

5

Quadratic Equations

5.1 Introduction :

An equation with one variable, in which the **highest power** of the variable is **two**, is known as **quadratic equation**.

For example :

(i) $3x^2 + 4x + 7 = 0$

(iii) $2x^2 - 50 = 0$

(ii) $4x^2 + 5x = 0$

(iv) $x^2 = 4$, etc.

1. The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a , b and c are all real numbers and $a \neq 0$.

e.g. equation $4x^2 + 5x - 6 = 0$ is a quadratic equation in standard form.

2. Every quadratic equation gives two values of the unknown variable used in it and these values are called **roots of the equation**.

3. **Discriminant** : For the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$; the expression $b^2 - 4ac$ is called discriminant and is, in general, denoted by the letter '**D**'.

Thus, discriminant $D = b^2 - 4ac$.

4. If a quadratic equation contains only two terms one square term and one first power term of the unknown, it is called **adfect quadratic equation**.

For example : (i) $4x^2 + 5x = 0$ (ii) $7x^2 - 3x = 0$, etc.

5. If the quadratic equation contains only the square of the unknown, it is called **pure quadratic equation**.

For example : (i) $x^2 = 4$ (ii) $3x^2 - 8 = 0$, etc.

5.2 To examine the nature of the roots :

Examining the roots of a quadratic equation means to know the type of its roots *i.e.* whether they are *real* or *imaginary*, *rational* or *irrational*, *equal* or *unequal*.

The nature of the roots of a quadratic equation depends entirely on the value of its discriminant $b^2 - 4ac$.

If for a quadratic equation $ax^2 + bx + c = 0$; where a , b and c are real numbers and $a \neq 0$, then discriminant :

- (i) $b^2 - 4ac = 0 \Rightarrow$ the roots are **real and equal**.
- (ii) $b^2 - 4ac > 0 \Rightarrow$ the roots are **real and unequal**.
- (iii) $b^2 - 4ac < 0 \Rightarrow$ the roots are **imaginary** (not real).

- Every number, whether it is rational or irrational, is a real number. *i.e.*
 - every rational number is a real number and
 - every irrational number is also a real number.
- Square root of a negative number is an imaginary number.
Thus : each of $\sqrt{-4}$, $\sqrt{-8}$, $2\sqrt{-5}$,, etc. is an imaginary number.

1 Without solving, examine the nature of the roots of the equations :
 (i) $5x^2 - 6x + 7 = 0$ (ii) $x^2 + 6x + 9 = 0$ (iii) $2x^2 + 6x + 3 = 0$

Solution :

(i) Comparing given quadratic equation $5x^2 - 6x + 7 = 0$ with equation $ax^2 + bx + c = 0$; we get : $a = 5$, $b = -6$ and $c = 7$.

$$\Rightarrow \text{Discriminant} = b^2 - 4ac = (-6)^2 - 4 \times 5 \times 7$$

$$= 36 - 140 = -104; \text{ which is negative.}$$

Since, a , b and c are real numbers; $a \neq 0$ and $b^2 - 4ac < 0$.

\therefore **The roots are not real *i.e.* the roots are imaginary.** **Ans.**

(ii) Comparing quadratic equation $x^2 + 6x + 9 = 0$ with $ax^2 + bx + c = 0$; we get : $a = 1$, $b = 6$ and $c = 9$

$$\Rightarrow b^2 - 4ac = (6)^2 - 4 \times 1 \times 9 = 36 - 36 = 0$$

Since; a , b and c are real numbers; $a \neq 0$ and $b^2 - 4ac = 0$.

\therefore **The roots are real and equal.** **Ans.**

(iii) Comparing $2x^2 + 6x + 3 = 0$ and $ax^2 + bx + c$, we get : $a = 2$, $b = 6$ and $c = 3$

$$b^2 - 4ac = (6)^2 - 4 \times 2 \times 3$$

$$= 36 - 24 = 12; \text{ which is positive.}$$

Since; a , b and c are real numbers; $a \neq 0$ and $b^2 - 4ac > 0$.

\therefore **The roots are real and unequal.** **Ans.**

2 Find the value of 'm', if the roots of the following quadratic equation are equal:
 $(4 + m)x^2 + (m + 1)x + 1 = 0$.

Solution :

For the given equation $(4 + m)x^2 + (m + 1)x + 1 = 0$;

$a = 4 + m$, $b = m + 1$ and $c = 1$

Since, the roots are equal

$$\therefore b^2 - 4ac = 0 \Rightarrow (m + 1)^2 - 4(4 + m) \times 1 = 0$$

$$\Rightarrow m^2 + 2m + 1 - 16 - 4m = 0$$

$$\Rightarrow m^2 - 2m - 15 = 0$$

On solving, we get : $m = 5$ or $m = -3$

Ans.

