 25 - 20 \\ -500 = 25 \times (-20) \end{cases}$$

$$\Rightarrow x(x + 25) - 20(x + 25) = 0$$

$$\Rightarrow \quad (x + 25)(x - 20) = 0$$

[Zero Product Rule]

Either $x + 25 = 0$, then $x = -25$ which is not possible

or $x - 20 = 0$, then $x = 20$

∴ No. of litres of petrol used by car B

$$= \frac{400}{x+5} = \frac{400}{20+5} = \frac{400}{25} = 16 \text{ litres}$$

Ans.

Q.21. The speed of a boat in still water is x km/hr and the speed of the stream is 3 km/hr.

(i) Write the speed of the boat upstream, in terms of x.

(ii) Write the speed of the boat downstream, in terms of x.

(iii) If the boat goes 15 km upstream and 22 km downstream in 5 hours, write an equation in x to represent the statement.

(iv) Solve the equation to evaluate x.

Sol. Speed of boat in still water = x km/hr

and, speed of stream = 3km/hr.

(i) Speed of boat upstream = (x − 3) km/hr.

(ii) Speed of boat downstream = (x + 3) km/hr.

(iii) Time taken in 15 km upstream and 22 km downstream = 5 hours.

According to the condition,

$$\frac{15}{x-3}+\frac{22}{x+3}=5$$

$$\Rightarrow \frac{15(x+3)+22(x-3)}{(x+3)(x-3)}=5$$

(iv) $$\Rightarrow \frac{15x+45+22x-66}{x^2-9}=5$$

$$\Rightarrow \qquad 37x-21=5x^2-45$$

$$\Rightarrow 5x^2-37x+21-45=0$$

$$\Rightarrow \qquad 5x^2-37x-24=0$$

$$\Rightarrow 5x^2-40x+3x-24=0$$

$$\begin{cases}-37=-40+3\\-120=-40\times3\end{cases}$$

$$\Rightarrow 5x(x-8)+3(x-8)=0$$

$$\Rightarrow \qquad (x-8)(5x+3)=0$$

[Zero Product Rule]

Either x − 8 = 0, then x = 8

or 5x + 3 = 0, then 5x = − 3

$$\Rightarrow x=-\frac{3}{5}$$

But it is not possible.

∴ x = 8 Ans.

Q. 22.The speed of an express train is x km/h and the speed of an ordinary train is 12 km/h less than that of the express train. If the ordinary train takes one hour longer than the express train to cover a distance of 240 km, find the speed of the express train. (2009)

Sol. $$\frac{240}{x-12}-\frac{240}{x}=1$$

$$\Rightarrow 240\left[\frac{1}{(x-12)}-\frac{1}{x}\right]=1$$

$$\Rightarrow 240\left[\frac{x-x+12}{x(x-12)}\right]=1$$

$$\Rightarrow 240\left[\frac{12}{x^2-12x}\right]=1$$

$$\Rightarrow 2880=x^2-12x$$

$$\Rightarrow x^2-12x-2880=0$$

$$x=\frac{12\pm\sqrt{144+11520}}{2}=\frac{12\pm108}{2}$$

= 60 km/h

Q. 23. The hypotenuse of a right triangle is 20 m. If the difference between the lengths of other sides be 4 m, find the other sides.

Sol. Hypotenuse of a right triangle = 20 m

Let one side = x

Then, second side = x − 4

According to the condition,

$$x^2+(x-4)^2=(20)^2$$

(Pythagoras Theorem)

$$\Rightarrow x^2+x^2-8x+16=400$$

$$\Rightarrow 2x^2-8x+16-400=0$$

$$\Rightarrow 2x^2-8x-384=0$$

$$\Rightarrow x^2-4x-192=0$$

(Dividing by 2)

$$\Rightarrow x^2-16x+12x-192=0$$

$$\begin{cases} \because -4 = -16 + 12 \\ -192 = -16 \times 12 \end{cases}$$

$\Rightarrow x(x-16) + 12(x-16) = 0$

$\Rightarrow \qquad (x-16)(x+12) = 0$

(Zero Product Rule)

Either $x - 16 = 0$, then $x = 16$

or $x + 12 = 0$, then $x = -12$, which is not possible.

$\therefore$ One side $= 16$ m

and second side $= x - 4 = 16 - 4 = 12$ m.

