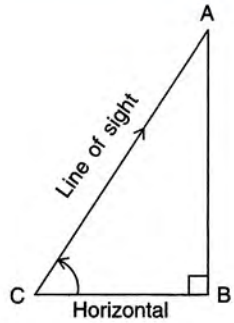
22 Heights and Distances

22.1 Angles of elevation and depression :

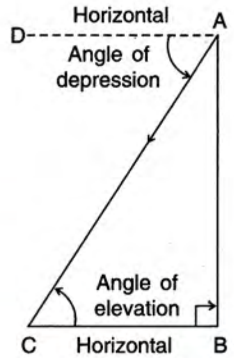


Let AB be a tower (or pillar or minar, *etc.*) standing on a level ground and a man, standing at any point C on the level ground, is viewing an object at A.

The line CA, joining his eye to the object, is called the **line of sight**.

The angle, which the line of sight makes with the horizontal is called the **angle of elevation.**

∴ In the given figure; **angle ACB** is the **angle of elevation.**



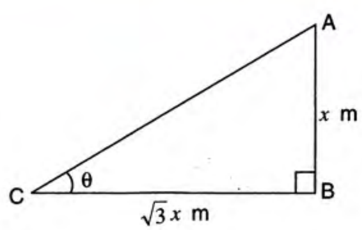
Similarly, if the man is at A and is viewing an object C on the level ground, then the angle, which the **line of sight** (AC) makes with horizontal, is called the **angle of depression**.

∴ In the given figure; angle DAC is **angle of depression.**

Angle of elevation of point A as seen from point C **is equal** to the angle of depression of point C as seen from point A. *i.e.*, $\angle ACB = \angle DAC$

1 The length of the shadow of a vertical tower is $\sqrt{3}$ times its height. Find the angle of elevation of the sun.

Solution :



Let the height of the tower be x m

∴ Length of its shadow = $\sqrt{3}x$ m

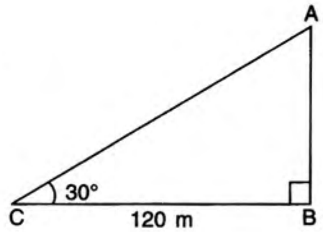
If θ is the angle of elevation of the sun, then

$$\tan\theta = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}} = \tan 30°$$

∴ $\theta = 30°$ **Ans.**

2 The angle of elevation of the top of a tower at a distance of 120 m from its foot on a horizontal plane is found to be 30°. Find the height of the tower.

Solution :



Let AB be the tower and C be the point of observation

$\therefore$ BC = 120 m

Since, angle of elevation is 30°

$\therefore \quad \angle ACB = 30°$

In Δ ABC, $\dfrac{AB}{120} = \tan 30°$

$\Rightarrow \quad$ **AB** $= 120 \times \dfrac{1}{\sqrt{3}} =$ **69·28 m** **Ans.**

> **3** A guard observes an enemy boat, from an observation tower at a height of 180 m above sea level, to be at an angle of depression of 29°.
> (i) Calculate, to the nearest metre, the distance of the boat from the foot of the observation tower.
> (ii) After some time, it is observed that the boat is 200 m from the foot of the observation tower. Calculate the new angle of depression.

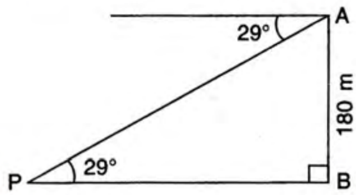
Solution :

(i) Let P be the boat and AB be the observation tower. In right triangle ABP,

$\tan 29° = \dfrac{AB}{PB}$

$\Rightarrow \quad 0{\cdot}5543 = \dfrac{180}{PB}$

$\therefore \quad$ **PB** $= \dfrac{180}{0{\cdot}5543} =$ **325 m (App.)**



Ans.

