

Equation of a Line

14.1 A Basic Concept :

In class IX, in the chapter of *graphs*, students have drawn lines for the given equations, like: 3x - 2y = 7, x + 5y = 8, x = 5, y + 3 = 0, etc. Such equations, which are of first degree in variables x and y both or only in x or only in y, are known as *linear equations* and each equation always represents a straight line.

In other words, every straight line can be represented by a linear equation.

Remember :

- (i) Any point, which satisfies the equation of a line, lies on that line.
- (ii) Any point, through which a line passes, will always satisfy the equation of that line.



- (i) Check, whether point (4, -2) lies on the line represented by equation 3x + 5y = 2 or not?
- (ii) The straight line represented by equation x 3y + 8 = 0 passes through (2, 4). Is this true?

Solution:

(i) Substituting x = 4 and y = -2 in the given equation, we get :

$$3 \times 4 + 5 \times -2 = 2$$
 \Rightarrow $12 - 10 = 2$, which is true.

:. Point (4, -2) satisfies the given equation and so it lies

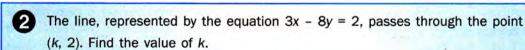
on the line represented by the equation
$$3x + 5y = 2$$

Ans.

(ii) Substituting x = 2 and y = 4 in the given equation, we get:

$$2 - 3 \times 4 + 8 = 0$$
 \Rightarrow $2 - 12 + 8 = 0$, which is not true.

... The line, represented by the equation x - 3y + 8 = 0, does not pass through the point (2, 4).



Solution:

Substituting x = k and y = 2 in the given equation, we get:

$$3k - 8 \times 2 = 2 \implies 3k - 16 = 2$$
 i.e. $k = 6$

Ans.

3

Does the line 3x = y + 1 bisect the line segment joining A (-2, 3) and B (4, 1)?

Solution :

The given line will bisect the join of AB, if the co-ordinates of the mid-point of AB satisfy the equation of the line.

Mid-point of A (-2, 3) and B (4, 1)

$$=\left(\frac{-2+4}{2}, \frac{3+1}{2}\right)=(1, 2)$$

Substituting x = 1 and y = 2 in the given equation, we get:

$$3 \times 1 = 2 + 1$$

 \Rightarrow 3 = 3, which is true.

.. The given line bisects the join of A and B.

Ans.

EXERCISE 14(A)

- 1. Find, which of the following points lie on the line x 2y + 5 = 0:
 - (i) (1, 3)
- (ii) (0, 5)
- (iii) (-5, 0)
- (iv) (5, 5)
- (v) (2, -1.5)
- (vi) (-2, -1.5)
- 2. State, true or false:
 - (i) the line $\frac{x}{2} + \frac{y}{3} = 0$ passes through the point (2, 3).
 - (ii) the line $\frac{x}{2} + \frac{y}{3} = 0$ passes through the point (4, -6).
 - (iii) the point (8, 7) lies on the line y-7=0
 - (iv) the point (-3, 0) lies on the line x + 3 = 0
 - (v) if the point (2, a) lies on the line 2x y = 3, then a = 5.
- 3. The line given by the equation $2x \frac{y}{3} = 7$ passes through the point (k, 6); calculate the value of k.
- 4. For what value of k will the point (3, -k) lie on the line 9x + 4y = 3?
- 5. The line $\frac{3x}{5} \frac{2y}{3} + 1 = 0$ contains the point (m, 2m-1); calculate the value of m.
- 6. Does the line 3x 5y = 6 bisect the join of (5, -2) and (-1, 2)?

- 7. (i) The line y = 3x 2 bisects the join of (a, 3) and (2, -5), find the value of a.
 - (ii) The line x 6y + 11 = 0 bisects the join of (8, -1) and (0, k). Find the value of k.
- 8. (i) The point (-3, 2) lies on the line ax + 3y + 6 = 0, calculate the value of a.
 - (ii) The line y = mx + 8 contains the point (-4, 4), calculate the value of m.
- 9. The point P divides the join of (2, 1) and (-3, 6) in the ratio 2 : 3. Does P lie on the line x 5y + 15 = 0?
- 10. The line segment joining the points (5, -4) and (2, 2) is divided by the point Q in the ratio 1:2. Does the line x-2y=0 contain Q?
- 11. Find the point of intersection of the lines 4x + 3y = 1 and 3x y + 9 = 0. If this point lies on the line (2k 1) x 2y = 4; find the value of k.

The above question can also be stated as: If the lines 4x + 3y = 1, 3x - y + 9 = 0 and (2k - 1) x - 2y = 4 are concurrent (pass through the same point), find the value of k.

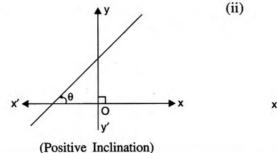
12. Show that the lines 2x + 5y = 1, x - 3y = 6 and x + 5y + 2 = 0 are concurrent.

14.2 Inclination of a line:

The inclination of a line is the angle θ which the part of the line (above x-axis) makes with x-axis.

If measured in anti-clockwise direction the inclination θ is positive and if measured in clockwise direction, the inclination θ is negative.

e.g. (i)



x'
(Negative Inclination)

- 1. Inclination (θ) of the x-axis and every line parallel to the x-axis is 0° .
- 2. Inclination (θ) of the y-axis and every line parallel to the y-axis is 90°.

14.3 Concept of Slope (or, gradient) :

The *slope* of any inclined plane is the ratio between the *vertical rise* of the plane and its *horizontal distance*.

i.e. slope of AC =
$$\frac{\text{vertical rise}}{\text{horizontal distance}} = \frac{\text{AB}}{\text{BC}}$$

C Horizontal distance

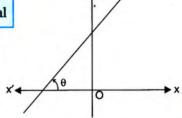
= $tan \theta$

where θ is the angle which the plane makes with the horizontal

14.4 Slope (or, gradient) of a straight line:

The slope of a straight line is the tangent of its inclination and is denoted by letter m.

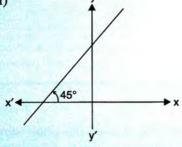
i.e. if the inclination of a line is θ , its slope $m = \tan \theta$.