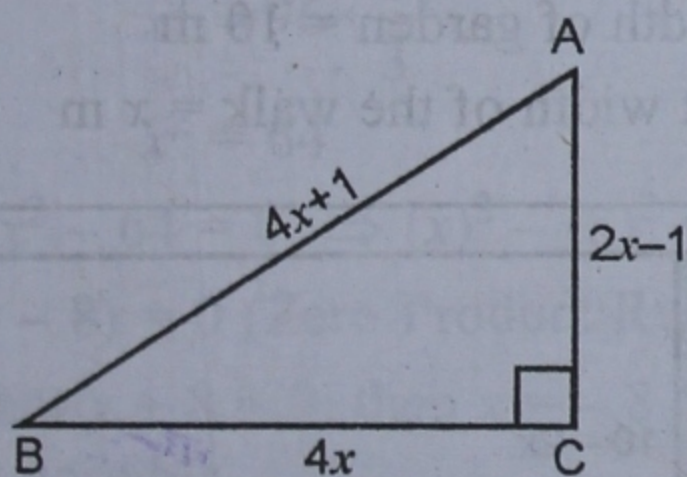Hence sides are 16m and 12m. **Ans.**

Q. 24. The lengths of the sides of a right triangle are $(2x-1)$ m, $(4x)$ m and $(4x+1)$ m, where $x > 0$, Find :

(i) the value of x

(ii) the area of the triangle.

Sol. Sides of a right triangle are $2x - 1$, $4x$ and $4x + 1$ (in metres)

Here, $4x + 1$ is the longest side

$\therefore (4x+1)^2 = (2x-1)^2 + (4x)^2$

[Pythagoras Theorem]

$\Rightarrow 16x^2 + 8x + 1 = 4x^2 - 4x + 1 + 16x^2$

$\Rightarrow 16x^2 + 8x + 1 - 4x^2 + 4x - 1 - 16x^2 = 0$

$\Rightarrow \quad -4x^2 + 12x = 0$

$\Rightarrow \qquad x^2 - 3x = 0$

[Dividing by -4]

$\Rightarrow \quad x(x-3) = 0$ \qquad [Zero Product Rule]

Either $x = 0$, which is not possible as $x > 0$

(i) or $x - 3 = 0$ then $x = 3$.

(ii) First side $= 2x - 1 = 2 \times 3 - 1$

$= 6 - 1 = 5\ m$

Second side $= 4x = 4 \times 3 = 12\ m$

Third side $= 4x + 1 = 4 \times 3 + 1$

$= 12 + 1 = 13 m$

Hence area of the triangle

$= \dfrac{1}{2}$ base $\times$ altitude $= \dfrac{1}{2} \times 5 \times 12$

$= 30\ m^2$ **Ans.**

Q. 25. Two squares have sides x cm and $(x + 5)$ cm.

The sum of their areas is 697 sq. cm.

(i) Express this as an algebraic equation in x.

(ii) Solve this equation to find the sides of the squares.

Sol. Side of first square $= x$ cm

And side of second square $= (x + 5)$ cm

Sum of areas $= 697$ sq. cm.

According to the condition,

$(x)^2 + (x+5)^2 = 697$

$\Rightarrow \quad x^2 + x^2 + 10x + 25 = 697$

$\Rightarrow 2x^2 + 10x + 25 - 697 = 0$

$\Rightarrow \qquad 2x^2 + 10x - 672 = 0$

(i) $\therefore x^2 + 5x - 336 = 0$ \quad [Dividing by 2]

$\Rightarrow x^2 + 21x - 16x - 336 = 0 \begin{cases} \because 5 = 21 - 16 \\ -336 = 21 \times (-16) \end{cases}$

$\Rightarrow x(x+21) - 16(x+21) = 0$

$\Rightarrow \qquad (x+21)(x-16) = 0$

[Zero Product Rule]

Either $x + 21 = 0$, then $x = -21$, which is not possible.

or $x - 16 = 0$, then $x = 16$

(ii) Side of first square $= 16$cm.

and side of second square $= 16 + 5$

$= 21$ cm **Ans.**

Q. 26. The ratio between the length and the breadth of a rectangular field is 3 : 2. If only the length is increased by 5 metres, the new area of the field will be 2600 sq. metres. What is the breadth of the rectangular field ?

Sol. Let the rectangular field $= x$ m

The length of the rectangular field $= 3x$

and breadth of the rectangular field $= 2x$

If length increased by 5 metres = $(3x + 5)$

and then new area of field will be

$\quad\quad = 2600$ sq. mtrs.

