9 Matrices

9.1 Matrix :

A matrix is a rectangular arrangement of numbers, arranged in rows and columns.

e.g. $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$, $\begin{bmatrix} 5 & 3 & 2 \end{bmatrix}$, etc.

1. Plural of matrix is *matrices.*
2. Each number or entity in a matrix is called its **element.**
3. In a matrix, the ***horizontal lines*** are called ***rows***; whereas the ***vertical lines*** are called ***columns***.

9.2 Order of a Matrix :

The order of a matrix = Number of rows in it × Number of columns in it;

i.e. if a matrix has ***m*** number of rows and ***n*** number of columns, its order is written as $m \times n$ and is read as *m* by *n*.

Consider the matrix $\begin{bmatrix} 2 & 1 & 5 \\ 3 & -2 & 7 \end{bmatrix}$ ← 1st row, ← 2nd row; ↑ 1st column, ↑ 2nd column, ↑ 3rd column

It has 2 rows and 3 columns; hence its **order = 2 × 3** (read as *2 by 3*)

While stating the order of a matrix, the number of rows is given first and then the number of columns.

Notation : Matrices, in general, are denoted by capital letters. For example, if *A is a* matrix with *m* rows and *n* columns, then it is written as $A_{m \times n}$.

Similarly, $B_{5 \times 3}$ means, a matrix *B* with 5 rows and 3 columns.

9.3 Elements of a Matrix :

Each number or entity in a matrix is called its **element.**

The total number of elements in a matrix is equal to the product of its number of rows and number of columns, *i.e.* if a matrix has 4 rows and 6 columns, the number of elements in it = 4 × 6 = 24.

Consider matrix A = $\begin{bmatrix} 2 & 4 & -3 \\ 0 & 1 & 2 \end{bmatrix}$

Since, matrix A has 2 rows and 3 columns,

so the number of elements in it = 2 × 3 = 6.

It must be noted here that if a matrix has 6 elements, then it may have :

(i) 1 row and 6 columns; as 1 × 6 = 6, or

(ii) 2 rows and 3 columns; as 2 × 3 = 6, or

(iii) 3 rows and 2 columns; as 3 × 2 = 6, or

(iv) 6 rows and 1 column; as 6 × 1 = 6.

Similarly, if a matrix has 8 elements, it may have :

(i) 1 row and 8 columns so that its order = 1 × 8 and number of elements in it = 1 × 8 = 8, or

(ii) 2 rows and 4 columns so that its order = 2 × 4 and number of elements in it = 2 × 4 = 8, or

(iii) 4 rows and 2 columns so that its order = 4 × 2 and number of elements in it = 4 × 2 = 8, or

(iv) 8 rows and 1 column so that its order = 8 × 1 and number of elements in it = 8 × 1.

9.4 Types of Matrices :

1. **Row Matrix :** A matrix which has *only one row* is called a *row matrix.*

   e.g. $[a \quad b]$ ← Single row (↑ 1st column, ↑ 2nd column)

   Since, this matrix has 1 row and 2 columns, its **order = 1 × 2** (*1 by* 2).

Similarly, $[a \quad b \quad c]$ is a row **matrix of order 1 × 3.**

A *row matrix* is also called a *row vector.*

2. **Column Matrix :** A matrix which has *only one column* is called a *column matrix.*

   e.g. $\begin{bmatrix} a \\ b \end{bmatrix}$ ← 1st row, ← 2nd row (↑ Single column)

   Since, this matrix has 2 rows and 1 column, its **order = 2 × 1** (2 *by* 1).

Similarly, $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is a column **matrix of order 3 × 1.**

A *column matrix* is also called a *column vector.*

3. **Square Matrix :** A matrix which has ***an equal number of rows and columns*** is called a *square matrix.*

   e.g. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ ← 1st row, ← 2nd row (↑ 1st column, ↑ 2nd column)

   Since, this matrix has 2 rows and 2 columns, its **order = 2 × 2** (2 *by* 2).

Similarly, $\begin{bmatrix} 5 & 7 & 4 \\ 2 & -1 & 0 \\ 0 & 3 & 4 \end{bmatrix}$ is a square matrix of order 3 × 3.

4. **Rectangular Matrix :** A matrix in which *the number of rows are not equal to the number of columns* is called a *rectangular matrix.*

   e.g. $\begin{bmatrix} 2 & 4 & 7 \\ 1 & 0 & 5 \end{bmatrix}$, $\begin{bmatrix} 3 & 1 \\ 6 & 2 \\ 1 & 7 \end{bmatrix}$

   Order is 2 × 3 **Order is 3 × 2**

5. **Zero** or **Null Matrix :** If each element of a matrix is zero, it is called a *zero matrix* or a *null matrix*.

   e.g. $\begin{bmatrix} 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, etc.

6. **Diagonal Matrix :** A square matrix which has all its elements zero each except those on the leading (or, principal) diagonal is called a *diagonal matrix.*

   e.g. $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $\begin{bmatrix} 5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, etc.

In a square matrix, the leading (principal) diagonal means the diagonal from top left to bottom right.

7. **Unit** or **Identity Matrix :** A diagonal matrix in which *each element of its leading diagonal is unity* (*i.e.* 1) is called ***a unit*** or ***identity matrix.*** It is denoted by **I**. In other words, it is a square matrix in which each element of its leading diagonal is equal to 1 and all other remaining elements of the matrix are zero each.

   e.g. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, etc.

