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Chapter 6

Quadratic Equations

POINTS TO REMEMBER

1. **Quadratic Equation :** An equation of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, is called a **quadratic equation**.
2. **Roots of Quadratic Equation :** A number say α is called a root of the equation $ax^2 + bx + c = 0$ if $a\alpha^2 + b\alpha + c = 0$. Value of α will satisfy the given quadratic equation.
3. **Zero Product Rule :** Let a and b be the two real numbers, then $ab = 0 \Rightarrow a = 0$ or $b = 0$ or both equal to zero. This is called zero product rule.
4. **Method for solving the quadratic equation : (A) By Factorization :**
   (*i*) Make the given equation free from fractions and radicals if any and then put it into the standard form $ax^2 + bx + c = 0$.
   (*ii*) Now factorise $ax^2 + bx + c$ into two linear factors.
   (*iii*) Put each factor equal to zero (by zero product rule)
   (*iv*) Now solve these linear equations, then we shall get two roots of the given quadratic equation.

(B) By Quadratic formula :

Let α and β be the roots of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, then

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and}$$

$$\beta = \frac{-b-\sqrt{a^2-4ac}}{2a}$$

Proof. $ax^2 + bx + c = 0$

$\Rightarrow 4a^2x^2 + 4abx + 4ac = 0$ (Multiplying by $4a$)

$\Rightarrow 4a^2x^2 + 4abx = -4ac$

Adding b^2 both sides, we get

$$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$$

$\Rightarrow (2ax)^2 + 2 \times 2ax.b + (b)^2 = b^2 - 4ac$

$\Rightarrow (2ax + b)^2 = b^2 - 4ac$

$\Rightarrow 2ax + b = \pm\sqrt{b^2-4ac}$ (Taking square root on both sides)

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

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If α and β be the roots of the given quadratic equation, then

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Note. $b^2 - 4ac$ is called the determinant of the quadratic equation $ax^2 + bx + c = 0$ and is denoted by D.

∴ $D = b^2 - 4ac$

5. Some different forms of the quadratic equation

(*i*) Equations of the form :

$\sqrt{ax+b} = (cx + d)$

We can find their solutions if $ax + b \geq 0$ and $cx + d \geq 0$.

Note. (*i*) We shall solve it first by squaring and then writing in the standard form.

(*ii*) We shall take only positive value of the square root.

(*ii*) Equations of the form

$\sqrt{ax^2 + bx + c} = (dx + c)$

We can find their solutions if $ax^2 + bx + c \geq 0$

Note. (*i*) We shall solve it first by squaring it and then write it in the standard form.

(*ii*) We shall take only positive value of square root.

EXERCISE 6 (A)

Q. 1. Find which of the following are the solutions of the equation $6x^2 - x - 2 = 0$?

(*i*) $\frac{1}{2}$ (*ii*) $-\frac{1}{2}$ (*iii*) $\frac{2}{3}$

Sol. Given equation is $6x^2 - x - 2 = 0$

(*i*) If $x = \frac{1}{2}$ is its solution, then it will satisfy the equation. Now, substituting the value of $x = \frac{1}{2}$ is the given equation.

Substituting $x = \frac{1}{2}$

$$= 6\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 2$$

$$= 6 \times \frac{1}{4} - \frac{1}{2} - 2$$

$$= \frac{3}{2} - \frac{1}{2} - 2 = \frac{3-1-4}{2} = \frac{-2}{2} = -1 \neq 0$$

∴ $x = \frac{1}{2}$ is not a root of the equation

$6x^2 - x - 2 = 0$

(*ii*) Given equation is $6x^2 - x - 2 = 0$

Substituting $x = -\frac{1}{2}$ in $6x^2 - x - 2 = 0$

$$6\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 2 = 0$$

$$6 \times \frac{1}{4} + \frac{1}{2} - 2 = 0$$

$$\frac{3}{2} + \frac{1}{2} - 2 = 0$$

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$$\frac{3+1-4}{2}=0$$

$$\frac{0}{2}=0 \Rightarrow 0=0$$

LHS = RHS

$\therefore x=-\frac{1}{2}$ is a root of equation

$6x^2-x-2=0$

(*iii*) $6x^2-x-2=0$

Substituting $x=\frac{2}{3}$ in equation

$$6x^2-x-2=0$$

$$6\left(\frac{2}{3}\right)^2-\left(\frac{2}{3}\right)-2=0$$

$$6\times\frac{4}{9}-\frac{2}{3}-2=0$$

$$\frac{8}{3}-\frac{2}{3}-2=0$$

$$\frac{8-2-6}{3}=0$$

$$\frac{8-8}{3}=0 \Rightarrow 0=0$$

LHS = RHS

$\therefore x=\frac{2}{3}$ is a root of equation

$6x^2-x-2=0$

Q. 2. Determine whether $x=\frac{-1}{3}$ and $x=\frac{2}{3}$ are the solutions of the equation $9x^2-3x-2=0$.

Sol. Given equation is $9x^2-3x-2=0$

(*i*) If $x=\frac{-1}{3}$ is its solution, then it will satisfy the equation. Now substituting the value of $x=\frac{-1}{3}$ is the given equation.

$$=9\left(-\frac{1}{3}\right)^2-3\left(-\frac{1}{3}\right)-2$$

$$=9\times\frac{1}{9}+3\times\frac{1}{3}-2$$

$$=\frac{9}{9}+\frac{3}{3}-2=\frac{9+9}{9}-2$$

$$=\frac{18}{9}-2=2-2=0$$

Hence, $x=\frac{-1}{3}$ is its solution.

(*ii*) Again, substituting the $x=\frac{2}{3}$ in the equation

$$9\left(\frac{2}{3}\right)^2-3\left(\frac{2}{3}\right)-2$$

$$=9\times\frac{4}{9}-3\times\frac{2}{3}-2$$

$$=\frac{36}{9}-\frac{6}{3}-2=\frac{36-18}{9}-2$$

$$=\frac{18}{9}-2=2-2=0$$

Hence, $x=\frac{2}{3}$ is also its solution.

$\therefore x=\frac{-1}{3}$ and $x=\frac{2}{3}$ are the solutions of the equation.

$9x^2-3x-2=0$ **Ans.**

Solve the following equations by factorisation :

Q. 3. $16x^2=25$

Sol. $16x^2=25$

$\Rightarrow 16x^2-25=0 \Rightarrow (4x)^2-(5)^2=0$

$\{\because a^2-b^2=(a+b)(a-b)\}$

$\Rightarrow (4x+5)(4x-5)=0$

[Zero Product Rule]

Either $4x+5=0$,

then $4x=-5 \Rightarrow x=-\frac{5}{4}$

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or $4x - 5 = 0$,

then $4x = 5 \Rightarrow x = \frac{5}{4}$

Hence, solution set is $\left\{\frac{5}{4}, -\frac{5}{4}\right\}$ **Ans.**

Q. 4. $x^2 + 2x = 24$

Sol. $x^2 + 2x = 24$

$\Rightarrow x^2 + 2x - 24 = 0$

$\Rightarrow x^2 + 6x - 4x - 24 = 0$

$\left\{\begin{matrix} \because 2 = 6 - 4 \\ -24 = 6 \times (-4) \end{matrix}\right\}$

$\Rightarrow x(x + 6) - 4(x + 6) = 0$

$\Rightarrow (x + 6)(x - 4) = 0$

[Zero Product Rule]

Either $x + 6 = 0$, then $x = -6$

or $x - 4 = 0$, then $x = 4$

Hence, solution set is $\{-6, 4\}$ **Ans.**

Q. 5. $x^2 - x = 156$

Sol. $x^2 - x = 156$

$\Rightarrow x^2 - x - 156 = 0$

$\Rightarrow x^2 - 13x + 12x - 156 = 0$

$\left\{\begin{matrix} \because -1 = -13 + 12 \\ -156 = -13 \times 12 \end{matrix}\right\}$

$\Rightarrow x(x - 13) + 12(x - 13) = 0$

$\Rightarrow (x - 13)(x + 12) = 0$

[Zero Product Rule]

Either $x - 13 = 0$, then $x = 13$

or $x + 12 = 0$, then $x = -12$

Hence, solution set is $\{13, -12\}$ **Ans.**

Q. 6. $x^2 - 11x = 42$

Sol. $x^2 - 11x = 42$

$\Rightarrow x^2 - 11x - 42 = 0$

$\Rightarrow x^2 - 14x + 3x - 42 = 0$

$\left\{\begin{matrix} \because -11 = -14 + 3 \\ -42 = -14 \times 3 \end{matrix}\right\}$

$\Rightarrow x(x - 14) + 3(x - 14) = 0$

$\Rightarrow (x - 14)(x + 3) = 0$

[Zero Product Rule]

Either $x - 14 = 0$, then $x = 14$

or $x + 3 = 0$, then $x = -3$

Hence, solution set is $\{14, -3\}$ **Ans.**

Q. 7. $x^2 - 7x + 10 = 0$

Sol. $x^2 - 7x + 10 = 0$

$\Rightarrow x^2 - 5x - 2x + 10 = 0$

$\left\{\begin{matrix} \because -7 = -5 - 2 \\ 10 = (-5) \times (-2) \end{matrix}\right\}$

$\Rightarrow x(x - 5) - 2(x - 5) = 0$

$\Rightarrow (x - 5)(x - 2) = 0$

(Zero Product Rule)

Either $x - 5 = 0$, then $x = 5$

or $x - 2 = 0$, then $x = 2$

Hence, solution set is $\{5, 2\}$ **Ans.**

Q. 8. $x^2 + 18x = 40$

Sol. $x^2 + 18x = 40$

$\Rightarrow x^2 + 18x - 40 = 0$

$\Rightarrow x^2 + 20x - 2x - 40 = 0$

$\left\{\begin{matrix} \because 18 = 20 - 2 \\ -40 = 20 \times (-2) \end{matrix}\right\}$

$\Rightarrow x(x + 20) - 2(x + 20) = 0$

$\Rightarrow (x + 20)(x - 2) = 0$

[Zero Product Rule]

Either $x + 20 = 0$, then $x = -20$

or $x - 2 = 0$, then $x = 2$.