Alternative Method :

In Δ ABP, $\angle APB + \angle PAB = 90° \Rightarrow \angle PAB = 90° - 29° = 61°$

$\therefore \quad \dfrac{BP}{AB} = \tan 61°$

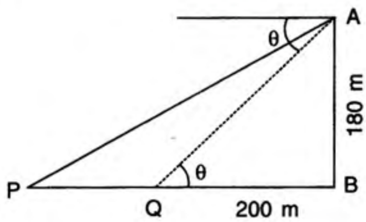
$\Rightarrow \quad$ **BP** $= 180 \times \tan 61° = 180 \times 1{\cdot}804 =$ **325 m (App.)** **Ans.**

(ii) Let Q be the new position of the boat and θ be the new angle of depression

$\therefore \quad \tan \theta = \dfrac{AB}{QB}$

$= \dfrac{180}{200} = 0{\cdot}9000$

$= \tan 41° \, 59'$

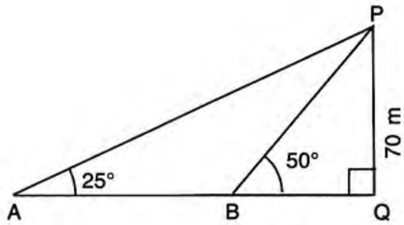


$\therefore \quad$ **θ = 41°59′** **Ans.**

> **4** Two people standing on the same side of a tower in a straight line with it, measure the angles of elevation of the top of the tower as 25° and 50° respectively. If the height of the tower is 70 m, find the distance between the two people. **[2004]**

Solution :

According to the given statement, the figure will be as shown alongside in which PQ is the tower so PQ = 70 m; and A and B be the positions of two people, such that ∠PAQ = 25° and ∠PBQ = 50°



In Δ PAQ, $\tan 25° = \dfrac{PQ}{AQ}$

$\Rightarrow \quad 0{\cdot}4663 = \dfrac{70}{AQ}$ *i.e.* $AQ = \dfrac{70}{0{\cdot}4663}$ m = 150·118 m

In Δ PBQ, $\tan 50° = \dfrac{PQ}{BQ}$

$\Rightarrow \quad 1{\cdot}1918 = \dfrac{70}{BQ}$ *i.e.* $BQ = \dfrac{70}{1{\cdot}1918}$ m = 58·735 m

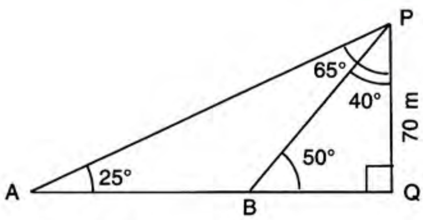
∴ **The distance between the two people**

= AB = AQ – BQ

= (150·118 – 58·735) m

= **91·38 m** **Ans.**

Alternative method :



In Δ PAQ, ∠A = 25°

⇒ ∠APQ = 90° – 25° = 65°

∴ $\tan 65° = \dfrac{AQ}{PQ}$

i.e. $2{\cdot}1445 = \dfrac{AQ}{70}$ ⇒ AQ = 2·1445 × 70 m = 150·115 m

In Δ PBQ, ∠PBQ = 50° ⇒ ∠BPQ = 90° – 50° = 40°

∴ $\tan 40° = \dfrac{BQ}{PQ}$

i.e. $0{\cdot}8391 = \dfrac{BQ}{70}$ ⇒ BQ = 0·8391 × 70 m = 58·737 m

∴ **The distance between the two people**

AB = AQ – BQ =(150·115 – 58·737) m = **91·38 m** **Ans.**

EXERCISE 22(A)

1. The height of a tree is $\sqrt{3}$ times the length of its shadow. Find the angle of elevation of the sun.
2. The angle of elevation of the top of a tower, from a point on the ground and at a distance of 160 m from its foot, is found to be 60°. Find the height of the tower.
3. A ladder is placed along a wall such that its upper end is resting against a vertical wall. The foot of the ladder is 2·4 m from the wall and the ladder is making an angle of 68° with the ground. Find the height, upto which the ladder reaches.
4. Two persons are standing on the opposite sides

of a tower. They observe the angles of elevation of the top of the tower to be 30° and 38° respectively. Find the distance between them, if the height of the tower is 50 m.

5. A kite is attached to a string. Find the length of the string, when the height of the kite is 60 m and the string makes an angle 30° with the ground.

6. A boy, 1·6 m tall, is 20 m away from a tower and observes the angle of elevation of the top of the tower to be (i) 45° (ii) 60°. Find the height of the tower in each case.

7. The upper part of a tree, broken over by the wind, makes an angle of 45° with the ground; and the distance from the root to the point where the top of the tree touches the ground, is 15 m. What was the height of the tree before it was broken ?

