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MATHEMATICS

Standard 9

(Semester II)

PLEDGE

India is my country.
All Indians are my brothers and sisters.
I love my country and I am proud of its rich and varied heritage.
I shall always strive to be worthy of it.
I shall respect my parents, teachers and all my elders and treat everyone with courtesy.
I pledge my devotion to my country and its people.
My happiness lies in their well-being and prosperity.

રાજ્ય સરકારની વિનામૂલ્યે યોજના હેઠળનું પુસ્તક

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PREFACE

The Gujarat State Secondary and Higher Secondary Education Board has prepared new syllabi in accordance with the new national syllabi prepared by the N.C.E.R.T. These syllabi are sanctioned by the Government of Gujarat.

It is pleasure for the Gujarat State Board of School Textbooks, to place before the students this textbook of **Mathematics** for **Standard 9 (Semester II)** prepared according to the new syllabus.

Before publishing the textbook, its manuscript has been fully reviewed by experts and teachers teaching at this level. Following suggestions given by teachers and experts, we have made necessary changes in the manuscript before publishing the textbook.

The Board has taken special care to ensure that this textbook is interesting, useful and free from errors. However, we welcome any suggestions, from people interested in education, to improve the quality of the textbook.

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FUNDAMENTAL DUTIES

It shall be the duty of every citizen of India

(A) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(B) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(C) to uphold and protect the sovereignty, unity and integrity of India;

(D) to defend the country and render national service when called upon to do so;

(E) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(F) to value and preserve the rich heritage of our composite culture;

(G) to protect and improve the natural environment including forests, lakes, rivers and wild life, and to have compassion for living creatures;

(H) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(I) to safeguard public property and to abjure violence;

(J) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

(K) to provide opportunities for education by the parent or the guardian, to his child or a ward between the age of 6-14 years as the case may be.

CONTENTS

CHAPTER 10

QUADRILATERALS

10.1 Introduction

We have learnt about triangles in the previous chapter using the terminology of the set theory. Now we shall study about quadrilaterals using the same terminology.

10.2 Plane Quadrilateral

We know that a triangle is the union of three line-segments determined by three non-collinear points.

Quadrilateral : A quadrilateral is the union of four line-segments determined by four distinct coplanar points of which no three are collinear and the line-segments intersect only at end points.

It is clear from the definition of a quadrilateral that for distinct coplanar points P, Q, R, S the following three conditions are essential to construct a quadrilateral :

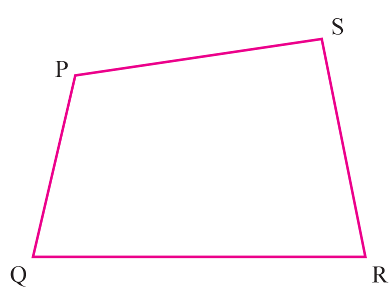


Figure 10.1

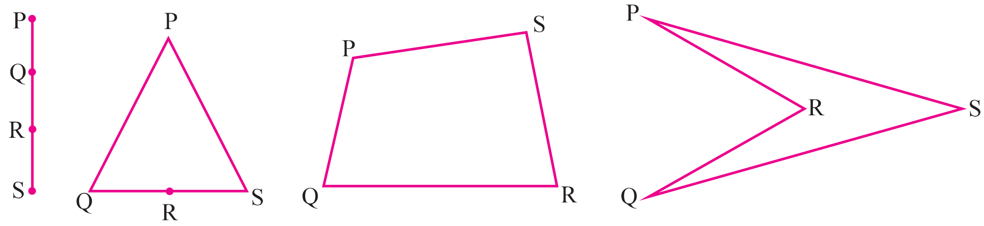
(i) P, Q, R and S are distinct and coplanar points.

(ii) No three of points P, Q, R and S are collinear.

(iii) Line-segments $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$ and $\overline{SP}$ intersect at their end points only. Then the union of $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$ and $\overline{SP}$ is the quadrilateral PQRS. We denote quadrilateral PQRS by $\square$ PQRS.

$\therefore \square PQRS = \overline{PQ} \cup \overline{QR} \cup \overline{RS} \cup \overline{SP}$

Now we see why above three conditions are essential :



(i) (ii) (iii) (iv)

Figure 10.2

If all the four points are collinear, we obtain line-segments as given in figure 10.2 (i).

If three out of four points are collinear, we may get a triangle as given in figure 10.2 (ii).

If no three points out of four points are collinear, we obtain a closed figure with four sides given in figure 10.2 (iii) and 10.2 (iv).

In our study, we will consider only quadrilaterals of type as in figure 10.2 (iii).

Convex quadrilateral : If in a quadrilateral, no side intersects the line containing its opposite side, then the quadrilateral is called a convex quadrilateral. The diagonals of a convex quadrilateral intersect each other.

We will refer to convex quadrilaterals as quadrilaterals in the rest of the chapter.

Quadrilaterals of type given in figure 10.2 (iv) are called **concave** quadrilaterals.

10.3 Parts of a Quadrilateral

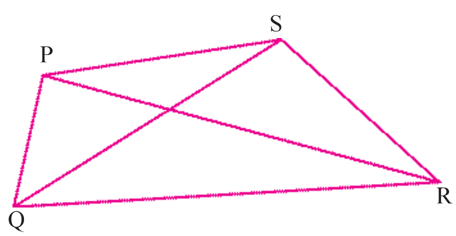


Figure 10.3

In the □ PQRS,

(i) Points P, Q, R, S are called the vertices of □ PQRS.

(ii) $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$, $\overline{SP}$ are called sides of □ PQRS.

(iii) $\angle SPQ$, $\angle PQR$, $\angle QRS$, $\angle RSP$ are called the angles of □ PQRS.

If there is no confusion, we denote these angles as $\angle P$, $\angle Q$, $\angle R$ and $\angle S$ respectively.

(iv) $\overline{PR}$ and $\overline{QS}$ are diagonals of □ PQRS.

It is clear that **the diagonals of a convex quadrilateral intersect each other.**

A quadralateral has 10 parts namely four sides, four angles and two diagonals.

Now we will learn about special pair of sides and angles of a quadrilateral.

(1) Two sides of a quadrilateral intersecting in a vertex are called adjacent sides.

As shown in figure 10.4, $\overline{PS}$ and $\overline{SR}$ have a common end point S. So, $\overline{PS}$ and $\overline{SR}$ are adjacent sides.

$\overline{PQ}$, $\overline{QR}$; $\overline{QR}$, $\overline{RS}$ and $\overline{PQ}$, $\overline{PS}$ are other pairs of adjacent sides of $\square$ PQRS.

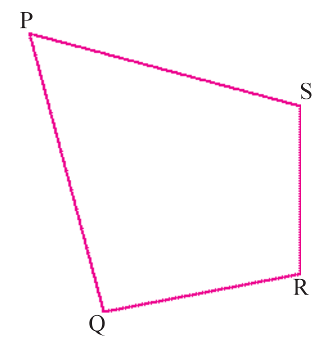


Figure 10.4

(2) The sides of a quadrilateral with no common end point are called opposite sides. The intersection of opposite sides is $\emptyset$.

Sides $\overline{PQ}$ and $\overline{SR}$ of $\square$ PQRS have no common end point, so $\overline{PQ}$ and $\overline{SR}$ are opposite sides of $\square$ PQRS. $\overline{PS}$ and $\overline{QR}$ is also another pair of **opposite sides.**

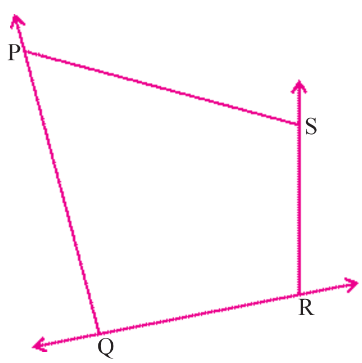


Figure 10.5

(3) If two angles of a quadrilateral intersect in a side of the quadrilateral, then these angles are called adjacent angles.

In figure 10.5, $\overline{QR}$ is the intersection of $\angle$Q and $\angle$R. Hence $\angle$Q and $\angle$R are adjacent angles of the quadrilateral. In this way, $\angle$Q and $\angle$R, $\angle$R and $\angle$S, $\angle$S and $\angle$P, $\angle$P and $\angle$Q are four pairs of the adjacent angles of $\square$ PQRS.

(4) If the intersection of two angles of a quadrilateral is not a side of the quadrilateral, then the two angles are called opposite angles. Two angles are opposite if and only if they are not adjacent. Intersection of two opposite angles consists of two vertices only.

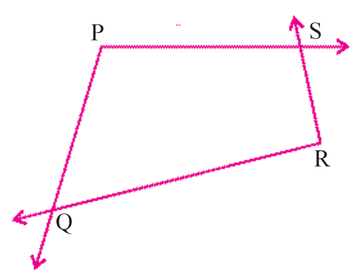


Figure 10.6

The intersection of two angles $\angle$P and $\angle$R does not contain any common side of the quadrilateral but consists of only two vertices Q and S. Hence $\angle$P and $\angle$R are opposite angles of the $\square$ PQRS. Thus (i) $\angle$P and $\angle$R (ii) $\angle$Q and $\angle$S are two pairs of opposite angles in $\square$ PQRS.

Now, with reference to □ PQRS it is clear from the above information that

(1) Every vertex of a quadrilateral is the common end point of two adjacent sides of the quadrilateral.

As in the figure 10.6, $\overline{PQ} \cap \overline{QR} = \{Q\}$, $\overline{QR} \cap \overline{RS} = \{R\}$, $\overline{SR} \cap \overline{SP} = \{S\}$, $\overline{SP} \cap \overline{PQ} = \{P\}$

(2) The union of the sides (line-segments) is a quadrilateral but the region enclosed by those line-segments is not a quadrilateral. (figure 10.6)

□ PQRS = $\overline{PQ} \cup \overline{QR} \cup \overline{RS} \cup \overline{SP}$

(3) All the vertices and sides of a quadrilateral are in the same plane. Thus a quadrilateral is a plane figure lying in a plane.

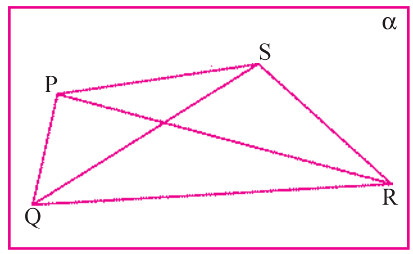


Figure 10.7

As shown in the figure 10.7, vertices P, Q, R, S are in the plane α and therefore $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$ and $\overline{SP}$ are also in plane α. Thus □ PQRS is a plane figure lying in the plane α.

(4) The sides and set of vertices of a quadrilateral are subsets of the quadrilateral.

In the figure 10.7, $\overline{PQ} \subset$ □ PQRS, $\overline{QR} \subset$ □ PQRS, $\overline{RS} \subset$ □ PQRS, $\overline{SP} \subset$ □ PQRS and $\{P, Q, R, S\} \subset$ □ PQRS.

(5) Angles and diagonals of a quadrilateral are not subsets of the quadrilateral.

In figure 10.7, $\angle P \not\subset$ □ PQRS, $\angle Q \not\subset$ □ PQRS, $\angle R \not\subset$ □ PQRS, $\angle S \not\subset$ □ PQRS, $\overline{PR} \not\subset$ □ PQRS, $\overline{QS} \not\subset$ □ PQRS.

(6) The plane containing a quadrilateral is partitioned into three mutually disjoint sets by the quadrilateral : (1) the quadrilateral (2) the interior of the quadrilateral (3) the exterior of the quadrilateral.

We get more clarity about naming of a quadrilateral from following examples :

(1) Name the quadrilateral with diagonals $\overline{AC}$ and $\overline{BD}$:

In the figure 10.8, the quadrilateral with diagonals $\overline{AC}$ and $\overline{BD}$ is □ ABCD. It can also be called □ ADCB.

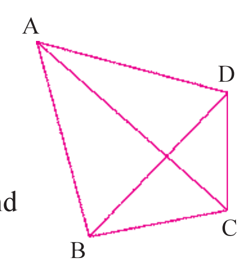


Figure 10.8

(2) Of which quadrilateral will $\overline{DF}$ and $\overline{GE}$ be the opposite sides and $\overline{DE}$ a diagonal ?

If $\overline{DF}$ and $\overline{GE}$ are the opposite sides of a quadrilateral and $\overline{DE}$ is the diagonal, then the quadrilateral is □ DGEF or □ DFEG.

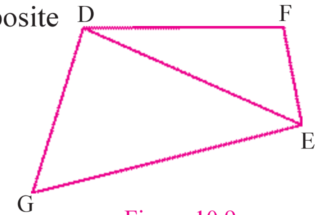


Figure 10.9

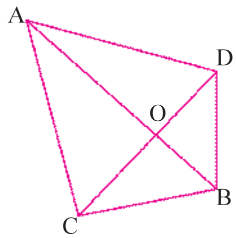


Figure 10.10

(3) If A–O–B and C–O–D and $\overline{AB} \cap \overline{CD} = \{O\}$, then which quadrilateral will be formed by A, B, C and D ?

If A–O–B and C–O–D and $\overline{AB} \cap \overline{CD} = \{O\}$, then □ ADBC or □ ACBD is formed.

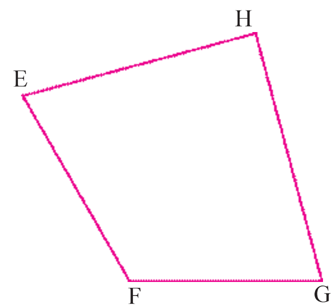


Figure 10.11

(4) Is □ EFGH = □ HGFE ? Give reasons.

Yes, □ EFGH = □ HGFE,

because

□ EFGH $= \overline{EF} \cup \overline{FG} \cup \overline{GH} \cup \overline{HE}$

$= \overline{HG} \cup \overline{GF} \cup \overline{FE} \cup \overline{EH}$

= □ HGFE as $\overline{HG} = \overline{GH}$, $\overline{EF} = \overline{FE}$ etc.

Thus, □ EFGH, □ HGFE, □ FGHE, □ GFEH, □ GHEF, □ FEHG and □ EHGF represent the same quadrilateral.

10.4 The Sum of the Measures of the Angles of a Quadrilateral

We know that the sum of the measures of all the angles of a triangle is 180. What should be sum of measures of all the angles of a quadrilateral ?

Drawing the diagonal $\overline{AC}$ of □ ABCD, we get Δ ABC and Δ ACD. Vertex C is in the interior of $\angle$DAB.

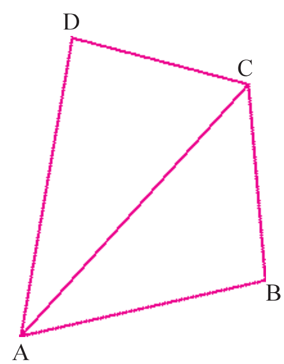


Figure 10.12

$m\angle DAC + m\angle CAB = m\angle DAB$. **(i)**

Similarly vertex A is in the interior of $\angle$BCD.

$\therefore\ m\angle BCA + m\angle ACD = m\angle BCD$ **(ii)**

In ΔABC, $m\angle CAB + m\angle ABC + m\angle BCA = 180$ **(iii)**

In ΔACD, $m\angle ACD + m\angle CDA + m\angle DAC = 180$ **(iv)**

From (iii) and (iv),

$m\angle CAB + m\angle ABC + m\angle BCA + m\angle ACD + m\angle CDA + m\angle DAC = 360$

From (i) and (ii),

$\therefore\ m\angle DAB + m\angle ABC + m\angle BCD + m\angle ADC = 360$

Thus, **the sum of the measures of the angles of a quadrilateral is 360.**

Example 1 : In □ ABCD, the measures of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are in proportion 2 : 4 : 5 : 4. Find the measure of each angle.

Solution : The measures of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ of □ ABCD are in proportion 2 : 4 : 5 : 4.

Let $m\angle A = 2x$, $m\angle B = 4x$, $m\angle C = 5x$ and $m\angle D = 4x$.

But in □ ABCD, $m\angle A + m\angle B + m\angle C + m\angle D = 360$

$\therefore\ 2x + 4x + 5x + 4x = 360$

$\therefore\ 15x = 360$

$\therefore\ x = \dfrac{360}{15} = 24$

$\therefore\ m\angle A = 2x = 48,\quad m\angle B = 4x = 96$

$m\angle C = 5x = 120,\quad m\angle D = 4x = 96$

EXERCISE 10.1

1. Describe the following for □ XYZW shown in the figure 10.13 :

(1) the sides (2) the angles (3) the diagonals

(4) pairs of adjacent sides

(5) pairs of opposite sides

(6) pairs of adjacent angles

(7) pairs of opposite angles

(8) $\overline{XW} \cap \overline{YZ}$ (9) $\overline{YX} \cap \overleftrightarrow{XW}$

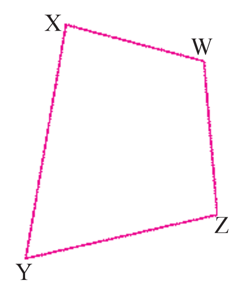


Figure 10.13

2. Is □ PQRS = □ PSQR ? Give reasons for your answer.

3. Solve the following :

(1) If in □ PQRS, $m\angle P = 2x$, $m\angle Q = 3x$, $m\angle R = 4x$ and $m\angle S = 6x$, then find the measure of each angle of □ PQRS.

(2) In □ ABCD, if $m\angle A = m\angle B = 70$, $m\angle C = 100$, find the measure of $\angle D$.

(3) In □ ABCD, the measures of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are in the proportion 2 : 5 : 6 : 7. Find the measure of each angle of □ ABCD.

(4) In □ ABCD, the measure of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are in proportion of 10 : 7 : 12 : 7. Find measure of each angle of □ ABCD.

4. For each of the following statements, state whether it is true or false :

(1) The angle of a quadrilateral is a subset of the quadrilateral.

(2) $\angle A$ and $\angle B$ are adjacent angles of □ ABCD.

(3) $\overline{GD}$ is a subset of □ DEFG.

(4) $\overline{AB}$ and $\overline{CD}$ are opposite sides of □ ABCD.

(5) $\overline{AC}$ is a diagonal of □ ABCD.

(6) If no three of E, F, G, H are collinear, then $\overline{EF} \cup \overline{FG} \cup \overline{GH} \cup \overline{HE} =$ □ EFGH.

(7) $\overline{ML}$ and $\overline{LN}$ are adjacent sides and $\overline{LO}$ is a diagonal, then MLON is a quadrilateral.

*

10.5 Types of Quadrilateral

We study different quadrilaterals given below :

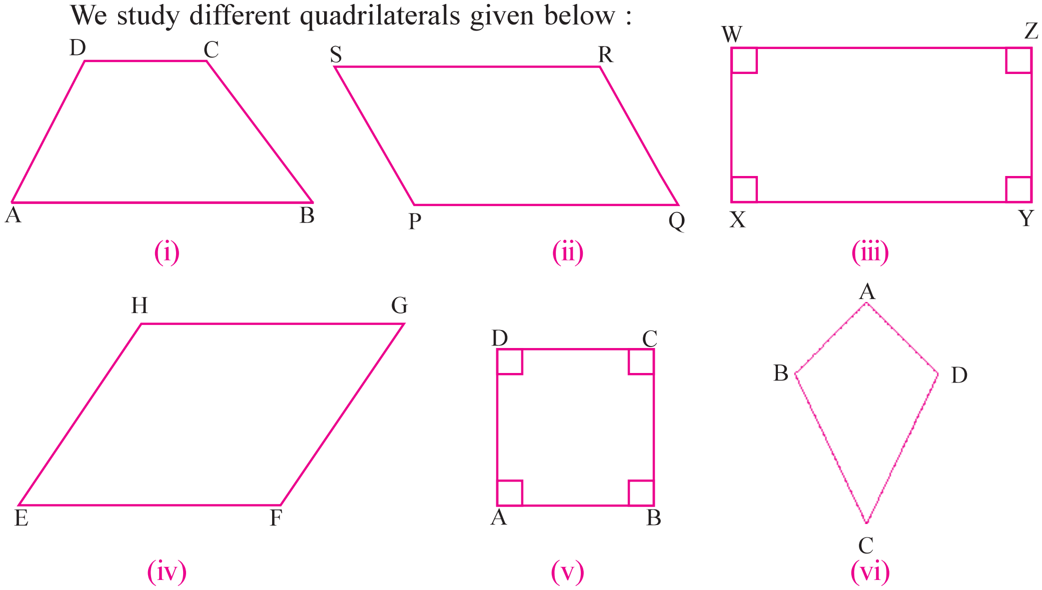


Figure 10.14

In figure 10.14 (i), in □ ABCD sides in only one pair of opposite sides $\overline{AB}$ and $\overline{CD}$ are parallel.

If in a quadrilateral, sides in only one pair of opposite sides are parallel to each other, then the quadrilateral is called a trapezium.

$\therefore$ □ ABCD is trapezium.

Sides in both the pairs of opposite sides are parallel in figure 10.14 (ii), (iii), (iv) and (v). Such quadrilaterals are called **parallelograms.**

Now let us get more information about each figure 10.14 (ii) to (v).

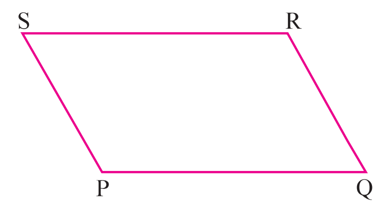


Figure 10.15

In a quadrilateral, if opposite sides are parallel to each other, then the quadrilateral is called a parallelogram.

In $\square$ PQRS, $\overline{SP} \parallel \overline{RQ}$ and $\overline{SR} \parallel \overline{PQ}$. Hence it is a parallelogram and it is denoted by $\square^m$ PQRS.

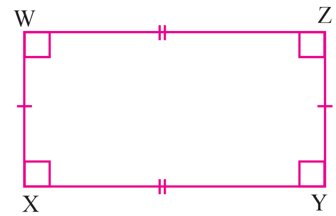


Figure 10.16

In $\square$ XYZW, $\overline{XW} \parallel \overline{ZY}$ and $\overline{XY} \parallel \overline{WZ}$.

So $\square$ XYZW is parallelogram, but also

$m\angle X = m\angle Y = m\angle Z = m\angle W = 90$.

$\square^m$ XYZW is known as a **rectangle**.

If all the angles of a parallelogram are right angles, then the parallelogram is called a rectangle.

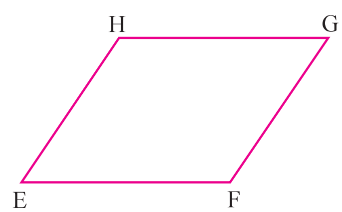


Figure 10.17

Here, we need to observe following facts :

(1) Each rectangle is parallelogram.

(2) All the four angles of a rectangle are congruent.

In $\square$ EFGH, $\overline{HE} \parallel \overline{GF}$, $\overline{HG} \parallel \overline{EF}$. $\square$ EFGH is a parallelogram. But in $\square^m$ EFGH, all sides are congruent. $\square$ EFGH is known as a **rhombus.**

If all the sides of a parallelogram are congruent, then it is called a rhombus.

Here we note the following facts :

(1) Each rhombus is a parallelogram.

(2) All the four sides of a rhombus are congruent.

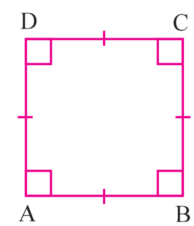


Figure 10.18

In $\square$ ABCD, since $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{CD}$, $\square$ ABCD is parallelogram. But here, $m\angle A = m\angle B = m\angle C = m\angle D = 90$ and also all the sides of $\square$ ABCD are congruent. So, $\square^m$ABCD is known as a **square.**

This $\square^m$ABCD is also a rectangle and $\square^m$ABCD is a rhombus also.

If all the side of a rectangle are congruent, then it is called a square.

We observe,

(1) A square is a parallelogram.

(2) Since all the four sides of a square are congruent, it is a rhombus too.

(3) Since each angle of a square is a right angle, a square is also a rectangle.

In figure 10.19, □ ABCD, AB = AD and BC = CD. So adjacent sides are congruent, but □ ABCD is not parallelogram. □ ABCD is known as a **kite**.

Note : Diagonals of a kite are not congruent but intersect each other at right angles.

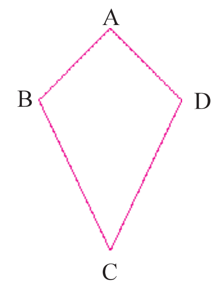


Figure 10.19

Example 2 : In a trapezium PQRS, if $\overline{PS} \parallel \overline{QR}$, $m\angle P : m\angle Q = 7 : 3$ and $m\angle R = 99$, then find the measures of all the remaining angles.

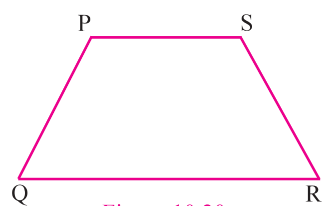


Figure 10.20

Solution : In □ PQRS, $\overline{QR} \parallel \overline{PS}$ and $\angle P$ and $\angle Q$ are the interior angles on one side of the transversal $\overleftrightarrow{PQ}$. Let $m\angle P = 7x$ and $m\angle Q = 3x$.

$\therefore\ m\angle P + m\angle Q = 180$

$\therefore\ 7x + 3x = 180$

$\therefore\ 10x = 180$

$\therefore\ x = 18$

$\therefore\ m\angle P = 7x = 7(18) = 126$

$\therefore\ m\angle Q = 3x = 3(18) = 54$

Now, in □ PQRS, $m\angle R + m\angle S = 180$ **($\overleftrightarrow{PS} \parallel \overleftrightarrow{RQ}$)**

$99 + m\angle S = 180$ **($m\angle R = 99$)**

$m\angle S = 180 - 99 = 81$

$\therefore\ m\angle S = 81$

EXERCISE 10.2

1. In a trapezium ABCD, $\overline{AB} \parallel \overline{CD}$. If $m\angle B = 60$ and $m\angle D = 100$, then find the measures of $\angle A$ and $\angle C$.

2. In a trapezium ABCD, $\overline{AB} \parallel \overline{DC}$. If $m\angle A = m\angle B = 60$, then find $m\angle C$ and $m\angle D$.

3. In a trapezium PQRS, $\overline{PQ} \parallel \overline{SR}$. If $m\angle P = 50$ and $m\angle R = 110$, then find $m\angle Q$ and $m\angle S$.

4. In a trapezium PQRS, if $\overline{PQ} \parallel \overline{RS}$, $m\angle S : m\angle P = 5 : 4$ and $m\angle Q = 72$, then find $m\angle R$, $m\angle S$, $m\angle P$.

5. In □ ABCD, the measures of the angles are in proportion 6 : 7 : 11 : 12. Find the measure of each angle of □ ABCD.

6. For each of the following statements, state whether it is true or false :

(1) Every square is a rectangle.
(2) Every rectangle is a parallelogram.
(3) Every rhombus is a square.
(4) Every trapezium is a parallelogram.
(5) Every rectangle is a trapezium.
(6) Every square is a rhombus.
(7) Every rhombus is a parallelogram.
(8) Every parallelogram is a rectangle.
(9) Every rectangle is a square.

*

10.6 Properties of Parallelograms

We have learnt about types of quadrilaterals. We have seen that a rectangle, a square, a rhombus are special types of parallelograms. A parallelogram is an important quadrilateral. Now we study some properties of parallelograms. We begin with proving following theorem asserting the congruence of triangles formed by each of its diagonals.

Theorem 10.1 : Two triangles formed by any diagonal of a parallelogram are congruent.

Data : Δ SPR and Δ QRP are formed by diagonal $\overline{PR}$ of □^{m} PQRS.

To Prove : $\Delta SPR \cong \Delta QRP$

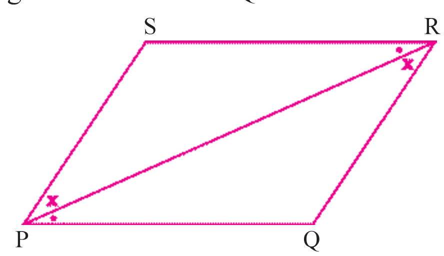


Figure 10.21

Proof : □ PQRS is parallelogram.

$\therefore$ $\overline{PS} \parallel \overline{QR}$ and $\overline{SR} \parallel \overline{PQ}$

$\overleftrightarrow{PS} \parallel \overleftrightarrow{QR}$ and $\overleftrightarrow{PR}$ is their transversal.

$\therefore$ $\angle SPR \cong \angle QRP$ **(alternate angles) (i)**

$\overleftrightarrow{SR} \parallel \overleftrightarrow{PQ}$ and $\overleftrightarrow{PR}$ is their transversal.

$\angle SRP \cong \angle QPR$ **(alternate angles) (ii)**

For correspondence SPR $\leftrightarrow$ QRP

$\angle SPR \cong \angle QRP$ **(by (i))**

$\angle SRP \cong \angle QPR$ **(by (ii))**

$\overline{PR} \cong \overline{PR}$

$\therefore$ The correspondence SPR $\leftrightarrow$ QRP is a congruence by ASA.

$\therefore$ Δ SPR $\cong$ Δ QRP

We know that if a correspondence between two triangles is a congruence, then corresponding sides and angles are congruent. Since two triangles formed by any one diagonal of a parallelogram are congruent; then it is obvious that opposite sides of the parallelogram are congruent. We accept this theorem without proof.

Theorem 10.2 : Opposite sides in a parallelogram are congruent.

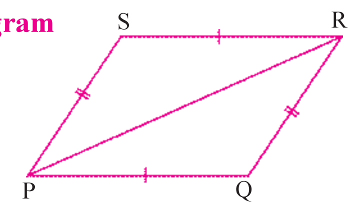


Figure 10.22

In $\square^m$ PQRS in figure 10.22, $\overline{PR}$ is diagonal.

$\therefore$ Δ SPR $\cong$ Δ QRP

$\therefore$ $\overline{SR} \cong \overline{QP}$ and $\overline{SP} \cong \overline{QR}$

Now if we construct a quadrilateral such that its opposite sides are congruent, then we get a parallelogram. This is the converse of the above theorem. We accept this theorem without proof.

Theorem 10.3 : If the sides in each pair of opposite sides in a quadrilateral are congruent, the quadrilateral is a parallelogram.

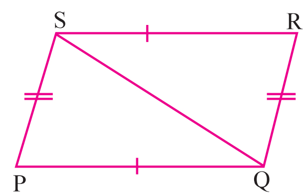


Figure 10.23

In figure 10.23, $\overline{SP} \cong \overline{QR}$ and $\overline{PQ} \cong \overline{SR}$.

So $\square$ PQRS is a parallelogram.

Example 3 : In $\square^m$ ABCD, AB = 10 *cm* and AD = 6 *cm*. Find the perimeter of $\square$ ABCD.

Soultion : In $\square^m$ ABCD, $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{CB}$

AB = DC = 10 *cm*, AD = BC = 6 *cm*

$\therefore$ The perimeter of $\square^m$ ABCD

= AB + BC + CD + AD

= 10 + 6 + 10 + 6 = 32 *cm*

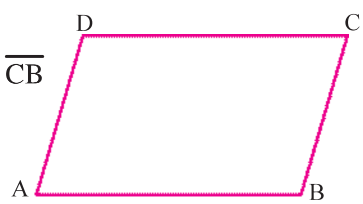


Figure 10.24

We construct a parallelogram and measure the opposite angles. We will find that they are congruent. We accept this theorem without proof.

Theorem 10.4 : Opposite angles in a parallelogram are congruent.

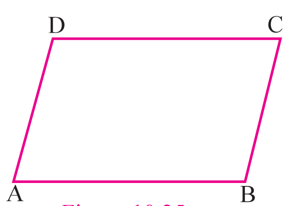


Figure 10.25

In figure 10.25, $\square$ ABCD is a parallelogram.

$\therefore$ $\angle B \cong \angle D$, $\angle A \cong \angle C$

If the opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. We accept this theorem without proof.

Theorm 10.5 : If in a quadrilateral, both the angles in each pair of opposite angles are congruent, then the quadrilateral is a parallelogram.

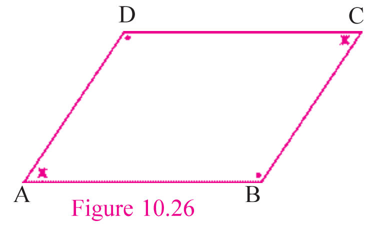


Figure 10.26

As shown in figure 10.26, for $\square$ ABCD, $\angle A \cong \angle C$ and $\angle B \cong \angle D$. So $\square$ ABCD is a parallelogram.

In a $\square^m$ PQRS, diagonals $\overline{SQ}$ and $\overline{PR}$ intersect each other at O. If we measure $\overline{SO}$, $\overline{OQ}$ and $\overline{OR}$, $\overline{PO}$ then we see that SO = OQ and PO = OR. So O is the midpoint of both $\overline{SQ}$ and $\overline{PR}$. So diagonals bisect each other at O. We accept this theorem without proof.

Theorem 10.6 : Diagonals of a parallelogram bisect each other.

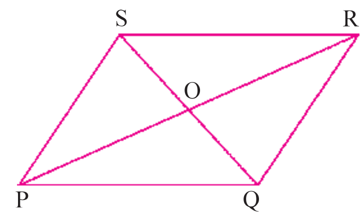


Figure 10.27

In figure 10.27, $\square$ PQRS is parallelogram. The diagonals $\overline{PR}$ and $\overline{SQ}$ bisect each other at O.

PO = OR and SO = OQ

Converse of this theorem is also true. We accept this theorem without proof.

Theorem 10.7 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

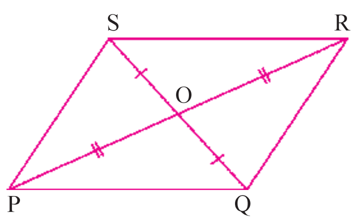


Figure 10.28

In the figure 10.28, the diagonals $\overline{PR}$ and $\overline{SQ}$ bisect each other at O. So $\overline{PO} \cong \overline{OR}$ and $\overline{SO} \cong \overline{OQ}$. $\square$ PQRS is a parallelogram.

Example 4 : In $\square^m$ ABCD, $m\angle A$ = 75 and $m\angle DBC$ = 60. Find $m\angle CDB$ and $m\angle ADC$.

Solution : $\square$ ABCD is a parallelogram.

$\overline{AD} \parallel \overline{BC}$ and $\overleftrightarrow{BD}$ is their transversal.

$\therefore \angle ADB \cong \angle DBC$ **(alternate angles)**

But $m\angle DBC = 60$

$\therefore m\angle ADB = 60$

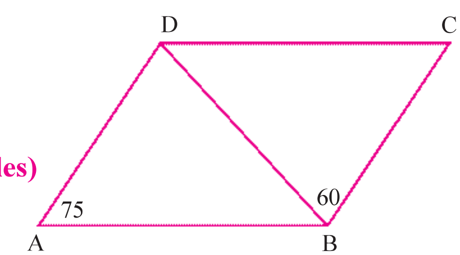


Figure 10.29

In ΔABD, $m\angle A + m\angle ADB + m\angle DBA = 180$

$75 + 60 + m\angle DBA = 180$

$\therefore\ m\angle DBA = 180 - 135 = 45$

$\overline{CD} \parallel \overline{AB}$ and $\overleftrightarrow{BD}$ is their transversal.

$\angle DBA \cong \angle CDB$ **(alternate angles)**

$\therefore\ m\angle DBA = m\angle CDB$

$\therefore\ m\angle CDB = 45$

$\therefore\ m\angle ADC = m\angle ADB + m\angle CDB = 60 + 45 = 105$

Example 5 : If an angle of a parallelogram is a right angle, then prove that the parallelogram is a rectangle.

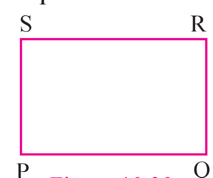


Figure 10.30

Solution : In $\square^m$ PQRS, $m\angle P = 90$

The opposite angles of a parallelogram are congruent.

$\therefore m\angle R = m\angle P = 90$

$\overleftrightarrow{PQ} \parallel \overleftrightarrow{SR}$ and $\overleftrightarrow{SP}$ is their transversal.

$\therefore\ \angle P$ and $\angle S$ are the interior angles on the same side of the transversal $\overleftrightarrow{SP}$.

$\therefore\ m\angle P + m\angle S = 180$

But $m\angle P = 90$. So $m\angle S = 90$

Hence $m\angle Q = 90$ **(opposite angles in a parallelogram)**

$m\angle P = m\angle Q = m\angle R = m\angle S = 90$

$\square^m$ PQRS is a rectangle.

An Important result (1) : Show that the diagonals of a rhombus are perpendicular to each other. Diagonals bisect the angles at the vertices.

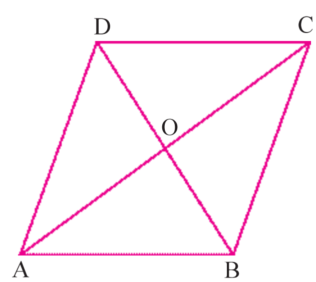


Figure 10.31

Solution : $\square$ ABCD is a rhombus.

So, AB = BC = CD = DA.

$\square$ ABCD is also a parallelogram.

$\therefore$ Diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other at O.

$\overline{AO} \cong \overline{OC}$, $\overline{DO} \cong \overline{OB}$ **(i)**

Now for the correspondence AOD $\leftrightarrow$ COD of ΔAOD and ΔCOD.

$\overline{AO} \cong \overline{CO}$

$\overline{OD} \cong \overline{OD}$ **(by (i))**

$\overline{AD} \cong \overline{CD}$ **(given)**

Thus, the correspondence AOD $\leftrightarrow$ COD is a congruence. **(SSS)**

$\therefore\ \Delta\, AOD \cong \Delta\, COD$ **(ii)**

$\angle AOD \cong \angle COD$ **(iii)**

But $m\angle AOD + m\angle COD = 180$ **(linear pair of angles)**

$\therefore 2m\angle AOD = 180$ **(by (iii))**

$\therefore m\angle AOD = 90$

$\therefore m\angle COD = 90$

$\therefore$ **Diagonals of a rhombus bisect each other at right angles.**

Also $\angle ODA \cong \angle ODC$ **(by (ii))**

but D – O – B.

$\therefore \angle BDA \cong \angle BDC$

$\therefore$ **Diagonal $\overline{BD}$ bisects $\angle D$.**

Similarly we can prove that $\overline{BD}$ bisects $\angle B$, diagonal $\overline{AC}$ bisects $\angle A$ and $\angle C$

An Important result (2) : Prove that the diagonals of a square are congruent and perpendicular to each other.

Solution : For the correspondence ADB $\leftrightarrow$ BCA of ΔADB and ΔBCA.

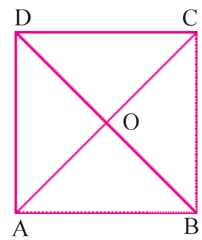


Figure 10.32

$\overline{AD} \cong \overline{BC}$ **(given)**

$\angle BAD \cong \angle ABC$ **(right angles)**

and $\overline{AB} \cong \overline{BA}$

$\therefore$ The correspondence ADB $\leftrightarrow$ BCA is a congruence. **(SAS)**

$\therefore \Delta\ ADB \cong \Delta\ BCA$

$\therefore \overline{DB} \cong \overline{CA}$

$\therefore$ Diagonals are congruent.

Note : For the rest of the proof refer to previous result (1).

EXERCISE 10.3

1. In $\square^m$ PQRS, $m\angle P : m\angle Q = 5 : 4$. Find the measure of each angle.

2. In $\square^m$ DEFG, if $m\angle DFG = 60$, then find $m\angle FDE$.

3. In $\square^m$ ABCD, $m\angle A - m\angle B = 30$. Find $m\angle C$ and $m\angle D$.

4. In $\square^m$ PQRS, $m\angle P = 3x$ and $m\angle Q = 6x$. Find the measures of all the angles.

5. Prove that in $\square^m$ ABCD, the bisectors of $\angle C$ and $\angle D$ intersect each other at right angles.

6. The diagonals of a rectangle PQRS intersect at O. If $m\angle POS = 54$, find the measure of $\angle OPS$.

7. $\square$ ABCD is a square. Find the measure of $\angle DCA$.

8. $\square$ ABCD is a rectangle. If $m\angle BAC = 30$, find the measure of $\angle DBC$.

9. $\square$ DEFG is a rhombus. $m\angle DFE = 50$. Find the measures of $\angle DFG$ and $\angle DGE$.

10. $\square$ ABCD is square. $\overline{AC}$ and $\overline{BD}$ intersect at O. Find the measure of $\angle AOB$.

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10.7 Another Condition for a Quadrilateral to be a Parallelogram

If we construct a quadrilateral in such a way that the sides in only one pair of opposite sides are congruent and parallel, then the quadrilateral is also a parallelogram. We accept this theorem stated below without proof :

Theorem 10.8 : If in a quadrilateral, one pair of opposite sides consists of congruent and parallel line-segments, then the quadrilateral is a parallelogram.

In □ ABCD, $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$.

$\therefore$ □ ABCD is a parallelogram.

Now, we shall apply above theorem to an illustration.

Example 6 : $\overline{AB}$ and $\overline{CD}$ are the sides of □^{m}ABCD and their midpoint are P and R respectively. $\overline{AR}$ intersect $\overline{DP}$ in the point S and $\overline{BR}$ intersects $\overline{CP}$ in the point Q. Prove that □ PQRS is a parallelogram.

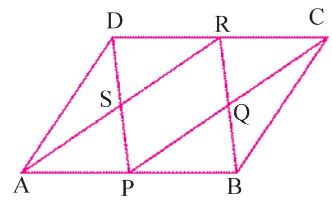

Figure 10.33

Solution : □ ABCD is a parallelogram.

$\therefore$ AB = CD

P and R are midpoints of $\overline{AB}$ and $\overline{CD}$ respectively.

AP = $\frac{1}{2}$ AB and CR = $\frac{1}{2}$ CD

$\therefore$ $\overline{AP} \cong \overline{CR}$ **(AB = CD) (i)**

Also, $\overline{AB} \parallel \overline{CD}$, A – P – B and C – R – D

$\therefore$ $\overline{AP} \parallel \overline{CR}$ **(ii)**

From (i) and (ii), $\overline{AP} \cong \overline{CR}$ and $\overline{AP} \parallel \overline{CR}$

□ APCR is a parallelogram.

$\therefore$ $\overline{AR} \parallel \overline{PC}$

$\therefore$ $\overline{SR} \parallel \overline{PQ}$ **(S $\in \overleftrightarrow{AR}$ and Q $\in \overleftrightarrow{PC}$) (iii)**

Similarly it can be proved that □ DRBP is a parallelogram.

$\therefore$ $\overline{BR} \parallel \overline{DP}$

$\therefore$ $\overline{RQ} \parallel \overline{SP}$ **(Q $\in \overleftrightarrow{BR}$ and S $\in \overleftrightarrow{PD}$) (iv)**

From (iii) and (iv), in □ PQRS, $\overline{SR} \parallel \overline{PQ}$ and $\overline{RQ} \parallel \overline{SP}$

$\therefore$ □ PQRS is a parallelogram.

An Important result : If the diagonals of a parallelogram are perpendicular to each other, then it is a rhombus.

Solution : In □^{m} ABCD diagonals bisect each other at O.

$\therefore$ OA = OC

Now for the correspondence AOD $\leftrightarrow$ COD of ΔAOD and ΔCOD.

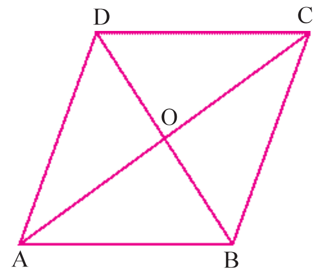

Figure 10.34

$\overline{OA} \cong \overline{OC}$

$\angle AOD \cong \angle COD$ **(right angles)**

$\overline{OD} \cong \overline{OD}$

$\therefore$ By SAS, the correspondence AOD $\leftrightarrow$ COD is a congruence.

$\therefore$ Δ AOD $\cong$ Δ COD

$\therefore$ AD = CD

but $\square$ ABCD is parallelogram.

AD = BC and CD = AB

AD = BC = CD = AB

$\square^m$ ABCD is a rhombus.

An Important result : If the diagonals of a parallelogram are congruent and intersect at right angles, then the parallelogram is a square.

Solution : For correspondence AOB $\leftrightarrow$ AOD of ΔAOB and ΔAOD,

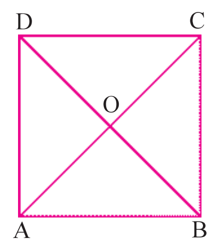

Figure 10.35

$\overline{AO} \cong \overline{AO}$

$\angle AOB \cong \angle AOD$ **(right angles)**

$\overline{OB} \cong \overline{OD}$

$\therefore$ By SAS, the correspondence AOB $\leftrightarrow$ AOD is congruence.

$\therefore$ AB = AD

But AB = CD and AD = BC

$\therefore$ AB = AD = CD = BC **(i)**

For the correspondence ABD $\leftrightarrow$ BAC of ΔABD and ΔBAC,

$\overline{AB} \cong \overline{BA}$

$\overline{AD} \cong \overline{BC}$

and $\overline{BD} \cong \overline{AC}$ **(given)**

By SSS, the correspondence ABD $\leftrightarrow$ BAC is a congruence.

$\therefore$ $\angle DAB \cong \angle CBA$

$\therefore$ $m\angle DAB = m\angle CBA$

But $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ and $\overleftrightarrow{AB}$ is a transeversal.

$m\angle DAB + m\angle CBA = 180$ **(interior angles on one side)**

$m\angle DAB = m\angle CBA = 90$ **(ii)**

From (i) and (ii) in $\square^m$ABCD, all the sides are congruent and all the angles are right angles.

$\therefore$ $\square^m$ ABCD is a square.

EXERCISE 10.4

1. Two sides of a rectangle have lengths 6 *cm* and 8 *cm*. Verify that the measures of the diagonals of the rectangle are same.

2. The perimeter of rectangle PQRS is 70 *cm*. If PQ : QR = 3 : 4, then find QR.

3. In rhombus ABCD, if AC = 10 *cm* and BD = 24 *cm*, then find the perimeter of rhombus ABCD.

4. $\square^m$ ABCD is neither a square nor a rhombus. Then prove that bisectors of its angles form a rectangle.

5. In $\square^m$ ABCD, $\overline{AP}$ and $\overline{CQ}$ are perpendicular from vertices A and C respectively to diagonal $\overline{BD}$. Prove that $\overline{AP} \cong \overline{CQ}$.

6. If the diagonals of a parallelogram are congruent, then prove that it is a rectangle.

7. $\square$ XYZW is a rectangle. If XY + YZ = 7 and XZ + YW = 10, then find XY.

*

10.8 The Mid-point Theorem

We studied the properties of a parallelogram. Using them we shall study some properties of triangles and parallel lines.

In Δ ABC, E and F are the midpoints of the sides $\overline{AB}$ and $\overline{AC}$ respectively. If we measure $\overline{EF}$ and $\overline{BC}$, then we see that EF = $\frac{1}{2}$BC. We accept the theorem stated below without proof.

Theorm 10.9 : The line-segment joining the midpoints of two sides of a triangle is parallel to the third side and its measure is half the measure of the third side.

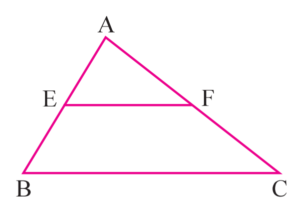

Figure 10.36

In Δ ABC, E and F are the midpoints of the sides $\overline{AB}$ and $\overline{AC}$ respectively. (i) $\overline{EF} \parallel \overline{BC}$ (ii) EF = $\frac{1}{2}$ BC.

We accept the following theorem without proof.

Theorem 10.10 A line passing through the midpoint of the one side and parallel to another side of a triangle bisects the third side of the triangle.

In Δ ABC, E is the midpoint of $\overline{AB}$. l is the line passing through E and parallel to $\overline{BC}$. l bisects $\overline{AC}$.

The following examples will help us in understanding the concept.

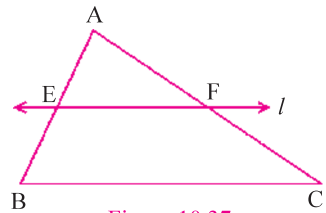

Figure 10.37

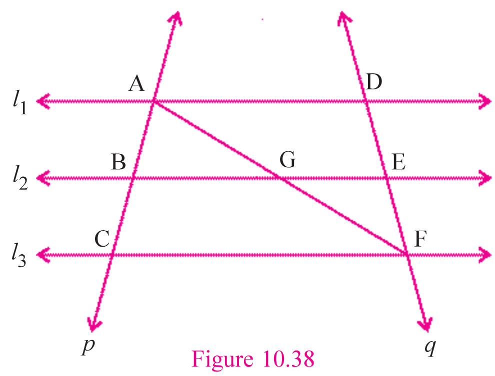

Figure 10.38

Example 7 : l_1, l_2 and l_3 are three parallel lines intersected by transversal p and q such that l_1, l_2 and l_3 cut off congruent intercepts $\overline{AB}$ and $\overline{BC}$ on p. Show that l_1, l_2 and l_3 cut off congruent intercepts $\overline{DE}$ and $\overline{EF}$ on q also.

Solution : We have AB = BC. **(given)**

Let $\overline{AF}$ intersect l_2 at G.

In Δ ACF, it is given that B is the midpoint of $\overline{AC}$ and $\overline{BG} \parallel \overline{CF}$ **($l_2 \parallel l_3$)**

$\therefore$ G is the midpoint of $\overline{AF}$

We apply the same theorem to Δ AFD. G is the midpoint of $\overline{AF}$. $\overline{GE} \parallel \overline{AD}$ and so by the theorem, E is the midpoint of $\overline{DF}$.

$\therefore$ $\overline{DE} \cong \overline{EF}$

In other words l_1, l_2 and l_3 cut off congruent intercepts on q also.

Example 8 : Δ ABC is an isosceles triangle with AB = AC and Let D, E and F be the midpoints of $\overline{BC}$, $\overline{CA}$ and $\overline{AB}$ respectively. Show that $\overline{AD} \perp \overline{EF}$ and $\overline{AD}$ bisects $\overline{EF}$.

Solution : In Δ ABC, D is the midpoint of $\overline{BC}$ and E is the midpoint of $\overline{AC}$.

$\therefore$ $\overleftrightarrow{DE} \parallel \overleftrightarrow{AB}$ and DE = $\frac{1}{2}$ AB **(i)**

Also AF = $\frac{1}{2}$ AB. **(ii)**

From (i) and (ii), DE = AF and $\overline{DE} \parallel \overline{AF}$. **(A–F–B)**

$\therefore$ $\square$ AFDE is a parallelogram.

$\therefore$ $\overline{AD}$ bisects $\overline{EF}$. **(iii)**

F and E are midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.

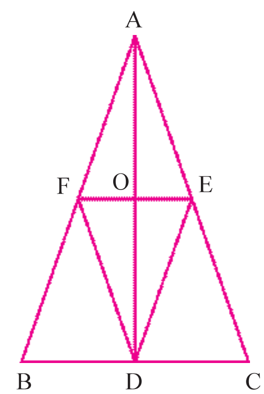

Figure 10.39

$\therefore$ AF = $\frac{1}{2}$ AB and AE = $\frac{1}{2}$ AC

But AB = AC **(given)**

$\therefore$ AE = AF **(iv)**

From (iii) and (iv), $\square$ AFDE is a rhombus.

$\therefore$ $\overline{AD} \perp \overline{EF}$

Example 9 : Δ ABC is a triangle right angled at B and P is the midpoint of $\overline{AC}$ $\overline{PQ} \parallel \overline{BC}$ and Q $\in$ $\overline{AB}$. Prove that (i) $\overline{PQ} \perp \overline{AB}$ (ii) Q is the midpoint of $\overline{AB}$ (iii) PB = PA = $\frac{1}{2}$AC

Solution : P is the midpoint of $\overline{AC}$ **(given)**

Also $\overleftrightarrow{PQ} \parallel \overleftrightarrow{BC}$

$\overline{PQ}$ intersects $\overline{AB}$ at Q.

$\angle$AQP $\cong$ $\angle$ABC.

But $m\angle$ABC = 90 **(given)**

$\therefore$ $m\angle$AQP = 90

$\therefore$ $\overline{PQ} \perp \overline{AB}$

In Δ ABC, P is the midpoint of $\overline{AC}$ and $\overline{PQ} \parallel \overline{BC}$. So Q is the midpoint of $\overline{AB}$.

$\therefore$ AQ = BQ

Now in ΔAPQ and ΔBPQ, consider the correspondence APQ $\leftrightarrow$ BPQ,

$\overline{AQ} \cong \overline{BQ}$

$\angle$AQP $\cong$ $\angle$BQP **(right angles)**

$\overline{PQ} \cong \overline{PQ}$

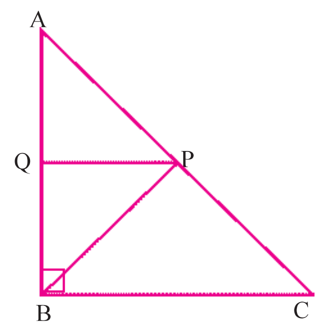

Figure 10.40

$\therefore$ The correspondence APQ $\leftrightarrow$ BPQ is a congruence by SAS.