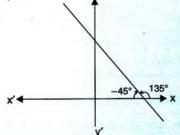
1. Slope of the x-axis is $m = \tan 0^{\circ} = 0$

- [Since, $\theta = 0^{\circ}$]
- 2. Slope of the y-axis is $m = \tan 90^{\circ} = \infty$ i.e. not defined [S
 - [Since, $\theta = 90^{\circ}$]
- 3. The slope of a line is *positive*, if it makes an acute angle in the anti-clockwise direction with x-axis.
- 4. The slope of a line is negative, if it makes an obtuse angle in the anticlockwise direction with the x-axis or an acute angle in the clockwise direction with the x-axis.

e.g. (i)



(ii)



Inclination $\theta = 45^{\circ}$

 $\therefore \text{ Slope} = \tan 45^{\circ} = 1$

Inclination $\theta = 135^{\circ} \text{ or } -45^{\circ}$

:. Slope = $\tan (-45^{\circ}) = -\tan 45^{\circ} = -1$.

14.5 The slope of a straight line passing through two given fixed points:

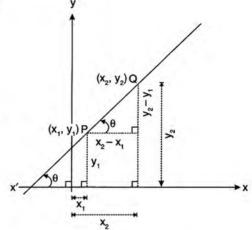
Let P (x_1, y_1) and Q (x_2, y_2) be any two fixed points.

Required to find, the slope of the line through P and Q.

As is clear from the adjoining diagram, the

As is clear from the adjoining diagram, the slope of the line passing through P and Q is

$$m = \tan \theta = \frac{y_2 - y_1}{x_2 - x_1}$$
 or, $m = \frac{y_1 - y_2}{x_1 - x_2}$



14.6 Parallel Lines :

Let AB and CD be two straight lines parallel to each other and having inclinations θ and α respectively.

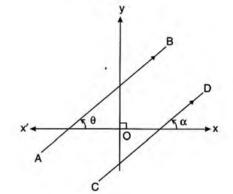
Since, the lines are parallel to each other, the corresponding angles are equal,

i.e.
$$\theta = \alpha \implies \tan \theta = \tan \alpha$$

i.e. slope of
$$AB = slope$$
 of CD .

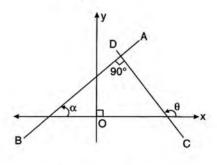
Therefore, if two lines are parallel their slopes are equal.

Conversely, if the slopes of two lines are equal, the lines are parallel to each other.



14.7 Perpendicular Lines :

Let AB and CD be two mutually perpendicular lines, and their inclinations be α and θ respectively.



As is clear from the diagram,

$$\theta = 90^{\circ} + \alpha$$
 [Ext. angle of a Δ = sum of its two int. opp. angles]

$$\Rightarrow$$
 tan $\theta = \tan (90^{\circ} + \alpha)$

$$\Rightarrow$$
 tan $\theta = -\cot \alpha$

[Since,
$$\tan (90^{\circ} + \alpha) = -\cot \alpha$$
]

$$\Rightarrow \qquad \tan \theta = -\frac{1}{\tan \alpha}$$

[Since,
$$\cot \alpha = \frac{1}{\tan \alpha}$$
]

$$\Rightarrow$$
 tan θ · tan $\alpha = -1$

$$\therefore m_1 \cdot m_2 = -1$$

[Where
$$m_1$$
 = slope of AB and m_2 = slope of CD]

Therefore, if two lines are mutually perpendicular, the product of their slopes is -1.

Conversely, if the product of the slopes of two lines is -1; the lines are mutually perpendicular.

- 1. If the slope of a line is 2,
 - (i) the slope of its parallel line is also 2 [So that : $m_1 = m_2$]
- and, (ii) the slope of its perpendicular is $-\frac{1}{2}$ [So that: $m_1 \times m_2 = 2 \times -\frac{1}{2} = -1$]
 - 2. Similarly, if the slope of a line is $-\frac{3}{4}$,
 - (i) the slope of its parallel is $-\frac{3}{4}$
- and, (ii) the slope of its perpendicular is $\frac{4}{3}$.
- 3. Since the slope of the x-axis is zero, therefore, the slope of every line parallel to the x-axis is also zero.
- 4. Since the slope of the y-axis is not defined. therefore, the slope of every line parallel to the y-axis is also not defined.

For:	x-axis	y-axis	Line parallel to x-axis	Line parallel to y-axis
1. Inctination (θ)	0°	90°	0°	90°
2. Slope (m)	0	∞ (not defined)	0	∞ '

Also, if the slope of a line is $-\frac{2}{3}$,

- (i) slope of its parallel = $-\frac{2}{3}$ and
- (ii) slope of its perpendicular = $\frac{3}{2}$.
- 4

Find the slope of the line segment whose inclination is:

(i) 60°

(ii) 52°

Solution:

(i) Slope = $\tan \theta$

$$= \tan 60^{\circ} = \sqrt{3}$$

[Given,
$$\theta = 60^{\circ}$$
]

Ans.

- (ii) Slope = $\tan 52^{\circ} = 1.2799$
- [From the trigonometrical table]
- Ans.

- 6 Find the inclination of the line whose slope is :
 - (i) 1

(ii) 2.9042

Solution:

(i) Since, slope = 1

$$\Rightarrow$$
 tan $\theta = 1 = \tan 45^{\circ} \Rightarrow \theta = 45^{\circ}$

Ans.

(ii) Since, slope = 2.9042

$$\Rightarrow \qquad \tan \theta = 2.9042 = \tan 71^{\circ}$$

[From the trigonometrical table]

 \Rightarrow

$$\theta = 71^{\circ}$$

Ans.

- 6 Find the slope of the line passing through the points A (-2, 3) and B (2, 7). Also find:
 - (i) the inclination of the line AB, (ii) slope of the line parallel to AB,
 - (iii) slope of the line perpendicular to AB.

Solution:

1

Let A (-2, 3) = (x_1, y_1) and B (2, 7) = (x_2, y_2)

:. Slope of AB =
$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 3}{2 + 2} = 1$$

Ans.

(i) Let inclination of line AB be θ

$$\therefore$$
 tan $\theta = 1 \implies \tan \theta = \tan 45^{\circ} \implies \theta = 45^{\circ}$

Ans.

(ii) Slope of the line parallel to AB = Slope of AB = 1

Ans.

(iii) Slope of the line perpendicular to AB

$$= -\frac{1}{\text{Slope of AB}} = -\frac{1}{1} = -1$$

Ans.

The line joining A (-3, 4) and B (2, -1) is parallel to the line joining C (1, -2) and D (0, x). Find x.