9.5 Transpose of a Matrix :

Transpose of a matrix is the matrix obtained on interchanging its rows and columns. If A is a matrix, then its transpose is denoted by A^t.

e.g. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 4 & 7 \end{bmatrix}$, then its transpose $A^t = \begin{bmatrix} 2 & 0 \\ 3 & 4 \\ 1 & 7 \end{bmatrix}$

9.6 Equality of Matrices :

Two matrices are said to be equal if :

(i) both the matrices have the same order,

(ii) the corresponding elements of both the matrices are equal.

i.e. if $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$; then $A = B$.

1 Find the values of x, y, a and b, if :

$$\begin{bmatrix} x-2 & y \\ a/2 & b+1 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 1 & 5 \end{bmatrix}$$

Solution :

$x - 2 = 0 \Rightarrow x = 2$

$y = 3$

$a/2 = 1 \Rightarrow a = 2$

$b + 1 = 5 \Rightarrow b = 4$

[If two matrices are equal, their corresponding elements are also equal.]

$\therefore$ $x = 2$, $y = 3$, $a = 2$ and $b = 4$ **Ans.**

9.7 Addition of Matrices :

Compatibility for addition of matrices :

Two matrices can be added together, if they are of the same order.

To add two matrices of the same order means to add the corresponding elements of both the matrices.

e.g. If $A = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$,

then $A + B = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2+3 & 1+2 \\ 5+1 & 6+4 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 6 & 10 \end{bmatrix}$

9.8 Subtraction of Matrices :

The same rule and method is used for the subtraction of matrices as is used for the addition of matrices.

i.e. If $A = \begin{bmatrix} 5 & 4 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 4 & 2 \end{bmatrix}$,

then $A - B = \begin{bmatrix} 5 & 4 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 5-3 & 4-0 \\ 2-4 & 1-2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -2 & -1 \end{bmatrix}$

2 Let $A = \begin{bmatrix} 5 & 4 \\ 3 & -2 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 0 \\ 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix}$, find :

(i) A + B and B + A

(ii) (A + B) + C and A + (B + C)

(iii) Is A + B = B + A ?

(iv) Is (A + B) + C = A + (B + C) ?

In each case, write the conclusion (if any) that you can draw.

Solution :

(i) $A + B = \begin{bmatrix} 5 & 4 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 5-3 & 4+0 \\ 3+1 & -2+4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ **Ans.**

$$B + A = \begin{bmatrix} -3 & 0 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 4 \\ 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -3+5 & 0+4 \\ 1+3 & 4-2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$$ **Ans.**

(ii) ∵ $$A + B = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$$

$$\Rightarrow (A + B) + C = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 4-3 \\ 4+0 & 2+2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 4 \end{bmatrix}$$ **Ans.**

∵ $$B + C = \begin{bmatrix} -3 & 0 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -3+1 & 0-3 \\ 1+0 & 4+2 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 1 & 6 \end{bmatrix}$$

$$\Rightarrow A + (B + C) = \begin{bmatrix} 5 & 4 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} -2 & -3 \\ 1 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-2 & 4-3 \\ 3+1 & -2+6 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 4 \end{bmatrix}$$ **Ans.**

(iii) **Yes**; A + B = B + A

Conclusion : Addition of matrices is commutative

(iv) **Yes**; (A + B) + C = A + (B + C)

Conclusion : Addition of matrices is associative.

Remember :

1. In addition or subtraction of the matrices, the order of the resulting matrix is the same as the order of matrices added or subtracted.
2. If A, B and C are the matrices of the same order, then :
   (i) $A + B = B + A$ *i.e. addition of matrices is commutative.*
   (ii) $A + (B + C) = (A + B) + C$ *i.e. addition of matrices is associative.*
   (iii) $A + X = B \Rightarrow X = B - A$

3 If $A = \begin{bmatrix} 5 & 4 \\ 3 & -1 \end{bmatrix}$; $B = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 2 \\ 1 & 0 \end{bmatrix}$; find :

(i) $A + C$ (ii) $B - A$ (iii) $A + B - C$.

Solution :

(i) $$A + C = \begin{bmatrix} 5 & 4 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5-3 & 4+2 \\ 3+1 & -1+0 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 4 & -1 \end{bmatrix}$$ **Ans.**

(ii) $B - A = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 4 \\ 3 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2-5 & 1-4 \\ 0-3 & 4+1 \end{bmatrix} = \begin{bmatrix} -3 & -3 \\ -3 & 5 \end{bmatrix}$ **Ans.**

(iii) $A + B - C = \begin{bmatrix} 5 & 4 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 7 & 5 \\ 3 & 3 \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 1 & 0 \end{bmatrix}$ [Evaluating $A + B$ then subtracting C]

$= \begin{bmatrix} 7+3 & 5-2 \\ 3-1 & 3-0 \end{bmatrix} = \begin{bmatrix} 10 & 3 \\ 2 & 3 \end{bmatrix}$ **Ans.**

4 If matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 \\ 7 & 4 \end{bmatrix}$; find transpose matrices A^t and B^t. If possible, find : (i) $A + A^t$ (ii) $B + B^t$

Solution :

$A^t = \begin{bmatrix} 2 & 4 \\ 1 & -3 \\ 3 & 2 \end{bmatrix}$ and $B^t = \begin{bmatrix} 3 & 7 \\ -2 & 4 \end{bmatrix}$ **Ans.**

(i) Since, the order of matrix A is 2×3 and that of A^t is 3×2;

$\therefore$ **$A + A^t$ is not possible.** **Ans.**

Two matrices are compatible for addition or subtraction; only when they have the same order.