According to the statement,

$\quad\quad (3x + 5) \times 2x = 2600$

$\Rightarrow\quad 6x^2 + 10x - 2600 = 0$

$\quad\quad 3x^2 + 5x - 1300 = 0$ (dividing by 2)

$\quad\quad 3x^2 + 65x - 60x - 1300 = 0$

$\quad\quad x(3x + 65) - 20(3x + 65) = 0$

$\quad\quad (3x + 65)(x - 20) = 0$

$\quad\quad\quad 3x + 65 = 0$

Either $3x = -65$

which is not possible because breadth can't be negative.

or $x - 20 = 0$

$\quad\quad\quad x = 20$

$\therefore$ breadth $= 2xm = 2 \times 20$ m $= 40$ m

Hence, the breadth of the rectangular field $= 40$ m **Ans.**

Q. 27. The perimeter of a rectangular plot of land is 114 metres and its area is 810 square metres.

(i) Take the length of plot as x metres. Use the perimeter 114 m to write the value of the breadth in terms of x.

(ii) Use the values of length, breadth and area to write an equation in x.

(iii) Solve the equation to find the length and breadth of the plot.

Sol. (i) Perimeter of a rectangle $= 114$ metres

Area of rectangle $= 810$ square metres.

Let length of rectangle $= x$ m

But perimeter $= 2$ (length + breadth)

$\Rightarrow\quad\quad 114 = 2(x + \text{breadth})$

$\Rightarrow\quad\quad 114 = 2x + 2\,\text{breadth}$

$\Rightarrow 2\,\text{breadth} = 114 - 2x$

$\therefore\quad$ Breadth $= \dfrac{114 - 2x}{2} = (57 - x)$ m.

(ii) Area $=$ length $\times$ breadth

$\Rightarrow\quad\quad 810 = x(57 - x)$

$\Rightarrow\quad\quad 810 = 57x - x^2$

$\Rightarrow x^2 - 57x + 810 = 0$

(iii) $x^2 - 57x + 810 = 0$

$\quad\quad x^2 - 30x - 27x + 810 = 0$

$\quad\quad \begin{cases} \because -57 = -30 - 27 \\ 810 = (-30)(-27) \end{cases}$

$\Rightarrow x(x - 30) - 27(x - 30) = 0$

$\Rightarrow\quad\quad (x - 30)(x - 27) = 0$

             [Zero Product Rule]

Either $x - 30 = 0$, then $x = 30$

or $x - 27 = 0$, then $x = 27$
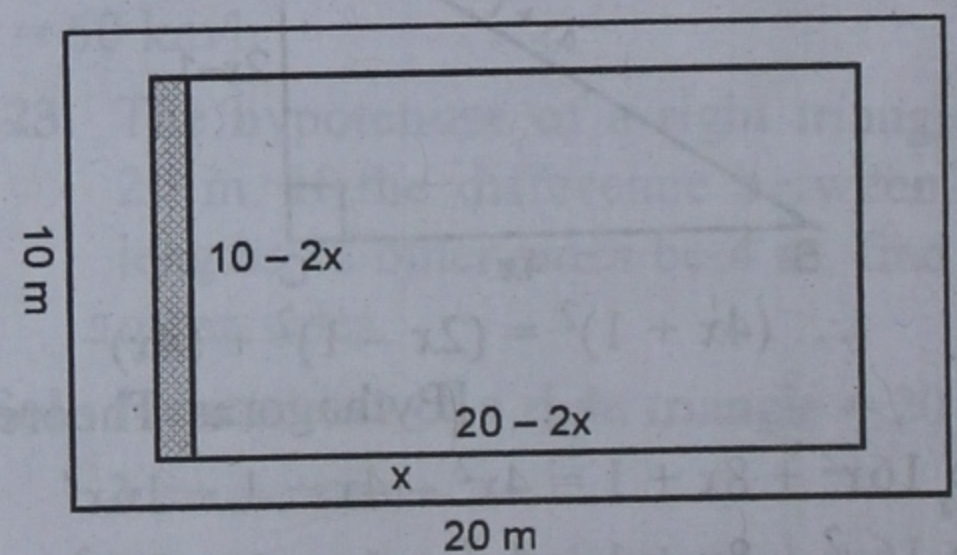
$\therefore$ Length of rectangle $= 30$ m

and breadth $= 27$ m **Ans.**

Q. 28. Within a rectangular garden 10 m wide and 20 m long, we wish to pave a walk around the borders of uniform width so as to leave an area of 96 m^2 for flowers. How wide should the walk be ?

