

# Unit 2

## Algebra

### Chapter 5

# Linear Inequations

#### POINTS TO REMEMBER

1. **Linear inequation** : A statement of inequality between two expressions involving a single variable  $x$  with highest power one, is called a linear inequation. The general forms of linear inequations are :

$$(i) ax + b > c \quad (ii) ax + b < c$$

$$(iii) ax + b \geq c \quad (iv) ax + b \leq c$$

where  $a, b, c$  are real numbers and  $a \neq 0$ .

2. **Replacement set or Domain of the Variable** : The set from which the values of the variable  $x$  are replaced in an inequation, is called the replacement set or the domain of the variable.

The replacement set is always given to us.

3. **Solution Set** : The set of all those values of  $x$  from the replacement set which satisfy the given inequation, is called the solution set of the inequation. Solution set is always a subset of the replacement set.

4. **Properties of Inequations** :

(i) Adding or subtracting the same number or expression to each side of an inequation does not change the inequality.

(ii) Multiplying or dividing each side of an inequation by the same positive number does not change the inequality.

(iii) Multiplying or dividing each side of an inequation by the same negative number reverses the inequality.

$$(iv) a < b \Leftrightarrow b > a$$

$$(v) a > b \Leftrightarrow b < a$$

5. **Some Special Sets of Numbers shown on Number Line** :

We should like to have a glimpse on how we represent sets of numbers on a number line.

#### EXERCISE 5

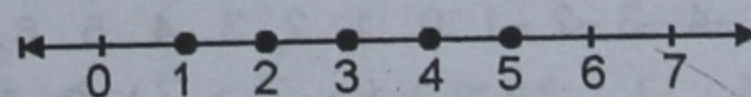
Solve each of the inequations given below and represent its solution set on a number line :

Q.1.  $2x - 7 < 4, x \in \{1, 2, 3, 4, 5, 6, 7\}$

Sol.  $2x - 7 < 4 \Rightarrow 2x < 4 + 7$

$$\Rightarrow 2x < 11 \Rightarrow x < \frac{11}{2}$$

$\therefore$  Solution set is  $\{1, 2, 3, 4, 5\}$  Ans.



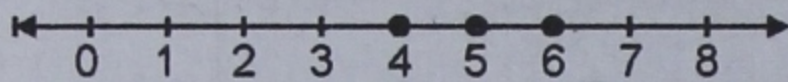
Q.2.  $2x - 3 > 3, x \in \{1, 2, 3, 4, 5, 6\}$

Sol.  $2x - 3 > 3 \Rightarrow 2x > 3 + 3$

$$\Rightarrow 2x > 6 \quad \Rightarrow x > \frac{6}{2}$$

$$\Rightarrow x > 3$$

$\therefore$  Solution set is  $\{4, 5, 6\}$  Ans.



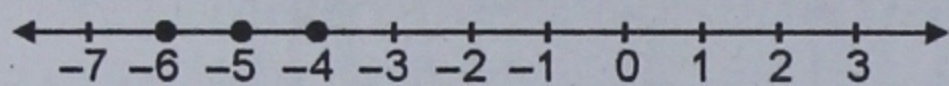
**Q.3.**  $9 \leq 1 - 2x, x \in \{-3, -4, -5, -6\}$ .

**Sol.**  $9 \leq 1 - 2x \quad \Rightarrow 2x \leq 1 - 9 \quad \Rightarrow$

$$\Rightarrow 2x \leq -8 \quad \Rightarrow x \leq -\frac{8}{2}$$

$$\Rightarrow x \leq -4$$

$\therefore$  Solution set is  $\{-4, -5, -6\}$  Ans.



**Q.4.**  $\frac{3x-5}{6} > \frac{1}{2}, x \in \{0, 1, 2, 3, 4, 5, 6\}$

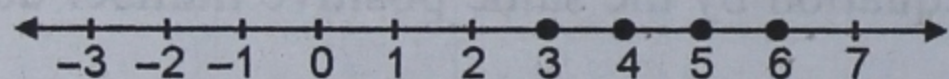
**Sol.**  $\frac{3x-5}{6} > \frac{1}{2} \Rightarrow 3x - 5 > \frac{1}{2} \times 6$

$$\Rightarrow 3x - 5 > 3 \quad \Rightarrow 3x > 3 + 5$$

$$\Rightarrow 3x > 8 \quad \Rightarrow x > \frac{8}{3}$$

$$\Rightarrow x > 2\frac{2}{3}$$

$\therefore$  Solution set is  $\{3, 4, 5, 6\}$  Ans.



**Q.5.**  $7x - 4(3 - x) \geq 3(2x - 5),$   
 $x \in \{-3, -2, -1, 0, 1, 2, 3\}$

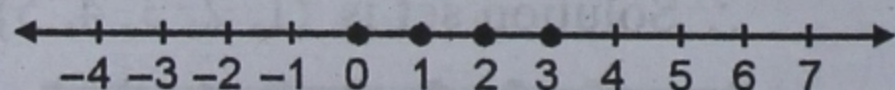
**Sol.**  $7x - 4(3 - x) \geq 3(2x - 5)$

$$\Rightarrow 7x - 12 + 4x \geq 6x - 15$$

$$\Rightarrow 7x + 4x - 6x \geq -15 + 12$$

$$\Rightarrow 5x \geq -3 \quad \Rightarrow x \geq -\frac{3}{5}$$

$\therefore$  Solution set is  $\{0, 1, 2, 3\}$  Ans.