$\therefore$ ΔAPQ $\cong$ ΔBPQ

$\therefore$ $\overline{PA} \cong \overline{PB}$

But P is the midpoint of $\overline{AC}$.

$\therefore$ PA = PB = $\frac{1}{2}$AC

Example 10 : In Δ ABC, $\overline{AD}$ is the median. E is the midpoint of $\overline{AD}$. $\overrightarrow{BE}$ intersects $\overline{AC}$ in F. Prove that AF = $\frac{1}{3}$AC.

Solution : Let $\overline{DK} \parallel \overline{BF}$ and K $\in$ $\overline{AC}$. In Δ ADK, E is the midpoint of $\overline{AD}$ and $\overline{EF} \parallel \overline{DK}$.

$\therefore$ F is the midpoint of $\overline{AK}$.

$\therefore$ AF = FK **(i)**

In Δ BCF, D is the midpoint of $\overline{BC}$ and $\overline{DK} \parallel \overline{BF}$

$\therefore$ K is the midpoint of $\overline{FC}$.

$\therefore$ FK = KC **(ii)**

From (i) and (ii), we have

AF = FK = KC

$\therefore$ AC = AF + FK + KC

$\therefore$ AC = AF + AF + AF

$\therefore$ AF = $\frac{1}{3}$ AC

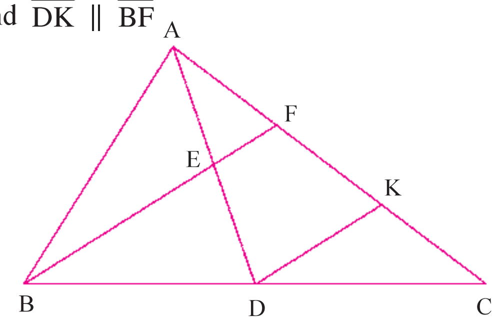

Figure 10.41

10.9 An Important Result

In a trapezium ABCD, $\overline{AB} \parallel \overline{CD}$. E and F are the midpoints of $\overline{AD}$ and $\overline{BC}$ respectively. Prove that $\overline{EF} \parallel \overline{AB}$ and EF = $\frac{1}{2}$ (AB + CD).

Solution : $\overrightarrow{DF}$ and $\overrightarrow{AB}$ intersect at P, so that A–B–P and D–F–P.

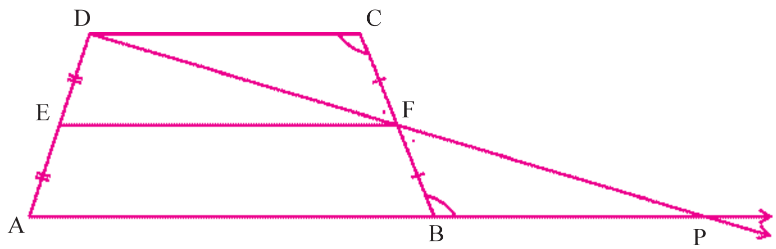

Figure 10.42

In the correspondence BPF $\leftrightarrow$ CDF of ΔBPF and ΔCDF.

$\angle$BFP $\cong$ $\angle$CFD **(Vertically opposite angles)**

$\overline{FB} \cong \overline{FC}$

and $\angle$FBP $\cong$ $\angle$FCD **(alternate angles made by transversal $\overleftrightarrow{BC}$ with $\overleftrightarrow{DC} \parallel \overrightarrow{BC}$)**

Thus, the correspondence BPF $\leftrightarrow$ CDF is a congruence **(ASA Theorem)**

So, $\overline{BP} \cong \overline{CD}$ and $\overline{PF} \cong \overline{DF}$. So, BP = CD and PF = DF.

So F is the midpoint of $\overline{DP}$. Now in Δ DAP, E is the midpoint of $\overline{DA}$ and F is the midpoint of $\overline{DP}$.

$\therefore$ $\overline{EF} \parallel \overline{AP}$ and EF = $\frac{1}{2}$AP

$\therefore$ $\overline{EF} \parallel \overline{AB}$ **(A – B – P)**

$\therefore$ EF = $\frac{1}{2}$AP = $\frac{1}{2}$(AB + BP)

$\therefore$ EF = $\frac{1}{2}$(AB + CD) **(BP = CD)**

Example 11 : In a trapezium PQRS, $\overline{PQ} \parallel \overline{SR}$ and PQ > SR. X and Y are midpoints of $\overline{SP}$ and $\overline{RQ}$ respectively. If SR = 12 and XY = 14.5, find PQ.

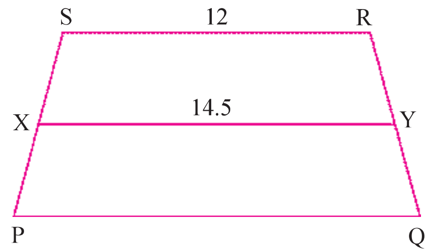

Figure 10.43

Solution : XY = $\frac{1}{2}$(SR + PQ)

$\therefore$ 14.5 = $\frac{1}{2}$(12 + PQ)

$\therefore$ 29 = 12 + PQ

$\therefore$ PQ = 17

EXERCISE 10.5

1. In Δ ABC, the points E and F are the midpoints of $\overline{AB}$ and $\overline{AC}$. If EF = 6.5, then find BC.

2. In Δ DEF, the points X and Y are the midpoints of $\overline{DE}$ and $\overline{DF}$ respectively. If EF = 20, then find XY.

3. The perimeter of Δ XYZ is 25. P, Q and R are the midpoints of $\overline{XY}$, $\overline{YZ}$ and $\overline{ZX}$ respectively. Find perimeter of Δ PQR.

4. In Δ ABC, D, E and F are the mid points of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively. If AB = 9, BC = 12, CA = 18, find the perimeters of $\square$ DBCF and Δ CFE.

5. In a trapezium ABCD, $\overline{AB} \parallel \overline{CD}$, AB > DC. P and Q are the midpoints of $\overline{AD}$ and $\overline{CB}$ respectively. If AB = 15 and DC = 7, find PQ.

6. In a trapezium PQRS, $\overline{PQ} \parallel \overline{SR}$, PQ > SR. X and Y are the midpoints of $\overline{SP}$ and $\overline{QR}$ respectively. If XY = 7.5 and PQ = 12, then find RS.

7. In Δ ABC, the points P and Q are on $\overline{AB}$ and $\overline{AC}$ such that AP = $\frac{1}{4}$AB and AQ = $\frac{1}{4}$AC. Prove that PQ = $\frac{1}{4}$ BC.

8. In an equilateral Δ ABC, M and N are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively. If MN = 4.5, find the perimeter of Δ ABC.

9. In Δ ABC, E, F and G are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively. If EF + EG = 14 and AB = 7, find the perimeter of Δ ABC.

10. In Δ PQR, A, B and C are the midpoints of $\overline{PQ}$, $\overline{QR}$, $\overline{RP}$ respectively. If AB : BC : CA = 3 : 4 : 5 and QR = 20, find perimeter of Δ PQR.

11. In Δ ABC, D, E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively. Prove that Δ ADF and Δ DBE, and Δ EFD and Δ FEC are congruent.

12. In Δ ABC, D, E and F are the midpoints of $\overline{BC}$, $\overline{CA}$ and $\overline{AB}$ respectively. Prove that $\overline{AD}$ and $\overline{EF}$ bisect each other.

13. In $\square$ ABCD, the midpoints of the sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ are P, Q, R and S respectively. Prove that $\square$ PQRS is a parallelogram.

14. If A, B, C, D are the midpoints of the sides $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$ and $\overline{SP}$ of a rectangle PQRS, then prove that $\square$ ABCD is a rhombus.

15. In an equilateral Δ ABC, P, Q and R are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$. Prove that Δ PQR is equilateral.

EXERCISE 10

1. Solve the following :

(1) $\square$ PQRS is a rhombus. If $m\angle QRS = 60$ and QS = 15, find the perimeter of the rhombus.

(2) $\square$ DEFG is a rhombus. If DF = 30 and EG = 16, find the perimeter of $\square$ DEFG.

(3) $\square$ PQRS is a rectangle. If its diagonals intersect each other at O and $m\angle POS = 120$, find the $m\angle QPO$.

(4) In a trapezium PQRS, $\overline{PS} \parallel \overline{QR}$, QR > PS and X and Y are the midpoints of $\overline{PQ}$ and $\overline{SR}$. If PS = 18, XY = 20, find QR.

(5) In a triangle PQR, $m\angle P = 75$, $m\angle Q = 60$, $m\angle R = 45$. Find the measures of the angles of the triangle formed by joining the midpoints of the sides of this triangle.

2. In $\square^m$ PQRS, A is a point on $\overline{PS}$ such that AP = $\frac{1}{3}$PS and B is a point on $\overline{QR}$ such that RB = $\frac{1}{3}$QR, prove that $\square$ APBR is a parallelogram.

3. Show that the quadrilateral, formed by joining the midpoints of the sides of a square in order is also a square.

4. The diagonals of a $\square$ PQRS are perpendicular to each other. Show that the quadrilateral formed by joining the midpoints of its sides is a rectangle.

5. $\square$ PQRS is a rhombus and A, B, C and D are the midpoints of $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$ and $\overline{SP}$ respectively. Prove that $\square$ ABCD is a rectangle.

6. In figure 10.44, in Δ PQR, $\overline{PA}$ is the median of Δ PQR and $\overline{AB} \parallel \overline{PQ}$. Prove that $\overline{QB}$ is a median Δ PQR.

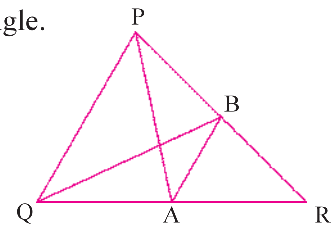

Figure 10.44

7. Select proper option (a), (b), (c) or (d) and write in the box given on the right so that the statement becomes correct :

(1) In $\square^m$ABCD, if $m\angle A : m\angle B = 2 : 3$, then $m\angle D$ is .... $\square$

(a) 72 (b) 108 (c) 60 (d) 90

(2) In $\square^m$ ABCD, if $m\angle B - m\angle C = 40$, then $m\angle A$ is ...... $\square$

(a) 70 (b) 110 (c) 55 (d) 35

(3) In $\square^m$ ABCD, $m\angle A : m\angle B = 1 : 3$, then $m\angle C$ is ...... $\square$

(a) 90 (b) 120 (c) 45 (d) 135

(4) If the diagonals of quadrilateral are not congruent and bisect each other at right angles, then the quadrilateral is a ......

(a) square (b) rectangle (c) trapezium (d) rhombus

(5) The diagonals of a quadrilateral are congruent and bisect each other but not at right angles. Then the quadrilateral is a ......

(a) rectangle (b) rhombus (c) square (d) parallelogram

(6) All the four sides of a quadrilateral are congruent but all the four angles are not congruent. Then the quadrilateral is a ....

(a) rhombus (b) square (c) rectangle (d) parallelogram

(7) All the four angles of a quadrilateral are congruent but all the four sides are not congruent. Then the quadrilateral is a ......

(a) rhombus (b) square (c) rectangle (d) trapezium

(8) A figure is formed by joining the midpoints of the sides of a quadrilateral. It is a ......

(a) square (b) rhombus (c) rectangle (d) parallelogram

(9) In rhombus PQRS if the diagonal PR = 8 and diagonal QS = 6, then perimeter of rhombus is ......

(a) 10 (b) 40 (c) 5 (d) 20

(10) The perimeter of rectangle ABCD is 36. If AB : BC = 4 : 5, then the length of $\overline{BC}$ is ......

(a) 8 (b) 16 (c) 10 (d) 9

(11) In ΔABC, D, E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively. If the perimeter of ΔDEF is 12, then the perimeter of ΔABC is ....

(a) 24 (b) 6 (c) 36 (d) 48

(12) ΔABC is an equilateral triangle. AB = 6. The points P, Q and R are midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively. The perimeter of $\square$ PBCR is ......

(a) 18 (b) 15 (c) 9 (d) 12

(13) In trapezium ABCD, $\overline{AD} \parallel \overline{BC}$, BC > AD. Points P and Q are midpoints of $\overline{AB}$ and $\overline{CD}$. If AD = 6 and BC = 8, then the measure of $\overline{PQ}$ is ......

(a) 14 (b) 7 (c) 4 (d) 3

(14) In trapezium PQRS, $\overline{PS} \parallel \overline{QR}$, QR > PS and points M and N are the midpoints of $\overline{PQ}$ and $\overline{SR}$. If QR = 16 and MN = 14, then the measure of $\overline{PS}$ is ......

(a) 44 (b) 9 (c) 12 (d) 4

(15) In $\square^m$ PQRS the bisectors of $\angle P$ and $\angle Q$ intersect at X. If $m\angle P = 70$, then $m\angle PXQ$ is ...... $\square$

(a) 90 (b) 35 (c) 55 (d) 110

(16) P and Q are the midpoints of $\overline{AB}$ and $\overline{AC}$ of Δ ABC. $\square$ PBCQ is a ...... $\square$

(a) square (b) rhombus (c) trapezium (d) rectangle

(17) $\square$ ABCD is a rhombus. If the diagonals $\overline{AC}$ and $\overline{BD}$ intersect at M, then $m\angle AMB$ is ...... $\square$

(a) 60 (b) 45 (c) 30 (d) 90

(18) $\square$ PQRS is a square. If PQ = 5, then QS is ...... $\square$

(a) 10 (b) 50 (c) $5\sqrt{2}$ (d) 15

(19) Perimeter of rhombus PQRS is 96, then PQ is ...... $\square$

(a) 24 (b) 48 (c) 12 (d) 6

*

Summary

In this chapter, we have learnt following points :

1. Plane quadrilateral and its parts
2. The sum of the measures of the angles of a quadrilateral
3. Types of quadrilateral
4. Properties of parallelograms and its theorems
5. Rhombus and its important result
   (i) Diagonals of a rhombus are perpendicular to each other and vice-versa
   (ii) Diagonals bisect the angle at vertices and vice-versa
6. Square and its properties
7. Diagonals of a square are congruent and perpendicular to each other and vice-versa.
8. The midpoint theorems for a triangle and vice-versa
9. For trapezium ABCD, $\overline{AB} \parallel \overline{CD}$ and E and F are midpoints of $\overline{AD}$ and $\overline{BC}$ then $EF = \frac{1}{2}(AB + CD)$.

CHAPTER 11

AREAS OF PARALLELOGRAMS AND TRIANGLES

11.1 Introduction

We have learnt earlier about areas of closed figures like triangles, quadrilaterals and circles. We know that **area is the 'measure' of the region enclosed by a closed figure in a plane. We know about units of area also.**

11.2 Interior of Triangle

We have learnt about interior of a triangle. The intersection of the interiors of all the three angles of a triangle is called the interior of the triangle. We also know that if we take the intersection of the interiors of any two angles of a triangle, then also we get the interior of the triangle.

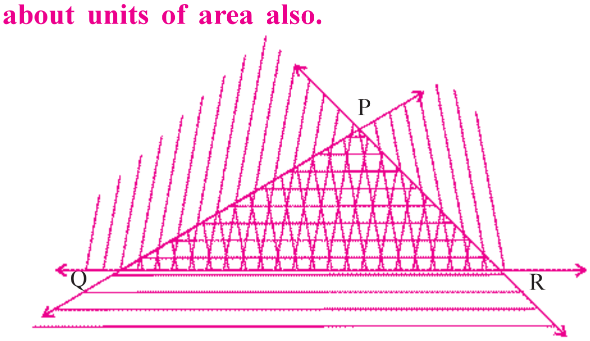

Figure 11.1

11.3 Triangular Region

For any ΔPQR, ΔPQR and interior of ΔPQR are two mutually disjoint sets. The union of these two sets is called the triangular region associated with ΔPQR.

Triangular region : The union of a triangle and its interior is called the triangular region associated with the given triangle. We denote the triangular region associated with the ΔPQR by Δ*PQR.

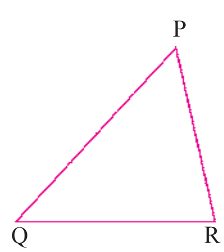

Figure 11.2 (i)

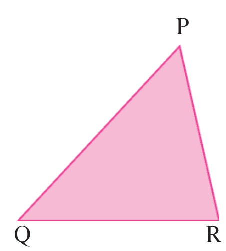

Figure 11.2 (ii)

ΔPQR is shown in figure 11.2(i) and triangular region Δ^*PQR as coloured region in figure 11.2(ii). $\Delta^*PQR = (\Delta PQR) \cup$ (interior of ΔPQR).

11.4 Interior of a Quadrilateral

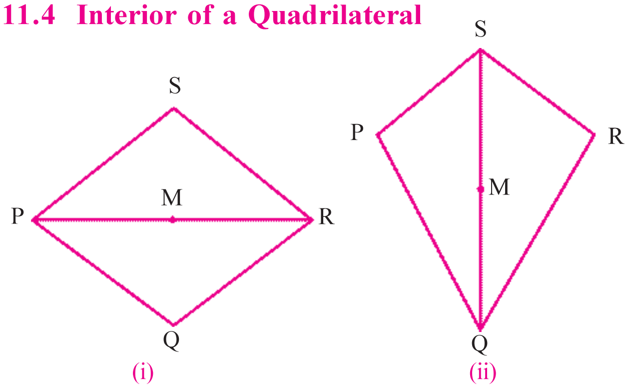

Figure 11.3

We have the concept of the interior of a triangle.

In figure 11.3 (i), we have a □ PQRS and $\overline{PR}$ is its diagonal. Then the interior of □ PQRS is the union of (1) The interior of Δ PSR (2) The interior of Δ PQR (3) The set of all the points M, such that P–M–R.

In figure 11.3 (ii), we have □ PQRS and $\overline{SQ}$ is its diagonal.

Then, the interior of □ PQRS is the union of (1) The interior of Δ PQS (2) The interior of Δ QRS (3) The set of all point M such that S–M–Q.

The intersection of the interiors of all the four angles of a quadrilateral is the interior of the quadrilateral.

If we take the intersection of the interiors of two opposite angles, then also we will get the interior of the quadrilateral.

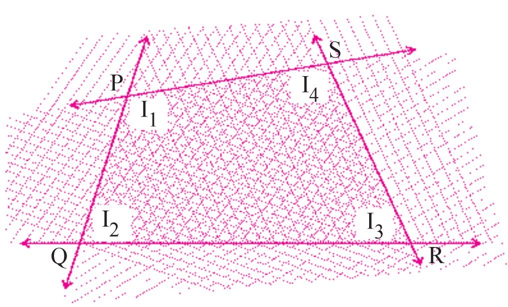

Figure 11.4

As in figure 11.4, let us denote interior of $\angle P$ by I_1, the interior of Q by I_2, the interior of R by I_3, the interior of S by I_4 and the interior of □ PQRS by I.

Then, $I = I_1 \cap I_2 \cap I_3 \cap I_4$

In □ PQRS, $\angle P$ and $\angle R$ are opposite angles. $\angle Q$ and $\angle S$ are opposite angles.

Then, $I = I_1 \cap I_3 = I_2 \cap I_4$

11.5 Quadrilateral Region

A quadrilateral and the interior of the quadrilateral are two mutually disjoint sets. The union of these two sets is called the quadrilateral region.

Figure 11.5 (i) shows □ PQRS and the coloured region in figure 11.5 (ii) shows the quadrilateral region of □ PQRS.

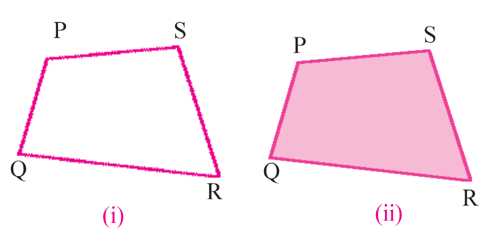

Figure 11.5

Quadrilateral region : The union of a quadrilateral and its interior is called the quadrilateral region associated with the given quadrilateral.

The quadrilateral region associated with $\square$ PQRS contains all the points of $\square$ PQRS as well as all the interior points of $\square$ PQRS. The quadrilateral region associated with $\square$ PQRS is denoted by $\square$* PQRS.

Thus, $\square$* PQRS = ($\square$ PQRS) $\cup$ (interior of $\square$ PQRS)

11.6 Postulates for Area

We know that area is a positive number and areas of congruent figures are equal. We shall take these natural ideas as postulates :

(1) The Postulate for Area : Corresponding to every triangular region, there is a unique positive number associated with it and it is called the area of the triangular region.

(2) Postulate for the Area of Congruent triangles : If two triangles are congruent, then the areas of their triangular regions are equal.

(3) Postulate for Addition of Areas : In Δ ABC, If B – D – C, then
area of Δ*ABC = area of Δ*ABD + area of Δ*ADC

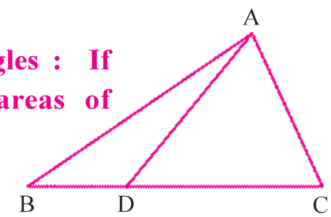

Figure 11.6

(Note that in the figure 11.6 interiors of Δ ABD and Δ ADC are mutually disjoint sets.)

If Δ* ABC is a union of several triangular regions, triangles having mutually disjoint interiors, then the area of Δ*ABC is the sum of the areas of these triangular regions. From now onwards, we shall denote the area of Δ*ABC by simply ABC and area of $\square$ *PQRS by PQRS.

11.7 Area of a Rectangle

We know the formula to find the area of a rectangle.

Area of rectangle = length $\times$ breadth

We shall accept this idea in the form of a postulate.

Postulate for the area of a rectangle : The area of any rectangular region is the product of the lengths of any two adjacent sides of the rectangle.

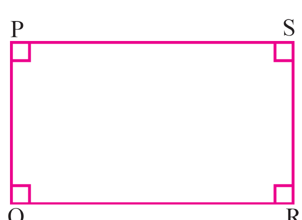

Figure 11.7

As shown in the figure 11.7, $\square$ PQRS is a rectangle. Taking its adjacent sides $\overline{PQ}$ and $\overline{QR}$, we have, area of the rectangle PQRS, PQRS = PQ $\times$ QR.

Note : For the sake of simplicity, we shall use triangle for **'triangular region'**, the words rectangle for **'rectangular region'** and side for the **'length of a side'** and similary quadrilateral for **'quadrilateral region'**.

Example 1 : The length of one side of a rectangle is thrice the length of its adjacent side. If the perimeter of the rectangle is 80 *cm*, find the area of the rectangle.

Solution : Let $\overline{DE}$ and $\overline{EF}$ be two adjacent sides of the rectangle DEFG. If the length of $\overline{DE}$ is x *cm*, then the length of $\overline{EF}$ is $3x$ *cm*. The perimeter of rectangle = 80 *cm*

$\therefore$ $2(x + 3x) = 80$

$\therefore$ $8x = 80$

$\therefore$ $x = 10$ *cm*

$\therefore$ $3x = 30$ *cm*

$\therefore$ DEFG = DE $\times$ EF

= $10 \times 30 = 300$ cm^2

$\therefore$ The area of the rectangle is 300 cm^2

Figure 11.8

11.8 The Area of a Right Triangle

The area of a right triangle is half the product of its sides forming the right angle.

In the figure 11.9, □ PQRS is a rectangle and $\overline{PR}$ is diagonal.

Δ PQR is a right triangle with base $\overline{QR}$ and $\overline{PQ}$ is its altitude.

But since Δ PQR $\cong$ Δ RSP, PQR = RSP

Also ΔPQR and ΔRSP have disjoint interiors.

$\therefore$ PQRS = PQR + RSP = PQR + PQR = 2 PQR

$\therefore$ PQR = $\frac{1}{2}$ PQRS

Now, PQRS = PQ $\times$ QR

Hence, PQR = $\frac{1}{2} \times$ QR $\times$ PQ

Hence, PQR = $\frac{1}{2}$ base $\times$ altitude

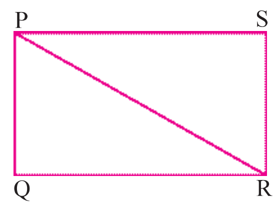

Figure 11.9

Example 2 : In a right triangle, the measure of one side is 12 *cm* and that of the hypotenuse is 13 *cm*. Find the area of the right triangle.

Solution : Let $\angle$B be the right angle in Δ ABC.

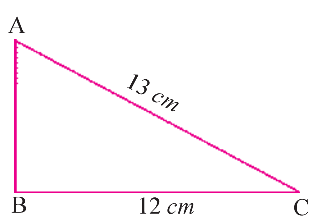

Figure 11.10

BC = 12 *cm* and AC = 13 *cm*.

In right triangle Δ ABC

$AC^2 = AB^2 + BC^2$

$\therefore$ $AB^2 = AC^2 - BC^2$

$= (13)^2 - (12)^2$

$= 169 - 144$

$= 25$

$\therefore$ AB $= 5$ *cm*

$\therefore$ Area of right triangle ABC $= \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 5 \times 12 = 30\ cm^2$

$\therefore$ The area of the right triangle is 30 cm^2.

11.9 Area of Triangle

The area of a triangle is one half the product of length of its altitude and the base corresponding to the altitude.

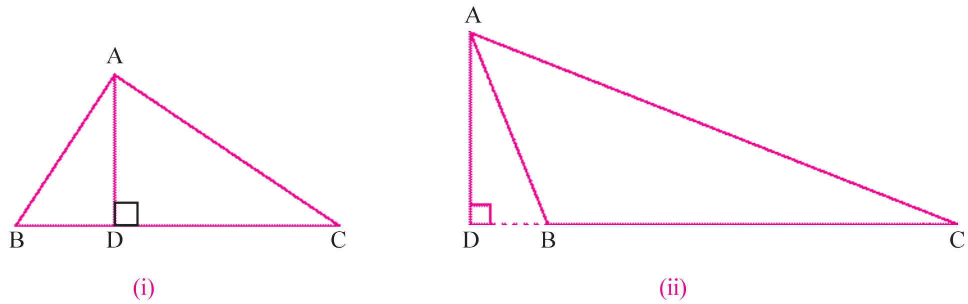

Figure 11.11

In figure 11.11 (i) $\overline{AD}$ is an altitude of Δ ABC, $\overline{BC}$ the corresponding base and B–D–C. Also ΔABC and ΔABD have disjoint interiors.

ABC = ABD + ADC **(postulate for addition of area)**

$= \frac{1}{2}$ AD $\times$ BD + $\frac{1}{2}$ AD $\times$ DC

$= \frac{1}{2}$ AD (BD + DC)

$\therefore$ ABC $= \frac{1}{2} \times$ AD $\times$ BC **(B – D – C)**

In figure 11.11 (ii), $\overline{AD}$ is the altitude to $\overleftrightarrow{BC}$ and it intersects $\overleftrightarrow{BC}$ in D such that D–B–C. $\overline{BC}$ is the base corresponding to the altitude $\overline{AD}$.

ΔABC and ΔADB have disjoint interiors.

$\therefore$ ADC = ADB + ABC

ABC = ADC – ADB **(postulate for addition of area)**

$= \frac{1}{2}$ AD $\times$ DC – $\frac{1}{2}$ AD $\times$ DB

$= \frac{1}{2}$ AD (DC – DB)

$= \frac{1}{2}$ AD $\times$ BC **(D – B – C)**

Every triangle has three altitudes and three corresponding bases so the **formula for area gives the area of the same triangle in three different ways.** However, for the same triangle, we get the same area by using any of these pairs of base and altitude.

11.10 Area of Parallelogram

A line-segment drawn from any vertex of a parallelogram and perpendicular to the line containing a side of the parallelogram which does not pass through that vertex, is called an altitude of the parallelogram and the side is called the base corresponding to the altitude.

In figure 11.12, sides $\overline{QR}$ and $\overline{SR}$ of $\square^m$ PQRS do not pass through vertex P. Line-segment $\overline{PM}$ passes through P and is perpendicular to $\overleftrightarrow{QR}$. So $\overline{QR}$ is the corresponding base and $\overline{PM}$ is the altitude.

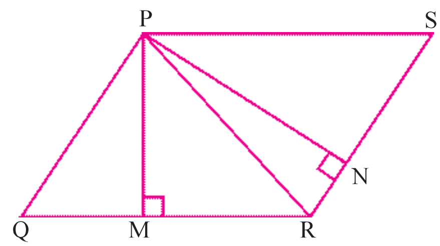

Figure 11.12

$\overline{PR}$ is a diagonal of $\square^m$ PQRS. Hence, Δ PQR $\cong$ Δ RSP. Also ΔPQR and ΔRSP have disjoint interiors. Thus area of $\square^m$ PQRS is twice the area of Δ PQR.

PQRS = 2 (PQR)

$= 2\left(\frac{1}{2}\text{PM} \times \text{QR}\right) = \text{PM} \times \text{QR}$

Hence, PQRS = altitude $\times$ corresponding base. Similarly, in figure 11.12, $\overline{SR}$ is also a side which does not pass through P. $\overline{PN}$ is the perpendicular line-segment from P to $\overleftrightarrow{SR}$. It is an altitude of $\square^m$ PQRS. Its corresponding base is $\overline{SR}$.

Since $\overline{PR}$ is the diagonal of $\square^m$ PQRS, Δ RSP $\cong$ Δ PQR. Hence the area of $\square^m$ PQRS is twice of Δ RSP.

As before PQRS = PN $\times$ SR

Thus, **the area of a parallelogram is the product of any of its altitude and its corresponding base.**

Note : Henceforth we will not mention about disjoint interiors, if it is obvious.

Example 3 : $\overline{EM}$ and $\overline{EN}$ are altitudes of $\square^m$ DEFG. Their corresponding bases are $\overline{DG}$ and $\overline{GF}$ respectively. If DG = 10 *cm*, EM = 8 *cm*, EN = 16 *cm*, find GF.

Solution : DEFG = EM $\times$ DG = EN $\times$ GF

$\therefore$ EM $\times$ DG = EN $\times$ GF

$\therefore$ 8 $\times$ 10 = 16 $\times$ GF

$\therefore$ GF $= \frac{8 \times 10}{16} = 5$

$\therefore$ GF = 5 *cm*

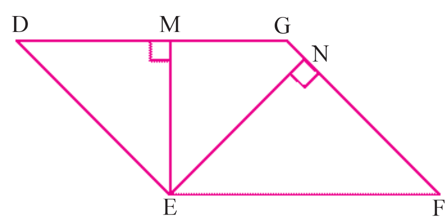

Figure 11.13

An Important Result : The area of a rhombus is half the product of its diagonals.

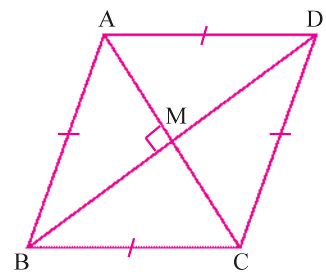

Figure 11.14

As shown in the figure 11.14, □ ABCD is a rhombus. Its diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other at right angles at point M.

Hence $\overline{BM}$ and $\overline{MD}$ are altitudes to base $\overline{AC}$ in Δ ABC and Δ ACD respectively.

$$\begin{aligned} \text{Now ABCD} &= \text{ABC} + \text{ACD} \\ &= \frac{1}{2}\,\text{AC} \times \text{BM} + \frac{1}{2}\,\text{AC} \times \text{MD} \\ &= \frac{1}{2}\,\text{AC}\,(\text{BM} + \text{MD}) \\ &= \frac{1}{2}\,\text{AC} \times \text{BD} \qquad \textbf{(B – M – D)} \end{aligned}$$

Example 4 : □ PQRS is a rhombus. The length of each side is 10 *cm*. If QS = 16 *cm*, find the area of □ PQRS.

Solution : □ PQRS is rhombus. Diagonals $\overline{SQ}$ and $\overline{PR}$ bisect each other at M at right angles.

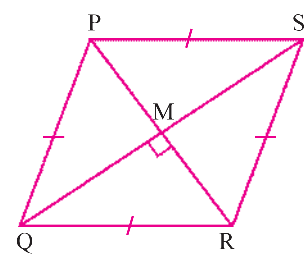

Figure 11.15

QS = 16 *cm* and M is the midpoint of $\overline{QS}$.

$\therefore$ QM = 8 *cm*

Now in right Δ PMQ,

$PM^2 = PQ^2 - QM^2 = (10)^2 - (8)^2 = 100 - 64 = 36$

$\therefore$ PM = 6 *cm*

$\therefore$ PR = 12 *cm*

$PQRS = \frac{1}{2} \times PR \times QS = \frac{1}{2} \times 12 \times 16 = 96\ cm^2$

$\therefore$ The area of the rhombus is 96 cm^2.

EXERCISE 11.1

1. State whether the following statements are true or false.

(1) A triangle and its triangular region are two disjoint sets.

(2) The intersection of a triangle and its interior is the empty set.

(3) If D, E and F are the midpoints of the sides of Δ PQR, then Δ^* DEF $\cup$ Δ^* PQR = Δ^* PQR.

(4) Every triangle is a subset of its triangular region.

(5) Interior of a triangle is a subset of its triangular region.

2. (1) In $\square^m$ ABCD, $\overline{CF} \perp \overline{AB}$ and $\overline{AE} \perp \overline{BC}$. If AB = 18 *cm*, AE = 10 *cm* and CF = 12 *cm*, find AD.

(2) If AD = 12 *cm*, CF = 20 *cm* and AE = 16 *cm*, find AB.

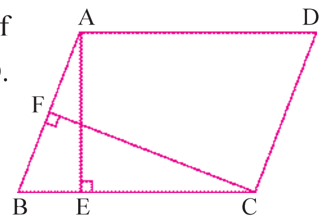

Figure 11.16

3. Let $\square^m$ ABCD be a parallelogram having area 250 cm^2. If E and F are the mid points of sides $\overline{AB}$ and $\overline{CD}$ respectively, then find the area of $\square$ AEFD.

4. In Δ ABC, $\overline{AD}$ is the altitude corresponding to base $\overline{BC}$. $\overline{BE}$ is the altitude corresponding to base $\overline{AC}$. If AD = 14, BC = 24 and AC = 35, find BE.

5. In Δ ABC, $\overline{BF}$ is the altitude to $\overline{AC}$ and $\overline{AE}$ is the altitude to $\overline{BC}$. If AC = 45 *cm*, BC = 15 *cm* and ABC = 225 cm^2, find BF and AE.

6. In $\square^m$ ABCD, $\overline{AM}$ and $\overline{BN}$ are altitudes and their corresponding bases are $\overline{BC}$ and $\overline{CD}$ respectively. If AM = 18, AB = 24, BC = 30, find BN.

7. Δ ABC is an equilateral triangle. If BC = 8, find ABC

8. In Δ ABC, P, Q and R are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively. If ABC = 64 cm^2. Find PQR, PQCR and PBCR.

9. In Δ ABC $m\angle B$ = 90, AB = 18 *cm*, BC = 24 *cm*, find ABC. Also find the measure of the altitude corresponding to $\overline{AC}$.

10. $\square$ ABCD is a rhombus. If AB = 25 and AC = 48, find ABCD.

*

11.11 Quadrilaterals on the Same Base and Between Two Parallel Lines

Let us observe the following figures 11.17 :

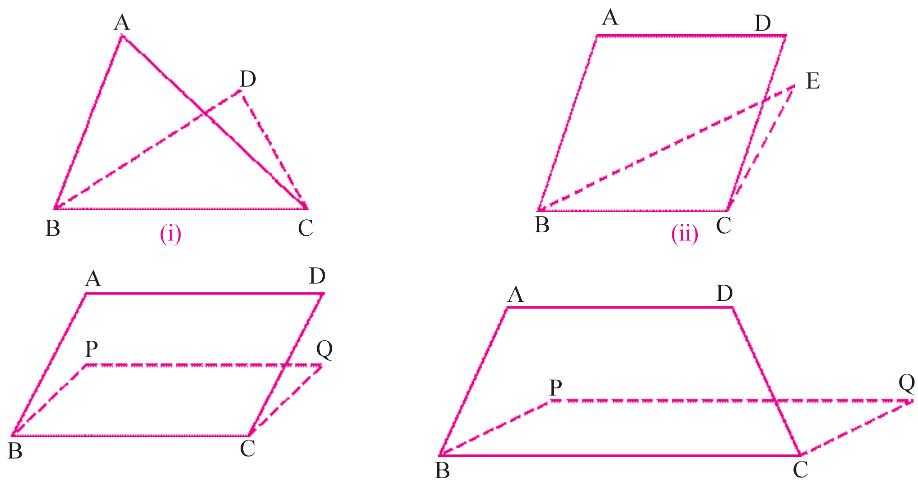

Figure 11.17

In figure 11.17 (i) ΔABC and ΔDBC have a common (same) base $\overline{BC}$. In figure 11.17 (ii) $\square^m$ ABCD and ΔEBC have the same base $\overline{BC}$. In figure 11.17 (iii) $\square^m$ ABCD and $\square^m$ PBCQ have the same base $\overline{BC}$. In figure 11.17 (iv) trapezium ABCD with $\overline{AD} \parallel \overline{BC}$ and $\square^m$ PBCQ have the same base $\overline{BC}$.

Now look at the following figure 11.18 :

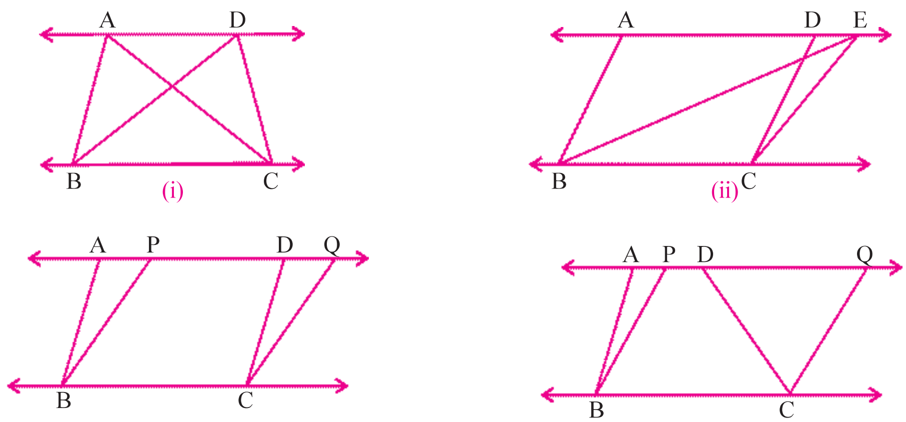

Figure 11.18

In figure 11.18(i), we observe that ΔABC and ΔDBC are on same base $\overline{BC}$ and lie between two parallel lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{AD}$. Vertices A and D of ΔABC and of ΔDBC are on the same side of the line containing the base $\overline{BC}$.

In figure 11.18(ii), $\square^m$ ABCD and ΔEBC are on same base $\overline{BC}$ and lie between two parallel lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{AD}$. Vertices A and D of $\square^m$ ABCD and vertex E of ΔEBC are on same line $\overleftrightarrow{AE}$ and are on the same side of the line containing the base $\overline{BC}$.

In figure 11.18(iii), $\square^m$ ABCD and $\square^m$ PBCQ are on same base $\overline{BC}$ and lie between two parallel lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{AQ}$. Vertices A and D of $\square^m$ ABCD and vertices P and Q of $\square^m$ PBCQ are on same line $\overleftrightarrow{AQ}$ and are on the same side of the line containing the base $\overline{BC}$.

In figure 11.18(iv), trapezium ABCD and $\square^m$ PBCQ are on same base $\overline{BC}$ and lie between two parallel lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{AQ}$. Vertices A and D of trapezium ABCD and vertices A and Q of $\square^m$ ABCQ are on same line $\overleftrightarrow{AQ}$ and are on the same side of the line containing the base $\overline{BC}$.

We observed that a triangle and a quadrilateral, two figures have same base and are between two parallel lines and the vertices (or vertex) lie on a line parallel to the base. What can we say about the areas of such figures ?

We shall study some theorems regarding the areas of figures lying between a pair of parallel lines.

Theorem 11.1 : Parallelograms having the same base and lying between a pair of parallel lines, have the same area.

Data : $\square^m$ ABCD and $\square^m$ ABEF have the same base $\overline{AB}$ and lie between a pair of parallel lines l and m.

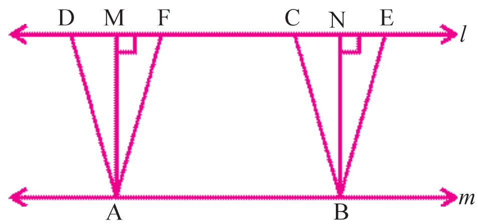

Figure 11.19

To prove : ABCD = ABEF

Proof : Let M and N be the feet of the perpendiculars from A and B respectively to l. We have $l \parallel m$.

AM and BN are perpendicular distances between l and m.

$\therefore$ AM = BN

Now ABCD = AM $\times$ CD

$\therefore$ ABCD = BN $\times$ CD **(AM = BN and AB = CD)**

Also ABEF = BN $\times$ EF = BN $\times$ CD **(EF = AB)**

$\therefore$ ABCD = ABEF

11.12 Triangles on the same Base and between a pair of Parallel Lines

Δ ABC and Δ PBC are on same base $\overline{BC}$ and lie between two parallel lines l and m.

Let us draw $\overline{CD} \parallel \overline{AB}$ and let D $\in l$. Let $\overline{CQ} \parallel \overline{BP}$ and let Q $\in l$.

$\therefore$ We get $\square^m$ ABCD and $\square^m$ PBCQ.

$\overline{AC}$ is diagonal of $\square^m$ ABCD. $\overline{PC}$ is diagonal of $\square^m$ PBCQ.

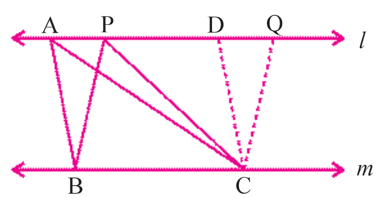

Figure 11.20

$\therefore$ ABC = $\frac{1}{2}$ ABCD and

PBC = $\frac{1}{2}$ PBCQ.

But ABCD = PBCQ

(on same base $\overline{BC}$ and between the pair of parallel lines l and m)

$\therefore$ $\frac{1}{2}$ ABCD = $\frac{1}{2}$ PBCQ

$\therefore$ ABC = PBC

We accept the theorem given below without proof.

Theorem 11.2 : Two triangles on the same base (or congruent bases) and lying between pair of parallel lines have same area.

The converse of theorem is also true and we accept the theorem without proof.

Theorem 11.3 : Two triangles having the same base (or congruent bases) and having their vertices (other than the base vertices) in the same half plane of the line containing the base (or congruent bases) and having equal areas lie between a pair of parallel lines.

Example 5 : Show that a median of a triangle divides a triangular region into two triangular regions with equal areas.

Solution : In Δ ABC, $\overline{AD}$ is the median.

$\therefore$ BD = DC

Let $\overline{AM} \perp \overline{BC}$

ABC $= \frac{1}{2}$ AM $\times$ BD

ADC $= \frac{1}{2}$ AM $\times$ CD

but BD = DC

$\therefore$ ABD = ADC

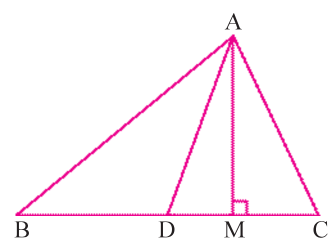

Figure 11.21

Example 6 : D, E and F are the midpoints of the sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively of ΔABC. Prove that $\square$BEFD, $\square$ECFD and $\square$EFAD have the same area.

Solution : In Δ ABC, D and F are the midpoints of the sides $\overline{AB}$ and $\overline{AC}$ respectively.

$\therefore$ DF $= \frac{1}{2}$ BC and $\overline{DF} \parallel \overline{BC}$

E is the midpoint of $\overline{BC}$.

$\therefore$ BE = EC $= \frac{1}{2}$BC = DF

$\therefore$ In $\square$BEFD, $\overline{BE} \cong \overline{DF}$ and $\overline{BE} \parallel \overline{DF}$ **(B – E – C)**

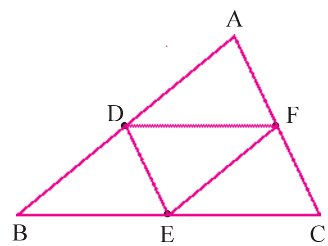

Figure 11.22

$\therefore$ $\square$BEFD is parallelogram.

Similarly, $\square$ECFD is also parallelogram.

Now $\square^m$ BEFD and $\square^m$ ECFD have the same base $\overline{FD}$ and lie between the pair of parallel lines $\overline{DF}$ and $\overline{BC}$.

$\therefore$ BEFD = ECFD

Similarly, it can be proved that

EFAD = ECFD

$\therefore$ BEFD = ECFD = EFAD

An Important Result : In a trapezium ABCD, $\overline{AB} \parallel \overline{DC}$. M is the foot of perpendicular from D to $\overline{AB}$ and A – M – B.

Then ABCD = $\frac{1}{2}$(AB + CD) × DM

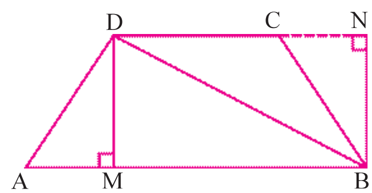

Figure 11.23

Solution : In trapezium ABCD, $\overline{AB} \parallel \overline{DC}$. M is the foot of the perpendicular from D to $\overline{AB}$ and A – M – B.

Let N be the foot of the perpendicular form B to $\overleftrightarrow{DC}$.

$\therefore$ DM and BN are perpendicular distances between parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$.

$\therefore$ DM = BN

Now, $\overline{DM}$ is the altitude of Δ ABD and $\overline{AB}$ is the corresponding base.

$\therefore$ ABD = $\frac{1}{2}$ AB × DM

Similarly, $\overline{BN}$ is the altitude and $\overline{CD}$ the corresponding base in Δ BCD.

$\therefore$ BCD = $\frac{1}{2}$ CD × BN

$\therefore$ BCD = $\frac{1}{2}$ CD × DM **(DM = BN)**

Now ABCD = ABD + BCD

$= \frac{1}{2}$ AB × DM + $\frac{1}{2}$ CD × DM

$= \frac{1}{2}$ (AB + CD) × DM

$\therefore$ ABCD $= \frac{1}{2}$ (AB + CD) × DM

Example 7 : If a triangle and a parallelogram are on the same base and lie between a pair of two parallel lines, then prove that the area of the triangle is equal to half the area of the parallelogram.

Solution : Let Δ PAB and $\square^m$ DABC have same base $\overline{AB}$ and lie between parallel lines $\overleftrightarrow{PC}$ and $\overleftrightarrow{AB}$.

Draw $\overline{QB} \parallel \overline{PA}$ and let $\overleftrightarrow{BQ}$ intersect $\overleftrightarrow{PC}$ at Q.

$\overline{PA} \parallel \overline{QB}$ and $\overline{AB} \parallel \overline{PQ}$

$\therefore$ $\square$PABQ is parallelogram.

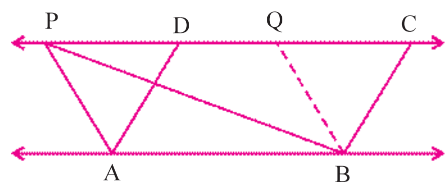

Figure 11.24

$\square^m$ ABCD and $\square^m$ ABQP are on the same base $\overline{AB}$ and lie between parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{PC}$.

$\therefore$ ABCD = ABQP **(i)**

In $\square^m$ ABQP, $\overline{PB}$ is a diagonal.

$\therefore$ PAB $= \frac{1}{2}$ ABQP

PAB $= \frac{1}{2}$ ABCD. **(from (i))**

EXERCISE 11.2

1. In a trapezium ABCD, $\overline{AD} \parallel \overline{BC}$ and M and N are the midpoints of $\overline{AB}$ and $\overline{CD}$ respectively. $\overline{AE} \perp \overline{BC}$ such that B–E–C. If BC = 16 *cm* and MN = 10 *cm* and AE = 6 *cm*, find ABCD.

2. In figure 11.25, $l \parallel m$. A, B, C, D, E and F are distinct points such that A, B $\in m$ and C, D, E, F $\in l$. The perpendicular distance between the lines l and m is 5 *cm* and AB = 10 *cm*. Answer the following :

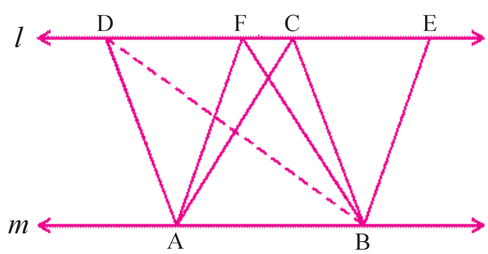

Figure 11.25

(1) Find the area of Δ ABD.

(2) Which other triangle has the same area as Δ ABD ? Why ?

(3) Find the area of $\square^m$ AFEB.

(4) Which other parallelogram has the same area as $\square^m$ AFEB ? Why ?

(5) Do Δ ADF and Δ BDF have the same area ? Why ?

(6) If DF = 3 *cm*, find the area of Δ ADF.

3. In Δ ABC, D is the midpoint of $\overline{BC}$ and E is the midpoint of $\overline{AD}$. Prove that BED $= \frac{1}{4}$ ABC.

4. Compute the area of the quadrilateral PQRS, where measures of sides are given in figure 11.26.

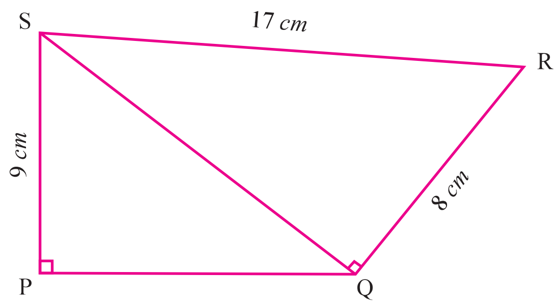

Figure 11.26

5. Compute the area of the trapezium ABCD using measures of sides given in figure 11.27.

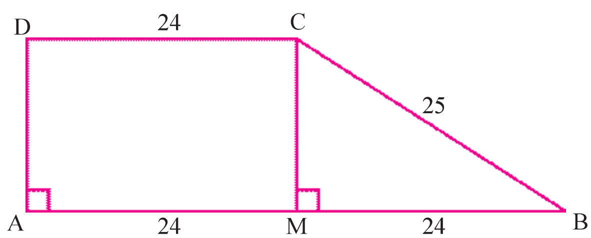

Figure 11.27

6. In the trapezium ABCD, AB = 14 *cm*, AD = BC = 10 *cm*, DC = x *cm* and distance between $\overline{AB}$ and $\overline{DC}$ is 8 *cm*. Find the value of x and area of the trapezium ABCD given in figure 11.28.

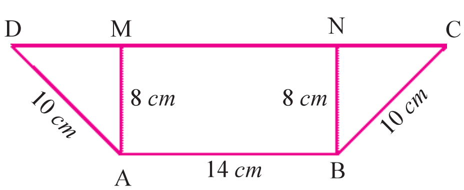

Figure 11.28

EXERCISE 11

1. If E, F, G and H are respectively the midpoints of the sides of a $\square^m$ PQRS, show that EFGH = $\frac{1}{2}$ (PQRS).

2. In figure 11.29, X is a point in the interior of a $\square^m$ PQRS. Show that,

(i) PXS + QXR = $\frac{1}{2}$ (PQRS)

(ii) PXQ + SXR = $\frac{1}{2}$ (PQRS)

(**Hint :** Draw a line through X parallel to $\overleftrightarrow{QR}$)

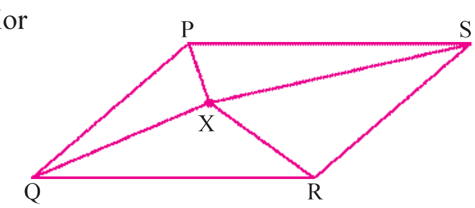

Figure 11.29

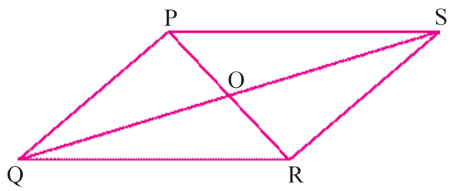

Figure 11.30

3. In figure 11.30, diagonals $\overline{PR}$ and $\overline{QS}$ intersect at O such that PO = OR. If SR = PQ, then show that (i) POQ = SOR (ii) PQR = SQR (iii) $\overleftrightarrow{PS} \parallel \overleftrightarrow{QR}$ and □ PQRS is parallelogram .

(**Hint :** Draw perpendicular to $\overline{QS}$ from P and R)

4. □^{m} PQRS and rectangle PQAB are on the same base $\overline{PQ}$ and have equal area. Show that the perimeter of the parallelogram is greater than that of the rectangle.

