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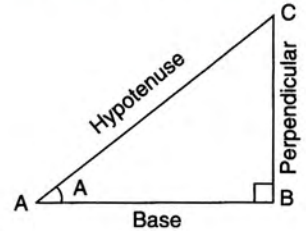
UNIT 5 :
Trigonometry**Trigonometrical Identities**

(Including Trigonometrical Ratios of Complementary Angles and Use of Four Figure Trigonometrical Tables)

21.1 Trigonometry :*Trigonometry means; the science which deals with the measurements of triangles.***21.2 Trigonometrical Ratios :**

There are six trigonometrical ratios relating to the three sides of a right-angled triangle (this has already been done by students in Class IX).

For an acute angle of a right-angled triangle :



- $$(1) \quad \text{sine (sin)} = \frac{\text{Perpendicular}}{\text{Hypotenuse}} \Rightarrow \sin A = \frac{BC}{AC}$$
- $$(2) \quad \text{cosine (cos)} = \frac{\text{Base}}{\text{Hypotenuse}} \Rightarrow \cos A = \frac{AB}{AC}$$
- $$(3) \quad \text{tangent (tan)} = \frac{\text{Perpendicular}}{\text{Base}} \Rightarrow \tan A = \frac{BC}{AB}$$
- $$(4) \quad \text{cotangent (cot)} = \frac{\text{Base}}{\text{Perpendicular}} \Rightarrow \cot A = \frac{AB}{BC}$$
- $$(5) \quad \text{secant (sec)} = \frac{\text{Hypotenuse}}{\text{Base}} \Rightarrow \sec A = \frac{AC}{AB}$$
- $$(6) \quad \text{cosecant (cosec)} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} \Rightarrow \text{cosec } A = \frac{AC}{BC}$$

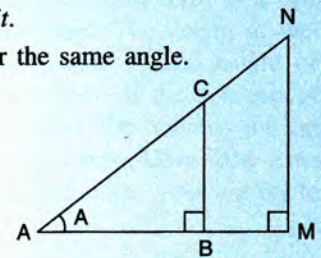
Remember :

- Each *trigonometrical ratio* is a *real number* and has *no unit*.
- The values of trigonometrical ratios are always the same for the same angle.

For Example :

$$\text{In right triangle ABC, } \sin A = \frac{BC}{AC}$$

$$\text{and in right triangle AMN, } \sin A = \frac{MN}{AN}$$



Since the angle A is same for both the triangles; we have $\sin A = \frac{BC}{AC} = \frac{MN}{AN}$

For the same reason : $\cos A = \frac{AB}{AC} = \frac{AM}{AN}$, $\tan A = \frac{BC}{AB} = \frac{MN}{AM}$ and so on.

21.3 Relations Between Different Trigonometrical Ratios :**1. Reciprocal relations :**

Since $\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}}$ and $\operatorname{cosec} A = \frac{\text{hypotenuse}}{\text{perpendicular}}$

$\Rightarrow \sin A$ and $\operatorname{cosec} A$ are reciprocals of each other

$$\text{i.e. } \sin A = \frac{1}{\operatorname{cosec} A} \quad \text{and} \quad \operatorname{cosec} A = \frac{1}{\sin A}$$

Similarly, (i) $\cos A$ and $\sec A$ are reciprocals of each other

$$\text{i.e. } \cos A = \frac{1}{\sec A} \quad \text{and} \quad \sec A = \frac{1}{\cos A}$$

(ii) $\tan A$ and $\cot A$ are reciprocals of each other

$$\text{i.e. } \tan A = \frac{1}{\cot A} \quad \text{and} \quad \cot A = \frac{1}{\tan A}$$

2. Quotient relations :

Since $\sin A = \frac{\text{perpendicular}}{\text{hypotenuse}}$ and $\cos A = \frac{\text{base}}{\text{hypotenuse}}$

$$\begin{aligned} \therefore \frac{\sin A}{\cos A} &= \frac{\text{perpendicular}}{\text{hypotenuse}} \times \frac{\text{hypotenuse}}{\text{base}} \\ &= \frac{\text{perpendicular}}{\text{base}} = \tan A \end{aligned}$$

Similarly, $\frac{\cos A}{\sin A} = \cot A$

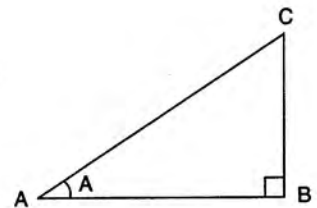
Hence, $\tan A = \frac{\sin A}{\cos A}$ and $\cot A = \frac{\cos A}{\sin A}$

3. Square relations :

In right-angled triangle ABC, with angle B = 90°;

$$\sin A = \frac{BC}{AC} \quad \text{and} \quad \cos A = \frac{AB}{AC}$$

$$\begin{aligned} \Rightarrow \sin^2 A + \cos^2 A &= \left(\frac{BC}{AC}\right)^2 + \left(\frac{AB}{AC}\right)^2 \\ &= \frac{BC^2 + AB^2}{AC^2} \\ &= \frac{AC^2}{AC^2} = 1 \end{aligned}$$



[As, $AB^2 + BC^2 = AC^2$]

$$\therefore \sin^2 A + \cos^2 A = 1$$

Similarly,

$$\begin{aligned}
 \text{(i) } 1 + \tan^2 A &= 1 + \left(\frac{BC}{AB}\right)^2 \\
 &= \frac{AB^2 + BC^2}{AB^2} = \frac{AC^2}{AB^2} \quad [\because AB^2 + BC^2 = AC^2] \\
 &= \left(\frac{AC}{AB}\right)^2 = \sec^2 A \quad [\because \sec A = \frac{AC}{AB}]
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) } 1 + \cot^2 A &= 1 + \left(\frac{AB}{BC}\right)^2 \\
 &= \frac{BC^2 + AB^2}{BC^2} = \frac{AC^2}{BC^2} \\
 &= \left(\frac{AC}{BC}\right)^2 = \operatorname{cosec}^2 A \quad [\because \operatorname{cosec} A = \frac{AC}{BC}]
 \end{aligned}$$