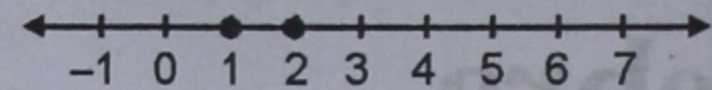
**Q.6.**  $11 - 3x > 2 + x, x \in \{1, 2, 3, 4, 5, 6\}$

**Sol.**  $11 - 3x > 2 + x$

$$\Rightarrow -3x - x > 2 - 11 \quad \Rightarrow -4x > -9$$

$$\Rightarrow 4x < 9 \quad \Rightarrow x < \frac{9}{4} \Rightarrow x < 2\frac{1}{4}$$

$\therefore$  Solution set is  $\{1, 2\}$  Ans.



**Q.7.**  $4 - 3x \geq 3x - 14, x \in \mathbb{N}$

**Sol.**  $4 - 3x \geq 3x - 14$

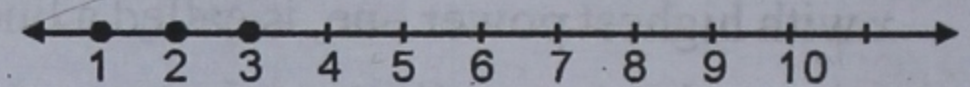
$$\Rightarrow -3x - 3x \geq -14 - 4$$

$$\Rightarrow -6x \geq -18 \quad \Rightarrow 6x \leq 18$$

$$\Rightarrow x \leq \frac{18}{6} \quad \Rightarrow x \leq 3.$$

$\therefore$  Solution set is  $\{1, 2, 3\}$   $\{\because x \in \mathbb{N}\}$

Ans.



**Q.8.**  $6 - 5x > 3 - 4x, x \in \mathbb{W}$

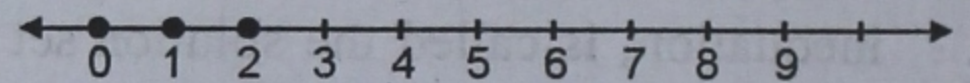
**Sol.**  $6 - 5x > 3 - 4x$

$$\Rightarrow -5x + 4x > 3 - 6$$

$$\Rightarrow -x > -3 \quad \Rightarrow x < 3$$

$\therefore$  Solution set is  $\{0, 1, 2\}$   $(\because x \in \mathbb{W})$

Ans.



**Q.9.**  $30 - 2(3x - 4) < 24, x \in \mathbb{W}$

**Sol.**  $30 - 2(3x - 4) < 24$

$$\Rightarrow 30 - 6x + 8 < 24$$

$$\Rightarrow -6x < 24 - 30 - 8$$

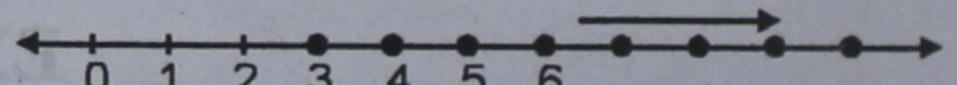
$$\Rightarrow -6x < 24 - 38 \Rightarrow -6x < -14$$

$$\Rightarrow 6x > 14 \quad \Rightarrow x > \frac{14}{6}$$

$$\Rightarrow x > \frac{7}{3} \quad \Rightarrow x > 2\frac{1}{3}$$

$\therefore$  Solution set is  $\{3, 4, 5, 6, \dots\}$ ,

$(\because x \in \mathbb{W})$  Ans.



**Q.10.**  $\frac{3}{5}x - \frac{2x-1}{3} > 1, x \in \mathbb{I}$

**Sol.**  $\frac{3}{5}x - \frac{2x-1}{3} > 1$

Multiplying by 15, the LCM of 5 and 3

$$15 \times \frac{3}{5}x - 15 \times \frac{2x-1}{3} > 1 \times 15$$

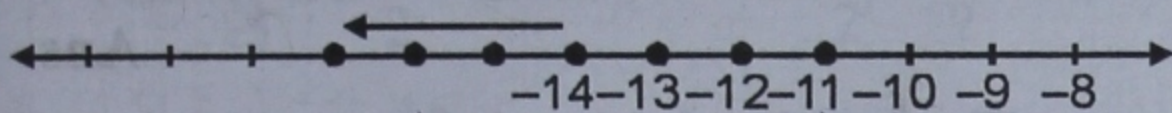
$$\Rightarrow 9x - 5(2x - 1) > 15$$

$$\Rightarrow 9x - 10x + 5 > 15$$

$$\Rightarrow -x > 15 - 5 \Rightarrow -x > 10$$

$$\Rightarrow x < -10$$

$\therefore$  Solution set is  $\{-11, -12, -13, \dots\}$   
 $(\because x \in \mathbb{I})$  Ans.