5. S is the midpoint of $\overline{QR}$ in Δ PQR and X is the midpoint of $\overline{QS}$. If Y is the midpoint of $\overline{PX}$, prove that QYX = $\frac{1}{8}$ (PQR)

6. Prove that the area of an equilateral triangle is equal to $\frac{\sqrt{3}}{4}l^2$, where l is the length of a side of the triangle.

7. A and B are any two points lying on the side $\overline{PS}$ and $\overline{PQ}$ respectively of a □^{m} PQRS. Show that AQR = BSR.

8. Δ ABC is equilateral triangle. If BC = 12 *cm*, find ABC

9. In ABC, P, Q, R are the midpoints of sides of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively. If ABC = 120 cm^2, find PQR, PQCR and PBCR.

10. Select proper option (a), (b), (c) or (d) and write in the box given on the right so that the statement becomes correct :

(1) In □^{m} ABCD, let $\overline{AM}$ be the altitude corresponding to the base $\overline{BC}$ and $\overline{CN}$ the altitude corresponding to the base $\overline{AB}$. If AB = 10 *cm*, AM = 6 *cm* and CN = 12 *cm*, then BC = ... *cm* □

(a) 20 (b) 10 (c) 12 (d) 5

(2) In □ ABCD, $\overline{AD} \parallel \overline{BC}$, $\overline{AM} \perp \overline{BC}$ such that B – M – C. If AD = 8 *cm*, BC = 12 *cm* and AM = 10 *cm*. ABCD = ... cm^2. □

(a) 100 (b) 50 (c) 200 (d) 400

(3) $\overline{AD}$ and $\overline{BE}$ are the altitudes of Δ ABC. If AD = 6 *cm*, BC = 16 *cm*, BE = 8 *cm*, then CA = ... *cm*. □

(a) 12 (b) 18 (c) 24 (d) 22

(4) $\overline{BE}$ and $\overline{CF}$ are the altitudes of Δ ABC. If BE = 10 *cm*, CA = 8 *cm*, AB = 16 *cm*, then CF = ... *cm*. □

(a) 2.5 (b) 5 (c) 10 (d) 6.4

(5) In $\square^m$ ABCD, $\overline{BC}$ is the base corresponding to the altitude $\overline{AM}$. If BC = 8 *cm* AM = 5 *cm*, then ABCD = ... cm^2. ☐

(a) 40 (b) 20 (c) 80 (d) 10

(6) In a $\square$ ABCD, $\overline{AB} \parallel \overline{CD}$, $\overline{DM}$ is the altitude on $\overline{AB}$. If AB = 15 *cm*, CD = 25 *cm* and DM = 10 *cm*, then ABCD = ... cm^2. ☐

(a) 400 (b) 250 (c) 100 (d) 200

(7) $\square$ ABCD is rhombus. If AC = 12 *cm* and BD = 15 *cm*, then the area of the rhombus ABCD = ... cm^2. ☐

(a) 90 (b) 180 (c) 45 (d) 360

(8) $\square$ ABCD is a rhombus If ABCD = 80 cm^2 and AC = 8 *cm*, then BD = ... *cm*. ☐

(a) 5 (b) 10 (c) 20 (d) 40

(9) If for $\square^m$ ABCD, ABCD = 48 cm^2, then ABC = ... cm^2. ☐

(a) 12 (b) 24 (c) 96 (d) 6

(10) In Δ ABC, P, Q and R are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively. If ABC = 60 cm^2, then PBCR = ... cm^2. ☐

(a) 15 (b) 30 (c) 45 (d) 75

*

Summary

In this chapter we have studied the following points :

1. Area of a figure is a number (in some units) associated with some part of the plane enclosed by that figure.
2. Two congruent figures have equal areas but the converse need not be true.
3. If a planer region formed by a figure T is made up of two non - overlaping planer regions formed by figures P and Q, then area of T = area of P + area of Q.
4. Area of a rectangle, area of a right triangle.
5. Area of a triangle is half the product of it base and the corresponding altitude.
6. Area of a parallelogram is product of its base and the corresponding altitude.
7. Parallelograms on a same base (or congruent bases) and lying between two parallel lines have equal area.
8. Parallelograms on the same base (or congruent bases) having equal areas lie between two parallel lines.
9. Triangles on the same base (or congruent bases) and lying between two parallel lines have equal area.
10. Triangles on the same base (or congruent bases) and having third vertex in the same semi plane of the line containing the base and having equal areas lie between the two parallel lines.
11. If a parallelogram and a triangle are on the same base and lie between a pair of parallel lines, then the area of the triangle is half the area of the parallelogram.
12. A median of a triangle divides it into two triangles of equal areas.

CHAPTER 12

CIRCLE

12.1 Introduction

Let us imagine about a routine scene of a village. A goat is tied up with a rope and the rope is fixed with a nail at some point on the ground. Now, think about the area that the goat can graze ! The boundary of that area and the fixed (nail) point gives us the idea of **a circle**. The length of the rope is **radius** and the nail where the rope is fixed is the **centre**.

We have already studied about a circle in earlier classes. Let us observe some circular objects in our neighbourhood. A circle is the edge of a wheel, edge of a button of a shirt, boundary of some coins, edge of full moon, etc.

Figure 12.1

12.2 Circle and its Related Terms

We can draw a circle by the use of a compass. Fix pointer at some fixed point O on a paper and fix the other end (where the pencil is inserted) at some distance and rotate this end through one revolution. The closed figure traced on the paper is a circle (figure 12.2). We have kept one point O fixed and that point is the **centre of the circle**. The circle is the arc traced by the

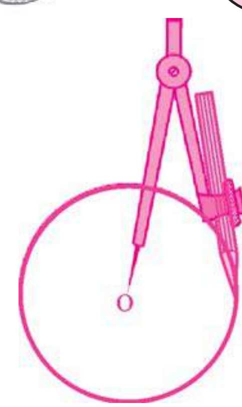

Figure 12.2

pencil. The distance of any boundary point P from the fixed point O is called radius of the circle. Now, we define a circle.

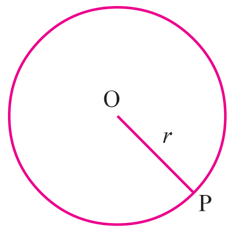

Figure 12.3

Circle : The set of points lying in a plane at a fixed positive distance from a fixed point in the plane is called a circle (figure 12.3).

If we denote the fixed point of the plane α, as O and fixed distance $r > 0$, then in the set form a circle can be defined as $\{P \mid OP = r, r > 0, P \in \alpha\}$.

Radius : The line-segment whose one end point is the centre and other end point is any of the points of the circle is called a radius of the circle. Its measure is also called radius and is denoted by *r*.

If O is the centre and r is the radius of a circle, then we denote the circle by $\odot(O, r)$.

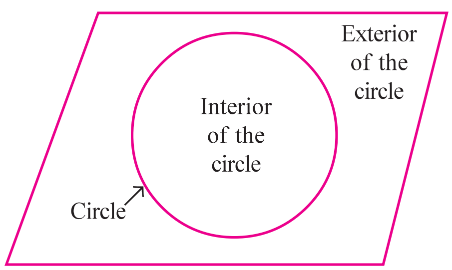

Figure 12.4

A circle divides plane into three parts,

(i) **Interior : The set of points whose distance from the centre of the circle is less than its radius is called the interior of the circle.**

(ii) **Circle : points on the circle.**

(iii) **Exterior : The set of points whose distance from the centre of the circle is greater than its radius is called the exterior of the circle.**

Circular region : Union of the set of the points of circle and its interior is called the circular region.

Chord : The line-segment both of whose end points are the elements of the circle is called a chord of the given circle. In figure 12.5, P, Q $\in$ $\odot(O, r)$. So $\overline{PQ}$ is a chord of $\odot(O, r)$.

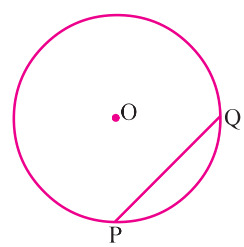

Figure 12.5

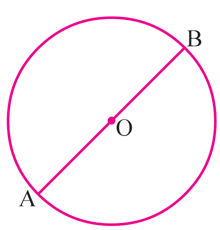

Figure 12.6

Diameter : If a chord of a circle passes through its centre, it is called a diameter of the circle (figure 12.6). $\overline{AB}$ is a diameter. **A diameter is the longest chord of the circle and has the length twice of its radius.** Length of the diameter is also called a diameter.

Arc : The set of points of a circle lying in each closed semi plane of a line passing through two distinct points of the circle is called an arc of the circle. The chord joining these two points is called the chord corresponding to the arc. The arc PQ, is denoted by $\overset{\frown}{PQ}$. (figure 12.7 and 12.8)

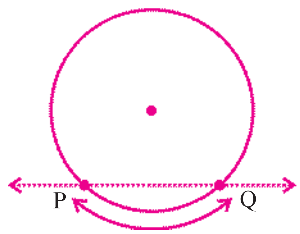

Figure 12.7

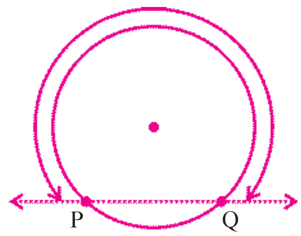

Figure 12.8

Minor arc : The set of points of a circle lying in the closed semi plane of the line containing a chord $\overline{PQ}$ and not containing the centre of the circle is called a minor arc of the circle (figure 12.7). We denote it by minor $\overset{\frown}{PQ}$.

Major arc : The set of points of a circle lying in the closed semi plane of the line containing a chord $\overline{PQ}$ and containing the centre of the circle is called a major arc (figure 12.8). $\overline{PQ}$ is not a diameter. We denote it by major $\overset{\frown}{PQ}$.

If a chord is a diameter of a circle, then arc corresponding to the chord is called a semi-circle arc.

We accept the following results about the length of an arc :

(i) If the measure of the angle subtended at the centre by minor $\overset{\frown}{AB}$ of $\odot(O, r)$ i.e. $m\angle AOB$ is α, then the length of minor $\overset{\frown}{AB}$ is $\frac{\pi r \alpha}{180}$.

(ii) The length of a semi circle arc of $\odot(O, r)$ is πr. we know 'length' of $\odot(O, r)$ i.e. its circumference is $2\pi r$.

(iii) If $\overline{AB}$ is the chord corresponding to major $\overset{\frown}{AB}$ of $\odot(O, r)$ and if $m\angle AOB = \alpha$, then the length of major $\overset{\frown}{AB}$ is $2\pi r - \frac{\pi r \alpha}{180}$.

Segment : The union of an arc and its corresponding chord of the circle is called a segment of the circle.

There are three types of segments : Minor segment, Major segment and Semi-circle segment.

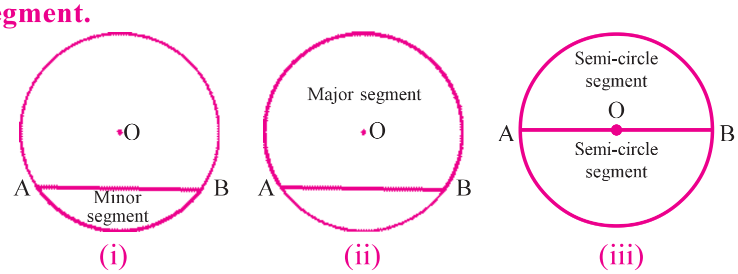

Figure 12.9

(i) **Minor segment : If an $\overset{\frown}{AB}$ is a minor arc, then $\overset{\frown}{AB} \cup \overline{AB}$ is called a minor segment** (figure 12.9 (i)).

(ii) **Major segment : If an $\overset{\frown}{AB}$ is a major arc, then $\overset{\frown}{AB} \cup \overline{AB}$ is called a major segment** (see figure 12.9 (ii)).

(iii) **Semi circle segment : If an $\overset{\frown}{AB}$ is a semi circle arc then $\overset{\frown}{AB} \cup \overline{AB}$ is called a semi-circle segment** (figure 12.9(iii)).

Sector : For the distinct points A and B of $\odot$(O, r), $\overset{\frown}{AB} \cup \overline{OA} \cup \overline{OB}$ is called a sector of the circle with centre O. As in case of a triangle, sector region OAB* is the corresponding region of sector OAB.

Minor sector, Major sector and Semi-circle sector.

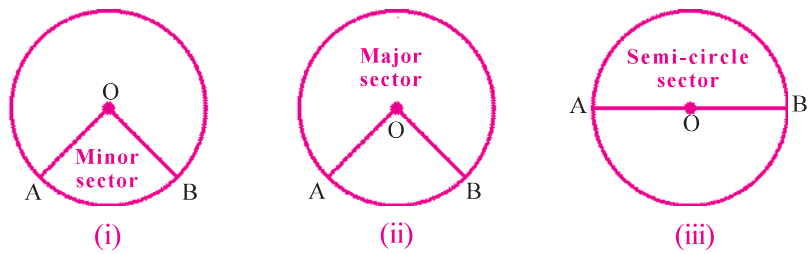

Figure 12.10

Congruent circles : Two or more than two circles having congruent radii and different centres are called congruent circles. (figure 12.11)

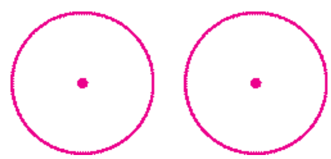

Figure 12.11

Concentric circles : If two or more than two circles in the same plane have the same centre and different radii, then they are called concentric circles. (figure 12.12)

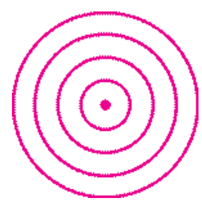

Figure 12.12

EXERCISE 12.1

1. Answer the following :

(1) If two circles having centres P and Q are concentric, then what can you say about P and Q ?

(2) If two circles having centres P and Q are congruent, then what can you say about their radii ?

(3) If P is in the interior and Q is in the exterior of the circle with centre O, which is larger between OP and OQ ?

2. State whether following statements are true or false. Give reasons for your answer.

(1) A line-segment joining the centre to any point of the circle is a diameter of the circle.

(2) An arc is a semi-circle arc, if its endpoints are the endpoints of a diameter.
(3) The set of points equidistant from a fixed point is called a circle.
(4) Union of two radii of a circle is a diameter of the circle.

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12.3 Angle Subtended by a Chord at a Point

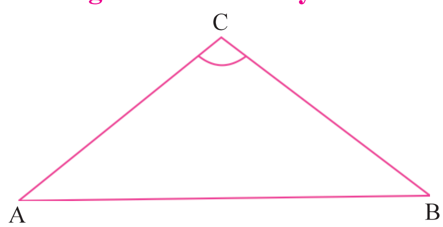
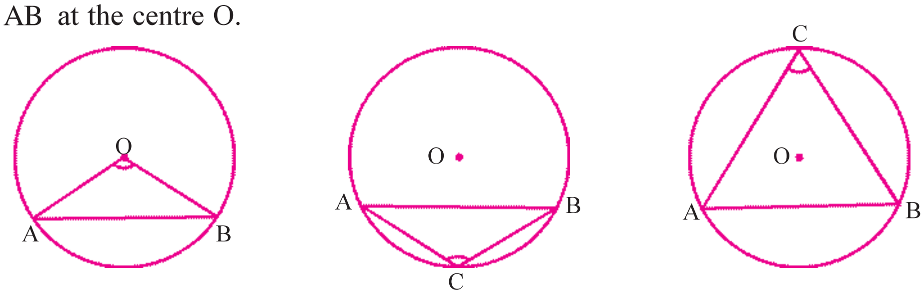


Figure 12.13

Angle subtended by a line segment : If the end points A and B of $\overline{AB}$ are joined to a third point C not lying on $\overleftrightarrow{AB}$, then $\angle ACB$ is called the angle subtended by, $\overline{AB}$ at C (figure 12.13).

The angle subtended by a chord (not a diameter) at the centre of the circle is called the angle subtended by the chord at the centre. If A and B lie on a circle (O, r) then $\angle AOB$ is called the angle subtended by chord $\overline{AB}$ at the centre O.



(i) (ii) (iii)

Figure 12.14

In figure 12.14 (i), $\angle AOB$ is the angle subtended by the chord $\overline{AB}$ at the centre O.

The angle subtended by a chord at any point of the arc is called the angle subtended by the chord on the arc.

In figure 12.14 (ii), $\angle ACB$ is the angle subtended by the chord $\overline{AB}$ on the minor $\widehat{AB}$.

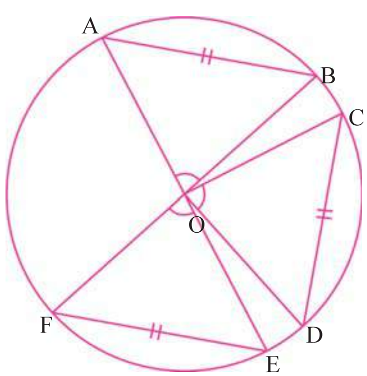


Figure 12.15

In figure 12.14 (iii), $\angle ACB$ is the angle subtended by the chord $\overline{AB}$ on major $\widehat{AB}$.

Activity : Draw a circle of desired radius on the plane paper.

Draw congruent chords in the circle. Measure angles subtended by them at the centre.

What can we say about the measures of such angles ? In fact, they are congruent angles. Let us prove this result as a theorem.

Theorem 12.1 : Congruent chords of a circle subtend congruent angles at the centre of the circle.

Data : Let O be the centre of the given circle and chords $\overline{AB} \cong \overline{CD}$.

To prove : $\angle AOB \cong \angle COD$

Proof : Consider the correspondence AOB $\leftrightarrow$ COD, for ΔAOB and ΔCOD,

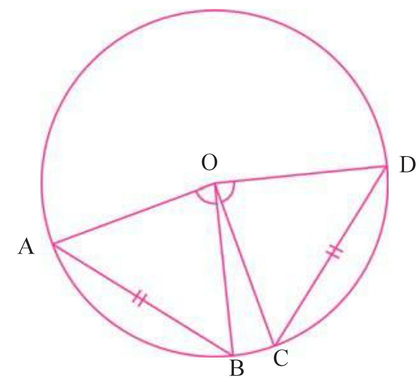


Figure 12.16

$\overline{AB} \cong \overline{CD}$ **(given)**

$\overline{OA} \cong \overline{OC}$ **(radii of the same circle)**

$\overline{OB} \cong \overline{OD}$ **(radii of the same circle)**

$\therefore$ The correspondence AOB $\leftrightarrow$ COD is a congruence. **(SSS)**

$\therefore \angle AOB \cong \angle COD$.

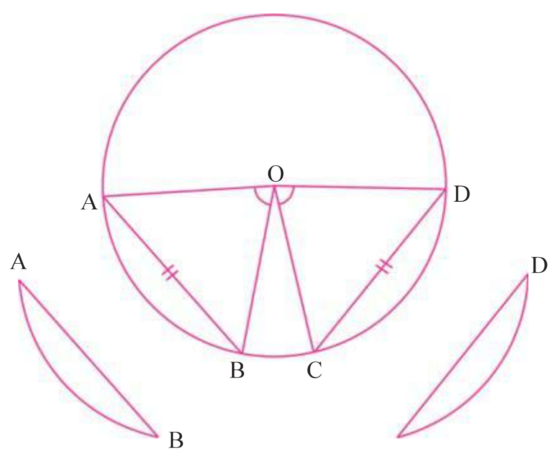


Figure 12.17

Activity : Draw a circle with centre O. Draw congruent angles $\angle AOB$ and $\angle COD$, where $\overline{AB}$ and $\overline{CD}$ are chords.

Now, cut regions enclosed by minor segments formed by $\overline{AB}$ and $\overline{CD}$. Place one segment on the other segment. Observe the result. They cover each other completely. So, the length of the chords have to be the same. This leads to the next theorem; the converse of theorem 12.1.

Theorem 12.2 : If the angles subtended by two chords at the centre of a circle are congruent, then the chords are congruent.

We accept this theorem without proof.

We note that theorms 12.1 and 12.2 are true for congruent circles also.

EXERCISE 12.2

1. Study figure 12.18 and answer the following questions :

(1) If $m\angle OCD=25$, then find $m\angle COD$.

(2) If the diameter of the circle is 10 *cm* and $m\angle COD=90$, then find CD.

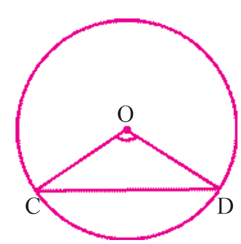


Figure 12.18

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12.4 Perpendicular drawn from the Centre to a Chord

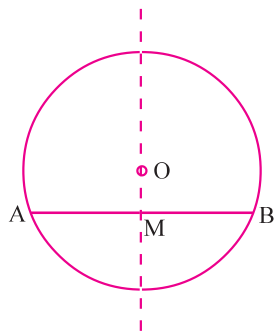

Figure 12.19

Activity : Draw a circle with centre O. Draw a chord $\overline{AB}$. Now fold the paper along the line through the centre O in such way that the portions of $\overline{AB}$ coincide with each other (i.e. point B falls on the point A). Let us cut $\overline{AB}$ at point M along the crease.

Observe that B coincides with A. What can you say about M ? Measure AM and BM. We can see that AM = MB. So M is the midpoint of $\overline{AB}$. This fact leads to the following theorem.

Theorem 12.3 : If a perpendicular is drawn to a chord from the centre of a circle, then it bisects the chord.

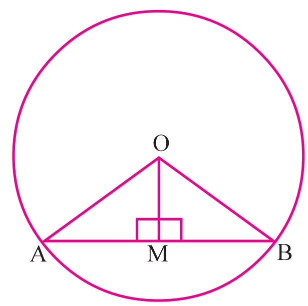

Figure 12.20

Data : Let O be the centre of the given circle. $\overline{AB}$ is a chord and $\overline{OM} \perp \overline{AB}$ and $M \in \overline{AB}$.

To prove : AM = BM.

Proof : Consider corrsepondence AOM $\leftrightarrow$ BOM for ΔAOM and ΔBOM.

$\overline{OA} \cong \overline{OB}$ **(radii)**

$\overline{OM} \cong \overline{OM}$ **(common segment)**

$\angle AMO \cong \angle BMO$ **(right angles)**

$\therefore$ The correspondence AOM $\leftrightarrow$ BOM is congruence. **(RHS theorem)**

$\therefore$ $\overline{AM} \cong \overline{BM}$

$\therefore$ AM = BM

$\therefore$ M is the midpoint of $\overline{AB}$.

$\therefore$ $\overline{OM}$ bisects chord $\overline{AB}$.

The converse of the theorem 12.3 is the theorem 12.4.

Theorem 12.4 : If a line from the centre of a circle bisects the chord, then it is perpendicular to the chord.

Data : Let O be the centre of the circle and l be the line through O bisecting the chord $\overline{AB}$ i.e. AM = BM.

To prove : $l \perp \overline{AB}$

Proof : In consider correspondence AOM $\leftrightarrow$ BOM for Δ AOM and Δ BOM.

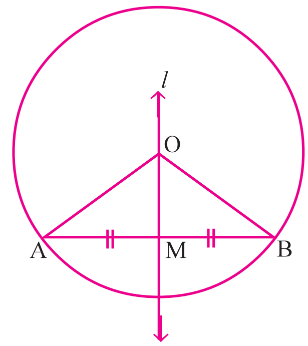

Figure 12.21

$\overline{AO} \cong \overline{BO}$ **(radii)**

$\overline{AM} \cong \overline{BM}$ **(given)**

$\overline{OM} \cong \overline{OM}$ **(common)**

The correspondence AOM $\leftrightarrow$ BOM is a congruence. **(SSS rule)**

$\therefore \angle AMO \cong \angle BMO$

But $m\angle AMO + m\angle BMO = 180$ as $\angle AMO$ and $\angle BMO$ form a linear pair.

$\therefore m\angle AMO = m\angle BMO = 90$.

$\therefore \overline{OM} \perp \overline{AB}$

$\therefore l \perp \overline{AB}$

12.5 Circle Through Three Distinct Points

We know that two distinct points are sufficient to determine unique line. A question arises that, how many points are sufficient to determine a unique circle ?

If one point is given, then how many circles can be drawn through this point ? Obviously, infinitely many circles can be drawn through a given point A, (see figure 12.22).

Now if two distinct points are given, then how many circles can be drawn passing through both the points ? Here also infinitely many circles can be drawn through the given points A and B, (see figure 12.23). Take two distinct points A and B and draw the perpendicular bisector l of $\overline{AB}$. Now the points on l are equidistant from A and B. So taking distinct points on l as the centres and distances of them from A or B as radii we can draw infinitely many circles passing through A and B (see figure 12.24).

Considering above fact if one point A is given, then taking B anywhere in the same plane, we can draw infinitely many circles passing through A.

If we take three distinct points, then we have to think about two cases.

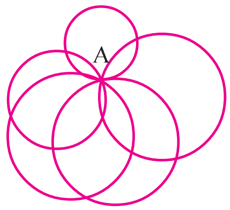

Figure 12.22

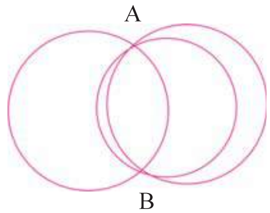

Figure 12.23

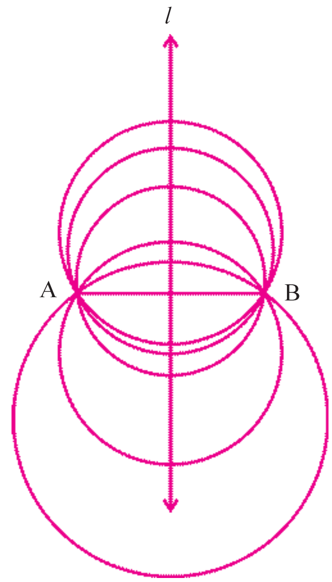

Figure 12.24

(i) collinear points and

(ii) non-collinear points.

If the points are collinear, the circle will not pass through all the three points. It will pass through two points and the remaining point lies in the interior or the exterior of the circle (figure 12.25 and 12.26).

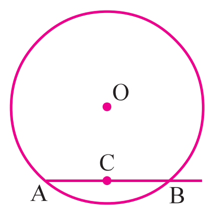

Figure 12.25

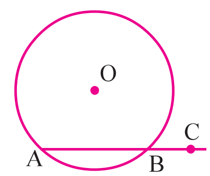

Figure 12.26

Now we take three distinct non-collinear points and we will try to draw a circle passing through them.

Let P, Q, R be three non-collinear points. To get a circle through P, Q, R, let us think in this way. Obviously, $\overline{PQ}$ and $\overline{QR}$ are going to be chords of the assumed circle. As we have learnt that the perpendicular bisector of a chord passes through the centre of the circle, perpendicular bisectors of $\overline{PQ}$ and $\overline{QR}$ both must pass through the centre of that assumed circle. Hence, the point of intersection of perpendicular bisectors of $\overline{PQ}$ and $\overline{QR}$ must be the centre of that assumed circle.

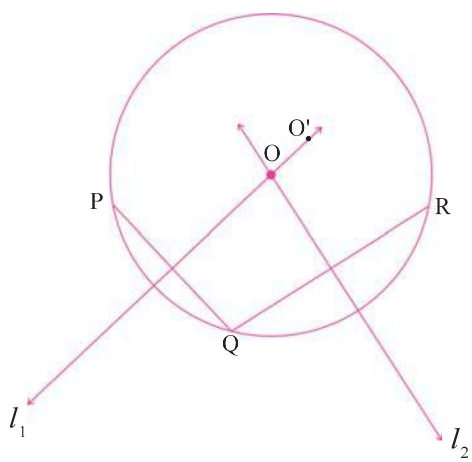

Figure 12.27

Draw perpendicular bisectors l_1 and l_2 of $\overline{PQ}$ and $\overline{QR}$ respectively. They intersect at a point say O. (figure 12.27). Here OP = OQ = OR.

i.e. O is equidistant from P, Q, R.

Now draw a circle with center O and radius OP. The circle passes through all the points P, Q and R.

Now take $O' \in l_1$, $O' \neq O$. Can we draw another circle passing through all the three points P, Q and R ? Obviously, our answer is no. Here O' is on the perpendicular bisector of $\overline{PQ}$ but not on the perpendicular bisector of $\overline{QR}$. So O' is equidistant from P and Q and so our circle, will pass through P and Q while $O'R \neq O'P$ (or $\neq O'Q$), so it will not pass through R. Thus, we observed that one and only one (unique) circle passes through three distinct non-collinear points.

The above discussion leads us to the following theorem. We accept it without proof.

Theorem 12.5 : There is a unique circle passing through three distinct non-collinear points.

A triangle has three vertices and they are non-collinear points, so from the above theorem we have a unique circle passing through the vertices of a triangle.

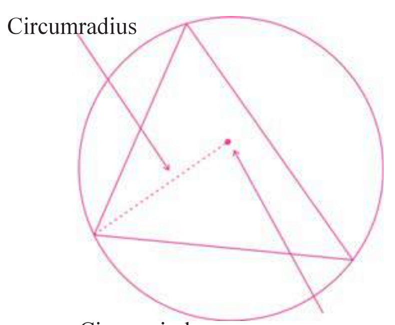

Figure 12.28

Circumcircle : A circle passing through the vertices of a triangle is called circumcircle of the triangle.

Circumcentre : The centre of the circumcircle of a triangle is called the circumcentre of the triangle.

Circumradius : The radius of the circumcircle of a triangle is called the circumradius of the triangle. It is usually denoted by R.

Example 1 : Draw the circle whose arc is given.

Solution : $\overset{\frown}{AB}$ is given. Let $C \in \overset{\frown}{AB}$. Join $\overline{AC}$ and $\overline{BC}$, Draw perpendicular bisectors of $\overline{AC}$ and $\overline{BC}$. They intersect at O.

Draw a circle with center O and radius OA. $\overset{\frown}{AB}$ is an arc of this Circle.

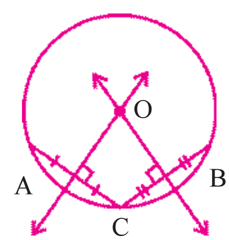

Figure 12.29

EXERCISE 12.3

1. Discuss the possible number of points of intersection of two distinct circles.
2. Explain how to find the centre of the circle of figure 12.30.

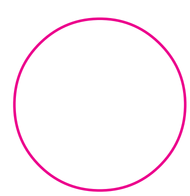
Figure 12.30

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12.6 Congruent Chords and their Distances from the Centre

Now we will make an observation about the distance of congruent chords from the centre of a circle.

Activity : Draw a circle with centre O and having arbitrary radius. Draw two congruent chords $\overline{AB}$ and $\overline{CD}$. Also draw $\overline{OM}$, $\overline{ON}$ perpendiculars to $\overline{AB}$ and $\overline{CD}$ respectively (figure 12.31).

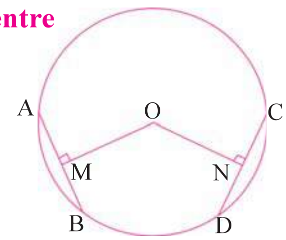


Figure 12.31

Now fold the figure in such a way that O will be on the crease, and C coincides with A, and D coincides with B. Now, where does N coincide ? Obviously, N coincides with M, i.e. OM = ON.

This activity leads us to the following theorem, which we accept without giving proof.

Theorem 12.6 : Congruent chords of a circle (or congruent chords of congruent circles) are equidistant from the centre of the of the circle (or centres).

Converse of this theorem is also true; we will do one activity to understand it.

Activity : Draw a circle with centre O. Draw two congruent segments $\overline{OM}$ and $\overline{ON}$ inside the circle.

Draw chords $\overline{AB}$ and $\overline{CD}$ perpendicular to $\overline{OM}$ and $\overline{ON}$ respectively (figure 12.31). Measure $\overline{AB}$ and $\overline{CD}$. We will observe that they are congruent.

Now we will state the converse of theorem 12.6, which we will accept without giving proof.

Theorem 12.7 : Chords equidistant from the centre of a circle (or centres of congruent circles) are congruent.

Example 2 : If two intersecting chords of a circle make congruent angles with the diameter passing through their point of intersection, then prove that chords are congruent.

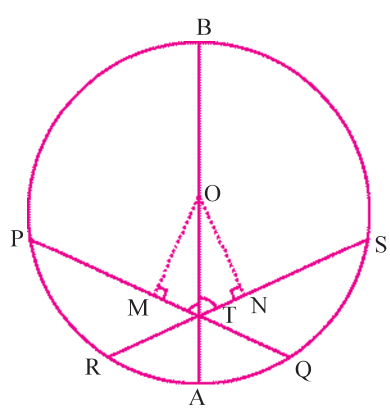


Figure 12.32

Solution : Take chords $\overline{PQ}$ and $\overline{RS}$ of a circle with centre O. Let $\overline{AB}$ be the diameter passing through T, the point of intersection of the given chords. Draw $\overline{OM}$ and $\overline{ON}$ perpendicular to $\overline{PQ}$ and $\overline{RS}$ respectively. We are given that

$\angle PTB \cong \angle STB$,

i.e. $\angle MTO \cong \angle NTO$

$(\overrightarrow{TP} = \overrightarrow{TM}$ and $\overrightarrow{TB} = \overrightarrow{TO})$ **(i)**

Now, consider the correspondance $MTO \leftrightarrow NTO$ for ΔMTO and ΔNTO.

$\angle OMT \cong \angle ONT$ **(right angles)**

$\angle MTO \cong \angle NTO$ **(given)**

$\overline{TO} \cong \overline{TO}$

$\therefore$ The correspondence MTO $\leftrightarrow$ NTO is a congruence. **(AAS)**

$\therefore$ $\overline{OM} \cong \overline{ON}$

$\therefore$ OM = ON

$\therefore$ $\overline{PQ} \cong \overline{RS}$

Example 3 : Find the length of the chord of $\odot$(O, 13) at distance 5 from the centre.

Solution : Let $\overline{OM}$ be perpendicular from centre O to chord $\overline{AB}$. M is the foot of perpendicular. Hence M is the midpoint of $\overline{AB}$.

OA = 13 and OM = 5 > 0. Hence O $\neq$ M,

for Δ OAM,

$\therefore$ $OA^2 = OM^2 + AM^2$

$\therefore$ $169 = 25 + AM^2$

$\therefore$ $AM^2 = 144$

$\therefore$ AM = 12

Also, $AM = MB = \frac{1}{2}AB$

$\therefore$ AB = 2AM = 24

$\therefore$ The length of the chord $\overline{AB}$ is 24.

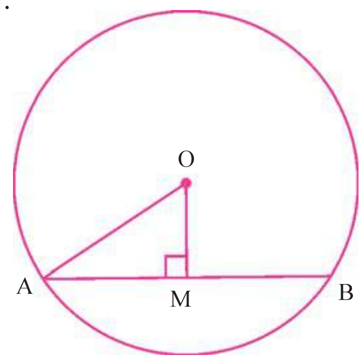


Figure 12.33

Example 4 : Lengths of two parallel chords of $\odot$(O,13) are 24 and 10. According as these chords are in different semi-planes or same semi-plane of the line containing the diameter parallel to these chords, find the distance between them.

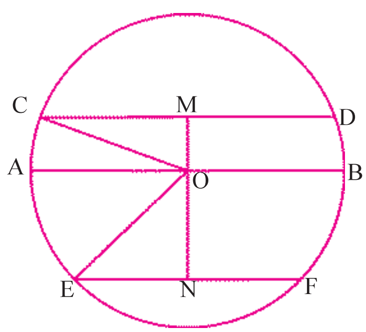


Figure 12.34

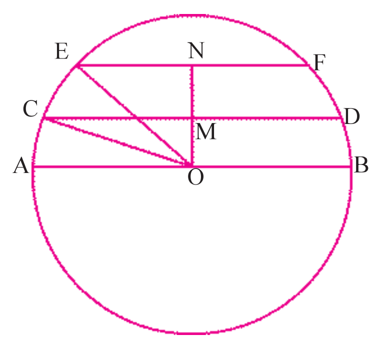


Figure 12.35

Solution : Let $\overline{CD}$ and $\overline{EF}$ be parallel chords. $\overline{AB}$ is the diameter parallel to them. CD = 24, EF = 10.

Perpendicular from O to $\overline{CD}$ is also perpendicular to $\overline{EF}$ as $\overline{CD} \parallel \overline{EF}$.

Let M and N be respectively the feet of perpendiculars from O to $\overline{CD}$ and $\overline{EF}$.

M is the midpoint of $\overline{CD}$ and N is the midpoint of $\overline{EF}$.

$\therefore$ CM = $\frac{1}{2}$ CD = 12, EN = $\frac{1}{2}$ EF = 5. Also radius r = 13.

For Δ OCM, $OC^2 = OM^2 + CM^2$

$\therefore$ $OM^2 = OC^2 - CM^2 = 169 - 144$

$\therefore$ $OM^2 = 25$

$\therefore$ OM = 5

Similarly, from Δ EON,

$\therefore$ $169 = 25 + ON^2$

$\therefore$ $ON^2 = 144$

$\therefore$ ON = 12

Now according to figure 12.34, $\overline{CD}$ and $\overline{EF}$ are on opposite sides of diameter $\overline{AB}$ and hence M–O–N.

$\therefore$ MN = OM + ON = 5 + 12 = 17

And according to figure 12.35, both the chords are on the same side of diameter $\overline{AB}$ and hence N - M - O. **(CD > EF)**

$\therefore$ OM + MN = ON

$\therefore$ 5 + MN = 12

$\therefore$ MN = 7

$\therefore$ If $\overline{CD}$ and $\overline{EF}$ are in different semi-planes of diameter $\overline{AB}$, then MN = 17 and if they are in the same semi-plane of diameter $\overline{AB}$, then MN = 7.

EXERCISE 12.4

1. Two congruent chords $\overline{AB}$ and $\overline{CD}$ which are not diameters, intersect at right angle in P. O is the centre of the circle. If M and N are the midpoints of $\overline{AB}$ and $\overline{CD}$ respectively, then prove that $\square$ OMPN is a square.

2. $\overline{AB}$ and $\overline{AC}$ are congruent chords of a circle with centre O. Feet of perpendiculars from O to $\overline{AB}$ and $\overline{AC}$ are D and E respectively. Prove Δ ADE is an isosceles triangle.

3. $\overline{AB}$ and $\overline{CD}$ are chords of a circle with radius r. AB = 2CD and the perpendicular distance of $\overline{CD}$ from the centre is twice perpendicular distance of $\overline{AB}$ from the centre. Prove that r = $\frac{\sqrt{5}}{2}$ CD.

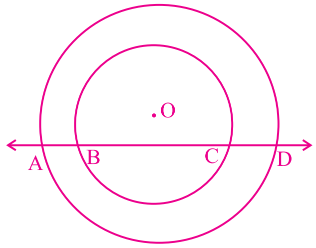


Figure 12.36

4. A line intersects two concentric circles at A, B, C and D. O is the centre, prove that $\overline{AB} \cong \overline{CD}$ (see figure 12.36).

5. If parallel chords $\overline{AB}$ and $\overline{CD}$ are in the same half-plane of a line containing a diameter parallel to them and AB = 8, CD = 6 and perpendicular distance between them is 1. Find the length of the diameter of the circle.

*

12.7 Angle Subtended by an Arc of a Circle

A chord other than diameter of a circle divides the circle into two subsets namely minor arc and major arc. **If chords of the same circle are congruent, then their coresponding arcs are also congruent.** (Here we will consider minor arc only).

Activity : Draw a circle with centre O on a piece of a paper. Draw two congruent chords $\overline{PQ}$ and $\overline{RS}$. Cut minor $\overset{\frown}{PQ}$ and place it on the minor $\overset{\frown}{RS}$. What do you observe ? $\overset{\frown}{PQ}$ will be exactly cover $\overset{\frown}{RS}$. This shows that $\overset{\frown}{PQ}$ and $\overset{\frown}{RS}$ are also congruent. This leads to the following result.

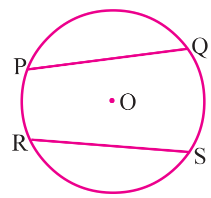


Figure 12.37

If two chords of a circle are congruent, then their corresponding arcs are also congruent and conversely, if two arcs of a circle are congruent then their corresponding chords are congruent.

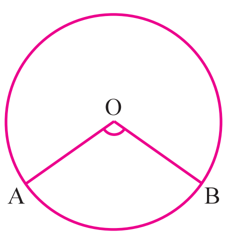


Figure 12.38

We define the angle subtended by an arc of a circle at the centre as the angle subtended by the corresponding chord of the arc at the centre. Here in figure 12.38, the angle subtended by the minor $\overset{\frown}{AB}$ is $\angle AOB$. In the same way, **we define the angle subtended by an arc at any point on the circle as the angle subtended by the corresponding chord of the arc at that point.**

From the property, **congruent chords of a circle subtend congruent angles at the centre, we can state that the congruent arcs also subtend congruent angles at the centre.**

Theorem 12.8 : The measure of the angle subtended by a minor arc of a circle at the centre is twice the measure of the angle subtended by the arc at any point on the remaining part of the circle.

Data : Minor $\overset{\frown}{AB}$ subtends $\angle AOB$ at the centre O of a circle and subtends $\angle APB$ at the remaining part of the circle.

To prove : $m\angle AOB = 2\ m\angle APB$

Proof : Select a point C on $\overrightarrow{PO}$, which is not on $\overline{PO}$. We consider three alternatives :

(i) O is in the interior of $\angle APB$.

(ii) O is in the exterior of $\angle APB$

(iii) O is on $\angle APB$.

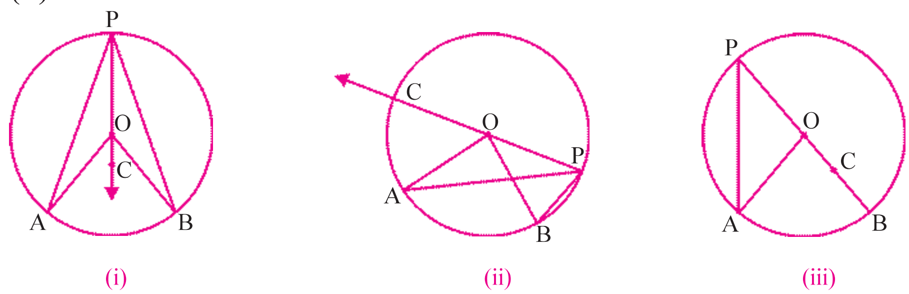


Figure 12.39

Let us consider alternatives (i) and (ii) to begin with.

For Δ AOP, $\angle AOC$ is an exterior angle.

$m\angle AOC = m\angle OPA + m\angle OAP$

But OA = OP,

$\therefore m\angle OPA = m\angle OAP$

$\therefore m\angle AOC = 2m\angle OPA$

Similarly, from consideration of Δ OPB, $m\ \angle BOC = 2m\angle OPB$.

According to alternative (i) (figure 12.39 (i)). O is in the interior of $\angle APB$ and C is also in the interior of $\angle AOB$.

$\therefore m\angle AOB = m\angle AOC + m\angle BOC$ **(C is in the interior of $\angle AOB$.)**

$= 2m\angle OPA + 2m\angle OPB$

$= 2\ (m\angle OPA + m\angle OPB)$

$= 2m\ \angle APB$ **(O is in the interior of $\angle APB$.)**

Similarly, if we consider alternative (ii) (see figure 12.39 (ii)), A is in the interior of $\angle BOC$ and $\angle OPB$.

$\therefore\ m\angle BOC = m\angle AOB + m\angle AOC$

$\therefore\ m\angle AOB = m\angle BOC - m\angle AOC$

$= 2m\angle OPB - 2m\ \angle OPA$

$= 2\ (m\angle OPB - m\angle OPA)$

Now A is an interior point of $\angle OPB$.

$m\angle OPA + m\angle APB = m\angle OPB$

$\therefore m\angle APB = m\angle OPB - m\angle OPA$

$\therefore m\angle AOB = 2m\angle APB$

As in alternative (iii) (see figure 12.39 (iii)). O is on an arm of $\angle APB$.

$\therefore m\angle AOB = m\angle OPA + m\angle OAP.$

$= 2m\angle APB.$

Hence in all the alternatives, $m\angle AOB = 2m\angle APB$.

If $\overline{AB}$ is a diameter and P is a point on semi circle $\overset{\frown}{AB}$, other than A or B, then $\angle APB$ is called an angle inscribed in semi-circle.

Corrollary : An angle inscribed in a semi-circle is a right angle.

Try to proove it !

Theorem 12.9 : Angles in the same segment of a circle are congruent. We will accept this theorem without proof.

Theorem 12.10 : If a line segment joining two distinct points A and B subtends congruent angles at two other points in the same semi plane of the line containing the line-segment, then all the four points lie on a circle whoes chord is $\overline{AB}$. (i.e. those four points are concyclic.)

Data : C and D are in the same semi plane of $\overleftrightarrow{AB}$ and $\angle ACB \cong \angle ADB$.

To prove : A, B, C, D lie on a circle or A, B, C, D are concyclic.

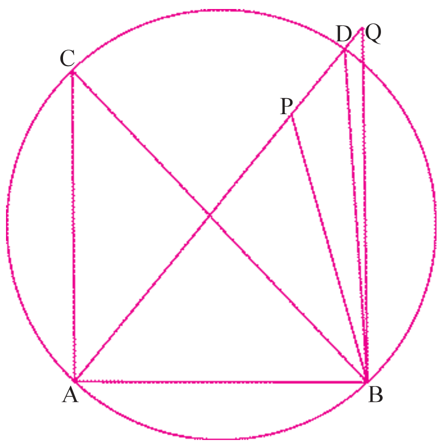


Figure 12.40

Proof : As A, B, C are non-collinear, there is a unique circle passing through A, B, C. This circle may pass or may not pass through D.

If the circle passes through D, then nothing remains to prove.

If the circle does not pass through D, draw $\overrightarrow{AD}$ such that circle intersects $\overrightarrow{AD}$ at P or Q. ($Q \in \overrightarrow{AD}$, $Q \notin \overline{AD}$) (figure 12.40)

Also $\angle ACB \cong \angle ADB$. **(given)**

$\therefore \angle ACB \cong \angle APB$ **(angle in the same segment of a circle)**

So $\angle APB \cong \angle ADB$.

$\therefore$ P = D. **(P $\in \overrightarrow{AD}$)**

Similarly we can prove that Q = D.

$\therefore$ D is on the circle.

$\therefore$ A, B, C, D are concyclic.

12.8 Cyclic Quadrilateral

If all the vertices of a quadrilateral lie on a circle, then that quadrilateral is called a cyclic quadrilateral.

Draw several circles of different radii and inscribe quadrilateral PQRS in each circle. Measuring the angles of the quadrilateral, can we observe some relation in their measures ? We can see that sum of the measures of opposite angles is 180. i.e. opposite angles are supplementary. This result is reflected in the next theorem which we accept without proof.

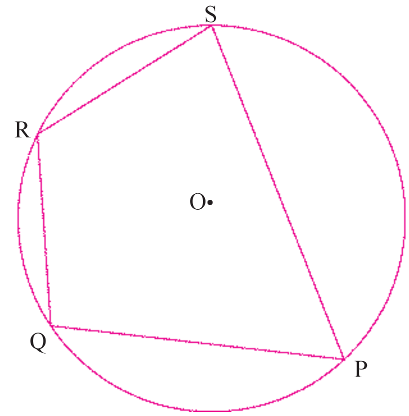


Figure 12.41

Theorem 12.11 : Opposite angles of a cyclic quadrilateral are supplementary.

The converse of this theorem is also true.

Theorem 12.12 : If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

We will accept above theorem also without proof.

Example 5 : If the non-parallel sides of a trapezium are congruent, then prove that the trapezium is cyclic.

Solution : In trapezium ABCD, $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \cong \overline{BC}$, AB > DC.

Draw $\overline{DM} \perp \overline{AB}$ and $\overline{CN} \perp \overline{AB}$ and M $\in \overline{AB}$, N $\in \overline{AB}$.

Consider the correspondence AMD $\leftrightarrow$ BNC for for Δ AMD and Δ BNC.

$\overline{AD} \cong \overline{BC}$ **(given)**

$\angle AMD \cong \angle BNC$ **(right angles)**

$\overline{DM} \cong \overline{CN}$ **($\overline{AB} \parallel \overline{CD}$)**

$\therefore$ The correspondence AMD $\leftrightarrow$ BNC is a congruence. **(RHS)**

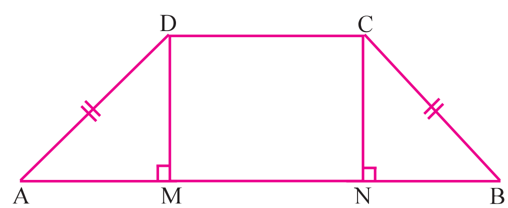


Figure 12.42

$\therefore$ $\angle MAD \cong \angle NBC$

$\angle DCB$ and $\angle ABC$ are supplementary.

(interior angles on the same side of the transversal)

$\therefore$ $\angle DCB$ and $\angle NBC$ are supplementary. **($N \in \overrightarrow{BA}$)**

$\therefore$ $\angle DCB$ and $\angle BAD$ are supplementary. **($\angle BAD = \angle MAD$ as $\overrightarrow{AB} = \overrightarrow{AM}$)**

Similarly, $\angle ADC$ and $\angle ABC$ are supplementary.

$\therefore$ The trapezium ABCD is cyclic.

Example 6 : $\overline{AC}$ and $\overline{BD}$ are different diameters of a circle. Prove $\square$ ABCD is a rectangle.

Solution : Diagonals $\overline{AC}$ and $\overline{BD}$ are different diameters of a circle.

$\angle ABC$ and $\angle ADC$ are inscribed in a semi-circle arc whose diameter is $\overline{AC}$.

$\therefore$ $m\angle ABC = m\angle ADC = 90$

Similarly $m\ \angle BAD = m\angle BCD = 90$

$\therefore$ $\square$ ABCD is a rectangle.

(**Note :** Diagonals of $\square$ABCD bisect each other and are congruent. Hence $\square$ ABCD is a rectangle.)

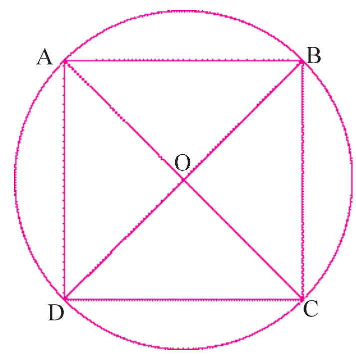


Figure 12.43

Example 7 : In figure 12.44, $\overline{AB}$ is a diameter. $m\angle PAB = 50$. Find $m\angle AQP$.

Solution : $m\angle APB = 90$, as $\overline{AB}$ is a diameter.

Also $m\angle PAB = 50$

$m\angle ABP = 90 - 50 = 40$

Being angles of same segment, $\overset{\frown}{AP} \cup \overline{AP}$

$\angle AQP \cong \angle ABP$.

$\therefore$ $m\angle AQP = 40$

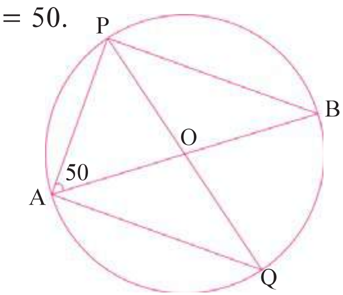


Figure 12.44

Example 8 : Prove that the quadrilateral formed (if possible) by the angle bisectors of any quadrilateral is cyclic.

Solution : PQRS is a quadrilateral in which the angle bisectors $\overrightarrow{PD}$, $\overrightarrow{QB}$, $\overrightarrow{RB}$ and $\overrightarrow{SD}$ of angles $\angle P$, $\angle Q$, $\angle R$ and $\angle S$ respectively form a quadrilateral ABCD. (see figure 12.45)

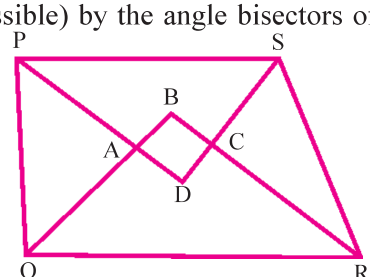


Figure 12.45

Now, $m\angle BAD = m\angle PAQ = 180 - m\angle APQ - m\angle AQP$

$= 180 - \frac{1}{2}(m\angle SPQ + m\angle PQR)$

Similarly $m\angle BCD = m\angle RCS = 180 - \frac{1}{2}(m\angle QRS + m\angle RSP)$

Therefore, $m\angle BAD + m\angle BCD$

$= 180 - \frac{1}{2}(m\angle SPQ + m\angle PQR) + 180 - \frac{1}{2}(m\angle QRS + m\angle RSP)$

$= 360 - \frac{1}{2}(m\angle SPQ + m\angle PQR + m\angle QRS + m\angle RSP)$

$= 360 - \frac{1}{2}(360) = 360 - 180 = 180$

Hence, a pair of opposite angles of □ ABCD is supplementary.