Solution:

Slope of AB = Slope of CD

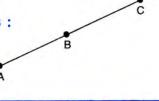
[Since AB // CD]

$$\Rightarrow \frac{-1-4}{2+3} = \frac{x+2}{0-1} \Rightarrow x = -1$$

Ans.

14.8 Condition for Collinearity of three points:

If three points A, B and C are collinear, *i.e.* they lie on the same straight line, then: slope of AB = slope of BC.



8 Given the points A (2, 3), B (-5, 0) and C (-2, a) are collinear. Find 'a'.

Solution:

Slope of AB = Slope of BC

[Since the points A, B and C are collinear]

$$\Rightarrow \frac{0-3}{-5-2} = \frac{a-0}{-2+5} \Rightarrow a = \frac{9}{7} = 1\frac{2}{7}$$

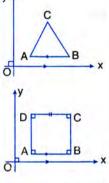
Ans.

EXERCISE 14(B)

- 1. Find the slope of the line whose inclination is:
 - (i) 0°
- (ii) 30°
- (iii) 72° 30′
- (iv) 46°
- 2. Find the inclination of the line whose slope is:
 - (i) 0
- (ii) $\sqrt{3}$
- (iii) 0.7646
- (iv) 1.0875

- 3. Find the slope of the line passing through the following pairs of points:
 - (i) (-2, -3) and (1, 2)
 - (ii) (-4, 0) and origin
 - (iii) (a, -b) and (b, -a)

- 4. Find the slope of the line parallel to AB if:
 - (i) A = (-2, 4) and B = (0, 6)
 - (ii) A = (0, -3) and B = (-2, 5)
- 5. Find the slope of the line perpendicular to AB if:
 - (i) A = (0, -5) and B = (-2, 4)
 - (ii) A = (3, -2) and B = (-1, 2)
- 6. The line passing through (0, 2) and (-3, -1) is parallel to the line passing through (-1, 5) and (4, a). Find a.
- 7. The line passing through (-4, -2) and (2, -3) is perpendicular to the line passing through (a, 5) and (2, -1). Find a.
- 8. Without using the distance formula, show that the points A (4, -2), B (-4, 4) and C (10, 6) are the vertices of a right-angled triangle.
- 9. Without using the distance formula, show that the points A (4, 5), B (1, 2), C (4, 3) and D (7, 6) are the vertices of a parallelogram.
- 10. (-2, 4), (4, 8), (10, 7) and (11, -5) are the vertices of a quadrilateral. Show that the quadrilateral, obtained on joining the mid-points of its sides, is a parallelogram.
- 11. Show that the points P (a, b+c), Q (b, c+a) and R (c, a+b) are collinear.
- 12. Find x, if the slope of the line joining (x, 2) and (8, -11) is $-\frac{3}{4}$.
- 13. The side AB of an equilateral triangle ABC is parallel to the x-axis. Find the slopes of all its sides.
- 14. The side AB of a square ABCD is parallel to the *x*-axis. Find the slopes of all its sides.



- Also, find:
 - (i) the slope of the diagonal AC,
- (ii) the slope of the diagonal BD.
- 15. A (5, 4), B (-3, -2) and C (1, -8) are the vertices of a triangle ABC. Find:
 - (i) the slope of the altitude of AB,
 - (ii) the slope of the median AD and
 - (iii) the slope of the line parallel to AC.
- 16. The slope of the side BC of a rectangle ABCD is $\frac{2}{3}$. Find:
 - (i) the slope of the side AB,
 - (ii) the slope of the side AD.
- 17. Find the slope and the inclination of the line AB if:
 - (i) A = (-3, -2) and B = (1, 2)
 - (ii) $A = (0, -\sqrt{3})$ and B = (3, 0)
 - (iii) A = $(-1, 2\sqrt{3})$ and B = $(-2, \sqrt{3})$
- 18. The points (-3, 2), (2, -1) and (a, 4) are collinear. Find a.
- 19. The points (K, 3), (2, -4) and (-K + 1, -2) are collinear. Find K.
- 20. Plot the points A (1, 1), B (4, 7) and C (4, 10) on a graph paper. Connect A and B, and also A and C.

Which segment appears to have the steeper slope, AB or AC ?

Justify your conclusion by calculating the slopes of AB and AC.

- 21. Find the value(s) of k so that PQ will be parallel to RS. Given:
 - (i) P (2, 4), Q (3, 6), R (8, 1) and S (10, k)
 - (ii) P (3, -1), Q (7, 11), R (-1, -1) and S (1, k)
 - (iii) P (5, -1), Q (6, 11), R (6, -4k) and S $(7, k^2)$

14.9 X-intercept :

If a line meets the x-axis at point A, then the distance of point A from the origin O (i.e. OA) is called x-intercept.

i.e. x-intercept = intercept made by the line on the x-axis = OA.

A x-intercept

14.10 Y-intercept :

If a line meets the y-axis at point B, then the distance of point B from the origin O

(i.e. OB) is called *y-intercept*. i.e. y-intercept = intercept made by the line on the y-axis = OB.

Also, if x-intercept of a point P is 5; P = (5, 0).

And, if y-intercept of a point Q is 5; Q = (0, 5).

In the same way, if x-intercept = -8; the corresponding point on the x-axis = (-8, 0) and if y-intercept is 6; the corresponding point on the y-axis = (0, 6) and so on.

14.11 Equation of a line :

[Various forms of the equations of straight lines]

Type 1: (Slope-Intercept Form). When slope (gradient) and intercept on the y-axis are given.

Let AB be a line which makes an angle θ° with x-axis and whose intercept OB on the y-axis is c (i.e. OB = c).

Required to find the equation of line AB.

As is clear from the figure, the slope of $AB = m = \tan \theta$.

Let P(x, y) be any point on the line AB.

Then in \triangle BPR,

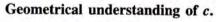
$$\tan \theta = \frac{y-c}{x}$$

i.e.
$$m = \frac{y-c}{x}$$

or,
$$mx = y - c$$

or, y = mx + c; which is the required equation of line AB.

:. Slope-intercept form of the equation of a line is y = mx + c.



As per the equation y = mx + c, the constant c is called the y-intercept.

- It is the ordinate of the point where the line intercepts the y-axis.
- 2. Also, it is the point on the line where x = 0.

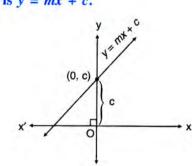
Type 2: (Point-Slope Form). When the slope of the line and a point in it are given.