**Q.11.**  $-3 < 2x - 1 < x + 4, x \in \mathbb{I}$

**Sol.**  $-3 < 2x - 1 < x + 4$

(i)  $-3 < 2x - 1 \Rightarrow -2x < -1 + 3$

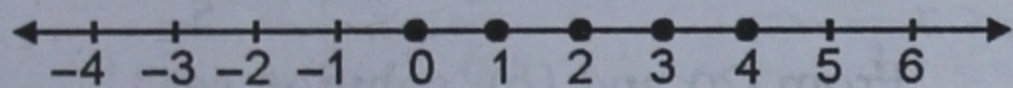
$$\Rightarrow -2x < 2 \Rightarrow 2x > -2$$

$$\Rightarrow x > \frac{-2}{2} \Rightarrow x > -1$$

(ii)  $2x - 1 < x + 4 \Rightarrow 2x - x < 4 + 1$

$$\Rightarrow x < 5$$

From (i) and (ii), solution set is  $\{0, 1, 2, 3, 4\}$  Ans.



**Q.12.**  $2 + 4x < 2x - 5 < 3x, x \in \mathbb{I}$

**Sol.**  $2 + 4x < 2x - 5 < 3x$

(i)  $2 + 4x < 2x - 5$

$$\Rightarrow 4x - 2x < -5 - 2 \Rightarrow 2x < -7$$

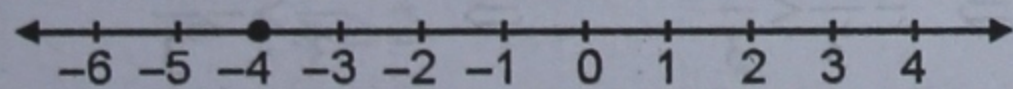
$$\Rightarrow x < -\frac{7}{2} \Rightarrow x < -3\frac{1}{2}$$

(ii)  $2x - 5 < 3x \Rightarrow -3x + 2x < 5$

$$\Rightarrow -x < 5 \Rightarrow x > -5$$

From (i) and (ii)

Solution set is  $\{-4\}$  Ans.



**Q.13.** Find the smallest value of  $x$ , which

satisfies the inequation  $2x + \frac{7}{2} > \frac{5x}{3} + 3,$

$x \in \mathbb{I}$

**Sol.**  $2x + \frac{7}{2} > \frac{5x}{3} + 3$

Multiplying by 6, the LCM of 2 and 3

$$6 \times 2x + 6 \times \frac{7}{2} > 6 \times \frac{5x}{3} + 6 \times 3$$

$$\Rightarrow 12x + 21 > 10x + 18$$

$$\Rightarrow 12x - 10x > 18 - 21$$

$$\Rightarrow 2x > -3 \Rightarrow x > -\frac{3}{2} \Rightarrow x > -1\frac{1}{2}$$

$\therefore x \in \mathbb{I}$  and it has smallest value  
 $x = 1$  Ans.

**Q.14.** If  $10 - 5x < 5(x + 6)$ , find the smallest value of  $x$ , when (i)  $x \in \mathbb{I}$  (ii)  $x \in \mathbb{W}$  (iii)  $x \in \mathbb{N}$

**Sol.**  $10 - 5x < 5(x + 6)$

$$\Rightarrow 10 - 5x < 5x + 30$$

$$\Rightarrow -5x - 5x < 30 - 10$$

$$\Rightarrow -10x < 20 \Rightarrow 10x > -20$$

$$\Rightarrow x > \frac{-20}{10} \Rightarrow x > -2$$

(i) When  $x \in \mathbb{I}$  and it has the smallest value.

$$\therefore x = -1$$

(ii) When  $x \in \mathbb{W}$  and it has the smallest value.

$$\therefore x = 0$$

(iii) When  $x \in \mathbb{N}$  and it has the smallest value.

$$\therefore x = 1$$
 Ans.

**Q.15.** Solve each of the inequations given below and graph the solution set on the number line :

(i)  $2x + 3 \leq 3x + 1, x \in \mathbb{R}$

(ii)  $\frac{5x-8}{3} \geq \frac{4x-7}{2}, x \in \mathbb{R}$

(iii)  $\frac{5x}{4} - \frac{4x-1}{3} > 1, x \in \mathbb{R}$

(iv)  $2 \leq 2x - 3 \leq 5, x \in \mathbb{R}$

(v)  $-1 \leq 3 + 4x < 23, x \in \mathbb{R}$

(vi)  $2x - 5 \leq 5x + 4 < 11, x \in \mathbb{R}$

(vii)  $-\frac{2}{3} < 1 + \frac{x}{3} \leq \frac{2}{3}, x \in \mathbb{R}$

(viii)  $-2 \leq \frac{1}{2} - \frac{2x}{3} < 1\frac{5}{6}, x \in \mathbb{I}$

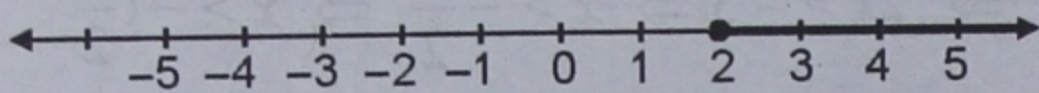
(ix)  $1 \geq 15 - 7x > 2x - 27, x \in \mathbb{N}$