EXERCISE 5(A)

1. Without solving, comment upon the nature of roots of each of the following equations :

- (i) $7x^2 - 9x + 2 = 0$ (ii) $6x^2 - 13x + 4 = 0$
 (iii) $25x^2 - 10x + 1 = 0$ (iv) $x^2 + 2\sqrt{3}x - 9 = 0$
 (v) $x^2 - ax - b^2 = 0$ (vi) $2x^2 + 8x + 9 = 0$

2. Find the value of 'p', if the following quadratic equations have equal roots :

- (i) $4x^2 - (p - 2)x + 1 = 0$
 (ii) $x^2 + (p - 3)x + p = 0$ [2013]

3. The equation $3x^2 - 12x + (n - 5) = 0$ has equal roots. Find the value of n .

4. Find the value of 'm', if the following equation has equal roots :

$$(m - 2)x^2 - (5 + m)x + 16 = 0$$

5. Find the value of k for which the equation $3x^2 - 6x + k = 0$ has distinct and real root.

[2015]

5.3 Solving quadratic equations by factorisation :

- Steps :** (i) Clear all fractions and brackets, if necessary.
 (ii) Transpose all the terms to the left hand side to get an equation in the form $ax^2 + bx + c = 0$.
 (iii) Factorise the expression on the left hand side.
 (iv) Put each factor equal to zero and solve.

Zero Product Rule : Whenever the product of two expressions is zero; at least one of the expressions is zero.

$$\begin{aligned} \text{Thus,} \quad & \text{if } (x + 3)(x - 2) = 0 \\ \Rightarrow & x + 3 = 0, \text{ or } x - 2 = 0 \\ \Rightarrow & x = -3, \text{ or } x = 2. \end{aligned}$$

- 3** Solve : (i) $2x^2 - 7x = 39$ (ii) $x^2 = 5x$ (iii) $x^2 = 16$

Solution :

(i) $2x^2 - 7x = 39$

$$\Rightarrow 2x^2 - 7x - 39 = 0$$

[Expressing as $ax^2 + bx + c = 0$]

$$\Rightarrow 2x^2 - 13x + 6x - 39 = 0$$

[Factorising the left hand side]

$$\Rightarrow x(2x - 13) + 3(2x - 13) = 0$$

$$(2x - 13)(x + 3) = 0$$

$$2x - 13 = 0, \text{ or } x + 3 = 0$$

[Zero Product Rule]

$$\Rightarrow x = \frac{13}{2}, \text{ or } x = -3$$

Ans.

(ii) $x^2 = 5x \Rightarrow x^2 - 5x = 0$

$$\Rightarrow x(x - 5) = 0$$

$$\Rightarrow x = 0, \text{ or } x - 5 = 0$$

$$\Rightarrow x = 0, \text{ or } x = 5$$

Ans.

(iii) $x^2 = 16 \Rightarrow x^2 - 16 = 0$

$$\Rightarrow (x + 4)(x - 4) = 0$$

$$\Rightarrow x + 4 = 0, \text{ or } x - 4 = 0$$

$$\Rightarrow x = -4, \text{ or } x = 4 \text{ Ans.}$$

Alternative method :

$$x^2 = 16$$

$$\Rightarrow x = \pm 4$$

$$\Rightarrow x = 4 \text{ or } x = -4 \text{ Ans.}$$

$$4 \text{ Solve : } \frac{x}{x-1} + \frac{x-1}{x} = 2\frac{1}{2}.$$

Solution :

$$\begin{aligned} \frac{x}{x-1} + \frac{x-1}{x} &= 2\frac{1}{2} \\ \Rightarrow \frac{x^2 + (x-1)^2}{x(x-1)} &= \frac{5}{2} \\ 2(x^2 + x^2 - 2x + 1) &= 5(x^2 - x) \\ \Rightarrow 4x^2 - 4x + 2 &= 5x^2 - 5x \\ \Rightarrow -x^2 + x + 2 &= 0 \\ \Rightarrow x^2 - x - 2 &= 0 && \text{[Changing the sign of each term]} \\ \Rightarrow (x-2)(x+1) &= 0 && \text{[On factorising]} \\ \Rightarrow x-2=0, \text{ or } x+1 &= 0 && \text{[Zero Product Rule]} \\ \Rightarrow x=2, \text{ or } x &= -1 && \text{Ans.} \end{aligned}$$

$$5 \text{ Find the quadratic equation whose solution set is } \{-2, 3\}.$$

Solution :

$$\begin{aligned} \text{Since, solution set} &= \{-2, 3\} \\ \Rightarrow \text{Roots are } -2 \text{ and } 3 \\ \Rightarrow x &= -2, \text{ or } x = 3 \\ \Rightarrow x + 2 &= 0, \text{ or } x - 3 = 0 \\ \Rightarrow (x + 2)(x - 3) &= 0 \\ \Rightarrow x^2 - 3x + 2x - 6 &= 0 \\ \Rightarrow x^2 - x - 6 &= 0; \text{ which is the required quadratic equation. Ans.} \end{aligned}$$

$$6 \text{ Use the substitution } x = 3y + 1 \text{ to solve for } y, \text{ if } 5(3y + 1)^2 + 6(3y + 1) - 8 = 0.$$

Solution :

$$\begin{aligned} 5(3y+1)^2 + 6(3y+1) - 8 &= 0 \\ \Rightarrow 5x^2 + 6x - 8 &= 0 && \text{[Putting } 3y + 1 = x\text{]} \\ \Rightarrow (x+2)(5x-4) &= 0 && \text{[On factorising]} \\ \Rightarrow x = -2, \text{ or } x &= \frac{4}{5} \\ \text{When } x = -2 &\Rightarrow 3y + 1 = -2 \Rightarrow y = -1 \\ \text{and, when } x = \frac{4}{5} &\Rightarrow 3y + 1 = \frac{4}{5} \Rightarrow y = -\frac{1}{15} \\ \therefore y = -1, \text{ or } y &= -\frac{1}{15} && \text{Ans.} \end{aligned}$$

- 7** Without solving the quadratic equation $3x^2 - 2x - 1 = 0$, find whether $x = 1$ is a solution (root) of this equation or not.