Hence,

$$\sin^2 A + \cos^2 A = 1; \quad 1 + \tan^2 A = \sec^2 A \quad \text{and} \quad 1 + \cot^2 A = \operatorname{cosec}^2 A.$$

Remember :

- | | | | | | |
|-------|---|---------------|---|-----|---|
| (i) | $\sin^2 A + \cos^2 A = 1$ | \Rightarrow | $\sin^2 A = 1 - \cos^2 A$ | and | $\cos^2 A = 1 - \sin^2 A$ |
| (ii) | $1 + \tan^2 A = \sec^2 A$ | \Rightarrow | $\sec^2 A - \tan^2 A = 1$ | and | $\sec^2 A - 1 = \tan^2 A$ |
| (iii) | $1 + \cot^2 A = \operatorname{cosec}^2 A$ | \Rightarrow | $\operatorname{cosec}^2 A - \cot^2 A = 1$ | and | $\operatorname{cosec}^2 A - 1 = \cot^2 A$ |

21.4 Trigonometric Identities :

When an equation, involving trigonometrical ratios of an angle A , is true for all values of A ; the equation is called a **trigonometrical identity**.

Each of the relations given above; viz. reciprocal relations, quotient relations and square relations; is a trigonometrical identity.

1 Prove the identity : $\tan A + \cot A = \sec A \cdot \operatorname{cosec} A$

Solution :

$$\begin{aligned}
 \text{L.H.S.} &= \tan A + \cot A \\
 &= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} = \frac{\sin^2 A + \cos^2 A}{\cos A \cdot \sin A} \\
 &= \frac{1}{\cos A \sin A} \quad [\because \sin^2 A + \cos^2 A = 1] \\
 &= \sec A \cdot \operatorname{cosec} A = \text{R.H.S.} \quad \left[\because \sec A = \frac{1}{\cos A} \text{ and } \operatorname{cosec} A = \frac{1}{\sin A} \right]
 \end{aligned}$$

Important :

In order to prove a trigonometrical identity : start with any side **left-hand-side** (L.H.S.) or **right-hand-side** (R.H.S.) and by applying trigonometrical relations reach to the other side, *i.e.*, if you start with L.H.S.; reach to R.H.S. and if you start with R.H.S. reach to L.H.S.

In general, *start with the more complicated side.*

Sometimes both the sides are complicated. In this situation, both the sides may be taken and reduced independently to the same result.

- 2** Prove that : (i) $\cos^4 A - \sin^4 A = 2 \cos^2 A - 1$
 (ii) $(1 + \cot A)^2 + (1 - \cot A)^2 = 2 \operatorname{cosec}^2 A$
 (iii) $\tan^4 A + \tan^2 A = \sec^4 A - \sec^2 A$

Solution :

- (i) **L.H.S.** = $(\cos^2 A)^2 - (\sin^2 A)^2$
 = $(\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$
 = $\cos^2 A - \sin^2 A$ [As, $\cos^2 A + \sin^2 A = 1$]
 = $\cos^2 A - (1 - \cos^2 A)$ [As, $\sin^2 A = 1 - \cos^2 A$]
 = $2 \cos^2 A - 1 = \mathbf{R.H.S.}$
- (ii) **L.H.S.** = $1 + \cot^2 A + 2 \cot A + 1 + \cot^2 A - 2 \cot A$
 = $2 + 2 \cot^2 A$
 = $2(1 + \cot^2 A)$
 = $2 \operatorname{cosec}^2 A$ [As, $1 + \cot^2 A = \operatorname{cosec}^2 A$]
 = **R.H.S.**
- (iii) **L.H.S.** = $\tan^2 A \cdot (\tan^2 A + 1)$
 = $(\sec^2 A - 1) \cdot \sec^2 A$ [As, $\sec^2 A = 1 + \tan^2 A$]
 = $\sec^4 A - \sec^2 A = \mathbf{R.H.S.}$

3 Prove that :

- (i) $\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A} = 2 \operatorname{cosec} A$ [2004, 2009]
- (ii) $\frac{1 + \cos A}{1 - \cos A} = (\operatorname{cosec} A + \cot A)^2$ (iii) $\frac{\cot A + \tan B}{\cot B + \tan A} = \cot A \tan B.$
- (iv) $\frac{\cos A \cot A}{1 - \sin A} = 1 + \operatorname{cosec} A$ [2006 type]

Solution :

- (i) **L.H.S.** = $\frac{\sin^2 A + (1 + \cos A)^2}{(1 + \cos A) \sin A}$
 = $\frac{\sin^2 A + 1 + \cos^2 A + 2 \cos A}{(1 + \cos A) \sin A}$
 = $\frac{1 + 1 + 2 \cos A}{(1 + \cos A) \sin A}$ [$\because \sin^2 A + \cos^2 A = 1$]
 = $\frac{2(1 + \cos A)}{(1 + \cos A) \sin A} = \frac{2}{\sin A} = 2 \operatorname{cosec} A = \mathbf{R.H.S.}$

$$\begin{aligned}
 \text{(ii) R.H.S.} &= \left(\frac{1}{\sin A} + \frac{\cos A}{\sin A} \right)^2 && \text{[Starting with the complicated side]} \\
 &= \frac{(1 + \cos A)^2}{\sin^2 A} = \frac{(1 + \cos A)^2}{1 - \cos^2 A} && [\because \sin^2 A = 1 - \cos^2 A] \\
 &= \frac{(1 + \cos A)(1 + \cos A)}{(1 + \cos A)(1 - \cos A)} = \frac{1 + \cos A}{1 - \cos A} = \text{L.H.S.}
 \end{aligned}$$