**Sol.** (i)  $2x + 3 \leq 3x + 1, x \in \mathbb{R}$

$$\Rightarrow 2x - 3x \leq 1 - 3 \Rightarrow -x \leq -2$$

$$\Rightarrow x \geq 2$$

$\therefore$  Solution set is  $\{x : x \geq 2, x \in \mathbb{R}\}$



$$(ii) \frac{5x-8}{3} \geq \frac{4x-7}{2}, x \in \mathbb{R}$$

$$(5x-8) \times 2 \geq (4x-7) \times 3$$

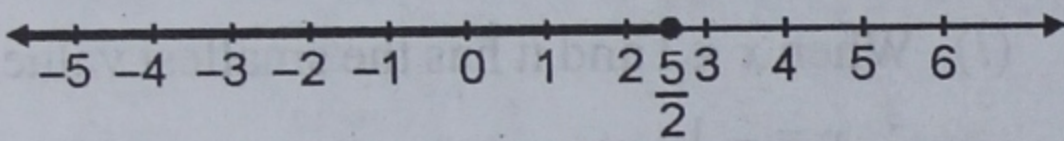
(By Cross multiplication)

$$\Rightarrow 10x - 16 \geq (12x - 21)$$

$$\Rightarrow 10x - 12x \geq -21 + 16$$

$$\Rightarrow -2x \geq -5 \Rightarrow 2x \leq 5 \Rightarrow x \leq \frac{5}{2}$$

$\therefore$  Solution set is  $\{x : x \leq \frac{5}{2}, x \in \mathbb{R}\}$



$$(iii) \frac{5x}{4} - \frac{4x-1}{3} > 1, x \in \mathbb{R}$$

Multiplying by 12, the LCM of 4 and 3

$$12 \times \frac{5x}{4} - 12 \times \frac{4x-1}{3} > 12 \times 1$$

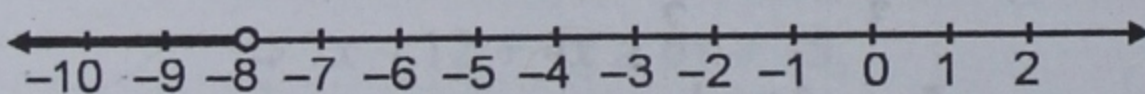
$$\Rightarrow 15x - 4(4x-1) > 12$$

$$\Rightarrow 15x - 16x + 4 > 12$$

$$\Rightarrow -x > 12 - 4$$

$$\Rightarrow -x > 8 \Rightarrow x < -8$$

Hence, solution set is  $\{x : x < -8, x \in \mathbb{R}\}$



$$(iv) 2 \leq 2x - 3 \leq 5, x \in \mathbb{R}$$

$$(a) 2 \leq 2x - 3 \Rightarrow 2 + 3 \leq 2x$$

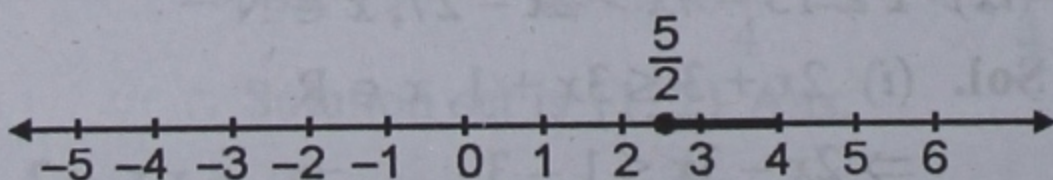
$$\Rightarrow 2x \geq 5 \Rightarrow x \geq \frac{5}{2}$$

$$(b) 2x - 3 \leq 5 \Rightarrow 2x \leq 5 + 3$$

$$\Rightarrow 2x \leq 8 \Rightarrow x \leq 4$$

From (a) and (b), solution set

$$= \{x : \frac{5}{2} \leq x \leq 4, x \in \mathbb{R}\}$$



(v) We have

$$-1 \leq 3 + 4x < 23, x \in \mathbb{R}$$

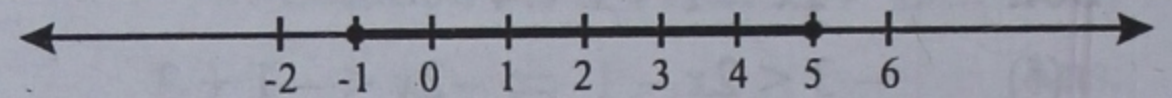
$$\Rightarrow -1 - 3 \leq 4x < 23 - 3 \Rightarrow -4 \leq 4x < 20$$

$$\Rightarrow 1 \leq x < 5, x \in \mathbb{R}$$

$\therefore$  Solution set =  $\{-1 \leq x < 5; x \in \mathbb{R}\}$

**Ans.**

The graph of the solution set is shown below:



$$(vi) 2x - 5 \leq 5x + 4 < 11, x \in \mathbb{R}$$

$$(a) 2x - 5 \leq 5x + 4 \Rightarrow 2x - 5x \leq 4 + 5$$

$$\Rightarrow -3x \leq 9 \Rightarrow 3x \geq -9$$

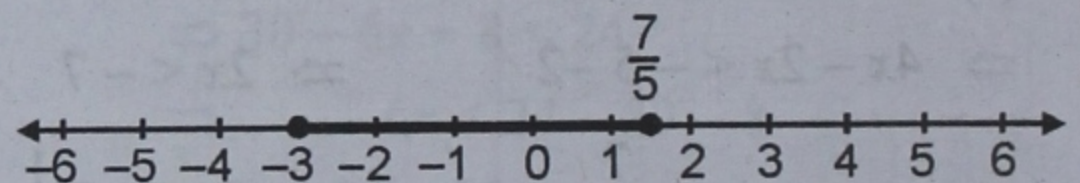
$$\Rightarrow x \geq -3$$

$$(b) 5x + 4 < 11 \Rightarrow 5x < 11 - 4$$

$$\Rightarrow 5x < 7 \Rightarrow x < \frac{7}{5}$$

From (a) and (b), solution set

$$= \{x : -3 \leq x < \frac{7}{5}, x \in \mathbb{R}\}$$



$$(vii) -\frac{2}{3} < 1 + \frac{x}{3} \leq \frac{2}{3}, x \in \mathbb{R}$$

$$(a) -\frac{2}{3} < 1 + \frac{x}{3} \Rightarrow -\frac{x}{3} < 1 + \frac{2}{3}$$

$$\Rightarrow -\frac{x}{3} < \frac{5}{3} \Rightarrow \frac{x}{3} > -\frac{5}{3}$$