Solution :

Substituting $x = 1$ in the given equation $3x^2 - 2x - 1 = 0$,

we get : $3(1)^2 - 2 \times 1 - 1 = 0$

$\Rightarrow 3 - 2 - 1 = 0$; *which is true.*

$\therefore x = 1$ is a solution of the given equation $3x^2 - 2x - 1 = 0$ **Ans.**

- 8** Without solving equation $x^2 - x + 1 = 0$; find whether $x = -1$ is a root of this equation or not.

Solution :

Substituting $x = -1$ in the given equation $x^2 - x + 1 = 0$,

we get : $(-1)^2 - (-1) + 1 = 0$

i.e. $1 + 1 + 1 = 0 \Rightarrow 3 = 0$; *which is not true.*

$\therefore x = -1$ is not a root of the given equation $x^2 - x + 1 = 0$ **Ans.**

- 9** Find the value of k for which $x = 2$ is a root (solution) of equation $kx^2 + 2x - 3 = 0$.

Solution :

Substituting $x = 2$ in the given equation $kx^2 + 2x - 3 = 0$; we get :

$$k(2)^2 + 2 \times 2 - 3 = 0$$

$\Rightarrow 4k + 4 - 3 = 0 \Rightarrow k = -\frac{1}{4}$ **Ans.**

- 10** If $x = 2$ and $x = 3$ are roots of the equation $3x^2 - 2mx + 2n = 0$; find the values of m and n .

Solution :

$x = 2$ is a root of the equation $3x^2 - 2mx + 2n = 0$

$\Rightarrow 3(2)^2 - 2m \times 2 + 2n = 0$

$\Rightarrow 12 - 4m + 2n = 0$

$\Rightarrow -4m + 2n = -12$ *i.e.* $2m - n = 6$ I

$x = 3$ is a root of the equation $3x^2 - 2mx + 2n = 0$

$\Rightarrow 3(3)^2 - 2m \times 3 + 2n = 0$

$\Rightarrow 27 - 6m + 2n = 0$

$\Rightarrow -6m + 2n = -27$ *i.e.* $6m - 2n = 27$ II

On solving equations I and II, we get :

$m = 7.5$ and $n = 9$ **Ans.**

- 11** If one root of the quadratic equation $2x^2 + ax - 6 = 0$ is 2, find the value of a . Also, find the other root.

Solution :

Since, $x = 2$ is a root of the given equation $2x^2 + ax - 6 = 0$

$$\Rightarrow 2(2)^2 + a \times 2 - 6 = 0 \quad \text{i.e. } 8 + 2a - 6 = 0 \quad \text{and } a = -1 \quad \text{Ans.}$$

Substituting $a = -1$, we get :

$$2x^2 + (-1)x - 6 = 0 \quad [\because 2x^2 + ax - 6 = 0]$$

$$\Rightarrow 2x^2 - x - 6 = 0$$

$$\Rightarrow 2x^2 - 4x + 3x - 6 = 0$$

$$\Rightarrow 2x(x - 2) + 3(x - 2) = 0$$

$$\Rightarrow (x - 2)(2x + 3) = 0 \quad \Rightarrow x = 2 \quad \text{or } x = \frac{-3}{2}$$

$$\Rightarrow \text{The other root} = \frac{-3}{2} \quad \text{Ans.}$$

- 12** Find the value of 'K' for which $x = 3$ is a solution of the quadratic equation, $(K + 2)x^2 - Kx + 6 = 0$
Hence, find the other root of the equation. [2015]

Solution :

$x = 3$ is a solution of equation $(K + 2)x^2 - Kx + 6 = 0$

$$\Rightarrow (K + 2) \times 9 - K \times 3 + 6 = 0$$

$$\Rightarrow 9K + 18 - 3K + 6 = 0 \quad \text{i.e. } 6K = -24 \quad \text{and } K = -4 \quad \text{Ans.}$$

For $K = -4$, $(K + 2)x^2 - Kx + 6 = 0$

$$\Rightarrow -2x^2 + 4x + 6 = 0 \quad \text{i.e. } x^2 - 2x - 3 = 0$$

$$\Rightarrow x^2 - 3x + x - 3 = 0 \quad \text{i.e. } x(x - 3) + 1(x - 3) = 0$$

$$\Rightarrow (x - 3)(x + 1) = 0 \quad \text{i.e. } x = 3 \quad \text{or } x = -1$$

Since, $x = 3$ is already given to be one root (solution) of the equation.