Alternative method :

$$\begin{aligned}
 \text{L.H.S.} &= \frac{1 + \cos A}{1 - \cos A} \times \frac{1 + \cos A}{1 + \cos A} && \text{[Multiplying and dividing by } (1 + \cos A)\text{]} \\
 &= \frac{(1 + \cos A)^2}{1 - \cos^2 A} \\
 &= \frac{(1 + \cos A)^2}{\sin^2 A} = \left(\frac{1 + \cos A}{\sin A} \right)^2 = \left(\frac{1}{\sin A} + \frac{\cos A}{\sin A} \right)^2 \\
 &= (\operatorname{cosec} A + \cot A)^2 = \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) L.H.S.} &= \frac{\frac{\cos A}{\sin A} + \frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B} + \frac{\sin A}{\cos A}} = \frac{\frac{\cos A \cos B + \sin A \sin B}{\sin A \cos B}}{\frac{\cos A \cos B + \sin A \sin B}{\sin B \cdot \cos A}} = \frac{\sin B \cos A}{\sin A \cos B} \\
 &= \left(\frac{\cos A}{\sin A} \right) \cdot \left(\frac{\sin B}{\cos B} \right) = \cot A \cdot \tan B = \text{R.H.S.}
 \end{aligned}$$

Alternative method :

$$\begin{aligned}
 \text{L.H.S.} &= \frac{\cot A + \tan B}{\frac{1}{\tan B} + \frac{1}{\cot A}} \\
 &= \frac{\cot A + \tan B}{\frac{\cot A + \tan B}{\tan B \cot A}} = \frac{(\cot A + \tan B) \cot A \tan B}{\cot A + \tan B} = \cot A \tan B = \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv) L.H.S.} &= \frac{\cos A \cot A}{1 - \sin A} = \frac{\cos A \times \frac{\cos A}{\sin A}}{1 - \sin A} = \frac{\cos^2 A}{\sin A(1 - \sin A)} \\
 &= \frac{(1 - \sin A)(1 + \sin A)}{\sin A(1 - \sin A)} && [\because \cos^2 A = 1 - \sin^2 A = (1 - \sin A)(1 + \sin A)] \\
 &= \frac{1 + \sin A}{\sin A} = \frac{1}{\sin A} + \frac{\sin A}{\sin A} = \operatorname{cosec} A + 1 = \text{R.H.S.}
 \end{aligned}$$

4 Prove that : $\frac{\sec A - \tan A}{\operatorname{cosec} A + \cot A} = \frac{\operatorname{cosec} A - \cot A}{\sec A + \tan A}$

Solution :

$$\text{Since } \sec^2 A - \tan^2 A = 1 \text{ and } \operatorname{cosec}^2 A - \cot^2 A = 1.$$

$$\begin{aligned} \therefore \quad & \sec^2 A - \tan^2 A = \operatorname{cosec}^2 A - \cot^2 A \\ \Rightarrow \quad & (\sec A - \tan A)(\sec A + \tan A) = (\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A) \\ \Rightarrow \quad & \frac{\sec A - \tan A}{\operatorname{cosec} A + \cot A} = \frac{\operatorname{cosec} A - \cot A}{\sec A + \tan A} \quad \text{Hence Proved.} \end{aligned}$$

Alternative method :

$$\begin{aligned} \text{L.H.S.} &= \frac{\sec A - \tan A}{\operatorname{cosec} A + \cot A} \\ &= \frac{\sec A - \tan A}{\operatorname{cosec} A + \cot A} \times \frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec} A - \cot A} \times \frac{\sec A + \tan A}{\sec A + \tan A} \\ &= \frac{(\sec^2 A - \tan^2 A)(\operatorname{cosec} A - \cot A)}{(\operatorname{cosec}^2 A - \cot^2 A)(\sec A + \tan A)} \\ &= \frac{1 \times (\operatorname{cosec} A - \cot A)}{1 \times (\sec A + \tan A)} = \frac{\operatorname{cosec} A - \cot A}{\sec A + \tan A} = \text{R.H.S.} \end{aligned}$$

5 Prove that : (i) $\sqrt{\frac{1 - \sin A}{1 + \sin A}} = \sec A - \tan A$

(ii) $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$

Solution :

$$\begin{aligned} \text{(i) L.H.S.} &= \sqrt{\frac{1 - \sin A}{1 + \sin A}} \times \frac{\sqrt{1 - \sin A}}{\sqrt{1 - \sin A}} \quad [\text{Multiplying and dividing by } \sqrt{1 - \sin A}] \\ &= \frac{1 - \sin A}{\sqrt{1 - \sin^2 A}} = \frac{1 - \sin A}{\cos A} \quad [\because 1 - \sin^2 A = \cos^2 A] \\ &= \frac{1}{\cos A} - \frac{\sin A}{\cos A} = \sec A - \tan A = \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} \text{(ii) L.H.S.} &= \frac{\tan A + \sec A - (\sec^2 A - \tan^2 A)}{\tan A - \sec A + 1} \quad [\because \sec^2 A - \tan^2 A = 1] \\ &= \frac{(\tan A + \sec A) - (\sec A + \tan A)(\sec A - \tan A)}{\tan A - \sec A + 1} \\ &= \frac{(\tan A + \sec A)(1 - \sec A + \tan A)}{\tan A - \sec A + 1} \\ &= \tan A + \sec A = \frac{\sin A}{\cos A} + \frac{1}{\cos A} = \frac{\sin A + 1}{\cos A} = \text{R.H.S.} \end{aligned}$$

Alternative method :

$$\text{L.H.S.} = \frac{\frac{\sin A}{\cos A} + \frac{1}{\cos A} - 1}{\frac{\sin A}{\cos A} - \frac{1}{\cos A} + 1} = \frac{\frac{\sin A + 1 - \cos A}{\cos A}}{\frac{\sin A - 1 + \cos A}{\cos A}}$$

$$\begin{aligned}
 &= \frac{\sin A + 1 - \cos A}{\sin A - 1 + \cos A} \\
 &= \frac{\sin A + 1 - \cos A}{\sin A - 1 + \cos A} \times \frac{1 + \sin A}{1 + \sin A} \quad [\text{Multiplying and dividing by } 1 + \sin A] \\
 &= \frac{(\sin A + 1 - \cos A)(1 + \sin A)}{\sin A - 1 + \cos A + \sin^2 A - \sin A + \sin A \cos A} \\
 &= \frac{(\sin A + 1 - \cos A)(1 + \sin A)}{-1 + \cos A + (1 - \cos^2 A) + \sin A \cos A} \quad [\because \sin^2 A = 1 - \cos^2 A] \\
 &= \frac{(\sin A + 1 - \cos A)(1 + \sin A)}{\cos A - \cos^2 A + \sin A \cos A} \\
 &= \frac{(\sin A + 1 - \cos A)(1 + \sin A)}{\cos A (1 - \cos A + \sin A)} = \frac{1 + \sin A}{\cos A} = \text{R.H.S.}
 \end{aligned}$$