Sol. Length of garden $= 20$ m

Width of garden $= 10$ m

Let width of the walk $= x$ m

$\therefore$ Inner length $= (20 - 2x)$ m

and   width $= (10 - 2x)$ m

According to the condition,

$\quad\quad (20 - 2x)(10 - 2x) = 96$

$\Rightarrow 200 - 40x - 20x + 4x^2 - 96 = 0$

$\Rightarrow 4x^2 - 60x + 200 - 96 = 0$

$\Rightarrow 4x^2 - 60x + 104 = 0$

$\Rightarrow\quad\quad x^2 - 15x + 26 = 0$

             (Dividing by 4)

$\Rightarrow\quad\quad x^2 - 13x - 2x + 26 = 0$

$\Rightarrow x(x - 13) - 2(x - 13) = 0$

$\Rightarrow \quad (x - 13)(x - 2) = 0$

(Zero Product Rule)

Either $x - 13 = 0$, then $x = 13$

But it is not possible.

or $x - 2 = 0$, then $x = 2$

Hence, width of walk = 2 m **Ans.**

29. The area of a right–angled triangle is 96 m². If the base is three times its altitude, find the base.

Sol. Area of right–angled triangle = 96m²

Let altitude of the triangle = x m

then base = $3x$ m

According to the condition,

$\dfrac{1}{2} \times$ base $\times$ altitude = Area

$\Rightarrow \dfrac{1}{2} \times 3x \times x = 96$

$\Rightarrow \dfrac{3}{2}x^2 = 96$

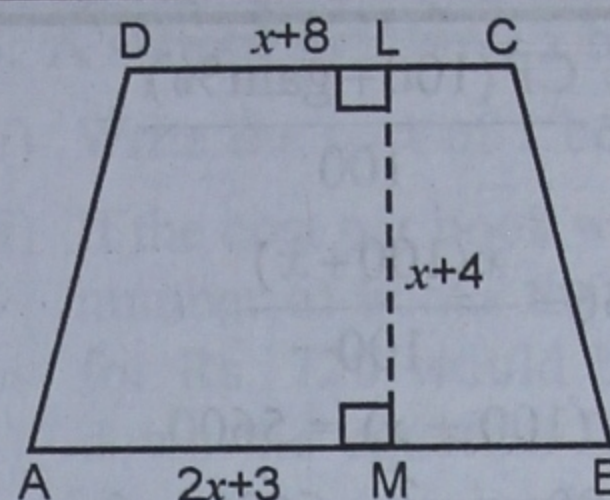$\Rightarrow \quad x^2 = 96 \times \dfrac{2}{3}$

$\Rightarrow \quad x^2 = 64$

$\Rightarrow x^2 - 64 = 0 \Rightarrow (x)^2 - (8)^2 = 0$

$(x + 8)(x - 8) = 0$ (Zero Product Rule)

Either $x + 8 = 0$, then $x = -8$, which is not possible.

or $x - 8 = 0$, then $x = 8$

$\therefore$ Base = $3x = 3 \times 8 = 24$ m. **Ans.**

30. The lengths of the parallel sides of a trapezium are $(x + 8)$ cm and $(2x + 3)$ cm, and the distance between them is $(x + 4)$ cm. If its area is 590 cm², find the value of x.

Sol. In trapezium ABCD, AB ∥ DC. First parallel side AB = $(2x + 3)$ cm and second parallel side CD = $(x + 8)$ cm

Altitude LM = $(x + 4)$ cm

Area = 590 cm²

According to the condition,

$\dfrac{1}{2}$ (sum of parallel sides) $\times$ height = Area

$\Rightarrow \dfrac{1}{2}(2x + 3 + x + 8) \times (x + 4) = 590$

$\Rightarrow \dfrac{1}{2}(3x + 11)(x + 4) = 590$

$\Rightarrow (3x + 11)(x + 4) = 1180$

$\Rightarrow 3x^2 + 12x + 11x + 44 - 1180 = 0$

$\Rightarrow 3x^2 + 23x - 1136 = 0$

$\Rightarrow 3x^2 + 71x - 48x - 1136 = 0$

$\begin{Bmatrix} \because -1136 \times 3 = -3408 \\ \therefore \quad 23 = 71 - 48 \\ -3408 = 71 \times (-48) \end{Bmatrix}$