\therefore The other root of the equation is $x = -1$. Ans.

EXERCISE 5(B)

Solve equations, number 1 to number 20, given below, using factorisation method :

1. $x^2 - 10x - 24 = 0$

2. $x^2 - 16 = 0$

3. $2x^2 - \frac{1}{2}x = 0$

4. $x(x - 5) = 24$

5. $\frac{9}{2}x = 5 + x^2$

6. $\frac{6}{x} = 1 + x$

7. $x = \frac{3x+1}{4x}$

8. $x + \frac{1}{x} = 2.5$

9. $(2x - 3)^2 = 49$

10. $2(x^2 - 6) = 3(x - 4)$

11. $(x + 1)(2x + 8) = (x + 7)(x + 3)$

12. $x^2 - (a + b)x + ab = 0$
13. $(x + 3)^2 - 4(x + 3) - 5 = 0$
14. $4(2x - 3)^2 - (2x - 3) - 14 = 0$
15. $\frac{3x-2}{2x-3} = \frac{3x-8}{x+4}$
16. $2x^2 - 9x + 10 = 0$, when :
(i) $x \in \mathbb{N}$ (ii) $x \in \mathbb{Q}$.
17. $\frac{x-3}{x+3} + \frac{x+3}{x-3} = 2\frac{1}{2}$
18. $\frac{4}{x+2} - \frac{1}{x+3} = \frac{4}{2x+1}$
19. $\frac{5}{x-2} - \frac{3}{x+6} = \frac{4}{x}$
20. $\left(1 + \frac{1}{x+1}\right)\left(1 - \frac{1}{x-1}\right) = \frac{7}{8}$
21. Find the quadratic equation, whose solution set is :
(i) $\{3, 5\}$ (ii) $\{-2, 3\}$
22. (i) Solve : $\frac{x}{3} + \frac{3}{6-x} = \frac{2(6+x)}{15}$; ($x \neq 6$)
(ii) Solve the equation $9x^2 + \frac{3x}{4} + 2 = 0$,
if possible, for real values of x .
23. Find the value of x , if $a + 1 = 0$ and $x^2 + ax - 6 = 0$.
24. Find the value of x , if $a + 7 = 0$;
 $b + 10 = 0$ and $12x^2 = ax - b$.
25. Use the substitution $y = 2x + 3$ to solve for x , if $4(2x + 3)^2 - (2x + 3) - 14 = 0$.
26. Without solving the quadratic equation $6x^2 - x - 2 = 0$, find whether $x = \frac{2}{3}$ is a solution of this equation or not.
27. Determine whether $x = -1$ is a root of the equation $x^2 - 3x + 2 = 0$ or not.
28. If $x = \frac{2}{3}$ is a solution of the quadratic equation $7x^2 + mx - 3 = 0$; find the value of m .
29. If $x = -3$ and $x = \frac{2}{3}$ are solutions of quadratic equation $mx^2 + 7x + n = 0$, find the values of m and n .
30. If quadratic equation $x^2 - (m + 1)x + 6 = 0$ has one root as $x = 3$; find the value of m and the other root of the equation.
31. Given that 2 is a root of the equation $3x^2 - p(x + 1) = 0$ and that the equation $px^2 - qx + 9 = 0$ has equal roots, find the values of p and q .
32. Solve : $\frac{x}{a} - \frac{a+b}{x} = \frac{b(a+b)}{ax}$.
33. Solve : $\left(\frac{1200}{x} + 2\right)(x - 10) - 1200 = 60$.
34. If -1 and 3 are the roots of $x^2 + px + q = 0$, find the values of p and q .

5.4 Solving quadratic equations using the formula :

The roots of the quadratic equation $ax^2 + bx + c = 0$; where $a \neq 0$ can be obtained by using the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Proof :

Given : $ax^2 + bx + c = 0$

$\Rightarrow 4a^2x^2 + 4abx + 4ac = 0$ [On multiplying each term by $4a$]

$\Rightarrow (2ax)^2 + 2 \times 2ax \times b + b^2 - b^2 + 4ac = 0$

$\Rightarrow (2ax + b)^2 - b^2 + 4ac = 0$

$\Rightarrow (2ax + b)^2 = b^2 - 4ac$

$$\Rightarrow 2ax + b = \pm\sqrt{b^2 - 4ac}$$

$$\Rightarrow 2ax = -b \pm \sqrt{b^2 - 4ac} \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Ans.}$$

13 Solve each of the following equations by using the formula :

(i) $5x^2 - 2x - 3 = 0$ (ii) $x^2 = 18x - 77$ (iii) $\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$.