(ii) Since, the order of matrix B is 2×2 and that of B^t is also 2×2;

$\therefore$ **$B + B^t$ is possible.**

and $B + B^t = \begin{bmatrix} 3 & -2 \\ 7 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 7 \\ -2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 3+3 & -2+7 \\ 7-2 & 4+4 \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 5 & 8 \end{bmatrix}$ **Ans.**

9.9 Additive Identity :

In ordinary numbers;

$3 + 0 = 3 = 0 + 3$; $0 + (-5) = (-5) + 0 = -5$ and so on.

i.e. if any number is added to zero or zero is added to any number, the number remains the same. Here, **0** is said to be the **additive identity in numbers**.

In a similar manner, if any matrix is added to null (zero) matrix of the same order or, a null matrix is added to a matrix of the same order; the matrix always remains unaltered. So, here, **null matrix** is said to be the **additive identity in matrices**.

e.g. (i) $\begin{bmatrix} 4 & 6 \\ 0 & 7 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ 0 & 7 \end{bmatrix}$

(ii) $[3 \quad -2] + [0 \quad 0] = [0 \quad 0] + [3 \quad -2] = [3 \quad -2]$ and so on.

9.10 Additive Inverse :

If A and B are two matrices of the same order such that :

$A + B = B + A$ = a null matrix; then A is said to be additive inverse of B and B is said to be additive inverse of A.

In fact, additive inverse of a matrix A is negative of matrix A *i.e.* $-A$.

e.g. If $A = \begin{bmatrix} 4 & -2 \\ -3 & 7 \end{bmatrix}$, then its additive inverse $= -A = \begin{bmatrix} -4 & 2 \\ 3 & -7 \end{bmatrix}$

and $A + (-A) = \begin{bmatrix} 4 & -2 \\ -3 & 7 \end{bmatrix} + \begin{bmatrix} -4 & 2 \\ 3 & -7 \end{bmatrix}$

$= \begin{bmatrix} 4-4 & -2+2 \\ -3+3 & 7-7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

If O is the zero (null) matrix of the same order as matrix A, then A + O = O + A = A and A + (– A) = (–A) + A = O

9.11 Solving Matrix Equations :

Let A and B be two matrices of the same order such that :

$$A + X = B;$$

where X is an unknown matrix of the same order as that of matrices A and B; then $X = B - A$.

5 If $A = \begin{bmatrix} 8 & 6 \\ -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 5 \\ 1 & 0 \end{bmatrix}$; then solve for 2 × 2 matrix X such that :

(i) $A + X = B$ (ii) $X - B = A$

Solution :

(i) $A + X = B \Rightarrow \boldsymbol{X = B - A}$ $= \begin{bmatrix} -3 & 5 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 8 & 6 \\ -2 & 4 \end{bmatrix}$

$= \begin{bmatrix} -3-8 & 5-6 \\ 1+2 & 0-4 \end{bmatrix} = \begin{bmatrix} -11 & -1 \\ 3 & -4 \end{bmatrix}$ **Ans.**

(ii) $X - B = A \Rightarrow \boldsymbol{X = A + B}$ $= \begin{bmatrix} 8 & 6 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -3 & 5 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 11 \\ -1 & 4 \end{bmatrix}$ **Ans.**

EXERCISE 9(A)

1. State, whether the following statements are true or false. If false, give a reason.
   (i) If A and B are two matrices of orders 3×2 and 2×3 respectively; then their sum $A + B$ is possible.
   (ii) The matrices $A_{2 \times 3}$ and $B_{2 \times 3}$ are conformable for subtraction.
   (iii) Transpose of a 2×1 matrix is a 2×1 matrix.
   (iv) Transpose of a square matrix is a square matrix.
   (v) A column matrix has many columns and only one row.

2. Given : $\begin{bmatrix} x & y+2 \\ 3 & z-1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix}$; find x, y and z.

3. Solve for a, b and c; if :
   (i) $\begin{bmatrix} -4 & a+5 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} b+4 & 2 \\ 3 & c-1 \end{bmatrix}$
   (ii) $\begin{bmatrix} a & a-b \\ b+c & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$

4. If $A = \begin{bmatrix} 8 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 \end{bmatrix}$; find :
   (i) $A + B$ (ii) $B - A$

5. If $A = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $C = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$, find :
   (i) $B + C$ (ii) $A - C$
   (iii) $A + B - C$ (iv) $A - B + C$

6. Wherever possible, write each of the following as a single matrix.
   (i) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 1 & -7 \end{bmatrix}$
   (ii) $\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 3 \\ 6 & -1 & 0 \end{bmatrix}$
   (iii) $\begin{bmatrix} 0 & 1 & 2 \\ 4 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$

7. Find, x and y from the following equations :
   (i) $\begin{bmatrix} 5 & 2 \\ -1 & y-1 \end{bmatrix} - \begin{bmatrix} 1 & x-1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -3 & 2 \end{bmatrix}$
   (ii) $\begin{bmatrix} -8 & x \end{bmatrix} + \begin{bmatrix} y & -2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \end{bmatrix}$

8. Given : $M = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$, find its transpose matrix M^t. If possible, find :
   (i) $M + M^t$ (ii) $M^t - M$

9. Write the additive inverse of matrices A, B and C :
   where $A = \begin{bmatrix} 6 & -5 \end{bmatrix}$; $B = \begin{bmatrix} -2 & 0 \\ 4 & -1 \end{bmatrix}$
   and $C = \begin{bmatrix} -7 \\ 4 \end{bmatrix}$.