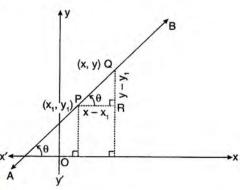
Let AB be a line having inclination θ and passing through the point P (x_1, y_1) .

Required to find the equation of line AB.

Clearly, slope of AB is $m = \tan \theta$ Consider a point Q (x, y) in the line AB. As shown in the diagram, in \triangle PQR,

$$\Rightarrow \frac{QR}{PR} = \tan \theta$$





$$\Rightarrow \frac{y - y_1}{x - x_1} = m$$

[Since, $\tan \theta = m$]

 \Rightarrow $y - y_1 = m (x - x_1)$; which is the required equation of line AB.

Type 3: (Two-Points Form).

When the co-ordinates of two points of the line are given.

Let the line AB pass through the points (x_1, y_1) and (x_2, y_2) .

Required to find the equation of line AB.

Slope of line AB =
$$\frac{y_2 - y_1}{x_2 - x_1} = m$$
.

Now, as the slope of line AB is m and it passes through the point (x_1, y_1) ; its equation is:

$$y - y_1 = m (x - x_1).$$

9 Find the

Find the equation of a line:

- (i) whose inclination is 45° and y-intercept is 5.
- (ii) with inclination = 60° and passing through (-2, 5).
- (iii) passing through the points (-3, 1) and (1, 5).

Solution:

- (i) Given, inclination $\theta = 45^{\circ}$ and y-intercept c = 5,
 - $\therefore \qquad \text{Slope } m = \tan \theta = \tan 45^\circ = 1$

Substituting the values of m and c in the equation y = mx + c, we get:

y = x + 5; which is the required equation.

Ans.

(ii) Given, $\theta = 60^{\circ}$

 \Rightarrow slope $m = \tan 60^{\circ} = \sqrt{3}$ and $(x_1, y_1) = (-2, 5)$.

Substituting in $y - y_1 = m (x - x_1)$, we get:

$$y - 5 = \sqrt{3}(x + 2)$$

 \Rightarrow y = $\sqrt{3}$ x + 2 $\sqrt{3}$ + 5, which is the required equation.

Ans.

(iii) Let $(-3, 1) = (x_1, y_1)$ and $(1, 5) = (x_2, y_2)$

$$\therefore \text{ Slope of the line} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{1 + 3} = 1.$$

Equation: $y - y_1 = m(x - x_1)$

$$\Rightarrow \qquad y-1=1\ (x+3)$$

or
$$y = x + 4$$

Ans.

10 Fi

Find the equation of the line whose x-intercept is 8 and y-intercept is -12.

Solution:

When x-intercept = 8; corresponding point on the x-axis = (8, 0)

When y-intercept = -12; corresponding point on the y-axis = (0, -12)

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Let
$$(8, 0) = (x_1, y_1)$$
 and $(0, -12) = (x_2, y_2)$

$$\Rightarrow$$
 Slope = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-12 - 0}{0 - 8} = \frac{3}{2}$

And, the required equation is: $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 0 = \frac{3}{2}(x - 8) \qquad \Rightarrow 2y = 3x - 24$$

Ans.

M Find the equation of the line whose slope is -3 and x-intercept is also -3.

Solution:

$$\therefore$$
 x-intercept = -3 \Rightarrow The corresponding point on the x-axis = (-3, 0).

Now, we have : m = -3 and $(x_1, y_1) = (-3, 0)$

$$\therefore$$
 Required equation is : $y - y_1 = m (x - x_1)$

$$\Rightarrow y - 0 = -3(x + 3) \Rightarrow$$

$$y = -3x - 9$$
 or $3x + y + 9 = 0$

Ans.

Find the equation of the line which passes through (2, 7) and whose y-intercept is 3.

Solution:

y-intercept =
$$3 \Rightarrow$$
 The corresponding point on the y-axis = $(0, 3)$

Now, we have : $(x_1, y_1) = (2, 7)$ and $(x_2, y_2) = (0, 3)$

$$\therefore \text{ Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 7}{0 - 2} = \frac{-4}{-2} = 2$$

And, required equation is: $y - y_1 = m(x - x_1)$

$$\Rightarrow$$
 y - 7 = 2 (x - 2) \Rightarrow y - 7 = 2x - 4 or y = 2x + 3

Ans.

Alternative method:

Given y-intercept = 3 i.e. c = 3

$$\therefore \qquad y = mx + c \quad \Rightarrow \quad y = mx + 3$$

Since, the line y = mx + 3 passes through (2, 7)

$$\therefore 7 = m \times 2 + 3 \implies m = 2$$

Hence, the required equation is : y = mx + 3 i.e. y = 2x + 3

Ans.

- Œ The equation of a line is 3x - 4y + 12 = 0. It meets the x-axis at point A and the y-axis at point B. Find:
 - (i) the co-ordinates of points A and B;
 - (ii) the length of intercept AB, cut by the line within the co-ordinate axes.

Solution:

(i) For A (the point on the x-axis); the value of y = 0

$$3x - 4y + 12 = 0 \Rightarrow 3x - 4 \times 0 + 12 = 0 \Rightarrow x = -4 \text{ and } A = (-4, 0)$$

Ans.

For B (the point on the y-axis); the value of x = 0

$$\therefore 3x - 4y + 12 = 0 \Rightarrow 3 \times 0 - 4y + 12 = 0 \Rightarrow y = 3 \text{ and } \mathbf{B} = (0, 3)$$
 Ans.

(ii) Let A = $(-4, 0) = (x_1, y_1)$ and B = $(0, 3) = (x_2, y_2)$

$$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 + 4)^2 + (3 - 0)^2} = \sqrt{25} = 5$$
 Ans.

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Write down the equation of the line whose gradient is $\frac{3}{2}$ and which passes through P, where P divides the line segment joining A (-2, 6) and B (3, -4) in

the ratio 2:3.

Solution:

For Point P:

$$(x_1, y_1) = A (-2, 6), (x_2, y_2) = B (3, -4) \text{ and } m_1 : m_2 = 2 : 3$$

$$P = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{2 \times 3 + 3 \times -2}{2 + 3}, \frac{2 \times -4 + 3 \times 6}{2 + 3}\right) = (0, 2)$$

For the required line : $m = \frac{3}{2}$ and $(x_1, y_1) = P(0, 2)$

Equation is $y - y_1 = m (x - x_1)$

$$\Rightarrow y - 2 = \frac{3}{2}(x - 0) \Rightarrow 2y - 4 = 3x \text{ i.e. } 3x - 2y + 4 = 0$$

Ans.