EXERCISE 21(A)

Prove the following identities :

1. $\frac{\sec A - 1}{\sec A + 1} = \frac{1 - \cos A}{1 + \cos A}$ [2007]
2. $\frac{1 + \sin A}{1 - \sin A} = \frac{\operatorname{cosec} A + 1}{\operatorname{cosec} A - 1}$
3. $\frac{1}{\tan A + \cot A} = \cos A \sin A$
4. $\tan A - \cot A = \frac{1 - 2 \cos^2 A}{\sin A \cos A}$
5. $\sin^4 A - \cos^4 A = 2 \sin^2 A - 1$
6. $(1 - \tan A)^2 + (1 + \tan A)^2 = 2 \sec^2 A$ [2005]
7. $\operatorname{cosec}^4 A - \operatorname{cosec}^2 A = \cot^4 A + \cot^2 A$
8. $\sec A (1 - \sin A) (\sec A + \tan A) = 1$
9. $\operatorname{cosec} A (1 + \cos A) (\operatorname{cosec} A - \cot A) = 1$
10. $\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \cdot \operatorname{cosec}^2 A$
11. $\frac{(1 + \tan^2 A) \cot A}{\operatorname{cosec}^2 A} = \tan A$
12. $\tan^2 A - \sin^2 A = \tan^2 A \cdot \sin^2 A$
13. $\cot^2 A - \cos^2 A = \cos^2 A \cdot \cot^2 A$
14. $(\operatorname{cosec} A + \sin A) (\operatorname{cosec} A - \sin A) = \cot^2 A + \cos^2 A$
15. $(\sec A - \cos A) (\sec A + \cos A) = \sin^2 A + \tan^2 A$
16. $(\cos A + \sin A)^2 + (\cos A - \sin A)^2 = 2$
17. $(\operatorname{cosec} A - \sin A) (\sec A - \cos A) (\tan A + \cot A) = 1$
18. $\frac{1}{\sec A + \tan A} = \sec A - \tan A$
19. $\operatorname{cosec} A + \cot A = \frac{1}{\operatorname{cosec} A - \cot A}$
20. $\frac{\sec A - \tan A}{\sec A + \tan A} = 1 - 2 \sec A \tan A + 2 \tan^2 A$
21. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$
22. $\sec^2 A \cdot \operatorname{cosec}^2 A = \tan^2 A + \cot^2 A + 2$
23. $\frac{1}{1 + \cos A} + \frac{1}{1 - \cos A} = 2 \operatorname{cosec}^2 A$
24. $\frac{1}{1 - \sin A} + \frac{1}{1 + \sin A} = 2 \sec^2 A$
25. $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2 \sec^2 A$
26. $\frac{\sec A}{\sec A + 1} + \frac{\sec A}{\sec A - 1} = 2 \operatorname{cosec}^2 A$
27. $\frac{1 + \cos A}{1 - \cos A} = \frac{\tan^2 A}{(\sec A - 1)^2}$ [2012]
28. $\frac{\cot^2 A}{(\operatorname{cosec} A + 1)^2} = \frac{1 - \sin A}{1 + \sin A}$
29. $\frac{1 + \sin A}{\cos A} + \frac{\cos A}{1 + \sin A} = 2 \sec A$

$$30. \frac{1 - \sin A}{1 + \sin A} = (\sec A - \tan A)^2$$

$$31. (\cot A - \operatorname{cosec} A)^2 = \frac{1 - \cos A}{1 + \cos A}$$

$$32. \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} = \left(\frac{\cos A}{1 + \sin A} \right)^2$$

$$33. \tan^2 A - \tan^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cdot \cos^2 B}$$

$$34. \frac{\sin A - 2 \sin^3 A}{2 \cos^3 A - \cos A} = \tan A$$

$$35. \frac{\sin A}{1 + \cos A} = \operatorname{cosec} A - \cot A \quad [2008]$$

$$36. \frac{\cos A}{1 - \sin A} = \sec A + \tan A$$

$$37. \frac{\sin A \tan A}{1 - \cos A} = 1 + \sec A$$

$$38. (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$$

$$39. \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

$$40. \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \operatorname{cosec} A - \cot A \quad [2000]$$

$$41. \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{\sin A}{1 + \cos A} \quad [2000, 2013]$$

$$42. \sqrt{\frac{1 - \sin A}{1 + \sin A}} = \frac{\cos A}{1 + \sin A}$$

$$43. 1 - \frac{\cos^2 A}{1 + \sin A} = \sin A \quad [2001]$$

$$44. \frac{1}{\sin A + \cos A} + \frac{1}{\sin A - \cos A} = \frac{2 \sin A}{1 - 2 \cos^2 A} \quad [2002]$$

$$45. \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{2 \sin^2 A - 1}$$

$$46. \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \frac{1 + \cos A}{\sin A}$$

$$47. \frac{\sin \theta \tan \theta}{1 - \cos \theta} = 1 + \sec \theta \quad [2006]$$

$$48. \frac{\cos \theta \cot \theta}{1 + \sin \theta} = \operatorname{cosec} \theta - 1$$

6 Prove that : (i) $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \cos A + \sin A$ [2015]

(ii) $(1 + \tan^2 A) + (1 + \frac{1}{\tan^2 A}) = \frac{1}{\sin^2 A - \sin^4 A}$

Solution :

$$\begin{aligned} \text{(i) L.H.S.} &= \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} \\ &= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}} \\ &= \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}} \\ &= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A} \\ &= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A} \\ &= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A} \\ &= \frac{(\cos A - \sin A)(\cos A + \sin A)}{\cos A - \sin A} = \cos A + \sin A = \text{R.H.S.} \end{aligned}$$

$$\begin{aligned}
 \text{(ii) L.H.S.} &= (1 + \tan^2 A) + \left(1 + \frac{1}{\tan^2 A}\right) \\
 &= \sec^2 A + (1 + \cot^2 A) \\
 &= \sec^2 A + \operatorname{cosec}^2 A \\
 &= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} = \frac{\sin^2 A + \cos^2 A}{\cos^2 A \sin^2 A} = \frac{1}{\cos^2 A \sin^2 A} \\
 &= \frac{1}{(1 - \sin^2 A) \sin^2 A} = \frac{1}{\sin^2 A - \sin^4 A} = \text{R.H.S.}
 \end{aligned}$$