10. Given $A = \begin{bmatrix} 2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 4 \end{bmatrix}$; find the matrix X in each of the following :
    (i) $X + B = C - A$
    (ii) $A - X = B + C$

11. Given $A = \begin{bmatrix} -1 & 0 \\ 2 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -3 \\ -2 & 0 \end{bmatrix}$; find the matrix X in each of the following :
    (i) $A + X = B$
    (ii) $A - X = B$
    (iii) $X - B = A$

9.12 Multiplication of a matrix by a scalar (real number) :

To multiply a matrix by a scalar means to multiply each of its elements by this scalar.

e.g. (i) $3\begin{bmatrix} 4 & 1 \end{bmatrix} = \begin{bmatrix} 3\times 4 & 3\times 1 \end{bmatrix} = \begin{bmatrix} 12 & 3 \end{bmatrix}$

(ii) $2\begin{bmatrix} 6 & -4 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2\times 6 & 2\times -4 \\ 2\times 2 & 2\times 0 \end{bmatrix} = \begin{bmatrix} 12 & -8 \\ 4 & 0 \end{bmatrix}$

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(iii) $\frac{1}{2}\begin{bmatrix} 6 & -4 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\times 6 & \frac{1}{2}\times -4 \\ \frac{1}{2}\times 2 & \frac{1}{2}\times 0 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$ and so on.

6 Given $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$, find : $A + 2B - 3C$.

Solution :

$$A + 2B - 3C = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + 2\begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} -3 & -9 \\ -6 & 10 \end{bmatrix}$$ **Ans.**

7 Given, matrix $A = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$ and matrix $B = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$; find matrix X such that : $A + 2X = B$.

Solution :

$$A + 2X = B \quad \Rightarrow \quad 2X = B - A$$

$$= \begin{bmatrix} -1 \\ 7 \end{bmatrix} - \begin{bmatrix} 5 \\ -3 \end{bmatrix} = \begin{bmatrix} -6 \\ 10 \end{bmatrix}$$

$$\Rightarrow \quad X = \frac{1}{2}\begin{bmatrix} -6 \\ 10 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\times -6 \\ \frac{1}{2}\times 10 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$$ **Ans.**

EXERCISE 9(B)

1. Evaluate :
   (i) $3[5 \quad -2]$
   (ii) $7\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$
   (iii) $2\begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 5 & 0 \end{bmatrix}$
   (iv) $6\begin{bmatrix} 3 \\ -2 \end{bmatrix} - 2\begin{bmatrix} -8 \\ 1 \end{bmatrix}$
2. Find x and y if :
   (i) $3[4 \quad x] + 2[y \quad -3] = [10 \quad 0]$
   (ii) $x\begin{bmatrix} -1 \\ 2 \end{bmatrix} - 4\begin{bmatrix} -2 \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -8 \end{bmatrix}$
3. Given $A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & -1 \\ 0 & 0 \end{bmatrix}$; find :
   (i) $2A - 3B + C$ (ii) $A + 2C - B$
4. If $\begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix} + 3A = \begin{bmatrix} -2 & -2 \\ 1 & -3 \end{bmatrix}$; find A.
5. Given $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$
   (i) find the matrix $2A + B$.
   (ii) find a matrix C such that :
   $$C + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

6. If $2\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 3 \\ y & 2 \end{bmatrix} = \begin{bmatrix} z & -7 \\ 15 & 8 \end{bmatrix}$; find the values of x, y and z.

7. Given $A = \begin{bmatrix} -3 & 6 \\ 0 & -9 \end{bmatrix}$ and A^t is its transpose matrix. Find :
   (i) $2A + 3A^t$ (ii) $2A^t - 3A$
   (iii) $\frac{1}{2}A - \frac{1}{3}A^t$ (iv) $A^t - \frac{1}{3}A$

8. Given $A = \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$.
   Solve for matrix X :
   (i) $X + 2A = B$ (ii) $3X + B + 2A = 0$
   (iii) $3A - 2X = X - 2B$.

9. If $M = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $N = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, show that :
   $3M + 5N = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

10. If I is the unit matrix of order 2 × 2; find the matrix M, such that :
   (i) $M - 2I = 3\begin{bmatrix} -1 & 0 \\ 4 & 1 \end{bmatrix}$
   (ii) $5M + 3I = 4\begin{bmatrix} 2 & -5 \\ 0 & -3 \end{bmatrix}$

11. If $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$, find the matrix M. **[2008]**

9.13 Multiplication of Matrices :

Compatibility for multiplication of matrices :

Two matrices A and B can be multiplied together to get the product matrix AB if, and only if, ***the number of columns* in *A*** (the left hand matrix) is **equal** to ***the number of rows* in B** (the right hand matrix).

Let matrix $A = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

Since, the number of columns in A = the number of rows in B = 2.

∴ Product matrix AB is possible.