$\therefore$ □ ABCD is cyclic.

EXERCISE 12.5

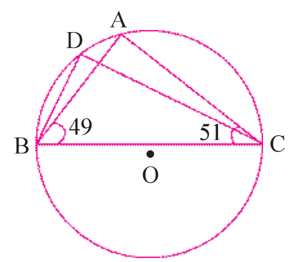


Figure 12.46

1. If D is on the major $\overset{\frown}{AB}$ of the circle with center O and $m\angle ADB = 45$, then find the measure of $\angle AOB$.

2. If $m\angle ABC = 49$, $m\angle ACB = 51$, find $m\angle BDC$. (Refer figure 12.46)

3. A chord of a circle is congruent to the radius of the circle. Find the measure of the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

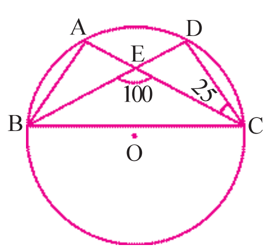


Figure 12.47

4. A, B, C and D are four points on a circle. $\overline{AC}$ and $\overline{BD}$ intersect at a point E such that $m\angle BEC = 100$ and $m\angle ECD = 25$. Find $m\angle BAC$. (see figure 12.47).

5. □ PQRS is a cyclic quadrilateral whose diagonals intersect at the point E. If $m\angle SQR = 70$, $m\angle QPR = 30$, find $m\angle QRS$. Further, if PQ = PR, find $m\angle ERS$.

6. Bisector of $\angle A$ intersects circumcircle of Δ ABC at D. If $m\angle BCD = 50$, then find $m\angle BAC$. (figure 12.48).

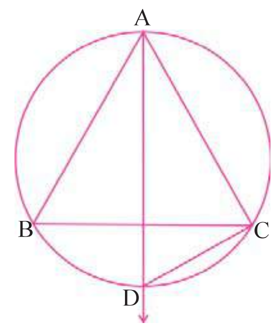


Figure 12.48

7. $\angle ABC$ is an angle inscribed in a semi-circle arc of $\odot(O, r)$. Δ ABC is isosceles and AB = $3\sqrt{2}$. Find area of the circle.

8. Prove that a cyclic parallelogram is a rectangle.

9. In a cyclic quadrilateral ABCD, $\overline{AB} \parallel \overline{CD}$. Prove that $\overline{AD} \cong \overline{BC}$.

10. If in a cyclic $\square$ABCD, $\overline{AD} \cong \overline{BC}$, prove $\overline{AB} \parallel \overline{CD}$.

EXERCISE 12

1. Congruent parallel chords $\overline{AB}$ and $\overline{CD}$ have mid points M and N respectively and the centre is O. $\overleftrightarrow{MN}$ intersects the circle in P and Q. Prove that PM = QN.

2. In Δ ABC, bisector of $\angle A$ passes through its circumcentre. Prove that AB = AC.

3. $\overline{AB}$ and $\overline{CD}$ are two parallel chords of a circle and AB = 24 *cm* and CD = 10 *cm*. If the perpendicular distance between them is 7 *cm*, then find the radius of the circle. Chords are in the same semiplane of the line containing the diameter parallel to them.

4. Chords $\overline{AB}$ and $\overline{CD}$ are parallel and they lie in the same semi plane of the line containing the diameter parallel to them. AB = 8 *cm*, CD = 6 *cm* and radius of the circle is 5 *cm*. Find the perpendicular distance between them.

5. $\overline{AC}$ and $\overline{BD}$ are different diameters of a circle. Prove that $\square$ABCD is a rectangle.

6. $\overline{AD}$ and $\overline{BE}$ are altitudes of Δ ABC. D $\in \overline{BC}$, E $\in \overline{AC}$. Prove that $\angle A$, $\angle B$, $\angle D$, $\angle E$ are angles of the same segment of a circle.

7. $\overline{AB}$ and $\overline{CD}$ are two parallel chords of a circle with centre O. If AB = 10, CD = 24 and distance between them is 17, then find its radius. (Chords are in different semi planes of the line containing the diameter parallel to them.)

8. Prove that the perpendicular bisector of a chord of a circle is the bisector of the corresponding arc of the circle.

9. If congruent chords of a circle with centre O are given, prove that $\overrightarrow{BO}$ is the bisector of $\angle ABC$, where $\overline{AB} \cong \overline{CB}$.

10. ΔABC is inscribed in a circle with centre O. If $m\angle BAC=30$, then prove that Δ OBC is an equilateral triangle.

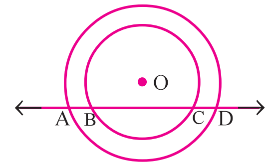


Figure 12.49

11. In the figure 12.49, AD = 12, BC = 8. Find AB, CD, AC and BD. (Here two circles are concentric.)

12. Select proper option (a), (b), (c) or (d) and write in the box given on the right so that the statement becomes correct :

(1) The centre of a circle lies ....

(a) in the interior of the circle. (b) in the exterior of the circle.
(c) on the circle. (d) anywhere in the plane.

(2) A point whose distance from the centre of a circle is less than its radius lies...

(a) in the interior of the circle. (b) in the exterior of the circle.
(c) on the circle (d) anywhere in the plane.

(3) The longest chord of a circle is......

(a) a line segment joining the centre and any point on the circle
(b) a chord joining the end points of a minor arc.
(c) a chord joining the end points of the major arc.
(d) a chord joining the end points of the semi circle arc.

(4) Line-segment joining the centre to any point on the circle is called ....

(a) a diameter (b) a chord (c) a line (d) a radius

(5) If a chord $\overline{AB}$ subtends an angle with measure 60 at the centre O, then Δ OAB is ....

(a) a right angled triangle (b) an obtuse angled triangle
(c) an equilateral triangle (d) an isosceles right angled triangle

(6) If a line-segment $\overline{AB}$ is a chord of a circle with centre O, then Δ OAB is always ....

(a) acute angled triangle (b) equilateral triangle
(c) obtuse angled triangle (d) isosceles triangle

(7) If the circle is a union of four disjoint congruent arcs, then the angle subtended by one of these arcs at the centre of the circle has measure ......

(a) 30 (b) 45 (c) 60 (d) 90

(8) The measure of the angle subtended by a chord of length equal to radius has measure ......

(a) 30 (b) 45 (c) 60 (d) 90

(9) If the measure of the angle between two radii of a circle is 50, then the region formed by these radii and the arc corresponding to this angle is ......

(a) a semi circle (b) a minor sector

(c) a major sector (d) the interior of the circle

(10) The perpendicular bisector of the chord of a circle passes through ......

(a) an end-point of the diameter (b) the mid-point of the diameter

(c) an end-point of the given chord (d) an end-point of an arc

(11) If the chord is at distance 3 *cm* from the centre of a circle having radius 5 *cm*, then the length of the chord is ......

(a) 4 *cm* (b) 6 *cm* (c) 8 *cm* (d) 10 *cm*

(12) The chord of the length 12 *cm* is at a distance 3 *cm* from the centre of a circle whose radius is ...... *cm*.

(a) $2\sqrt{5}$ (b) $3\sqrt{5}$ (c) $4\sqrt{5}$ (d) $6\sqrt{5}$

(13) Number of circle / circles passing through three distinct non-collinear points is / are ......

(a) zero (b) one (c) three (d) infinite

(14) Number of circles passing through a single given point are ......

(a) two (b) four (c) three (d) infinite

(15) A, B, C are three distinct non-collinear points. The point of intersection of the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$ is ......

(a) the centre of a circle passing through only B.

(b) the centre of a circle passing through only A.

(c) the centre of the circle passing through all A, B, C.

(d) the centre of a circle passing through none of A, B, C.

(16) A line passing through the centres of two circles intersecting in two distinct points is not ......

(a) a line bisecting the common chord.

(b) a line perpendicular to the common chord.

(c) a line which is the perpendicular bisector of the common chord.

(d) a line passing through one of the end points of the common chord.

(17) If 50 and 100 are the measures of the angles of a cyclic quadrilateral, then the remaining angles are of measure ...... and ....

(a) 130, 80 (b) 100, 50 (c) 100, 130 (d) 80, 50

(18) □ PQRS is a cyclic quadrilateral in which $m\angle SQR = 65$ and $m\angle QPR = 30$, then $m\angle QRS =$ ....

(a) 85 (b) 95 (c) 115 (d) 150

(19) In a cyclic quadrilateral ABCD, $m\angle CAB=45$ and $m\angle ABC=100$, then $m\angle ADB$ = ... ☐

(a) 55 (b) 105 (c) 80 (d) 35

(20) If $\overline{AB}$ is a diameter of the circle and P is on the semi-circle, and if $m\angle PAB = 40$, then $m\angle PBA$ is .... ☐

(a) 30 (b) 40 (c) 50 (d) 90

(21) A circle passes through the vertices of an equilateral ΔABC. The measure of the angle subtended by the side $\overline{AB}$ at the centre of the circle has measure .... ☐

(a) 30 (b) 60 (c) 90 (d) 120

*

Summary

In this chapter we have studied the following points :

1. We have defined a circle, its centre and radius, different terms related to the circle and congruent circles.
2. Congruent chords of a circle subtend congruent angles at the centre of the circle and its converse is true.
3. The perpendicular drawn from the centre of the circle to a chord bisects the chord and its converse is true.
4. A unique circle passes through three non-collinear distinct points.
5. Congruent chords of a circle are equidistant from the centre of circle and its converse is true.
6. If two arcs are congruent, then their corresponding chords are also congruent and conversely.
7. Congruent arcs of a circle subtend congruent angles at the centre of the circle.
8. The angle subtended by an arc at the centre has measure twice the measure of the angle subtended by it at any point on the remaining part of the circle.
9. Angles in the same segment of a circle are congruent.
10. Angle in a semicircle is a right angle.
11. If a line-segment joining two points subtends congruent angles at two other points lying on the same side of the line containing the line-segment, the four points lie on a circle.
12. The pair of opposite angles of a cyclic quadrilateral are supplementary and its converse is also true.

CHAPTER 13

CONSTRUCTIONS

13.1 Introduction

In earlier chapters, the necessary rough diagrams drawn were just sufficient to represent the given situation. There was no precision required in the drawing of different figures. But in different walks of life, precise drawing is essential. For example in furniture design, fashion design, machine drawing, constructions of buildings etc, the geometrical figures must be in the precise form and with accurate measure. So, we shall learn some constructions with the help of a straight edge and compass only. Here we shall also see the mathematical justification for the procedure adopted for the constructions, which will also use the ideas discussed in the earlier chapters. Also such constructions will help us to develop the skill of correctness in our mathematical understanding.

13.2 Basic Constructions

We have learnt how to construct a circle, the perpendicular bisector of a line-segment, the bisector of a given angle and also the angles of measure 30, 45, 60, 90 and 120 with the help of straight edge and compass only. The justification of these constructions was not discussed there. In this chapter, mathematical justification is also given at the end of each constructions. It will justify the validity and correctness of the steps taken for the constructions.

Construction 1 : To construct the bisector of a given angle.

Data : $\angle ABC$ is given.

To construct : To construct the bisector of $\angle ABC$.

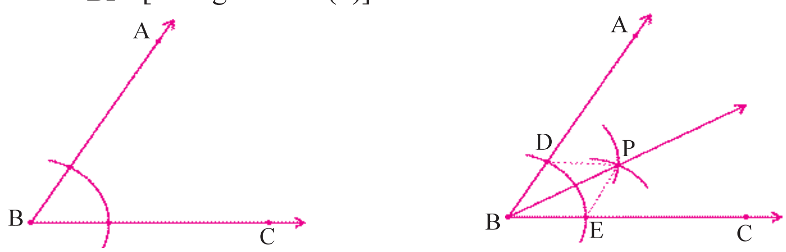
Steps of Construction :

(1) Taking B as a centre and an arbitrary radius, draw an arc intersecting both the arms $\overrightarrow{BA}$ and $\overrightarrow{BC}$ of $\angle ABC$ at D and E respectively.

(2) Draw arcs having equal radius with length more than $\frac{1}{2}$ DE by taking D and E as a centres.

These arcs intersect each other at some point P.

(3) Draw $\overrightarrow{BP}$. [see figure 13.1 (ii)]



(i) Figure 13.1 (ii)

Thus $\overrightarrow{BP}$ is the required bisector of $\angle ABC$.

Now we justify our method of construction.

Draw $\overline{PD}$ and $\overline{PE}$.

For the correspondence BEP $\leftrightarrow$ BDP of ΔBEP and ΔBDP.

$\overline{BE} \cong \overline{BD}$ **(radii of the same circle)**

$\overline{EP} \cong \overline{DP}$ **(congruent radii)**

$\overline{BP} \cong \overline{BP}$ **(common line-segment)**

$\therefore$ The correspondence BEP $\leftrightarrow$ BDP is a congruence. **(SSS)**

$\therefore \angle EBP \cong \angle DBP$

$\therefore$ $\overrightarrow{BP}$ is the bisector of $\angle ABC$.

Construction 2 : To construct the perpendicular bisector of a given line-segment.

Data : $\overline{AB}$ is given.

To construct : The perpendicular bisector of $\overline{AB}$.

Steps of Construction :

(1) Draw arcs with equal radius having length more than $\frac{1}{2}$ AB taking as centres A and B in upper and lower half-planes of $\overleftrightarrow{AB}$.

(2) Let these arcs intersect, each other at points P and Q.

(3) Draw $\overleftrightarrow{PQ}$, which intersects $\overline{AB}$ at point say M.

Thus $\overleftrightarrow{PQ}$ is the perpendicular bisector of $\overline{AB}$.

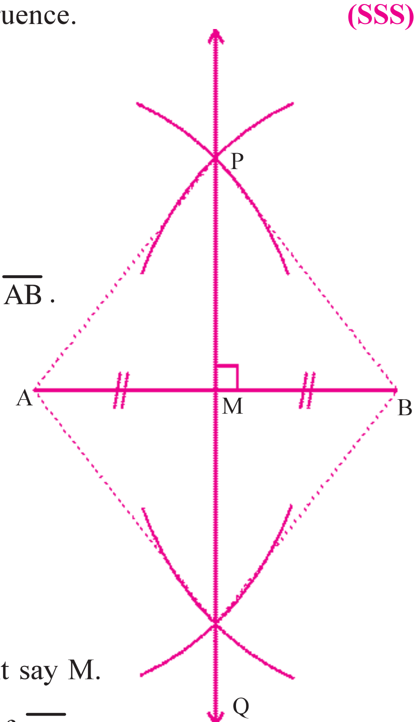


Figure 13.2

Now, we justify our method of constructions.

Join A and B with both P and Q to form $\overline{AP}$, $\overline{AQ}$, $\overline{BP}$ and $\overline{BQ}$.

For correspondence PAQ $\leftrightarrow$ PBQ of ΔPAQ and ΔPBQ.

$\overline{AP} \cong \overline{BP}$ **(radii of the congruent circles)**

$\overline{AQ} \cong \overline{BQ}$ **(radii of the cogruent circles)**

$\overline{PQ} \cong \overline{PQ}$ **(common line-segment)**

$\therefore$ The correspondence PAQ $\leftrightarrow$ PBQ is a congruence. **(SSS)**

$\therefore \angle APQ \cong \angle BPQ$

Hence $\angle APM \cong \angle BPM$ as P–M–Q

Now for correspondence PMA $\leftrightarrow$ PMB of ΔPMA and ΔPMB

$\overline{AP} \cong \overline{BP}$ **(radii of the congruent circles)**

$\angle APM \cong \angle BPM$ **(proved)**

$\overline{PM} \cong \overline{PM}$ **(common line-segment)**

$\therefore$ The correspondence PMA $\leftrightarrow$ PMB is a congruence. **(SAS)**

$\therefore \overline{AM} \cong \overline{BM}$ and $\angle AMP \cong \angle BMP$ **(i)**

As $\angle AMP$ and $\angle BMP$ form a linear pair of angles, they are supplementary angles and they are congruent also.

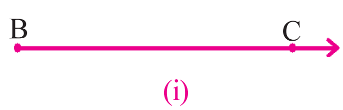
$\therefore m\angle AMP = m\angle BMP = 90$ **(ii)**

From (i) and (ii), we can say that $\overleftrightarrow{PQ}$ is the perpendicular bisector of $\overline{AB}$.

Construction 3 : To construct an angle having measure 60 at the initial point of a given ray.

Data : $\overrightarrow{BC}$ with initial point B is given.

(figure 13.3(i))



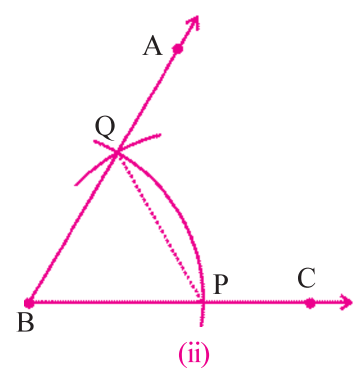


Figure 13.3

To construct : To construct $\overrightarrow{BA}$ such that $m\angle ABC = 60$.

Steps of Construction :

(1) Draw an arc with B as centre and arbitrary radius. Let this arc intersect $\overrightarrow{BC}$ at P.

(2) With centre at P and keeping the same radius as before, draw an arc to intersect the previous arc at a point, say Q.

(3) Draw $\overrightarrow{BA}$ passing through the point Q. (see figure 13.3 (ii))

Thus, we have $\angle ABC$ of measure 60.

Now, we justify our method of constructions.

Draw $\overline{PQ}$.

In ΔBPQ, $\overline{BP} \cong \overline{BQ} \cong \overline{PQ}$ **(radii of the same circle or congruent circles)**

ΔBPQ is an equilateral triangle and hence it is an equiangular triangle.

$m\angle QBP = 60$ and hence $m\angle ABC = 60$ **(Q $\in \overrightarrow{BA}$ and P $\in \overrightarrow{BC}$)**

One can construct any angle having measure which is a multiple of 15 using constructions 1 and 3. Of course we remember that measure of an angle lies between 0 and 180 !

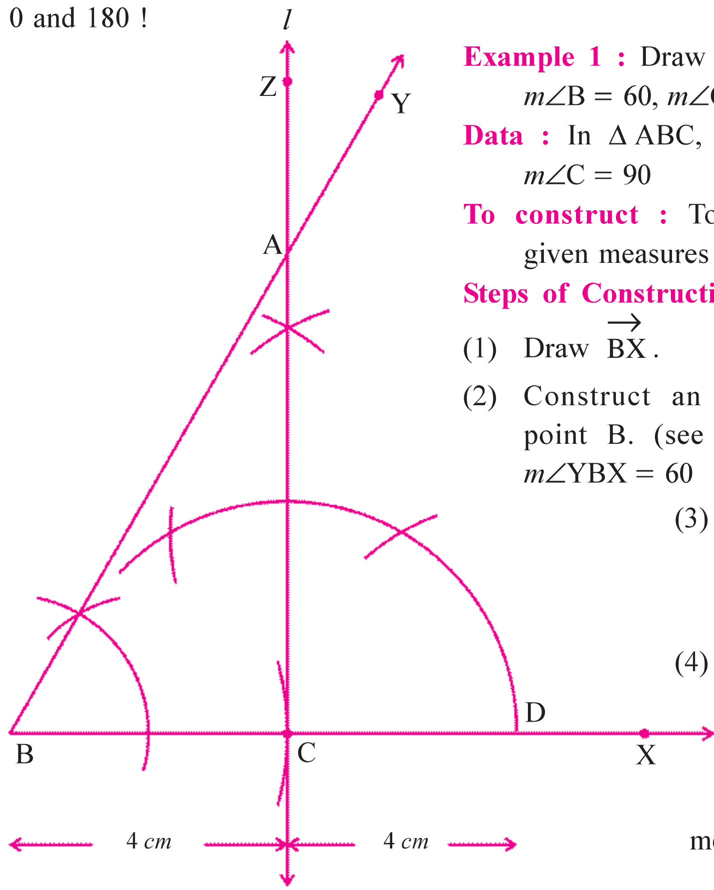


Figure 13.4

Example 1 : Draw Δ ABC where BC = 4 *cm*, $m\angle B = 60$, $m\angle C = 90$

Data : In Δ ABC, BC = 4 *cm*, $m\angle B = 60$, $m\angle C = 90$

To construct : To construct Δ ABC having given measures for side and angles.

Steps of Construction :

(1) Draw $\overrightarrow{BX}$.

(2) Construct an angle of measure 60 at point B. (see construction 3) such that $m\angle YBX = 60$

(3) Mark points C and D on $\overrightarrow{BX}$ such that BC = 4 *cm* and CD = 4 *cm*.

(4) Draw $\angle BCZ$ such that $m\angle BCZ = 90$. $\overleftrightarrow{CZ}$ intersects $\overrightarrow{BY}$ at A.

Then Δ ABC with given measure is constructed.

EXERCISE 13.1

1. Draw $\overline{AB}$ having length 10 *cm*. Construct its perpendiculer bisector $\overleftrightarrow{PQ}$, which intersects $\overline{AB}$ at M. Measure $\overline{AM}$ and $\overline{BM}$.
2. Construct an angle having measure 120 by using a pair of compass and a straight edge only.

3. Construct an angle having measure 30 by using a pair of compass and a straight edge only.

4. Construct an angle having measure (1) 15 (2) 90 (3) 150 by using a pair of compass and a straight edge only.

5. Construct an equilateral triangle having length of each side 6 *cm* by using a pair of compass and a straight edge only.

6. Construct Δ PQR, where $m\angle Q = 60$, $m\angle R = 90$ and QR = 5 *cm* by using a pair of compass and a straight edge only.

7. Construct Δ XYZ, where YZ = 4 *cm*, $m\angle X = 60$, $m\angle Z = 90$.

*

13.3 Some Constructions related to Triangles

Now we will construct triangles using the constructions learnt in our earlier classes and in this chapter.

We know that a triangle has six parts i.e. three sides and three angles. Because of the postulates and theorems of congruence of triangles, some definite three parts of a triangle determine the triangle completely. We shall now see how to construct a triangle when some definite relations among measures of angles and measures of sides are given. You may have noted that at least three parts of a triangle have to be given for the constructions of a triangle, but not all combinations of three parts are sufficient for our purpose. For example, if two sides and not included angle are given, then it is not possible to construct such a triangle. When we are given the measure of an angle for such constructions, we shall construct the angle with the help of a compass. We shall not use a protractor.

Construction 4 : To construct a triangle, given the base, one base angle and the sum of measures of two sides.

Data : Base QR, $m\angle PRQ$ and PQ + PR are given.

To construct : To construct ΔPQR with given measures.

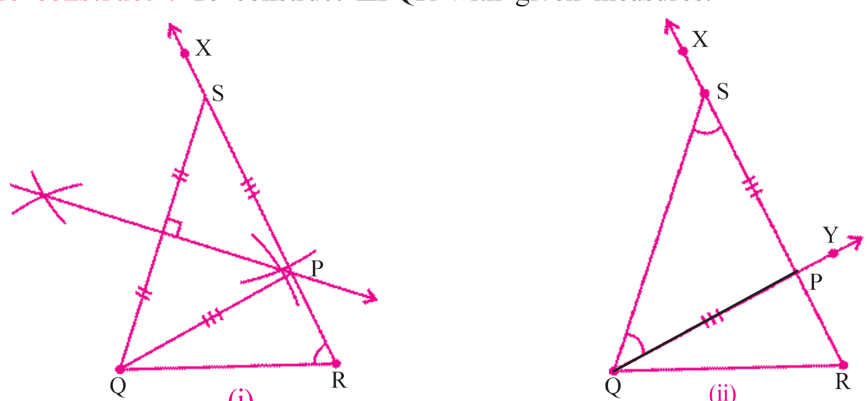


Figure 13.5

Steps of Construction :

(1) Draw $\overline{QR}$ having given measure.

(2) $\overrightarrow{RX}$ can be constructed such that $m\angle QRX$ is equal to the given $m\angle PRQ$.

(3) Select S on $\overrightarrow{RX}$ such that RS = PQ + PR.

(4) Draw $\overline{QS}$.

(5) Now to get P on $\overline{RS}$ such that PQ = PS, construct the perpendicular bisector of $\overline{QS}$, which intersects $\overline{RS}$ at P [see Figure 13.5 (i)] or Draw ∠SQY, whose measure is equal to $m\angle$RSQ. Let $\overrightarrow{QY}$ intersect $\overrightarrow{RX}$ at P (see figure 13.5 (ii)).

Then ΔPQR is the required triangle with given measures.

Now we justify our method of constructions.

In ΔPQS, PQ = PS. **(by construction)**

Then PR = RS – PS = RS –PQ

PR + PQ = RS

[if $m\angle$PSQ = $m\angle$PQS, then also PQ = PS]

Example 2 : Construct Δ ABC such that BC = 3 *cm*, $m\angle$BCA = 75 and AB + AC = 8 *cm*.

Data : In Δ ABC, BC = 3 *cm*, $m\angle$BCA = 75 and AB + AC = 8 *cm*.

To construct : To construct Δ ABC with given measures.

Steps of Construction :

(1) Draw $\overline{BC}$ such that BC = 3 *cm*.

(2) Draw $\overrightarrow{CX}$ such that $m\angle$BCX = 75 [using constructions 3 and 1].

(3) Take a point D on $\overrightarrow{CX}$ such that CD = 8 *cm*.

(4) Draw $\overline{BD}$.

(5) Draw the perpendicular bisector of $\overline{BD}$ which intersects $\overline{CD}$ at A.

(6) Draw $\overline{BA}$.

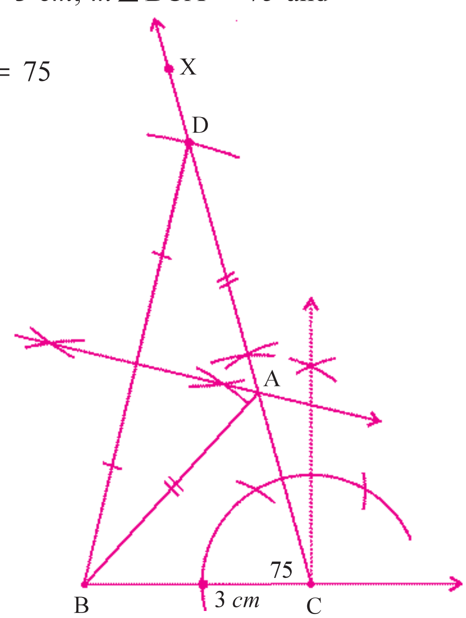


Figure 13.6

Then Δ ABC is the required triangle with given measures.

Construction 5 : To construct a triangle given its base, a base angle and the difference of the other two sides

Data : In Δ ABC, BC, $m\angle$ABC and AB–AC or AC–AB are given.

To construct : To construct Δ ABC with given measures.

Steps of Construction :

Case (1) Let AB > AC and AB–AC be given,

(1) Draw $\overline{BC}$ of given measure.

(2) Construct $\overrightarrow{BX}$ such that $m\angle CBX$ equal to given $m\angle ABC$.

(3) Select D on $\overrightarrow{BX}$ such that BD = AB – AC.

(4) Draw $\overline{CD}$.

(5) Draw the perpendicular bisector of $\overline{CD}$, which intersects $\overrightarrow{BX}$ at the point A.

(6) Draw $\overline{AC}$. (see Figure 13.7)

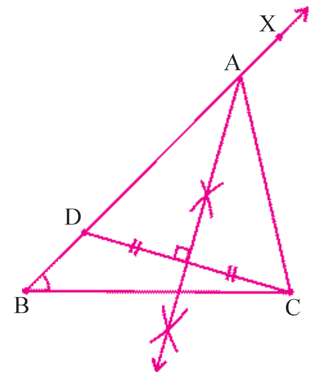


Figure 13.7

Then ΔABC is the required triangle with given measures.

Case (2) : Let AC > AB, AC – AB be given.

(1) Draw $\overline{BC}$ of given measure.

(2) Construct $\overrightarrow{BX}$ such that $m\angle CBX$ equal to given $m\angle ABC$.

(3) Draw $\overrightarrow{BY}$, opposite ray of $\overrightarrow{BX}$.

(4) Select $D \in \overrightarrow{BY}$ such that BD = AC – AB.

(5) Draw $\overline{CD}$.

(6) Draw the perpendicular bisector of $\overline{CD}$ which intersects $\overrightarrow{BX}$ at the point A.

(7) Draw $\overline{AC}$. (see figure 13.8)

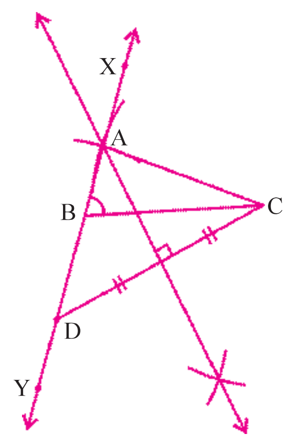


Figure 13.8

Select the point D in such a way that, if the base angle ∠B is given and the side whose one of the end point is B is greater side (AB) then A–D–B, if that side (AB) is less, then A–B–D.

Then Δ ABC is the required triangle with given measures.

Now we justify our method of construction.

Case (1) $\overline{BC}$ and ∠B of given measures are drawn

∴ AD = AC, as A is on the perpendicular bisector of $\overline{CD}$.

Now AD = AB – BD

∴ AC = AB – BD

∴ BD = AB – AC

Thus $\overline{BD}$ representes AB – AC.

Case (2) AC = AD as A is on the perpendicular bisector of $\overline{CD}$.

∴ AC = AB + BD

∴ BD = AC – AB

∴ $\overline{BD}$ represents AC – AB

Example 3 : Construct ΔPQR, where $QR = 6$ *cm*. $m\angle PRQ = 30$, $PQ - PR = 3$ *cm*.

Data : In ΔPQR, $QR = 6$ *cm*. $m\angle PRQ = 30$, $PQ - PR = 3$ *cm*.

To construct : To construct ΔPQR with given measures.

Steps of Construction :

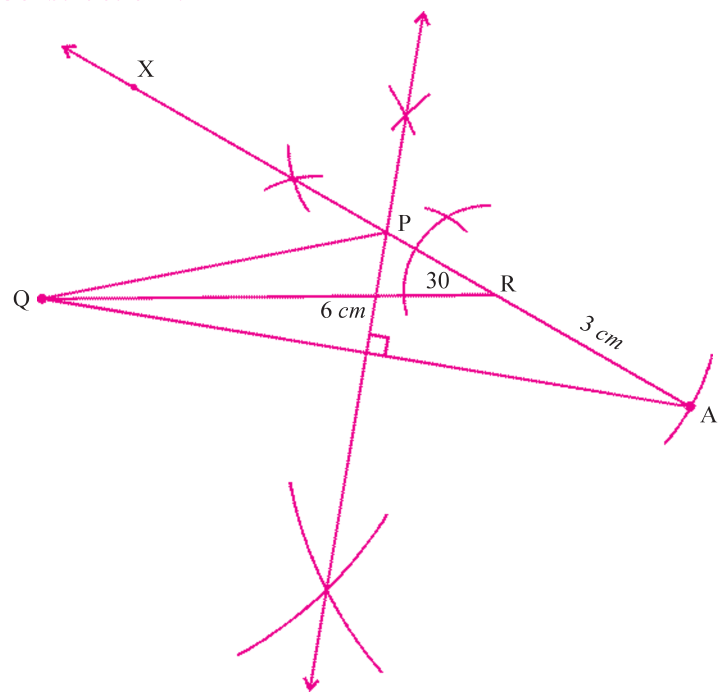


Figure 13.9

(1) Draw $\overline{QR}$ of length 6 *cm*.

(2) Draw $\overrightarrow{RX}$ such that $m\angle QRX = 30$ (Construction of an angle of measure 30)

(3) Take a point A on the ray opposite to $\overrightarrow{RX}$ such that $RA = 3$ *cm*. (Why ?)

(4) Draw $\overline{QA}$.

(5) Draw the perpendicular bisector of $\overline{QA}$, which intersects $\overrightarrow{RX}$ at P

(6) Draw $\overline{PQ}$.

Thus ΔPQR with given conditions is constructed.

Example 4 : Construct ΔDEF such that $EF = 5$ *cm*, $m\angle DFE = 30$, $DF - DE = 2$ *cm*

Data : In ΔDEF, $EF = 5$ *cm*, $m\angle DFE = 30$, $DF - DE = 2$ *cm*.

Construction 5 : To construct ΔDEF with given measures.

Steps of Construction :

(1) Draw $\overline{EF}$ of length 5 *cm*.

(2) Draw $\overrightarrow{FX}$ such that $m\angle EFX = 30$.
(Construction of an angle of measure 30)

(3) Take a point C on $\overrightarrow{FX}$ such that FC = 2 *cm*.

(4) Draw $\overline{EC}$.

(5) Draw the perpendicular bisector of $\overline{EC}$ which intersects $\overrightarrow{FX}$ at D.

(6) Draw $\overline{DE}$.

Then ΔDEF is constructed in accordance with given conditions.

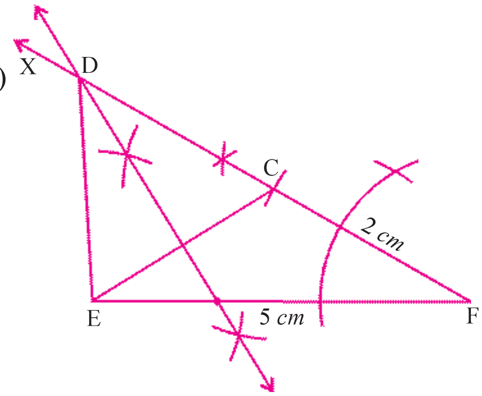


Figure 13.10

Construction 6 : To construct a triangle, given its perimeter and its two base angles.

Data : In ΔPQR, $m\angle Q$, $m\angle R$ and PQ + QR + RP are given.

To construct : To construct ΔPQR with given conditions.

Steps of Construction :

(1) Draw $\overline{XY}$ such that XY = PQ + QR + RP.

(2) Construct $\angle AXY$ and $\angle BYX$ such that $m\angle AXY = m\angle Q$ and $m\angle BYX = m\angle R$.

(3) Draw bisectors of $\angle AXY$ and $\angle BYX$, and they intersect at P.

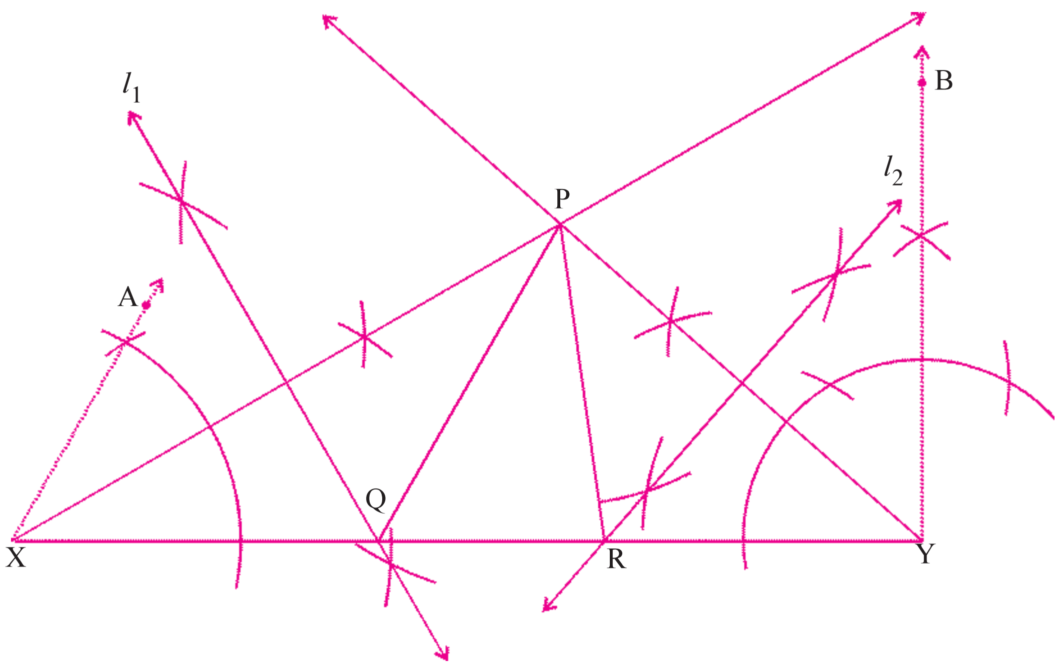


Figure 13.11

(4) Draw the perpendicular bisector, l_1 and l_2 of $\overline{PX}$ and $\overline{PY}$ respectively intersecting $\overline{XY}$ at Q and R respectively.

(5) Draw $\overline{PQ}$ and $\overline{PR}$.

Thus, ΔPQR with given conditions is constructed.

Now, we justify our method of construction.

$m\angle PYR = \frac{1}{2}\, m\angle R$ and $m\angle PXQ = \frac{1}{2}\, m\angle Q$

Line l_1 is the perpendicular bisector of $\overline{PX}$.

$\therefore\ \overline{PQ} \cong \overline{XQ}$ and similarly $\overline{PR} \cong \overline{RY}$

$\therefore\ PQ = QX$ and $PR = RY$

$\therefore\ m\angle PXQ = m\angle QPX = \frac{1}{2}\, m\angle PQR$

$\therefore\ m\angle PQR = 2\, m\angle PXQ = m\angle AXQ = m\angle AXY$

Similarly $m\angle PRQ = m\angle BYR = m\angle BYX$

Also $XY = XQ + QR + RY = PQ + QR + PR$

Example 5 : Construct ΔPQR Such that $m\angle Q = 60$, $m\angle R = 90$ and $PQ + QR + RP = 12$ *cm*.

Data : In ΔPQR, $m\angle Q = 60$, $m\angle R = 90$ and $PQ + QR + RP = 12$ *cm*

To construct : To construct ΔPQR with given conditions.

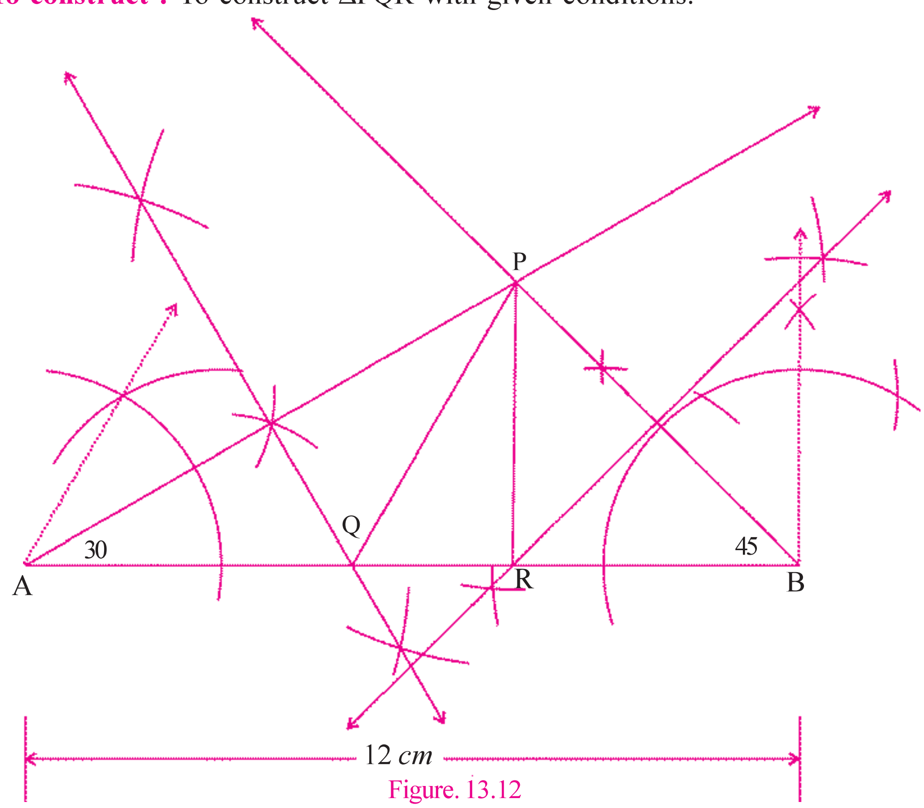


Figure. 13.12

Steps of Construction :

(1) Draw $\overline{AB}$ of length 12 *cm*.

(2) Construct ΔPAB with $m\angle A = 30$, $m\angle B = 45$ whose arms intersect at P.

(3) Construct the perpendicular bisectors of $\overline{AP}$ and $\overline{BP}$ which intersect $\overline{AB}$ at Q and R respectively.

(4) Draw $\overline{PQ}$ and $\overline{PR}$.

Thus, ΔPQR of given measures is constructed.

EXERCICE 13

1. Construct ΔABC such that BC = 6 *cm*. $m\angle B = 60$, AB + CA = 9 *cm*. Write the steps of the constuction.

2. Construct ΔPQR where PQ = 7 *cm*. $m\angle P = 30$, RP – QR = 3 *cm*. Write the steps of the construction.

3. Construct ΔABC in which $m\angle B = 30$ and $m\angle C = 30$, AB + BC + CA = 12 *cm*. Also write the steps of the construction.

4. Construct and write the steps of the construction for ΔPQR in which QR = 8 *cm* $m\angle Q = 45$ and PR – PQ = 2 *cm*.

*

Summary

In this chapter we have done the following constructions with the help of straight edge (ruler) and compass only :

1. To bisect a given angle.

2. To draw the perpendicular bisector of a line segment.

3. To draw an angle with measure 60.

4. To draw an angle having measure a multiple of 15.

5. To draw a triangle, whose base, a base angle and sum of other two sides are given.

6. To draw a triangle, whose base, a base angle and difference of other two sides are given.

7. To draw a triangle, given its two base angles and perimeter.

●

CHAPTER 14

HERON'S FORMULA

14.1 Introduction

In the previous classes, we have studied about the figures of different shapes such as a triangle, a square, a rectangle, a rhombus, a trapezium etc. Moreover, we had found out the areas of regions enclosed by the figures and also calculated the perimeters of them. For example, if we want to find out the perimeter of any floor of a room of our school or home, it is obvious that we walk around the boundary of that room. The total distance covered by us is considered as perimeter of that room and the floor of that room will have an area also.

So if the floor of our room is rectangular and its length is l and breath is b, then total distance covered will be $2(l+b)$ i.e. its perimeter and its area is lb.

How can we find the area of a triangle ? We know the following result about area.

Area $= \frac{1}{2} \times$ base $\times$ altitude **(i)**

For a right angled triangle we can use the above formula directly because an altitude from the vertex to the base of the triangle will be a side of the triangle. For example, in the right angled Δ ABC, $m\angle B = 90$, AB $= 3$ *cm*, BC $= 4$ *cm*, length of the hypotenuse AC $= 5$ *cm*. Then the area of the triangle is given by $\frac{1}{2} \times AB \times BC$ where AB is the altitude and BC is the base of the triangle.

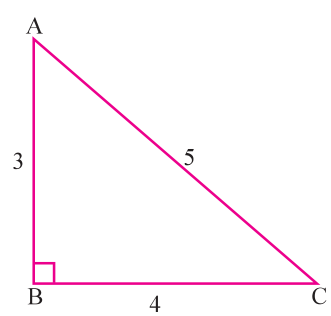


Figure 14.1

Area $= \frac{1}{2} \times 4 \times 3 = 6$ cm^2

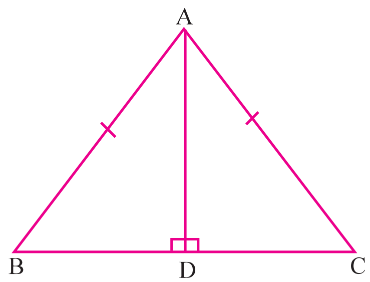


Figure 14.2

Let us find out the area of an isosceles triangle with the help of the above formula. In Δ ABC, let AB = AC. Now draw the perpendicular from the vertex A to the base $\overline{BC}$ which intersects $\overline{BC}$ at D. Thus, Δ*ABC is divided into two triangular regions, Δ*ABD and Δ*ACD.

$m\angle ADB = m\angle ADC = 90$

Now if AB = 5 *cm*, then AC is also 5 *cm* and let BC = 6 *cm*. Altitude from A divides $\overline{BC}$ in two congruent line-segments $\overline{BD}$ and $\overline{DC}$. Thus BD + DC = BC, so that BD = DC = 3 *cm* (figure 14.2)

Now, apply Pythagoras' theorem to the right angled Δ ADB

$AB^2 = BD^2 + AD^2$

$\therefore\ 5^2 = (3)^2 + AD^2$

$\therefore\ 25 - 9 = AD^2$

$\therefore\ AD^2 = 16$

$\therefore$ AD = 4 *cm* = length of the altitude

$\therefore$ By (i), area of the isosceles Δ ABC = $\frac{1}{2} \times 6 \times 4 = 12\ cm^2$

Similarly, we want to find the area of an equilateral Δ ABC, where the length of each side is 12 *cm*. For this triangle, if we draw a perpendicular from the vertex A to the base $\overline{BC}$ which intersects $\overline{BC}$ at D, then $\overline{AD}$ is an altitude of ΔABC. Here D is the midpoint of $\overline{BC}$.

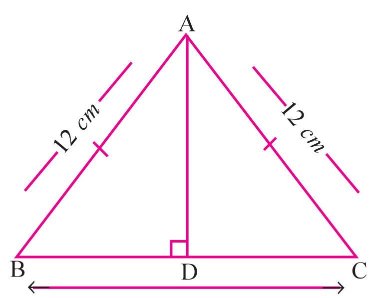


Figure 14.3

Thus, BD = DC = 6 *cm* (figure 14.3)

For right angled ΔADB, $AB^2 = BD^2 + AD^2$

$\therefore\ (12)^2 = AD^2 + (6)^2$

$\therefore\ AD^2 = 144 - 36$

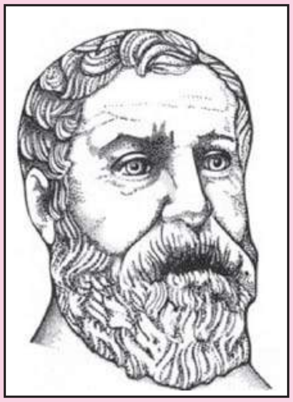
$\therefore\ AD^2 = 108$

$\therefore\ AD = 6\sqrt{3}\ cm$

$\therefore$ The area of equilateral Δ ABC is given by, $\frac{1}{2} \times AD \times BC = \frac{1}{2} \times 6\sqrt{3} \times 12$

$\therefore$ The area of $\Delta ABC = 36\sqrt{3}\ cm^2$

14.2 Heron's Formula



Heron (10AD - 75 AD)

Heron was born in about 10 A.D. possibly in Alexandria in Egypt. He worked in applied mathematics. His work on mathematical and physical subjects are so numerous and varied that he is considered to be an encyclopedic writer in these fields. His geometrical works deal largely with problems on mensuration written in three books. Book I deals with the area of squares, rectangles, triangles, trapezoids (trapezia), various other specialised quadrilaterals, the regular polygons, circles, surfaces of cylinders, cones, spheres etc. In this book, Heron has derived the famous formula for the area of a triangle in terms of its three sides.

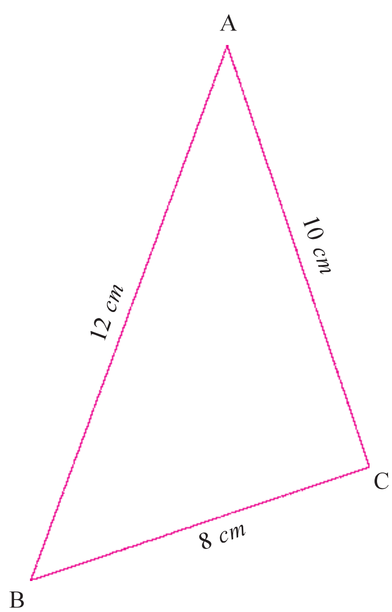


Figure 14.4

For an isosceles, equilateral and right angled triangle, we can draw the perpendiculars from the vertex to the base and we can find their lengths. Then we can find the area of the triangle by using the formula $\frac{1}{2} \times$ base $\times$ altitude. But if we have a scalene triangle, then we do not have any clue to find the length of an altitude (i.e. perpendicular from a vertex to the base of the triangle).

For an example, in Δ ABC, Let AB = 12 *cm*, BC = 8 *cm* and AC = 10 *cm*. Now there is a problem as to how can we calculate the area of this triangle ? For this, a formula is given by Heron, which is known as **Heron's formula**. It is as follows :

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ **(ii)**

Here a, b, c are the lengths of the sides of the triangle and s is semiperimeter of the triangle.

Thus, perimeter $= a + b + c = 2s$

$\therefore\ s = \frac{a+b+c}{2}$

So, if the length of the altitude is not given and it is not easy to find it, then this formula (ii) will be helpful to find the area of the triangle. So for the above example,

$s = \frac{12+10+8}{2} = 15\ cm$

Area of Δ ABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15(15-12)(15-10)(15-8)}$

$= \sqrt{15(3)(5)(7)} = 15\sqrt{7}\ cm^2$

Let us solve following examples to understand the application of Heron's formula.

Example 1 : Find the area of the triangle whose sides have lengths 15, 15, 12 *cm*.

Solution : Here, $s = \frac{a+b+c}{2} = \frac{15+15+12}{2} = \frac{42}{2} = 21\ cm$

$\therefore$ The area of Δ ABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-15)(21-15)(21-12)}$

$= \sqrt{21 \times 6 \times 6 \times 9}$

$= 18\sqrt{21}\ cm^2$

(Do you have any other alternative method ?)

Example 2 : The lengths of the sides of a triangular park are in proportion 3 : 5 : 7 and its perimeter is 450 metre, then find out the area of this park. Also find the cost of fencing it with barbed wire at the rate of ₹ 25 per metre by leaving a space of 5 metre wide for a gate on all the sides.

Solution : The sides are in the proportion 3 : 5 : 7. Suppose the lengths of the sides of the triangular park are $3x$, $5x$ and $7x$. ($x > 0$).

Now, perimeter of triangular park = 450 metre

$\therefore 3x + 5x + 7x = 450$

$\therefore 15x = 450$

$\therefore x = 30$ metre

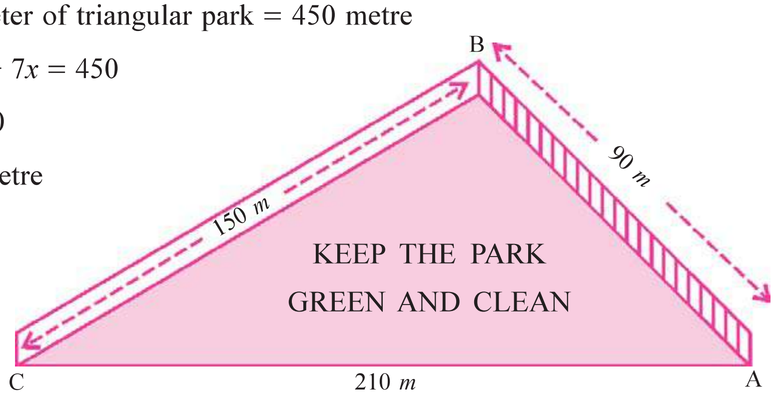


Figure 14.5

Thus, for Δ ABC, AB $= c = 3x$ metre $= 3(30) = 90$ metre

BC $= a = 5x$ metre $= 5(30) = 150$ metre

AC $= b = 7x$ metre $= 7(30) = 210$ metre

Now, $s = \frac{a+b+c}{2} = \frac{90+150+210}{2} = \frac{450}{2} = 225$ metre

$$\therefore \text{ The area of } \Delta \text{ ABC} = \sqrt{225(225-90)(225-150)(225-210)}$$
$$= \sqrt{225(135)(75)(15)}$$
$$= \sqrt{15\times15\times15\times9\times25\times3\times15}$$
$$= \sqrt{(15)^4\times(5)^2\times(3)^2\times3}$$
$$= (15)^2 \times 5 \times 3 \times \sqrt{3}$$
$$= 3375\sqrt{3}\ m^2$$

Now, for the fencing, 5 metre space is left on each side of the triangular park. Then total space left will be $5 \times 3 = 15\ m$. Hence the total length for the fencing = length of the wire needed for fencing = Permeter of the triangular park – length of the gates

= 450 metre – 15 metre = 435 metre

$\therefore$ Total cost of fencing $= 435 \times 25$

= ₹ 10875

Example 3 : Find the area of the triangle ΔABC where AB = 5 *cm*, BC = 8 *cm* and AC = 9 *cm* . Find the length of the perpendicular drawn from A to $\overline{BC}$

Solution : Here, $s = \frac{a+b+c}{2} = \frac{5+8+9}{2} = 11\ cm$

$$\therefore \text{ The area of } \Delta \text{ ABC} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{11(11-8)(11-9)(11-5)}$$
$$= \sqrt{11\times3\times2\times6}$$
$$= \sqrt{11\times(6)^2}$$
$$= 6\sqrt{11}\ cm^2$$

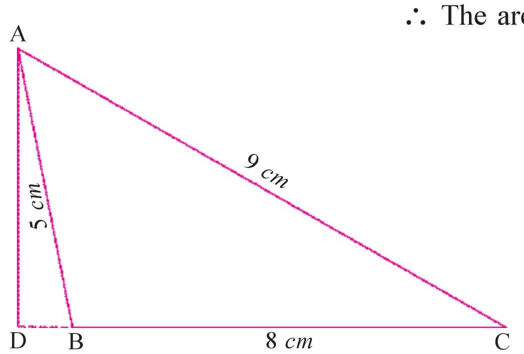


Figure 14.6

Here, $\overline{AD} \perp \overline{BC}$ (see figure 14.6)

Now we have, area of ΔABC

$= \frac{1}{2} \times$ base $\times$ altitude of Δ ABC

$= \frac{1}{2} \times 8 \times$ AD

$\therefore\ 6\sqrt{11} = 4$ AD

$\therefore$ AD $= \frac{6\sqrt{11}}{4} = \frac{3}{2}\sqrt{11}$ *cm*

$\therefore$ The length of the perpendicular from A to base $\overline{BC} = \frac{3}{2}\sqrt{11}$ *cm*

EXERCISE 14.1

1. Find the area of the equilateral triangle having length of each side 6 units.

2. Find the area of the right angled triangle whose hypotenuse has the length 17 *cm* and has length of its base 15 *cm*.