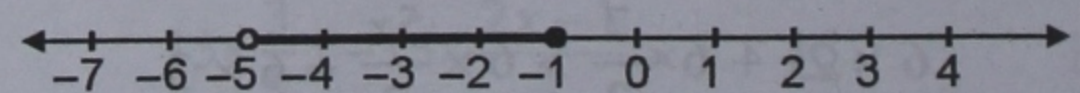
$$\Rightarrow x > -5$$

$$(b) 1 + \frac{x}{3} \leq \frac{2}{3} \Rightarrow \frac{x}{3} \leq \frac{2}{3} - 1$$

$$\Rightarrow \frac{x}{3} \leq -\frac{1}{3} \Rightarrow x \leq -1$$

From (a) and (b), solution set

$$= \{x : -5 < x \leq -1, x \in \mathbb{R}\}$$



$$(viii) -2 \leq \frac{1}{2} - \frac{2x}{3} < 1\frac{5}{6}, x \in I$$

$$(a) -2 \leq \frac{1}{2} - \frac{2x}{3} \Rightarrow \frac{2x}{3} \leq \frac{1}{2} + 2$$

$$\Rightarrow \frac{2}{3}x \leq \frac{5}{2} \Rightarrow x \leq \frac{5}{2} \times \frac{3}{2}$$

$$\Rightarrow x \leq \frac{15}{4} \Rightarrow x \leq 3\frac{3}{4}$$

$$(b) \frac{1}{2} - \frac{2}{3}x < 1\frac{5}{6}$$

$$\Rightarrow \frac{-2}{3}x < 1\frac{5}{6} - \frac{1}{2}$$

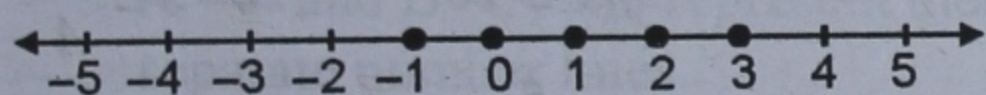
$$\Rightarrow -\frac{2}{3}x < \frac{11}{6} - \frac{1}{2}$$

$$\Rightarrow -\frac{2}{3}x < \frac{11-3}{6}$$

$$\Rightarrow -\frac{2}{3}x < \frac{8}{6} \Rightarrow \frac{2}{3}x > -\frac{8}{6}$$

$$\Rightarrow x > -\frac{8}{6} \times \frac{3}{2} \Rightarrow x > -2$$

From (a) and (b), solution set is  $\{-1, 0, 1, 2, 3\}$ , ( $\because x \in I$ )



$$(ix) 1 \geq 15 - 7x > 2x - 27, x \in N$$

$$(a) 1 \geq 15 - 7x \Rightarrow 7x \geq 15 - 1$$

$$\Rightarrow 7x \geq 14 \Rightarrow x \geq \frac{14}{7}$$

$$\Rightarrow x \geq 2$$

$$(b) 15 - 7x > 2x - 27$$

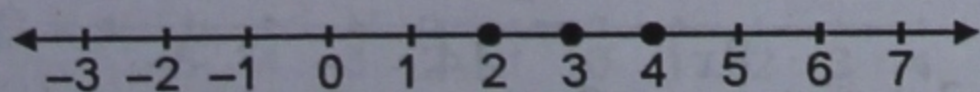
$$\Rightarrow -7x - 2x > -27 - 15$$

$$\Rightarrow -9x > -42 \Rightarrow 9x < 42$$

$$\Rightarrow x < \frac{42}{9} \Rightarrow x < \frac{14}{3}$$

$$\Rightarrow x < 4\frac{2}{3}$$

From (a) and (b) solution set is  $\{2, 3, 4\}$



**Q.16.** Find the range of values of  $x$ , which satisfy  $-\frac{1}{3} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$ . Graph the values of  $x$  on the real line.