Solution :

(i) Comparing $5x^2 - 2x - 3 = 0$ with $ax^2 + bx + c = 0$, we get :

$$a = 5, b = -2 \text{ and } c = -3;$$

$$\text{and so, } x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 5 \times -3}}{2 \times 5} \quad [\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}]$$

$$= \frac{2 \pm \sqrt{64}}{10} = \frac{2 \pm 8}{10} = \frac{2+8}{10}, \text{ or } \frac{2-8}{10} = 1, \text{ or } -\frac{3}{5} \quad \text{Ans.}$$

(ii) $x^2 = 18x - 77 = 0 \Rightarrow x^2 - 18x + 77 = 0$

Comparing with $ax^2 + bx + c = 0$, we get : $a = 1, b = -18$ and $c = 77$

$$\therefore x = \frac{18 \pm \sqrt{(-18)^2 - 4 \times 1 \times 77}}{2 \times 1} \quad [\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}]$$

$$= \frac{18 \pm \sqrt{16}}{2} = \frac{18 \pm 4}{2}, \text{ or } \frac{18-4}{2} = 11, \text{ or } 7 \quad \text{Ans.}$$

(iii) $\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0 \Rightarrow a = \sqrt{3}, b = 11$ and $c = 6\sqrt{3}$

$$\therefore x = \frac{-11 \pm \sqrt{(11)^2 - 4 \times \sqrt{3} \times 6\sqrt{3}}}{2 \times \sqrt{3}} \quad [\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}]$$

$$= \frac{-11 \pm \sqrt{49}}{2\sqrt{3}} = \frac{-11 \pm 7}{2\sqrt{3}}, \text{ or } \frac{-11-7}{2\sqrt{3}}$$

$$= \frac{-4}{2\sqrt{3}}, \text{ or } \frac{-18}{2\sqrt{3}}$$

$$= -\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}, \text{ or } \frac{-9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \quad \text{[Rationalizing the denominators]}$$

$$= -\frac{2\sqrt{3}}{3}, \text{ or } -3\sqrt{3} \quad \text{Ans.}$$

14 Solve each of the following equations for x and give, in each case, your answer correct to 2 decimal places :

(i) $x^2 - 10x + 6 = 0$

(ii) $3x^2 + 5x - 9 = 0$

Solution :

(i) $x^2 - 10x + 6 = 0 \Rightarrow a = 1, b = -10$ and $c = 6$

$$\begin{aligned} \therefore b^2 - 4ac &= (-10)^2 - 4 \times 1 \times 6 \\ &= 100 - 24 = 76 \end{aligned}$$

$$\Rightarrow \sqrt{b^2 - 4ac} = \sqrt{76} = 8.718$$

$$\therefore x = \frac{10 \pm 8.718}{2 \times 1}$$

$$[\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}]$$

$$= \frac{10 + 8.718}{2}, \text{ or } \frac{10 - 8.718}{2}$$

$$= 9.359, \text{ or } 0.641$$

$$= \mathbf{9.36, \text{ or } 0.64}$$

[Correct to 2 decimal places] **Ans.**

(ii) $3x^2 + 5x - 9 = 0 \Rightarrow a = 3, b = 5$ and $c = -9$

$$\begin{aligned} \therefore b^2 - 4ac &= (5)^2 - 4 \times 3 \times -9 \\ &= 25 + 108 = 133 \end{aligned}$$

$$\Rightarrow \sqrt{b^2 - 4ac} = \sqrt{133} = 11.533$$

$$\therefore x = \frac{-5 \pm 11.533}{2 \times 3}$$

$$[\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}]$$

$$= \frac{-5 + 11.533}{6}, \text{ or } \frac{-5 - 11.533}{6}$$

$$= 1.089, \text{ or } -2.756$$

$$= \mathbf{1.09, \text{ or } -2.76}$$

[Correct to 2 decimal places] **Ans.**

15 Solve the following equation :

$$x - \frac{18}{x} = 6. \text{ Give your answer correct to two significant figures. } \quad [2011]$$

Solution :

$$\begin{aligned} x - \frac{18}{x} = 6 &\Rightarrow x^2 - 18 = 6x \\ &\Rightarrow x^2 - 6x - 18 = 0 \end{aligned}$$

Comparing with $ax^2 + bx + c = 0$, we get : $a = 1, b = -6$ and $c = -18$.

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned}
 &= \frac{6 \pm \sqrt{36 - 4 \times 1 \times -18}}{2 \times 1} = \frac{6 \pm 10 \cdot 392}{2} \\
 &= \frac{16 \cdot 392}{2} \quad \text{or} \quad \frac{-4 \cdot 392}{2} \\
 &= 8 \cdot 196 \quad \text{or} \quad -2 \cdot 196 = 8 \cdot 2 \quad \text{or} \quad -2 \cdot 2
 \end{aligned}$$

Ans.

5.5 Equations Reducible to Quadratic Equations :**16** Solve : (i) $2x^4 - 5x^2 + 3 = 0$ (ii) $(x^2 + 3x)^2 - (x^2 + 3x) - 6 = 0, x \in \mathbb{R}$ **Solution :**

(i) $2x^4 - 5x^2 + 3 = 0$

$$\Rightarrow 2y^2 - 5y + 3 = 0 \quad \text{[Taking } x^2 = y]$$

$$\Rightarrow (y - 1)(2y - 3) = 0 \quad \text{[On factorising]}$$

$$\Rightarrow y = 1, \text{ or } y = \frac{3}{2}$$

$$\text{When } y = 1 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

$$\text{and, when } y = \frac{3}{2} \Rightarrow x^2 = \frac{3}{2} \Rightarrow x = \pm \sqrt{\frac{3}{2}} = \pm \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \pm \frac{\sqrt{6}}{2}$$

$$\therefore \text{ Required solution} = 1, -1, \frac{\sqrt{6}}{2}, \text{ or } -\frac{\sqrt{6}}{2}$$

Ans.