And, $AB = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Step 1 : Multiply every element of 1st row of matrix A with corresponding element of 1st column of B and add them to get the *first element of the 1st row of the product matrix AB.*

$$AB = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times1+4\times3 & \\ & \end{bmatrix}$$

$$= \begin{bmatrix} 15 & \\ & \end{bmatrix}$$

Step 2 : Multiply every element of 1st row of matrix A with corresponding elements of 2nd column of B and add them to get the *second element of the 1st row of product matrix AB.*

$$AB = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 15 & 3\times2+4\times4 \\ & \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 22 \\ & \end{bmatrix}$$

Step 3 : In the similar manner, multiply the elements of 2nd row of A with corresponding elements of the 1st column of B and get the first element of the *second row of AB*.

$$AB = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 15 & 22 \\ 5\times1+0\times3 & \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 22 \\ 5 & \end{bmatrix}$$

Step 4 : Finally, multiply the elements of the second row of matrix A with corresponding elements of second column of matrix B to get the second element of the second row of AB.

$$AB = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 15 & 22 \\ 5 & 10+0 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 22 \\ 5 & 10 \end{bmatrix}$$

$\therefore$ Product of matrices A and $B = AB$

$$= \begin{bmatrix} \text{1st row of } A\times\text{1st column of } B & \text{1st row of } A\times\text{2nd column of } B \\ \text{2nd row of } A\times\text{1st column of } B & \text{2nd row of } A\times\text{2nd column of } B \end{bmatrix}$$

8 If $A = \begin{bmatrix} -2 & 3 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$; find : (i) AB (ii) BA.

(iii) Is AB = BA ?

(iv) Write the conclusion that you draw from the result obtained above in (iii).

Solution :

(i) **AB** $= \begin{bmatrix} -2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} -2\times1 + 3\times3 & -2\times2 + 3\times5 \\ 4\times1 + 1\times3 & 4\times2 + 1\times5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ 7 & 13 \end{bmatrix}$ **Ans.**

(ii) ***BA*** $= \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 4 & 1 \end{bmatrix}$

$= \begin{bmatrix} \text{1st row of } B\times\text{1st column of } A & \text{1st row of } B\times\text{2nd column of } A \\ \text{2nd row of } B\times\text{1st column of } A & \text{2nd row of } B\times\text{2nd column of } A \end{bmatrix}$

$= \begin{bmatrix} 1\times-2 + 2\times4 & 1\times3 + 2\times1 \\ 3\times-2 + 5\times4 & 3\times3 + 5\times1 \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 14 & 14 \end{bmatrix}$ **Ans.**

(iii) No, AB $\neq$ BA **Ans.**

(iv) **Conclusion :** Matrix multiplication is not commutative. **Ans.**

9 Let $A = \begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 \\ 4 & 6 \end{bmatrix}$; find the matrix AB. Write the conclusion, if any, that you can draw from the result obtained.

Solution :

$$AB = \begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} -12+12 & -18+18 \\ 8-8 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \textbf{Ans.}$$

The result obtained is AB = 0, zero matrix.

Conclusion :

The product of two non-zero matrices can be a zero matrix.

10 If $A = \begin{bmatrix} 4 & -4 \\ -3 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 5 \\ 3 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$ show that AB = AC. Write the conclusion, if any, that you can draw from the result obtained above.

Solution :

$$AB = \begin{bmatrix} 4 & -4 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 6 & 5 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 24-12 & 20+0 \\ -18+9 & -15+0 \end{bmatrix} = \begin{bmatrix} 12 & 20 \\ -9 & -15 \end{bmatrix} \quad .....I$$

$$AC = \begin{bmatrix} 4 & -4 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 8+4 & 12+8 \\ -6-3 & -9-6 \end{bmatrix} = \begin{bmatrix} 12 & 20 \\ -9 & -15 \end{bmatrix} \quad .....II$$

From I and II, we get : **AB = AC**

Conclusion :

AB = AC $\Rightarrow$ Matrices B and C are not equal and matrix A is not a zero-matrix, even then AB = AC.

Conversely, if AB = AC, it does not imply that B = C. That is in AB = AC, we cannot cancel matrix A from both the sides.

In other words, *cancellation law is not applicable in matrix multiplication.*

9.14 Identity Matrix for Multiplication :

In matrix multiplication, the unit matrix *I* is known as the identity matrix for multiplication since, on multiplying any matrix with the identity matrix of the same order, the matrix remains unaltered.

i.e. if I is the unit matrix and A is any matrix of the same order as that of I, then :

$$A \times I = A = I \times A.$$

For Example :

Let $A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$

then $A \times I = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2\times1+3\times0 & 2\times0+3\times1 \\ 4\times1+6\times0 & 4\times0+6\times1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} = A$

and $I \times A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 1\times2+0\times4 & 1\times3+0\times6 \\ 0\times2+1\times4 & 0\times3+1\times6 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} = A$

$\therefore A \times I = A = I \times A.$

11 If $A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$, evaluate $A^2 - 3A + 2I$, where I is a unit matrix of order 2.

Solution :

A unit matrix of order 2 means; a unit matrix of order 2 × 2.

Here, $A^2 = A\cdot A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 4+1 & -2-3 \\ -2-3 & 1+9 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ -5 & 10 \end{bmatrix}$

$$\therefore \quad A^2 - 3A + 2I = \begin{bmatrix} 5 & -5 \\ -5 & 10 \end{bmatrix} - 3\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -5 \\ -5 & 10 \end{bmatrix} - \begin{bmatrix} 6 & -3 \\ -3 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5-6+2 & -5+3+0 \\ -5+3+0 & 10-9+2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}$$ **Ans.**

Important :

If the order of matrix A is $m \times n$ and the order of matrix B is $n \times p$ then the product AB is possible (*i.e.* matrices are *conformable or compatible* for multiplication) as the number of columns of first matrix A are equal to the number of rows of second matrix B. And, the order of product matrix C, obtained is $m \times p$ (*i.e.* number of rows of first matrix × number of columns of second matrix).

$$A_{m\times n} \times B_{n\times p} = C_{m\times p}$$

i.e. (i) $A_{3\times5} \times B_{5\times2} = (AB)_{3\times2}$

(ii) $P_{4\times3} \times Q_{3\times2} = (PQ)_{4\times2}$ and so on.