8. The angle of elevation of the top of an unfinished tower at a point distance 80 m from its base is 30°. How much higher must the tower be raised so that its angle of elevation at the same point may be 60° ?

9. At a particular time, when the sun's altitude is 30°, the length of the shadow of a vertical tower is 45 m. Calculate :
   (i) the height of the tower,
   (ii) the length of the shadow of the same tower, when the sun's altitude is :
   (a) 45° (b) 60°.

10. Two vertical poles are on either side of a road. A 30 m long ladder is placed between the two poles. When the ladder rests against one pole, it makes angle 32°24′ with the pole and when it is turned to rest against another pole, it makes angle 32°24′ with the road. Calculate the width of the road.

11. Two climbers are at points A and B on a vertical cliff face. To an observer C, 40 m from the foot of the cliff, on the level ground, A is at an elevation of 48° and B of 57°. What is the distance between the climbers ?

12. A man stands 9 m away from a flag-pole. He observes that angle of elevation of the top of the pole is 28° and the angle of depression of the bottom of the pole is 13°. Calculate the height of the pole.

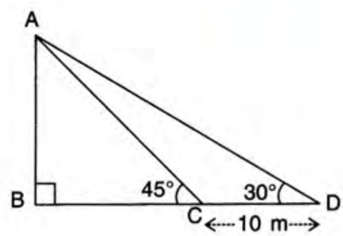
13. From the top of a cliff 92 m high, the angle of depression of a buoy is 20°. Calculate, to the nearest metre, the distance of the buoy from the foot of the cliff.

5 The length of the shadow of a vertical tower on level ground increases by 10 m, when the altitude of the sun changes from 45° to 30°. Calculate the height of the tower, correct to two decimal places. **[2006]**

Solution :

Let AB be the tower. BC be its shadow when the sun's altitude is 45°, *i.e.* ∠ACB = 45°.

When the sun's altitude changes to 30°, the length of the shadow is BD and ∠ADB = 30°



Clearly, DC = 10 m.

In triangle ABC,

$$\tan 45° = \frac{AB}{BC} \Rightarrow 1 = \frac{AB}{BC} \Rightarrow BC = AB$$

In triangle ABD,

$$\tan 30° = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BD} \Rightarrow BD = AB\sqrt{3}$$

Since $BD - BC = DC$

$AB\sqrt{3} - AB = 10$ $[\because BC = AB \text{ and } BD = AB\sqrt{3}]$

$$AB(\sqrt{3}-1) = 10$$

$$\textbf{AB} = \frac{10}{\sqrt{3}-1} = \frac{10}{1.732-1} = \frac{10}{0.732} = \textbf{13.66 m} \qquad \textbf{Ans.}$$

OR,

$$\textbf{AB} = \frac{10}{\sqrt{3}-1} = \frac{10}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{10(1.732+1)}{3-1} = 5 \times 2.732 = \textbf{13.66 m} \qquad \textbf{Ans.}$$

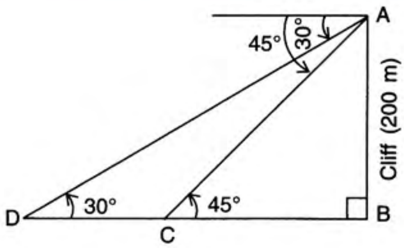
6 An observer on the top of a cliff; 200 m above the sea-level, observes the angles of depression of the two ships to be 45° and 30° respectively. Find the distance between the ships, if the ships are :

(i) on the same side of the cliff,

(ii) on the opposite sides of the cliff.

Solution :

(i) *Ships on the same side of the cliff.*



In the figure, cliff AB = 200 m,

C and D are the positions of two ships.

Required to find : Length of CD.