3. Find the area of the triangle with the length of the sides 36 *cm*, 48 *cm* and 60 *cm*.

4. If the lengths of the sides of a triangle are in proportion 3 : 4 : 5 and the perimeter of the triangle is 120 metre, then find the area of the triangle.

5. An isosceles triangle has perimeter 30 *cm* and length of its congruent sides is 12 *cm*. Find the area of the triangle.

6. The triangular side walls of a flyover have been used for advertisements. The sides of the walls have lengths 100*m*, 35*m* and 105*m*. The rent per year for the advertisements is ₹ 4000 per m^2. A company hired one of its walls for 2 months. How much rent did it pay ? ($\sqrt{34} \cong 5.83$)

7. Find the area of the triangle with the lengths of the sides 5 *cm*, 7 *cm* and 10 *cm*. Also find the length of the altitude drawn from the vertex to the side whose length is 10 *cm*.

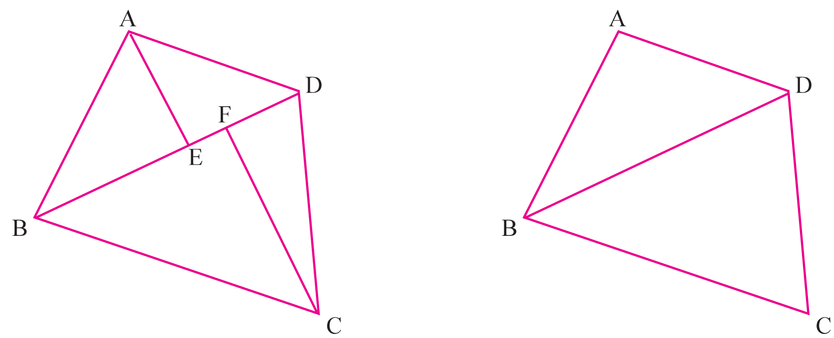
*

14.3 Application of Heron's Formula in Finding Area of Quadrilaterals

For a quadrilateral ABCD, if we join two opposite vertices, then we get a diagonal and if we draw the perpendiculars from remaining two vertices to the diagonals, then we have a formula to find the area of the quatrilateral as

Area of the quadrilateral = $\frac{1}{2}$ ***(length of a diagonal) (sum of the length of perpendiculars drawn to the diagonal from other two vertices)***

But it is a difficult and tedious process. So instead of it, if we draw a diagonal then quadrilateral region can be divided into two triangular regions and then we can use the fact that area of the quadrilateral = sum of the areas of both triangles. Both these cases are shown in the figure 14.7.



(i) Figure 14.7 (ii)

In figure 14.7 (i) we have the diagonal $\overline{BD}$ and the altitudes are $\overline{AE}$ and $\overline{CF}$. So by finding their lengths (i.e. AE and CF) we can use the result. In figure 14.7 (ii) by a single diagonal we get two triangles and by Heron's formula we can find the area of both the triangles and then take the sum of them. Thus we get the area of the quadrilateral. It will be easier to find the area of a quadrilateral in this manner.

Let us understand this discussion by the following examples.

Example 4 : In quadrilateral ABCD, AB = 3 *cm*, BC = 4 *cm*, CD = 6 *cm* and DA = 5 *cm* and the length of the diagonal $\overline{AC}$ is 5 *cm*. Find the area of $\square$ABCD.

Solution : Here diagonal $\overline{AC}$ partitions $\square$* ABCD in two triangular regions : Δ*ACD and Δ*ABC. For ΔACD,

$$s = \frac{AD+DC+AC}{2} = \frac{5+6+5}{2} = 8\ cm$$

Now the area of ΔACD $= \sqrt{8(8-5)(8-6)(8-5)}$

$= \sqrt{8(3)(2)(3)}$

$= 12\ cm^2$

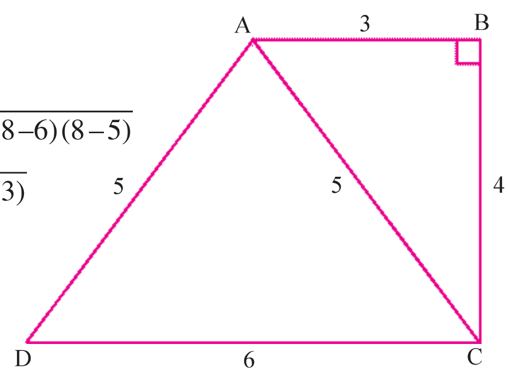


Figure 14.8

For ΔABC, $s = \frac{AB + BC + AC}{2}$

$= \frac{3+4+5}{2} = 6\ cm$

Now the area of ΔABC $= \sqrt{6(6-3)(6-4)(6-5)}$

$= \sqrt{6(3)(2)(1)} = 6\ cm^2$

$\therefore$ Area of $\square$ ABCD $=$ Area of Δ ACD + Area of Δ ABC

$= 12 + 6$

$= 18\ cm^2$

See that ΔABC is a right angled triangle. ΔADC is an isosceles triangle. So there is no need to use of Heron's formula. Do it by yourself.

Example 5 : A park is in the shape of a quadrilateral ABCD, where $m\angle C = 90$. Lengths of the sides are AB = 11 *m*; BC = 3 *m*, CD = 4 *m*, AD = 8 *m*. Then find the area of the park.

Solution : Here, for the quadrilateral ABCD, $m\angle C = 90$, and $\overline{BD}$ = diagonal. (figure 14.9). Thus for right angled Δ BCD, see that we $\overline{BD}$ is the hypotenuse.

$\therefore BD^2 = CD^2 + BC^2 = (4)^2 + (3)^2 = 25$

$\therefore$ BD = 5 = length of the diagonal

Now the area of quadrilateral ABCD

= The area of Δ BCD + The area of Δ ABD

$\therefore$ The area of Δ BCD

$= \frac{1}{2} \times \text{base} \times \text{altitude}$

$= \frac{1}{2} \times BC \times CD$

$= \frac{1}{2} \times 3 \times 4$

$= 6\ m^2$

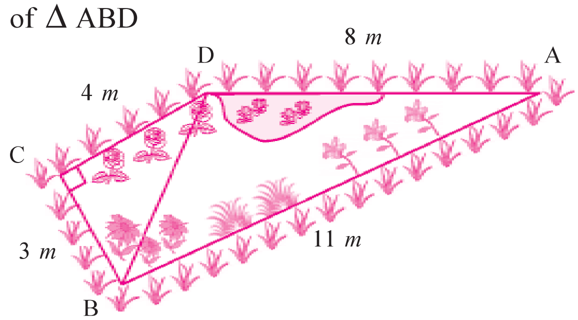


Figure 14.9

Now, for the area of Δ ABD,

$$s = \frac{AB+BD+AD}{2} = \frac{11+5+8}{2} = 12\ m$$

$\therefore$ Area of Δ ABD $= \sqrt{12(12-5)(12-8)(12-11)}$

$= \sqrt{12 \times 7 \times 4 \times 1}$

$= \sqrt{4 \times 3 \times 7 \times 4}$

$= 4\sqrt{21}\ m^2$

$\therefore$ Area of quadrilateral ABCD $= 6 + 4\sqrt{21}\ m^2$

EXERCISE 14.2

1. Find the area of the quadrilateral ABCD where AB = 7 *cm*, BC = 6 *cm*, CD = 12 *cm* and AD = 15 *cm* and the length of the diagonal $\overline{AC}$ is 11 *cm*.
2. Find the area of the quadrilateral ABCD where AB = 8 *m*, BC = 15 *m* and CD = 13 *m*, DA = 12 *m*, $m\angle B = 90$.

3. If the perimeter of a quadrilateral ABCD is 92 *m* and the perimeter of Δ ABD is 90 *m*, then find the length of the diagonal $\overline{BD}$. Also find the area of the quadrilateral ABCD where AB = 40 *m*, BC = 15 *m*, CD = 28 *m*, DA = 9 *m*.

4. If the lengths of the diagonals of a quadrilateral field are 40 *m* and 24 *m* and they bisect each other at right angles, then find its area.

5. If the lengths of the sides of a parallelogram are 13 *cm* and 10 *cm* and the length of one of its diagonal is 9 *cm,* then find its area.

*

EXERCISE 14

1. Find the area of regular hexagon ABCDEF (figure 14.10) where the length of each side is 4 *cm* and O is the midpoint of the diagonals $\overline{FC}$, $\overline{DA}$ and $\overline{BE}$ and their lengths are 8 *cm*.

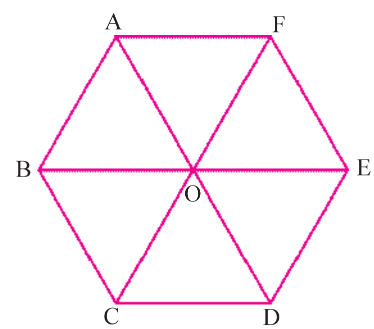


Figure 14.10

2. Find the area of the quadrilateral ABCD, where AB = 9 *cm*, BC = 10 *cm*, CD = 12 *cm*, DA = 11 *cm* and $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

3. A bulk of triangular tiles of the length 3 *cm*, 4 *cm* and 5 *cm* is to be used for the flooring of a room with area 216 cm^2. Find how many tiles should be used for the flooring. Find the total cost of polishing the tiles at the rate of ₹ 2.75 per cm^2.

4. An umbrella is to be made by stitching 8 triangular pieces of cloth with lengths 17 *cm*, 17 *cm* and 16 *cm*. Find how much cloth is required for the umbrella.

5. Find the area of the triangle whose length of the sides are 6 *cm*, 8 *cm* and 10 *cm*.

6. If the length of the sides of a triangle are in proportion 25 : 17 : 12 and its perimeter is 540 *m*, then find the lengths of the largest and smallest altitudes.

7. In figure 14.11, BC = 5 *cm*, CD = 3 *cm*, CF = 6 *cm*. Find the area occupied by the prism on the prism table.

8. The base of a triangular field is twice to its altitude and the cost of cultivating the field is ₹ 30 per hectre and the total cost is ₹ 480. Find the length of the base and altitude of that trianguler field. (10000 m^2 = 1 Hector)

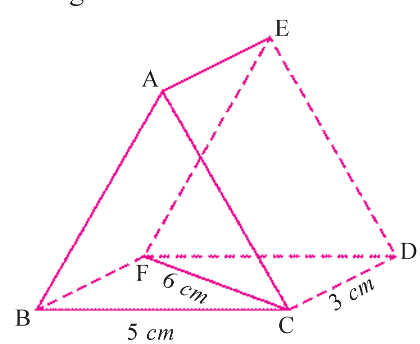


Figure 14.11

9. If the length of the side of a square is 5 *m* and it is converted into a rhombus whose major diagonal has length 8 *m,* then, find the length of the other diagonal and also find the area of the rhombus.

10. If the area of a rhombus is 100 cm^2 and the length of one of its digonal is 8 *cm*, then find the length of the other diagonal.

11. Both of the parallel sides of a trapezium are 8 *cm* and 16 *cm*. Non-parallel sides are congruent, each being 10 *cm*. Then find the area of the trapezium

12. Select proper option (a), (b), (c) or (d) and write in the box given on the right so that the statement becomes correct :

(1) For the Δ ABC, semiperimeter is ...... where AB = 8 *cm*, BC = 6 *cm*, AC = 10 *cm*. $\square$

(a) 24 (b) 20 (c) 12 (d) 16

(2) For a $\square^m$ ABCD, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{BC} \parallel \overleftrightarrow{DA}$. If AB = 8 *cm* and BC = 10 *cm* the perimeter of the $\square^m$ ABCD is ...... *cm* $\square$

(a) 18 (b) 20 (c) 36 (d) 56

(3) If the perimeter of a trapezium is 50 *cm* and the lengths of non-parallel sides are equal to 12 *cm,* then the sum of parallel sides is .... $\square$

(a) 13 *cm* (b) 26 *cm* (c) 28 *cm* (d) 30 *cm*

(4) If the area of a rhombus is 54 cm^2 and the lengths of one of its diagonal is 9 *cm*, then the length of its other diagonal is ...... *cm*. $\square$

(a) 9 (b) 12 (c) 27 (d) 90

(5) If the lengths of the sides of a triangle are in proportion 3 : 4 : 5 then the area of the triangle is ...... sq units where perimeter of the triangle is 144. $\square$

(a) 64 (b) 364 (c) 564 (d) 864

(6) If the base of an isosceles triangle has length 10 *cm* and its perimeter is 28 *cm,* then the length of each congruent side is ...... *cm*. $\square$

(a) 38 (b) 18 (c) 9 (d) 19

(7) If the lengths of the sides of a triangle are 8 *cm*, 11 *cm* and 13 *cm*, then area of the triangle is .... $(cm)^2$. $\square$

(a) 44 (b) 43 (c) 42.82 (d) $8\sqrt{30}$

(8) If the length of the base of a triangle is 12 *cm* and the length of the altitude to that base is 8 *cm,* then the area of the triangle is ...... $(cm)^2$. $\square$

(a) 12 (b) 24 (c) 36 (d) 48

(9) If the area of an equilateral triangle is $2\sqrt{3}$ cm^2, then the length of each side of the triangle is ...... *cm*. $\square$

(a) $\sqrt{2}$ (b) $2\sqrt{3}$ (c) $2\sqrt{2}$ (d) $3\sqrt{2}$

(10) In a Δ ABC, $\overline{CD}$ is the altitude of Δ ABC where AD = 4 *cm*, CD = 5 *cm* and BD = 5 *cm*. Also the area of a square is the same as the area of Δ ABC. Then length of each side of the square is ...... *cm*. $\square$

(a) $\frac{3\sqrt{2}}{5}$ (b) $\frac{3}{2}$ (c) $\frac{3\sqrt{10}}{2}$ (d) $\frac{3\sqrt{5}}{2}$

(11) In a square ABCD, length of each side is 7 *cm*. Then length of its diagonal is ...... *cm* $\square$

(a) $\sqrt{2}$ (b) 7 (c) $7\sqrt{2}$ (d) $2\sqrt{7}$

(12) In quadrilateral ABCD, the lengths of each side is shown in the figure 14.12 then the length of the diagonal $\overline{AC}$ is ...... *m*. $\square$

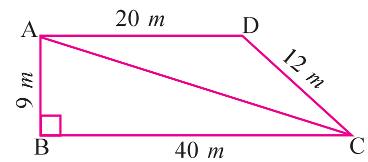


Figure 14.12

(a) 40 (b) 9 (c) 49 (d) 41

*

Summary

In this chapter we have studied the following points :

1. If the lengths of the sides of a triangle are *a*, *b* and *c*, then the perimeter of Δ ABC is $a + b + c = 2s$ and its semiperimeter is $s = \frac{a+b+c}{2}$.
2. The area of a triangle is given by Heron's formula and it is $\sqrt{s(s-a)(s-b)(s-c)}$.
3. To find the area of a quadrilateral whose sides and one diagonal are given. By a diagonal the quadrilateral region is partitioned into two triangular regions and then by Heron's formula we can find the area of each of the triangles. The sum of areas of both triangles gives us the area of quadrilateral.

CHAPTER 15

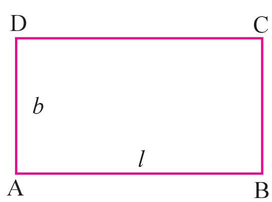
SURFACE AREA AND VOLUME

15.1 Introduction

We have learnt about plane figures like a rectangle, a square, a circle etc. We have also studied how to find out their perimeters and area in earlier classes. Now, we will learn about congruent figures made by cutting from cardboard sheet and stacking them up in a vertical pile. By this process we shall obtain a **'solid'**. We have already studied in earlier classes about cuboid, cube etc. We will now learn here about solids in detail.

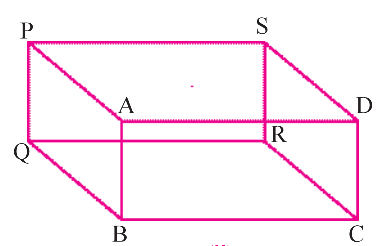
15.2 Introduction of a Cuboid and a Cube

We know about a rectangle and a square and formulae to find their areas and perimeters.



(i)

(i) Area = $l \times b$ Perimeter = $2(l + b)$



(ii)

Figure 15.1

Cuboid : A cuboid is a solid bounded by six rectangular plane regions.

Figure 15.1 (ii) represents a cuboid. We will study some solids.

In figure 15.1 (ii) □ ABCD, □ PQRS; □ SRCD, □ PQBA; □ PADS, □ QBCR are six faces of the cuboid. Each face is a rectangle. □ PADS and □ QBCR are **top and bottom faces** respectively. Also they are **opposite faces**. Similarly □ PQBA and □ SRCD; □ ABCD and □ PQRS are pairs of opposite faces. □ PQBA and □ ABCD are **adjacent faces**. Can you name another pair of adjacent faces from the figure ?

$\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$; $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$, $\overline{SP}$; $\overline{PA}$, $\overline{QB}$, $\overline{RC}$, $\overline{SD}$ are twelve **edges** of the cuboid. Adjacent faces intersect in an edge in one side of a rectangle only. Since opposite sides of a rectangle are congruent, BC = AD = QR = PS, AB = DC = SR = PQ, QB = PA = CR = SD.

A, B, C, D, P, Q, R and S are **vertices** of cuboid.

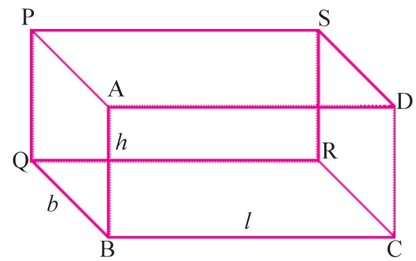


Figure 15.2

We can take any face of a cuboid as base of the cuboid. In this case, the four faces which meet the base are called **the lateral faces of cuboid.** In our cuboid type of classroom, four walls are faces of cuboid.

When we take, a rectangle, a face of a cuboid, as the base, then its length and breadth are known as the length and breadth of the cuboid. Any two lateral faces intersect in a line-segment called height of the cuboid. In figure 15.2 the rectangle QBCR is a base of cube. BC is the length l and QB is the breadth b. Intersection of faces □ ABCD and □ PQBA is $\overline{AB}$. Its length AB is the height of the cuboid.

The length, breadth and height of the cuboid are denoted by l, b and h respectively.

Cube : A cuboid whose length, breadth and height are equal is called a cube.

15.3 Surface Area of a Cuboid and Cube

We take a bundle of many congruent rectangular sheets of paper. The shape of this bundle is a cuboid. It is also called a **rectangular parallelopiped.**

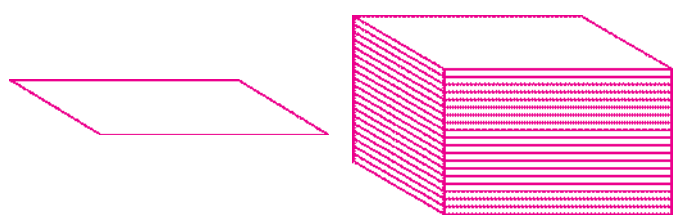


Figure 15.3

Activity (1) :

First, we take an empty chalk-box. Open all the sides of the chalk-box carefully and arrange all the faces of the chalk-box on the table as given in the figure 15.4. Name all the faces.

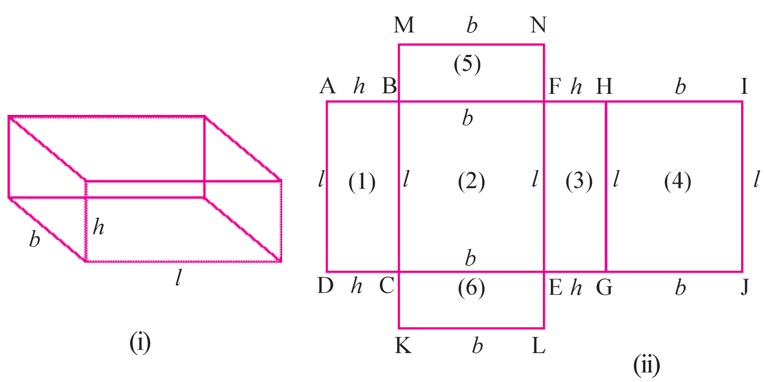


Figure 15.4

Area of the face ABCD = Area of the face FEGH = $l \times h$

Area of the face BCEF = Area of the face HGJI = $l \times b$

Area of the face CKLE = Area of the face BMNF = $b \times h$

Total surface area of a cuboid = Sum of the areas of all its six faces

$= 2\,(l \times h) + 2\,(l \times b) + 2\,(b \times h)$

$= \mathbf{2\,(lb + bh + hl)}$

Note : To find out the surface area of a cuboid, the length, breadth and height must be expressed in the same units.

Example 1 : If the dimensions of a cuboid are 20 $cm \times$ 15 $cm \times$ 10 cm, find its total surface area.

Solution : Total surface area $= 2\,(lb + bh + hl)$

$= 2\,(20 \times 15 + 15 \times 10 + 10 \times 20)$

$= 2\,(300 + 150 + 200)$

$= 2\,(650)$

$= 1300\ cm^2$

Surface Area of a Cube : For a cube, we have $l = b = h$. All the six faces of a cube are squares of the same size.

Total surface area of a cube $= 2\,(l \times l + l \times l + l \times l)$

$= 2\,(l^2 + l^2 + l^2)$

$= 6l^2$

$= 6\,(\text{length of cube})^2$

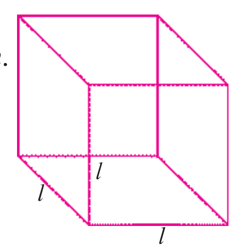


Figure 15.5

15.4 Lateral Surface Area of Cuboid and Cube :

Now we find the sum of the areas of the four faces of a cuboid excluding top and bottom faces. This sum is called the lateral surface area of the cuboid or the cube.

Lateral surface area of a cuboid

= Area of the face ABCD + Area of the face FBCG + Area of the face EFGH + Area of the face EADH.

$= l \times h + h \times b + l \times h + b \times h$

$= 2\,(l \times h) + 2\,(h \times b)$

$= \mathbf{2h\,(l + b) = h \cdot 2\,(l + b)}$

= **Height $\times$ Perimeter of base**

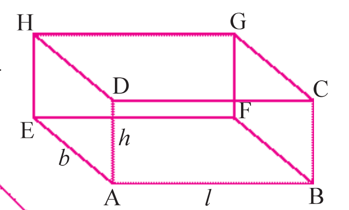


Figure 15.6

Cube : Lateral surface area of a cube

$= l^2 + l^2 + l^2 + l^2$

$= \mathbf{4l^2}$

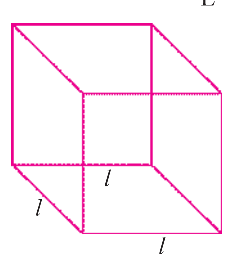


Figure 15.7

Example 2 : A cubical box has each edge having length 12 *cm* and another cuboidal box has edges 15 *cm* long, 12 *cm* wide and 8 *cm* high. (i) Which box has the smaller total surface area and by how much amount ? (ii) Which box has the greater lateral surface area and by how much amount ?

Solution : (i) Let the total surface areas of the cubical and cuboidal boxes be S_1 and S_2.

$S_1 = 6\,(l)^2 = 6\,(12)^2 = 6\,(144) = 864\ cm^2$

$S_2 = 2(lb + bh + hl)$

$= 2\,(15 \times 12 + 12 \times 8 + 8 \times 15)$

$= 2\,(180 + 96 + 120)$

$= 2\,(396)$

$= 792\ cm^2$

$\therefore\ S_1 - S_2 = 864 - 792 = 72\ cm^2$

$\therefore$ The cuboidal box has smaller surface area and is smaller by 72 cm^2

(ii) Let the lateral surface areas of the cubical and cuboid boxes be L_1 and L_2.

$L_1 = 4\,(l)^2$
$= 4\,(12)^2$
$= 4\,(144)$
$= 576\ cm^2$

$L_2 = 2h\,(l + b)$
$= 2 \times 8\,(15 + 12)$
$= 432\ cm^2$

$L_1 - L_2 = 576 - 432$
$= 144\ cm^2$

Thus, the cubical box has greater lateral surface area and is greater by 144 cm^2.

Example 3 : Kanjibhai had built closed cubical water tank with lid for his factory. The length, breadth and height of the tank are 2.5 *m*, 1.5 *m* and 1 *m* respectively. He wants to cover outer surface of the tank (excluding the base) with square tiles of side 25 *cm*. Find out the number of tiles and total cost, if the rate of the tiles is ₹ 480 per dozen.

(1 dozen = 12 units)

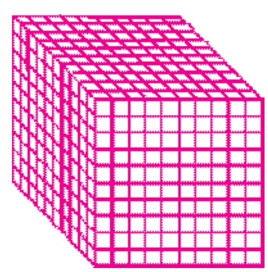


Figure 15.8

Solution : First we should find out total surface area of five outer faces of tank.

Length of the tank = 2.5 *m* = 250 *cm*

Breadth of the tank = 1.5 *m* = 150 *cm*

Height of the tank = 1 *m* = 100 *cm*

$\therefore$ Surface Area (excluding base) $= l \times b + 2(b \times h) + 2\,(h \times l)$

$= [250 \times 150 + 2\,(150 \times 100) + 2\,(100 \times 250)]$

$= (37500 + 30000 + 50000)$

$= 117500\ cm^2$

Area of each square tile = $(25 \times 25)\ cm^2$

$\therefore$ Number of tiles required = $\dfrac{\text{area of the tank}}{\text{area of one tile}} = \dfrac{117500}{25 \times 25}$ = 188 tiles

Since cost of 12 tiles is ₹ 480, cost of 188 tiles = $\dfrac{480 \times 188}{12}$ = ₹ 7520

$\therefore$ Number of tiles required is 188 and total cost is ₹ 7520.

Note : In fact $\frac{250}{25} \times \frac{150}{25}$ tiles are required for top.

$\therefore$ Total numbers of tiles required for top = $10 \times 6 = 60$

Similarly total numbers of tiles required for sides

$= 2\left(\frac{150}{25} \times \frac{100}{25} + \frac{250}{25} \times \frac{100}{25}\right)$

$= 2(6 \times 4 + 10 \times 4) = 128$

$\therefore$ Total number of tiles required is 128 + 60 = 188.

If l or b or h is not a multiple of 25 then tiles would have to be broken ! Not a practical solution.

Example 4 : A hall for prayer in a school is 10 m long, 8 m wide and 5 m high. It has two doors each measuring $(3 \times 1.5)\ m^2$ and Four windows, each measuring $(2 \times 2)\ m^2$. Find the total expense for whitewashing the interior walls. The rate of whitewashing is ₹ 6 per m^2.

Solution : Area of four walls = (Lateral surface area of cuboidal hall)

$= 2h\,(l + b)$

$= 2 \times 5\,(10 + 8)$

$= 180\ m^2$

Area of two doors $= 2\,(3 \times 1.5) = 9\ m^2$

Area of four windows $= 4\,(2 \times 2) = 16\ m^2$

Area to be whitewashed = (Area of four walls with door and windows) – (Area of doors + Area of windows)

$= (180 - (9 + 16)) = 155\ m^2$

The rate of whitewashing is ₹ 6 per m^2.

$\therefore$ cost of whitewashing $= (155 \times 6)$

= ₹ 930

$\therefore$ The cost of whitewashing is ₹ 930.

EXERCISE 15.1

1. Fill in the blanks in each row in the following table from given information :

No.	length	breadth	height	lateral surface area	Total surface area
(1)	18 *cm*	10 *cm*	5 *cm*	.........cm^2	.........cm^2
(2)	3 *m*	3 *m*	3 *m*	.........m^2	.........m^2
(3)	1 *m*	75 *cm*	50 *cm*	.........cm^2	.........cm^2

2. A small indoor green house (herberium) is made entirely of glass panes (including base) held together with tape. It is 40 *cm* long, 30 *cm* wide and 25 *cm* high.

(1) What is the area of the glass panes used ?

(2) Find the cost of glass painting of four walls of the green-house. The rate of glass-painting is ₹ 500 per m^2.

3. Find the area of the four walls and ceiling of a room, whose length is 10 *m*, breadth is 8 *m* and height is 5 *m*. Also find the cost of whitewashing the walls and ceiling, at the rate of ₹ 15 per m^2.

4. The floor of a rectangular hall has a perimeter of 300 *m*. Its height is 10 *m*. There are two doors of 5 $m \times 3$ *m* and four windows of 3 $m \times 1.5$ *m*. Find the cost of painting of its four walls at the rate of ₹ 30 per m^2.

5. A cubical box is 15 *cm* long and another cuboidal box is 25 *cm* long, 20 *cm* wide and 10 *cm* high.

(1) Which box has the smaller lateral area and by how much ?

(2) Which box has the greater total surface area and by how much ?

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15.5 Surface Area of a Right Circular Cylinder

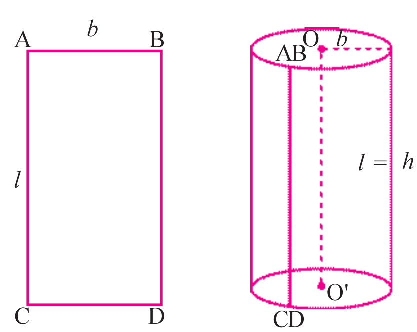

Figure 15.9

We know about a cylinder and formula to find its area.

Activity (1) : A cylinder is generated by the revolution of a rectangle about one of its sides. This cylinder is called a **right circular cylinder.**

Top and bottom of a right circular cylinder are parallel circular region.

In figure 15.9, breadth of the rectangle CD namely (b) becomes the circumference of the base. The radius of the base is the radius of the cylinder. The length of the rectangle (l) becomes the height (h) of the cylinder.

The line-segment joining the two centres of circular ends is perpendicular to base. This is the height (h) of cylinder. If the line-segment is not perpendicular to base, then what is the situation ? Let us see.

Activity (2) : If we take a number of coins of five rupees and stack them vertically up, then we get a right circular cylinder (figure 15.10).

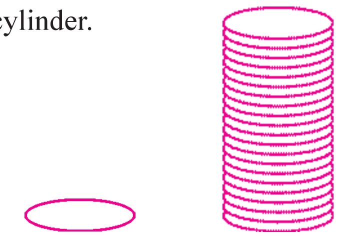

Figure 15.10

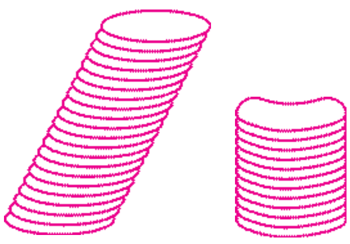

Figure 15.11

Keep in mind that stack of coins has been kept at right angle to the base and the base is circular.

Figure 15.11 does not represent right circular cylinder.

Note : In our study, a cylinder would mean a right circular cylinder.

Activity (3) : Now, we take a sufficiently large coloured rectangular paper, whose length is just enough to go round the cylinder and whose breadth is equal to the height of the cylinder (see figure 15.12).

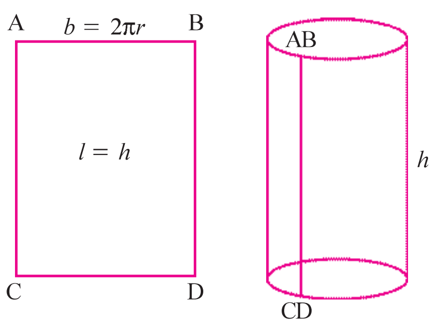

Figure 15.12

The rectangular region ABDC gives us curved surface of the cylinder. The breadth (b) of the rectangle is equal to the circumference of the circular base of the cylinder which is equal to $2\pi r$. The length (l) of the rectangle is the height (h) of the cylinder.

$\therefore$ Curved surface area of the cylinder = Area of the rectangle

= length $\times$ breadth

= perimeter of the base of the cylinder $\times$ height of the cylinder

= $2\pi r \times h = 2\pi rh$

$\therefore$ **Curved surface area of the cylinder = $2\pi rh$**

If the top and the bottom of the cylinder are also to be covered, since both the ends are circular and radius of the circular base of the cylinder is r, area of the circular ends is $2\pi r^2$

$\therefore$ **Total surface area of the cylinder = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$**

Example 5 : The diameter and the height of a closed cylindrical water tank are 1 m and 14 m respecively. Find the total cost for painting the lateral surface area of this tank, if the cost per m^2 is ₹ 25.

Solution : Here, radius = $\dfrac{\text{diameter}}{2} = \dfrac{1}{2}$ m, height = 14 metre

$\therefore$ Lateral surface area of the cylindrical tank = $2\pi rh$

$$= \left(2 \times \frac{22}{7} \times \frac{1}{2} \times 14\right) = 44\ m^2$$

Cost of the painting per 1 m^2 = ₹ 25

$\therefore$ Cost of the painting 44 m^2 = (44×25) = ₹ 1100

$\therefore$ Total cost for painting lateral surface is ₹ 1100.

Example 6 : The diameter of a 140 cm long roller is 80 cm. Find the area covered by roller in 600 complete revolutions to level the ground.

Solution : The roller is a right circular cylinder of height h = 140 cm and radius of its base is 40 cm.

Area covered by the roller in one revolution

= The curved surface area of the roller

$= 2\pi rh$

$= \left(2 \times \frac{22}{7} \times 40 \times 140\right)$

$= 35,200\ cm^2$

$\therefore$ The area covered by the roller in 600 revolution $= (35200 \times 600)$

$= 21120000\ cm^2$

$= \dfrac{21120000}{10000}\ m^2$

$= 2112\ m^2$

EXERCISE 15.2

1. Fill in the blanks in the following table using the information given about a cylinder :

No.	Value of π	Radius of base	Height	Curved surface area	Total surface area
(1)	$\frac{22}{7}$	14 *cm*	20 *cm*	...... cm^2	...... cm^2
(2)	$\frac{22}{7}$	...... *cm*	14 *cm*	616 cm^2	...... cm^2
(3)	3.14	15 *cm*	30 *cm*	...... cm^2	...... cm^2

2. The radius and the height of a cylindrical tank with lid are 28 *cm* and 1 *m* respectively. Find the cost of painting the outer surface of the cylindrical tank at the rate of ₹ 1 per cm^2. (Neglect the area of the bottom.)

3. The curved surface area of a cylinder is 3696 cm^2. If the radius of the cylinder is 14 *cm*, find the height of the cylinder.

4. The height of a cylinder is 28 *cm* and curved surface area is 2816 cm^2. Find its diameter.

5. The radius and the height of a cylinder are equal to 50 *cm*. Find the total surface area. (π = 3.14)

6. 50 circular plates each of diameter 14 *cm* and thickness 0.5 *cm* are placed one above the other to form a right circular cylinder. Find the total surface area.

7. The inner diameter of a circular well is 4.2 *m*. It is 20 *m* deep. Find (i) the inner curved surface area (ii) the cost of plastering this curved surface at the rate of ₹ 50 per m^2.

*

15.6 Surface Area of a Right Circular Cone

In our day-to-day life we often see objects like an ice-cream cone, a conical tent, a conical vessel, a clown's cap, etc. We get an idea about a cone from observation of these objects.

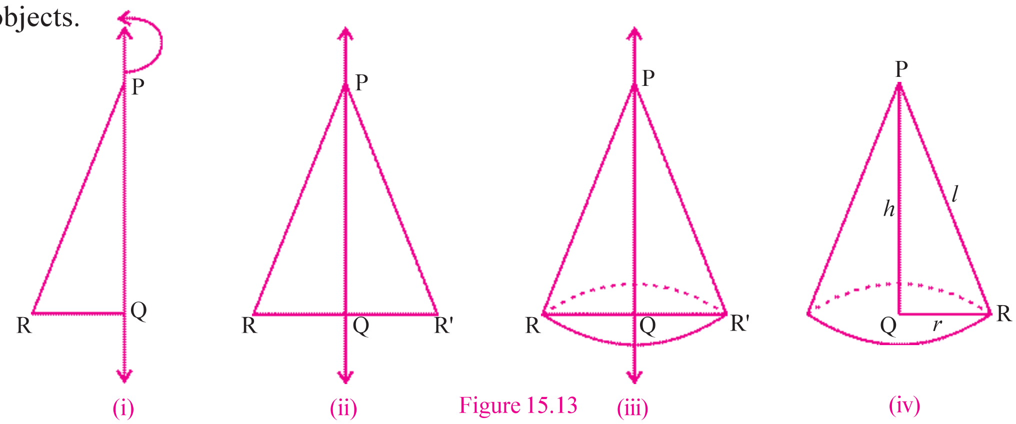

Figure 15.13

Activity : In figure 15.13 (i) P is a fixed point. $\overleftrightarrow{PQ}$ is fixed line and $\overleftrightarrow{PR}$ is a revolving line. $\angle$PQR is right angle. Now we revolve Δ*PQR around the $\overleftrightarrow{PQ}$. If we revolve Δ*PQR about $\overleftrightarrow{PQ}$ we get a **right circular cone** (figure 15.13 (iii)). We get a solid cone with a circular base having centre at Q and radius RQ. $\overline{PQ}$ is perpendicular line-segment joining vertex P and centre Q of the circular base of the cone.

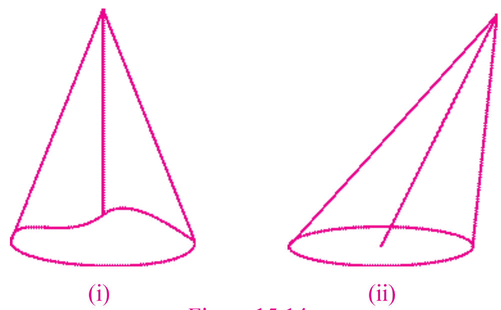

Figure 15.14

PQ is the height of the cone, denoted by h. Radius of the circular base is called the radius of the cone and is denoted by r. PR is the slant height of the cone and is denoted by l.

In ΔPQR, $m\angle$Q = 90. Since $l^2 = h^2 + r^2$, $l = \sqrt{h^2+r^2}$

Observe that figure 15.14 does not represent a right circular cone. In our study, a cone would mean a right circular cone.

Activity : Cut out a neatly made paper cone (figure 15.15 (i)) along the slant height $\overline{PR}$ and spread it on a table. We will find that the spread out (figure 15.15 (ii)) figure is a sector of a circle of radius equal to the slant height (l) of the cone and whose length of arc is equal to circumference of the circular base of the cone.

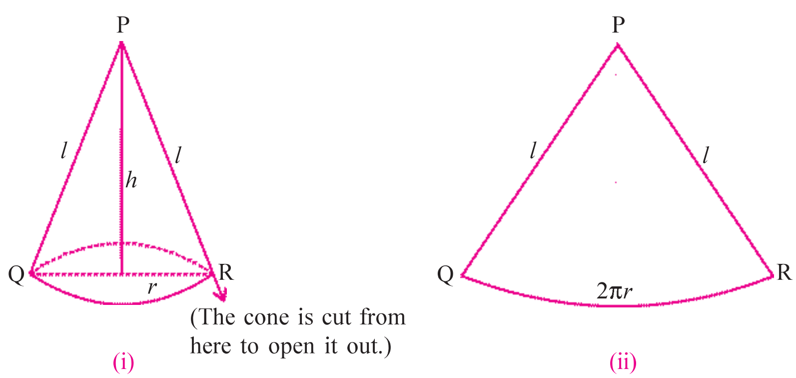

Figure 15.15

We assume that area of a sector of a circle with radius r and arc length l is $\frac{1}{2}lr$.

Curved surface area of the cone = area of the sector PQR.

$$= \frac{1}{2} \times (\text{length of arc}) \times (\text{radius})$$

$$= \frac{1}{2} \times (2\pi r) \times l = \pi rl$$

Total surface area of the cone = curved surface area + area of the circular base

$= \pi rl + \pi r^2$

$= \pi r\,(l + r)$

The curved surface area of a cone is also called the lateral surface area of the cone.

Example 7 : Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find the radius of the base and total surface area.

Solution : We have curved surface area = 308 cm^2, slant height l = 14 cm

$\therefore\ \pi rl = 308$

$\therefore\ \frac{22}{7} \times r \times 14 = 308$

$\therefore\ r = \frac{308 \times 7}{14 \times 22} = 7\ cm$

Total surface area $= \pi rl + \pi r^2$

$= \left(308 + \frac{22}{7} \times 7 \times 7\right)$

$= (308 + 154) = 462\ cm^2$

The radius of the base is 7 cm. The total surface area is 462 cm^2.

Example 8 : The radius and the slant height of a cone are in the ratio 4 : 7. If its curved surface area is 792 cm^2, find its radius.

Solution : Let r be the radius and l be the slant height of the cone.

$\therefore\ r : l = 4 : 7$. So let $r = 4x$ and $l = 7x$, $x > 0$

Now, curved surface area = 792 cm^2

$\therefore\ \pi rl = 792$

$\therefore\ \frac{22}{7} \times 4x \times 7x = 792$

$\therefore\ 88 \times x^2 = 792$

$\therefore\ x^2 = \frac{792}{88} = 9$

$\therefore\ x = 3$ **($x > 0$)**

$\therefore\ r = 4x = 12\ cm$

$\therefore$ The radius is 12 cm.

Example 9 : How many metres of cloth 2 m wide will be required to make a conical tent having the radius of base 7 m and height 24 m.

Solution : radius r = 7 m, height h = 24 m

$$\therefore\ l = \sqrt{r^2 + h^2} = \sqrt{(7)^2 + (24)^2}$$
$$= \sqrt{49 + 576}$$
$$= \sqrt{625}$$
$$= 25\ m$$

$\therefore$ The curved surface area of the cone $= \pi r l$

$$= \left(\frac{22}{7} \times 7 \times 25\right) = 550\ m^2$$

The area of the cloth used $= 550\ m^2$

The width of the cloth $= 2\ m$

$\therefore$ Length of the cloth used $= \dfrac{\text{Area}}{\text{Width}} = \dfrac{550}{2} = 275\ m$

$\therefore$ The length of cloth required is 275 m.

Example 10 : A corn cob (figure 15.16) shaped some what like a cone, has the radius of its broadest end as 2.1 cm and length (height) as 20 cm. If each 1 cm^2 of the surface of the cob carries an average of four grains, find how many grains you would find on the entire cob.

Solution : Since the grains of corn are found only on the curved surface of the corn cob, we would need to know the curved surface area of the corn cob to find the total number of grains on it. In this question, we are given the height of the cone, so we need to find its slant height.

Figure 15.16

Here, $l = \sqrt{r^2 + h^2} = \sqrt{(2.1)^2 + (20)^2}$

$$= \sqrt{404.41} = 20.11\ cm \text{ (approx)}$$

Therefore, the curved surface area of the corn cob $= \pi r l$

$$= \frac{22}{7} \times 2.1 \times 20.11$$
$$= 132.726$$
$$= 132.73\ cm^2 \text{ (approx)}$$

Number of grains of corn on 1 cm^2 of the surface of the corn cob = 4

$\therefore$ Number of grains on the entire curved surface of the cob $= 132.73 \times 4$

$$= 530.92 = 531 \text{ (approx)}$$

So, there would be approximately 531 grains of corn on the cob.

EXERCISE 15.3

1. Fill the blanks in the following table from the given information for the cone :

No.	Radius of base	Height	Slant height	Lateral surface area	Total surface area
(1)	...... *cm*	9 *cm*	15 *cm*	...... $\pi\ cm^2$	...... $\pi\ cm^2$
(2)	7 *cm*	---	9 *cm*	...... $\pi\ cm^2$	...... $\pi\ cm^2$
(3)	3 *cm*	4 *cm*	...... *cm*	...... $\pi\ cm^2$	...... $\pi\ cm^2$

2. A conical tent is 12 *m* high and the radius of its base is 5 *m*. Find (i) the slant height (ii) the cost of the canvas required to make, if the cost of 1 m^2 canvas is ₹ 100. ($\pi = 3.14$)

3. A joker's cap is in the form of a right circular cone of base radius 7 *cm* and height 24 *cm*. Find the area of the sheet of paper required to make 15 such caps.

4. The slant height of a closed cone is seven times the radius of its base. If the radius of the base is 3 *cm*, find the total surface area. ($\pi = 3.14$)

5. How many conical tents, each of height 4 *m* and radius of base 3 *m*, can be prepared from cloth 282.60 m^2. ($\pi = 3.14$)

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15.7 Surface Area of a Sphere

The shape of cricket ball, a tennis ball, a football and a volleyball is a sphere.

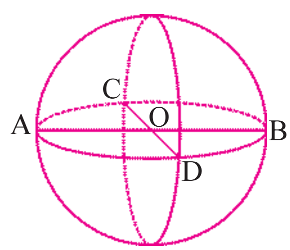

Figure 15.17

Sphere : The set of all points in space, which are equidistant from a fixed point is called a sphere.

The fixed point is called the centre of the sphere and the constant distance is its radius. The diameter is a line-segment passing through the centre of the sphere with the endpoints on the sphere.

AO = BO = OD = OC (radii of the same sphere)

AB = CD (diameters of the same sphere)

Activity : If we pass a string along the diameter of circular disc and rotate it, we get a solid figure called a sphere.

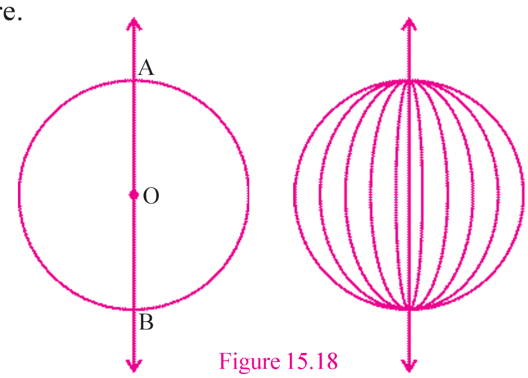

Figure 15.18

The surface area of a sphere having radius r is $4\pi r^2$.

If we divide a sphere into two equal parts by a plane passing through the centre, then what we get is called a **hemisphere.**

Lateral surface area of the outer side of the hemisphere = $2\pi r^2$. Lateral surface consists of the outer surface of the hemisphere and the circular plane surface.

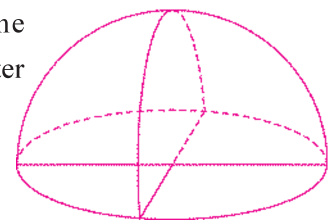

Figure 15.19

Total surface area of solid hemisphere

= Lateral surface area of the hemisphere + Area of the circular base.

$= 2\pi r^2 + \pi r^2 = 3\pi r^2$

$\therefore$ **total surface area of solid hemisphere = $3\pi r^2$**

Example 11 : If the ratio of total surface area of a closed solid hemishpere and surface area of a sphere is 25 : 108, find the ratio of their radii in the same order

Solution : Suppose the radius of the closed hemisphere is r_1 and the radius of the sphere is r_2. Suppose their surface areas are A_1 and A_2. Then

$A_1 = 3\pi r_1^2$, and $A_2 = 4\pi r_2^2$

$$\frac{A_1}{A_2} = \frac{3\pi r_1^2}{4\pi r_2^2}$$

$$\therefore \frac{25}{108} = \frac{3\pi r_1^2}{4\pi r_2^2}$$

$$\therefore \frac{25\times 4}{108\times 3} = \frac{r_1^2}{r_2^2}$$

$$\therefore \frac{r_1^2}{r_2^2} = \frac{25}{81}$$

$$\therefore \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{5}{9}\right)^2$$

$$\therefore \frac{r_1}{r_2} = \frac{5}{9}$$

$\therefore$ The ratio of their radii in the same order is 5 : 9.

Example 12 : A sphere, a cylinder and a cone have same radius and same height. Find the ratio of the areas of their curved surfaces.

Solution : Let r be the common radius of the sphere, the cone and the cylinder.

Then, the height of the cone = the height of the cylinder = the height of the sphere = $2r$

Let l be the slant height of the cone.

Then, $l = \sqrt{r^2 + h^2}$

$= \sqrt{r^2 + 4r^2} = \sqrt{5r^2} = \sqrt{5}\, r$

Let S_1 = the curved surface area of the sphere = $4\pi r^2$

S_2 = the curved surface area of the cylinder = $2\pi r \times 2r = 4\pi r^2$

S_3 = the curved surface area of the cone = $\pi r l = \pi r \times \sqrt{5}\, r = \sqrt{5}\,\pi r^2$

$\therefore S_1 : S_2 : S_3 = 4\pi r^2 : 4\pi r^2 : \sqrt{5}\,\pi r^2 = 4 : 4 : \sqrt{5}$

The ratio of their curved surface areas is $4 : 4 : \sqrt{5}$.

*

EXERCISE 15.4

1. Fill the blanks in the following table from the given information for the sphere :

No.	Value of π	Radius	Diameter	Total surface area of sphere	Lateral surface area of hollow hemisphere	Surface area of solid hemisphere
(1)	$\frac{22}{7}$	5.6 *cm*	...... *cm*	...... cm^2	...... cm^2	...... cm^2
(2)	3.14	10 *cm*	...... *cm*	...... cm^2	...... cm^2	...... cm^2
(3)	$\frac{22}{7}$	...... *cm*	...... *cm*	154 cm^2	...... cm^2	.... cm^2

2. The radius of a spherical balloon increases from 14 *cm* to 21 *cm* as air is pumped into it. Find the ratio of the surface areas of the balloon in the two situations.

3. The internal and external radii of a hollow hemispherical vessel are 15 *cm* and 16 *cm* respectively. The cost of painting 1 cm^2 of the surface is ₹ 7. Find the total cost of painting the vessel all over. (ignore the area of edges)

4. The total surface area of the solid hemisphere is 462 cm^2. Find the radius of hemisphere.

5. The diameter of hemisphrical lid is 2 metre. 500 hemispherical lids are prepared in a factory. Find the expense to paint outer surface of lids at ₹ 20 per m^2. (π = 3.14)

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15.8 Volume of a Cuboid

We have already learnt about volume of cuboid, cube etc. in previous classes. We also know that solid objects occupy space. The measure of this occupied space is called the volume of the object.

If the object is hollow, then interior part can be filled with air or liquid that will fill the space of its container. The volume of air or liquid that can fill this interior is called capacity of the container.

There is a cuboid of length l, breadth b and height h in figure 15.20. The area of the rectangular base PQRS is $(l \times b)$.

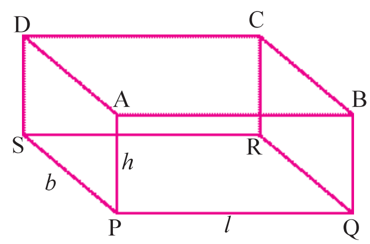
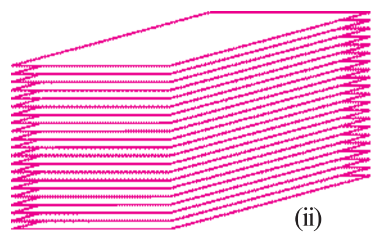


Figure 15.20

If we take rectangular sheets congruent to the base PQRS of the cuboid and stack them up, we get a cuboid of height h given in the figure 15.21(ii),

(i)



(ii)

Figure 15.21

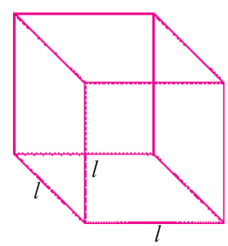


Figure 15.22

The measure of the space occupied by the cuboid (V)

$= \text{Area of the rectangular sheet} \times h = (l \times b) \times h$

$\therefore$ **Volume of cuboid $= l \times b \times h$**

$=$ Area of the base $\times$ height

Volume of the cube with sides of length $l = l \times l \times l = l^3$

Note : For the calculation of volume, the length, breadth and height must be expressed in the same units.

Example 13 : The capacity of a cuboidal tank is 60,000 litres. Find the breadth of the tank if its length and depth are 4 m and 1.5 m respectively.