(ii) $(x^2 + 3x)^2 - (x^2 + 3x) - 6 = 0$

$$\Rightarrow y^2 - y - 6 = 0 \quad \text{[Taking } x^2 + 3x = y]$$

$$\Rightarrow (y - 3)(y + 2) = 0 \quad \text{[On factorising]}$$

$$\Rightarrow y = 3, \text{ or } y = -2$$

$$y = 3 \Rightarrow x^2 + 3x = 3$$

$$\Rightarrow x^2 + 3x - 3 = 0$$

$$\Rightarrow x = \frac{-3 \pm \sqrt{(3)^2 - 4 \times 1 \times -3}}{2 \times 1} \Rightarrow x = \frac{-3 \pm \sqrt{21}}{2}$$

$$\text{and } y = -2 \Rightarrow x^2 + 3x = -2 \Rightarrow x^2 + 3x + 2 = 0$$

$$\Rightarrow x = \frac{-3 \pm \sqrt{(3)^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$= \frac{-3 \pm 1}{2} = \frac{-3+1}{2} \quad \text{or} \quad \frac{-3-1}{2} = -1 \text{ or } -2$$

$$\therefore \text{ Required solution is : } \frac{-3 + \sqrt{21}}{2}, \frac{-3 - \sqrt{21}}{2}, -1, \text{ or } -2$$

Ans.

17 Solve : $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}$, $x \neq 0$ and $x \neq 1$.

Solution :

$$\text{Let } \sqrt{\frac{x}{1-x}} = y \Rightarrow \sqrt{\frac{1-x}{x}} = \frac{1}{y}$$

\therefore Given equation reduces to :

$$\begin{aligned} y + \frac{1}{y} &= \frac{13}{6} \Rightarrow 6y^2 + 6 = 13y \\ &\Rightarrow 6y^2 - 13y + 6 = 0 \\ &\Rightarrow (2y - 3)(3y - 2) = 0 \end{aligned}$$

[On factorising]

$$\Rightarrow y = \frac{3}{2}, \text{ or } y = \frac{2}{3}$$

$$\text{When } y = \frac{3}{2} \Rightarrow \sqrt{\frac{x}{1-x}} = \frac{3}{2} \Rightarrow \frac{x}{1-x} = \frac{9}{4}$$

$$\Rightarrow 4x = 9 - 9x \Rightarrow x = \frac{9}{13}$$

$$\text{and } y = \frac{2}{3} \Rightarrow \sqrt{\frac{x}{1-x}} = \frac{2}{3} \Rightarrow \frac{x}{1-x} = \frac{4}{9}$$

$$\Rightarrow 9x = 4 - 4x \Rightarrow x = \frac{4}{13}$$

\therefore Required solution is : $\frac{9}{13}$, or $\frac{4}{13}$

Ans.

EXERCISE 5(C)

1. Solve, each of the following equations, using the formula :

(i) $x^2 - 6x = 27$ (ii) $x^2 - 10x + 21 = 0$

(iii) $x^2 + 6x - 10 = 0$ (iv) $x^2 + 2x - 6 = 0$

(v) $3x^2 + 2x - 1 = 0$ (vi) $2x^2 + 7x + 5 = 0$

(vii) $\frac{2}{3}x = -\frac{1}{6}x^2 - \frac{1}{3}$ (viii) $\frac{1}{15}x^2 + \frac{5}{3} = \frac{2}{3}x$

(ix) $x^2 - 6 = 2\sqrt{2}x$ (x) $\frac{4}{x} - 3 = \frac{5}{2x+3}$

(xi) $\frac{2x+3}{x+3} = \frac{x+4}{x+2}$

(xii) $\sqrt{6}x^2 - 4x - 2\sqrt{6} = 0$

(xiii) $\frac{2x}{x-4} + \frac{2x-5}{x-3} = 8\frac{1}{3}$

(xiv) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$

2. Solve each of the following equations for x and give, in each case, your answer correct to one decimal place :

(i) $x^2 - 8x + 5 = 0$

(ii) $5x^2 + 10x - 3 = 0$

3. Solve each of the following equations for x and give, in each case, your answer correct to 2 decimal places :

(i) $2x^2 - 10x + 5 = 0$

(ii) $4x + \frac{6}{x} + 13 = 0$

(iii) $x^2 - 3x - 9 = 0$ [2007]

(iv) $x^2 - 5x - 10 = 0$ [2013]