Hence, solution set is $\{-20, 2\}$ **Ans.**

Q. 9. $x^2 + 17 = 18x$

Sol. $x^2 + 17 = 18x$

$\Rightarrow x^2 - 18x + 17 = 0$

$\Rightarrow x^2 - x - 17x + 17 = 0$

$\left\{\begin{matrix} \because -18 = -1 - 17 \\ 17 = (-1)(-17) \end{matrix}\right\}$

$\Rightarrow x(x - 1) - 17(x - 1) = 0$

$\Rightarrow (x - 1)(x - 17) = 0.$

[Zero Product Rule]

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Either $x - 1 = 0$, then $x = 1$

or $x - 17 = 0$, then $x = 17$

Hence, solution set is $\{1, 17\}$ **Ans.**

Q. 10. $3x^2 = 5x$

Sol. $3x^2 = 5x$

$\Rightarrow 3x^2 - 5x = 0$

$\Rightarrow x(3x - 5) = 0$ [Zero Product Rule]

Either $x = 0$

or $3x - 5 = 0$

then $3x = 5 \Rightarrow x = \frac{5}{3}$

Hence, solution set is $\left[0, \frac{5}{3}\right]$ **Ans.**

Q. 11. $(x+3)(x-3) = 27$

Sol. $(x+3)(x-3) = 27$

$\Rightarrow x^2 - 9 = 27 \Rightarrow x^2 = 27 + 9$

$\Rightarrow x^2 = 36$

$\Rightarrow x^2 - 36 = 0$

$\Rightarrow \{(x)^2 - (6)^2\} = 0$

$\Rightarrow (x+6)(x-6) = 0$

$\{\because a^2 - b^2 = (a+b)(a-b)\}$

[Zero Product Rule]

Either $x + 6 = 0$, then $x = -6$

or $x - 6 = 0$, then $x = 6$

Hence, solution set is $\{6, -6\}$ **Ans.**

Q. 12. $x^2 - 30x + 216 = 0$

Sol. $x^2 - 30x + 216 = 0$

$\Rightarrow x^2 - 18x - 12x + 216 = 0$

$\Rightarrow x(x - 18) - 12(x - 18) = 0$

$\left\{\begin{matrix}\because -30 = -18 - 12 \\ 216 = (-18)(-12)\end{matrix}\right\}$

$\Rightarrow (x - 18)(x - 12) = 0$

[Zero Product Rule]

Either $x - 18 = 0$, then $x = 18$

or $x - 12 = 0$, then $x = 12$

Hence, solution set is $\{12, 18\}$ **Ans.**

Q. 13. $12x^2 + 29x + 14 = 0$

Sol. $12x^2 + 29x + 14 = 0$ $\left\{\begin{matrix}\because 12 \times 14 = 168 \\ \therefore 29 = 21 + 8 \\ 168 = 21 \times 8\end{matrix}\right\}$

$\Rightarrow 12x^2 + 21x + 8x + 14 = 0$

$\Rightarrow 3x(4x + 7) + 2(4x + 7) = 0$

$\Rightarrow (4x + 7)(3x + 2) = 0.$

[Zero Product Rule]

Either $4x + 7 = 0$, then $4x = -7$

$\Rightarrow x = -\frac{7}{4}$

or $3x + 2 = 0$,

then $3x = -2 \Rightarrow x = -\frac{2}{3}$

Hence, solution set is $\left\{-\frac{7}{4}, -\frac{2}{3}\right\}$ **Ans.**

Q. 14. $2x^2 - 7x = 39$

Sol. $2x^2 - 7x = 39$

$\Rightarrow 2x^2 - 7x - 39 = 0$

$\left\{\begin{matrix}\because 2 \times (-39) = -78 \\ \therefore -7 = -13 + 6 \\ 78 = -13 \times 6\end{matrix}\right\}$

$\Rightarrow 2x^2 - 13x + 6x - 39 = 0$

$\Rightarrow x(2x - 13) + 3(2x - 13) = 0$

$\Rightarrow (2x - 13)(x + 3) = 0$

[Zero Product Rule]

Either $2x - 13 = 0$, then $2x = 13$

$\Rightarrow x = \frac{13}{2}$

or $x + 3 = 0$, then $x = -3$

Hence, solution set is $\left\{\frac{13}{2}, -3\right\}$ **Ans.**

Q. 15. $10x^2 = 9x + 7$

Sol. $10x^2 = 9x + 7$

$\Rightarrow 10x^2 - 9x - 7 = 0$ $\left\{\begin{matrix}\because 10 \times (-7) = -70 \\ \therefore -9 = -14 + 5 \\ -70 = -14 \times 5\end{matrix}\right\}$

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$\Rightarrow \quad 10x^2 - 14x + 5x - 7 = 0$

$\Rightarrow 2x(5x-7) + 1(5x-7) = 0$

$\Rightarrow \quad (5x-7)(2x+1) = 0$

[Zero Product Rule]

Either $5x - 7 = 0$, then $5x = 7$

$\Rightarrow \quad x = \frac{7}{5}$

or $\quad 2x + 1 = 0$,

then $\quad 2x = -1 \quad \Rightarrow \quad x = -\frac{1}{2}$

Hence, solution set is $\left\{\frac{7}{5}, -\frac{1}{2}\right\}$ **Ans.**

Q. 16. $15x^2 - 28 = x$

Sol. $15x^2 - x - 28 = 0$

$\Rightarrow 15x^2 - 21x + 20x - 28 = 0$

$$\left\{\begin{array}{l}\because 15\times(-28) = -420\\ \therefore -1 = -21+20\\ -420 = -21\times 20\end{array}\right\}$$

$\Rightarrow 3x(5x-7) + 4(5x-7) = 0$

$\Rightarrow \quad (5x-7)(3x+4) = 0$

[Zero Product Rule]

Either $5x - 7 = 0$, then $5x = 7$

$\Rightarrow \quad x = \frac{7}{5}$

or $\quad 3x + 4 = 0$,

then $\quad 3x = -4 \quad \Rightarrow \quad x = \frac{-4}{3}$

Hence, solution set is $\left\{\frac{7}{5}, \frac{-4}{3}\right\}$ **Ans.**

Q. 17. $8x^2 + 15 = 26x$

Sol. $8x^2 + 15 = 26x$

$\Rightarrow \quad 8x^2 - 26x + 15 = 0$

$\Rightarrow 8x^2 - 6x - 20x + 15 = 0$

$$\left\{\begin{array}{l}\because 8\times 15 = 120\\ \therefore -26 = -6-20\\ 120 = (-6)(-20)\end{array}\right\}$$

$\Rightarrow 2x(4x-3) - 5(4x-3) = 0$

$\Rightarrow \quad (4x-3)(2x-5) = 0$

[Zero Product Rule]

Either $4x - 3 = 0$, then $4x = 3$

$\Rightarrow \quad x = \frac{3}{4}$

or $\quad 2x - 5 = 0$,

then $\quad 2x = 5 \quad \Rightarrow \quad x = \frac{5}{2}$

Hence, solution set is $\left\{\frac{3}{4}, \frac{5}{2}\right\}$ **Ans.**

Q. 18. $3x^2 + 8 = 10x$

Sol. $3x^2 + 8 = 10x$

$\Rightarrow \quad 3x^2 - 10x + 8 = 0$

$\Rightarrow 3x^2 - 4x - 6x + 8 = 0$

$$\left\{\begin{array}{l}\because 3\times 8 = 24\\ \therefore -10 = -4-6\\ 24 = (-4)(-6)\end{array}\right\}$$

$\Rightarrow x(3x-4) - 2(3x-4) = 0$

$\Rightarrow \quad (3x-4)(x-2) = 0$

[Zero Product Rule]

Either $3x - 4 = 0$, then $3x = 4$

$\Rightarrow \quad x = \frac{4}{3}$

or $\quad x - 2 = 0$, then $x = 2$

Hence, solution set is $\left\{2, \frac{4}{3}\right\}$ **Ans.**

Q. 19. $x(6x - 11) = 35$

Sol. $x(6x - 11) = 35$

$\Rightarrow \quad 6x^2 - 11x - 35 = 0$

$\Rightarrow \quad 6x^2 - 21x + 10x - 35 = 0$

$$\left\{\begin{array}{l}\because 6\times(-35) = -210\\ \therefore -11 = -21+10\\ -210 = -21\times 10\end{array}\right\}$$

$\Rightarrow 3x(2x-7) + 5(2x-7) = 0$

$\Rightarrow \quad (2x-7)(3x+5) = 0$

(Zero Product Rule)

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Either $2x - 7 = 0$, then $2x = 7$

$\Rightarrow \quad x = \frac{7}{2}$

or $\quad 3x + 5 = 0$,

then $\quad 3x = -5 \Rightarrow x = -\frac{5}{3}$

Hence, solution set is $\left\{\frac{7}{2}, -\frac{5}{3}\right\}$ **Ans.**

Q. 20. $6x(3x - 7) = 7(7 - 3x)$

Sol. $6x(3x - 7) = 7(7 - 3x)$

$\Rightarrow 18x^2 - 42x = 49 - 21x$

$\Rightarrow 18x^2 - 42x - 49 + 21x = 0$

$\Rightarrow 18x^2 - 21x - 49 = 0$

$\Rightarrow 18x^2 - 42x + 21x - 49 = 0$

$$\left[\begin{array}{l} \because 18\times(-49) = -882 \\ \therefore -21 = -42 + 21 \\ -882 = -42\times 21 \end{array}\right]$$

$\Rightarrow 6x(3x - 7) + 7(3x - 7) = 0$

$\Rightarrow \quad (3x - 7)(6x + 7) = 0$

[Zero Product Rule]

Either $3x - 7 = 0$, then $3x = 7$

$\Rightarrow \quad x = \frac{7}{3}$

or $\quad 6x + 7 = 0$,

then $\quad 6x = -7 \Rightarrow x = \frac{-7}{6}$

Hence, solution set is $\left\{\frac{7}{3}, \frac{-7}{6}\right\}$ **Ans.**

Q. 21. $2x^2 - 9x + 10 = 0$, when

(*i*) $x \in N$ $\quad$ (*ii*) $x \in Q$.

Sol. $2x^2 - 9x + 10 = 0$

$\Rightarrow 2x^2 - 5x - 4x + 10 = 0$

$$\left[\begin{array}{l} \because 2\times 10 = 20 \\ \therefore -9 = -5 - 4 = 20 \\ 20 = -5\times(-4) \end{array}\right]$$

$\Rightarrow x(2x - 5) - 2(2x - 5) = 0$

$\Rightarrow \quad (2x - 5)(x - 2) = 0$

Either $\quad x - 2 = 0, 2x - 5 = 0$

[Zero Product Rule]

$\Rightarrow \quad x = 2 \quad$ or $\quad x = \frac{5}{2}$

(*i*) When $x \in N$:

As $2 \in N$ and $\frac{5}{2} \notin N$, so the solution set $= \{2\}$.

(*ii*) When $x \in Q$:

As $2 \in Q$ and $\frac{5}{2} \in Q$, so the solution set

$= \left\{2, \frac{5}{2}\right\}$ **Ans.**

Q. 22. $4x^2 - 9x - 100 = 0$, when $x \in Q$.

Sol. $4x^2 - 9x - 100 = 0$

$\Rightarrow 4x^2 - 25x + 16x - 100 = 0$

$$\left[\begin{array}{l} \because 4\times -100 = -400 \\ \therefore -9 = -25 + 16 \\ -400 = -25\times 16 \end{array}\right]$$

$\Rightarrow x(4x - 25) + 4(4x - 25) = 0$

$\Rightarrow (4x - 25)(x + 4) = 0$

[Zero Product Rule]

$\Rightarrow 4x - 25 \Rightarrow 4x = 25 \Rightarrow x = \frac{25}{4}$

or

$\Rightarrow \quad x + 4 \Rightarrow x = -4$

When $x \in Q$, then solution set is

$\left\{-4, \frac{25}{4}\right\}$ **Ans.**

Q. 23. $3x^2 + 11x + 10 = 0$, when $x \in I$

Sol. $3x^2 + 11x + 10 = 0$

$\Rightarrow 3x^2 + 6x + 5x + 10 = 0$

$$\left[\begin{array}{l} \because 3\times 10 = 30 \\ \therefore 11 = 6 + 5 \\ 30 = 6\times 5 \end{array}\right]$$

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$\Rightarrow 3x(x+2)+5(x+2)=0$

$\Rightarrow (x+2)(3x+5)=0$

[Zero Product Rule]

Either $x+2=0$, then $x=-2$

or $3x+5=0$, then $3x=-5$

$\Rightarrow x=-\frac{5}{3}$

When $x \in I$, then solution set is $\{-2\}$ **Ans.**

Q. 24. $x+\frac{1}{x}=3\frac{1}{3}$, $x \neq 0$

Sol. $x+\frac{1}{x}=3\frac{1}{3}$

$\Rightarrow x+\frac{1}{x}=\frac{10}{3} \Rightarrow \frac{x^2+1}{x}=\frac{10}{3}$