Solution : Let the breadth of the tank be b metres. We know 1000 litres = 1 m^3

We have, V = 60,000 litres

$$= \frac{60000}{1000}\ m^3 = 60\ m^3$$

$l = 4\ m,\ h = 1.5\ m$

$$\text{Breadth} = \frac{\text{volume}}{\text{length} \times \text{height}} = \frac{60 \times 10}{4 \times 15} = 10\ m$$

$\therefore$ The breadth is 10 m.

Example 14 : A cube of edge 6 *cm* is immersed completely in a cuboidal vessel containing water and water does not overflow. If the dimensions of the base are 12 *cm* and 10 *cm*, find the rise in the water level in the vessel.

Solution : The edge of the given cube = 6 *cm*

The volume of the cube = $(6)^3 = 216\ cm^3$

If the cube is immersed in the vessel, then the water level rises.

Let the rise in the water level be *a cm*.

The volume of the cube = The volume of the water replaced by it

$\therefore$ The volume of the cube = The volume of the cuboid with dimensions $12\ cm \times 10\ cm \times a\ cm$

$\therefore 216 = 12 \times 10 \times a$

$\therefore a = \frac{216}{12 \times 10} = 1.8\ cm$

$\therefore$ The rise in the water level is 1.8 *cm*

Example 15 : A pit of length 20 *m* and breadth 15 *m* is dug 10 *m* deep. The earth taken out of it is spread evenly all around it to form a platform on a square ground of length 50 *m*. Find the height of the platform.

Solution : The volume of the earth taken out of the pit = The volume of the platform

The length of pit = 20 *m*, The breadth of pit = 15 *m*, The height of pit = 10 *m*

The length of the platform on a square ground = 50 *m*

$\therefore$ The volume of the earth spread from the pit = $l \times b \times h = (20 \times 15 \times 10)\ m^3$

Let x be the height of platform.

The volume of the earth spread to form the platform = $(50 \times 50 \times x)\ m^3$

$\therefore 20 \times 15 \times 10 = 50 \times 50 \times x$

$$\therefore x = \frac{20 \times 15 \times 10}{50 \times 50} = \frac{6}{5} = 1.20\ m$$

$\therefore$ The height of the platform formed on square base is = 1.20 *m*.

EXERCISE 15.5

1. A chalk-box measures 10 *cm* × 8 *cm* × 6 *cm*. What will be the volume of a packet containing 6 such boxes ?
2. A co-operative society has cuboidal water tank having dimensions 4 *m* × 3 *m* × 2 *m*. How many litres of water can it hold ?
3. A cuboidal vessel is 8 *m* long and 6 *m* wide. How much height should it have in order to hold 30,000 litres of liquid ?
4. A village, having a population of 5000, consumes 200 litres of water per head per day. It has a tank having dimensions 25 *m* × 20 *m* × 10 *m*. For how many days will the water of this tank last ?

5. A godown measures 45 $m \times 30$ $m \times 15$ m. Find the maximum number of wooden crates each measuring 2.5 $m \times 1$ $m \times 0.75$ m that can be stored in godown.

6. If the areas of three adjacent faces of a cuboid are 16 cm^2, 12 cm^2 and 27 cm^2, find the volume of the cuboid.

7. A cuboidal well of dimension 55 $m \times 20$ $m \times 7$ m is dug and the earth obtained from digging is evenly spread out to form a platform having rectangle base 22 $m \times 14$ m. Find the height of the platform.

8. A metallic sheet is of the rectangular shape with dimensions 50 $cm \times 40$ cm. From each one of its corner, a square of 5 cm is cut off. An open box is made of the remaining sheet. Find the volume of the box.

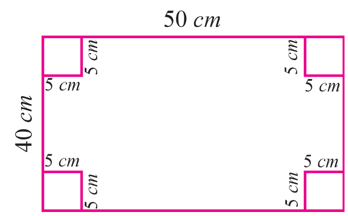


Figure 15.23

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15.9 Volume of Cylinder

Let us take circular sheets of radius r and stack them up vertically to form a right circular cylinder of height h.

Then volume of the cylinder = Measure of the space occupied by the cylinder

= area of each circular sheet $\times$ height

$= \pi r^2 \times h$

$= \boldsymbol{\pi r^2 h}$

Example 16 : The circumference of the base of a cylinder is 165 cm and its height is 40 cm. Find the volume of the cylinder.

Solution : Let r be the radius of the cylinder. Now circumference is 165 cm.

$\therefore\ 2\pi r = 165$

$\therefore\ 2 \times \frac{22}{7} \times r = 165$

$r = \frac{165 \times 7}{2 \times 22} = \frac{105}{4}$ cm

Also the height of the cylinder = 40 cm

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times \frac{105}{4} \times \frac{105}{4} \times 40$

$= 86625$ cm^3

$\therefore$ The volume of the cylinder is 86625 cm^3.

Example 17 : A solid cylinder has total surface area 462 cm^2. Its curved surface area is one-third of its total surface area. Find the volume of the cylinder.

Solution : Let r be the radius and h be the height of cylinder.

Total surface area $= 2\pi rh + 2\pi r^2$

The curved surface area $= 2\pi rh$

$\therefore$ The curved surface area $= \frac{1}{3}$ (Total surface area)

$$= \frac{1}{3} \times 462 = 154$$

$\therefore 2\pi rh = 154$

Now total surface area $= 462\ cm^2$

$\therefore 2\pi rh + 2\pi r^2 = 462$

$\therefore 154 + 2\pi r^2 = 462$

$\therefore 2\pi r^2 = 308$

$\therefore 2 \times \frac{22}{7} \times r^2 = 308$

$\therefore r^2 = \frac{308 \times 7}{2 \times 22} = 7 \times 7$

$\therefore r = 7\ cm$

Now $2\pi rh = 154$

$\therefore \frac{2 \times 22}{7} \times 7 \times h = 154$

$\therefore h = \frac{154}{2 \times 22} = \frac{7}{2}\ cm$

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times \frac{7}{2}$$

$$= 539\ cm^3$$

$\therefore$ Volume of the cylinder is 539 cm^3.

Example 18 : A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform with rectangular base having dimension $(22 \times 14)\ m^2$. Find the height of the platform.

Solution : The volume of the earth taken out of the well

= The volume of the cylinder of radius $\frac{7}{2}$ m and height 20 m

$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 20 = 770 \ m^3$

Let the height of the platform be equal to x metres.

$\therefore$ The volume of platform = The volume of the earth taken out of the well

$\therefore 22 \times 14 \times x = 770$

$\therefore x = \frac{770}{22 \times 14} \ m$

$\therefore x = \frac{5}{2} = 2.5 \ m$

$\therefore$ The height of the platform is 2.5 m.

Example 19 : The pillars of a temple are cylindrically shaped (see figure 15.24). If each pillar has a circular base of radius 20 cm and height 10 m, how much concrete mixture would be required to build 14 such pillars ?

Solution : Since the concrete mixture to be used to build up the pillars is going to occupy the entire space of the pillar, what we need to find here is the volume of the cylinders.

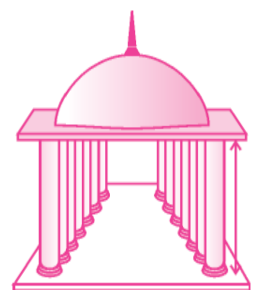

Figure 15.24

The radius of the base of the cylinder = 20 cm

The height of the cylindrical pillar = 10 m = 1000 cm

So, the volume of each cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 20 \times 20 \times 1000$

$= \frac{8800000}{7} \ cm^3$

$= \frac{8.8}{7} \ m^3$ (since 1000000 cm^3 = 1 m^3)

Therefore, the volume of 14 pillars = The volume of each cylinder × 14

$= \frac{8.8}{7} \times 14 = 17.6 \ m^3$

So, 14 pillar would require 17.6 m^3 concrete mixture.

EXERCISE 15.6

1. The circumference of the base of a cylindrical vessel is 220 *cm* and height is 35 *cm*. How many litres of water can it hold ?

2. If the diameter and the height of a carrom coin are 4 *cm* and 0.5 *cm* respectively, find the volume of the cylinder made up of such 12 carrom coins stacked on each other. ($\pi = 3.14$)

3. The capacity of a cylindrical cistern at a petrol pump is 38,500 litres. If its diameter is 3.5 *m*, find its height.

4. Find the height of a cylindrical tank having radius 3 *m* to supply 1413 litres of water to each of 60 houses of a society ? ($\pi = 3.14$)

5. The curved surface area of a cylinder is 440 cm^2 and its height is 7 *cm*. Find the volume of the cylinder.

6. A soft drink is available in two packs : (i) a tin can with a rectangular base of length 6 *cm* and width 5 *cm*, having a height of 20 *cm* and (ii) a cylindrical tin with circular base of radius 3.5 *cm* and height 20 *cm*. Which container has greater capacity and by how much amount ?

7. How many completely full bags of wheat can be emptied into a cylindrical drum of radius 1.4 *m* and height 7 *m*, if the space required for wheat in each bag is 0.4312 m^3.

8. The radius and height of a cylinder are in the ratio 5 : 7 and its volume is 550 cm^3. Find its radius.

9. The curved surface area of a cylindrical pillar is 264 m^2 and its volume is 924 m^3. Find the radius and the height of the pillar.

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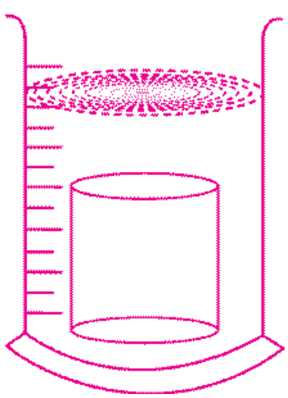
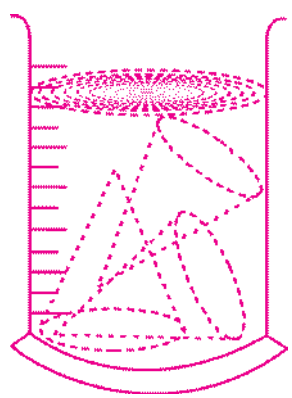
15.10 Volume of a Cone

We understand the formula for volume of a cone through an activity.



Marked cylinder - 1



Marked cylinder - 2

Figure 15.25

Both the marked cylinders are of the same size. Both are filled with water upto the same mark. We have certain number of cylinders and cones having the same height and radii of the base. We measure the increase in the level of water, when a cyinder is immersed in the first cylinder without overflow and a cone is immersed in the second cylinder. We observe that the level of water in the second is lower than that in the first cylinder. According to Archimedes principle the levels equal only when three cones are immersed in the second cylinder. Thus, we deduce that when a cylinder and a cone have same height and same radii of the base, then the volume of 1 cylinder = the volume of 3 cones

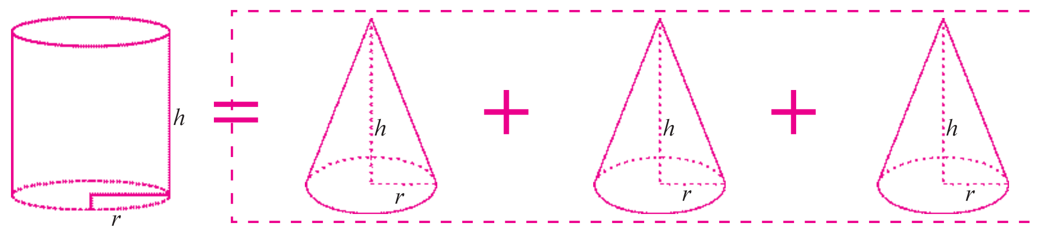


Figure 15.26

3 × the volume of a cone = the volume of cylinder (with the same height and radius)

$\therefore$ 3 × the volume of the cone = $\pi r^2 h$

$\therefore$ **The volume of a cone = $\frac{1}{3}\pi r^2 h$**

Example 20 : A conical vessel whose internal radius is 5 *cm* and height 24 *cm* is full of water. The water is poured completely into an empty cylindrical vessel with internal radius 10 *cm*. Find the height to which the water level increases.

Solution :

	cone	**cylinder**
Radii	$r_1 = 5\ cm$	$r_2 = 10\ cm$
Height	$h_1 = 24\ cm$	$h_2 = ?$

Suppose water rises up to the height of h_2 *cm* in cylindrical vessel.

Clearly, the volume of water in the conical vessel = the volume of water in the cylindrical vessel

Now, the volume of a cone = $\frac{1}{3}\pi r^2 h$ and the volume of a cylinder = $\pi r^2 h$

$\therefore \frac{1}{3}\pi r_1^2 h_1 = \pi r_2^2 h_2$

$\therefore \pi r_1^2 h_1 = 3\pi r_2^2 h_2$

$\therefore 5 \times 5 \times 24 = 3 \times 10 \times 10 \times h_2$

$\therefore h_2 = \frac{5\times5\times24}{3\times10\times10} = 2\ cm$

$\therefore$ The increase in the height of water level in cylindrical vessel is 2 *cm*.

Example 21 : A conical tent is to accommodate 22 persons. Each person must get 4 m^2 of the space on the ground and 30 m^3 of air to breath. Find the height of the tent.

Solution : Let h be the height and r be the radius of base of the cone. The tent can accommodate 22 persons and each person requires 4 m^2 of the space on the ground and 30 m^3 of air.

Required area of the base = $\pi r^2 = (22 \times 4) = 88\ m^2$

Volume of the cone = $\frac{1}{3}\pi r^2 h = (22 \times 30) = 660\ m^3$

$$\therefore \frac{\frac{1}{3}\pi r^2 h}{\pi r^2} = \frac{660}{88}$$

$$\therefore \frac{h}{3} = \frac{15}{2}$$

$$\therefore h = \frac{45}{2} = 22.5\ m$$

$\therefore$ The height of the tent is 22.5 m.

EXERCISE 15.7

1. Find the volume of a right circular cone with :

(1) radius 4 cm, height 14 cm

(2) radius 7 cm, height 12 cm

(3) height 12 cm, slant height 15 cm. ($\pi = 3.14$)

2. Find the volume of a cone having radius of its base 15 cm and height twice that of its radius of the base. ($\pi = 3.14$)

3. There are 15 conical heaps of wheat, each of them having diameter 70 cm and height 24 cm, in the farm of Ramjibhai. To stock the wheat in a cylindrical container of the same radius, what should be its height ?

4. A cone of a radius and height 21 cm is filled with water. If water from the cone is poured into a cylinder of radius 21 cm, find the height of the cylinder.

5. Find the volume of a cone having diameter of the base 18 m and height 7 m.

6. The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find (i) the height of the cone, (ii) the slant height of the cone, (iii) the curved surface area of the cone.

*

15.11 Volume of Sphere

We accept that volume of a sphere = $\frac{4}{3}\pi r^3$ and volume of a hemisphere = $\frac{2}{3}\pi r^3$

Example 22 : The volume of two spheres are in the ratio 125 : 27. Find the difference of their surface areas, if sum of their radii is 8 *cm*.

Solution : Let the radii of the two spheres be r_1 *cm* and r_2 *cm*.

$\therefore \frac{V_1}{V_2} = \frac{125}{27}$

$\therefore \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{125}{27}$. $\therefore \frac{r_1^3}{r_2^3} = \frac{5^3}{3^3}$

$\therefore \frac{r_1}{r_2} = \frac{5}{3}$

$\therefore r_1 = \frac{5}{3}r_2$

Now, $r_1 + r_2 = 8$

$\therefore \frac{5}{3}r_2 + r_2 = 8$

$\therefore \frac{8}{3}r_2 = 8$

$\therefore r_2 = 3$ *cm*. Also $r_1 = \frac{5}{3}r_2 = 5$ *cm*

$S_1 = 4\pi r_1^2 = 4\pi (5)^2 = 100\pi\ cm^2$; $S_2 = 4\pi r_2^2 = 4\pi (3)^2 = 36\pi\ cm^2$

$\therefore S_1 - S_2 = 100\pi\ cm^2 - 36\pi\ cm^2 = 64\pi\ cm^2$

$\therefore$ The difference of their surface areas is $64\pi\ cm^2$

EXERCISE 15.8

1. Find the volume of the sphere whose radius is :
   (1) 6 *cm* ($\pi = 3.14$) (2) 7 *cm* (3) 10.5 *cm*
2. Find the volume of the hemisphere having the radius (1) 14 *cm* (2) 21 *cm*.
3. A hemispherical tank has inner diameter 4.2 *m*. Find its capacity in litres.
4. A sphere of radius 10 *cm* is immersed in a cylinder filled with water. The level of water rises by $\frac{10}{3}$ *cm*. Find the radius of the cylinder.
5. A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their radii and heights.

EXERCISE 15

1. Find the ratio of the total surface area of a cylinder to its curved surface area, given that its height and radius are 35 *cm* and 14 *cm* respectively.

2. A solid cylinder has total surface area of 1386 cm^2. Its curved surface area is one-ninth of its total surface area. Find the radius and height of the cylinder.

3. Find the ratio of the surface areas of two cones if their radii of the bases are equal and slant heights are in the ratio 2 : 3.

4. The lateral surface area of a cylinder is equal to the curved surface area of a cone. If the radius be the same, find the ratio of the height of the cylinder to the slant height of the cone.

5. A cube of edge 15 *cm* is immersed completely in a cuboidal vessel containing water. If the dimensions of the base are 18 *cm* and 15 *cm*, find the water level rise in vessel.

6. A rectangular sheet of paper 44 $cm \times$ 22 *cm* is rolled along its length to form a cylinder. Find the volume of the cylinder so formed.

7. Select proper option (a), (b), (c) or (d) and write in the box given on the right so that the statement becomes correct :

(1) The surface area of a cube of length 2 *cm* is .... cm^2. ☐

(a) 4 (b) 16 (c) 24 (d) 8

(2) The surface area of a cuboid of 5 $cm \times 4$ $cm \times 3$ *cm* is .... cm^2. ☐

(a) 60 (b) 47 (c) 24 (d) 94

(3) The expense to paint outer surface area (excluding top and base) of cuboidal tank of dimensions 30 $m \times 10$ $m \times 5$ *m* at the rate of ₹ 150 m^2 is .... ☐

(a) ₹ 1,05,000 (b) ₹ 75,000 (c) ₹ 60,000 (d) ₹ 1,50,000

(4) The radius and height of a cylinder are equal to x *cm*. The total surface area is .... cm^2. ☐

(a) $2\pi x^3$ (b) $2\pi x^2$ (c) $4\pi x^2$ (d) $4\pi x^3$

(5) The diameter of a cylinder is 7 *cm* and the area of its curved surface is 1320 cm^2. The height of the cylinder is .... *cm*. ☐

(a) 120 (b) 60 (c) 30 (d) 150

(6) The height of a cylinder is 35 *cm* and the area of its curved surface is 3520 cm^2. Then the radius of the cylinder is .... *cm* ☐

(a) 32 (b) 16 (c) 8 (d) 4

(7) The curved surface area of a cone with the radius of its base 2 *cm* and the slant height 5 *cm* is .... cm^2 ☐

(a) 15π (b) 12π (c) 18π (d) 10π

(8) The radius and the slant height of a cone are equal of x *cm*. The total surface area is .... cm^2. ☐

(a) $2\pi x^2$ (b) πx^2 (c) $2\pi x$ (d) πx

(9) The ratio of the radii of two cones is 2 : 3 and the ratio of their slant heights is 9 : 4. Then the ratio of their curved surface areas is .... ☐

(a) 3 : 2 (b) 1 : 2 (c) 1 : 3 (d) 2 : 3

(10) The surface area of a sphere is same as the curved surface area of a right circular cylinder, whose height and diameter are 12 *cm* each. The radius of sphere is .... *cm*. ☐

(a) 3 (b) 4 (c) 6 (d) 12

(11) If the surface area of a sphere is 616 cm^2, then its radius is .... *cm*. ☐

(a) 6 (b) 7 (c) 8 (d) 5

(12) If the ratio of radii of two spheres is 2 : 5, then the ratio of their curved surfaces areas is .... ☐

(a) 8 : 125 (b) 4 : 25 (c) 25 : 4 (d) 125 : 8

(13) The areas of curved surface of a sphere and cylinder having equal radii are equal. Then the height of cylinder is .... times the radius of the sphere. ☐

(a) 2 (b) 4 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

(14) The ratio of surface areas of two cubes is 4 : 9. The ratio of their volumes is .... ☐

(a) 2 : 3 (b) 64 : 27 (c) 27 : 64 (d) 8 : 27

(15) Total surface area of a cube is 216 cm^2. Hence, its volume is .... cm^3 ☐

(a) 36 (b) 216 (c) 12 (d) 6

(16) The ratio of the volume of cube and the volume of another cube having the length of side twice the length of the first cube is .... ☐

(a) 1 : 2 (b) 1 : 4 (c) 1 : 8 (d) 1 : 6

(17) The radius and the height of a cylinder are equal. If its diameter is 10 *cm*, then its volume is .... cm^3. ☐

(a) 5π (b) 25π (c) 125π (d) 10π

(18) The volume of a cone having radius and height equal to 1 *cm* is .... cm^3. ☐

(a) 3π (b) $\frac{1}{3}\pi$ (c) π (d) 2π

(19) The radii and heights of a cylinder and a cone are equal. The volume of the cone = ... $\times$ the volume of the cylinder. ☐

(a) $\frac{1}{4}$ (b) 4 (c) 3 (d) $\frac{1}{3}$

(20) The volume of a cone with radius 1 *cm* and the height thrice the radius is .... cm^3. ☐

(a) π (b) 3π (c) 9π (d) 6π

(21) The circumference of the base of a cone is 44 *cm* and its height 3 *cm*, then its volume is .... cm^3. ☐

(a) 44 (b) 66 (c) 132 (d) 154

(22) The volume and the surface area of a sphere are numerically equal, then the radius of the sphere is .... *cm*. ☐

(a) 2 (b) 4 (c) 6 (d) 3

*

Summary

In this chapter we have studied the following points :

1. The surface area of a cuboid = $2(lb + bh + lh)$
2. The surface area of a cube = $6l^2$
3. The curved surface area of a cylinder = $2\pi rh$
4. The total surface area of a cylinder = $2\pi r(r + h)$
5. The curved surface area of a cone = πrl
6. The total surface area of a cone = $\pi r(l + r)$
7. The surface area of a sphere = $4\pi r^2$
8. The surface area of a hemisphere = $2\pi r^2$
9. The total surface area of a hemisphere = $3\pi r^2$
10. The volume of a cuboid = lbh
11. The volume of a cube = l^3
12. The volume of a cylinder = $\pi r^2 h$
13. The volume of a cone = $\frac{1}{3}\pi r^2 h$
14. The volume of a sphere = $\frac{4}{3}\pi r^3$
15. The volume of a hemisphere = $\frac{2}{3}\pi r^3$

1 litre = 1000 cm^3

1 m^3 = 1000 litre = 1 kilolitre

CHAPTER 16

STATISTICS

16.1 Introduction

Everyday we come across a lot of information in the form of facts, numerical figures, tables, graphs etc. They are provided by newspapers, television media, magazines and other means of communications. These may relate to a batsman's average in cricket or bowling averages, profit-loss account of a company, temperatures of cities, expenditures in various sectors of a five year plan; percentage polling and so on. **These facts or figures, which are numerical or otherwise, collected with a certain purpose are called data**. Data is the plural form of the Latin word "datum".

The solutions to the problems pertaining to the basic sciences, sociology, agriculture, industry, management, administration etc. are sought today with the help of statistics. Though statistics is an old subject, it has become more prevalent from the beginning of the 20th century. When the administrators of any firm or department began to realise difficulties to bring about the solution to the problems, then the help from mathematicians and statisticians was sought. They collected data regarding the problems, analysed the collected data regarding the problems, scientifically evaluated the situation by constructing new principles based on mathematics and derived conclusions. When these conclusions proved to be very effective, the principles of statistics became very popular and progressive. **Thus statistics is a science dealing with the scientific methods of collecting, arranging, reducing, analysing the data and drawing proper and correct conclusions with the help of scientific principles.**

We have noticed that the base of statistics is data. For the solution of some problems or for certain predictions, the basic and important thing in statistics is data. In this chapter, we shall learn about data and other details regarding it.

16.2 Collection of Data

Let us start to collect data by the following activity.

Activity : We divide the students of our class into five groups. Assign each group the task to collect the data for one of the following information :

(i) Weight of 30 students of our class.

(ii) Number of family members in the families of 20 students of this class.

(iii) Height of 25 plants in or around our school.

(iv) Height of 20 students of our class.

(v) Total income of the family of 20 students of our class.

Now let us observe the results the students have collected.

How do they collect the data in each group ?

(i) Did they get the information from each and every student, house to house or personally contacted the head of the family for obtaining the information ?

(ii) Did they get the information from some source like school record available ?

For activities (i) to (iv) when the information is collected by the investigator himself or herself with a definite objective in his or her mind, the data obtained is called a **primary data.**

In activity (v), when the information was gathered from a source which is already stored in the school, the data obtained is called a **secondary data**. **Such data which has been collected by someone else in another context needs to be used with great care ensuring that the source is reliable.**

If the observations of the given data are expressed numerically, then it is said to be a **quantitative data** and if they are expressed non-numerically in qualitative form, then it is said to be a **qualitative data**. For example heights and weights of n students is a quantitative data, whereas the set of n observations obtained by tossing a balance coin n times is called a qualitative data.

EXERCISE 16.1

1. Classify the following data as primary data or secondary data :

(1) Number of students in the class.

(2) Election results obtained from print media or television news channels.

(3) Literacy rate figures obtained from educational survey.

(4) Number of trees in the school campus.

(5) Amount of telephone bills of our home for last one year.

(6) Profit or loss of any company obtained from its annual report.

(7) Temperature of the city for the last month.

*

16.3 Presentation of Data

As soon as the work related to collect the data is over, the investigator has to find out ways to represent them in the form which is meaningful, easily understood and gives its main features at a glance. Sometimes the data available from sample survey is so large and extensive that it is difficult to derive conclusion from it, if it is not reduced or classified properly.

Let us find various ways of representing the data through illustrations

Range : The difference between the largest observation and the smallest observation is called range of the quantitative data.

As for example, consider the runs scored by Yusuf Pathan in 10 innnings as given : 37, 52, 25, 18, 22, 30, 54, 11, 41, 47.

The data in this form is called a raw data.

From the above data we can find the highest and the lowest number of runs. It is less time consuming if these were arranged in ascending or descending order. Let us arrange these numbers in ascending order as 11, 18, 22, 25, 30, 37, 41, 47, 52, 54

Now we can clearly see that the lowest score is 11 and highest score is 54.

$\therefore$ The range of this data is $54 - 11 = 43$.

When the number of observations in an experiment is large, the presentation of data in ascending or descending order is quite time consuming.

Moreover range does not give a clear picture of data. For example in above illustration the range is 43. But 43 is also the range in the following examples.

(i) 1, 44

(ii) 1001, 1044

(iii) 1, 2, 3, 4, 5, ..., 44

If the data is large, instead of arranging them in increasing or decreasing order, we prepare a table as follows.

The marks obtained by 30 students out of 100 students of class IX are as follows :

15	85	50	30	80	50	35	70	55	90
75	60	99	70	40	70	35	60	50	40
60	55	35	85	60	40	70	90	40	90

The number of students who have obtained certain number of marks is called the **frequency** of those marks. For example, 2 students got 85 marks. So the frequency of observation 85 is 2. To make the data more easily understandable, we write it in a table, as given below :

Table 16.1

Marks	15	30	35	40	50	55	60	70	75	80	85	90	99	Total
No. of students (i.e. the frequency)	1	1	3	4	3	2	4	4	1	1	2	3	1	30

Table 16.1 is called an **frequency distribution table for ungrouped data** or simply a **frequency distribution table**.

Still an easier approach to prepare a table is to use telly marks. When an observation comes for the first time, we mark | against the class. For the observation occuring second time, we put || against the class in which it occurs. For a group of five observations symbol ~~||||~~ is used. For six observations we write ~~||||~~ | against the class containing the observation and so on.

The marks (out of 30) by 60 students of class IX in mathematics are as follows :

6	22	17	9	24	13	17	13	15	18	13	2	21	27	30
15	1	3	10	24	29	6	6	25	28	26	10	4	22	26
19	14	26	18	25	21	7	15	25	18	6	4	9	11	12
14	18	20	17	10	1	21	19	25	15	7	5	12	23	21

For such a large amount of data, we convert it into groups like 1 – 5, 6 – 10, 11 – 15, ..., 26 – 30 (since our data is from 1 to 30). These groups are called **classes** or **class intervals.**

The size of classes is called **class-size** or **class width** or **class length**, which is 5 here. In each of these classes the least possible observation of the class is called **lower class limit** of the class and the largest possible observation of the class is called the **upper class limit.**

Upper class limit of class 1-5 is 5.

Upper class limit of class 21-25 is 25 etc.

Lower class limit of class 6-10 is 6.

Lower class limit of class 16-20 is 16 etc.

Table 16.2

Marks (class)	Telly mark	Number of students
1 – 5	卌 ||	07
6 – 10	卌 卌 |	11
11 – 15	卌 卌 ||	12
16 – 20	卌 卌	10
21 – 25	卌 卌 |||	13
26 – 30	卌 ||	07
		Total 60

By representing the data in this form simplifies and condenses data and enables us to observe certain important features at a glance.

This type of table is called a **frequency distribution table for a grouped data**.

Example 1 : The data regarding the quantity of tea being served in each cup (in ml) in 50 different hotels are as follows :

106	107	76	82	109	107	115	93	187	95
123	125	111	92	86	70	126	68	130	129
139	119	115	128	100	180	84	99	113	204
111	141	130	123	90	115	98	110	78	90
107	81	131	75	84	104	110	80	118	82

Prepare frequency distribution table.

Solution : Here the minimum observation is 68 and maximum observation is 204. So, range is 204 – 68 = 136

Generally we divide the grouped data in 6 to 8 classes.

Let us take classes of equal length 20 i.e. 60 – 79, 80 – 99, ..., 200 – 219

Class	Telly mark	Frequency
60 – 79	卌	05
80 – 99	卌 卌 ||||	14
100 – 119	卌 卌 卌 ||	17
120 – 139	卌 卌	10
140 – 159	|	01
160 – 179		00
180 – 199	||	02
200 – 219	|	01
		Total 50

Now consider following situation :

The following distribution table shows the weight of 40 students of class IX :

Weight (in kg)	Number of students
31 – 35	9
36 – 40	5
41 – 45	14
46 – 50	3
51 – 55	2
56 – 60	3
61 – 65	2
66 – 70	1
71 – 75	1
	Total 40

Now, suppose two new students having weight 35.5 kg and 40.5 kg are admitted to this class. Then to which class should they belong ? We cannot add them to 35 – 40 or 41 – 45.

Why ? Because there is a gap between the upper and the lower limits of two consecutive classes. So, we have to devide the intervals in such a manner that the upper end-point of a class is same as the lower end-point of the next class. For this we have to find the difference between the upper limit of a class and the lower limit of its succeeding class. Then we add half of this difference to each of the upper limit and subtract the same from each of the lower limit.

For example : Consider the classes 31 – 35 and 36 – 40.

The lower limit of 36 – 40 is 36.

The upper limit of 31 – 35 is 35.

The difference is 36 – 35 = 1 and so half of it is $\frac{1}{2} = 0.5$

So, the new class intervals formed using 31 – 35 is 30.5 – 35.5 (31 – 0.5 and 35 + 0.5).

Similarly, the new class formed using the class 36 – 40 is 35.5 – 40.5 and so on.

If we take this type of class-intervals, another problems arise. 35.5 is a candidate for both classes 30.5 – 35.5 and 35.5 – 40.5. So to which class should 35.5 belong ?

By convention, we consider 35.5 in the class 35.5 – 40.5 and not in 30.5 – 35.5.

So, the new weights 35.5 and 40.5 would be included in 35.5 – 40.5 and 40.5 – 45.5 respectively. So the new frequency distribution table is shown below :

Class	Frequency
30.5 – 35.5	9
35.5 – 40.5	6
40.5 – 45.5	15
45.5 – 50.5	3
50.5 – 55.5	2
55.5 – 60.5	3
60.5 – 65.5	2
65.5 – 70.5	1
70.5 – 75.5	1
	Total 42

Such a frequency distribution table is called continuous frequency distribution table. 30.5, 35.5,..., etc. are called **lower boundary points of classes** 30.5 – 35.5, 35.5 – 40.5 respectively. 35.5 is the **upper boundary point of class** 30.5 – 35.5 and 40.5 is the upper boundary point of class 35.5 – 40.5 etc. Note that **the upper boundary point of a class is the same as the lower boundary** point of the next class.

*

EXERCISE 16.2

1. The monthly expenses in rupees of 50 students selected at random from a hostel are given below :

551	863	1180	709	903	852	757	790	972	535
425	760	1040	936	748	649	490	652	642	777
944	770	752	879	921	765	873	942	878	869
794	796	579	858	665	867	590	874	658	732
603	718	672	857	626	781	707	773	669	766.

Prepare frequency distribution table in which one of the classes is 425 – 524. What is the range of the data ?

2. The relative humidity (in %) of a certain city for a period of 30 days was recorded as follows :

98.1	98.0	99.2	90.3	88.5	93.5	92.0	98.1	94.2	95.1
89.5	92.3	97.1	93.5	92.7	95.1	97.2	93.3	95.2	96.5
96.2	92.1	84.9	90.2	95.7	89.3	97.3	96.1	92.1	98.0

(i) Construct a grouped frequency distribution table with classes 84 – 86, 86 – 88 ... etc.

(ii) What is the range of this data ?

3. During *Vanche Gujarat* 100 books were given to each of 100 schools. After two months, the number of books that were read in each school was recorded as :

85	67	28	32	65	65	69	33	98	96
76	42	32	38	42	40	40	69	95	92
75	83	76	85	85	62	37	65	63	49
89	65	73	81	48	52	64	76	83	92
95	68	55	79	81	83	59	82	75	82
86	90	44	62	31	32	38	42	39	86
85	56	56	23	40	77	83	85	30	87
69	83	86	50	45	39	84	75	66	83
92	75	89	66	91	38	88	89	93	29
53	69	90	55	66	49	52	83	34	56

Prepare a frequency distribution table with classes 20 – 29, 30 – 39, ... etc. Also find number of schools where more than 50 % books were read.

4. The heights of 50 students, measured to the nearest centimeters have been found to be as follows :

165	160	154	162	168	165	157	162	150	151
162	164	171	165	158	154	156	172	160	170
150	158	161	175	162	168	166	170	165	164
155	152	153	156	158	162	160	161	173	175
161	159	162	167	148	159	158	153	154	160

(i) Represent the above data by a grouped frequency distribution table taking the class intervals as 160 – 165, 165 – 170,... etc.

(ii) What do we conclude about the heights from the table ?

5. An experiment to study the effect of new medicine for making the patients unconscious before operation is performed on 50 rats. Each rat was injected with a standard dose and the time taken by each rat to become conscious is noted in minutes (correct upto one decimal point) and the following data were obtained :

45.0	58.2	55.1	52.2	61.7	52.9	70.4	62.5	71.3	50.1
84.9	60.9	35.4	64.3	75.7	48.5	41.3	53.8	66.8	37.4
32.4	50.7	82.3	71.8	66.4	49.7	51.7	56.0	88.8	64.7
77.9	41.4	52.7	53.4	57.9	51.7	55.6	44.1	85.4	67.3
87.3	52.5	40.7	48.7	60.0	66.0	77.3	46.5	54.3	52.6

Prepare a frequency distribution table from above data.

6. A study was conducted to find out the concentration of radium in air in part per million (ppm) in a certain city. The data obtained for 30 days are as follows :

0.03	0.08	0.08	0.09	0.04	0.17	0.16	0.05	0.02	0.06
0.15	0.16	0.12	0.06	0.09	0.13	0.22	0.09	0.08	0.02
0.12	0.08	0.08	0.19	0.12	0.08	0.06	0.08	0.02	0.08

(i) Make a grouped frequency distribution table for these data with class intervals as 0.00 – 0.04, 0.04 – 0.08 and so on.

(ii) For how many days, was the concentration of radium more than 0.11 parts per million ?

7. A company manufactures car tape-recorders of a particular type. The proper functioning record of 40 such tape-recorders were recorded as follows :

2.5	3.0	3.5	3.2	2.2	4.1	3.5	4.5	3.5	3.9
3.1	3.4	3.7	3.2	4.6	3.7	2.5	4.7	3.4	3.3
3.0	3.0	4.2	2.8	3.6	3.8	3.9	3.1	3.2	3.1
3.2	3.4	4.5	3.8	3.2	2.6	3.5	4.2	3.2	3.5

Construct a grouped frequency distribution table for these data, using class intervals of length 0.5 starting from the interval 2.0 – 2.5.

8. The distances (in 100 metres) covered by 40 students from their residence to their school were found as follows :

6	4	15	20	25	10	14	8	12	3
19	10	12	17	18	15	32	18	16	6
17	19	17	18	13	15	12	15	18	5
12	14	12	19	16	15	15	20	6	15

Construct a grouped frequency distribution table with class length 5, taking the first interval 0 – 5 (5 not included). What main feature do we observe from this tabular representation ?

9. A random sample of 25 ball bearings is selected from the population of ball bearing manufactured by a company. The data regarding the measures of their diameters in *cm* are as follows :

0.738	0.743	0.736	0.735	0.726	0.728	0.736	0.724	
0.742	0.739	0.745	0.742	0.728	0.725	0.734	0.733	
0.732	0.739	0.738	0.727	0.727	0.734	0.730	0.731	0.740

Prepare a frequency distribution from these data with six classes of equal class length.

16.4 Graphical Representation of Data

We have seen that an ungrouped data is not useful in drawing conclusions. Solution to many problems are sought with the help of grouped data and frequency distribution. If the frequency distributions are represented graphically, many characteristic properties of the given data are observed at first sight. It is well said that **"one picture is better than thousand words."** We will study following graphs to study discrete and continuous distributions.

Before drawing the graphs we shall keep following things in mind :

Due to reduction of a graph actually 1 *cm* does not look 1 *cm* but we understand that five units is same as 1 *cm*.

Usually comparison among the individual data are best shown by means of graphs. We will study these graphs : **(1) Bar diagrams (2) Histograms of uniform width and histograms of varying width (3) Frequency polygons**

(1) Bar diagram : Bar diagram is a pictorial representation of data in which usually bars of uniform width are drawn with equal spacing between them. We represent the variable on X-axis. The frequency of the variable is shown on Y-axis and the heights of the bars are proportionate to the frequency of the variable. This graph is used for discrete grouped data.

Example 2 : The number of students studying in colleges in different faculties of some city in the academic year 2009-2010 are given below. Represent given data by a bar diagram.

Faculty	Number of students
Medical	140
Engineering	210
Science	700
Commerce	950
Arts	810
Law	320

Solution : We will represent faculty on X-axis and number of students on Y-axis. Using the scale 1 *cm* = 100 students, we will draw bars of equal width and appropriate heights corresponding to the number of students of different faculties. For example there are 210 students in engineering faculty so as per our scale of 1 *cm* = 100 students, the height of the bar for the students of engineering will be 2.1 *cm* along Y-axis.

Similarly for other faculties we can calculate the heights of bars.

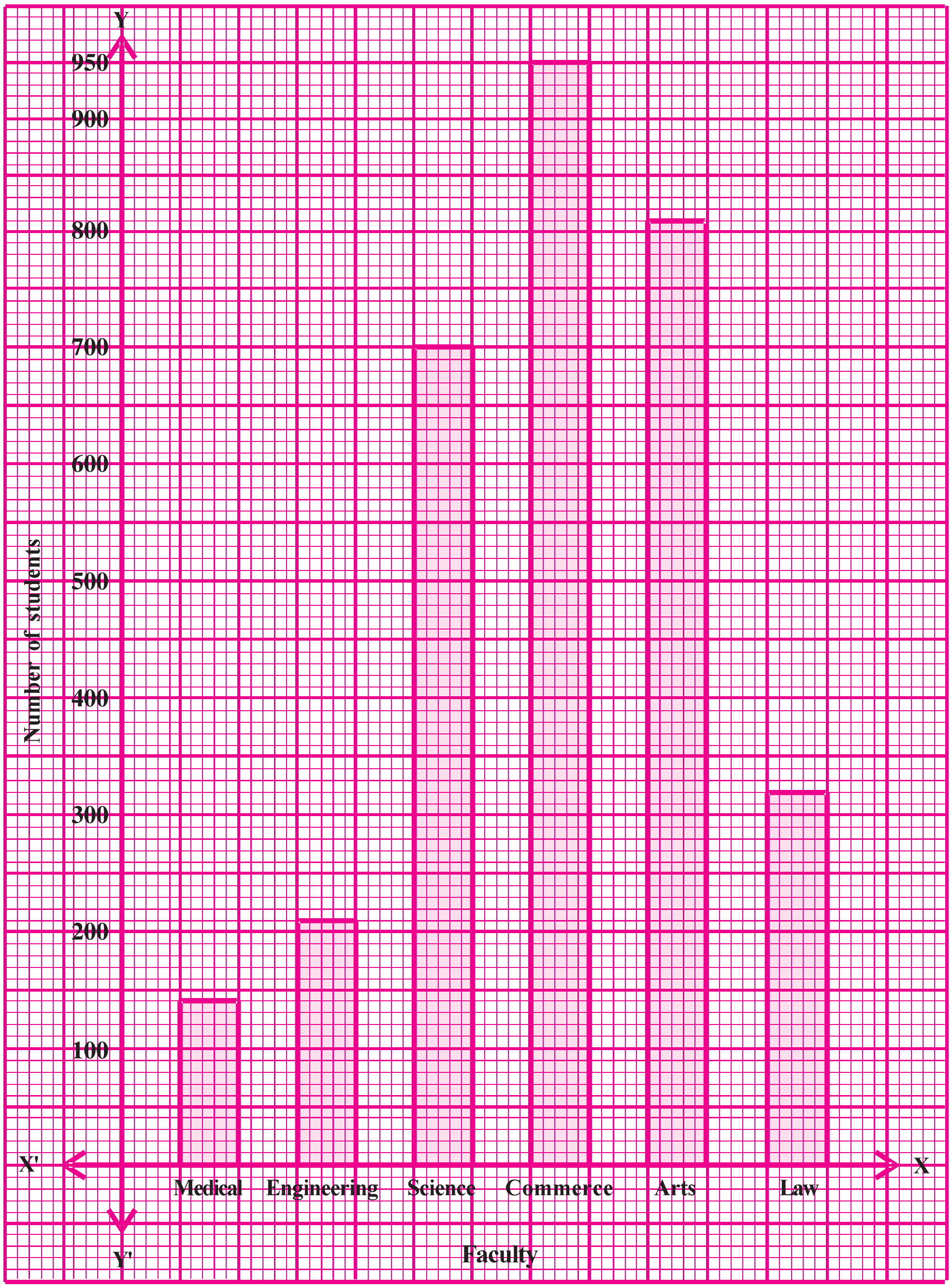


Figure 16.1

Bar diagram showing the number of students in different faculties of the colleges in a city for the year 2009-2010.

Example 3 : The data regarding the number of visits to a mall or to a multiplex by 50 families of a city during Diwali week are as under :

Number of visits	0	1	2	3	4	5	6	Total
Number of families	12	11	9	6	8	3	1	50

Draw the bar diagram.

Solution : Let us represent number of visits on X-axis and number of families on Y-axis. Scale 1 *cm* = 1 family. (figure 16.2)

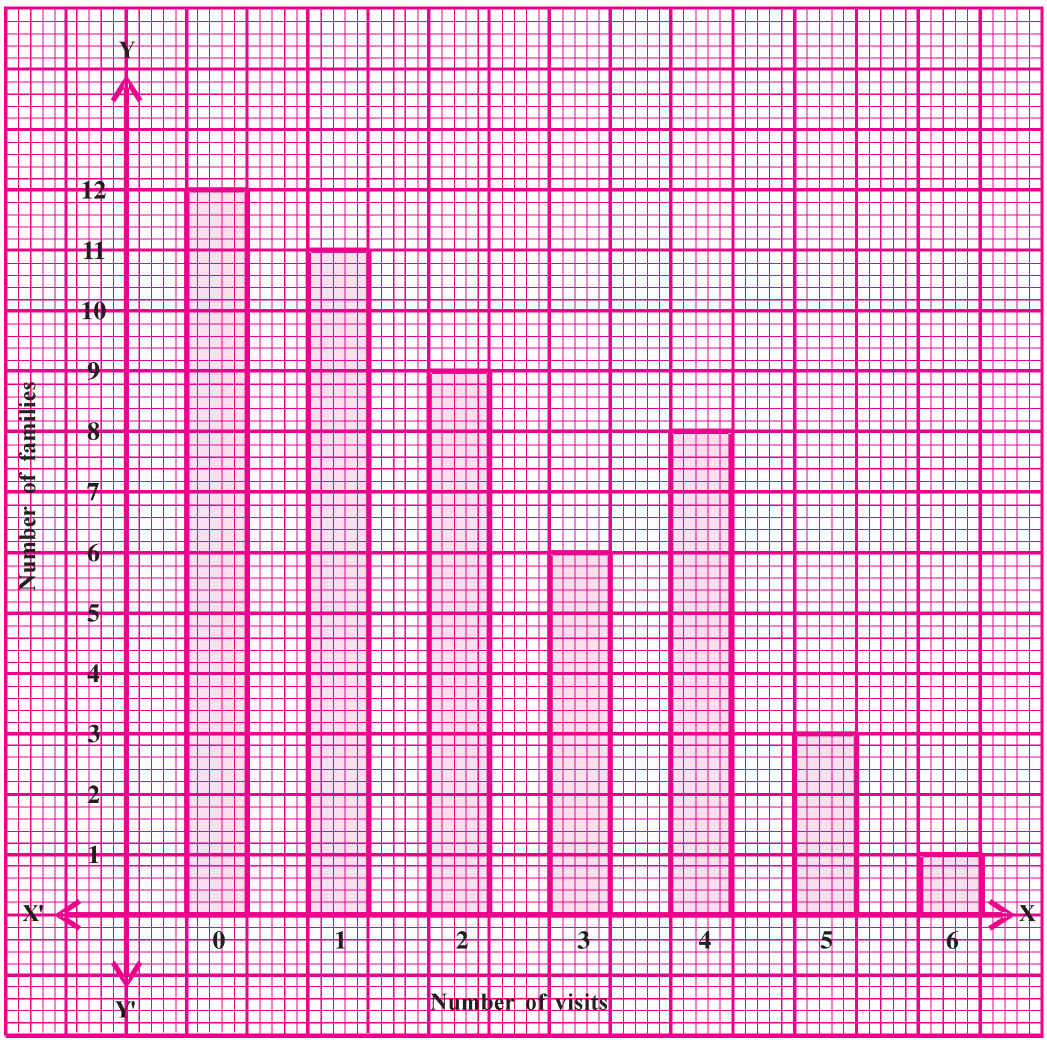


Figure 16.2

Bar diagram showing number of visits to a mall or to a multiplex during Diwali week and number of families

Activity : Continuing with the same five data of activity-1, represent the data by suitable bar graphs.

(2) Histogram : This is similar to bar graphs, but it is used for continuous grouped data with classes. For example consider the frequency distribution in table 16.3 representing the weights of 40 students.

Table 16.3

Weight (in kg)	Number of students
30.5 – 35.5	10
35.5 – 40.5	7
40.5 – 45.5	17
45.5 – 50.5	3
50.5 – 55.5	1
55.5 – 60.5	2
	Total 40

Now let us represent the above data graphically as follows :

To plot histogram, we shall take the boundary points on X-axis and frequency on Y-axis.

(i) We will represent the weight on X-axis on a suitable scale like 1 *cm* = 5 kg. Also the leading class starts from 30.5 and not zero. **We show it on the graph by marking kink or break on the X-axis.**

(ii) We will represent the frequency (i.e. number of students) on Y-axis with suitable scale. Since the maximum frequency is 17, we need to choose the scale to accomodate this maximum frequency.

(iii) Now we draw a rectangle (or rectangular bar) with width equal to the class-length and height according to the frequencies of the corresponding class-intervals. For example the rectangle for the class-intervals 30.5 – 35.5 will have the width 1 *cm* and length (height) 10 *cm*. (figure 16.3)

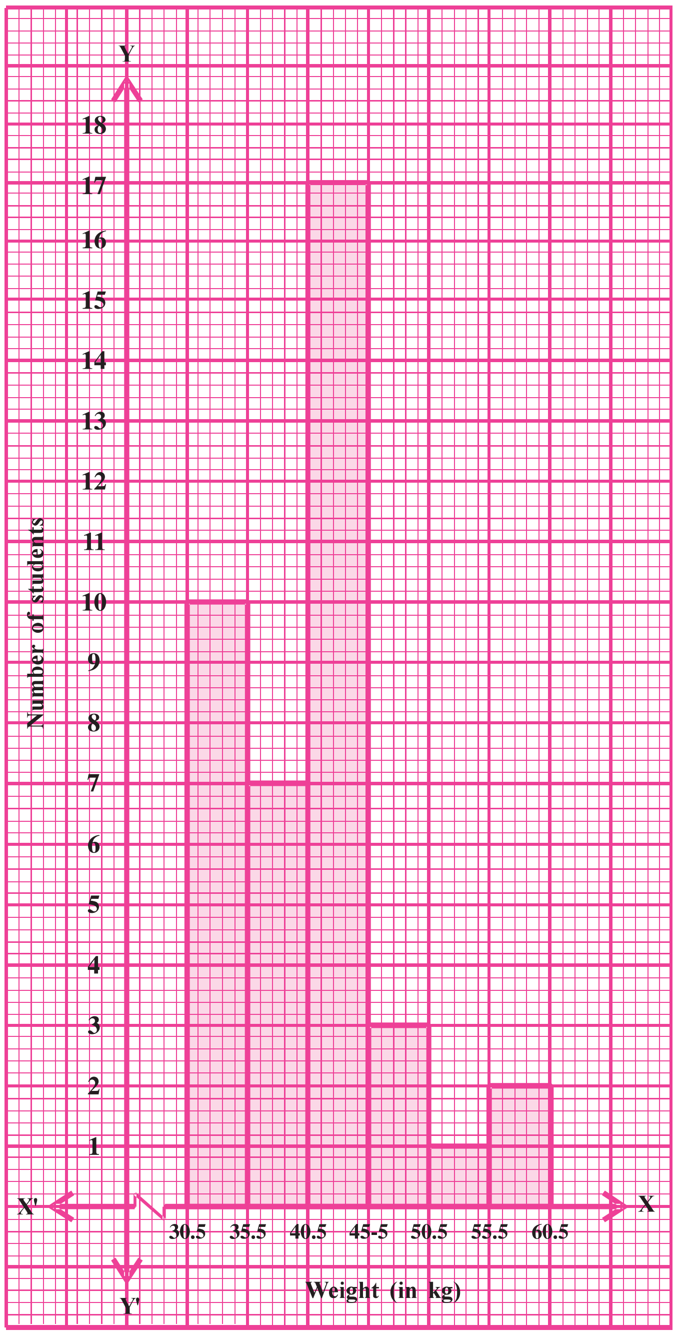


Figure 16.3

Histogram showing number of students and their weights (in kg)

Now let us consider another example in which the class length is not same.

Example 4 : The frequency distribution table is given as follows :

Class	10 – 15	15 – 20	20 – 30	30 – 40	40 – 55	55 – 75	75 – 100
Frequency	4	7	10	14	15	12	5

A student draws the histogram for above distribution as shown in figure 16.4.