Also, consider $A = \begin{bmatrix} 2 & 0 \\ 5 & 3 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Here, A is a 3×2 matrix and B is a 2×1 matrix, therefore, product AB is possible; and the order of the product matrix = No. of rows in A × No. of columns in $B = 3 \times 1$.

$$AB = \begin{bmatrix} 2 & 0 \\ 5 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2\times1 + 0\times3 \\ 5\times1 + 3\times3 \\ 4\times1 + 1\times3 \end{bmatrix} = \begin{bmatrix} 2 \\ 14 \\ 7 \end{bmatrix};$$ **which is a 3 × 1 matrix**

12 If $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, is the product AB possible ?

Give a reason. If yes, find AB. [2011]

Solution :

Since, order of matrix $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ is 2×2 *i.e.* A has two rows and two columns

and order of matrix $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ is 2×1 *i.e.* B has two rows and one column.

Hence, the number of columns in matrix A is same as the number of rows in matrix B; therefore the product AB is possible. **Ans.**

$$AB = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times2+5\times4 \\ 4\times2-2\times4 \end{bmatrix} = \begin{bmatrix} 26 \\ 0 \end{bmatrix}$$ **Ans.**

Remember :

1. For any three matrices A, B and C which are compatible for multiplication; $ABC = A\,(BC)$.
   Similarly, $BAC = B(AC)$; $CAB = C(AB)$ and so on.
2. In general $AB \neq BA$ *i.e. product of matrices is not commutative.*
3. $(AB)\,C = A\,(BC)$ *i.e. product of matrices is associative.*
4. If $A \neq 0$ and $AB = AC$, then it is not necessary that $B = C$.
5. If $AB = 0$, then it is not necessary that $A = 0$ or $B = 0$.
6. If $A = 0$ or $B = 0$, then $AB = 0 = BA$.
7. (i) $A\,(B + C) = AB + AC$ *i.e.* in matrices, multiplication is *distributive over addition.*
   (ii) $(A + B)\,C = AC + BC$.

In the same way, $A\,(B - C) = AB - AC$ and $(A - B)\,C = AC - BC$.

13 Let $A = \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$; find :

(i) $(A + B)\,(A - B)$ (ii) $A^2 - B^2$ Is $(A + B)\,(A - B) = A^2 - B^2$?

Solution :

(i) $\because \quad A + B = \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3+1 & 2+0 \\ 0+1 & 5+2 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 7 \end{bmatrix}$

and, $\quad A - B = \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3-1 & 2-0 \\ 0-1 & 5-2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix}$

$\therefore \; (A + B)(A - B) = \begin{bmatrix} 4 & 2 \\ 1 & 7 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4\times2+2\times-1 & 4\times2+2\times3 \\ 1\times2+7\times-1 & 1\times2+7\times3 \end{bmatrix} = \begin{bmatrix} 6 & 14 \\ -5 & 23 \end{bmatrix}$ **Ans.**

(ii) $\because \quad A^2 = A \times A = \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 3\times3+2\times0 & 3\times2+2\times5 \\ 0\times3+5\times0 & 0\times2+5\times5 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & 25 \end{bmatrix}$

and, $\quad B^2 = B \times B = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1\times1+0\times1 & 1\times0+0\times2 \\ 1\times1+2\times1 & 1\times0+2\times2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix}$

$\therefore \quad A^2 - B^2 = \begin{bmatrix} 9 & 16 \\ 0 & 25 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 16 \\ -3 & 21 \end{bmatrix}$ **Ans.**

From the results of parts (i) and (ii), it is clear that :

$(A + B)(A - B) \neq A^2 - B^2$ **Ans.**

14 Given : $\begin{bmatrix} 3 & -8 \\ 9 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$, find x and y.

Solution :

$\begin{bmatrix} 3 & -8 \\ 9 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 3x-8y \\ 9x+4y \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$

$\Rightarrow \quad 3x - 8y = -2$ and $9x + 4y = 8$

On solving, we get : $x = \frac{2}{3}$ and $y = \frac{1}{2}$ **Ans.**

15 If B and C are two matrices such that $B = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 17 & 7 \\ -4 & -8 \end{bmatrix}$, find the matrix A so that BA = C.

Solution :

Let order of matrix A = $m \times n$

$\therefore$ BA = C $\Rightarrow$ $B_{2\times2} \cdot A_{m\times n} = C_{2\times2}$ $\Rightarrow$ order of matrix A = $m \times n = 2 \times 2$

Now, take matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given : BA = C $\Rightarrow$ $\begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 17 & 7 \\ -4 & -8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+3c & b+3d \\ -2a & -2b \end{bmatrix} = \begin{bmatrix} 17 & 7 \\ -4 & -8 \end{bmatrix}$

$\Rightarrow$ $a + 3c = 17$ ....I $-2a = -4$ II

$b + 3d = 7$ ....III and $-2b = -8$ IV

On solving equations I and II, we get : $a = 2$ and $c = 5$

On solving equations III and IV, we get : $b = 4$ and $d = 1$

$\therefore$ $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 5 & 1 \end{bmatrix}$ **Ans.**

16 Find the matrix M, such that $M \times \begin{bmatrix} 3 & 6 \\ -2 & -8 \end{bmatrix} = [-2 \quad 16]$.

Solution :

First of all, we must find the order of matrix M.