$\Rightarrow 3x^2+3=10x$

[By cross multiplication]

$\Rightarrow 3x^2-10x+3=0$

$\Rightarrow 3x^2-9x-x+3=0$

$\left\{\begin{array}{l}\because 3\times 3=9\\ \therefore -10=-9-1\\ 9=(-9)\times(-1)\end{array}\right\}$

$\Rightarrow 3x(x-3)-1(x-3)=0$

$\Rightarrow (x-3)(3x-1)=0$

[Zero Product Rule]

Either $x-3=0$, then $x=3$

or $3x-1=0$, then $3x=1$

$\Rightarrow x=\frac{1}{3}$

Hence, solution set is $\left\{3, \frac{1}{3}\right\}$ **Ans.**

Q. 25. $5x-\frac{35}{x}=18$

Sol. $5x-\frac{35}{x}=18$

$\Rightarrow \frac{5x^2-35}{x}=\frac{18}{1}$

$\Rightarrow 5x^2-35=18x$

[By cross multiplication]

$\Rightarrow 5x^2-18x-35=0$

$\left\{\begin{array}{l}\because 5\times(-35)=-175\\ \therefore -18=-25+7\\ -175=-25\times 7\end{array}\right\}$

$\Rightarrow 5x^2-25x+7x-35=0$

$\Rightarrow 5x(x-5)+7(x-5)=0$

$\Rightarrow (x-5)(5x+7)=0$

[Zero Product Rule]

Either $x-5=0$, then $x=5$

or $5x+7=0$, then $5x=-7$

$\Rightarrow x=\frac{-7}{5}$

Hence, solution set is $\left\{5, \frac{-7}{5}\right\}$ **Ans.**

Q. 26. $10x-\frac{1}{x}=3$

Sol. $10x-\frac{1}{x}=3 \quad \Rightarrow 10x^2-1=3x$

$\Rightarrow 10x^2-3x-1=0$

$\Rightarrow 10x^2-5x+2x-1=0$

$\Rightarrow 5x(2x-1)+1(2x-1)=0$

$(2x-1)(5x+1)=0,$

Either $2x-1=0$, then $x=\frac{1}{2}$

or $5x+1=0$, then $x=-\frac{1}{5}$

Hence $x=\frac{1}{2}, -\frac{1}{5}$ **Ans.**

$\therefore$ Solution set $=\left\{\frac{1}{2}, -\frac{1}{5}\right\}$.

Q. 27. $3a^2x^2+8abx+4b^2=0$, $a \neq 0$

Sol. $3a^2x^2+8abx+4b^2=0$

$\left\{\begin{array}{l}\because 3a^2\times 4b^2=12a^2b^2\\ \therefore 8ab=6ab+2ab\\ 12a^2b^2=6ab\times 2ab\end{array}\right\}$

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$\Rightarrow \quad 3a^2x^2 + 6abx + 2abx + 4b^2 = 0$

$\Rightarrow 3ax\,(ax + 2b) + 2b\,(ax + 2b) = 0$

$\Rightarrow \quad (ax + 2b)\,(3ax + 2b) = 0$

[Zero Product Rule]

Either $ax + 2b = 0$,

then $\quad ax = -\,2b$

$\Rightarrow \quad x = \frac{-2b}{a}$

or $\quad 3ax + 2b = 0$,

then $\quad 3ax = -\,2b$

$\Rightarrow \quad x = \frac{-2b}{3a}$

Hence, solution set is $\left\{\frac{-2b}{a}, \frac{-2b}{3a}\right\}$ **Ans.**

Q. 28. $4x^2 - 4ax + (a^2 - b^2) = 0$, where $a, b \in R$.

Ans. $4x^2 - 4ax + (a^2 - b^2) = 0$

$4x^2 - 2(a + b)x - 2(a - b)x + (a^2 - b^2) = 0$

$\Rightarrow 4x^2 - 2(a + b)x - 2(a - b)x + (a + b)(a - b) = 0$

$\Rightarrow 2x\,[2x - (a + b)] - (a - b)(2x - (a + b)) = 0$

$\Rightarrow (2x - (a - b))\,(2x - (a + b)) = 0$

Either $\quad 2x - (a - b) = 0$

$\Rightarrow \quad 2x = (a - b)$

$\Rightarrow \quad x = \frac{(a - b)}{2}$

or $\quad 2x - (a + b) = 0$

$\Rightarrow \quad 2x = (a + b)$

$\Rightarrow \quad x = \frac{(a + b)}{2}$

When $ab \in R$, then solution set is

$\left\{\frac{(a + b)}{2}, \frac{(a - b)}{2}\right\}$

Q. 29. $5x^2 - 12x - 9 = 0$,

When (*i*) $x \in$ **I** (*ii*) $x \in$ **Q.**

Sol. $\quad 5x^2 - 12x - 9 = 0$

$\Rightarrow 5x^2 - 15x + 3x - 9 = 0$

$\left\{\begin{array}{l} \because 5 \times (-9) = -45 \\ \therefore -12 = -15 + 3 \\ -45 = -15 \times 3 \end{array}\right\}$

$\Rightarrow 5x\,(x - 3) + 3\,(x - 3) = 0$

$\Rightarrow \quad (x - 3)\,(5x + 3) = 0$

[Zero Product Rule]

Either $x - 3 = 0$, then $x = 3$

or $\quad 5x + 3 = 0$, then $5x = -\,3$

$\Rightarrow \quad x = \frac{-3}{5}$

(*i*) When $x \in$ **I**, then solution set is $\{3\}$

(*ii*) When $x \in$ Q, then solution set is $\left\{3, \frac{-3}{5}\right\}$ **Ans.**

Q. 30. $2x^2 - 11x + 15 = 0$,

when (*i*) $x \in$ **N** (*ii*) $x \in$ **I**

Sol. $\quad 2x^2 - 11x + 15 = 0$

$\Rightarrow 2x^2 - 6x - 5x + 15 = 0$

$\left\{\begin{array}{l} \because 2 \times 15 = 30 \\ -11 = -6 + (-5) \\ 30 = (-6)(-5) \end{array}\right\}$

$\Rightarrow 2x\,(x - 3) - 5\,(x - 3) = 0$

$\Rightarrow \quad (x - 3)\,(2x - 5) = 0$

[Zero Product Rule]

Either $x - 3 = 0$, then $x = 3$

or $\quad 2x - 5 = 0$, then $2x = 5$

$\Rightarrow \quad x = \frac{5}{2}$

(*i*) When $x \in$ N, then solution set is $\{3\}$

(*ii*) When $x \in$ I, then solution set is $\{3\}$

$\{\because \frac{5}{2} \notin I\}$ **Ans.**

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Q. 31. $\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$

Sol. $\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x^2 + 9x + 2x + 6\sqrt{3} = 0$

$\left\{\begin{array}{l} \because \sqrt{3} \times 6\sqrt{3} = 18 \\ \therefore 11 = 9 + 2 \\ 18 = 9 \times 2 \end{array}\right\}$

$\Rightarrow \sqrt{3}x(x + 3\sqrt{3}) + 2(x + 3\sqrt{3}) = 0$

$\Rightarrow (x + 3\sqrt{3})(\sqrt{3}x + 2) = 0$

[Zero Product Rule]

Either $x + 3\sqrt{3} = 0$, then $x = -3\sqrt{3}$

or $\sqrt{3}x + 2 = 0$, then $\sqrt{3}x = -2$

$\Rightarrow x = -\dfrac{2}{\sqrt{3}}$

Hence, solution set is $\left\{-3\sqrt{3}, -\dfrac{2}{\sqrt{3}}\right\}$ **Ans.**

Q. 32. $2\sqrt{5}x^2 - 3x - \sqrt{5} = 0$

Sol. $2\sqrt{5}x^2 - 3x - \sqrt{5} = 0$

$\Rightarrow 2\sqrt{5}x^2 - 5x + 2x - \sqrt{5} = 0$

$\left\{\begin{array}{l} \because 2\sqrt{5} \times (-\sqrt{5}) = -10 \\ -3 = -5 + 2 \\ -10 = -5 \times 2 \end{array}\right\}$

$\Rightarrow \sqrt{5}x(2x - \sqrt{5}) + 1(2x - \sqrt{5}) = 0$

$\Rightarrow (2x - \sqrt{5})(\sqrt{5}x + 1) = 0$

[Zero Product Rule]

Either $2x - \sqrt{5} = 0$, then $2x = \sqrt{5}$

$\Rightarrow x = \dfrac{\sqrt{5}}{2}$

or $\sqrt{5}x + 1 = 0$, then $\sqrt{5}x = -1$

$\Rightarrow x = -\dfrac{1}{\sqrt{5}}$

Hence, solution set is $\left\{\dfrac{\sqrt{5}}{2}, \dfrac{-1}{\sqrt{5}}\right\}$ **Ans.**

Q. 33. $x^2 - (1 + \sqrt{2})x + \sqrt{2} = 0$

Sol. $x^2 - (1 + \sqrt{2})x + \sqrt{2} = 0$

$\Rightarrow x^2 - x - \sqrt{2}x + \sqrt{2} = 0$

$\Rightarrow x(x - 1) - \sqrt{2}(x - 1) = 0$

$\Rightarrow (x - 1)(x - \sqrt{2}) = 0$

[Zero Product Rule]

Either $x - 1 = 0$, then $x = 1$

or $x - \sqrt{2} = 0$, then $x = \sqrt{2}$

Hence, solution set is $\{1, \sqrt{2}\}$ **Ans.**

Q. 34. $\dfrac{x+1}{x-1} = \dfrac{3x-7}{2x-5}$

Sol. $\dfrac{x+1}{x-1} = \dfrac{3x-7}{2x-5}$

$\Rightarrow (x + 1)(2x - 5) = (3x - 7)(x - 1)$

[By Cross multiplication]

$\Rightarrow 2x^2 - 5x + 2x - 5$

$= 3x^2 - 3x - 7x + 7$

$\Rightarrow 2x^2 - 5x + 2x - 5 - 3x^2$

$+ 3x + 7x - 7 = 0$

$\Rightarrow -x^2 + 7x - 12 = 0$

$\Rightarrow x^2 - 7x + 12 = 0$

$\Rightarrow x^2 - 3x - 4x + 12 = 0$

$\left\{\begin{array}{l} \because -7 = -3 - 4 \\ 12 = (-3)(-4) \end{array}\right\}$

$\Rightarrow x(x - 3) - 4(x - 3) = 0$

$\Rightarrow (x - 3)(x - 4) = 0$

[Zero Product Rule]

Either $x - 3 = 0$, then $x = 3$

or $x - 4 = 0$, then $x = 4$

Hence, solution set is $\{3, 4\}$ **Ans.**

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Q. 35. $\frac{3x+1}{7x+1}=\frac{5x+1}{7x+5}$

Sol. $\frac{3x+1}{7x+1}=\frac{5x+1}{7x+5}$

$\Rightarrow (3x+1)(7x+5)=(5x+1)(7x+1)$ [By Cross Multiplication]

$\Rightarrow 21x^2+15x+7x+5 = 35x^2+5x+7x+1$

$\Rightarrow 21x^2+22x+5-35x^2-12x-1=0$

$\Rightarrow -14x^2+10x+4=0$

$\Rightarrow 7x^2-5x-2=0$ [Dividing by – 2]