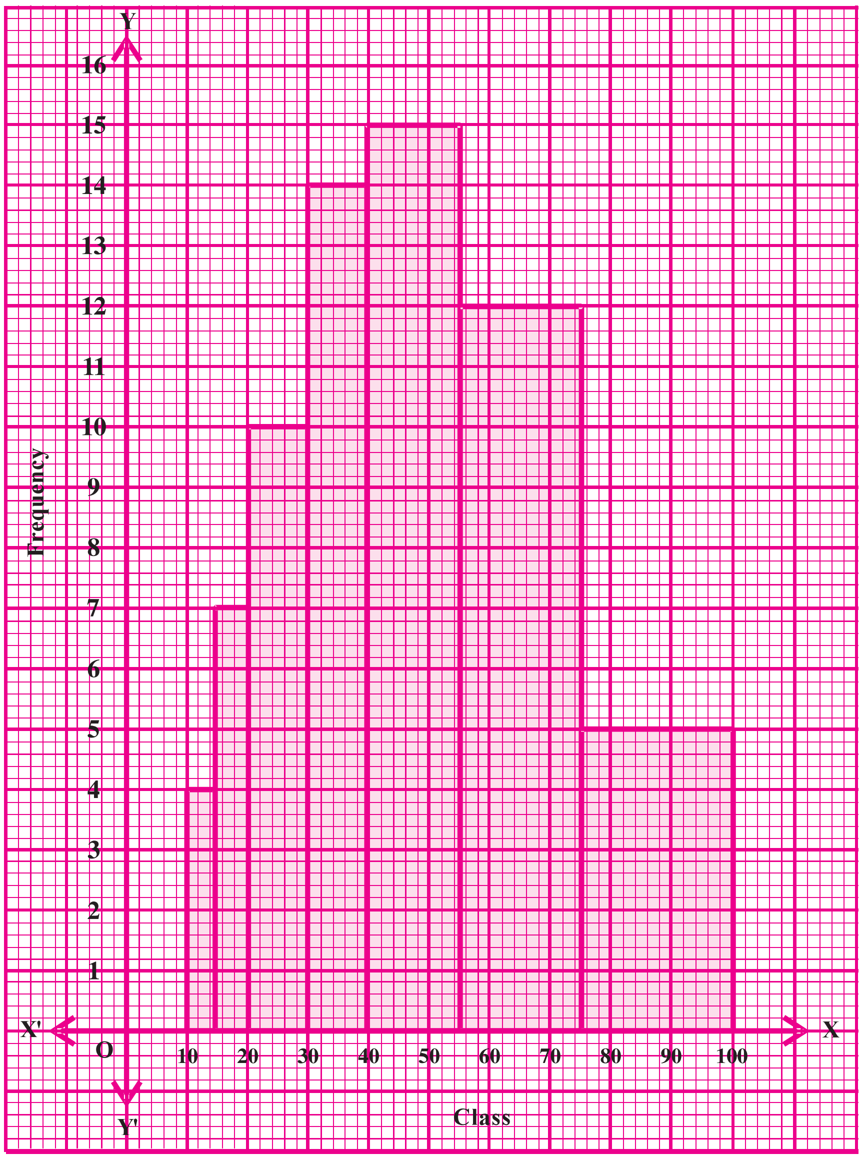


Figure 16.4

Histogram showing class and frequency

From above graph, do we think that it correctly represents the data ? No, the graph gives us a misleading picture. **The area of the rectangles should be proportional to the frequencies in a histogram.** In the previous example, this problem did not arise, because the widths of all the rectangles were equal. But here the widths of the rectangles are varying so the histogram drawn in figure 16.4 by the student does not give correct picture of the data. For example the greatest frequency occurs in the interval 40 – 55, which is not proper.

Solution : So we make certain modifications in the length of rectangles so that the areas are again proportional to the frequencies.

The steps to be followed are as under :

1. Select a class-interval with the minimum class length. In the above example the minimum class length is 5.
2. The length of the rectangles are then modified to be proportionate according to the class length 5.

$$\text{Proportionate frequency} = \frac{\text{frequency of a given class} \times \text{minimum class length}}{\text{class length of given class}}$$

For example, for class 55 - 75, the minimum class length is 5 and frequency of 55 - 75 is 12, then proportionate frequency = $\frac{12 \times 5}{20} = 3$

For example, when the class length is 15, the frequency is 15, so when the class length is 5, the length of rectangle = $\frac{15}{15} \times 5 = 5$

Similarly, proceeding in this manner, we get the following table 16.4

Table 16.4

Class boundary points	Frequency	Width of class	Length of rectangle
10.0 – 15.0	4	5	$\frac{4}{5} \cdot 5 = 4$
15.0 – 20.0	7	5	$\frac{7}{5} \cdot 5 = 7$
20.0 – 30.0	10	10	$\frac{10}{10} \cdot 5 = 5$
30.0 – 40.0	14	10	$\frac{14}{10} \cdot 5 = 7$
40.0 – 55.0	15	15	$\frac{15}{15} \cdot 5 = 5$
55.0 – 75.0	12	20	$\frac{12}{20} \cdot 5 = 3$
75.0 – 100.0	5	25	$\frac{5}{25} \cdot 5 = 1$

Since we have calculated these lengths for a class-length 5 in each case, we may call these lengths as **"Proportionate frequency for class-interval 5".**

So, the correct histogram with varying width is given in figure 16.5.

Figure 16.5

Histogram showing class and frequency

(3) Frequency polygon : This is yet another way of representing frequencies visually and it is called a frequency polygon.

Consider the histogram represented by figure 16.5. Let us join the midpoints of the upper sides of the adjacent rectangles of this histogram by means of line-segments. Let us call these points B, C, D, E, F, G, H (figure 16.6). To complete the polygon, we assume that there is a class interval with frequency zero before 9.5 – 15.5 and after 75.5 – 100.5, and their mid points are A and I respectively. ABCDEFGHI is the frequency polygon corresponding to the data shown in example 4.

Figure 16.6

Frequency polygon showing class and frequency

Example 5 : Consider the marks out of 100, obtained by 51 students of a class in a test, as given in table 16.5.

Table 16.5

Class	Number of students (Frequency)
0 – 10	5
10 – 20	10
20 – 30	4
30 – 40	6
40 – 50	7
50 – 60	3
60 – 70	2
70 – 80	2
80 – 90	3
90 – 100	9
	Total 51

Draw the histogram and the frequency polygon for above data.

Solution : Let us first draw the histogram for this data and mark the midpoints of the upper sides of the rectangles as B, C, D, E, F, G, H, I, J, K respectively. Here first class is 0 – 10. So to find the class preceding 0 – 10, we extend the horizontal axis in the negative direction and find the midpoint of the imaginary class-interval (–10) – 0. The first end point i.e. B is joined to this midpoint with zero frequency in the negative direction

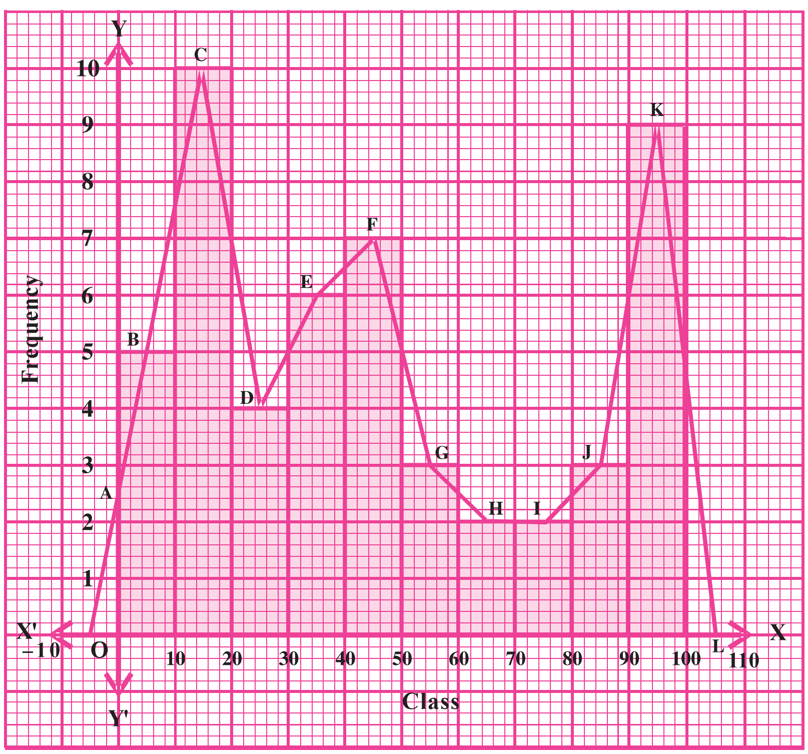


Figure 16.7

Frequency polygon showing class and frequency

of the horizontal axis. The point where this line-segment meets the vertical axis is marked as A. Let L be the midpoint of the class succeeding the last class of the given data. Then OABCDEFGHIJKL is the frequency polygon, as shown in figure 16.7. Frequency polygon can also be drawn independently without drawing histograms. For this we require midpoints of the class-intervals used in the data. These midpoints of the classes are called **class-marks**. (or **central values**)

$$\text{Class mark of a class} = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Example 6 : In a company of 40 employees wage per hour (in ₹) is as follows :

Wage per hour (in ₹)	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Number of employees	2	8	12	10	6	2

Draw frequency polygon without drawing the histogram of this data.

Solution : For the above example we have to find the classmark (central value) of each class as follows :

Wage per hour	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Class mark hour (in ₹)	15	25	35	45	55	65
Number of employes (frequency)	2	8	12	10	6	2

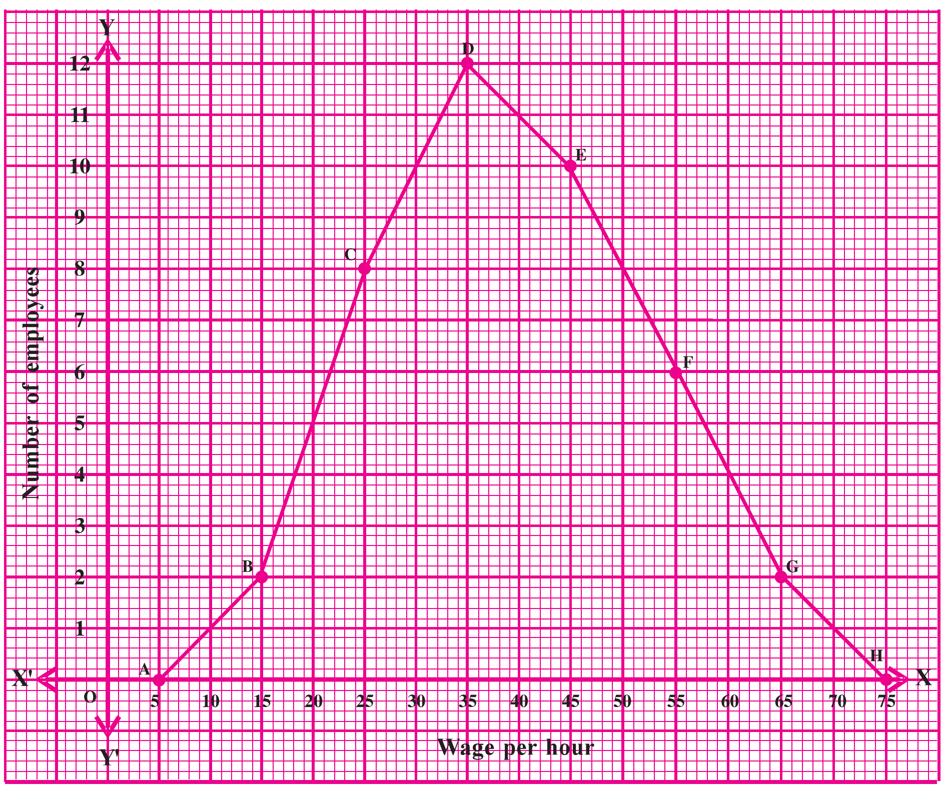


Figure 16.8

Frequency polygon showing number of employees and wage per hour

The graph is drawn in figure 16.8.

On X-axis we will take central values with a scale of 1 *cm* = ₹ 5 and on Y-axis we will take number of employees with a scale of 1 *cm* = 5 employees. Now we can draw the frequency polygon. Plotting and joining the points B (15, 2), C (25, 8), D (35, 12), E (45, 10), F (55, 6), G (65, 2) by line-segments. We also take central value of class 0 – 10 (just before 10 – 20) with zero frequency and class 70 – 80 (just after 60 – 70) with zero frequency that is A (5, 0) and H (75, 0). So, the resulting polygon will be ABCDEFGH (figure 16.8).

Frequency polygons are used when the data is continuous and very large. It is very usuful for comparing two different sets of data of the same nature.

If continuous grouped data is given in classes using upper limits and lower limits, we convert them into classes with boundary points in order to draw histogram.

Example 7 : The length of 40 leaves of a plant are measured correct to one millimeter and the obtained data are represented in the following table :

Length (in *mm*)	Number of leaves
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

Draw the histogram for above data.

Solution : Here we have to transform classes with limit points in classes with boundary points.

Length (in *mm*)	Number of leaves
117.5 – 126.5	3
126.5 – 135.5	5
135.5 – 144.5	9
144.5 – 153.5	12
153.5 – 162.5	5
162.5 – 171.5	4
171.5 – 180.5	2

Now by taking suitable scale on both the axis such as 1 *cm* = 9 *mm* (length of a leaf) on X-axis and 1 *cm* = 1 leaf on Y-axis, the histogram is as in figure 16.9.

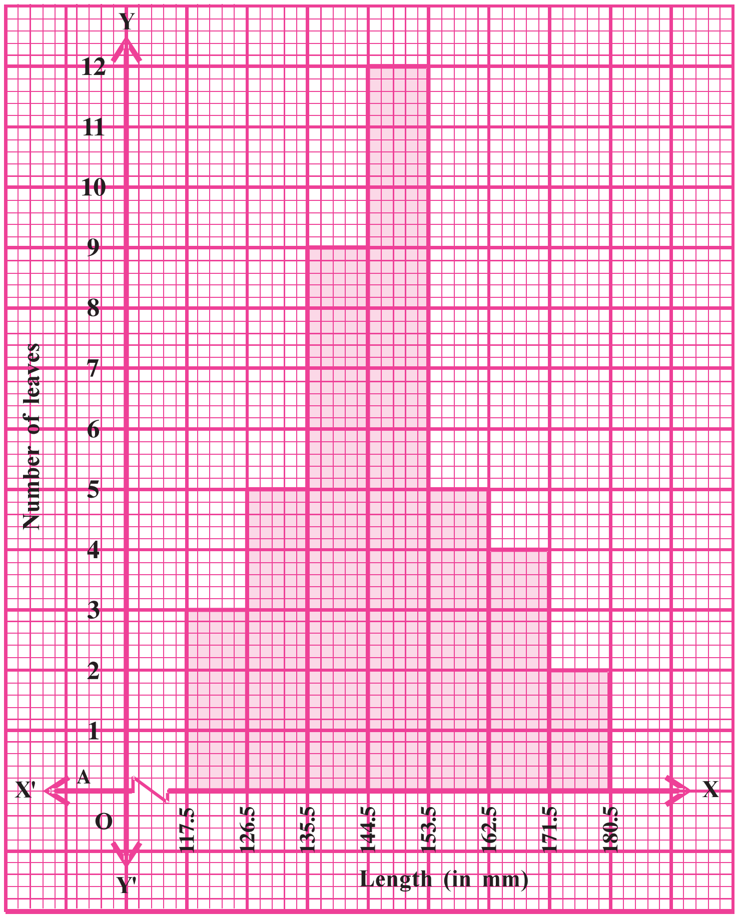


Figure 16.9

Histogram showing number of leaves and length

EXERCISE 16.3

1. The details of export (in crore ₹) of a country for the last seven years are given below. Represent the data by bar diagram.

Year	2001	2002	2003	2004	2005	2006	2007
Export (in crore ₹)	1000	1200	1300	1500	1600	1700	1900

2. The number of boy students from standard 8 to 12 of a school are as follows. Draw bar diagram for the data.

Standard	8	9	10	11	12
Number of boy students	100	90	85	75	60

3. The production of wheat of a state for five years is given below. Represent the data by bar diagram.

Year	2003	2004	2005	2006	2007
Production of wheat (in metric tons)	25,000	30,000	37,000	33,000	42,000

4. A survey conducted by an organisation for the cause of illness and death among the male between the ages 15–44 (in years) world wide, found the following figures (in %) :

Sr. no.	**Cause**	**Male facality rate**
1	Cardio vascular condition	4.7
2	By smoking	31.8
3	By unhygenic food	25.4
4	Neuropsychiatric condition	21.3
5	Accident	12.3
6	Other cause	4.5

Represent the above data by bar diagram.

5. The following table gives the life period of 400 neno bulbs (lamps) :

Life time (in hour)	**Number of bulbs**
400 – 500	10
500 – 600	56
600 – 700	60
700 – 800	80
800 – 900	74
900 – 1000	68
1000 – 1100	52

Draw the histogram for above data. How many bulbs have life time more than 800 hours ?

6. 100 surnames were randomly picked up from a telephone directory and the frequency distribution of the number of letters in the English alphabet in the surname was found as follows :

Number of letters	1 – 4	4 – 6	6 – 8	8 – 12	12 – 20
Number of surnames	5	35	40	16	4

Draw the histogram for the above data.

7. Draw the histogram of the following frequency distribution :

Class	10 – 20	20 – 40	40 – 70	70 – 110	110 – 160
Frequency	10	24	39	60	50

8. The runs scored by Sachin and Sehvag in the first 60 balls in a cricket match are given below :

Number of balls	Sachin	Sehvag
1 – 6	2	5
7 – 12	1	6
13 – 18	8	2
19 – 24	9	10
25 – 30	4	5
31 – 36	5	6
37 – 42	6	3
43 – 48	10	4
49 – 54	6	8
55 – 60	2	10

Represent the data for both the players on different graphs by frequency polygons.

(Hint : First let the classes be transformed into classes with boundary points.**)**

*

16.5 Measures of Central Tendency

If the number of observations is very large, the data are condensed by classification in the form of frequency distribution. The frequency distribution is represented graphically by drawing bar graphs, histogram and frequency polygons. The main objective of statistical analysis is to obtain a measure which represents the summary or essence of the observations of data. The value of this measure lies between or in the middle of the smallest and the largest value of the observations of the data. Hence it is called the **measure of central tendency or average of the data.**

Consider the situation when two students Max and Mohan received their test copies. The test had five sections, each carying 10 marks. The scores were as follows :

Section	A	B	C	D	E
Max's Score	10	7	9	8	7
Mohan's Score	5	10	10	7	10

Both of them found their averages.

Max's average score = $\frac{41}{5}$ = 8.2. Mohan's average score = $\frac{42}{5}$ = 8.4

Since Mohan's average score was more than Max's average score, Mohan claimed that his performance was better than Mohan's performance. But Mohan asked to arrange their scores in ascending order as follows :

Max's score	7	7	8	9	10
Mohan's score	5	7	10	10	10

Mohan found his middle score was 10 which was higher than Max's middle score 8. So Mohan claimed that his performance is better than Max's performance. Mohan found another strategy that he got score 10 (3 times) more often as compared to Max's score and Max scoreed 10 marks only once.

Now, to solve their problem, let us see the three measures which they had adopted.

The average score that Max found is the **mean**. The **"middle"** score that Mohan found is the "**median**". The most often scored marks by Mohan is the **"mode".**

Mean : The mean or average of a number of observations is the sum of the values of all the observations divided by the total number of observations. It is denoted by $\bar{x}$ (read as x bar).

So, if $x_1, x_2, x_3,...,x_n$ are observations, then the mean of these observations is

$$\bar{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$$

We use the Greek symbol Σ (read as sigma) for summation. Instead of writing $x_1 + x_2 + ... + x_n$, we write $\sum_{i=1}^{n} x_i$, which is read as "the sum of x_i as i varies from 1 to n".

$$\text{So, } \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Example 8 : Find the mean of the observations 2, 5, 6, 11, 11, 12, 13, 14.

Solution : Here eight observations are given. Let us take $x_1 = 2$, $x_2 = 5$, $x_3 = 6$, $x_4 = 11$, $x_5 = 11$, $x_6 = 12$, $x_7 = 13$ and $x_8 = 14$.

So the mean $\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8}{8}$

$$= \frac{2+5+6+11+11+12+13+14}{8}$$

$$= \frac{74}{8}$$

$$= 9.25$$

Example 9 : Five students have spent their time for reading during the last weeks recorded as 10, 7, 13, 20 and 15 hours. Find the mean time spent by the students during the week.

Solution : We know that $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ (here $n = 5$)

$$= \frac{10+7+13+20+15}{5}$$

$$= \frac{65}{5}$$

$$= 13$$

So the mean time spent by the students for reading is 13 hours per week.

Example 10 : Mohan Bagan made goals in five football matches. The goals recorded as : 7, 3, 5, 6, 4. Find the mean of the goals made by him.

Solution : We know that $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

$$= \frac{7+3+5+6+4}{5}$$

$$= \frac{25}{5}$$

$$= 5$$

Mohan Bagan made 5 goals on average in each match.

To simplify calculations, let A be any real number. A is subtrated from all observations. Then the mean is

$$A + \frac{\text{sum of deviations from assumed number}}{\text{number of observations}}$$

So if assumed number is A and sum of all deviations from observations is Σd_i,

then $\bar{x} = A + \frac{\sum d_i}{n}$ **($d_i = x_i - A$)**

Example 11 : The following observations represent the heights (in *cm*) of students : 120, 115, 117, 123, 122, 122, 119, 125, 121, 116. Find the mean.

Solution : Here numbers are large. Addition would be tiring task. So we make the following table :

Here suppose A is 122 (not necessary that A be one of the observations)

Height (in *cm*) x_i	**Deviation** $d_i = x_i - \text{A}$
120	–2
115	–7
117	–5
123	1
122	0
122	0
119	–3
125	3
121	–1
116	–6
$n = 10$	$\Sigma d_i = -20$

$$\therefore \quad \overline{x} = \text{A} + \frac{\Sigma d_i}{n}$$

$$= 122 + \frac{(-20)}{10}$$

$$= 122 - 2$$

$$= 120$$

Now when discrete grouped frequency distribution is given i.e. x_i and f_i are given, then the mean is defined as

$$\overline{x} = \frac{x_1 f_1 + x_2 f_2 + x_3 f_3 + \ldots + x_k f_k}{f_1 + f_2 + f_3 + \ldots + f_k}$$

$$\therefore \quad \overline{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}, \text{ where } n = \sum_{i=1}^{k} f_i$$

Example 12 : Find the mean of the marks obtained by 30 students of class IX of a school, given in example 2.

Solution :

Marks (x_i)	Number of students (f_i)	$f_i x_i$
10	1	10
20	1	20
36	3	108
40	4	160
50	3	150
56	2	112
60	4	240
70	4	280
72	1	72
80	1	80
88	2	176
92	3	276
95	1	95
	$n = \Sigma f_i = 30$	$\Sigma f_i x_i = 1779$

In this case of a grouped frequency distribution, we can use the formula

$$\bar{x} = \frac{\sum_{i=1}^{k} f_i \; x_i}{n} = \frac{1779}{30} = 59.3$$

Median (M) : After arranging the observations in ascending or descending order, the number which is obtained in the middle is called the **median**. It is denoted by M.

Note that **if the number of observations n is odd then $\left(\frac{n+1}{2}\right)^{\text{th}}$ observation is the median and if the number of observations n is even, then median**

$$M = \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{observation} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{observation}}{2}$$

For example if observations are 13, 3, 9, 20, 18, 16, 19, then arrange them in ascending order as 3, 9, 13, 16 , 18, 19, 20. Here seven observations are given. Therefore $\left(\frac{7+1}{2}\right)^{\text{th}}$ that is 4th observation is median. Here 4th observation is 16. So M = 16.

Let observations be 32, 14, 8, 11, 12, 16, 5, 35. Here eight observations are given (i.e. even). Arrange them in ascending order or decending order. We arrange them in decending order as 35, 32, 16, 14, 12 , 11, 8, 5. So the median is the average of 4th observation and 5th observation i.e. average of 14 and 12. So, M = $\frac{14+12}{2}$ = 13.

Mode (Z) : The observation which is repeated most often in an ungrouped data is called the mode of the data. It is denoted by Z. If there are two or more observations in the data that are repeated most often (and the same number of times), each such number is a mode. A data with exactly two modes is called bimodal, while one with more than two modes is called multimodal.

Example 13 : Find mean, median and mode for odd numbers between 36 and 49.

The odd numbers between 36 and 49 are 37, 39, 41, 43, 45, 47

$$\therefore \ \bar{x} = \frac{37+39+41+43+45+47}{6} = \frac{252}{6} = 42$$

The number in ascending order are : 37, 39, 41, 43, 45, 47

Here $n = 6$ is even.

$$\text{Hence M} = \frac{\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \text{observation}}{2}$$

$$= \frac{\text{Third observation + Fourth observation}}{2} = \frac{41+43}{2} = 42$$

Since no number is repeated in the data, the data has no mode.

Example 14 : The marks out of 20 obtained by 10 students are as follows. Find the mode of the following data :

8, 12, 5, 13, 12, 8, 9, 12, 8, 10

Solution : We arrange the marks in the increasing order :

5, 8, 8, 8, 9, 10, 12, 12, 12, 13

Here 8 and 12 both occur frequently i.e. three times. So, the modes are 8 and 12. (**Bimodal data**)

Example 15 : The temperature from 8 a.m. to 8 p.m. on a day every hour is noted as follows : (approximatly in complete degree)

23°, 25°, 25°, 29°, 27°, 27°, 23°, 27°, 29°, 28°, 23°, 25°. Find the mode.

Solution : Arrange the temperature in the following form :

23°, 23°, 23°, 25°, 25°, 25°, 27°, 27°, 27°, 28°, 29°, 29°

Here 23°, 25°, 27° occur frequently i.e. three times each.

So, the modes are 23°, 25°, 27°. (**multi modal data**)

Example 16 : The observations of the given data, in ascending order are : 31, 33, $a + 2$, $a + 6$, 45 and 49 where a is a constant. If the median of the data is 39 find the value of a and mean of the data.

Solution : The number of observations is 6 (i.e. even).

$$\therefore \text{M} = \frac{\text{Third observation + Fourth observation}}{2}$$

$$\therefore 39 = \frac{a+2+a+6}{2}$$

$\therefore 78 = 2a + 8$

$\therefore 2a = 70$

$\therefore a = 35$

$\therefore a + 2 = 37,\ a + 6 = 41$

$$\therefore \text{Mean } \bar{x} = \frac{31+33+37+41+45+49}{6}$$

$$= \frac{236}{6} = 39.33$$

Properties of Mean :

(1) Subtraction of the mean from each observation gives the 'deviation' with respect to the mean. The sum of all such deviations is always zero. i.e. $\Sigma (x_i - \bar{x}) = 0$.

(2) The greatest and the lowest observations have strong influence on the mean. The mean can be considered to be a stable measure, if the range of data is small.

(3) For a given data :

(a) If a number a is added to each observation, then the mean is increased by a.

(b) If a number a is subtracted from each observation, then the mean is decreased by a.

(c) If every observation is multiplied by a ($a \neq 0$), the mean gets multiplied by a.

(d) If every observation is divided by a ($a \neq 0$), the mean gets divided by a.

(4) If the mean of n observations of one data is $\bar{x}$, the sum of n observations is $n\bar{x}$. If the mean of m observations of another data is $\bar{y}$, the sum of m observations is $m\bar{y}$. Hence the sum of $(m + n)$ observations is $(n\bar{x} + m\bar{y})$.

The combined mean of all observations of two given grouped data is $\frac{n\bar{x}+n\bar{y}}{m+n}$.

*

EXERCISE 16

1. The following number of goals were scored by a team in a series of 10 matches : 3, 3, 4, 5, 7, 1, 3, 3, 4, 3. Find mean, median and mode of these scores.

2. In a Ramanujan mathematics test of 15 students, the following marks (out of 100) recorded here :

45, 52, 62, 54, 39, 48, 55, 96, 98, 40, 55, 60, 45, 40, 55.

Find the mean, median and mode of this data.

3. Find the mean salary of 80 workers of a factory from the following table :

Salary (in ₹)	Number of workers
2500	16
3500	12
4500	10
5500	14
6500	10
7000	4
8000	3
9000	10
10000	1
	Total 80

4. Find the mean of the following frequency distribution :

Value of the variable	11	12	13	14	15	16	17	18	19	20
Frequency	26	28	18	19	22	25	30	32	40	45

5. The mean of 20 observations is 31. In this data, one observation was taken by mistake as 52 instead of 25. Find the correct mean.

6. The mean of 25 observations is 10.2. While calculating the mean one observation was taken by mistake as (–10) instead of 10. Find the correct mean.

7. The height of five students are 140, 143, 150, 137, 145 *cm*. Find the mean and median of this data.

8. The marks obtained by 10 students in a test of 20 marks are as follows : 14, 19, 7, 20, 11, 8, 13, 14, 14, 17. Find the mean, median and mode of this data.

9. The following observations have been arranged in acending order : 26, 33, 38, 44, $x + 1$, $x + 3$, 53, 57, 62, 67. If the median of the data is 51, find x.

10. If the mean of following 10 observations is 37, then find the value of x.

28, 52, 34, x, 30, 62, 50, 54, 30, 20

11. If the mean and sum of n observations are 5 and 50, find the value of n.

12. The frequency distribution of descrete frequency distribution is as follows :

Variable (x_i)	0	1	2	3	4	5
Frequency	92	40	–	36	32	20

If the mean is 1.744, then find the missing frequency.

13. Find the mean of the frequency distribution :

x_i (Variable)	4	12	20	28	36	44
Frequency	8	7	16	24	15	7

14. Select proper option (a), (b), (c) or (d) and write in the box given on the right so that the statement becomes correct :

(1) Total number of classes in our school is ...... data.

(a) primary (b) secondary (c) quantitative (d) qualitative

(2) Total number of books in our library is ...... data.

(a) primary (b) secondary (c) quantitative (d) qualitative

(3) Inflation rate figure obtained from print media is ..... data.

(a) primary (b) secondary (c) numerical (d) qualitative

(4) Profit and loss account of the company obtained from company report is ...... data.

(a) primary (b) secondary (c) numerical (d) qualitative

(5) The marks obtained (out of 50) by 10 students in a test are 13, 25, 42, 11, 40, 33, 49, 37, 19, 27. The range of this data is ...... ·

(a) 14 (b) 38 (c) 36 (d) 49

(6) If daily wages of 5 workers in a factory are 45, 32, 59, 37 and 52, then Mean of this data is ....

(a) 45 (b) 32 (c) 31 (d) 63

(7) The upper limit of the class 41 – 50 is ....

(a) 41 (b) 50 (c) 45 (d) 91

(8) The lower limit of the class 20 – 29 is ....

(a) 49 (b) 9 (c) 29 (d) 20

(9) The frequency of 7 in the data 3, 7, 5, 6, 7, 5, 7, 9, 4, 7 is ....

(a) 1 (b) 2 (c) 3 (d) 4

(10) In continuous frequency distribution table, the observation 20 will be in the class ....

(a) 0 – 10 (b) 10 – 20 (c) 20 – 30 (d) 30 – 40

(11) The class mark (central value) of the class 25 – 30 is ......

(a) 25.5 (b) 27.5 (c) 29.5 (d) 30.5

(12) The class mark of the class 45 – 55 is ....

(a) 55 (b) 45 (c) 50 (d) 47.5

(13)

Class	0–10	10–20	20–40	40–70	70–100
Frequency	3	5	14	12	6

Then the proportionate frequency of class 70 – 100 is ....

(a) 3 (b) 6 (c) 2 (d) 1

(14) From above example the height of the rectangle for the class 20 – 40 in histogram is ....

(a) 14 (b) 7 (c) 6 (d) 3

(15) The width of the class 30 – 45 is ....

(a) 30 (b) 75 (c) 45 (d) 15

(16) The width of the class 55.5 – 60.5 is ....

(a) 10 (b) 5 (c) 2.5 (d) 7

(17) The mean of 7, 10, 16, 20, 27 is ....

(a) 16 (b) 15 (c) 10 (d) 20

(18) The average of 4, 9, 18, 21, 30, 16, 30, 16 is ....

(a) 17 (b) 18 (c) 20 (d) 24

(19) The mean for the following frequency distribution is ....

x_i	5	7	8	9	10
f_i	2	8	3	5	2

(a) 6.50 (b) 10.75 (c) 14.75 (d) 7.75

(20)

x_i	10	15	20	25	30
f_i	7	8	9	4	2

The mean is ....

(a) 17.66 (b) 15.66 (c) 17.5 (d) 15.5

(21) The median of the observations 17, 23, 9, 32, 14, 27, 11 is ....

(a) 32 (b) 9 (c) 17 (d) 11

(22) The median of the observations 54, 32, 19, 36, 29, 44, 21, 47 is ....

(a) 32 (b) 36 (c) 39 (d) 34

(23) The median of the observations 26, 13, 7, 31, 21, 17 is ....

(a) 21 (b) 17 (c) 20 (d) 19

(24) The median of the data 76, 81, 68, 92, 88 is ....

(a) 81 (b) 88 (c) 76 (d) 68

(25) The mode of the data 9, 8, 11, 3, 8, 15, 8, 9, 10, 14 is ....

(a) 9 (b) 11 (c) 8 (d) 10

(26) The salaries of five workers is ₹ 9000 each, then the mean, median, and mode of this data is ₹ ....

(a) 5000 (b) 6000 (c) 8000 (d) 9000

(27) The mode of the observations 1, 3, 2, 5, 3, 7, 2 is ....

(a) 1 (b) 3 (c) 2 (d) 2 and 3

(28) The mode of observations 7, 13, 15, 11, 13, 13, 7, 7, 19, 20, 15, 15 is ....

(a) 15 (b) 13 (c) 7 and 13 (d) 7, 13, 15

(29) The data of example 28 is ....

(a) having no mode (b) unimodal (c) bimodal (d) multimodal

(30) Given that $1 + 3 + ... + (2n - 1) = n^2$, then the mean of first n odd numbers is ....

(a) $2n + 1$ (b) $2n - 1$ (c) n (d) n^2

(31) If all the observations 3, 7, 9, 18, 21, 32 are multiplied by 3, then the new mean is ....

(a) 15 (b) 90 (c) 45 (d) 60

(32) If we add (–7) to each of the observation 8, 17, 25, 28, 32, then the new mean = ...

(a) 22 (b) 15 (c) 8 (d) 1

(33) If we divide all the observations 18, 33, 36, 39, 44 by 2, then the new mean = ...

(a) 34 (b) 29 (c) 22 (d) 17

(34) If for the observations 5, 37, 29, 18 we replace 5 by (–5), then the new mean = ...

(a) 22.25 (b) 19.75 (c) 21.75 (d) 20.25

(35) In the observations –2, –9, 31, 28, 41, 13, if we write 9 instead of (–9), then the new mean = ...

(a) 20 (b) 17 (c) 13 (d) 22

(36) If all the observations 33, 17, 23, 28, 42, 37 are increased by 4, then new mean = ...

(a) 28 (b) 30 (c) 32 (d) 34

(37) If all the observations 8, 13, 9, 15, 12 are multiplied by (–5), then the new mean = ...

(a) 11.4 (b) –11.5 (c) –57 (d) 57

*

Summary

In this chapter we have studied the following points :

1. Facts or figures, collected with a certain purpose, are called data.

2. Statistics is the area of study dealing with the presentation, analysis and interpretation of data.

3. Data are of two types (i) primary data and (ii) secondary data.

4. Data can be presented graphically in the form of bar graphs, histograms and frequency polygons.

5. The three measures of central tendency for ungrouped data are :

(i) **Mean :** The number obtained by dividing the sum of values of observations of data by the number of observations is called the mean of the data. It is denoted by $\overline{x}$ and $\overline{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

(ii) The mean for grouped frequency distribution is given by

$$\overline{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n} \text{ ; where } n = \sum_{i=1}^{k} f_i$$

(iii) **Median (M) :** It is the value of middle-most observation (s).

If n is odd, then M = the value of $\left(\dfrac{n+1}{2}\right)^{\text{th}}$ observation

If n is even, then M = Mean of the values of $\left(\dfrac{n}{2}\right)^{\text{th}}$ and $\left(\dfrac{n}{2}+1\right)^{\text{th}}$ observations.

(iv) **Mode (Z) :** The mode is the most frequently occuring observation.

CHAPTER 17

PROBABILITY

"It is not certain that everything is uncertain."
"Contradiction is not a sign of falsity nor the lack of concentration a sign of truth." – ***Pascal***

17.1 Introduction

The words such as 'probably', 'chances', 'most probably', 'doubtful' are often used in day-to-day language.

(1) The weather forecaster on T.V. might say "There will be heavy rains in Jamnagar and South Gujarat within two days" based on forecast models.

(2) On railway station we hear the announcements such as : "The Lok-shakti express from Dadar (Mumbai) to Ahmedabad is expected to arrive 10 minutes late than its scheduled time." There are probable predictions.

(3) There is a 70-30 chance of India winning a toss in today's match.

(4) Most probably Nikita will stand first in board examination in our school.

(5) Chances are less that the price of onion will go down.

These words signify the likelihood or chances of something happening or not happening. But the word **'probability'** is not another word of possibility. In case of uncertainty, we may also like to know the degree of uncertainty. Before setting up manufacturing plant, the enterpreneur would like to know how the product will sell. Before going on picnic it would help us to know the chances of rain etc. The theory of probability helps in such matters. The theory attempts to analyse mathematically the possible outcomes of happening whose actual result can not be predicted with certainty. It provides us with the measure of uncertainty in an uncertain situation.

Though probability started with gambling, it has been used extensively in the field of physics, commerce, science, biological sciences, medical science, weather forcasting etc.

17.2 Probability – an Experimental Approach

In previous classes, we have had a glimpse of probability when we performed experiments like tossing a coin, playing cards, throwing of dice etc. and observed their out-comes. We will now learn to measure the chances of occurrence of particular out-comes in an experiment.

Blaise Pascal (1623-1662)

Pierre de Fermat (Born : 17 Aug. 1601 Died : 12 Jan. 1665, France)

The concept of probability developed in a very strange manner. In 1654, a gambler Chevalier de Mere approached the well-known 17th century French philosopher and mathematician Blaise Pascal regarding certain dice problems. Pascal became interested in these problems, studied them and discussed them with another French mathematician, Pierre de Fermat. Both Pascal and Fermat solved the problems independently. This work was the beginning of Probability Theory.

The first book on the subject was written by the Italian mathematician, J. Cardan (1501-1576). The title of the book was 'Book on Games of Chance' (Liber de Ludo Aleae), published in 1663. Notable contributions were also made by mathematicians J. Bernoulli (1654-1705), P. Laplace (1749-1827), A. A. Markov (1856-1922) and A. N. Kolmogorov (born 1903).

Activity 1 : Take any balanced coin, toss it five times and note down the number of times head and tail come up. Record the observations in the following table :

Table 17.1

Number of times the coin is tossed	Number of times head (H) comes up	Number of times tail (T) comes up
5		

Now write down the value of the following fractions :

$$\frac{\text{Number of times head comes up}}{\text{Total number of times the coin is tossed}} \quad \text{and} \quad \frac{\text{Number of times tail comes up}}{\text{Total number of times the coin is tossed}}$$

Now toss the coin ten times in the same way and record the observations as above. Again find the value of the fractions mentioned above.

Repeat the same experiments by increasing the number of trials 20 times, 25 times and record the number of times head and tail come up and also find the corresponding fractions.

We will find that when the number of tosses is very large, the value of the fractions comes closer and closer to 0.5.

Activity 2 : Divide the class in groups of 3 or 4 students. Let a student in each group toss a coin 25 times. Another student in each group will record the observations regarding the heads and tails. Note that the coin given to each group should be a balanced coin. By a balanced coin we mean when tossed the coin has equal chances of a head or a tail.

Now prepare a table like table 17.2.

Table 17.2

Group (i)	Number of heads (ii)	Number of tails (iii)	$\frac{\text{Total number of heads}}{\text{Total number of times the coin is tossed}}$ (iv)	$\frac{\text{Total number of tails}}{\text{Total number of times the coin is tossed}}$ (v)
1	9	16	$\frac{9}{25} = 0.36$	$\frac{16}{25} = 0.64$
2	12	13	$\frac{12+9}{25+25} = \frac{21}{50} = 0.42$	$\frac{13+16}{25+25} = \frac{29}{50} = 0.58$
3	17	8	$\frac{9+12+17}{25+25+25} = \frac{38}{75} = 0.51$	$\frac{16+13+8}{25+25+25} = \frac{37}{75} = 0.49$
4	15	10	$\frac{9+12+17+15}{25+25+25+25} = \frac{53}{100} = 0.53$	$\frac{16+13+8+10}{25+25+25+25} = \frac{47}{100} = 0.47$
.	.	.	...	...
.	.	.	...	...
.	.	.	...	...

First the group 1 will write down its observations and calculate the fractions. Then group 2 will write down its observations, but will calculate the fractions for the combined (cummulative) data of group 1 and group 2. Repeat the same for other groups. These fractions are called cummulative fractions.

We have noted the first four rows based on the observations given by this class.

What do we observe in the table ? We will find that as the total number of tosses increases, the value of the fractions in column (iv) and (v) comes closer and closer to 0.5.

Activity 3 : Throw a balanced die 15 times and note down the number of times the numbers 1, 2, 3, 4, 5, 6 come up. Record the observations in table 17.3.

Table 17.3

Number of times a die is thrown	Number of times the scores turn up					
	1	2	3	4	5	6
15						

Then find the value of the fractions :

$$\frac{\text{Number of times 1 turned up}}{\text{Total number of times the die is thrown}}$$

$$\frac{\text{Number of times 2 turned up}}{\text{Total number of times the die is thrown}}$$

.

.

$$\frac{\text{Number of times 6 turned up}}{\text{Total number of times the die is thrown}}$$

Now throw the die 30 times and record the observations and calculate the fractions as above.

From above activities, as the number of throws of the die increases, we will find that the value of each fraction calculated comes closer and closer to $\frac{1}{6}$.

To check this, we can perform a group activity in the class as activity 2. Divide the students of the class in four to five groups. One student in each group will throw a die ten times. The observations should be noted and cummulative fractions should be calculated.

We will record the value of the fraction for the number 3 in table 17.4.

Table 17.4

Group (i)	Total number of times a die is thrown by the group (ii)	$\frac{\text{Cummulative number of times 3 turned up}}{\text{Total number of times the die is thrown}}$ (iii)
1.	---	---
2.	---	---
3.	---	---
4.	---	---
5.	---	---

The above table can be extended to write down fractions for the other numbers.

What do we observe in this table ?

We will find that as the total number of throws of the die increases, the fraction in column (iii) moves closer and closer to $\frac{1}{6}$.

Activity 4 : Toss two balanced coins simultaneously twenty times and record the observations in the table given below :

Table 17.5

Number of times the two coins are tossed	Number of times one head comes up	Number of times two heads come up	Number of times two tails come up
20	---	---	---

Now calculate the value of fractions :

$$A = \frac{\text{Number of times one head comes up}}{\text{Total number of times two coins are tossed}}$$

$$B = \frac{\text{Number of times two heads come up}}{\text{Total number of times two coins are tossed}}$$

$$C = \frac{\text{Number of times two tails come up}}{\text{Total number of times two coins are tossed}}$$

[Note : 'two tails comes up' is same as 'no head comes up']

In activity 1 each toss of a coin is called a trial. In activity 3 each throw of a die is a trial and in activity 4 toss of two coins is also trial. So, a trial is an action which results in one or more outcomes. So, an **event** for an experiment is the collection of some outcomes of the experiment.

From above activities, let us now see what probability is ? Here from what we directly observe as the outcomes of our trials, we find the experimental or empirical probability.

Let n be the total number of trials. The empirical probability denoted by P(E) of an event E happening, is given by

$$P(E) = \frac{\text{Number of trials in which the event occured}}{\text{Total number of trials}}$$

For our convenience we will write probability instead of empirical probability.

Example 1 : A coin is tossed 100 times in which 56 times head comes up and 44 times tail comes up. Calculate the probability for each event.

Soultion : Here the coin is tossed 100 times. Therefore the total number of trials is 100. Let us call the events of getting a head and getting a tail as E and F respectively. Then the number of times E happens. i.e. the number of times a head comes up is 56.

$$\text{So, the probability of E} = \frac{\text{Number of times head comes up}}{\text{Total number of trials}}$$

i.e. $P(E) = \frac{56}{100} = 0.56$

$$\text{Similarly, the probability of the event of getting tail} = \frac{\text{Number of times tail comes up}}{\text{Total number of trials}}$$

i.e. $P(F) = \frac{44}{100} = 0.44$

Note that in above example P(E) + P(F) = 0.56 + 0.44 = 1. Here E and F are the only two possible outcomes of each trial.

Example 2 : In cricket Sachin hits a century in 12 innings out of 60 innings. Find the probability that he did not hit century.

Solution : Let the event that Sachin hit a century 12 times be called event A.

$\therefore$ Number of trials Sachin did not hit century out of 60 innings = 60 – 12 = 48

Let B be the event that Sachin did not hit century.

$$\therefore P(B) = \frac{\text{Number of innings in which Sachin did not hit century}}{\text{Total number of innings he played}}$$

$P(B) = \frac{48}{60} = \frac{4}{5} = 0.80$

Example 3 : Two coins are lossed 1000 times and we get two heads 225 times, one head 500 times and no head 275 times. Find the probability of occurrence of each of these events.

Solution : Let us denote the events of getting two heads, one head and no head by A, B and C respectively. So,

$$P(A) = \frac{225}{1000} = 0.225$$

$$P(B) = \frac{500}{1000} = 0.500$$

$$P(C) = \frac{275}{1000} = 0.275$$

Here also note that, P(A) + P(B) + P(C) = 0.225 + 0.500 + 0.275 = 1

and A, B, C are the only outcomes of the trial.

When a coin is tossed and the head turns up, we say event H has occured. Similarly when a coin is tossed and the tail turns up, we say event T has occured. If a coin is tossed twice or two coins are tossed simultaneously and two heads turn up, we say event HH has occured. Similarly when a coin is tossed thrice and head, head and tail turn up respectively we say the event HHT has occured etc.

Example 4 : A balanced coin is tossed thrice, find the probabilities of the following events :

(i) Occurrence of event H all the three times.

(ii) Occurrence of event H twice and T once.

(iii) Occurrence of H once and T twice.

(iv) Occurrence of T all the three times.

(v) Occurrence of T four times.

(vi) Atmost three heads occur.

Solution : The outcomes of an event that a balance coin is tossed thrice are

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

Here total number of outcomes is 8.

(i) Let A be the event that H occur all the three times. Then this event can occur in only one way, HHH.

$$\therefore\ P(A) = \frac{\text{Number of outcomes containing three heads}}{\text{Total number of outcomes}} = \frac{1}{8}$$

(i) Let B be the event that H comes up twice and T comes once. This event can occur in three ways : HHT, HTH and THH.

$$\therefore\ P(B) = \frac{3}{8}$$

(iii) Let C be the event that H comes once and T twice. This event can also occur in three ways : HTT, THT, TTH.

$\therefore$ P(C) = $\frac{3}{8}$

(iv) Let D be the event that T comes all three times.

The event can occur in one way : TTT. So P(D) = $\frac{1}{8}$

Here also we note that P(A) + P(B) + P(C) + P(D) = $\frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$

$= \frac{8}{8} = 1$

(v) Let E be the event that T occurs four times which is not possible for this example. So number of outcomes is zero.

P(E) = 0

(vi) Let F be the event that H occurs atmost three times. This is a certain event because all eight outcomes has atmost three heads.

$\therefore$ P(F) = $\frac{8}{8} = 1$

Example 5 : A die is thrown 100 times with the frequencies for the outcomes 1, 2, 3, 4, 5 and 6 as given in table 17.6.

Table 17.6

Outcome	1	2	3	4	5	6
Frequency	18	14	11	17	18	22

Find the probability of getting each outcome.

Solution : Let E_i denote the event of getting the outcome i, where i = 1, 2, 3, 4, 5, 6. Then probability of getting outcome

$$P(E_i) = \frac{\text{Frequency of } i}{\text{Total number of times the die is thrown}}$$

$$\therefore \quad P(E_1) = \frac{18}{100} = 0.18$$

Similarly, $P(E_2) = \frac{14}{100} = 0.14$

$$P(E_3) = \frac{11}{100} = 0.11$$

$$P(E_4) = \frac{17}{100} = 0.17$$

$$P(E_5) = \frac{18}{100} = 0.18$$

$$P(E_6) = \frac{22}{100} = 0.22$$

Note that $P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6)$

$= 0.18 + 0.14 + 0.11 + 0.17 + 0.18 + 0.22 = 1$

Note : From above examples note that

(i) The probability of each event lies between 0 and 1 including 0 and 1.

(ii) The sum of all the probabilities is 1, if the events are all the possible events and having no common outcome.

(iii) For example in example 5, E_1, E_2, E_3, E_4, E_5, E_6 are all the possible outcomes of the trial.

(iv) The probability of an impossible event is zero while probability of certain event is one.

An object is chosen at random means out of all objects, object is selected without any prejudice and pre-condition.

Example 6 : On one page of a telephone directory, there were 200 telephone numbers. The frequency distribution of their unit place digit (for example in the number 230627, the unit place digit is 7) is given in the table 17.7.

Table 17.7

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	22	26	22	22	20	10	14	28	16	20

Without looking at any page, a number is choosen at random. What is the probability that the digit in its unit place is 5, 7 or 9 ?

Solution : (i) The probability of digit 5 in the unit place

$$= \frac{\text{frequency of 5}}{\text{Total number of selected telephone numbers}} = \frac{10}{200} = 0.05$$

(ii) The probability of digit 7 in the unit place $= \frac{28}{200} = 0.14$

(iii) The probability of digit 9 in the unit place $= \frac{20}{200} = 0.1$

Example 7 : 1500 family with two children were selected randomly, and the following data were recorded :

Number of girls in family	2	1	0
Number of families	475	814	211

Compute the probability of a family chosen at random having,

(i) 2 girls (ii) 1 girl (iii) No girl.

Solution : Here total number of families is 1500.

(i) The probability of two girls in the selected family $= \frac{475}{1500} = 0.3167$

(ii) The probability of 1 girl in the selected family = $\frac{814}{1500}$ = 0.5427

(iii) The probability of no girl in the selected family = $\frac{211}{1500}$ = 0.1406

Example 8 : An organisation selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in the family. The information gathered is listed in the table below.

Monthly income (in ₹)	Vehicles per family			
	0	1	2	More than 2
Less than 10000	10	160	25	0
10000 - 13000	0	305	27	2
13000 - 16000	1	535	29	1
16000 - 19000	2	469	59	25
19000 or more	1	579	82	88

Suppose a family is chosen at random. Find the probability that the family chosen is

(1) earning ₹ 13000-16000 per month and owns exactly 2 vechicles.
(2) earning ₹ 19000 or more per month and owns exactly 1 vehicle.
(3) earning less than ₹ 10000 per month and does not own any vehicle.
(4) earning ₹ 19000 or more per month and owns more than 2 vehicles
(5) owns not more than 1 vehicle.

Solution : Here total number of families is 2400.

(1) Probability of a chosen family earning ₹ 13000-16000 per month and owning exactly 2 vehicles = $\frac{29}{2400}$ = 0.0121

(2) Probability of a chosen family earning ₹ 19000 or more per month and owning exactly 1 vehicle = $\frac{579}{2400}$ = 0.2413

(3) Probability of a chosen family earning less than ₹ 10000 per month and does not own any vehicle = $\frac{10}{2400}$ = 0.0004

(4) Probability of a chosen family earning ₹ 19000 or more per month and owning more than 2 vehicles = $\frac{88}{2400}$ = 0.3667

(5) Probability of a chosen family owning not more than 1 vehicle

$$= \frac{\text{Number of families having 0 vehicle + number of families having 1 vehicle}}{\text{Total number of families}}$$

$$= \frac{10+0+1+2+1+160+305+535+469+579}{2400} = \frac{2062}{2400} = 0.8592$$

Example 9 : A teacher wanted to analyse the performance of students of two sections in mathematics test of 100 marks. Looking at their performance, he found that a few students got less than 20 marks and a few got 70 or more marks. So, he decided to group them into classes in lengths of varying sizes as follwos :

Marks	0–20	20–30	30–40	40–50	50–60	60–70	70 & above	Total
No. of students	7	10	10	20	20	15	8	90

(i) Find that probability that a randomly selected student obtained less than 20 % in the mathematics test.

(ii) Find the probability that a randomly selected student obtained 60 or more marks.

Solution : Here total number of students is 90.

(i) Let A be the event that a student obtained less than 20 % in mathematics test

$$\therefore \text{ P(A)} = \frac{\text{Number of students with less than 20 marks}}{\text{Total number of students}} = \frac{7}{90} = 0.0778$$

(ii) Let B be the event that a student obtained 60 or more marks.

Here number of students who obtained 60 or more marks = 15 + 8 = 23

$$\therefore \text{ P(B)} = \frac{\text{Number of students who obtained 60 or more marks}}{\text{Total number of students}} = \frac{23}{90} = 0.2556$$

Example 10 : The blood groups of 30 students of class IX are recorded as follows :

Blood group	**Number of students**
A+	9
B–	6
O+	12
AB+	3
	Total 30

Find the probability that a student of this class, selected at random has blood group : (i) AB+ (ii) O+ (iii) Neither O+ nor AB+

Solution : Here total number of students of the class is 30.

(i) Let A be the event that a student selected at random has blood group AB+.

$\therefore$ P (A) = $\frac{3}{30}$ = 0.10

(ii) Let B be the event that a student selected at random has blood group O+.

$\therefore$ P (B) = $\frac{12}{30}$ = 0.400

(iii) Let C be the event that a student selected at random has blood group neither O+ nor AB+.

In event C total number of students having blood group neither O+ nor AB+ is 9 + 6 = 15.

$\therefore$ P (C) = $\frac{15}{30}$ = 0.50

Note that the student having blood group neither O+ nor AB+ is same as the student having blood group either A+ or B–.

*

EXERCISE 17

1. The record of a weather station shows that out of the past 250 consecutive days, its weather forcasts were correct on 175 days.