In right-angled triangle ABC,

$$\tan 45° = \frac{200}{BC} \Rightarrow 1 = \frac{200}{BC} \Rightarrow BC = 200 \text{ m.}$$

In right-angled triangle ABD,

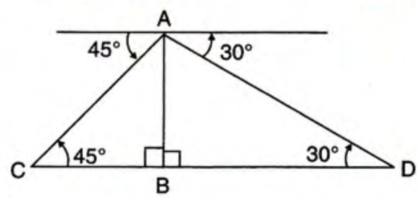
$$\tan 30° = \frac{200}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{200}{BD} \Rightarrow BD = 200\sqrt{3} \text{ m} = 200 \times 1.732 = 346.4 \text{ m}$$

∴ **Distance between the ships** = CD

= BD – BC

= 346.4 m – 200 m = **146.4 m** **Ans.**

(ii) *When the ships are on the opposite sides of the cliff;*



The distance between the ships = CD

= BD + BC

= 346.4 m + 200 m = **546.4 m** **Ans.**

7 A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this will the car reach the observation tower ?

Solution :

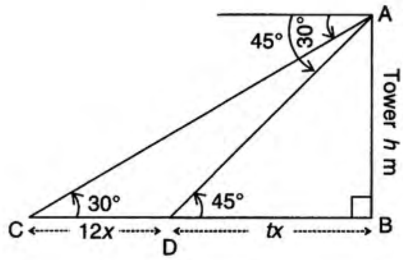
Let AB be the tower and C be the initial position of the car. After 12 minutes, the car reaches the position D.

Clearly, $\angle ACB = 30°$ and $\angle ADB = 45°$.

Let the speed of the car be x m/minute and it will take t minutes to reach the observation tower.

$\therefore$ CD = $12x$ m [Since, distance = time × speed]

and DB = $t\,x$ m



In Δ ABD, $\dfrac{h}{tx} = \tan 45° = 1 \quad \therefore h = t\,x$

In Δ ACB, $\dfrac{h}{12\,x + tx} = \tan 30°$

$\Rightarrow \quad \dfrac{t\,x}{(12+t)\,x} = \dfrac{1}{\sqrt{3}}$ (Since $h = tx$)

$\therefore \quad \sqrt{3}\,t = 12 + t$

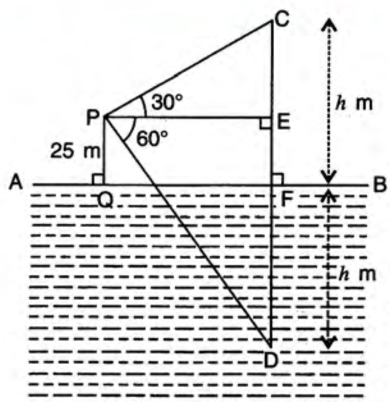
$\Rightarrow \quad \sqrt{3}\,t - t = 12$

$\Rightarrow \quad t(\sqrt{3} - 1) = 12 \Rightarrow t = \dfrac{12}{\sqrt{3}-1}$ = **16·39 minutes** **Ans.**

8 The angle of elevation of a stationary cloud from a point 25 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°. What is the height of the cloud above that lake-level ?

Solution :

Let AB be the surface of the lake, P be the point of observation which is 25 m above the lake *i.e.* PQ = 25 m.



If C be the cloud and D be its reflection in the lake then according to the properties of reflection, the height of cloud C above the lake-level is equal to the depth of its image D below the lake level *i.e.* CF = DF = h m (let)

As is clear from the figure, $\angle CPE = 30°$

$\angle DPE = 60°$, CE = CF − EF = $(h - 25)$ m

and, DE = DF + EF = $(h + 25)$ m

Clearly, $\tan 30° = \dfrac{CE}{PE}$ [In Δ CPE]

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{h-25}{PE}$ and $PE = \sqrt{3}(h - 25)$ .....(I)

And, $\tan 60° = \dfrac{DE}{PE}$ [In Δ DPE]

$\Rightarrow \quad \sqrt{3} = \dfrac{h+25}{PE}$ and $PE = \dfrac{h+25}{\sqrt{3}}$ .....(II)

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From I and II, we have :

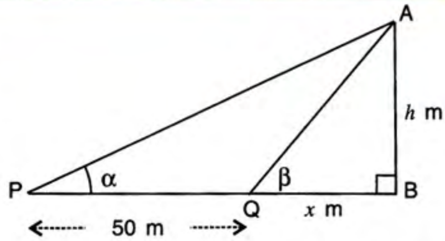
$$\sqrt{3}(h-25) = \frac{h+25}{\sqrt{3}} \quad i.e. \quad 3(h-25) = h+25$$

$$\Rightarrow \quad 3h - 75 = h + 25 \quad i.e. \quad h = 50$$

∴ **The height of the cloud above the lake-level $x = 50$ m** **Ans.**

9 From a point on the ground, the angle of elevation of the top of a vertical tower is found to be such that its tangent is $\frac{3}{5}$. On walking 50 m towards the tower, the tangent of the new angle of elevation of the top of the tower is found to be $\frac{4}{5}$. Find the height of the tower.