4. Solve each of the following equations for x , giving your answer correct to 3 decimal places:

(i) $3x^2 - 12x - 1 = 0$

(ii) $x^2 - 16x + 6 = 0$

(iii) $2x^2 + 11x + 4 = 0$

5. Solve :

(i) $x^4 - 2x^2 - 3 = 0$ (ii) $x^4 - 10x^2 + 9 = 0$

6. Solve :

(i) $(x^2 - x)^2 + 5(x^2 - x) + 4 = 0$

(ii) $(x^2 - 3x)^2 - 16(x^2 - 3x) - 36 = 0$

7. Solve :

(i) $\sqrt{\frac{x}{x-3}} + \sqrt{\frac{x-3}{x}} = \frac{5}{2}$

(ii) $\left(\frac{2x-3}{x-1}\right) - 4\left(\frac{x-1}{2x-3}\right) = 3$

(iii) $\left(\frac{3x+1}{x+1}\right) + \left(\frac{x+1}{3x+1}\right) = \frac{5}{2}$

8. Solve the equation $2x - \frac{1}{x} = 7$. Write your answer correct to two decimal places. [2006]

9. Solve the following equation and give your answer correct to 3 significant figures :

$5x^2 - 3x - 4 = 0$ [2012]

10. Solve for x using the quadratic formula. Write your answer correct to two significant figures.

$(x-1)^2 - 3x + 4 = 0$. [2014]

18 Find the solution set of the equation $3x^2 - 8x - 3 = 0$; when :

(i) $x \in \mathbb{Z}$ (integers) (ii) $x \in \mathbb{Q}$ (rational numbers).

Solution :

$$\begin{aligned} 3x^2 - 8x - 3 = 0 &\Rightarrow 3x^2 - 9x + x - 3 = 0 \\ &\Rightarrow 3x(x-3) + 1(x-3) = 0 \\ &\Rightarrow (x-3)(3x+1) = 0 \\ &\Rightarrow x = 3, \text{ or } x = -\frac{1}{3} \end{aligned}$$

(i) When $x \in \mathbb{Z}$, the solution set = $\{3\}$

Ans.

(ii) When $x \in \mathbb{Q}$, the solution set = $\{3, -\frac{1}{3}\}$

Ans.

19 Solve : $(2x - 3)^2 = 25$.

Solution :

$$\begin{aligned} (2x - 3)^2 = 25 &\Rightarrow 4x^2 - 12x + 9 - 25 = 0 \\ &\Rightarrow 4x^2 - 12x - 16 = 0 \\ &\Rightarrow x^2 - 3x - 4 = 0 \\ &\Rightarrow (x-4)(x+1) = 0 \\ &\Rightarrow x = 4, \text{ or } x = -1 \end{aligned}$$

Ans.

Alternative method :

$$\begin{aligned} (2x - 3)^2 = 25 &\Rightarrow 2x - 3 = \pm 5 \\ \text{Now, } 2x - 3 = 5 &\Rightarrow 2x = 8 \text{ and } x = 4 \\ \text{And, } 2x - 3 = -5 &\Rightarrow 2x = -2 \text{ and } x = -1 \\ \therefore &x = 4, \text{ or } x = -1 \end{aligned}$$

Ans.

20 Solve for x : $4(x - \frac{1}{x})^2 + 8(x + \frac{1}{x}) = 29$. $x \neq 0$.

Solution :

$$\text{Let } x + \frac{1}{x} = y$$

$$\therefore (x + \frac{1}{x})^2 - (x - \frac{1}{x})^2 = 4 \quad \Rightarrow \quad y^2 - (x - \frac{1}{x})^2 = 4$$

$$\text{and } (x - \frac{1}{x})^2 = y^2 - 4$$

$$\therefore 4(x - \frac{1}{x})^2 + 8(x + \frac{1}{x}) = 29 \quad \Rightarrow \quad 4(y^2 - 4) + 8y = 29$$

$$\Rightarrow 4y^2 - 16 + 8y = 29$$

$$\Rightarrow 4y^2 + 8y - 45 = 0$$

$$\Rightarrow 4y^2 + 18y - 10y - 45 = 0 \quad \text{i.e. } 2y(2y + 9) - 5(2y + 9) = 0$$

$$\Rightarrow (2y + 9)(2y - 5) = 0 \quad \text{i.e. } y = -\frac{9}{2} \text{ or } y = \frac{5}{2}$$

$$y = -\frac{9}{2} \Rightarrow x + \frac{1}{x} = -\frac{9}{2} \quad \text{i.e. } 2x^2 + 9x + 2 = 0$$

$$\Rightarrow x = \frac{-9 \pm \sqrt{(9)^2 - 4 \times 2 \times 2}}{2 \times 2} = \frac{-9 \pm \sqrt{65}}{4}$$

$$y = \frac{5}{2} \Rightarrow x + \frac{1}{x} = \frac{5}{2} \quad \text{i.e. } 2x^2 - 5x + 2 = 0$$

$$\Rightarrow 2x^2 - 4x - x + 2 = 0 \quad \text{i.e. } 2x(x - 2) - 1(x - 2) = 0$$

$$\Rightarrow (x - 2)(2x - 1) = 0 \quad \text{i.e. } x = 2 \text{ or } x = \frac{1}{2}$$

$$\therefore \text{Solution} = \frac{-9 \pm \sqrt{65}}{4}, 2, \text{ or } \frac{1}{2}$$

Ans.