$$\text{Sol. } -\frac{1}{3} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$$

$$(a) -\frac{1}{3} \leq \frac{x}{2} - \frac{4}{3}$$

$$\Rightarrow -\frac{x}{2} \leq -\frac{4}{3} + \frac{1}{3} \Rightarrow -\frac{x}{2} \leq -\frac{3}{3}$$

$$\Rightarrow -\frac{x}{2} \leq -1 \Rightarrow \frac{x}{2} \geq 1$$

$$\Rightarrow x \geq 2$$

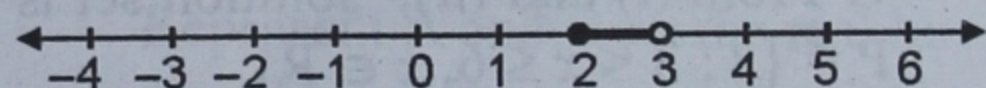
$$(b) \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6} \Rightarrow \frac{x}{2} - \frac{4}{3} < \frac{1}{6}$$

$$\Rightarrow \frac{x}{2} < \frac{1}{6} + \frac{4}{3} \Rightarrow \frac{x}{2} < \frac{1+8}{6}$$

$$\Rightarrow \frac{x}{2} < \frac{9}{6} \Rightarrow x < \frac{9}{6} \times 2$$

$$\Rightarrow x < 3$$

$\therefore$  From (a) and (b), solution set is  $\{x : 2 \leq x < 3, x \in R\}$



**Q.17.** Solve the following inequation and graph the solution on the number line.

$$-2\frac{2}{3} \leq x + \frac{1}{3} < 3\frac{1}{3}; x \in R$$

Graph the values of  $x$  on the number line.

$$\text{Sol. Given } -2\frac{2}{3} \leq x + \frac{1}{3} < 3\frac{1}{3}; x \in R$$

$$-\frac{8}{3} \leq x + \frac{1}{3} < \frac{10}{3}$$

Multiplying by 3, L.C.M. of fractions, we get

$$-8 \leq 3x + 1 < 10$$

$$-8 - 1 \leq 3x + 1 - 1 < 10 - 1$$

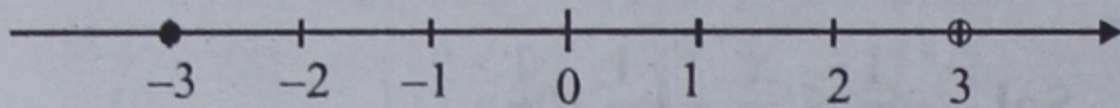
[Add -1]

$$-9 \leq 3x < 9$$

$$-3 \leq x < 3$$

[Dividing by 3]

Hence, the solution set is  $\{x : x \in \mathbb{R}, -3 \leq x < 3\}$



The graph of the solution set is shown by the thick portion of the number line. The solid circle  $-3$  indicates that the number  $-3$  is included among the solutions whereas the open circle at  $3$  states that  $3$  is not included among the solutions.

18. Given :  $P = \{x : 5 < 2x - 1 \leq 11, x \in \mathbb{R}\}$   
and  $Q = \{x : -1 \leq 3 + 4x < 23, x \in \mathbb{I}\}$ ,  
where  $\mathbb{R} = \{\text{real numbers}\}$  and  
 $\mathbb{I} = \{\text{integers}\}$ .

Represent  $P$  and  $Q$  on the number line.

Write down the elements of  $P \cap Q$ .

Sol.  $P = \{x : 5 < 2x - 1 \leq 11, x \in \mathbb{R}\}$

$Q = \{x : -1 \leq 3 + 4x < 23, x \in \mathbb{I}\}$

(i)  $5 < 2x - 1 \Rightarrow -2x < -1 - 5$   
 $\Rightarrow -2x < -6 \Rightarrow 2x > 6$   
 $\Rightarrow x > 3$

(ii)  $2x - 1 \leq 11 \Rightarrow 2x \leq 11 + 1$

$\Rightarrow 2x \leq 12 \Rightarrow x \leq \frac{12}{2} \Rightarrow x \leq 6$

$\therefore$  From (i) and (ii), solution set is

$P = \{x : 3 < x \leq 6, x \in \mathbb{R}\}$

Again (i)  $-1 \leq 3 + 4x$

$\Rightarrow -4x \leq 3 + 1 \Rightarrow -4x \leq 4$

$\Rightarrow 4x \geq -4 \Rightarrow x \geq -1$

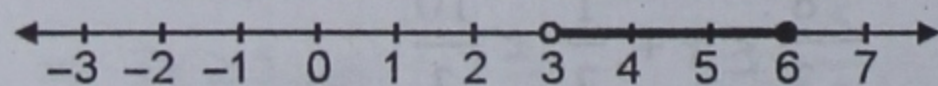
(ii)  $3 + 4x < 23 \Rightarrow 4x < 23 - 3$

$\Rightarrow 4x < 20 \Rightarrow x < \frac{20}{4}$

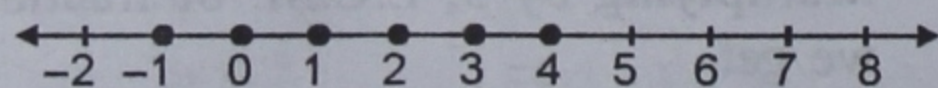
$\Rightarrow x < 5$

From (i) and (ii), solution set is

$Q = \{-1, 0, 1, 2, 3, 4\}, \quad \{\because x \in \mathbb{I}\}$



$P = \{x : 3 < x \leq 6, x \in \mathbb{R}\}$



$Q = \{-1, 0, 1, 2, 3, 4\}$

$\therefore P \cap Q = \{4\}$  Ans.