7 If $\tan A + \sin A = m$ and $\tan A - \sin A = n$;
 prove that : $m^2 - n^2 = 4\sqrt{mn}$.

Solution :

$$\begin{aligned}
 m^2 - n^2 &= (m + n)(m - n) \\
 &= (\tan A + \sin A + \tan A - \sin A)(\tan A + \sin A - \tan A + \sin A) \\
 &= (2 \tan A)(2 \sin A) \\
 &= 4 \tan A \sin A \qquad \dots\text{(I)}
 \end{aligned}$$

$$\begin{aligned}
 4\sqrt{mn} &= 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)} \\
 &= 4\sqrt{\tan^2 A - \sin^2 A} \\
 &= 4\sqrt{\frac{\sin^2 A}{\cos^2 A} - \sin^2 A} \\
 &= 4 \sin A \sqrt{\sec^2 A - 1} \qquad \left[\because \frac{1}{\cos^2 A} = \sec^2 A\right] \\
 &= 4 \sin A \cdot \tan A \qquad \dots\text{(II)} \qquad \left[\because \sec^2 A - 1 = \tan^2 A\right]
 \end{aligned}$$

$$\therefore m^2 - n^2 = 4\sqrt{mn} \qquad \text{[From I and II]}$$

Hence Proved.

8 If $x = a \sec A \cos B$, $y = b \sec A \sin B$ and $z = c \tan A$; show that :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Solution :

$$\begin{aligned}
 \text{L.H.S.} &= \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} \\
 &= \frac{(a \sec A \cos B)^2}{a^2} + \frac{(b \sec A \sin B)^2}{b^2} - \frac{(c \tan A)^2}{c^2}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{a^2 \sec^2 A \cos^2 B}{a^2} + \frac{b^2 \sec^2 A \sin^2 B}{b^2} - \frac{c^2 \tan^2 A}{c^2} \\
 &= \sec^2 A \cos^2 B + \sec^2 A \sin^2 B - \tan^2 A \\
 &= \sec^2 A (\cos^2 B + \sin^2 B) - \tan^2 A \\
 &= \sec^2 A - \tan^2 A \qquad [\because \cos^2 B + \sin^2 B = 1] \\
 &= 1 = \text{R.H.S.} \qquad [\because \sec^2 A - \tan^2 A = 1]
 \end{aligned}$$

EXERCISE 21(B)

1. Prove that :

(i) $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$

[2003]

(ii) $\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A} + \frac{\cos^3 A - \sin^3 A}{\cos A - \sin A} = 2$

(iii) $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = \sec A \operatorname{cosec} A + 1$

(iv) $\left(\tan A + \frac{1}{\cos A}\right)^2 + \left(\tan A - \frac{1}{\cos A}\right)^2$
 $= 2\left(\frac{1 + \sin^2 A}{1 - \sin^2 A}\right)$

(v) $2 \sin^2 A + \cos^4 A = 1 + \sin^4 A$

(vi) $\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$

(vii) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$$= \frac{1}{\tan A + \cot A}$$

(viii) $(1 + \tan A \cdot \tan B)^2 + (\tan A - \tan B)^2$
 $= \sec^2 A \sec^2 B$

(ix) $\frac{1}{\cos A + \sin A - 1} + \frac{1}{\cos A + \sin A + 1}$
 $= \operatorname{cosec} A + \sec A$

2. If $x \cos A + y \sin A = m$ and
 $x \sin A - y \cos A = n$, then prove that :

$$x^2 + y^2 = m^2 + n^2$$

3. If $m = a \sec A + b \tan A$ and
 $n = a \tan A + b \sec A$, then prove that :

$$m^2 - n^2 = a^2 - b^2$$

4. If $x = r \sin A \cos B$, $y = r \sin A \sin B$ and
 $z = r \cos A$, then prove that :

$$x^2 + y^2 + z^2 = r^2$$

5. If $\sin A + \cos A = m$ and
 $\sec A + \operatorname{cosec} A = n$, show that :

$$n(m^2 - 1) = 2m$$

6. If $x = r \cos A \cos B$, $y = r \cos A \sin B$ and
 $z = r \sin A$, show that :

$$x^2 + y^2 + z^2 = r^2$$

7. If $\frac{\cos A}{\cos B} = m$ and $\frac{\cos A}{\sin B} = n$,

show that :

$$(m^2 + n^2) \cos^2 B = n^2.$$

21.5 Trigonometrical Ratios of Complementary Angles :

For an acute angle A,

(i) $\sin(90^\circ - A) = \cos A$,

(ii) $\cos(90^\circ - A) = \sin A$,

(iii) $\tan(90^\circ - A) = \cot A$,

(iv) $\cot(90^\circ - A) = \tan A$,

(v) $\sec(90^\circ - A) = \operatorname{cosec} A$ and

(vi) $\operatorname{cosec}(90^\circ - A) = \sec A$.