Solution :



Let AB be the tower whose height is h m. P be the first point of observation such that if the angle of elevation of the top of the tower at point P is α; then $\tan \alpha = \frac{3}{5}$.

Let Q be the other point of observation obtained on moving 50 m towards the tower (*i.e.* PQ = 50 m) such that if the angle of elevation of the top of the tower at point Q is β; then $\tan \beta = \frac{4}{5}$.

Assume BQ to be x m.

In Δ APB; $\tan \alpha = \frac{AB}{PB} \Rightarrow \frac{3}{5} = \frac{h}{x+50}$

$\Rightarrow 5h = 3x + 150$ ....(I)

In Δ AQB; $\tan \beta = \frac{AB}{QB} \Rightarrow \frac{4}{5} = \frac{h}{x}$

$\Rightarrow 5h = 4x$ ....(II)

On solving I and II, we get : $h = 120$

∴ **The height of the tower = 120 m** **Ans.**

10 A vertical pole and a vertical tower are on the same level ground. From the top of the pole the angle of elevation of the top of the tower is 60° and the angle of depression of the foot of the tower is 30°. Find the height of the tower if the height of the pole is 20 m. [2008]

Solution :

According to the given statement, the diagram will be as shown on the next page; where BC is the pole and AE is the tower

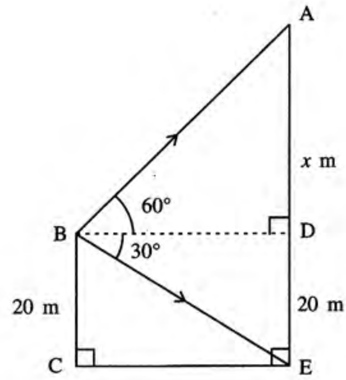
Clearly, BC = 20 m = DE, ∠ABD = 60° and ∠EBD = 30°

Let AD = x m and so AE = $(x + 20)$ m

In right-angled Δ ABD,

$$\tan 60° = \frac{AD}{BD} \Rightarrow \sqrt{3} = \frac{x}{BD} \Rightarrow BD = \frac{x}{\sqrt{3}} \quad ...I$$

In right-angled Δ EBD,



$$\tan 30° = \frac{DE}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{20}{BD}$$

$$\Rightarrow \quad BD = 20\sqrt{3} \qquad ...II$$

From equations I and II, we get :

$$\frac{x}{\sqrt{3}} = 20\sqrt{3}$$

$$\Rightarrow \quad x = 60$$

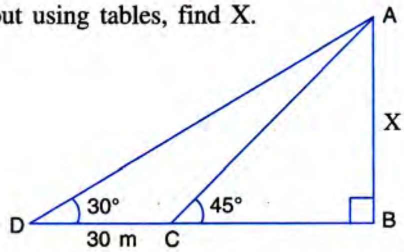
∴ **Height of the tower** = AE = $(x + 20)$m

= (60 + 20) m = **80 m** **Ans.**

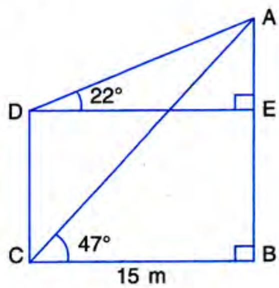
EXERCISE 22(B)

1. In the figure, given below, it is given that AB is perpendicular to BD and is of length X metres. DC = 30 m, ∠ADB = 30° and ∠ACB = 45°.

Without using tables, find X.