21 Solve : $\frac{a}{ax-1} + \frac{b}{bx-1} = a + b$, where $a + b \neq 0$, $ab \neq 0$.

Solution :

$$\frac{a}{ax-1} + \frac{b}{bx-1} = a + b \Rightarrow \frac{a}{ax-1} - b + \frac{b}{bx-1} - a = 0$$

$$\text{i.e.} \quad \frac{a - abx + b}{ax-1} + \frac{b - abx + a}{bx-1} = 0$$

$$\Rightarrow (a + b - abx) \left[\frac{1}{ax-1} + \frac{1}{bx-1} \right] = 0$$

$$\Rightarrow a + b - abx = 0, \quad \text{or} \quad \frac{1}{ax-1} + \frac{1}{bx-1} = 0$$

$$\begin{aligned} \Rightarrow -abx &= -a - b, & \text{or} & & \frac{1}{ax-1} &= -\frac{1}{bx-1} \\ \Rightarrow abx &= a + b, & \text{or} & & bx - 1 &= -ax + 1 \\ \Rightarrow x &= \frac{a+b}{ab}, & \text{or} & & x &= \frac{2}{a+b} \end{aligned}$$

Ans.

EXERCISE 5(D)

Solve each of the following equations :

1. $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0;$
 $x \neq 3, x \neq -\frac{3}{2}$

2. $(2x + 3)^2 = 81$

3. $a^2x^2 - b^2 = 0$

4. $x^2 - \frac{11}{4}x + \frac{15}{8} = 0$

5. $x + \frac{4}{x} = -4; x \neq 0$

6. $2x^4 - 5x^2 + 3 = 0$ Take $x^2 = y$

7. $x^4 - 2x^2 - 3 = 0$

8. $9(x^2 + \frac{1}{x^2}) - 9(x + \frac{1}{x}) - 52 = 0$

Let $x + \frac{1}{x} = y \Rightarrow x^2 + \frac{1}{x^2} + 2 = y^2$

$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2.$

\therefore Given equation reduces to :

$9(y^2 - 2) - 9y - 52 = 0$

i.e. $9y^2 - 9y - 70 = 0$

$\Rightarrow (3y - 10)(3y + 7) = 0$

$\Rightarrow y = \frac{10}{3}, \text{ or } y = -\frac{7}{3}.$

$y = \frac{10}{3} \Rightarrow x + \frac{1}{x} = \frac{10}{3},$

solve it to get $x = 3, \text{ or } \frac{1}{3}.$

Similarly $y = -\frac{7}{3} \Rightarrow x + \frac{1}{x} = -\frac{7}{3},$

solve it to get $x = \frac{-7 \pm \sqrt{13}}{6}.$

\therefore The solution is $3, \frac{1}{3}, \frac{-7 \pm \sqrt{13}}{6}$

9. $2(x^2 + \frac{1}{x^2}) - (x + \frac{1}{x}) = 11$

10. $(x^2 + \frac{1}{x^2}) - 3(x - \frac{1}{x}) - 2 = 0$

Let $x - \frac{1}{x} = y \Rightarrow x^2 + \frac{1}{x^2} = y^2 + 2$

11. $(x^2 + 5x + 4)(x^2 + 5x + 6) = 120$

Take $x^2 + 5x = y$

12. Solve each of the following equations, giving answer upto two decimal places.

(i) $x^2 - 5x - 10 = 0$ [2005]

(ii) $3x^2 - x - 7 = 0$ [2004]

13. Solve: $(\frac{x}{x+2})^2 - 7(\frac{x}{x+2}) + 12 = 0; x \neq -2.$

14. Solve :

(i) $x^2 - 11x - 12 = 0; \text{ when } x \in \mathbb{N}$

(ii) $x^2 - 4x - 12 = 0; \text{ when } x \in \mathbb{I}$

(iii) $2x^2 - 9x + 10 = 0; \text{ when } x \in \mathbb{Q}.$

15. Solve :

$(a + b)^2 x^2 - (a + b)x - 6 = 0; a + b \neq 0.$

Take : $(a + b)x = y$

16. Solve : $\frac{1}{p} + \frac{1}{q} + \frac{1}{x} = \frac{1}{x+p+q}$

Take : $(\frac{1}{p} + \frac{1}{q}) + (\frac{1}{x} - \frac{1}{x+p+q}) = 0$

17. Solve :

(i) $x(x + 1) + (x + 2)(x + 3) = 42$

(ii) $\frac{1}{x+1} - \frac{2}{x+2} = \frac{3}{x+3} - \frac{4}{x+4}$

18. For each equation, given below, find the value of 'm' so that the equation has equal roots. Also, find the solution of each equation :

(i) $(m - 3)x^2 - 4x + 1 = 0$

(ii) $3x^2 + 12x + (m + 7) = 0$

(iii) $x^2 - (m + 2)x + (m + 5) = 0$

19. Without solving the following quadratic equation, find the value of 'p' for which the roots are equal.

$px^2 - 4x + 3 = 0$ [2010]

20. Without solving the following quadratic equation, find the value of 'm' for which the given equation has real and equal roots.

$x^2 + 2(m - 1)x + (m + 5) = 0$ [2012]