9 Find the value of x , if :
 $\cos x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$.

Solution :

$$\begin{aligned}\cos x &= \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2} \\ &= \frac{\sqrt{3} + \sqrt{3}}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} = \cos 30^\circ \quad \therefore x = 30^\circ \quad \text{Ans.}\end{aligned}$$

10 Given $\cos 38^\circ \sec (90^\circ - 2A) = 1$; find the value of angle A .

Solution :

$$\begin{aligned}\cos 38^\circ \sec (90^\circ - 2A) = 1 &\Rightarrow \cos 38^\circ \operatorname{cosec} 2A = 1 \quad [\because \sec (90^\circ - \theta) = \operatorname{cosec} \theta] \\ &\Rightarrow \cos 38^\circ \times \frac{1}{\sin 2A} = 1 \quad [\because \operatorname{cosec} \theta = \frac{1}{\sin \theta}] \\ &\Rightarrow \sin 2A = \cos 38^\circ \\ &= \cos (90^\circ - 52^\circ) \\ &\Rightarrow \sin 2A = \sin 52^\circ \quad [\because \cos (90^\circ - \theta) = \sin \theta] \\ \therefore 2A = 52^\circ \text{ and } A = 26^\circ &\quad \text{Ans.}\end{aligned}$$

EXERCISE 21(C)

1. Show that :

(i) $\tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ = 1$

(ii) $\sin 42^\circ \sec 48^\circ + \cos 42^\circ \operatorname{cosec} 48^\circ = 2$

(iii) $\frac{\sin 26^\circ}{\sec 64^\circ} + \frac{\cos 26^\circ}{\operatorname{cosec} 64^\circ} = 1$

2. Express each of the following in terms of angles between 0° and 45° :

(i) $\sin 59^\circ + \tan 63^\circ$

(ii) $\operatorname{cosec} 68^\circ + \cot 72^\circ$

(iii) $\cos 74^\circ + \sec 67^\circ$

3. Show that :

(i) $\frac{\sin A}{\sin (90^\circ - A)} + \frac{\cos A}{\cos (90^\circ - A)} = \sec A \operatorname{cosec} A$

(ii) $\sin A \cos A - \frac{\sin A \cos (90^\circ - A) \cos A}{\sec (90^\circ - A)}$
 $- \frac{\cos A \sin (90^\circ - A) \sin A}{\operatorname{cosec} (90^\circ - A)} = 0$

4. For triangle ABC , show that :

(i) $\sin \frac{A+B}{2} = \cos \frac{C}{2}$

(ii) $\tan \frac{B+C}{2} = \cot \frac{A}{2}$

5. Evaluate :

(i) $3 \frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

(ii) $3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \sin 59^\circ \sec 31^\circ$.
[2013]

(iii) $\frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ$ [2007]

(iv) $\tan (55^\circ - A) - \cot (35^\circ + A)$

(v) $\operatorname{cosec} (65^\circ + A) - \sec (25^\circ - A)$

(vi) $2 \frac{\tan 57^\circ}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2} \cos 45^\circ$

(vii) $\frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2 \frac{\sin^2 75^\circ}{\cos^2 15^\circ}$

(viii) $\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8 \sin^2 30^\circ$

(ix) $14 \sin 30^\circ + 6 \cos 60^\circ - 5 \tan 45^\circ$. [2004]

6. A triangle ABC is right angled at B; find the value of $\frac{\sec A \cdot \operatorname{cosec} C - \tan A \cdot \cot C}{\sin B}$

7. Find (in each case, given below) the value of x , if :

(i) $\sin x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

(ii) $\sin x = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

(iii) $\cos x = \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

(iv) $\tan x = \frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$

(v) $\sin 2x = 2 \sin 45^\circ \cos 45^\circ$

(vi) $\sin 3x = 2 \sin 30^\circ \cos 30^\circ$

(vii) $\cos (2x - 6^\circ) = \cos^2 30^\circ - \cos^2 60^\circ$

8. In each case, given below, find the value of angle A, where $0^\circ \leq A \leq 90^\circ$.

(i) $\sin (90^\circ - 3A) \cdot \operatorname{cosec} 42^\circ = 1$

(ii) $\cos (90^\circ - A) \cdot \sec 77^\circ = 1$

9. Prove that :

(i) $\frac{\cos (90^\circ - \theta) \cos \theta}{\cot \theta} = 1 - \cos^2 \theta$

(ii) $\frac{\sin \theta \cdot \sin (90^\circ - \theta)}{\cot (90^\circ - \theta)} = 1 - \sin^2 \theta$

10. Evaluate :

$\frac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\operatorname{cosec}^2 10^\circ - \tan^2 80^\circ}$ [2010]

11. Evaluate :

$\sin^2 34^\circ + \sin^2 56^\circ + 2 \tan 18^\circ \tan 72^\circ - \cot^2 30^\circ$ [2014]

21.6 Using the Trigonometrical Tables :

(i.e., to find the trigonometrical ratios of acute angles other than 0° , 30° , 45° and 60°)

The *trigonometrical tables* give the values of *natural sines*, *cosines* and *tangents* to **four decimal places**. A trigonometrical table consists of three parts :

(i) a *column* on the extreme left which contains degrees from 0° to 89° .

(ii) *ten columns* headed by $0'$, $6'$, $12'$, $18'$, $24'$, $30'$, $36'$, $42'$, $48'$ and $54'$ respectively.

(iii) *five columns* headed by $1'$, $2'$, $3'$, $4'$ and $5'$ respectively.

Note : When one degree (1°) is divided into sixty equal parts, each part is called **one minute** ($1'$).

\therefore One degree = 60 minute i.e. $1^\circ = 60'$.

11 Find : $\sin 36^\circ 51'$.

Solution :

See the table given for natural sines :

x°	$0'$	$6'$	$12'$	$18'$	$24'$	$30'$	$36'$	$42'$	$48'$	$54'$	$1'$	$2'$	$3'$	$4'$	$5'$
											Difference to add				
36	0.5878							5990							7

Since $\sin 36^\circ 51' = \sin (36^\circ 48' + 3')$

From table,

$$\sin 36^\circ 48' = 0.5990 \quad [\text{See the number in the row against } 36^\circ \text{ \& in the column headed } 48']$$

$$\text{diff for } 3' = 0.0007 \quad (\text{To add}) \quad [\text{See the number in the same row and under } 3']$$

$$\therefore \sin 36^\circ 51' = 0.5997 \quad \text{Ans.}$$

12 Find : $\tan 53^\circ 38'$.

Solution :

See the table for natural tangents :

x°	0'	6' 12' 18'	24' 30' 36'	42' 48' 54'	1' 2' 3' 4' 5' Difference to add
53	1.3270		3564		16

$$\text{Since } \tan 53^\circ 38' = \tan (53^\circ 36' + 2')$$

$$\therefore \tan 53^\circ 36' = 1.3564 \quad [\text{From table}]$$

$$\text{diff for } 2' = 0.0016 \quad [\text{To add}]$$

$$\therefore \tan 53^\circ 38' = 1.3580 \quad \text{Ans.}$$

13 Find : $\cos 62^\circ 27'$.

Solution :

See the table for natural cosines :

x°	0'	6' 12' 18'	24' 30' 36'	42' 48' 54'	1' 2' 3' 4' 5' Difference to subtract
62	0.4695		4633		8

$$\text{Since } \cos 62^\circ 27' = \cos (62^\circ 24' + 3')$$

$$\therefore \cos 62^\circ 24' = 0.4633 \quad [\text{From table}]$$

$$\text{diff for } 3' = 0.0008 \quad [\text{To subtract}]$$

$$\therefore \cos 62^\circ 27' = 0.4625 \quad \text{Ans.}$$

Note : The trigonometrical tables can also be used to find an acute angle.