2. Find the height of a tree when it is found that on walking away from it 20 m, in a horizontal line through its base, the elevation of its top changes from 60° to 30°.
3. Find the height of a building, when it is found that on walking towards it 40 m in a horizontal line through its base the angular elevation of its top changes from 30° to 45°.
4. From the top of a light house 100 m high, the angles of depression of two ships are observed as 48° and 36° respectively. Find the distance between the two ships (in the nearest metre) if :
   (i) the ships are on the same side of the light house,
   (ii) the ships are on the opposite sides of the light house. **[2010]**
5. Two pillars of equal heights stand on either side of a roadway, which is 150 m wide. At a point in the roadway between the pillars the elevations of the tops of the pillars are 60° and 30°; find the height of the pillars and the position of the point.
6. From the figure, given below, calculate the length of CD.



7. The angle of elevation of the top of a tower is observed to be 60°. At a point, 30 m vertically above the first point of observation, the elevation is found to be 45°. Find :
   (i) the height of the tower,
   (ii) its horizontal distance from the points of observation.
8. From the top of a cliff, 60 metres high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60°. Find the height of the tower.
9. A man on a cliff observes a boat, at an angle of depression 30°, which is sailing towards the shore to the point immediately beneath him. Three minutes later, the angle of depression of the boat is found to be 60°. Assuming that the boat sails at a uniform speed, determine :
   (i) how much more time it will take to reach the shore ?

(ii) the speed of the boat in metre per second, if the height of the cliff is 500 m.

10. A man in a boat rowing away from a lighthouse 150 m high, takes 2 minutes to change the angle of elevation of the top of the lighthouse from 60° to 45°. Find the speed of the boat.

11. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 m away from the bank, he finds the angle of elevation to be 30°. Find :

(i) the height of the tree, correct to 2 decimal places,

(ii) the width of the river.

12. The horizontal distance between two towers is 75 m and the angular depression of the top of the first tower as seen from the top of the second, which is 160 m high, is 45°. Find the height of the first tower.

13. The length of the shadow of a tower standing on level plane is found to be $2y$ metres longer when the sun's altitude is 30° than when it was 45°. Prove that the height of the tower is $y(\sqrt{3} + 1)$ metres.

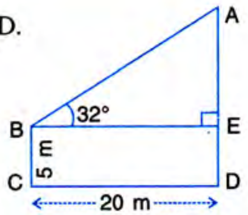
14. An aeroplane flying horizontally 1 km above the ground and going away from the observer is observed at an elevation of 60°. After 10 seconds, its elevation is observed to be 30°; find the uniform speed of the aeroplane in km per hour.

15. From the top of a hill, the angles of depression of two consecutive kilometre stones, due east, are found to be 30° and 45° respectively. Find the distances of the two stones from the foot of the hill. **[2007]**

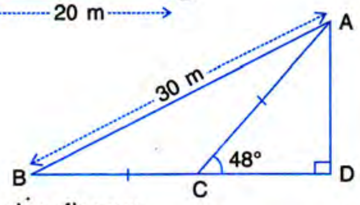
EXERCISE 22(C)

1. Find AD.

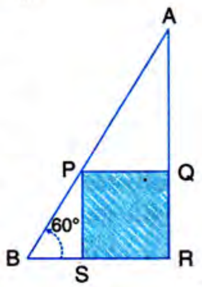
(i)



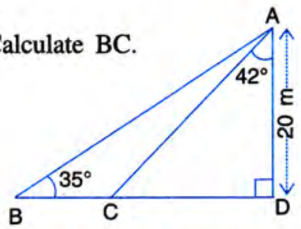
(ii)



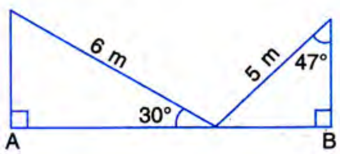
2. In the following diagram, AB is a floor-board; PQRS is a cubical box with each edge = 1 m and ∠B = 60°. Calculate the length of the board AB.



3. Calculate BC.



4. Calculate AB.



5. The radius of a circle is given as 15 cm and chord AB subtends an angle of 131° at the centre C of the circle. Using trigonometry, calculate :

(i) the length of AB;

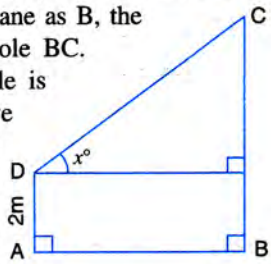
(ii) the distance of AB from the centre C.

6. At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is $\frac{5}{12}$. On walking 192 metres towards the tower; the tangent of the angle is found to be $\frac{3}{4}$. Find the height of the tower.