$\Rightarrow x(3x + 71) - 16(3x + 71) = 0$

$\Rightarrow (3x + 71)(x - 16) = 0$

[Zero Product Rule]

Either $3x + 71 = 0$, then $3x = -71$

$\Rightarrow \quad x = -\dfrac{71}{3}$

Which is not possible.

or $x - 16 = 0$, then $x = 16$ **Ans.**

Q. 31. A man buys an article for Rs. x and sells it for Rs. 56 at a gain of x%. Find the value of x.

Sol. C.P. of article = Rs. x

Gain = x%

S.P. = Rs. 56

According to the condition,

$$SP = \frac{CP(100 + gain\%)}{100}$$

$$\Rightarrow 56 = \frac{x(100 + x)}{100}$$

$\Rightarrow x(100 + x) = 5600$

$\Rightarrow 100x + x^2 - 5600 = 0$

$\Rightarrow x^2 + 100x - 5600 = 0$

$\Rightarrow x^2 + 140x - 40x - 5600 = 0$

$$\begin{cases} \because 100 = 140 - 40 \\ -5600 = 140 \times (-40) \end{cases}$$

$\Rightarrow x(x + 140) - 40(x + 140) = 0$

$\Rightarrow (x + 140)(x - 40) = 0$

(Zero Product Rule)

Either $x + 140 = 0$, then $x = -140$, which is not possible.

or $x - 40 = 0$,

then $x = 40$ **Ans.**

Q. 32. Rohit is on tour and he has Rs. 360 for his expenses. If he exceeds his tour by 4 days, he must cut down his daily expenses by Rs. 3. For how many days Rohit is on tour ?

Sol. Let the number of days of tour programme = x

Amount = Rs. 360

$\therefore$ Expenses for each day = $\dfrac{360}{x}$

In second case, no. of days = $x + 4$

Then, expenses of each day = $\dfrac{360}{x + 4}$

Now, according to the condition,

$$\frac{360}{x} - \frac{360}{x + 4} = 3$$

$$\Rightarrow 360 \left[\frac{1}{x} - \frac{1}{x + 4} \right] = 3$$

$$\Rightarrow 360 \frac{(x + 4 - x)}{x^2 + 4x} = 3$$

$\Rightarrow 360 \times 4 = 3x^2 + 12x$

$\Rightarrow 1440 = 3x^2 + 12x$

$\Rightarrow 3x^2 + 12x - 1440 = 0$

$\Rightarrow x^2 + 4x - 480 = 0$

(Dividing by 3)

$\Rightarrow x^2 + 24x - 20x - 480 = 0$

$\Rightarrow x(x + 24) - 20(x + 24) = 0$

$\Rightarrow (x + 24)(x - 20) = 0$

Either, $x + 24 = 0$, then $x = -24$, but it is not possible as it is in negative.

or $x - 20 = 0$, then $x = 20$

$\therefore$ Rohit is on tour = 20 days.

Q. 33. Rs. 6400 were divided equally among x persons. Had this money been divided equally among $(x + 14)$ persons, each would have got Rs. 28 less. Find the value of x.

Sol. Total money = Rs. 6400

In first case, no. of persons = x

$\therefore$ Amount to each person = Rs. $\dfrac{6400}{x}$

In second case, no. of persons = $x + 14$

Amount to each person = Rs $\left(\dfrac{6400}{x + 14} \right)$

According to the condition,

$$\frac{6400}{x} - \frac{6400}{x + 14} = 28$$

$$\Rightarrow 6400 \left[\frac{1}{x} - \frac{1}{x + 14} \right] = 28$$

$$\Rightarrow 6400 \left[\frac{x + 14 - x}{x(x + 14)} \right] = 28$$

$$\Rightarrow \frac{6400 \times 14}{x(x + 14)} = \frac{28}{1}$$

$\Rightarrow 28x(x + 14) = 89600$

$$\Rightarrow x(x + 14) = \frac{89600}{28} = 3200$$

$\Rightarrow x^2 + 14x - 3200 = 0$

$\Rightarrow x^2 + 64x - 50x - 3200 = 0$

$$\begin{cases} \because 14 = 64 - 50 \\ -3200 = 64 \times (-50) \end{cases}$$

$\Rightarrow x(x + 64) - 50(x + 64) = 0$

$\Rightarrow (x + 64)(x - 50) = 0$

[Zero Product Rule)