- Q.19. Solve each of the following inequations and graph the solution set on the number line:

(i)  $5x - 11 \leq 7x - 5 < 9$

(ii)  $2x - 1 \geq x + \frac{7-x}{3} > 2$

Sol. (i)  $5x - 11 \leq 7x - 5 < 9$

(a)  $5x - 11 \leq 7x - 5$

$\Rightarrow 5x - 7x \leq -5 + 11$

$\Rightarrow -2x \leq 6 \Rightarrow 2x \geq -6$

$\Rightarrow x \geq -\frac{6}{2} \Rightarrow x \geq -3$

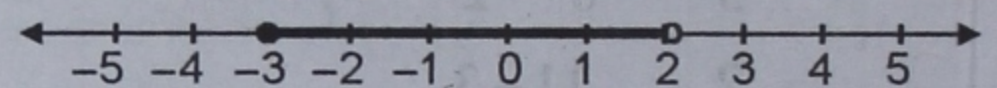
(b)  $7x - 5 < 9$

$\Rightarrow 7x < 9 + 5$

$\Rightarrow 7x < 14 \Rightarrow x < \frac{14}{7}$

$\Rightarrow x < 2$ .

From (a) and (b), solution set is  
 $\{x : -3 \leq x < 2, x \in \mathbb{R}\}$



(ii)  $2x - 1, \geq x + \frac{7-x}{3} > 2$

(a)  $2x - 1 \geq x + \frac{7-x}{3}$

Multiplying by 3, we get

$6x - 3 \geq 3x + 7 - x$

$\Rightarrow 6x - 3x + x \geq 7 + 3$

$\Rightarrow 4x \geq 10 \Rightarrow x \geq \frac{10}{4}$

$\Rightarrow x \geq \frac{5}{2}$

(b)  $x + \frac{7-x}{3} > 2$

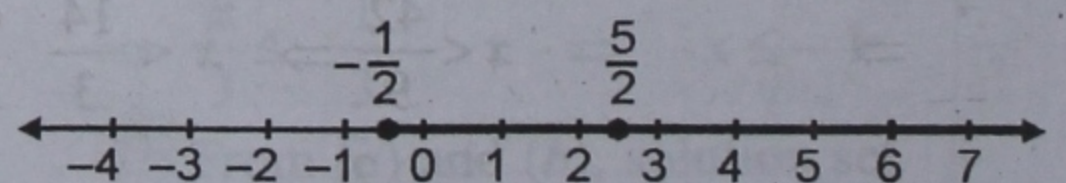
Multiplying by 3, we get

$3x + 7 - x > 6 \Rightarrow 2x > 6 - 7$

$\Rightarrow 2x > -1 \Rightarrow x > -\frac{1}{2}$

From (a) and (b), solution set is

$\{x : x \geq \frac{5}{2}, x \in \mathbb{R}\}$



- Q. 20. Solve the inequation and represent the solution set on the number line.

$-3 + x \leq \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x$ , where  $x \in \mathbb{I}$ .

**Sol. Given :**

$$-3 + x \leq \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x, \text{ where } x \in \mathbb{I}$$

$$\Rightarrow -3 + x \leq \frac{8x}{3} + 2 \quad \dots(i)$$

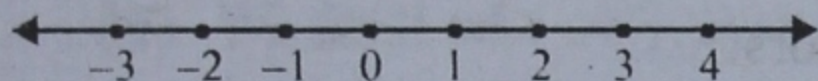
$$\text{and } \frac{8x}{3} + 2 \leq \frac{14}{3} + 2x \quad \dots(ii)$$

$$\Rightarrow -5 \leq \frac{5x}{3} \text{ and } \frac{2x}{3} \leq \frac{8}{3} \Rightarrow x \geq -3 \text{ and } x \leq 4$$

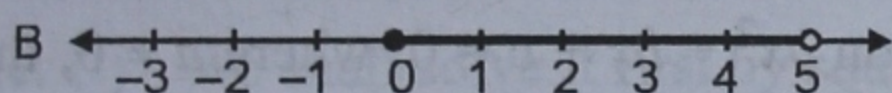
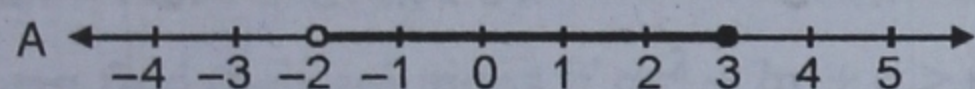
$$\therefore -3 \leq x \leq 4$$

$$\text{Solution set} = \{-3, -2, -1, 0, 1, 2, 3, 4\}$$

Number line



**Q.21.** The given diagram represents two sets A and B on the number line :



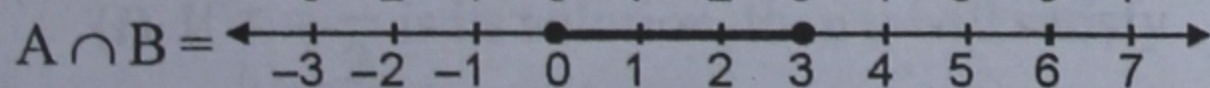
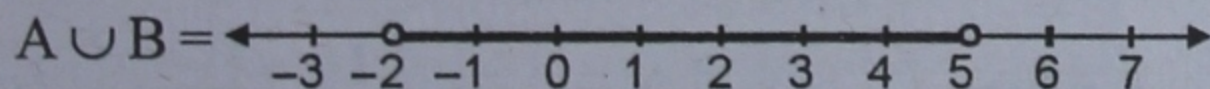
- (i) Write down A and B in set builder form.  
 (ii) Write down  $A \cup B$ ,  $A \cap B$ ,  $A' \cap B$ ,  $A - B$  and  $B - A$  and represent them on separate number lines.