7. A vertical tower stands on horizontal plane and is surmounted by a vertical flagstaff of height h metre. At a point on the plane, the angle of elevation of the bottom of the flagstaff is α and that of the top of flagstaff is β. Prove that the height of the tower is

$$\frac{h \tan \alpha}{\tan \beta - \tan \alpha}$$

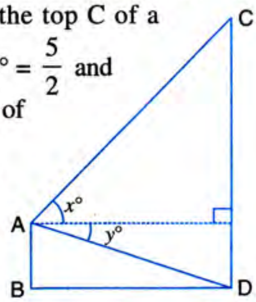
8. With reference to the given figure, a man stands on the ground at point A, which is on the same horizontal plane as B, the foot of the vertical pole BC. The height of the pole is 10 m. The man's eye is 2 m above the ground. He observes the angle of elevation of C, the top of the



pole, as $x°$, where $\tan x° = \frac{2}{5}$. Calculate : (i) the distance AB in metres; (ii) angle of elevation of the top of the pole when he is standing 15 metres from the pole. Give your answer to the nearest degree.

9. The angles of elevation of the top of a tower from two points on the ground at distances a and b metres from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is $\sqrt{ab}$ metre.

10. From a window A, 10 m above the ground the angle of elevation of the top C of a tower is $x°$, where $\tan x° = \frac{5}{2}$ and the angle of depression of the foot D of the tower is $y°$, where $\tan y° = \frac{1}{4}$. (See the given figure). Calculate the height CD of the tower in metres. **[2000]**



11. A vertical tower is 20 m high. A man standing at some distance from the tower knows that the cosine of the angle of elevation of the top of the tower is 0·53. How far is he standing from the foot of the tower ? **[2001]**

12. A man standing on the bank of a river observes that the angle of elevation of a tree on the opposite bank is 60°. When he moves 50 m away from the bank, he finds the angle of elevation to be 30°. Calculate : (i) the width of the river and (ii) the height of the tree. **[2003]**

13. A 20 m high vertical pole and a vertical tower are on the same level ground in such a way that the angle of elevation of the top of the tower, as seen from the foot of the pole, is 60° and the angle of elevation of the top of the pole as seen from the foot of the tower is 30°. Find : (i) the height of the tower. (ii) the horizontal distance between the pole and the tower.

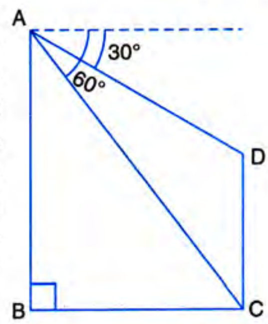
14. A vertical pole and a vertical tower are on the same level ground in such a way that from the top of the pole the angle of elevation of the top of the tower is 60° and the angle of depression of the bottom of the tower is 30°. Find : (i) the height of the tower, if the height of the pole is 20 m; (ii) the height of the pole, if the height of the tower is 75 m.

15. From a point, 36 m above the surface of a lake, the angle of elevation of a bird is observed to be 30° and angle of depression of its image in the water of the lake is observed to be 60°. Find the actual height of the bird above the surface of the lake.

16. A man observes the angle of elevation of the top of a building to be 30°. He walks towards it in a horizontal line through its base. On covering 60 m, the angle of elevation changes to 60°. Find the height of the building correct to the nearest metre. **[2001]**

17. As observed from the top of a 80 m tall lighthouse, the angles of depression of two ships, on the same side of the light house in horizontal line with its base, are 30° and 40° respectively. Find the distance between the two ships. Give your answer correct to the nearest metre. **[2012]**

18. In the given figure, from the top of a building AB = 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find :



(i) the horizontal distance between AB and CD.

(ii) the height of the lamp post.

19. An aeroplane, at an altitude of 250 m, observes the angles of depression of two boats on the opposite banks of a river to be 45° and 60° respectively. Find the width of the river. Write the answer correct to the nearest whole number. **[2014]**

20. The horizontal distance between two towers is 120 m. The angle of elevation of the top and angle of depression of the bottom of the first tower as observed from the top of the second tower is 30° and 24° respectively. Find the height of the two towers. Give your answer correct to 3 significant figures. **[2015]**