Let the order of matrix M be $a \times b$

i.e. $M_{a\times b} \times \begin{bmatrix} 3 & 6 \\ -2 & -8 \end{bmatrix}_{2\times2} = [-2 \quad 16]_{1\times2}$

(Ist matrix) (2nd matrix) (resulting matrix)

Since, the product of matrices is possible, only when the number of columns in the first matrix is equal to the number of rows in the second.

$\therefore b = 2$

Also, the no. of rows of product (resulting) matrix is equal to the no. of rows of first matrix.

$\therefore a = 1$ $\Rightarrow$ **Order of matrix $M = a \times b = 1 \times 2$.**

Let $M = [x \quad y]$

$\therefore [x \quad y] \begin{bmatrix} 3 & 6 \\ -2 & -8 \end{bmatrix} = [-2 \quad 16]$

$\Rightarrow [3x-2y \quad 6x-8y] = [-2 \quad 16]$ $\Rightarrow$ $3x - 2y = -2$ and $6x - 8y = 16$

On solving, we get : $x = -4$ and $y = -5$

$\therefore$ $M = [-4 \quad -5]$ **Ans.**

17 Given : $\begin{bmatrix} 8 & -2 \\ 1 & 4 \end{bmatrix} \cdot X = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$

Write down : (i) the order of the matrix X (ii) the matrix X.

Solution :

(i) Let the order of matrix X be $a \times b$.

$$\therefore \begin{bmatrix} 8 & -2 \\ 1 & 4 \end{bmatrix}_{2\times 2} \cdot X_{a \times b} = \begin{bmatrix} 12 \\ 10 \end{bmatrix}_{2\times 1} \quad \Rightarrow \quad a = 2 \text{ and } b = 1.$$

$\therefore$ *The order of the matrix* $X = a \times b = 2 \times 1$ **Ans.**

(ii) Let $X = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\therefore \begin{bmatrix} 8 & -2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \end{bmatrix} \Rightarrow \begin{bmatrix} 8x-2y \\ x+4y \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$$

$\Rightarrow 8x - 2y = 12$ and $x + 4y = 10$

On solving, we get : $x = 2$ and $y = 2$

$\therefore$ The matrix $X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ **Ans.**

18 State with reason, whether the following are true or false. A, B and C are matrices of order 2×2.

(i) $A \cdot B = B \cdot A$ (ii) $A \cdot (B \cdot C) = (A \cdot B) \cdot C$

(iii) $(A + B)^2 = A^2 + 2AB + B^2$ (iv) $A \cdot (B + C) = A \cdot B + A \cdot C$

Solution :

(i) **False,** as *matrix multiplication is not commutative.* **Ans.**

(ii) **True,** as *matrix multiplication is always associative.* **Ans.**

(iii) **False,** as *laws of algebra are not applicable to matrices.* **Ans.**

(iv) **True,** as in the case of matrices **the multiplication is always distributive over addition.** **Ans.**

EXERCISE 9(C)

1. Evaluate : if possible :

(i) $\begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix}$

(iii) $\begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ (iv) $\begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -1 & 3 \end{bmatrix}$

If not possible, give a reason.

2. If $A = \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$ and I is a unit matrix of order 2×2, find :

(i) AB (ii) BA (iii) AI

(iv) IB (v) A^2 (vi) B^2A

3. If A = $\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$, find x and y when $A^2 = B$. **[2015]**

4. Find x and y, if :

(i) $\begin{bmatrix} 4 & 3x \\ x & -2 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$

(ii) $\begin{bmatrix} x & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$

5. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, find :

(i) $(AB)\,C$ (ii) $A\,(BC)$

Is $A\,(BC) = (AB)\,C$?

6. Given $A = \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix}$, is the following possible :

(i) AB (ii) BA (iii) A^2.

7. Let A = $\begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$, B = $\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ and C = $\begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$. Find $A^2 + AC - 5B$. **[2014]**

8. If $M = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and I is a unit matrix of the same order as that of M; show that :

$$M^2 = 2M + 3I.$$

9. If $A=\begin{bmatrix} a & 0 \\ 0 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 0 & -b \\ 1 & 0 \end{bmatrix}$, $M=\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $BA = M^2$, find the values of a and b.

10. Given $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, find :

(i) $A - B$ (ii) A^2

(iii) AB (iv) $A^2 - AB + 2B$

11. If $A = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$, find :

(i) $(A + B)^2$ (ii) $A^2 + B^2$

(iii) Is $(A + B)^2 = A^2 + B^2$?

12. Find the matrix A, if $B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ and $B^2 = B + \frac{1}{2}A$.

13. If $A = \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}$ and $A^2 = I$, find a and b.

14. If $A = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}$; then show that :

(i) $A(B + C) = AB + AC$

(ii) $(B - A)C = BC - AC.$

15. If $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, simplify : $A^2 + BC$.

16. Solve for x and y :

(i) $\begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ 14 \end{bmatrix}$

(ii) $\begin{bmatrix} x+y & x-4 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} -7 & -11 \end{bmatrix}$

(iii) $\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}$. **[2014]**

17. In each case given below, find :

(a) the order of matrix M.