**Sol.** From the given number line, we find that

$$(i) A = \{x : -2 < x \leq 3, x \in \mathbb{R}\} \text{ and } B = \{x : 0 \leq x < 5, x \in \mathbb{R}\}.$$

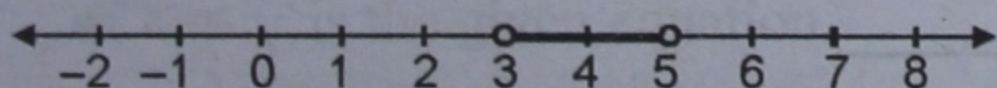
$$(ii) A \cup B = \{x : -2 < x < 5, x \in \mathbb{R}\} \text{ and } A \cap B = \{x : 0 \leq x \leq 3, x \in \mathbb{R}\}$$

Now number lines of



$$\text{Now } A' = \{x : x > 3\}$$

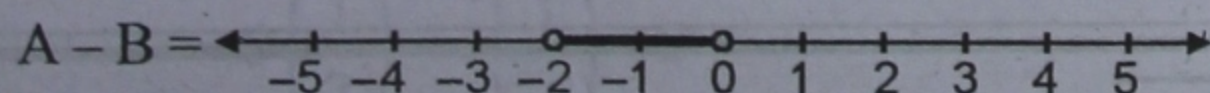
$$\therefore A' \cap B = \{x : 3 < x < 5, x \in \mathbb{R}\}$$



$$A - B = \{x : 2 < x < 0, x \in \mathbb{R}\} \text{ and}$$

$$B - A = \{x : 3 < x < 5, x \in \mathbb{R}\}$$

Now number lines of



Number of line of  $B - A$  is same as

$A' \cap B$  as given above.

**Q. 22.**  $A = \{x : 11x - 5 > 7x + 3, x \in \mathbb{R}\}$  and  $B = \{x : 18x - 9 \geq 15 + 12x, x \in \mathbb{R}\}$

Find the range of set  $A \cap B$  and represent it on a number line

**Sol.**  $A = \{x : 11x - 5 > 7x + 3, x \in \mathbb{R}\}$

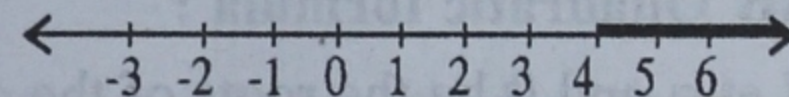
$$B = \{x : 18x - 9 \geq 15 + 12x, x \in \mathbb{R}\}$$

$$\begin{aligned} \text{Now, } A &= 11x - 5 > 7x + 3 \\ &= 11x - 7x > 3 + 5 = 4x > 8 \\ &= x > 2, \quad x \in \mathbb{R} \end{aligned}$$

$$\begin{aligned} B &= 18x - 9 \geq 15 + 12x \\ &= 18x - 12x \geq 15 + 9 \\ &= 6x \geq 24 \\ &= x \geq 4 \quad (x \in \mathbb{R}) \end{aligned}$$

$$\therefore A \cap B = x \geq 4, x \in \mathbb{R}$$

Hence Range of  $A \cap B = \{x : x \geq 4, x \in \mathbb{R}\}$  and its graph will be.



**Q.23.** Given :  $A = \{x : 5x - 4 \geq 6, x \in \mathbb{R}\}$  and  $B = \{x : 5 - x > 1, x \in \mathbb{R}\}$

Represent A and B on the real line.

Find (i)  $A \cap B$  (ii)  $A' \cap B$ .

**Sol.**  $A = \{x : 5x - 4 \geq 6, x \in \mathbb{R}\}$

$$B = \{x : 5 - x > 1, x \in \mathbb{R}\}$$

In set A,  $5x - 4 \geq 6$

$$\Rightarrow 5x \geq 6 + 4 \Rightarrow 5x \geq 10$$

$$\Rightarrow x \geq \frac{10}{5} \Rightarrow x \geq 2$$

$$\therefore A = \{x : x \geq 2, x \in \mathbb{R}\}$$

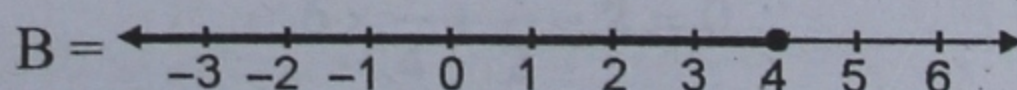
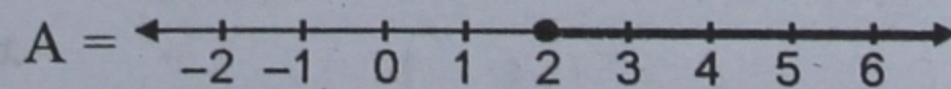
In set B,  $5 - x > 1$

$$\Rightarrow -x > 1 - 5 \Rightarrow -x > -4$$

$$\Rightarrow x < 4$$

$$\therefore B = \{x : x < 4, x \in \mathbb{R}\}.$$

Now, number line of A and B are given.



Now (i)  $A \cap B = \{x : 2 \leq x < 4, x \in \mathbb{R}\}$

and (ii)  $A' = \{x : x < 2, x \in \mathbb{R}\}$

$$\therefore A' \cap B = \{x : x < 2, x \in \mathbb{R}\} \text{ Ans.}$$