14.12 Equally Inclined Lines :

Equally inclined lines means the lines which make equal angles with both the co-ordinate axes. The given figure shows two equally inclined lines AB and CD.

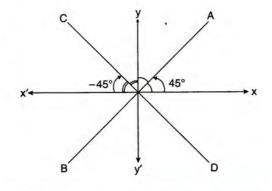
As is clear from the figure:

For AB: Inclination $\theta = 45^{\circ}$,

 \therefore Slope = tan 45° = 1.

For CD: Inclination $\theta = -45^{\circ}$,

:. Slope = $\tan (-45^{\circ}) = -1$.





Find the equations of the lines which pass through the point (-2, 3) and are equally inclined to the co-ordinate axes.

Solution:

As is clear from the figure, there are two lines AB and CD, equally inclined to the co-ordinate axes.

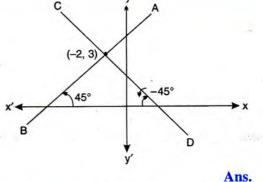
 $m = \tan 45^{\circ} = 1$ For line AB:

and
$$(x_1, y_1) = (-2, 3)$$
.

Its equation: $y - y_1 = m(x - x_1)$

$$\Rightarrow \qquad y-3=1\ (x+2)$$

$$\Rightarrow \qquad \qquad y = x + 5$$



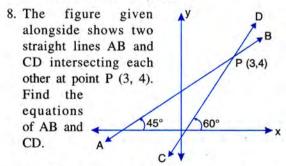
 $m = \tan (-45^{\circ}) = -1 \text{ and } (x_1, y_1) = (-2, 3)$ For line CD:

Its equation is $y - 3 = -1 (x + 2) \Rightarrow x + y = 1$

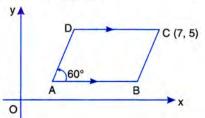
Ans.

EXERCISE 14(C)

- 1. Find the equation of a line whose : y-intercept = 2 and slope = 3,
- 2. Find the equation of a line whose : y-intercept = -1 and inclination = 45°,
- 3. Find the equation of the line whose slope is $-\frac{4}{3}$ and which passes through (-3, 4).
- 4. Find the equation of a line which passes through (5, 4) and makes an angle of 60° with the positive direction of the x-axis.
- 5. Find the equation of the line passing through:
 - (i) (0, 1) and (1, 2) (ii) (-1, -4) and (3, 0)
- 6. The co-ordinates of two points P and Q are (2, 6) and (-3, 5) respectively. Find:
 - (i) the gradient of PQ;
 - (ii) the equation of PQ;
 - (iii) the co-ordinates of the point where PQ intersects the x-axis.
- 7. The co-ordinates of two points A and B are (-3, 4) and (2, -1). Find:
 - (i) the equation of AB;
 - (ii) the co-ordinates of the point where the line AB intersects the *y*-axis.



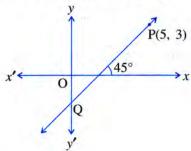
- 9. In $\triangle ABC$, A = (3, 5), B = (7, 8) and C = (1, -10). Find the equation of the median through A.
- 10. The following figure shows a parallelogram ABCD whose side AB is parallel to the x-axis, $\angle A = 60^{\circ}$ and vertex C = (7, 5). Find the equations of BC and CD.



- 11. Find the equation of the straight line passing through origin and the point of intersection of the lines x + 2y = 7 and x y = 4.
- 12. In triangle ABC, the co-ordinates of vertices A, B and C are (4, 7), (-2, 3) and (0, 1) respectively. Find the equation of median through vertex A.

Also, find the equation of the line through vertex B and parallel to AC.

- 13. A, B and C have co-ordinates (0, 3), (4, 4) and (8, 0) respectively. Find the equation of the line through A and perpendicular to BC.
- 14. Find the equation of the perpendicular dropped from the point (-1, 2) onto the line joining the points (1, 4) and (2, 3).
- 15. Find the equation of the line, whose :
 - (i) x-intercept = 5 and y-intercept = 3
 - (ii) x-intercept = -4 and y-intercept = 6
 - (iii) x-intercept = -8 and y-intercept = -4
- 16. Find the equation of the line whose slope is $-\frac{5}{6}$ and x-intercept is 6.
- 17. Find the equation of the line with x-intercept 5 and a point on it (-3, 2).
- 18. Find the equation of the line through (1, 3) and making an intercept of 5 on the y-axis.
- 19. Find the equations of the lines passing through point (-2, 0) and equally inclined to the co-ordinate axes.
- 20. The line through P(5, 3) intersects y-axis at Q.
 - (i) Write the slope of the line.
 - (ii) Write the equation of the line.
 - (iii) Find the co-ordinates of Q. [2012]



- 21. Write down the equation of the line whose gradient is $-\frac{2}{5}$ and which passes through point P, where P divides the line segment joining A (4, -8) and B (12, 0) in the ratio 3:1.
- 22. A (1, 4), B (3, 2) and C (7, 5) are vertices of a triangle ABC. Find:
- (i) the co-ordinates of the centroid of triangle ABC.
- (ii) the equation of a line, through the centroid and parallel to AB. [2002]
- 23. A (7, -1), B (4, 1) and C (-3, 4) are the vertices of a triangle ABC. Find the equation of a line through the vertex B and the point P in AC; such that AP: CP = 2:3.

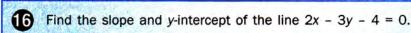
Remember :

- 1. Equation of the x-axis is y = 0.
- 2. Equation of the y-axis is x = 0.
- 3. Equation of a line parallel to the x-axis and at a distance 'a' from it, is y = a.
- 4. Equation of a line parallel to the y-axis and at a distance 'b' from it, is x = b.

14.13 To find the slope and y-intercept of a given line :

Steps:

- 1. Convert the given equation of the line in the form y = mx + c.
- 2. Then, the coefficient of x is slope (m) and the constant term with its proper sign is y-intercept (c).



Solution:

Given equation is 2x - 3y - 4 = 0

$$\Rightarrow \qquad -3y = -2x + 4 \quad \Rightarrow \quad y = \frac{2}{3}x - \frac{4}{3}$$

:. Slope (m) of the given line = $\frac{2}{3}$ and its y-intercept (c) = $-\frac{4}{3}$ Ans.