$\Rightarrow 7x^2-7x+2x-2=0$

$\left\{\begin{array}{l}\because 7\times(-2)=-14\\ \therefore -5=-7+2\\ -14=-7\times 2\end{array}\right\}$

$\Rightarrow 7x(x-1)+2(x-1)=0$

$\Rightarrow (x-1)(7x+2)=0$ [Zero Product Rule]

Either $x-1=0$, then $x=1$

or $7x+2=0$, then $7x=-2$

$\Rightarrow x=\frac{-2}{7}$

Hence, solution set is $\left\{1,\frac{-2}{7}\right\}$ **Ans.**

Q. 36. $\frac{5}{(2x+1)}+\frac{6}{(x+1)}=3$

Sol. $\frac{5}{2x+1}+\frac{6}{x+1}=3$

$\frac{5(x+1)+6(2x+1)}{(2x+1)(x+1)}=3$

$\Rightarrow \frac{5x+5+12x+6}{2x^2+2x+x+1}=\frac{3}{1}$

$\Rightarrow \frac{17x+11}{2x^2+3x+1}=\frac{3}{1}$

$\Rightarrow (2x^2+3x+1)\times 3=(17x+11)\times 1$ [By Cross Multiplication]

$\Rightarrow 6x^2+9x+3=17x+11$

$\Rightarrow 6x^2+9x+3-17x-11=0$

$\Rightarrow 6x^2-8x-8=0$

$\Rightarrow 3x^2-4x-4=0$ (Dividing by 2)

$\Rightarrow 3x^2-6x+2x-4=0$

$\left\{\begin{array}{l}\because 3\times(-4)=-12\\ \therefore -4=-6+2\\ -12=-6\times 2\end{array}\right\}$

$\Rightarrow 3x(x-2)+2(x-2)=0$

$\Rightarrow (x-2)(3x+2)=0$ [Zero Product Rule]

Either $x-2=0$, then $x=2$

or $3x+2=0$, then $3x=-2$

$\Rightarrow x=-\frac{2}{3}$

Hence, solution set is $\left\{2,\frac{-2}{3}\right\}$ **Ans.**

Q. 37. $\frac{2x}{x-4}+\frac{2x-5}{x-3}=\frac{25}{3}$

Sol. $\frac{2x}{x-4}+\frac{2x-5}{x-3}=\frac{25}{3}$

$\frac{2x^2-6x+2x^2-8x-5x+20}{(x-4)(x-3)}=\frac{25}{3}$

$\Rightarrow \frac{4x^2-19x+20}{(x-4)(x-3)}=\frac{25}{3}$

$\Rightarrow 3(4x^2-19x+20) = 25(x-4)(x-3)=0$

$\Rightarrow 12x^2-57x+60 = 25x^2-75x-100x+300=0$

$\Rightarrow 12x^2-57x+60 = 25x^2-175x+300=0$

$\Rightarrow 12x^2-57x+60-25x^2 +175x-300=0$

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$\Rightarrow -13x^2 + 118x - 240 = 0$

$\Rightarrow 13x^2 - 118x + 240 = 0$

$\Rightarrow 13x^2 - 78x - 40x + 240 = 0$

$\Rightarrow 13x(x-6) - 40(x-6) = 0$

$\Rightarrow (x-6)(13x-40) = 0$

Either $x - 6 = 0$, then $x = 6$

or $13x - 40 = 0$, then $13x = 40 \Rightarrow x = \frac{40}{13}$

$\therefore$ Solution set $= \left\{6, \frac{40}{13}\right\}$ **Ans.**

Q. 38. $\frac{x+3}{x-2} - \frac{1-x}{x} = 4\frac{1}{4}$

Sol. $\frac{x+3}{x-2} - \frac{1-x}{x} = 4\frac{1}{4}$

$\Rightarrow \frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$

$\Rightarrow \frac{x(x+3) - (1-x)(x-2)}{x(x-2)} = \frac{17}{4}$

$\Rightarrow \frac{x^2 + 3x - x + 2 + x^2 - 2x}{x^2 - 2x} = \frac{17}{4}$

$\Rightarrow \frac{2x^2 + 2}{x^2 - 2x} = \frac{17}{4}$

$(2x^2 + 2) \times 4 = 17 \times (x^2 - 2x)$

[By Cross Multiplication]

$8x^2 + 8 = 17x^2 - 34x$

$8x^2 - 17x^2 + 8 + 34x = 0$

$\Rightarrow -9x^2 + 34x + 8 = 0$

$\Rightarrow 9x^2 - 34x - 8 = 0$ [Dividing by – 1]

$\Rightarrow 9x^2 - 36x + 2x - 8 = 0$

$\left\{\begin{array}{l} \because 9 \times (-8) = -72 \\ \therefore -34 = -36 + 2 \\ -72 = -36 \times 2 \end{array}\right\}$

$\Rightarrow 9x(x-4) + 2(x-4) = 0$

$\Rightarrow (x-4)(9x+2) = 0$

[Zero Product Rule]

Either $x - 4 = 0$, then $x = 4$

or $9x + 2 = 0$, then $9x = -2$

$\Rightarrow x = \frac{-2}{9}$

Hence, solution set is $\left\{4, \frac{-2}{9}\right\}$ **Ans.**

Q. 39. $\frac{1}{(x-2)} + \frac{2}{(x-1)} = \frac{6}{x}$

Sol. $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$

$\frac{1 \times (x-1) + 2(x-2)}{(x-2)(x-1)} = \frac{6}{x}$

$\Rightarrow \frac{x - 1 + 2x - 4}{x^2 - x - 2x + 2} = \frac{6}{x}$

$\Rightarrow \frac{3x-5}{x^2 - 3x + 2} = \frac{6}{x}$

$\Rightarrow x(3x-5) = 6(x^2 - 3x + 2)$

[By cross multiplication]

$\Rightarrow 3x^2 - 5x = 6x^2 - 18x + 12$

$\Rightarrow 3x^2 - 5x - 6x^2 + 18x - 12 = 0$

$\Rightarrow -3x^2 + 13x - 12 = 0$

$\Rightarrow 3x^2 - 13x + 12 = 0$

[Dividing by – 1]

$\Rightarrow 3x^2 - 9x - 4x + 12 = 0$

$\left\{\begin{array}{l} \because 3 \times 12 = 36 \\ \therefore -13 = -9 - 4 \\ 36 = (-9) \times (-4) \end{array}\right\}$

$\Rightarrow 3x(x-3) - 4(x-3) = 0$

$\Rightarrow (x-3)(3x-4) = 0$

[Zero Product Rule]

Either $x - 3 = 0$, then $x = 3$

or $3x - 4 = 0$, then $3x = 4$

$\Rightarrow x = \frac{4}{3}$

Hence, solution set is $\left\{3, \frac{4}{3}\right\}$ **Ans.**

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Q. 40. $x^4 - 10x^2 + 9 = 0.$

Sol. Putting $x^2 = y$, the given equation becomes $y^2 - 10y + 9 = 0.$

Now, $y^2 - 10y + 9 = 0$

$\Leftrightarrow \quad y^2 - 9y - y + 9 = 0$

$\Leftrightarrow \quad y(y-9) - 1(y-9) = 0$

$\Leftrightarrow \quad (y-9)(y-1) = 0$

$\Leftrightarrow \quad y - 1 = 0 \text{ or } y - 9 = 0$

either $y = 1$ or $y = 9$

Now, $y = 9 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

and $y = 1 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$\therefore$ Solution set is $\{1, -1, 3, -3\}$

Q. 41. $4y^4 - 33y^2 + 8 = 0$

Sol. Putting $y^2 = x$, the given equation becomes $4x^2 - 33x + 8 = 0$

Now, $4x^2 - 33x + 8 = 0$

$\Leftrightarrow \quad 4x^2 - 32x - x + 8 = 0$

$\Leftrightarrow \quad 4x(x-8) - 1(x-8) = 0$

$\Leftrightarrow \quad (x-8)(4x-1) = 0$

$\Leftrightarrow \quad x = 8 \text{ or } 4x = 1 = x = \frac{1}{4}$

Now, $x = 8 \Rightarrow y^2 = 8 = x = 2\sqrt{2}, -2\sqrt{2}$

and $x = \frac{1}{4} \Rightarrow y^2 = \frac{1}{4} = \frac{1}{2}, -\frac{1}{2}$

Q. 42. $x^{2/3} + x^{1/3} - 2 = 0$

Sol. Putting $x^{1/3} = y$, the given equation becomes :

$$y^2 + y - 2 = 0.$$

$\Leftrightarrow \quad y^2 + 2y - y - 2 = 0$

$\Leftrightarrow \quad y(y+2) - 1(y+2) = 0$

$\Leftrightarrow \quad (y+2)(y-1) = 0$

$\Leftrightarrow \quad y + 2 = 0 \text{ or } y - 1 = 0$

$\Leftrightarrow \quad y = -2 \text{ or } y = 1$

Now, $y = -2 \Leftrightarrow x^{1/3} = -2$

$\Rightarrow \quad x = -2^3 = -8$

and $y = 1 \Rightarrow x^{1/3} = 1 \Rightarrow x = (1)^3 = 1$

$\therefore$ Solution set is $\{-8, 1\}$ **Ans.**

Q. 43. $2\left(\frac{x}{x+1}\right)^2 - 5\left(\frac{x}{x+1}\right) + 2 = 0, \; x \neq -1$

Sol. Putting $\left(\frac{x}{x+1}\right) = y$ in the given equation, it becomes $2y^2 - 5y + 2 = 0.$

Now, $2y^2 - 5y + 2 = 0$

$\Leftrightarrow \quad 2y^2 - 4y - 1y + 2 = 0$

$\Leftrightarrow \quad 2y(y-2) - 1(y-2) = 0$

$\Leftrightarrow \quad (y-2)(2y-1) = 0$

$\Leftrightarrow \quad y - 2 = 0 \text{ or } 2y - 1 = 0$

$\Leftrightarrow \quad y = 2 \text{ or } y = \frac{1}{2}.$

Now, $y = 2 \Rightarrow \frac{x}{x+1} = 2$

$\Rightarrow \quad x = 2x + 2 \Rightarrow x = -2$

and $y = \frac{1}{2} = \frac{x}{x+1} = \frac{1}{2}$

$\Rightarrow \quad 2x = x + 1 \Rightarrow x = 1$

$\therefore$ Solution set $= \{-2, 1\}.$

Q. 44. $2^{(2x+3)} - 57 = 65(2^x - 1).$

Sol. The given equation can be written as

$$2^{2x}.2^3 - 57 = 65(2^x - 1)$$

$\Rightarrow \quad (2^x)^2.8 - 57 = 65(2^x) - 65 \quad \ldots(i)$

Putting $2^x = y$ in (i), we get :

$$8y^2 - 57 = 65y - 65$$

$\Rightarrow \quad 8y^2 - 65y - 57 + 65 = 0$

$\Rightarrow \quad 8y^2 - 65y + 8 = 0$

$\Rightarrow \quad 8y^2 - 64y - y + 8 = 0$

$\Rightarrow \quad 8y(y-8) - 1(y-8) = 0$

$\Rightarrow \quad (y-8)(8y-1) = 0$

(Factorising left side)

$\Rightarrow \quad y - 8 = 0 \text{ or } 8y - 1 = 0$

(Zero-product rule)

$\Rightarrow \quad y = 8 \text{ or } y = \frac{1}{8}$

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$\Rightarrow\ 2^x = 8$ or $2^x = \frac{1}{8}$ $(\because y = 2^x)$

$\Rightarrow\ 2^x = 2^3$ or $2^x = 2^{-3}$

$\Rightarrow\ x = 3$ or $x = -3$

Hence, the roots of the given equation are $\{3, -3\}$.