14 Find θ ; if $\sin \theta = 0.5798$.

Solution :

From the table of natural sines find the angle whose sine is just smaller than 0.5798.

x°	0'	6' 12' 18'	24' 30' 36'	42' 48' 54'	1' 2' 3' 4' 5' Difference to add
35	0.5736		5793		5

From the table, it is clear that;

$$\sin 35^\circ 24' = 0.5793$$

$$\sin \theta - \sin 35^\circ 24' = 0.5798 - 0.5793 = 0.0005$$

$$\text{From the table; diff of } 2' = 0.0005$$

$$\therefore \theta = 35^\circ 24' + 2' = 35^\circ 26'$$

Ans.

15 Use tables to find, θ if : (i) $\cos \theta = 0.4457$ (ii) $\tan \theta = 0.8516$.

Solution :

(i) See the table for natural cosines.

x°	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	1'	2'	3'	4'	5'	
											Difference to subtract					
63	0.4540						4446								10	13

$$\text{Given, } \cos \theta = 0.4457$$

$$\cos 63^\circ 36' = 0.4446 \quad \text{[From table]}$$

$$\text{diff. in values} = 0.0011 \quad [0.4457 - 0.4446 = 0.0011]$$

$$\text{From table, diff of } 4' = 0.0011 \quad \text{[To subtract]}$$

$$\therefore \theta = 63^\circ 36' - 4' = 63^\circ 32' \quad \text{(Ans)} \quad \text{[Greater is the value of } \cos \theta, \text{ lesser is } \theta]$$

(ii) Similarly, given that $\tan \theta = 0.8516$

And, from the table of natural tangents, we observe :

$$\tan 40^\circ 24' = 0.8511$$

$$\text{diff. in values} = 0.0005 \quad [0.8516 - 0.8511 = 0.0005]$$

$$\text{Since, diff. for } 1' = 0.0005 \quad \text{[From table]}$$

$$\therefore \theta = 40^\circ 24' + 1' = 40^\circ 25' \quad \text{Ans.}$$

EXERCISE 21(D)

1. Use tables to find sine of :

(i) 21° (ii) $34^\circ 42'$

(iii) $47^\circ 32'$ (iv) $62^\circ 57'$

(v) $10^\circ 20' + 20^\circ 45'$

2. Use tables to find cosine of :

(i) $2^\circ 4'$ (ii) $8^\circ 12'$

(iii) $26^\circ 32'$ (iv) $65^\circ 41'$

(v) $9^\circ 23' + 15^\circ 54'$

3. Use trigonometrical tables to find tangent of :

(i) 37° (ii) $42^\circ 18'$

(iii) $17^\circ 27'$

4. Use tables to find the acute angle θ , if the value of $\sin \theta$ is :

(i) 0.4848 (ii) 0.3827

(iii) 0.6525

5. Use tables to find the acute angle θ , if the value of $\cos \theta$ is :

(i) 0.9848 (ii) 0.9574

(iii) 0.6885

6. Use tables to find the acute angle θ , if the value of $\tan \theta$ is :

(i) 0.2419 (ii) 0.4741

(iii) 0.7391

EXERCISE 21(E)

1. Prove the following identities :

$$(i) \frac{1}{\cos A + \sin A} + \frac{1}{\cos A - \sin A} \\ = \frac{2 \cos A}{2 \cos^2 A - 1}$$

$$(ii) \operatorname{cosec} A - \cot A = \frac{\sin A}{1 + \cos A}$$

$$(iii) 1 - \frac{\sin^2 A}{1 + \cos A} = \cos A$$

$$(iv) \frac{1 - \cos A}{\sin A} + \frac{\sin A}{1 - \cos A} = 2 \operatorname{cosec} A$$

$$(v) \frac{\cot A}{1 - \tan A} + \frac{\tan A}{1 - \cot A} = 1 + \tan A + \cot A$$

$$(vi) \frac{\cos A}{1 + \sin A} + \tan A = \sec A$$

$$(vii) \frac{\sin A}{1 - \cos A} - \cot A = \operatorname{cosec} A$$

$$(viii) \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{\cos A}{1 - \sin A}$$

$$(ix) \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \frac{\cos A}{1 - \sin A}$$

$$(x) \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{\sin A}{1 + \cos A}$$

$$(xi) \frac{1 + (\sec A - \tan A)^2}{\operatorname{cosec} A (\sec A - \tan A)} = 2 \tan A$$

$$(xii) \frac{(\operatorname{cosec} A - \cot A)^2 + 1}{\sec A (\operatorname{cosec} A - \cot A)} = 2 \cot A$$

$$(xiii) \cot^2 A \left(\frac{\sec A - 1}{1 + \sin A} \right) + \sec^2 A \left(\frac{\sin A - 1}{1 + \sec A} \right) = 0$$

$$(xiv) \frac{(1 - 2 \sin^2 A)^2}{\cos^4 A - \sin^4 A} = 2 \cos^2 A - 1$$

$$(xv) \sec^4 A (1 - \sin^4 A) - 2 \tan^2 A = 1$$

$$(xvi) \operatorname{cosec}^4 A (1 - \cos^4 A) - 2 \cot^2 A = 1$$

$$(xvii) (1 + \tan A + \sec A) (1 + \cot A - \operatorname{cosec} A) = 2$$

2. If $\sin A + \cos A = p$
and $\sec A + \operatorname{cosec} A = q$, then prove that :
 $q(p^2 - 1) = 2p$.

3. If $x = a \cos \theta$ and $y = b \cot \theta$, show that :

$$\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$$

4. If $\sec A + \tan A = p$, show that :

$$\sin A = \frac{p^2 - 1}{p^2 + 1}$$

5. If $\tan A = n \tan B$ and $\sin A = m \sin B$, prove that :

$$\cos^2 A = \frac{m^2 - 1}{n^2 - 1}$$

6. (i) If $2 \sin A - 1 = 0$, show that :

$$\sin 3A = 3 \sin A - 4 \sin^3 A \quad [2001]$$

(ii) If $4 \cos^2 A - 3 = 0$, show that :
 $\cos 3A = 4 \cos^3 A - 3 \cos A$

7. Evaluate :

$$(i) 2 \left(\frac{\tan 35^\circ}{\cot 55^\circ} \right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ} \right)^2 - 3 \left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ} \right)$$