Given two straight lines 3x - 2y = 5 and 2x + ky + 7 = 0. Find the value of k for which the given lines are :

(i) parallel to each other.

(ii) perpendicular to each other.

Ans.

Solution:

$$3x - 2y = 5 \implies y = \frac{3}{2}x - \frac{5}{2} \implies \text{its slope} = \frac{3}{2}$$

$$2x + ky + 7 = 0 \implies y = -\frac{2}{k}x - \frac{7}{k} \implies \text{its slope} = \frac{-2}{k}$$

(i) Lines are parallel
$$\Rightarrow \frac{3}{2} = \frac{-2}{k} \Rightarrow k = \frac{-4}{3}$$

(ii) Lines are perpendicular
$$\Rightarrow \frac{3}{2} \times \frac{-2}{k} = -1 \Rightarrow k = 3$$
 Ans.

13

Find the equation of the line passing through (2, -1) and parallel to the line 2x - y = 4.

Solution:

The given line is 2x - y = 4. Converting it into the form : y = mx + c; we get :

$$y = 2x - 4$$

 \Rightarrow Slope of the given line = 2 = slope of the required parallel line

Hence, for the required parallel line; m = 2 and $(x_1, y_1) = (2, -1)$.

 \therefore Equation is: $y - y_1 = m(x - x_1)$

$$\Rightarrow \qquad y+1=2(x-2) \Rightarrow y=2x-5$$

x-5

Ans.



Find the equation of the line which passes through the point (-2, 3) and is perpendicular to the line 2x + 3y + 4 = 0

Solution:

$$2x + 3y + 4 = 0$$
 $\Rightarrow 3y = -2x - 4$ $\Rightarrow y = -\frac{2}{3}x - \frac{4}{3}$

 \therefore Slope of the given line = $-\frac{2}{3}$ and slope of its perpendicular line = $\frac{3}{2}$

 \therefore Hence, req. equation of perpendicular line is $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 3 = \frac{3}{2}(x + 2) \qquad m = \frac{3}{2} \text{ and}$$

$$\Rightarrow 2y - 6 = 3x + 6 \qquad (x_1, y_1) = (-2, 3)$$
or, $2y = 3x + 12$
Ans.



Given two points A (-5, 2) and B (1, -4), find:

- (i) mid-point of AB;
- (ii) slope of AB;
- (iii) slope of perpendicular to AB; (iv) equation of the perpendicular bisector of AB.

Solution:

(i) Mid-point P of AB =
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

= $\left(\frac{-5 + 1}{2}, \frac{2 - 4}{2}\right)$
= $(-2, -1)$ Ans.

(ii) Slope of AB =
$$\frac{y_2 - y_1}{x_2 - x_1}$$
 = $\frac{-4 - 2}{1 + 5}$ = -1 Ans.

(iii) Slope of
$$\perp$$
 to AB = $-\frac{1}{\text{slope of AB}} = -\frac{1}{-1} = 1$ Ans.

(iv) For the perpendicular bisector (i.e. the line passing through the mid-point of AB and perpendicular to it):

$$m = 1$$
 and $(x_1, y_1) = P(-2, -1)$

$$\therefore$$
 Equation is : $y - y_1 = m (x - x_1)$

$$\Rightarrow \qquad y+1=1\ (x+2) \quad \Rightarrow \quad y=x+1$$

Ans.



ABCD is a rhombus. The co-ordinates of A and C are (3, 6) and (-1, 2) respectively. Find the equation of BD.

Solution:

In a rhombus, the diagonals bisect each other at right angle at point P.

$$\therefore$$
 P = the mid-point of AC = $\left[\frac{3+(-1)}{2}, \frac{6+2}{2}\right] = (1, 4)$

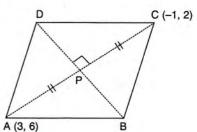
Since, slope of AC =
$$\frac{2-6}{-1-3} = \frac{-4}{-4} = 1$$

$$\therefore$$
 Slope of diagonal BD = $-\frac{1}{1} = -1$

Hence, for BD:
$$m = -1, (x_1, y_1) = P(1, 4)$$

$$\Rightarrow$$
 Its equation is : $y - y_1 = m (x - x_1)$

$$\Rightarrow y - 4 = -1 (x - 1) \Rightarrow x + y = 5$$



Ans.

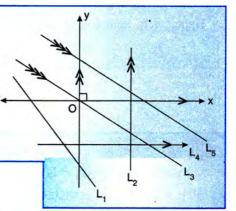
22 Match the equations A, B, C, D and E with the lines L₁, L₂, L₃, L₄ and L₅, whose graphs are roughly drawn in the given diagram.

$$A = 2x + y = 0;$$
 $B = 2x + y = 20$

$$C \equiv x = 8$$
:

$$D \equiv y = -12$$

$$E = 2x + 3y + 12 = 0$$



Solution:

By substituting x = 0 and y = 0 in equation

$$2x + y = 0$$
 we get: $2 \times 0 + 0 = 0$; which is true.

$$\therefore$$
 A = 2x + y = 0 passes through the origin; hence A = L₃

Ans.

Now, slope of 2x + y = 0 is -2 and slope of 2x + y = 20 is also -2; therefore A = 2x + y = 0 and B = 2x + y = 20 are parallel to each other. In the diagram L_3 and L_5 are parallel and $A = L_3$; therefore $B = L_5$

Since, the equation of a line parallel to y-axis is of the form x = a constant;

$$\therefore C = L_2$$
 Ans.

Since, the equation of a line parallel to x-axis is of the form y = a constant.

$$\therefore D = L_4$$
 Ans.

Now, for E = 2x + 3y + 12 = 0; $y = 0 \implies 2x + 0 + 12 = 0 \implies x = -6$ i.e. x-intercept = -6. Similarly, y-intercept = -4.

As for E = 2x + 3y + 12 = 0, the intercepts with the axes are negative; $E = L_1$ Ans.