Q. 45. $2^{2x} - 3 \times 2^{x+2} + 32 = 0$

Sol. $2^{2x} - 3\,(2^x)\,2^2 + 32 = 0$

$\Rightarrow (2^x)^2 - 3 \times 4\,(2^x) + 32 = 0$

$\Rightarrow (2^x)^2 - 12\,(2^x) + 32 = 0$

Let $2^x = y$, then

$y^2 - 12\,y + 32 = 0$

$\Rightarrow y^2 - 8y - 4\,y + 32 = 0$

$\Rightarrow y\,(y - 8) - 4\,(y - 8) = 0$

$\Rightarrow (y - 8)\,(y - 4) = 0$

Either $y - 8 = 0$, then $y = 8$

or $y - 4 = 0$, then $y = 4$

(*i*) If $y = 8$, then $2^x = y = 8 = 2^3$

$\therefore x = 3$

(*ii*) and if $y = 4$, then $2^x = y = 4 = 2^2$

$\therefore x = 2$

Hence, $x = 2, 3$ **Ans.**

Q. 46. $\sqrt{x+15} = x+3$

Sol. $\sqrt{x+15} = x+3$

Squaring both sides,

$\left(\sqrt{x+15}\right)^2 = (x+3)^2$

$\Rightarrow\ x + 15 = x^2 + 2 \times x \times 3 + 9$

$\Rightarrow x + 15 = x^2 + 6x + 9$

$\Rightarrow x^2 + 6x + 9 - x - 15 = 0$

$\Rightarrow\ x^2 + 5x - 6 = 0$

$\Rightarrow\ x^2 + 6x - x - 6 = 0$ $\left\{\begin{array}{l}\because\ 5 = 6 - 1 \\ \therefore -6 = 6 \times (-1)\end{array}\right\}$

$\Rightarrow x\,(x + 6) - 1\,(x + 6) = 0$

$\Rightarrow\ (x + 6)\,(x - 1) = 0$

[Zero Product rule)

Either $x + 6 = 0$, then $x = -6$

or $x - 1 = 0$, then $x = 1$

Check (*i*) If $x = -6$, then

L.H.S. $= \sqrt{x+15} = \sqrt{-6+15} = \sqrt{9} = 3$

R.H.S. $= x + 3 = -6 + 3 = -3$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore x = -6$ is not its solution.

(*ii*) If $x = 1$, then

L.H.S. $= \sqrt{x+15} = \sqrt{1+15} = \sqrt{16} = 4$

R.H.S. $= x + 3 = 1 + 3 = 4$

$\therefore$ L.H.S. = R.H.S.

Hence, solution set is $\{1\}$ **Ans.**

Q. 47. $\sqrt{2x+9} = (13 - x)$

Sol. $\sqrt{2x+9} = (13 - x)$

Squaring both sides,

$\left(\sqrt{2x+9}\right)^2 = (13 - x)^2$

$\Rightarrow 2x + 9 = 169 - 2 \times 13 \times x + x^2$

$\Rightarrow 2x + 9 = 169 - 26x + x^2$

$\Rightarrow x^2 - 26x + 169 - 2x - 9 = 0$

$\Rightarrow\ x^2 - 28x + 160 = 0$

$\Rightarrow\ x^2 - 20x - 8x + 160 = 0$

$\left\{\begin{array}{l}\because -28 = -20 - 8 \\ 160 = (-20) \times (-8)\end{array}\right\}$

$\Rightarrow x\,(x - 20) - 8\,(x - 20) = 0$

$\Rightarrow\ (x - 20)\,(x - 8) = 0$

[Zero Product Rule]

Either $x - 20 = 0$, then $x = 20$

or $x - 8 = 0$, then $x = 8$

Check : (*i*) When $x = 20$, then

L.H.S. $= \sqrt{2x+9} = \sqrt{2 \times 20 + 9}$

$= \sqrt{40+9} = \sqrt{49} = 7$

R.H.S. $= 13 - x = 13 - 20 = -7$

$\therefore$ L.H.S. $\neq$ R.H.S.

$\therefore x = 20$ is not its solution.

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(*ii*) when $x = 8$, then

L.H.S. $= \sqrt{2x+9} = \sqrt{2\times8+9}$

$= \sqrt{16+9} = \sqrt{25} = 5$

R.H.S. $= 13 - x = 13 - 8 = 5$

$\because$ L.H.S. = R.H.S.

$\therefore$ $x = 8$ is its solution

Hence, solution set is $\{8\}$ **Ans.**

Q. 48. $\sqrt{3x^2 - 2} = (2x - 1)$

Sol. $\sqrt{3x^2 - 2} = (2x - 1)$

Squaring both sides,

$\left(\sqrt{3x^2 - 2}\right)^2 = (2x - 1)^2$

$\Rightarrow 3x^2 - 2 = 4x^2 - 2 \times 2x \times 1 + 1$

$\Rightarrow 4x^2 - 4x + 1 - 3x^2 + 2 = 0$

$\Rightarrow \quad x^2 - 4x + 3 = 0$

$\Rightarrow x^2 - x - 3x + 3 = 0 \quad \left\{\begin{matrix}\because -4 = -1 - 3 \\ 3 = (-1)\times(-3)\end{matrix}\right\}$

$\Rightarrow x(x - 1) - 3(x - 1) = 0$

$\Rightarrow \quad (x - 1)(x - 3) = 0$

[Zero Product Rule]

Either $x - 1 = 0$, then $x = 1$

or $\quad x - 3 = 0$, then $x = 3$

Check : (*i*) When $x = 1$, then

L.H.S. $= \sqrt{3x^2 - 2} = \sqrt{3(1)^2 - 2}$

$= \sqrt{3-2} = \sqrt{1} = 1$

R.H.S. $2x - 1 = 2 \times 1 - 1 = 2 - 1 = 1$

$\because$ L.H.S. = R.H.S.

$\therefore$ $x = 1$ is its solution

(*ii*) When $x = 3$, then

L.H.S. $= \sqrt{3x^2 - 2} = \sqrt{3\times3^2 - 2}$

$= \sqrt{3\times9-2}$

$= \sqrt{27-2} = \sqrt{25} = 5$

R.H.S. $= 2x - 1 = 2 \times 3 - 1$

$= 6 - 1$

$= 5$

$\because$ L.H.S. = R.H.S.

$\therefore$ $x = 5$ is also its solution

Hence, its solution set is $\{1, 3\}$ **Ans.**

Q. 49. $\sqrt{3x^2 + x + 5} = (x - 3)$

Sol. $\sqrt{3x^2 + x + 5} = (x - 3)$

Squaring both sides,

$\left(\sqrt{3x^2 + x + 5}\right)^2 = (x - 3)^2$

$\Rightarrow \quad 3x^2 + x + 5 = x^2 - 2 \times x \times 3 + 9$

$\Rightarrow 3x^2 + x + 5 - x^2 + 6x - 9 = 0$

$\Rightarrow \quad 2x^2 + 7x - 4 = 0$

$\Rightarrow 2x^2 + 8x - x - 4 = 0 \left\{\begin{matrix}\because 2\times(-4) = -8 \\ \therefore 7 = 8 - 1 \\ -8 = 8\times(-1)\end{matrix}\right\}$

$\Rightarrow 2x(x + 4) - 1(x + 4) = 0$

$\Rightarrow \quad (x + 4)(2x - 1) = 0$

[Zero Product Rule]

Either $x + 4 = 0$, then $x = -4$

or $\quad 2x - 1 = 0$, then $2x = 1$

$\Rightarrow \quad x = \frac{1}{2}$

Check : (*i*) When $x = -4$, then

L.H.S. $= \sqrt{3x^2 + x + 5}$

$= \sqrt{3(-4)^2 + (-4) + 5}$

$= \sqrt{3\times16 - 4 + 5}$

$= \sqrt{48 - 4 + 5} = \sqrt{49} = 7$

R.H.S. $= x - 3 = -4 - 3$

$= -7$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore$ $x = -4$ is not its solution.

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(*ii*) When $x = \frac{1}{2}$, then

L.H.S. $= \sqrt{3x^2 + x + 5}$

$= \sqrt{3\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) + 5}$

$= \sqrt{\frac{3}{4} + \frac{1}{2} + 5} = \sqrt{\frac{3+2+20}{4}}$

$= \sqrt{\frac{25}{4}} = \frac{5}{2}$

R.H.S. $= x - 3 = \frac{1}{2} - 3 = \frac{1-6}{2} = -\frac{5}{2}$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore x = -\frac{1}{2}$ is not also its solutions.

Hence, solution set is 1 **Ans.**

Q.50. Find the quadratic equation whose solution set is :

(*i*) $\{2, -3\}$ (*ii*) $\left\{-3, \frac{2}{5}\right\}$

(*iii*) $\left\{\frac{2}{5}, -\frac{1}{2}\right\}$

Sol. (*i*) $\because$ Solution set is (2, – 3)

$\therefore x = 2$ and $x = -3$

$\Rightarrow x - 2 = 0$ and $x + 3 = 0$

$\Rightarrow (x - 2)(x + 3) = 0$

$\Rightarrow x^2 + 3x - 2x - 6 = 0$

$\Rightarrow x^2 + x - 6 = 0$ **Ans.**

(*ii*) $\because$ Solution set is $\left\{-3, \frac{2}{5}\right\}$

$\therefore x = -3$ and $x = \frac{2}{5}$

$\Rightarrow x + 3 = 0$ and $x - \frac{2}{5} = 0$

$\Rightarrow (x + 3)\left(x - \frac{2}{5}\right) = 0$

$\Rightarrow (x + 3)\left(\frac{5x-2}{5}\right) = 0$

$\Rightarrow (x + 3)(5x - 2) = 0$

$\Rightarrow 5x^2 - 2x + 15x - 6 = 0$

$\Rightarrow 5x^2 + 13x - 6 = 0$ **Ans.**

(*iii*) $\because$ Solution set is $\left\{\frac{2}{5}, -\frac{1}{2}\right\}$

$\therefore x = \frac{2}{5}$ and $x = -\frac{1}{2}$

$\Rightarrow \left(x - \frac{2}{5}\right) = 0$ and $\left(x + \frac{1}{2}\right) = 0$

$\Rightarrow \left(x - \frac{2}{5}\right)\left(x + \frac{1}{2}\right) = 0$

$\Rightarrow \left(\frac{5x-2}{5}\right)\left(\frac{2x+1}{2}\right) = 0$

$\Rightarrow (5x - 2)(2x + 1) = 0$

$\Rightarrow 10x^2 + 5x - 4x - 2 = 0$

$\Rightarrow 10x^2 + x - 2 = 0$ **Ans.**

EXERCISE 6 (B)

Solve each of the following equations by using formula :

Q.1. $5x^2 - 2x - 3 = 0$

Sol. $5x^2 - 2x - 3 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 5, b = -2, c = -3$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 5 \times (-3)}}{2 \times 5}$

$= \frac{2 \pm \sqrt{4 + 60}}{10} = \frac{2 \pm \sqrt{64}}{10} = \frac{2 \pm 8}{10}$