[2011]

$$(ii) \sec 26^\circ \sin 64^\circ + \frac{\operatorname{cosec} 33^\circ}{\sec 57^\circ}$$

$$(iii) \frac{5 \sin 66^\circ}{\cos 24^\circ} - \frac{2 \cot 85^\circ}{\tan 5^\circ}$$

$$(iv) \cos 40^\circ \operatorname{cosec} 50^\circ + \sin 50^\circ \sec 40^\circ$$

$$(v) \sin 27^\circ \sin 63^\circ - \cos 63^\circ \cos 27^\circ$$

$$(vi) \frac{3 \sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$$

[2000]

$$(vii) 3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \operatorname{cosec} 31^\circ$$

[2002]

$$(viii) \frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$$

[2003]

8. Prove that :

$$(i) \tan (55^\circ + x) = \cot (35^\circ - x)$$

$$(ii) \sec (70^\circ - \theta) = \operatorname{cosec} (20^\circ + \theta)$$

$$(iii) \sin (28^\circ + A) = \cos (62^\circ - A)$$

$$(iv) \frac{1}{1 + \cos (90^\circ - A)} + \frac{1}{1 - \cos (90^\circ - A)}$$

$$= 2 \operatorname{cosec}^2 (90^\circ - A)$$

$$(v) \frac{1}{1 + \sin (90^\circ - A)} + \frac{1}{1 - \sin (90^\circ - A)}$$

$$= 2 \sec^2 (90^\circ - A)$$

9. If A and B are complementary angles, prove that :

- (i) $\cot B + \cos B = \sec A \cos B (1 + \sin B)$
 (ii) $\cot A \cot B - \sin A \cos B - \cos A \sin B = 0$
 (iii) $\operatorname{cosec}^2 A + \operatorname{cosec}^2 B = \operatorname{cosec}^2 A \operatorname{cosec}^2 B$
 (iv) $\frac{\sin A + \sin B}{\sin A - \sin B} + \frac{\cos B - \cos A}{\cos B + \cos A} = \frac{2}{2 \sin^2 A - 1}$

10. Prove that :

- (i) $\frac{1}{\sin A - \cos A} - \frac{1}{\sin A + \cos A} = \frac{2 \cos A}{2 \sin^2 A - 1}$
 (ii) $\frac{\cot^2 A}{\operatorname{cosec} A - 1} - 1 = \operatorname{cosec} A$
 (iii) $\frac{\cos A}{1 + \sin A} = \sec A - \tan A$
 (iv) $\cos A (1 + \cot A) + \sin A (1 + \tan A) = \sec A + \operatorname{cosec} A$
 (v) $(\sin A - \cos A) (1 + \tan A + \cot A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}$
 (vi) $\sqrt{\sec^2 A + \operatorname{cosec}^2 A} = \tan A + \cot A$
 (vii) $(\sin A + \cos A) (\sec A + \operatorname{cosec} A) = 2 + \sec A \operatorname{cosec} A$
 (viii) $(\tan A + \cot A) (\operatorname{cosec} A - \sin A) (\sec A - \cos A) = 1$
 (ix) $\cot^2 A - \cot^2 B = \frac{\cos^2 A - \cos^2 B}{\sin^2 A \sin^2 B} = \operatorname{cosec}^2 A - \operatorname{cosec}^2 B$

11. If $4 \cos^2 A - 3 = 0$ and $0^\circ \leq A \leq 90^\circ$, then prove that :

- (i) $\sin 3 A = 3 \sin A - 4 \sin^3 A$
 (ii) $\cos 3 A = 4 \cos^3 A - 3 \cos A$

$$4 \cos^2 A - 3 = 0 \Rightarrow \cos^2 A = \frac{3}{4}$$

$$\text{and } \cos A = \frac{\sqrt{3}}{2} \Rightarrow A = 30^\circ$$

$$(i) \sin 3 A = \sin 90^\circ = 1$$

$$\text{and, } 3 \sin A - 4 \sin^3 A$$

$$= 3 \sin 30^\circ - 4 \sin^3 30^\circ$$

$$= 3 \times \frac{1}{2} - 4 \times \left(\frac{1}{2}\right)^3$$

$$= \frac{3}{2} - \frac{1}{2} = 1$$

$$\therefore \sin 3 A = 3 \sin A - 4 \sin^3 A.$$

12. Find A, if $0^\circ \leq A \leq 90^\circ$ and :

- (i) $2 \cos^2 A - 1 = 0$
 (ii) $\sin 3 A - 1 = 0$
 (iii) $4 \sin^2 A - 3 = 0$
 (iv) $\cos^2 A - \cos A = 0$
 (v) $2 \cos^2 A + \cos A - 1 = 0$

13. If $0^\circ < A < 90^\circ$; find A, if :

- (i) $\frac{\cos A}{1 - \sin A} + \frac{\cos A}{1 + \sin A} = 4$
 (ii) $\frac{\sin A}{\sec A - 1} + \frac{\sin A}{\sec A + 1} = 2$

14. Prove that :

$$(\operatorname{cosec} A - \sin A) (\sec A - \cos A) \sec^2 A = \tan A$$

[2011]

15. Prove the identity $(\sin \theta + \cos \theta) (\tan \theta + \cot \theta)$

$$= \sec \theta + \operatorname{cosec} \theta.$$

[2014]