Either $x + 64 = 0$, then $x = -64$. But it is not possible.

or $x - 50 = 0$, then $x = 50$

∴ $x = 50$ **Ans.**

34. Some students planned a picnic. The budget for the food was Rs. 480. As eight of them failed to join the party, the cost of the food for each member increased by Rs. 10. Find how many students went for the picnic. **(2008)**

Sol. Let total number of students = x

and total budget for food = Rs. 480

∴ Each slope = Rs. $\dfrac{480}{x}$

∵ 8 students did not join then

∴ Remaining students = $x - 8$

∴ Each share = Rs. $\dfrac{480}{x-8}$

According to the condition,

$\dfrac{480}{x-8} - \dfrac{480}{x} = 10$

$\Rightarrow 480\left(\dfrac{1}{x-8} - \dfrac{1}{x}\right) = 10 \Rightarrow 480\left[\dfrac{x-x+8}{x(x-8)}\right] = 10$

$\Rightarrow \dfrac{480 \times 8}{x(x-8)} = 10 \Rightarrow 3840 = 10 \times x\,(x-8)$

$\Rightarrow 3840 = 10x^2 - 80x$

$\Rightarrow 10x^2 - 80x - 3840 = 0$ (Dividing by 10)

$x^2 - 8x - 384 = 0$

$x^2 - 24x + 16x - 384 = 0$

$[\because -24 \times 16 = -384 - 24 + 16 = -8]$

$\Rightarrow x\,(x - 24) + 16\,(x - 24) = 0$

$\Rightarrow (x - 24)\,(x + 16) = 0$

Either $x - 24 = 0$, then $x = 24$

or $x + 16 = 0$, then $x = -16$ But it is not possible

∴ Total number of students = 24

and number of students who went for picnic
= 24 − 8 = 16 **Ans.**

Q. 35. A shopkeeper buys x books for Rs. 720.

(i) Write the cost of 1 book in terms of x.

(ii) If the cost per book were Rs. 5 less, the number of books that could be bought for Rs. 720 would be 2 more. Write down the equation in x for the above situation and solve it to find x. **(2006)**

Sol. (i) Total amount = Rs. 720

No. of books = x

∴ Price of each book = Rs. $\dfrac{720}{x}$

(ii) In second case,

No. of books = $x + 2$

∴ Price of each book = Rs. $\dfrac{720}{x+2}$

According to the condition,

$\dfrac{720}{x} - \dfrac{720}{x+2} = 5$

∴ $720\left(\dfrac{1}{x} - \dfrac{1}{x+2}\right) = 5$

$\Rightarrow 720\left(\dfrac{x+2-x}{x(x+2)}\right) = 5$

$\Rightarrow \dfrac{720 \times 2}{x^2 + 2x} = 5$

$\Rightarrow 5\,(x^2 + 2x) = 720 \times 2$

$\Rightarrow x^2 + 2x = \dfrac{720 \times 2}{5}$

$\Rightarrow x^2 + 2x = 144 \times 2$

$\Rightarrow x^2 + 2x = 288$

$\Rightarrow x^2 + 2x - 288 = 0$

Hence, equation is $x^2 + 2x - 288 = 0$

$\Rightarrow x^2 + 18x - 16x - 288 = 0$

$\left\{\begin{array}{l}\because 2 = 18 - 16 \\ -288 = 18 \times (-16)\end{array}\right\}$

$\Rightarrow x\,(x + 18) - 16(x + 18) = 0$

$\Rightarrow (x + 18)\,(x - 16) = 0$

[Zero Product Rule]

Either $x + 18 = 0$, then $x = -18$, which is not possible.

or $x - 16 = 0$, then $x = 16$ **Ans.**

Q. 36. A piece of cloth costs Rs. 35. If the length of the piece would have been 4 m longer and each metre costs Re. 1 less, the cost would have remained unchanged. How long is the piece ?