Hence $x_1 = \frac{2+8}{10} = \frac{10}{10} = 1$

and $x_2 = \frac{2-8}{10} = \frac{-6}{10} = \frac{-3}{5}$

Hence, solution set is $\left\{1, \frac{-3}{5}\right\}$ **Ans.**

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Q. 2. $9x^2 + 7x - 2 = 0$

Sol. $9x^2 + 7x - 2 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 9, b = 7, c = -2$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{(7)^2 - 4 \times 9 \times (-2)}}{2 \times 9}$$

$$= \frac{-7 \pm \sqrt{49 + 72}}{18}$$

$$= \frac{-7 \pm \sqrt{121}}{18} = \frac{-7 \pm 11}{18}$$

Hence $x_1 = \frac{-7+11}{18} = \frac{4}{18} = \frac{2}{9}$

and $x_2 = \frac{-7-11}{18} = \frac{-18}{18} = -1$

Hence, solution set is $\left\{\frac{2}{9}, -1\right\}$ **Ans.**

Q. 3. $4 - 11x = 3x^2$

Sol. $4 - 11x = 3x^3 \Rightarrow 3x^2 + 11x - 4 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 3, b = 11, c = -4$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-11 \pm \sqrt{(11)^2 - 4 \times 3 \times (-4)}}{2 \times 3}$$

$$= \frac{-11 \pm \sqrt{121 + 48}}{6} = \frac{-11 \pm \sqrt{169}}{6}$$

$$= \frac{-11 \pm 13}{6}$$

Hence $x_1 = \frac{-11+13}{6} = \frac{2}{6} = \frac{1}{3}$

and $x_2 = \frac{-11-13}{6} = \frac{-24}{6} = -4$

Hence, solution set is $\left\{\frac{1}{3}, -4\right\}$ **Ans.**

Q. 4. $25x^2 + 30x + 7 = 0$

Sol. Here, $a = 25, b = 30, c = 7$

$D = b^2 - 4ac = (30)^2 - 4 \times 25 \times 7$

$= 900 - 700 = 200$

$$\because x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-30 \pm \sqrt{200}}{2 \times 25}$$

$$= \frac{-30 \pm \sqrt{100 \times 2}}{50} = \frac{-30 \pm 10\sqrt{2}}{50}$$

$$= \frac{-3 + \sqrt{2}}{5}$$

$$\therefore x_1 = \frac{-3 + \sqrt{2}}{5} \text{ and } x_2 = \frac{-3 - \sqrt{2}}{5}$$

Hence, $x = \frac{-3 + \sqrt{2}}{5}, \frac{-3 - \sqrt{2}}{5}$ **Ans.**

Q. 5. $5x^2 - 19x + 17 = 0$

Sol. Here, $a = 5, b = -19, c = 17$

$\therefore \quad D = b^2 - 4ac$

$= (-19)^2 - 4 \times 5 \times 17$

$= 361 - 340 = 21$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-(-19) \pm \sqrt{21}}{2 \times 5} = \frac{19 \pm \sqrt{21}}{10}$$

$$= \frac{19 \pm \sqrt{21}}{10}$$

$$\therefore x_1 = \frac{19 \pm \sqrt{21}}{10} \text{ and } x_2 = \frac{19 - \sqrt{21}}{10}$$

Hence, $x = \frac{19 + \sqrt{21}}{10}, \frac{19 - \sqrt{21}}{10}$ **Ans.**

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Q. 6. $3x^2 - 8x + 2 = 0$

Sol. $3x^2 - 8x + 2 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 3, b = -8, c = 2$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 3 \times 2}}{2 \times 3}$$

$$= \frac{8 \pm \sqrt{64 - 24}}{6} = \frac{8 \pm \sqrt{40}}{6}$$

$$= \frac{8 \pm \sqrt{4 \times 10}}{6} = \frac{8 \pm 2\sqrt{10}}{6}$$

$$= \frac{4 \pm \sqrt{10}}{3} \quad \text{(Dividing by 2)}$$

$$\therefore x_1 = \frac{4 + \sqrt{10}}{3} \text{ and } x_2 = \frac{4 - \sqrt{10}}{3}$$

Hence, solution set is

$$\left\{\frac{4 + \sqrt{10}}{3}, \frac{4 - \sqrt{10}}{3}\right\}$$ **Ans.**

Q. 7. $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

Sol. $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = \sqrt{3}, b = 10, c = -8\sqrt{3}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{(10)^2 - 4 \times \sqrt{3} \times (-8\sqrt{3})}}{2 \times \sqrt{3}}$$

$$= \frac{-10 \pm \sqrt{100 + 96}}{2\sqrt{3}} = \frac{-10 \pm \sqrt{196}}{2\sqrt{3}}$$

$$= \frac{-10 \pm 14}{2\sqrt{3}}$$

$$\therefore x_1 = \frac{-10 + 14}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\text{and } x_2 = \frac{-10 - 14}{2\sqrt{3}} = \frac{-24}{2\sqrt{3}}$$

$$= \frac{-12 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{-12\sqrt{3}}{3}$$

$$= -4\sqrt{3}$$

Hence, solution set is

$$\left\{\frac{2}{\sqrt{3}}, -4\sqrt{3}\right\}$$ **Ans.**

Q. 8. $2x^2 + \sqrt{7}x - 7 = 0$

Sol. $2x^2 + \sqrt{7}x - 7 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 2, b = \sqrt{7}, c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\sqrt{7} \pm \sqrt{(\sqrt{7})^2 - 4 \times 2 \times (-7)}}{2 \times 2}$$

$$= \frac{-\sqrt{7} \pm \sqrt{7 + 56}}{4}$$

$$= \frac{-\sqrt{7} \pm \sqrt{63}}{4} = \frac{\sqrt{7} \pm \sqrt{9 \times 7}}{4}$$

$$= \frac{-\sqrt{7} \pm 3\sqrt{7}}{4}$$

$$\therefore x_1 = \frac{-\sqrt{7} + 3\sqrt{7}}{4} = \frac{2\sqrt{7}}{4} = \frac{\sqrt{7}}{2}$$

$$\text{and } x_2 = \frac{-\sqrt{7} - 3\sqrt{7}}{4} = \frac{-4\sqrt{7}}{4} = -\sqrt{7}$$

Hence, solution set is $\left\{-\sqrt{7}, \frac{\sqrt{7}}{2}\right\}$ **Ans.**

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Q.9. $6x^2 - 31x = 105$

Sol. $6x^2 - 31x = 105$

$\Rightarrow 6x^2 - 31x - 105 = 0$

Comparing it with $ax^2 + bx = 0$, we get

$a = 6, b = -31, c = -105$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-31) \pm \sqrt{(-31)^2 - 4 \times 6 \times (-105)}}{2 \times 6}$$

$$= \frac{31 \pm \sqrt{961 + 2520}}{12} = \frac{31 \pm \sqrt{3481}}{12}$$

$$= \frac{31 \pm 59}{12}$$

$$\therefore \quad x_1 = \frac{31+59}{12} = \frac{90}{12} = \frac{15}{2}$$

$$\text{and } x_2 = \frac{31-59}{12} = \frac{-28}{12} = -\frac{7}{3}$$

Hence, solution set is $\left\{\frac{15}{2}, \frac{-7}{3}\right\}$ **Ans.**

Q.10. $\frac{x+3}{2x+3} = \frac{x+1}{3x+2}$

Sol. $\frac{x+3}{2x+3} = \frac{x+1}{3x+2}$

$(x + 3)(3x + 2) = (x + 1)(2x + 3)$

[By cross multiplication]

$\Rightarrow 3x^2 + 2x + 9x + 6 = 2x^2 + 3x + 2x + 3$

$\Rightarrow 3x^2 + 11x + 6 = 2x^2 + 5x + 3$

$\Rightarrow 3x^2 + 11x + 6 - 2x^2 - 5x - 3 = 0$

$\Rightarrow x^2 + 6x + 3 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 1, b = 6, c = 3$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{(6)^2 - 4 \times 1 \times 3}}{2 \times 1}$$

$$= \frac{-6 \pm \sqrt{36 - 12}}{2} = \frac{-6 \pm \sqrt{24}}{2}$$

$$= \frac{-6 \pm \sqrt{4 \times 6}}{2} = \frac{-6 \pm 2\sqrt{6}}{2}$$

$= -3 \pm \sqrt{6}$ [Dividing by 2]

$\therefore x_1 = -3 + \sqrt{6}$ and $x_2 = -3 - \sqrt{6}$

Hence, solution set is

$\{-3 + \sqrt{6}, -3 - \sqrt{6}\}$ **Ans.**

Q.11. $\frac{1}{(x+1)} + \frac{2}{(x+2)} = \frac{4}{(x+4)}$

Sol. $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$

$$\Rightarrow \frac{x+2+2x+2}{(x+1)(x+2)} = \frac{4}{x+4}$$

$$\Rightarrow \frac{3x+4}{x^2 + 2x + x + 2} = \frac{4}{x+4}$$

$$\Rightarrow \frac{3x+4}{x^2 + 3x + 2} = \frac{4}{x+4}$$

$\Rightarrow (3x + 4)(x + 4) = 4(x^2 + 3x + 2)$

[By cross multiplications]

$\Rightarrow 3x^2 + 12x + 4x + 16 = 4x^2 + 12x + 8$

$\Rightarrow 3x^2 + 16x + 16 - 4x^2 - 12x - 8 = 0$

$\Rightarrow -x^2 + 4x + 8 = 0$

$\Rightarrow x^2 - 4x - 8 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 1, b = -4, c = -8$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-8)}}{2 \times 1}$$

$$= \frac{4 \pm \sqrt{16 + 32}}{2} = \frac{4 \pm \sqrt{48}}{2}$$

$$= \frac{4 \pm \sqrt{16 \times 3}}{2} = \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$$

$\therefore x_1 = 2 + 2\sqrt{3}$ and $x_2 = 2 - 2\sqrt{3}$

Hence, solution set is

$\{2 + 2\sqrt{3}, 2 - 2\sqrt{3}\}$ **Ans.**

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Q.12. $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}$

Sol. $\frac{x-1}{x-2}+\frac{x-3}{x-4}=\frac{10}{3}$

$\Rightarrow \frac{(x-1)(x-4)+(x-3)(x-2)}{(x-2)(x-4)}=\frac{10}{3}$

$\Rightarrow \frac{x^2-4x-x+4+x^2-2x-3x+6}{x^2-4x-2x+8}=\frac{10}{3}$

$\Rightarrow \frac{2x^2-10x+10}{x^2-6x+8}=\frac{10}{3}$

$\Rightarrow 3(2x^2-10x+10)=10(x^2-6x+8)$

[By cross multiplication]

$\Rightarrow 6x^2-30x+30=10x^2-60x+80$

$\Rightarrow 6x^2-30x+30-10x^2+60x-80=0$

$\Rightarrow -4x^2+30x-50=0$

$\Rightarrow 2x^2-15x+25=0$ [Dividing by –2]

Comparing it with $ax^2+bx+c=0$, we get

$a=2, b=-15, c=25$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-(-15)\pm\sqrt{(-15)^2-4\times2\times25}}{2\times2}$

$=\frac{15\pm\sqrt{225-200}}{4}=\frac{15\pm\sqrt{25}}{4}=\frac{15\pm5}{4}$

$\therefore\ x_1=\frac{15+5}{4}=\frac{20}{4}=5$

and $x_2=\frac{15-5}{4}=\frac{10}{4}=\frac{5}{2}$

Hence, solution set is $\left\{5,\frac{5}{2}\right\}$ **Ans.**

Q. 13. $x^2-10x+6=0$ **(2001)**

Sol. Here, $a=1, b=-10, c=6$

$D=b^2-4ac=(-10)^2-4\times1\times6$

$=100-24=76$

$\because\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-10)\pm\sqrt{76}}{2\times1}$