EXERCISE 14(D)

1. Find the slope and y-intercept of the line:

(i) y = 4

(ii) ax - by = 0

(iii) 3x - 4y = 5

- 2. The equation of a line is x y = 4. Find its slope and y-intercept. Also, find its inclination.
- 3. (i) Is the line 3x + 4y + 7 = 0 perpendicular to the line 28x - 21y + 50 = 0?
 - (ii) Is the line x 3y = 4 perpendicular to the line 3x - y = 7?
 - (iii) Is the line 3x + 2y = 5 parallel to the line x + 2y = 1?
 - (iv) Determine x so that the slope of the line through (1, 4) and (x, 2) is 2.
- 4. Find the slope of the line which is parallel to:

(i)
$$x + 2y + 3 = 0$$

(i)
$$x + 2y + 3 = 0$$
 (ii) $\frac{x}{2} - \frac{y}{3} - 1 = 0$

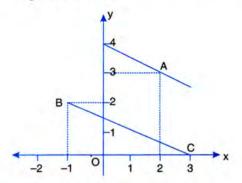
5. Find the slope of the line which is perpendicular to:

(i)
$$x - \frac{y}{2} + 3 = 0$$
 (ii) $\frac{x}{3} - 2y = 4$

(ii)
$$\frac{x}{3} - 2y = 4$$

- 6. (i) Lines 2x by + 5 = 0 and ax + 3y = 2are parallel to each other. Find the relation connecting a and b.
 - (ii) Lines mx + 3y + 7 = 0 and 5x ny 3= 0 are perpendicular to each other. Find the relation connecting m and n.
- 7. Find the value of p if the lines, whose equations are 2x - y + 5 = 0 and px + 3y = 4are perpendicular to each other.
- 8. The equation of a line AB is 2x 2y + 3 = 0.
 - (i) Find the slope of the line AB.
 - (ii) Calculate the angle that the line AB makes with the positive direction of the x-axis.
- 9. The lines represented by 4x + 3y = 9 and px - 6y + 3 = 0 are parallel. Find the value of p.
- 10. If the lines y = 3x + 7 and 2y + px = 3 are perpendicular to each other, find the value of p. [2006]
- 11. The line through A(-2, 3) and B(4, b) is perpendicular to the line 2x - 4y = 5. Find the value of b. [2012]
- 12. Find the equation of the line passing through (-5, 7) and parallel to:
 - (i) x-axis
- (ii) y-axis
- 13. (i) Find the equation of the line passing through (5, -3) and parallel to x - 3y = 4.

- (ii) Find the equation of the line parallel to the line 3x + 2y = 8 and passing through the point (0, 1). [2007]
- 14. Find the equation of the line passing through (-2, 1) and perpendicular to 4x + 5y = 6.
- 15. Find the equation of the perpendicular bisector of the line segment obtained on joining the points (6, -3) and (0, 3).
- 16. In the following diagram, write down:
 - (i) the co-ordinates of the points A, B and C.
 - (ii) the equation of the line through A and parallel to BC.



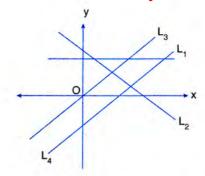
- 17. B (-5, 6) and D (1, 4) are the vertices of rhombus ABCD. Find the equations of diagonals BD and AC.
- 18. A = (7, -2) and C = (-1, -6) are the vertices of square ABCD. Find the equations of diagonals AC and BD.
- 19. A (1, -5), B (2, 2) and C (-2, 4) are the vertices of triangle ABC. find the equation of:
 - (i) the median of the triangle through A.
 - (ii) the altitude of the triangle through B.
 - (iii) the line through C and parallel to AB.
- 20. (i) Write down the equation of the line AB, through (3, 2) and perpendicular to the line 2y = 3x + 5.
 - (ii) AB meets the x-axis at A and the y-axis at B. Write down the co-ordinates of A and B. Calculate the area of triangle OAB, where O is the origin.
- 21. The line 4x 3y + 12 = 0 meets x-axis at A. Write the co-ordinates of A.
 - Determine the equation of the line through A and perpendicular to 4x - 3y + 12 = 0.
- 22. The point P is the foot of perpendicular from A (-5, 7) to the line 2x - 3y + 18 = 0. Determine:

- (i) the equation of the line AP
- (ii) the co-ordinates of P
- 23. The points A, B and C are (4, 0), (2, 2) and (0, 6) respectively. Find the equations of AB and BC.

If AB cuts the y-axis at P and BC cuts the x-axis at Q, find the co-ordinates of P and O.

24. Match the equations A, B, C and D with the lines L₁, L₂, L₃ and L₄, whose graphs are roughly drawn in the given diagram.

$$A \equiv y = 2x;$$
 $B \equiv y - 2x + 2 = 0;$
 $C \equiv 3x + 2y = 6;$ $D \equiv y = 2$



25. Find the value of a for which the points A(a, 3), B(2, 1) and C(5, a) are collinear. Hence, find the equation of the line.

EXERCISE 14(E)

1. Point P divides the line segment joining the points A (8, 0) and B (16, -8) in the ratio 3:5. Find its co-ordinates of point P. Also, find the equation of the line through P

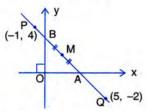
and parallel to 3x + 5y = 7.

2. The line segment joining the points A (3, -4)and B (-2, 1) is divided in the ratio 1:3 at point P in it. Find the co-ordinates of P. Also, find the equation of the line through P and perpendicular to the line 5x - 3y = 4.

3. A line 5x + 3y + 15 = 0 meets y-axis at point P. Find the co-ordinates of point P. Find the equation of a line through P and perpendicular to x - 3y + 4 = 0.

4. Find the value of k for which the lines kx - 5y + 4 = 0 and 5x - 2y + 5 = 0 are perpendicular to each other [2003]

5. A straight line passes through the points P(-1, 4) and Q(5, -2). It intersects the co-ordinate axes at points A and B. M is the midpoint of the segment AB. Find:



- (i) The equation of the line.
- (ii) The co-ordinates of A and B.
- (iii) The co-ordinates of M.

[2003]

6. (1, 5) and (-3, -1) are the co-ordinates of vertices A and C respectively of rhombus ABCD. Find the equations of the diagonals AC and BD.

- 7. Show that A (3, 2), B (6, -2) and C (2, -5) can be the vertices of a square.
 - (i) Find the co-ordinates of its fourth vertex D, if ABCD is a square.
 - (ii) Without using the co-ordinates of vertex D, find the equation of side AD of the square and also the equation of diagonal BD.

8. A line through origin meets the line x = 3y + 2 at right angles at point X. Find the co-ordinates of X.

9. A straight line passes through the point (3, 2) and the portion of this line, intercepted between the positive axes, is bisected at this point. Find the equation of the line.

10. Find the equation of the line passing through the point of intersection of 7x + 6y = 71 and 5x - 8y = -23; and perpendicular to the line 4x - 2y = 1.