Sol. Let the length of cloth $= x$ m

$\therefore$ Cost of 1 m $=$ Rs. $\dfrac{35}{x}$;

Cost of 1 m in second case $=$ Rs. $\left(\dfrac{35}{x} - 1\right)$

According to the question,

$$(x+4)\left(\dfrac{35}{x} - 1\right) = 35$$

$$\Rightarrow \dfrac{(x+4)(35-x)}{x} = \dfrac{35}{1}$$

$$\Rightarrow (x+4)(35-x) = 35\,x$$

$$\Rightarrow 35\,x - x^2 + 140 - 4\,x = 35\,x$$

$$\Rightarrow -x^2 + 35\,x - 4\,x - 35\,x + 140 = 0$$

$$\Rightarrow -x^2 - 4\,x + 140 = 0$$

$$\Rightarrow x^2 + 4\,x - 140 = 0$$

$$\Rightarrow x^2 + 14\,x - 10\,x - 140 = 0$$

$$\Rightarrow x(x+14) - 10(x+14) = 0$$

$$\Rightarrow (x+14)(x-10) = 0$$

Either $x + 14 = 0$, then $x = -14$

But, it can not be possible.

or $x - 10 = 0$, then $x = 10$

$\therefore$ 10 m long is the piece of cloth. **Ans.**

Q. 37. A fruit-seller bought x apples for Rs. 1200.

(i) Write the cost price of each apple in terms of x.

(ii) If 10 of the apples were rotten and he sold each of the rest at Rs. 3 more than the cost price of each, write the selling price of $(x - 10)$ apples.

(iii) If he made a profit of Rs. 60 in this transaction, form an equation in x and solve it to evaluate x.

Sol. (i) Total amount $=$ Rs. 1200

No. of apples $= x$

$\therefore$ Cost price of each apple $=$ Rs. $\dfrac{1200}{x}$

(ii) No. of rotten apples $= 10$

$\therefore$ Balance apples $= x - 10$

$\therefore$ Selling price of $(x - 10)$ apples

$= $ Rs. $(x - 10)\left(\dfrac{1200}{x} + 3\right)$

(iii) Profit $=$ Rs. 60

But S.P. $-$ C.P. $=$ Profit

$$\Rightarrow (x-10)\left(\dfrac{1200}{x} + 3\right) - 1200 = 60$$

$$\Rightarrow 1200 + 3x - \dfrac{12000}{x} - 30 - 1200 = 60$$

$$\Rightarrow 3x - \dfrac{12000}{x} - 30 - 60 = 0$$

$$\Rightarrow 3x - \dfrac{12000}{x} - 90 = 0$$

$$\Rightarrow 3x^2 - 12000 - 90x = 0$$

$$\Rightarrow 3x^2 - 90x - 12000 = 0$$

$$\Rightarrow x^2 - 30x - 4000 = 0$$

                    [Dividing by 3]

$$\Rightarrow x^2 - 80x + 50x - 4000 = 0$$

$$\left\{\begin{matrix}\because -30 = -80 + 50 \\ -4000 = -80 \times 50\end{matrix}\right\}$$

$$\Rightarrow x(x - 80) + 50(x - 80) = 0$$

$$\Rightarrow (x - 80)(x + 50) = 0$$

               [Zero Product Rule]

Either $x - 80 = 0$, then $x = 80$

or $x + 50 = 0$, then $x = -50$, which is not possible.

Hence $x = 80$ **Ans.**

Q. 38. Vibha and Sanya distribute Rs. 100 each in charity. Vibha distributes money to 5 more people than Sanya and Sanya gives

each Re. 1 more than Vibha. How many people are recipients of the charity ?

Sol. Amount of charity of each = Rs. 100

Let no. of recipients in case of Sanya = x

$\therefore$ no. of recipients in case of Vibha

$$= x + 5$$

Now, according to the condition,

$$\frac{100}{x} - \frac{100}{x + 5} = 1$$

$$\Rightarrow 100 \left(\frac{1}{x} - \frac{1}{x + 5} \right) = 1$$

$$\Rightarrow 100 \frac{(x + 5 - x)}{x(x + 5)} = 1$$

$$\Rightarrow 500 = x^2 + 5x$$

$$\Rightarrow x^2 + 5x - 500 = 0$$

$$\Rightarrow x^2 + 25x - 20x - 500 = 0$$

$$\Rightarrow x(x + 25) - 20(x + 25) = 0$$

$$\Rightarrow (x + 25)(x - 20) = 0 \qquad \text{(Zero Product Rule)}$$

Either $x + 25 = 0$, then $x = -25$

which is not possible.

or $x - 20 = 0$, then $x = 20$

Hence, total number of recipients

$$= x + x + 5 = 20 + 20 + 5 = 45 \text{ Ans.}$$