$=\frac{10\pm\sqrt{4\times19}}{2}=\frac{10\pm2\sqrt{19}}{2}$

$=5\pm\sqrt{19}$

$\therefore\ x_1=5+\sqrt{19}=5+4\cdot36=9\cdot36$

$x_2=5-\sqrt{19}=5-4\cdot36=0\cdot64$ **Ans.**

Q. 14. $x^2-5x-10=0$

Sol. Given Equation is : $x^2-5x-10=0$

On comparing with, $ax^2+bx+c=0$

$a=1, b=-5, c=-10$

$\because\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-10)}}{2\times1}$

$\therefore\ x=\frac{5\pm\sqrt{25+40}}{2}$

$\Rightarrow x=\frac{5\pm\sqrt{65}}{2}=\frac{5\pm8.06}{2}$

$\Rightarrow =\frac{5+8.06}{2}=\frac{13.06}{2}=6.53$

$=\frac{5-8.06}{2}=\frac{-3.06}{2}=-1.53$

$\therefore\ x=6.53, x=-1.53$ **Ans.**

Q. 15. $2x^2-6x+3=0$

Sol. $2x^2-6x+3=0$

Comparing it with $ax^2+bx+c=0$, we get

$a=2, b=-6, c=3$

$\therefore\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-(-6)\pm\sqrt{(-6)^2-4\times2\times3}}{2\times2}$

$=\frac{6\pm\sqrt{36-24}}{4}=\frac{6\pm\sqrt{12}}{4}$

$=\frac{6\pm\sqrt{4\times3}}{4}=\frac{6\pm2\sqrt{3}}{4}=\frac{3\pm\sqrt{3}}{2}$

(Dividing by 2)

$\therefore\ x_1=\frac{3+\sqrt{3}}{2}$ and $x_2=\frac{3-\sqrt{3}}{2}$

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Hence, solution set is $\left\{\frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}\right\}$ **Ans.**

Q. 16. $3x^2 - 32x + 12 = 0$

Sol. Here, $a = 3, b = -32, c = 12$

$D = b^2 - 4ac = (-32)^2 - 4 \times 3 \times 12$
$= 1024 - 144 = 880$

$\because x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-32) \pm \sqrt{880}}{2 \times 3}$

$= \frac{32 \pm \sqrt{4 \times 220}}{6} = \frac{32 \pm 2\sqrt{220}}{6} = \frac{16 \pm \sqrt{220}}{3}$

$\therefore x_1 = \frac{16 + 14.83}{3} = \frac{30{\cdot}83}{3} = 10{\cdot}28$

$x_2 = \frac{16 - \sqrt{220}}{3} = \frac{16 - 14{\cdot}83}{3} = \frac{1{\cdot}17}{3} = 0{\cdot}39$

Q. 17. $2x - \frac{1}{x} = 7.$

Sol. We have $2x - \frac{1}{x} = 7 \Rightarrow 2x^2 - 1 = 7x$

$\Rightarrow 2x^2 - 7x - 1 = 0$ ....(i)

Comparing (i) with $ax^2 + bx + c$, we get,
$a = 2, b = -7, c = -1$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2) \times (-1)}}{2 \times 2}$

$= \frac{7 \pm \sqrt{49 + 8}}{4} = \frac{7 \pm \sqrt{57}}{4}$

$\Rightarrow x = \frac{7 + \sqrt{57}}{4}$ or $x = \frac{7 - \sqrt{57}}{4}$

$\Rightarrow x = \frac{7 + 7.55}{4}$ or $x = \frac{7 - 7.55}{4}$

$\Rightarrow x = \frac{14.55}{4}$ or $x = \frac{-0.55}{4}$

$\Rightarrow x = 3.64$ or $x = -0.14$ **Ans.**

Q. 18. $x^2 - 3x - 9 = 0$

Sol. The given equation is $x^2 - 3x - 9 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get
$a = 1, b = -3, c = -9$

By using formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we obtain

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times (-9)}}{2 \times 1}$

$\Rightarrow x = \frac{3 \pm \sqrt{9 + 36}}{2} = \frac{3 \pm \sqrt{45}}{2} = \frac{3 \pm \sqrt{9 \times 5}}{2}$

$\Rightarrow x = \frac{3 \pm 3\sqrt{5}}{2} = \frac{3 \pm 3 \times 2.236}{2}$ $(\because \sqrt{5} = 2.236)$

$\Rightarrow x = \frac{3 \pm 6.708}{2} = \frac{9.708}{2}$ or $-\frac{3.708}{2}$

$x = 4.85$ or -1.85 (correct to two decimal places)

Q. 19. $5x(x + 2) = 3$

Sol. $5x(x + 2) = 3$

$\Rightarrow 5x^2 + 10x = 3 \Rightarrow 5x^2 + 10x - 3 = 0$

Here $a = 5, b = 10, c = -3$

$D = b^2 - 4ac = (10)^2 - 4 \times 5 \times (-3)$
$= 100 + 60 = 160$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{160}}{2 \times 5}$

$= \frac{-10 \pm \sqrt{16 \times 10}}{10} = \frac{-10 \pm 4\sqrt{10}}{10}$

$= \frac{-10 \pm 4(3.162)}{10} = \frac{-10 \pm 12.648}{10}$

$\therefore x_1 = \frac{-10 + 12.648}{10} = \frac{2.648}{10} = 0.2648 = 0.265$

$x_2 = \frac{-10 - 12.648}{10} = \frac{-22.648}{10} = -2.2648$

$\therefore x = 0.26, -2.26$ **Ans.**

Q. 20. Solve the following quadratic equation and give the answer correct to two significant figures. **(2009)**

$4x^2 - 7x + 2 = 0$

Sol. $4x^2 - 7x + 2 = 0$

$x = \frac{7 \pm \sqrt{49 - 32}}{8} = \frac{7 \pm \sqrt{17}}{8} \Rightarrow x = \frac{7 \pm 4.12}{8}$

Taking (+ ve) sign $x = \frac{11.12}{8} = 1.39$

Taking (– ve) sign $x = \frac{2.88}{8} = 0.36$

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EXERCISE 6-C

Discuss the nature of the roots of each of the following equations without actually solving it :

Q. 1. $x^2 - 8x + 7 = 0$

Sol. In equation

$x^2 - 8x + 7 = 0$

$a = 1, b = -8, c = 7$

$\therefore$ D $= b^2 - 4ac$

$= (-8)^2 - 4 \times 1 \times 7 = 64 - 28 = 36 = (6)^2$

$\because$ D > 0 and is a perfect square

$\therefore$ Roots of rational and distinct Ans.

Q. 2. $6x^2 + 7x - 10 = 0$

Sol. In equation

$6x^2 + 7x - 10 = 0$

$a = 6, b = 7, c = -10$

$\therefore$ D $= b^2 - 4ac$

$= (7)^2 - 4 \times 6 \times (-10)$

$= 49 + 240 = 289 = (17)^2$

$\because$ D > 0 and is a perfect square

$\therefore$ Roots are rational and distinct Ans.

Q. 3. $25x^2 + 30x + 7 = 0$

Sol. In equation

$25x^2 + 30x + 7 = 0$

$a = 25, b = 30$ and $c = 7$

$\therefore$ D $= b^2 - 4ac$

$= (30)^2 - 4 \times 25 \times 7$

$= 900 - 700 = 200 = 100 \times 2 = \left(10\sqrt{2}\right)^2$

$\because$ D > 0 and is not a perfect square

$\therefore$ Roots are irrational and distinct Ans.

Q. 4. $15x^2 - 28 = x$

Sol. In equation $15x^2 - 28 = x$

or $15x^2 - x - 28 = 0$

Here $a = 15, b = -1, c = -28$

$\therefore$ D $= b^2 - 4ac$

$= (-1)^2 - 4 \times 15 \times (-28)$

$= 1 + 1680 = 1681 = (41)^2$

$\because$ D > 0 and is a perfect square

$\therefore$ Roots are rational and distinct Ans.

Q. 5. $16x^2 = 24x + 1$

Sol. In equation

$16x^2 = 24x + 1$

$\Rightarrow 16x^2 - 24x - 1 = 0$

$a = 16, b = -24, c = -1$

$\therefore$ D $= b^2 - 4ac$

$= (-24)^2 - 4 \times 16 \times (-1)$

$= 576 + 64 = 640 = 64 \times 10 = \left(8\sqrt{10}\right)^2$

$\therefore$ D > 0 and not a perfect square

$\therefore$ Roots are irrational and distinct. Ans.

Q. 6. $2x^2 - 2\sqrt{6}\,x + 3 = 0$

Sol. In equation $2x^2 - 2\sqrt{6}\,x + 3 = 0$

$a = 2, b = -2\sqrt{6}, c = 3$

$\therefore$ D $= b^2 - 4ac = \left(-2\sqrt{6}\right)^2 - 4 \times 2 \times 3$

$= 24 - 24 = 0$

$\because$ D = 0

$\therefore$ Roots are real and equal Ans.

Q. 7. $2x^2 + 2x + 3 = 0$

Sol. In equation

$2x^2 + 2x + 3 = 0$

$a = 2, b = 2, c = 3$

$\because$ D $= b^2 - 4ac$

$= (2)^2 - 4 \times 2 \times 3 = 4 - 24 = -20$

$= -(4 \times 5) = -\left(2\sqrt{5}\right)^2$

$\therefore$ D < 0 and not a perfect square

$\therefore$ Roots are not real and unequal

or roots are imaginary and unequal. Ans.

Q. 8. $2x^2 - 5x - 4 = 0$

Sol. In equation,

$2x^2 - 5x - 4 = 0$

$a = 2, b = -5, c = -4$

$\therefore$ D $= b^2 - 4ac = (-5)^2 - 4 \times 2 \times (-4)$

$= 25 + 32 = 57 = \left(\sqrt{57}\right)^2$

$\because$ D > 0 and not a perfect square

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$\therefore$ Roots are irrational and unequal. Ans.

Q. 9. $5x^2 - 13x - 6 = 0$

Sol. In equation,

$5x^2 - 13x - 6 = 0$

$a = 5, b = -13, c = -6$

$\therefore$ $D = b^2 - 4ac = (-13)^2 - 4 \times 5 \times (-6)$
$= 169 + 120 = 289 = (17)^2$

$\therefore$ $D > 0$ and is a perfect square.

$\therefore$ Roots are rational and unequal Ans.

Q. 10. $9x^2 - 6x + 1 = 0$

Sol. In equation,

$9x^2 - 6x + 1 = 0$

$a = 9, b = -6, c = 1$

$\therefore$ $D = b^2 - 36 = (-6)^2 - 4 \times 9 \times 1$
$= 36 - 36 = 0$

$\because$ $D = 0$

$\therefore$ Roots are real and equal Ans.

Q. 11. $3x^2 - 2x + 5 = 0$

Sol. In the equation,

$3x^2 - 2x + 5 = 0$

$a = 3, b = -2, c = 5$

$\therefore D = b^2 - 4ac = (-2)^2 - 4 \times 3 \times 5$

$= 4 - 60 = -56 = -(4 \times 14) = -\left(2\sqrt{14}\right)^2$

$\therefore$ $D < 0$ and is not a perfect square

$\therefore$ Roots are imaginary and unequal. Ans.