11. Find the equation of the line which is perpendicular to the line $\frac{x}{a} - \frac{y}{b} = 1$ at the point where this line meets y-axis.

- 12. O (0, 0), A (3, 5) and B (-5, -3) are the vertices of triangle OAB. Find:
 - (i) the equation of median of triangle OAB through vertex O.
 - (ii) the equation of altitude of triangle OAB through vertex B.
- 13. Determine whether the line through points (-2, 3) and (4, 1) is perpendicular to the line 3x = y + 1.

Does line 3x = y + 1 bisect the line segment joining the two given points?

- 14. Given a straight line $x \cos 30^{\circ} + y \sin 30^{\circ} = 2$. Determine the equation of the other line which is parallel to it and passes through (4, 3).
- 15. Find the value of k such that the line (k-2) x + (k+3) y 5 = 0 is:
 - (i) perpendicular to the line 2x y + 7 = 0
 - (ii) parallel to it.
- 16. The vertices of a triangle ABC are A (0, 5), B (-1, -2) and C (11, 7). Write down the equation of BC. Find:
 - (i) the equation of line through A and perpendicular to BC.
 - (ii) the co-ordinates of the point P, where the perpendicular through A, as obtained in (i), meets BC.
- 17. From the given figure, find:
 - (i) the coordinates of A, B and C.
 - A, B and C.

 (ii) the equation of the line x through A and parallel to BC.

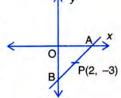
 [2005]
- 18. P(3, 4), Q(7, -2) and R(-2, -1) are the vertices of triangle PQR. Write down the equation of the median of the triangle through R. [2004]
- 19. A(8, -6), B(-4, 2) and C(0, -10) are vertices of a triangle ABC. If P is the mid-point of AB and Q is the mid-point of AC, use co-ordinate geometry to show that PQ is parallel to BC. Give a special name to quadrilateral PBCQ.
- 20. A line AB meets the x-axis at point A and y-axis at point B. The point P(-4, -2) divides the line segment AB internally such that AP: PB = 1: 2. Find:
 - (i) the co-ordinates of A and B.
 - (ii) equation of line through P and perpendicular to AB.
- 21. A line intersects x-axis at point (-2, 0) and cuts off an intercept of 3 units from the positive side of y-axis. Find the equation of the line.
- 22. Find the equation of a line passing through the point (2, 3) and having the x-intercept of 4 units. [2002]
- 23. The given figure (not drawn to scale) shows two straight lines AB and CD. If equation of the line AB is:
 - y = x + 1 and equation of line CD is:

- $y = \sqrt{3} x 1$. Write down the inclination of lines AB and CD; also, find the angle θ between AB and CD.
- 24. Write down the equation of the line whose gradient is $\frac{3}{2}$ and which passes through P, where P divides the line segment joining A(-2, 6) and B(3, -4) in the ratio 2: 3.

[2001]

- 25. The ordinate of a point lying on the line joining the points (6, 4) and (7, -5) is -23. Find the co-ordinates of that point.
- 26. Point A and B have co-ordinates (7, -3) and (1, 9) respectively. Find:
 - (i) the slope of AB.
 - (ii) the equation of perpendicular bisector of the line segment AB.
 - (iii) the value of 'p' of (-2, p) lies on it. [2008]
- 27. A and B are two points on the x-axis and y-axis respectively. P(2, -3) is the mid point of AB. Find the

 (i) co-ordinates of A

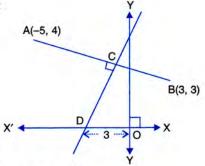


- and B
- (ii) slope of line AB
- (iii) equation of line AB. [2010]
- 28. The equation of a line is 3x + 4y 7 = 0. Find:
 - (i) the slope of the line.
 - (ii) the equation of a line perpendicular to the given line and passing through the intersection of the lines x y + 2 = 0 and 3x + y 10 = 0. [2010]
- 29. ABCD is a parallelogram where A(x, y), B(5, 8), C(4, 7) and D(2, -4). Find:
 - (i) co-ordinates of A
 - (ii) equation of diagonal BD. [2011]
- 30. Given equation of line L_1 is y = 4.
 - (i) Write the slope of line L_2 if L_2 is the bisector of angle O.
 - angle O.

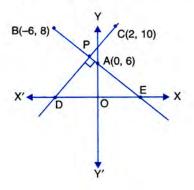
 (ii) Write the L₁ P L₂

 co-ordinates of point P.
 - (iii) Find the $x' \in O$ $\longrightarrow x$ equation of L_2 .

31. Find:

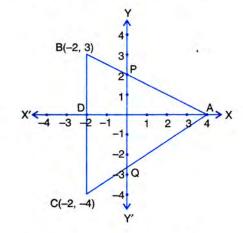


- (i) equation of AB
- (ii) equation of CD
- 32. Find the equation of the line that has x-intercept = -3 and is perpendicular to 3x + 5y = 1.
- 33. A straight line passes through the points P(-1, 4) and Q(5, -2). It intersects x-axis at point A and y-axis at point B. M is the midpoint of the line segment AB. Find:
 - (i) the equation of the line.
 - (ii) the co-ordinates of points A and B.
 - (iii) the co-ordinates of point M.
- 34. In the given figure, line AB meets y-axis at point A. Line through C(2, 10) and D intersects line AB at right angle at point P. Find:



- (i) equation of line AB.
- (ii) equation of line CD.
- (iii) co-ordinates of points E and D.

- 35. A line through point P(4, 3) meets x-axis at point A and the y-axis at point B. If BP is double of PA, find the equation of AB.
- 36. Find the equation of line through the intersection of lines 2x y = 1 and 3x + 2y = -9 and making an angle of 30° with positive direction of x-axis.
- 37. Find the equation of the line through the points A(-1, 3) and B(0, 2). Hence, show that the points A, B and C(1, 1) are collinear.
- 38. Three vertices of a parallelogram ABCD taken in order are A(3, 6), B(5, 10) and C(3, 2), find:
 - (i) the co-ordinates of the fourth vertex D.
 - (ii) length of diagonal BD.
 - (iii) equation of side AB of the parallelogram ABCD. [2015]
- 39. In the figure, given, ABC is a triangle and BC is parallel to the y-axis. AB and AC intersect the y-axis at P and Q respectively.



- (i) Write the co-ordinates of A.
- (ii) Find the length of AB and AC.
- (iii) Find the ratio in which Q divides AC.
- (iv) Find the equation of the line AC.

[2015]