(b) the matrix M.

(i) $M \times \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \times M = \begin{bmatrix} 13 \\ 5 \end{bmatrix}$

18. If $A = \begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$; find the value of x, given that : $A^2 = B$.

19. If A = $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$.

Find : $AB - 5C$. **[2015]**

20. If A and B are any two 2×2 matrices such that $AB = BA = B$ and B is not a zero matrix, what can you say about the matrix A ?

21. Given $A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ and that $AB = A + B$; find the values of a, b and c.

22. If $P = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, then compute:
    (i) $P^2 - Q^2$ (ii) $(P + Q)(P - Q)$
    Is $(P + Q)(P - Q) = P^2 - Q^2$ true for matrix algebra ?

23. Given the matrices :
    $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$ and
    $C = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$. Find :
    (i) ABC (ii) ACB.
    State whether $ABC = ACB$.

24. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 1 \\ 1 & 1 \end{bmatrix}$ and
    $C = \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix}$, find each of the following and state if they are equal :
    (i) $CA + B$ (ii) $A + CB$

25. If $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ -11 \end{bmatrix}$, find the matrix X such that $AX = B$.

26. If $A = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$, find $(A - 2I)(A - 3I)$.

27. If $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}$, Find :
    (i) $A^t . A$ (ii) $A . A^t$
    where A^t is the transpose of matrix A.

28. If $M = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$, show that : $6M - M^2 = 9I$; where I is a 2×2 unit matrix.

29. If $P = \begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}$ and $Q = \begin{bmatrix} 3 & x \\ y & 2 \end{bmatrix}$, find x and y such that PQ = null matrix.

30. Evaluate :
$$\begin{bmatrix} 2\cos 60° & -2\sin 30° \\ -\tan 45° & \cos 0° \end{bmatrix} \begin{bmatrix} \cot 45° & \operatorname{cosec} 30° \\ \sec 60° & \sin 90° \end{bmatrix}$$

31. State, with reason, whether the following are true or false. A, B and C are matrices of order 2×2.
    (i) $A + B = B + A$
    (ii) $A - B = B - A$
    (iii) $(B . C) . A = B . (C . A)$
    (iv) $(A + B) . C = A . C + B . C$
    (v) $A . (B - C) = A . B - A . C$
    (vi) $(A - B) . C = A . C - B . C$
    (vii) $A^2 - B^2 = (A + B)(A - B)$
    (viii) $(A - B)^2 = A^2 - 2A . B + B^2$

EXERCISE 9(D)

1. Find x and y, if :
$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4 \begin{bmatrix} 2 \\ y \end{bmatrix}$$
[2003]

2. Find x and y, if :
$$[3x \quad 8] \begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix} - 3\,[2 \quad -7] = 5[3 \quad 2y]$$

3. If $[x \quad y] \begin{bmatrix} x \\ y \end{bmatrix} = [25]$ and $[-x \quad y] \begin{bmatrix} 2x \\ y \end{bmatrix} = [-2]$; find x and y, if :
    (i) $x, y \in W$ (whole numbers)
    (ii) $x, y \in Z$ (integers)

4. Given $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \cdot X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$. Write :
    (i) the order of the matrix X.
    (ii) the matrix X. **[2012]**

5. Evaluate :
$$\begin{bmatrix} \cos 45° & \sin 30° \\ \sqrt{2}\cos 0° & \sin 0° \end{bmatrix} \begin{bmatrix} \sin 45° & \cos 90° \\ \sin 90° & \cot 45° \end{bmatrix}$$

6. If $A = \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -5 \\ 6 \end{bmatrix}$ and $3A \times M = 2B$; find matrix M.

7. If $\begin{bmatrix} a & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$, find the values of a, b and c.

8. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$; find :

   (i) $A(BA)$ (ii) $(AB)B$.

9. Find x and y, if : $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$. **[2013]**

10. If matrix $X = \begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ and $2X - 3Y = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$, find the matrix '$X$' and matrix '$Y$'.

11. Given $A = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, find the matrix X such that : $A + X = 2B + C$. **[2005]**

12. Find the value of x, given that $A^2 = B$, $A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$ **[2005]**

13. If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ and I is the identity matric of the same order and A^t is the transpose of matrix A, find $A^t \cdot B + BI$. **[2011]**

14. Given $A = \begin{bmatrix} 2 & -6 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$. Find the matrix X such that $A + 2X = 2B + C$. **[2013]**

15. Let $A = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$. Find $A^2 - A + BC$ **[2006]**

16. Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$. Find $A^2 + AB + B^2$ **[2007]**

17. If $A = \begin{bmatrix} 3 & a \\ -4 & 8 \end{bmatrix}$, $B = \begin{bmatrix} c & 4 \\ -3 & 0 \end{bmatrix}$ $C = \begin{bmatrix} -1 & 4 \\ 3 & b \end{bmatrix}$ and $3A - 2C = 6B$, find the values of a, b and c.

18. Given $A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$ and $BA = C^2$. Find the values of p and q. **[2008]**

19. Given $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ and $D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. Find : $AB + 2C - 4D$ **[2010]**

20. Evaluate : $\begin{bmatrix} 4\sin 30° & 2\cos 60° \\ \sin 90° & 2\cos 0° \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$. **[2010]**

21. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find $A^2 - 5A + 7I$. **[2012]**