Q. 12. $x^2 + 2x\sqrt{3} - 1 = 0$

Sol. In the equation

$x^2 + 2x\sqrt{3} - 1 = 0$ or

$x^2 + 2\sqrt{3}\,x - 1 = 0$

$a = 1, b = 2\sqrt{3}, c = -1$

$\therefore$ $D = b^2 - 4ac = \left(2\sqrt{3}\right)^2 - 4 \times 1 \times (-1)$
$= 12 + 4 = 16 = (4)^2$

$\therefore$ $D > 0$ and is a perfect square.

$\therefore$ Roots are rational and unequal Ans.

Find the values of k for which each of the following equations has equal roots.

Q. 13. $9x^2 + kx + 1 = 0$

Sol. In the equation

$9x^2 + kx + 1 = 0$

$a = 9, b = k, c = 1$

$\therefore$ $D = b^2 - 4ac = (k)^2 - 4 \times 9 \times 1$
$= k^2 - 36$

$\because$ Roots are equal.

$\therefore$ $D = 0$

$\Rightarrow$ $k^2 - 36 = 0$ $\Rightarrow$ $k^2 - (6)^2 = 0$

$\Rightarrow$ $(k + 6)(k - 6) = 0$

Either $k + 6 = 0$, then $k = -6$

or $k - 6 = 0$, then $k = 6$

$\therefore$ $k = 6, -6$ Ans.

Q. 14. $x^2 - 2kx + 7k - 12 = 0$

Sol. In the equation

$x^2 - 2kx + 7k - 12 = 0$

$\Rightarrow$ $x^2 - 2kx + (7k - 12) = 0$

Here $a = 1, b = -2k, c = 7k - 12$

$\therefore$ $D = b^2 - 4ac$
$= (-2k)^2 - 4 \times 1 \times (7k - 12)$
$= 4k^2 - 28k + 48$

$\because$ Roots are equal.

$\therefore$ $D = 0$

$\Rightarrow$ $4k^2 - 28k + 48 = 0$

$\Rightarrow$ $k^2 - 7k + 12 = 0$ (Dividing by 4)

$\Rightarrow$ $k^2 - 3k - 4k + 12 = 0$

$\Rightarrow$ $k(k - 3) - 4(k - 3) = 0$

$\Rightarrow$ $(k - 3)(k - 4) = 0$

Either $k - 3 = 0$, then $k = 3$

or $k - 4 = 0$, then $k = 4$

$\therefore$ $k = 4, 3$ Ans.

Q. 15. $(3k + 1)x^2 + 2(k + 1)x + k = 0$

Sol. In the equation

$(3k + 1)x^2 + 2(k + 1)x + k = 0$

Here $a = 3k + 7, b = 2(k + 1), c = k$

$\therefore$ $D = b^2 - 4ac$
$= [2(k + 1)]^2 - 4 \times (3k + 1) \times k$
$= 4(k^2 + 2k + 1) - 4k(3k + 1)$
$= 4k^2 + 8k + 4 - 12k^2 - 4k = -8k^2 + 4 + 4k$

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$= -8k^2 + 4k + 4$

$\because$ Roots are equal

$\therefore$ D = 0

$-8k^2 + 4k + 4 = 0$ (Dividing by – 4)

$\Rightarrow 2k^2 - k - 1 = 0$

$= 2k^2 - 2k + k - 1 = 0$

$= 2k(k - 1) + 1(k - 1)$

$= 0 \ (k - 1)(2k + 1)$

Either $k - 1 = 0$, then $k = 1$

or $2k + 1 = 0$ then $2k = -1$

$\Rightarrow k = -\frac{1}{2}$

Hence $k = 1, -\frac{1}{2}$ Ans.

Q. 16. $x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$

Sol. In equations

$x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$

$a = 1, b = -2(5 + 2k)$ and $c = 3(7 + 10k)$

$\therefore D = b^2 - 4ac$

$= [-2(5 + 2k)]^2 - 4 \times 1 \times 3(7 + 10k)$

$= 4(25 + 20k + 4k^2) - 12(7 + 10k)$

$= 100 + 80k + 16k^2 - 84 - 120k$

$= 16k^2 - 40k + 16$

$\because$ Roots are equal

$\therefore$ D = 0

$\Rightarrow 16k^2 - 40k + 16 = 0$ (Dividing by 8)

$\Rightarrow 2k^2 - 5k + 2 = 0$

$\Rightarrow 2k^2 - 4k - k + 2 = 0$

$\Rightarrow 2k(k - 2) - 1(k - 2) = 0$

$\Rightarrow (k - 2)(2k - 1) = 0$

Either $k - 2 = 0$, then $k = 2$

or $2k - 1 = 0$, then $2k = 1$

$\Rightarrow k = \frac{1}{2}$

Hence $k = 2, \frac{1}{2}$ Ans.

Q. 17. $(k + 1)x^2 + 2(k + 3)x + (k + 8) = 0$

Sol. In the equation

$(k + 1)x^2 + 2(k + 3)x + (k + 8) = 0$

$a = k + 1, b = 2(k + 3)$ and $c = k + 8$

$\therefore D = b^2 - 4ac$

$= [2(k + 3)]^2 - 4(k + 1)(k + 8)$

$= 4(k^2 + 6k + 9) - 4(k^2 + 9k + 8)$

$= 4k^2 + 24k + 36 - 4k^2 - 36k - 32$

$= -12k + 4$

$\because$ Roots are equal

D = 0

$\Rightarrow -12k + 4 = 0 \quad \Rightarrow -12k = -4$

$\Rightarrow k = \frac{-4}{-12} = \frac{1}{3}$

Hence $k = \frac{1}{3}$ Ans.

Q. 18. $kx^2 + kx + 1 = -4x^2 - x$

Sol. $kx^2 + kx + 1 = -4x^2 - x$

$\Rightarrow kx^2 + kx + 1 + 4x^2 + x = 0 \Rightarrow$
$kx^2 + 4x^2 + kx + x + 1 = 0 \quad \Rightarrow x^2(k + 4) + x(k + 1) + 1 = 0$

Here $a = k + 4, b = k + 1, c = 1$

$\therefore D = b^2 - 4ac$

$= (k + 1)^2 - 4 \times (k + 4) \times 1$

$= k^2 + 2k + 1 - 4k - 16$

$= k^2 - 2k - 15$

$\because$ Roots are equal

$\therefore$ D = 0

$\Rightarrow k^2 - 2k - 15 = 0$

$\Rightarrow k^2 - 5k + 3k - 15 = 0$

$\Rightarrow k(k - 5) + 3(k - 5) = 0$

$\Rightarrow (k - 5)(k + 3) = 0$

Either $k - 5 = 0$, then $k = 5$ or $k + 3 = 0$, then $k = -3$

Hence $k = 5, -3$ Ans.

Q. 19. $3kx^2 = 4(kx - 1)$

Sol. $3kx^2 = 4(kx - 1)$

$\Rightarrow 3kx^2 = 4kx - 4 \Rightarrow 3kx^2 - 4kx + 4 = 0$

Here $a = 3k, b = -4k, c = 4$

$\therefore D = b^2 - 4ac$

$= (-4k)^2 - 4 \times 3k \times 4 = 16k^2 - 48k$

$\because$ Roots are equal

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$\therefore$ D = 0 $\Rightarrow$ $16k^2 - 48k = 0$

$\Rightarrow$ $k^2 - 3k = 0$ (Dividing by 16)

$\Rightarrow$ $k(k - 3) = 0$

Either $k = 0$

or $k - 3 = 0$, then $k = 3$

Hence $k = 0, 3$ Ans.

Q. 20. If $a, b, c \in$ R, show that the roots of the equation $(a - b) x^2 + (b + c - a) x - c = 0$ are rational

Sol. $a, b, c \in$ R

$(a - b) x^2 + (b + c - a) x - c = 0$

Here A = $a - b$, B = $(b + c - a)$ and C = $-c$

D = B² − 4AC

$= (b + c - a)^2 - 4 (a - b) (- c)$

$= b^2 + c^2 + a^2 + 2bc - 2ca - 2ab + 4ac - 4bc = a^2 + b^2 + c^2 - 2bc + 2ac - 2ab$

$= (a + c - b)^2$

$\because$ D is a perfect square and D ≥ 0

$\therefore$ Roots are rational.

Hence proved.

Q. 21. If the roots of the equation $(c^2 - ab) x^2 - 2(a^2 - bc) x + (b^2 - ac) = 0$ are real and equal, show that either $(a^3 + b^3 + c^3 = 3abc)$ or $a = 0$

Sol. In the equation,

$(c^2 - ab) x^2 - 2(a^2 - bc) x + (b^2 - ac) = 0$

A = $c^2 - ab$, B = $-2 (a^2 - bc)$ and C = $b^2 - ac$

$\therefore$ D = B² − 4AC

$= [-2 (a^2 - bc)]^2 - 4 (c^2 - ab) (b^2 - ac)$

$= 4(a^4 + b^2c^2 - 2a^2bc) - 4 (b^2c^2 - ac^3 - ab^3 + a^2bc)$

$= 4a^4 + 4b^2c^2 - 8a^2bc - 4b^2c^2 + 4ac^3 + 4ab^3 - 4a^2bc$

$= 4a^4 - 12a^2bc + 4ac^3 + 4ab^3$

$= 4a [a^3 + b^3 + c^3 - 3abc]$

$\because$ Roots are real and equal

$\therefore$ D = 0

$\therefore$ $4a (a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow a(a^3 + b^3 + c^3 - 3abc) = 0$ (Dividing by 4)

Either $a = 0$

or $a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow$ $a^3 + b^3 + c^3 = 3abc$

Hence $a^3 + b^3 + c^3 = 3abc$ or $a = 0$

Q. 22. If a, b, c are rational, prove that the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are also rational.

Sol. a, b, c are rational in the equation,

$(b - c) x^2 + (c - a) x + (a - b) = 0$

Here A = $b - c$, B = $c - a$ and C = $a - b$

$\therefore$ D = B² − 4AC

$\Rightarrow$ D = $(c - a)^2 - 4 (b - c) (a - b)$

$= c^2 + a^2 - 2ca - 4(ab - b^2 - ac + bc)$

$= c^2 + a^2 - 2ca - 4ab + 4b^2 + 4ca - 4bc$

$= a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ca$

$= (a)^2 + (2b)^2 + (c)^2 - 4ab - 4bc + 2ca$

$= (a - 2b + c)^2$

$\therefore$ D is a perfect square and D ≥ 0

$\therefore$ Roots are rational.

Hence proved.

Q. 23. Prove that the equation $3x^2 + 7x + 8 = 0$ is not true for any real value of x.

Sol. In the equation,

$3x^2 + 7x + 8 = 0$

$a = 3, b = 7, c = 8$

$\therefore$ D = $b^2 - 4ac$ = $(7)^2 - 4 \times 3 \times 8$

$= 49 - 96 = - 47$

$\because$ D < 0

$\therefore$ Roots are imaginary or roots are not real.

Hence, value of x is not real.

Hence proved.

Q. 24. Show that the equation $x^2 + ax - 1 = 0$ has real and distinct roots for all real values of a.

Sol. In the equation

$x^2 + ax - 1 = 0$

Here A = 1, B = a, C = − 1

$\therefore$ D = B² − 4AC

$= a^2 - 4 \times 1 (-1) = a^2 + 4$

$\because$ D > 0 (as a^2 and 4 are both positive)

$\therefore$ Roots are real and distinct for all values of 'a'.

Hence proved.

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