(i) What is the probability that on a given day it was correct ?

(ii) What is the probability that it was not correct on a given day ?

2. A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table shows the results of 1000 cases.

Distance (in km)	Less than 4000	4000 to 9000	9001 to 14000	More than 14000
Frequency	20	210	325	445

If you buy a tyre of this company, what is the probability that

(i) it will need to be replaced before it has covered 4000 km ?

(ii) it will be replaced after 9000 km ?

(iii) it will need to be replaced after it has covered distance somewhere between 4000 km and 14000 km ?

3. The percentage of marks obtained by a student in the monthly unit tests are given below :

Unit test	I	II	III	IV	V
% of marks obtained	68	72	75	70	65

Find the probability that the student gets more than 70 % marks and in between 60 % to 70 % marks in unit test.

4. An insurance company selected 1000 drivers at random in a particular city to find the relationship between age and accidents. The data are given in the following table :

Age of driver (in years)	Accidents in one year				
	0	1	2	3	More than
18 – 29	220	80	55	30	17
30 – 50	252	63	30	11	9
Above 50	180	23	17	8	5

Find the probability of the following events for a driver chosen at random from the city :

(i) Being 18 – 29 years of age and doing exactly 3 accidents in one year.

(ii) Being 30 – 50 years of age and doing one or more accidents in one year.

(iii) Doing no accident in one year.

5. The following frequency distribution table gives the weight of 40 students of a class :

Weight (in kg)	Number of students
31 – 35	9
36 – 40	5
41 – 45	14
46 – 50	3
51 – 55	3
56 – 60	2
61 – 65	2
66 – 70	1
71 – 75	1
	Total 40

(i) Find the probability that the weight of a student in the class lies in the interval 46 – 50 kg.

(ii) What is the probability that the weight of a student is 30 kg ?

(iii) What is the probability that the weight of a student is more than 30 kg ?

6. Fifty seeds were selected at random from each of 5 bags of seeds and were kept under standardised conditions favourable to germination. After 20 days, the number of seeds which have germinated in each collection were counted and recorded as follows :

Bag	1	2	3	4	5
Number of seeds germinated	40	48	40	35	45

What is the probability of germination of

(i) more than 40 seeds in a bag ?

(ii) 49 seeds in a bag ?

(iii) more than 35 seeds in a bag ?

7. Twelve bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg) :

5.0, 4.97, 5.05, 5.03, 5.08, 5.0, 4.98, 4.99, 5.04, 5.07, 5.06, 4.96

Find the probability that any of these bags chosen at random contains (i) more than 5 kg of flour (ii) exactly 5 kg of flour.

8. Two balance dice are tossed 50 times. The sum of integers obtained on the dice is noted below :

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	3	9	8	8	4	5	1	3	7	2	0

Find the probability that

(i) The sum of integers is more than 9.

(ii) The sum of integers is exactly 7.

(iii) The sum of integers is less than 6.

9. The distance covered by (in km) 40 students from their residence to their school in rural area is as follows :

Distance (in km)	**Number of students**
0 – 5	5
5 – 10	11
10 – 15	11
15 – 20	9
20 – 25	1
25 – 30	1
30 – 35	2
	Total 40

What is the probability that the distance of a student from residence to school is

(i) more than 20 km.

(ii) less than or equal to 15 km.

(iii) between 10 – 15 km.

(iv) between 10 – 20 km.

10. From a well-shuffled pack of 52 cards one card is selected at random. Find the probability that the card is

(i) an ace of heart.

(ii) a club card.

(iii) a face card.

(iv) a queen or a king.

11. A die is tossed once. Then find the probability that the number appearing on the die is even.

12. A die is tossed once. Then find the probability that the number appearing on the die is prime.

13. A survey of 500 families having girls is as follows :

Number of girls	0	1	2
Number of families	75	275	150

Find the probability of a family chosen randomly

(i) having one girl. (ii) having two girls (iii) atleast one girl.

14. A survey of 1000 students is conducted for their I.Q. is as follows :

I.Q.	Below 30	30 – 50	50 – 60	60 – 70	More than 70
Number of students	120	230	300	190	160

Find the probability of

(i) I.Q. between 50 – 60 (ii) I.Q. more than 70

(iii) I.Q. 50 or below 50 (iv) I.Q. between 60 – 70

(v) I.Q. more than 50

15. The marks obtained in mathematics out of 50 by 50 students of a class are as follows :

Marks	Below 20	20 – 30	30 – 40	40 – 50
Number of students	6	11	20	13

Find the probability of a student getting

(i) marks between 20 and 40. (ii) marks above 40.

(iii) marks less than or equal to 30. (iv) marks between 30 and 40.

(v) marks above 20.

16. Select proper option (a), (b), (c) or (d) and write in the box given on the right so that the statement becomes correct :

(1) The probability of getting number 5 on a balance die is .... $\square$

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{6}$

(2) The probability of getting both heads when two balanced coins are tossed is .... $\square$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

(3) The probability of any event (other than impossible and certain event) always lies between ....

(a) 1 and 2 (b) 0 and 1 (c) 0 and 2 (d) – 1 and 1

(4) The probability of one card, selected from a pack of 52 cards is a jack is ....

(a) $\frac{1}{52}$ (b) $\frac{2}{52}$ (c) $\frac{1}{13}$ (d) $\frac{1}{17}$

(5) The probability of getting 51 marks out of 50 marks is ....

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

(6) The probability of the event "the sun rises in the east" is ....

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

*

Summary

In this chapter, we have studied the following points :

1. An event for an experiment is the collection of 'some' outcomes of the experiment.

2. The empirical (or experimental) probability P(E) of an event E is given by

$$P(E) = \frac{\text{Number of times event occurs}}{\text{Total number of trials}}$$

3. The probability of an event lies between 0 and 1 (0 and 1 inclusive).

CHAPTER 18

LOGARITHM

18.1 Introduction

Previously we have learnt about powers and exponents. Also we have learnt about the properties of exponents.

For, a, $b \in R^+$, x, $y \in R$

(i) $a^x \cdot a^y = a^{x+y}$ (ii) $\dfrac{a^x}{a^y} = a^{x-y}$

(iii) $(a^x)^y = a^{xy}$ (iv) $(ab)^x = a^x \cdot b^x$ (v) $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

18.2 Logarithm

John Napier was born in 1550. He died on 4th April, 1667 in Edinburgh. A mathematician ***John Napier*** introduced the concept of logarithm for the first time in 17th century. Later, ***Henry Briggs***, a British mathematician born in Feb. 1561 in Yorkshire – England, prepared and published logarithm tables. He died on 26th January, 1663 in Oxford – England. Logarithm tables made complicated numerical calculations both – easy and fast. Today with the advent of desk calculators and computers, the work of numerical calculations has become easier and faster, thus reducing the usefulness of logarithm tables. All the while they are useful for calculations in the study of science and mathematics.

Definition : Let $a \in R^+ - \{1\}$ $y \in R^+$, $x \in R$ and let $a^x = y$. Then the value of x is called logarithm of y to the base a. It is denoted by $\log_a y$ (read as log y to the base a).

$\therefore$ $a^x = y$ if and only if $x = \log_a y$

From the above definition we can conclude that,

(i) we can obtain the logarithm of only positive real numbers.

(ii) for any $a \in R^+ - \{1\}$, $\log_a 1 = 0$, since $a^0 = 1$.

(iii) for every $a \in R^+ - \{1\}$, $\log_a a = 1$, since $a^1 = a$

(iv) for every $x \in R^+$, $y \in R^+$, $\log_a x = \log_a y$ if and only if $x = y$.

18.3 Properties of Logarithm

We will assume following properties of logarithm :

(1) If $a \in R^+ - \{1\}$, then $a^{\log_a x} = x$ ($x \in R^+$) and $\log_a a^x = x$ ($x \in R$).

Theorem 1 : Product rule

Let $a \in R^+ - \{1\}$.

Then for $x, y \in R^+$, $\log_a (xy) = \log_a x + \log_a y$

Corollary : If $x_1, x_2, x_3, ..., x_n \in R^+$ and $a \in R^+ - \{1\}$, then

$\log_a (x_1 x_2 x_3 ... x_n) = \log_a x_1 + \log_a x_2 + ... + \log_a x_n$

Theorem 2 : Quotient Rule

If $a \in R^+ - \{1\}$, and $x, y \in R^+$, $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

Corollary : $\log_a \left(\frac{1}{y}\right) = -\log_a y$; $a \in R^+ - \{1\}, y \in R^+$

Theorem 3 : Rule for the logarithm of a power

If $a \in R^+ - \{1\}$, $x \in R^+$, $n \in R$, then $\log_a x^n = n \log_a x$.

Example 1 : Simplify

(i) $\log_3 \left(\frac{17}{25}\right) + \log_3 \left(\frac{600}{119}\right) - \log_3 \left(\frac{8}{7}\right)$ (ii) $4\log_a \left(\frac{2}{7}\right) - 3\log_a \left(\frac{3}{49}\right) - \log_a \left(\frac{14}{9}\right)$

(iii) $\log_2 \left(\frac{\sqrt[3]{16}}{4}\right) + \log_3 \left(\frac{\sqrt{27}}{81}\right)$

Solution : (i) $\log_3 \left(\frac{17}{25}\right) + \log_3 \left(\frac{600}{119}\right) - \log_3 \left(\frac{8}{7}\right)$

$= \log_3 \left(\frac{17}{25} \times \frac{600}{119}\right) - \log \left(\frac{8}{7}\right)$

$= \log_3 \left(\frac{17}{25} \times \frac{600}{119} \div \frac{8}{7}\right)$

$= \log_3 \left(\frac{17}{25} \times \frac{600}{119} \times \frac{7}{8}\right)$

$= \log_3 3 = 1$

(ii) $4\log_a\left(\frac{2}{7}\right) - 3\log_a\left(\frac{3}{49}\right) - \log_a\left(\frac{14}{9}\right)$

$= \log_a\left(\frac{2}{7}\right)^4 - \log_a\left(\frac{3}{49}\right)^3 - \log_a\left(\frac{14}{9}\right)$

$= \log_a\left(\frac{2^4}{7^4}\right) - \log_a\left(\frac{3^3}{(49)^3}\right) - \log_a\left(\frac{14}{9}\right)$

$= \log_a\left[\frac{2\times2\times2\times2}{7\times7\times7\times7}\times\frac{49\times49\times49}{3\times3\times3}\times\frac{9}{14}\right] = \log_a\left(\frac{56}{3}\right)$

(iii) $\log_2\left(\frac{\sqrt[3]{16}}{4}\right) + \log_3\left(\frac{\sqrt{27}}{81}\right)$

$= \log_2\left(\frac{(2^4)^{\frac{1}{3}}}{2^2}\right) + \log_3\left(\frac{(3^3)^{\frac{1}{2}}}{3^4}\right)$

$= \log_2\left(\frac{2^{\frac{4}{3}}}{2^2}\right) + \log_3\left(\frac{3^{\frac{3}{2}}}{3^4}\right)$

$= \log_2\left(2^{-\frac{2}{3}}\right) + \log_3\left(3^{-\frac{5}{2}}\right)$

$= \left(-\frac{2}{3}\right).\ \log_2 2 + \left(-\frac{5}{2}\right)\log_3 3$

$= -\frac{2}{3} - \frac{5}{2}$ **($\log_a a = 1$)**

$= -\frac{19}{6}$

Example 2 : Simplify : (i) $\log_a \frac{x^2}{yz} + \log_a \frac{y^2}{xz} + \log_a \frac{z^2}{xy}$ (ii) $\frac{(\log_3 81)(\log_2 64)}{\log_5 125}$

Solution : (i) $\log_a \frac{x^2}{yz} + \log_a \frac{y^2}{xz} + \log_a \frac{z^2}{xy}$

$= \log_a\left(\frac{x^2}{yz}\times\frac{y^2}{xz}\times\frac{z^2}{xy}\right)$

$= \log_a 1 = 0$

(ii) $\frac{(\log_3 81)(\log_2 64)}{\log_5 125} = \frac{(\log_3 3^4)(\log_2 2^6)}{(\log_5 5^3)}$

$$= \frac{(4\log_3 3)(6\log_2 2)}{3\log_5 5}$$

$$= \frac{4 \times 6}{3} \qquad (\because \log_a a = 1)$$

$$= 8$$

18.4 Common Logarithm

Since we write numbers in the decimal system, calculations become simple if we use the logarithm to the base 10. The logarithm to the base 10 is called common logarithm. In the rest of this chapter, we will simply write $\log x$ instead of $\log_{10} x$. To find $\log x$ for positive x, let us study the following table :

Number x	0.0001	0.001	0.01	0.1	1	10	100	1000
x written as power of 10	10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}	10^{3}
Logarithm of x (to the base 10)	-4	-3	-2	-1	0	1	2	3

Here each x is an integral power of 10. So, it is easy to find log x. When x is not an integral power of 10, to find logarithm (to the base 10), first we write x as a product of an integral power of 10 and a number between 1 and 10. This is done because the logarithm tables have been prepared only for numbers between 1 and 10. It is convenient to find the logarithm of any positive number using this form.

(1) $108.9 = \frac{108.9}{100} \times 100 = 1.089 \times 10^2$

(2) $75.32 = \frac{75.32}{10} \times 10 = 7.532 \times 10^1$

(3) $0.54 = 0.54 \times 10 \times \frac{1}{10} = 5.4 \times 10^{-1}$

(4) $0.000279 = 0.000279 \times 10000 \times \frac{1}{10000} = 2.79 \times 10^{-4}$

(5) $0.0000163 = 0.0000163 \times 100000 \times \frac{1}{100000} = 1.63 \times 10^{-5}$

(6) $456723 = \frac{456723}{100000} \times 100000 = 4.56723 \times 10^5$

In each of the above examples, we have divided or multiplied by an appropriate power of 10 to get a non-zero digit to the left of decimal point and then multiplied or divided by a power of 10 to make both sides equal, leading to the representation of the given numbers in the required form.

In general, any positive number n can be put in the form $n = t \times 10^p$, where $1 \leq t < 10$ and p is an integer. We shall call this representation of a positive number as presentation of number in the standard form.

If the standard form of a number is 8.97×10^6, its decimal form is $8.97 \times 1000000 = 8970000$.

A positive number expressed in its decimal form can be expressed in its standard form by applying the following rules :

(1) To shift the decimal point p places to the left, multiply by 10^p.

(2) To shift the decimal point p place to the right, multiply by 10^{-p}.

Example 3 : Write the following numbers in the standard form :

(1) 703251 (2) 3279 (3) 89.99 (4) 603.328 (5) 0.001938 (6) 0.0000168

Solution : (1) $703251 = 7.03251 \times 10^5$ (2) $3279 = 3.279 \times 10^3$

(3) $89.99 = 8.999 \times 10^1$ (4) $603.328 = 6.03328 \times 10^2$

(5) $0.001938 = 1.938 \times 10^{-3}$ (6) $0.0000168 = 1.68 \times 10^{-5}$

Example 4 : Write the following numbers in decimal form :

(1) 3.72×10^2 (2) 45.793×10^4 (3) 1.798×10^{-3} (4) 728.32×10^{-5}

(5) 83.596×10^{-2}

Solution : (1) $3.72 \times 10^2 = 372$ (2) $45.793 \times 10^4 = 457930$

(3) $1.798 \times 10^{-3} = 0.001798$ (4) $728.32 \times 10^{-5} = 0.0072832$

(5) $83.596 \times 10^{-2} = 0.83596$

18.5 The Characteristic and Mantissa of Logarithm

Let the standard form of a positive number n be $t \times 10^p$, where $1 \leq t < 10$ and p is an integer.

$$\begin{aligned} \therefore \log n &= \log (t \times 10^p) \\ &= \log t + \log 10^p \\ &= \log t + p \log 10 \\ &= \log t + p \end{aligned}$$

Since $1 \leq t < 10$, we have $\log 1 \leq \log t < \log 10$. i.e. $0 \leq \log t < 1$. We note that $\log n = \log t + p$ consist of two parts : (1) p and (2) $\log t$.

Here p is called the **characteristic** and $\log t$ is called the **mantissa** of $\log n$.

For example : $83.628 = 8.3628 \times 10^1$, $p = 1$

$894.82 = 8.9482 \times 10^2$, $p = 2$

$0.0329 = 3.29 \times 10^{-2}$, $p = -2$

$0.000487 = 4.87 \times 10^{-4}$, $p = -4$

$279389 = 2.79389 \times 10^5$, $p = 5$

From above examples, we note that –

(1) When the integral part of a number is non-zero, p is one less than the number of digits in the integral part.

(2) When the integral part of the number is zero, $p = -(n + 1)$, where n is the number of zeros beween the decimal point and the first non-zero digit of the number.

18.6 Use of Logarithmic Tables

Ready tables of **logarithms** and **antilogarithms** shortly called **logtables** and **antilogtables** are available. The logtables consist of three parts : In the first part, there is one column, the first column from left, which contains two digit numbers from 10 to 99. Next there are ten columns headed by numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The last part called **'mean difference'** has nine columns headed by numbers from 1 to 9.

The antilogtables are of the same type, except that the first column contains numbers froms 0.00 to 0.99.

Suppose we start with a two digit number 81 and wish to find log 81. Here 81 = 81 + 0. Its characteristic is 1. The mantissa can be obtained from logtables. Look for the number formed by first two digits in the first column. For this, find 81 in the first column and look at row against it. At the intersection of this row and the column headed by 0 is the number 9085. The mantissa of log 81 is 0.9085. Hence, log 81 = 1 + 0.9085 = 1.9085.

To obtain the mantissa of the logarithm of a three digit number, first find the number formed by the first two digits of the given number in the column to the extreme left of the logtables. Look at the row against this number. In this row, the number in the column headed by the third digit of the given number gives the mantissa. For example to find mantissa of log 723, look at the row against 72 in the first column and in the column headed by 3. The number 8591 appears there. Hence mantissa of log 723 is 0.8591. Since the characteristic of log 723 is 2, we have log 723 = 2.8591.

For finding the logarithm of a number with four digits, the columns of mean difference will also be used. For examples suppose we want to find the mantissa of log 3986. The number 3986 is divided into three parts 39, 8 and 6. Now look for 39 in the first column. Then find the number in the row against 39 in the column headed by 8. This is 5999. Finally look for the number in the same row in the column headed by 6 among the columns of mean differences. This number is 7. Adding 7 to 5999, we get 6006. Hence the mantissa of log 3986 is 0.6006. Since the characteristic of 3986 is 3, log 3986 = 3.6006.

Note that the logtables are used to find the mantissa of the logarithm of a number. Our logtables are four digits tables and so for finding the mantissa of the logarithm of a number with more than four digits. We approximate the number to a four digit number. For this, form the number formed by first four digits of the given number. If the fifth digit of the given number is less than 5, this four digit number is the required approximation. If the fifth digit is 5 or greater, then add 1 to the last digit of the four digit number obtained by truncation. The characteristic of the logarithm of a given number is obtained in the usual way. The mantissa is the mantissa of the logarithm of the four digit number which approximates the given number. For example, let $x = 5.79881$. Then the characteristic of log x is 0. The four digit approximation of x is 5.799. Hence the mantissa of log x = the mantissa of log 5.799 = 0.7634. Hence log 5.79881 = 0.7634.

When the characteristic of a logarithm is a negative number $-n$ it is denoted by $\bar{n}$ (read as n bar). For example, log (0.002675) = $\bar{3}.4273$.

18.7 Use of Antilogtables

The antilogarithm is used to get the number from its logarithm. The first column from the left of the antilogtables contain numbers from 0.00 to 0.99. In all other respects, antilogtables are similar to logtables. The antilogs are also used in the same way as logtables.

Since the logtable gives only the mantissa part of the logarithm of a number, the antilog table will give a number corresponding to the mantissa part only. Then by using characteristic the actual number for the given logarithm can be obtained. For example, suppose we want to find antilog (1.5278). From antilogtables, we find that antilog 0.5278 = 3.371 (Meaning that log 3.371 = 0.5278). Hence, antilog 1.5278 = 3.371×10^1 = 33.71. Also antilog $\bar{3}.5278 = 3.371 \times 10^{-3} = 0.003371$. Note that power of 10 is (–1) means no zero between decimal point and first non-zero digit. (–3) means two zeroes between decimal point and first non-zero digit etc.

In fact antilog is obtained from first four digits after decimal point (the truncated four digit number). If the characteristic is p, we multiply antilog obtained by 10^p.

Example 5 : Find the value using logtable and antilogtables :

(1) 49.673×9.4891

(2) $\dfrac{(329)^{\frac{5}{2}} \times 9826}{(67.891)^3}$

(3) $\sqrt{\dfrac{(8432)^2 \times (0.1259)}{(27.478)^5}}$

(4) $\sqrt[3]{\dfrac{(7776)^2 \times 0.3564}{(92.3428)^4}}$

(5) $\sqrt[8]{87.992}$

(6) $(41.23)^3$

(7) $(0.01237)^4$

Solution : (1) Suppose $x = 49.673 \times 9.4891$

$\therefore \log x = \log (49.673) + \log (9.4891)$

$= 1.6961 + 0.9772 = 2.6733$

$\therefore$ antilog (log x) = antilog (2.6733)

$\therefore x = 471.3$

(2) Suppose $x = \dfrac{(329)^{\frac{5}{2}} \times 9826}{(67.891)^3}$

$\therefore$ $\log x = \log (329)^{\frac{5}{2}} + \log (9826) - \log (67.891)^3$

$= \frac{5}{2} \log (329) + \log (9826) - 3 \log (67.89)$

$= \frac{5}{2} (2.5172) + 3.9924 - 3 (1.8318)$

$= 6.2930 + 3.9924 - 5.4954$

$= 4.7900$

$\therefore$ antilog ($\log x$) = antilog (4.7900)

$\therefore$ $x = 61660$

(3) Suppose $x = \sqrt{\dfrac{(8432)^2 \times (0.1259)}{(27.478)^5}}$

$\therefore$ $\log x = \log \left[\dfrac{(8432)^2 \times (0.1259)}{(27.478)^5}\right]^{\frac{1}{2}}$

$= \frac{1}{2} \{\log (8432)^2 + \log (0.1259) - \log (27.478)^5\}$

$= \frac{1}{2} \{2\log (8432) + \log (0.1259) - 5\log (27.478)\}$

$= \frac{1}{2} \{2(3.9259) + \bar{1}.1000 - 5(1.4391)\}$

$= \frac{1}{2} \{(7.8518) + \bar{1}.1000 - 7.1955\}$

$= \frac{1}{2} (\bar{1}.7563\}$

$= \frac{1}{2} \{\bar{2} + 1.7563\} = \bar{1}.8782$

$\therefore$ antilog ($\log x$) = antilog ($\bar{1}.8782$)

$\therefore$ $x = 0.7554$

(4) Suppose $x = \sqrt[3]{\dfrac{(7776)^2 \times 0.3564}{(92.3428)^4}}$

$$\log x = \frac{1}{3}\{\log (7776)^2 + \log (0.3564) - \log (92.3428)^4\}$$

$$= \frac{1}{3}\{2\log (7776) + \log (0.3564) - 4\log (92.3428)\}$$

$$= \frac{1}{3}\{2(3.8908) + \bar{1}.5519 - 4(1.9654)\}$$

$$= \frac{1}{3}\{7.7816 + \bar{1}.5519 - 7.8616\}$$

$$= \frac{1}{3}\{\bar{1}.4719\}$$

$$= \frac{1}{3}\{\bar{3} + 2.4719\} = \bar{1}.8240$$

$\therefore$ antilog (log x) = antilog ($\bar{1}$.8240)

$\therefore$ $x = 0.6668$

(5) Suppose $x = \sqrt[8]{87.992}$

$\therefore$ $\log x = \frac{1}{8} \log (87.992)$

$= \frac{1}{8}(1.9444) = 0.2431$

$\therefore$ antilog (log x) = antilog (0.2431)

$\therefore$ $x = 1.750$

(6) Suppose $x = (41.23)^3$

$\therefore$ $\log x = 3 \log (41.23)$

$= 3\,(1.6152) = 4.8456$

$\therefore$ antilog (log x) = antilog (4.8456)

$\therefore$ $x = 70080$

(7) Suppose $x = (0.01237)^4$

$\therefore$ $\log x = 4 \log (0.01237)$

$= 4\,(\bar{2}.0923)$

$= \bar{8}.3692$

$\therefore$ antilog (log x) = antilog ($\bar{8}$.3692)

$\therefore$ $x = 0.00000002340$

EXERCISE 18

1. Find the value of following (using logtables) :

(1) $3.8217 \times 23.469 \times 0.2987$

(2) $47.37 \times 1.921 \times 771$

(3) $(0.3215) \times 7.92 \times 87.69$

(4) $\dfrac{(23.76)^2 \times (41.82)}{(11.372)^3}$

(5) $\dfrac{3.98 \times 8.76 \times 0.1718}{0.03 \times 0.526 \times 8.43}$

(6) $\dfrac{\sqrt{91.82}}{\sqrt[3]{43.39}}$

(7) $(51.32)^5$

(8) $\sqrt[4]{\dfrac{(8237)^3 \times (1.9821)}{(47.13)^4}}$

(9) $\sqrt[6]{\dfrac{(921)^5 \times (44.44)^2}{(37 \cdot 78)^3}}$

(10) $(53.83)^{\frac{1}{4}} \times (87.23)^{\frac{1}{2}}$

2. Select proper option (a), (b), (c) or (d) and write in the box given on the right so that the statement becomes correct :

(1) The decimal form of the number 8.97×10^4 = ... ☐

(a) 897000 (b) 89700 (c) 8970000 (d) 897

(2) The decimal form of the number 3.8269×10^{-4} = ... ☐

(a) 0.0038269 (b) 0.38269 (c) 0.038269 (d) 0.00038269

(3) The standard form of the number 9382 = ... ☐

(a) 9.382×10^2 (b) 9.382×10^{-2} (c) 9.382×10^3 (d) 9.382×10^{-3}

(4) The standard form of the number 773259 = ... ☐

(a) 7.73259×10^{-6} (b) 7.73259×10^6 (c) 7.73259×10^{-5} (d) 7.73259×10^5

(5) The standard form of the number 0.03711 = ... ☐

(a) 3.711×10^2 (b) 3.711×10^{-2} (c) 3.711×10^{-5} (d) 3.711×10^5

(6) The standard form of the number 0.00023821 = ... ☐

(a) 2.382×10^{-4} (b) 2.3821×10^4 (c) 23.821×10^4 (d) 2382.1×10^{-7}

(7) The characteristic of the number log 55231 = ... ☐

(a) 5 (b) 4 (c) 3 (d) 2

(8) The characteristic of the number log 8989340 = ... ☐

(a) 8 (b) 9 (c) 6 (d) 5

(9) The characteristic of the number log 0.003942 = ...

(a) 3 (b) 2 (c) −3 (d) −2

(10) The characteristic of the number log 0.13879 = ...

(a) 0 (b) −2 (c) 1 (d) −1

*

Summary

In this chapter we have studied the following points :

1. $a^x = y$ if and only if $x = \log_a y$; where $a \in R^+ - \{1\}$, $x \in R$, $y \in R^+$.
2. $a^{\log_a x} = x$ ($x \in R^+$) and $\log_a a^x = x$, $x \in R$, $a \in R^+ - \{1\}$.
3. **Product rule :** for $x, y \in R^+$, $a \in R^+ - \{1\}$, $\log_a xy = \log_a x + \log_a y$
4. **Quotient rule :** for $x, y \in R^+$, $a \in R^+ - \{1\}$, $\log_a \frac{x}{y} = \log_a x - \log_a y$
5. **Power law for logarithm :**

   For $a \in R^+ - \{1\}$, $x \in R^+$, $n \in R$, $\log_a x^n = n\log_a x$
6. For positive number n, we can put it as $n = t \times 10^p$; where $1 \le t < 10$ and $p \in Z$. This is called standard form of n.
7. For positive number n, if the standard form of n is $n = t \times 10^p$, where $1 \le t < 10$ and $p \in Z$ then $\log n = \log t + p$. p is called the characteristic and $\log t$ is called the mantissa.
8. To find logarithm of any number, $n \in N$, first we will find the characteristic and then the mantissa from logarithmic table.

ANSWERS

(Answers to only problems involving some calculations are given.)

Exercise 10.1

1. (1) Sides : $\overline{XY}$, $\overline{YZ}$, $\overline{ZW}$, $\overline{WX}$ (2) Angles : $\angle X$, $\angle Y$, $\angle Z$, $\angle W$
   (3) Diagonals : $\overline{XZ}$, $\overline{YW}$ (4) $\overline{XY}$ and $\overline{YZ}$, $\overline{XY}$ and $\overline{XW}$, $\overline{YZ}$ and $\overline{ZW}$, $\overline{ZW}$ and $\overline{WX}$
   (5) $\overline{XY}$ and $\overline{ZW}$, $\overline{YZ}$ and $\overline{XW}$ (6) $\angle X$ and $\angle Y$, $\angle Y$ and $\angle Z$, $\angle Z$ and $\angle W$, $\angle W$ and $\angle X$ (7) $\angle X$ and $\angle Z$, $\angle Y$ and $\angle W$ (8) $\emptyset$ (9) {X}
2. No, because if one is a quadrilateral, then the other is not.
3. (1) $m\angle P = 48$, $m\angle Q = 72$, $m\angle R = 96$, $m\angle S = 144$ (2) $m\angle D = 120$
   (3) $m\angle A = 36$, $m\angle B = 90$, $m\angle C = 108$, $m\angle D = 126$
   (4) $m\angle A = 100$, $m\angle B = 70$, $m\angle C = 120$, $m\angle D = 70$
4. (1) False (2) True (3) True (4) True (5) True (6) False (7) False

Exercise 10.2

1. $m\angle A = 80$, $m\angle C = 120$ **2.** $m\angle C = 120$, $m\angle D = 120$
3. $m\angle Q = 70$, $m\angle S = 130$ **4.** $m\angle R = 108$, $m\angle S = 100$, $m\angle P = 80$
5. $m\angle A = 60$, $m\angle B = 70$, $m\angle C = 110$, $m\angle D = 120$
6. (1) True (2) True (3) False (4) False (5) False (6) True (7) True (8) False (9) False

Exercise 10.3

1. $m\angle P = 100$, $m\angle Q = 80$, $m\angle R = 100$, $m\angle S = 80$ **2.** $m\angle FDE = 60$
3. $m\angle C = 105$ and $m\angle D = 75$ **4.** $m\angle P = 60$, $m\angle Q = 120$, $m\angle R = 60$, $m\angle S = 120$
6. $m\angle OPS = 63$ **7.** $m\angle DCA = 45$ **8.** $m\angle DBC = 60$
9. $m\angle DFG = 50$, $m\angle DGE = 40$ **10.** $m\angle AOB = 90$

Exercise 10.4

2. QR = 20 *cm* **3.** 52 *cm* **7.** XY = 4 or XY = 3

Exercise 10.5

1. BC = 13 **2.** XY = 10 **3.** 12.5 **4.** Perimeter of □ DBCF is 31.5, Perimeter of ΔCFE is 19.5 **5.** PQ = 11 **6.** RS = 3 **8.** 27 **9.** 35 **10.** 48

Exercise 10

1. (1) 60 (2) 68 (3) $m\angle QPO = 60$ (4) QR = 22 (5) 45, 75, 60
7. (1) b (2) a (3) c (4) d (5) a (6) a (7) c (8) d (9) d (10) c (11) a (12) b (13) b (14) c (15) a (16) c (17) d (18) c (19) a

Exercise 11.1

1. (1) False (2) True (3) True (4) True (5) True

2. (1) AD = 21.6 *cm* (2) AB = 9.6 *cm* **3.** 125 cm^2 **4.** BE = 9.6

5. BF = 45 *cm* and AE = 30 *cm* **6.** BN = 22.5 **7.** ABC = $16\sqrt{3}$ cm^2

8. PQR = 16 cm^2, PQCR = 32 cm^2, PBCR = 48 cm^2

9. ABC = 216 cm^2, altitude corresponding to $\overline{AC}$ = 14.4 *cm* **10.** 336 sq unit

Exercise 11.2

1. 60 cm^2 **2.** (1) 25 cm^2 (2) ΔAFB and ΔACB (3) AFEB = 50 cm^2

(4) $\square^m$ ABCD (5) Yes (6) ADF = 7.5 cm^2 **4.** 114 cm^2

5. 252 cm^2 **6.** 160 cm^2 and x = 26

Exercise 11

8. ABC = $36\sqrt{3}$ cm^2 **9.** PQR = 30 cm^2, PQCR = 60 cm^2, PBCR = 90 cm^2

10. (1) a (2) a (3) a (4) b (5) a (6) d (7) a (8) c (9) b (10) c

Exercise 12.1

1. (1) P = Q (2) Equal (3) OQ **2.** (1) False (2) True (3) False (4) False

Exercise 12.2

1. (1) $m\angle$COD = 130 (2) CD = $5\sqrt{2}$ *cm*

Exercise 12.4

5. Diameter = 10

Exercise 12.5

1. 90 **2.** $m\angle$BDC = 80 **3.** 150, 30 **4.** $m\angle$BAC = 75 **5.** $m\angle$QRS = 80, $m\angle$ERS = 5 **6.** $m\angle$BAC = 100 **7.** r = 3, Area of the circle = 9π sq units

Exercise 12

3. r = 13 **4.** 1 *cm* **7.** Radius = 13 **11.** AB = CD = 2, AC = BD = 10

12. (1) a (2) a (3) d (4) d (5) c (6) d (7) d (8) c (9) b (10) b (11) c (12) b (13) b (14) d (15) c (16) d (17) a (18) a (19) d (20) c (21) d

Exercise 14.1

1. $9\sqrt{3}$ sq units **2.** 60 cm^2 **3.** 864 cm^2 **4.** 600 m^2 **5.** $9\sqrt{15}$ cm^2

6. ₹ 11,66,000 **7.** Length of altitude $\frac{2\sqrt{66}}{5}$ *cm*

Exercise 14.2

1. $(6\sqrt{10} + 4\sqrt{266})$ cm^2 **2.** $12(5 + \sqrt{42})$ m^2 **3.** 306 m^2 **4.** 480 m^2

5. $24\sqrt{14}$ cm^2

Exercise 14

1. $24\sqrt{3}\ m^2$ **2.** $42\sqrt{6}\ cm^2$ **3.** 36 tiles, ₹ 594 **4.** 960 cm^2 **5.** 24 cm^2

6. 150 m, 72 m **7.** $4\sqrt{14}\ cm^2$ **8.** base 800 m, altitude 400 m **9.** 24 m^2, 6 m

10. BD = 25 cm **11.** $24\sqrt{21}\ cm^2$

12. (1) c (2) c (3) b (4) b (5) d (6) c (7) d (8) d (9) c (10) c (11) c (12) d

Exercise 15.1

1. (1) 280 cm^2, 640 cm^2 (2) 36 m^2, 54 m^2 (3) 17500 cm^2, 32500 cm^2

2. (1) 5900 cm^2 (2) ₹ 175 **3.** 260 m^2, ₹ 3900

4. ₹ 88,560 **5.** (1) Areas of both boxes are equal.
(2) Total surface area of cuboid is more by 550 cm^2.

Exercise 15.2

1. (1) curved surface area 1760 cm^2, total surface area 2292 cm^2
(2) r = 7 cm, total surface area 924 cm^2 (3) curved surface area 2826 cm^2, total surface area 4239 cm^2

2. ₹ 20,064 **3.** h = 42 cm **4.** Diameter = 32 cm **5.** 31400 cm^2 **6.** 1408 cm^2

7. (1) 264 m^2 (2) ₹ 13,200

Exercise 15.3

1. (1) $180\ \pi\ cm^2$, $324\ \pi\ cm^2$ (2) $h = 4\sqrt{2}\ cm$, $63\ \pi\ cm^2$, $112\ \pi\ cm^2$
(3) l = 5, $15\ \pi\ cm^2$, $24\ \pi\ cm^2$ **2.** l = 13, 204.10 cm^2, ₹ 20,410

3. l = 25 8250 cm^2 **4.** l = 21, r = 3 226.28 cm^2 **5.** l = 5, 47.1 m^2, number of tents 6

Exercise 15.4

1. (1) 11.2 cm, 394.24 cm^2, 197.12 cm^2, 295.68 cm^2
(2) 20, 1256, 628, 942
(3) r = 3.5 cm, Diameter = 7 cm, 77 cm^2, 115.5 cm^2

2. 4 : 9 **3.** ₹ 21,164 **4.** r = 7 cm **5.** ₹ 62,800

Exercise 15.5

1. 480 cm^3, 2880 cm^3 **2.** 24000 litres **3.** 0.625 m **4.** 5 days **5.** 10800 crates

6. 5184 cm^3 **7.** h = 25 m **8.** 6000 cm^3

Exercise 15.6

1. r = 35, 134.750 litre **2.** 75.36 cm^3 **3.** h = 4 m **4.** h = 3 m

5. 2200 cm^3 **6.** (1) volume of cuboid = 600 cm^3 (2) volume of cylinder = 770 cm^3, capacity of cylinder is more by 170 cm^3 **7.** number of bags 100 **8.** radius = 5 cm

9. r = 7, h = 6

Exercise 15.7

1. (1) 234.66 cm^3 (2) 616 cm^3 (3) 1018.28 cm^3 **2.** 7065 cm^3 **3.** 120 cm
4. 7 cm **5.** 594 m^3 **6.** (1) 48 cm (2) 50 cm (3) 2200 cm^3

Exercise 15.8

1. (1) 904.32 cm^3 (2) 1437.33 cm^3 (3) 4851 cm^3
2. (1) 5749.33 cm^3 (2) 19404 cm^3 **3.** 19404 litre **4.** 20 cm **5.** 1 : 2

Exercise 15

1. 7 : 5 **2.** $r = 14$, $h = 1.75$ cm **3.** 2 : 3 **4.** $\frac{h}{l} = \frac{1}{2}$ **5.** $h = 12.5$ cm **6.** 1694 cm^3
7. (1) c (2) d (3) c (4) c (5) b (6) b (7) d (8) a (9) a (10) c (11) b (12) b (13) a (14) d (15) b (16) c (17) c (18) b (19) d (20) a (21) c (22) d

Exercise 16.2

1. Range of Data = 755 **2.** (ii) Range of Data = 14.3 **3.** 73 read more than 50 % **6.** (ii) concentration more than 0.11 for 10 days

Exercise 16

1. Mean $(\overline{x})$ = 3.6, Median (M) = 3, Mode (Z) = 3
2. Mean $(\overline{x})$ = 56.27, Median (M) = 54, Mode (Z) = 55
3. Average Salary = ₹ 5262.50 **4.** $\overline{x} = 16.133$ **5.** Correct Mean $(\overline{x})$ = 29.65
6. Correct Mean $(\overline{x})$ = 11 **7.** $\overline{x} = 143$, M = 143 **8.** $\overline{x} = 13.7$, M = 14, Z = 14
9. $x = 49$ **10.** $x = 10$ **11.** $n = 10$ **12.** $f = 30$ **13.** $\overline{x} = 25.4026$
14. (1) a (2) b (3) b (4) b (5) b (6) a (7) b (8) d (9) d (10) c (11) b (12) c (13) c (14) b (15) d (16) b (17) a (18) b (19) d (20) a (21) c (22) d (23) d (24) a (25) c (26) d (27) d (28) d (29) d (30) c (31) c (32) b (33) d (34) b (35) a (36) d (37) c

Exercise 17

1. (i) 0.7 (ii) 0.3 **2.** (i) 0.02 (ii) 0.77 (iii) 0.535 **3.** (i) 0.6 (ii) 0.4
4. (ii) 0.03 (ii) 0.113 (iii) 0.652 **5.** (i) 0.075 (ii) 0 (iii) 1 **6.** (i) 0.4 (ii) 0 (iii) 0.8
7. (i) 0.5 (ii) 0.17 **8.** (i) 0.18 (ii) 0.1 (iii) 0.56 **9.** (i) 0.1 (ii) 0.675 (iii) 0.275 (iv) 0.5
10. (i) 0.02 (ii) 0.25 (iii) 0.23 (iv) 0.15 **11.** 0.5 **12.** 0.5
13. (i) 0.55 (ii) 0.3 (iii) 0.85 **14.** (i) 0.3 (ii) 0.16 (iii) 0.35 (iv) 0.19 (v) 0.65
15. (i) 0.62 (ii) 0.26 (iii) 0.34 (iv) 0.4 (v) 0.88
16. (1) d (2) c (3) b (4) c (5) a (6) b

Exercise 18

1. (1) 26.79 (2) 70170 (3) 223.2 (4) 16.06 (5) 45.03 (6) 2.727 (7) 356000000 (8) 21.77 (9) 170.2 (10) 25.29
2. (1) b (2) d (3) c (4) d (5) b (6) a (7) b (8) c (9) c (10) d

TERMINOLOGY (In Gujarati)

AAS (Angle Angle Side)	ખૂખૂબા
Acute Angle	લઘુકોણ
Algebraic Expression	બૈજિક પદાવલિ
Alternate Angles	યુગ્મકોણ
Altitude	વેધ
Angle Bisector	ખૂણાઓનો દ્વિભાજક
Antecedent	પૂર્વપદ
Antilogarithm	પ્રતિ લઘુગણક
Approximate Value	સન્નિકટ કિંમત
Arc	ચાપ
Area	ક્ષેત્રફળ
ASA (Angle Side Angle)	ખૂબાખૂ
Associative Law	જૂથનો નિયમ
At least	ઓછામાં ઓછું
Axes	અક્ષો
Axiom / Postulate	પૂર્વધારણા
Balanced Die	સમતોલ પાસો
Bar Diagram	લંબાલેખ
Base	આધાર
Base	પાયો
Bisector	દ્વિભાજક
Bisector of a Line-segment	રેખાખંડનો દ્વિભાજક
Capacity	ક્ષમતા
Cartesian Product	કાર્તેઝિય ગુણાકાર
Central Tendency	મધ્યવર્તી સ્થિતિમાન
Centroid	મધ્યકેન્દ્ર
Characteristic	પૂર્ણાંશ
Circle	વર્તુળ
Circumcentre	પરિકેન્દ્ર
Circumcircle	પરિવૃત્ત
Circumference	પરિઘ

Circumradius	પરિત્રિજ્યા
Class	વર્ગ
Class-interval	વર્ગલંબાઈ
Coefficient	સહગુણક
Collinear Points	સમરેખ બિંદુઓ
Commutative Law	ક્રમનો નિયમ
Complement of a Set	પૂરક ગણ
Complementary Angles	કોટિકોણ
Concave Quadrilateral	અંતર્મુખ ચતુષ્કોણ
Concentric Circles	સમકેન્દ્રી વર્તુળો
Congruence of Triangles	ત્રિકોણની એકરૂપતા
Congruent Angles	એકરૂપ ખૂણા
Consecutive Sides	ક્રમિક બાજુઓ
Construction	રચના
Continuous	સતત
Converse	પ્રતીપ
Convex Quadrilateral	બહિર્મુખ ચતુષ્કોણ
Co-ordinate Plane	યામ-સમતલ
Coplanar Lines	સમતલીય રેખાઓ
Coplanar Points	સમતલીય બિંદુઓ
Correspondence	સંગતતા
Corresponding Angles	અનુકોણ
Cube	સમઘન
Cube Root	ઘનમૂળ
Cubic	ત્રિઘાત
Cuboid	લંબઘન
Cumulative Frequency	સંચયી આવૃત્તિ
Cyclic Quadrilateral	ચક્રીય ચતુષ્કોણ
Cylinder	નળાકાર
Data	માહિતી
Decimal Expansion	દશાંશ વિસ્તરણ
Denominator	છેદ
Deviation	વિચલન
Diagonal	વિકર્ણ
Direct Proof	પ્રત્યક્ષ સાબિતી

Disjoint Set	અલગ ગણ
Distance	અંતર
Distributive Law	વિભાજનનો નિયમ
Dividend Polynomial	ભાજ્ય બહુપદી
Divisor Polynomial	ભાજક બહુપદી
Equal Sets	સમાન ગણ
Equation	સમીકરણ
Equiangular Triangle	સમકોણ ત્રિકોણ
Equilateral Triangle	સમબાજુ ત્રિકોણ
Equivalent Set	સામ્ય ગણ
Event	ઘટના
Exponent	ઘાતાંક
Exterior Angle	બહિષ્કોણ
Face	પૃષ્ઠ
Factor	અવયવ
Finite Set	સાન્ત ગણ
Foot of Perpendicular	લંબપાદ
Frequency	આવૃત્તિ
Frequency Distribution Table	આવૃત્તિ વિતરણ કોષ્ટક
Frequency Polygon	આવૃત્તિ બહુકોણ
Great Circle	દીર્ઘવૃત્ત
Head	છાપ
Hemishpere	અર્ધગોળો
Histogram	સ્તંભાલેખ
Hollow Sphere	પોલો ગોળો
Identity	નિત્યસમ
Incentre	અંતઃકેન્દ્ર
Incircle	અંતઃવૃત્ત
Included Angle	અંતર્ગત ખૂણો
Indirect Proof	અપ્રત્યક્ષ સાબિતી
Inequality	અસમાનતા
Infinite Set	અનંત ગણ
Inradius	અંતઃત્રિજ્યા
Interior Angles	અંતઃકોણ

Interior Opposite Angles	અંત:સમ્મુખકોણ
Intersection	છેદગણ
Irrational Number	અસંમેય સંખ્યા
Isosceles Triangle	સમદ્વિબાજુ ત્રિકોણ
Kite	પતંગાકાર
Lateral Surfaces	પાર્શ્વપૃષ્ઠો
Line	રેખા
Line-segment	રેખાખંડ
Linear	સુરેખ
Linear Pair of Angles	રૈખિકજોડના ખૂણા
Logarithm	લઘુગણક
Lower Limit	અધ:સીમા
Lower Limit point	અધ:સીમા બિંદુ
Major Arc	ગુરુચાપ
Major Segment	ગુરુવૃત્તખંડ
Mantissa	અપૂર્ણાંશ
Mean	મધ્યક
Measure	માપ
Median	મધ્યસ્થ
Mid Value	મધ્યકિંમત
Minor Arc	લઘુચાપ
Minor Segment	લઘુવૃત્તખંડ
Mode	બહુલક
Non-collinear Points	અસમરેખ બિંદુઓ
Non-terminating and Non-recurring	અનંત અને અનાવૃત્ત
nth root	n-મૂળ
Null Set	ખાલીગણ
Numerator	અંશ
Observation	અવલોકન
Obtuse Angle	ગુરુકોણ
One-One Correspondence	એક-એક સંગતતા
Opposite Angles	સામસામેના ખૂણા
Opposite Sides	સામસામેની બાજુઓ
Ordered Pair	ક્રમયુક્ત જોડ

Origin	ઊગમબિંદુ
Orthocentre	લંબકેન્દ્ર
Parallel	સમાંતર
Parallelogram	સમાંતરબાજુ ચતુષ્કોણ
Perimetre	પરિમિતિ
Perpendicular Bisector	લંબદ્વિભાજક
Perpendicular Line	લંબરેખા
Point	બિંદુ
Primary Data	પ્રાથમિક માહિતી
Probability	સંભાવના
Quadrant	ચરણ
Quadratic	દ્વિઘાત
Quadrilateral	ચતુષ્કોણ
Quadrilateral Region	ચતુષ્કોણીય પ્રદેશ
Qualitative Data	ગુણાત્મક માહિતી
Quantitative Data	સંખ્યાત્મક માહિતી
Quotient Polynomial	ભાગાકાર બહુપદી
Random	યાદચ્છિક
Range	વિસ્તાર
Rational Number	સંમેય સંખ્યા
Rationalization	સંમેયીકરણ
Raw Data	કાચી માહિતી
Ray	કિરણ
Rectangle	લંબચોરસ
Remainder Polynomial	શેષ બહુપદી
Remainder Theorem	શેષ પ્રમેય
Rhombus	સમબાજુ ચતુષ્કોણ
RHS (Right Angle Hypotenuse Side)	કાકબા
Right Angle	કાટકોણ
Right Angled Triangle	કાટકોણ ત્રિકોણ
SAS (Side Angle Side)	બાખૂબા
Scalene Triangle	વિષમભુજ ત્રિકોણ
Secondary Data	ગૌણ માહિતી
Sector of a Circle	વૃત્તાંશ

Segment of a Circle	વૃત્તખંડ
Set	ગણ
Singleton	એકાકી ગણ
Skew Lines	વિષમતલીય રેખાઓ
Slant Height	ત્રાંસી ઊંચાઈ
Space	અવકાશ
Sphere	ગોળો
SSS (Side Side Side)	બાબાબા
Step	સોપાન
Suplimentary Angles	પૂરકકોણ
Surd	કરણી
Tail	કાંટો
Terminating Recurring	સાન્ત અને આવૃત્ત
Transversal	છેદિકા
Trapezium	સમલંબ ચતુષ્કોણ
Triangle	ત્રિકોણ
Undefined Term	અવ્યાખ્યાયિત પદ
Union Set	યોગ ગણ
Universal Set	સાર્વત્રિક ગણ
Universal Truth	સ્વયંસિદ્ધ સત્યપ
Upper Limit	ઊર્ધ્વસીમા
Upper Limit Point	ઊર્ધ્વસીમાબિંદુ
Variable	ચલ
Vertex	શિરોબિંદુ
Vertical Line	શિરોલંબ રેખા
Vertically Opposite Angle	અભિકોણ
Volume	ઘનફળ
Zeroes	શૂન્યો

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0826	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	26	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1616	1647	1675	1903	1931	1959	1987	2014	3	6	6	11	14	17	20	22	25
16	2041	2066	2095	2122	2148	2175	2201	2227	2253	2279	3	5	6	11	13	16	16	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2610	2833	2656	2676	2900	2923	2945	2967	2969	2	4	7	9	11	13	16	16	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3656	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4163	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	6	10	11	13	15
27	4314	4330	4346	4362	4376	4393	4409	4425	4440	4456	2	3	5	6	6	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4696	4713	4726	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4766	4800	4614	4629	4843	4657	4671	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4989	4983	4997	5011	5024	5038	1	3	4	8	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	6	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5476	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5566	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5966	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	6	9
44	6435	6345	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6616	1	2	3	4	5	6	7	6	9
46	6626	6637	6646	6656	6665	6675	6664	6693	6702	6712	1	2	3	4	5	6	7	7	6
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6926	6937	6946	6955	6954	6972	6961	1	2	3	4	4	5	6	7	6
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

LOGARITHMS

	0	1	2	3	4	5	6	7	6	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7480	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	8	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7834	7642	7649	7657	7864	7672	7679	7886	7694	7701	1	1	2	3	4	4	5	8	7
59	7709	7716	7723	7731	7788	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7769	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
82	7924	7931	7938	7945	7952	7959	7966	7973	7880	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8138	8142	8149	8156	8162	8189	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8883	8370	8376	8382	1	1	2	3	3	4	4	5	6
89	8388	8395	6401	6407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8483	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8833	8639	8645	8651	8857	8663	8689	8875	8881	8886	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8614	8820	8825	8831	8337	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
76	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8882	8987	8993	8988	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9038	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
61	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
62	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9198	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9480	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
69	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9809	9614	9619	9824	9628	9833	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9885	9589	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9788	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9843	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
	0	1	2	3	4	5	8	7	8	9	1	2	3	4	5	6	7	8	9

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	8	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1066	1069	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1188	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1367	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1561	0	1	1	1	2	2	3	3	3
.20	1585	1289	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1667	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1776	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.26	1905	1910	1914	1919	1923	1926	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1964	1959	1963	1968	1972	1977	1962	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2016	2023	2026	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2146	2153	2158	2163	2166	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2362	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2760	2786	2793	2799	2605	2612	1	1	2	3	3	4	4	5	6
.45	2616	2625	2831	2836	2844	2651	2856	2864	2671	2677	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3063	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	8	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	8	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.58	3631	3639	3648	3656	3664	3673	3681	3890	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.80	3981	3990	3999	4009	4018	4027	4036	4048	4055	4064	1	2	3	4	5	6	6	7	8
.81	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.82	4169	4178	4188	4198	4207	4217	4227	4236	4246	4258	1	2	3	4	5	6	7	8	9
.83	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	8	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.86	4571	4581	4592	4803	4813	4624	4634	4845	4656	4867	1	2	3	4	5	8	7	9	10
.67	4877	4688	4899	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	8	7	8	9	10
.89	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5184	5176	5188	5200	5212	5224	5236	1	2	4	5	8	7	8	10	11
.72	5248	5280	5272	5284	5297	5309	5321	5333	5348	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5463	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5886	5902	5918	5929	5943	5957	5970	5964	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	8180	6194	6209	6223	8237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	8368	6383	6397	8412	8427	8442	1	3	4	6	7	9	10	12	13
.81	8457	6471	6486	6501	8516	6531	6546	6561	8577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6715	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6639	6855	6871	6887	8902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7856	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	12	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8168	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8358	8375	8395	8414	8433	8453	8472	6492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9018	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9228	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9663	9705	9727	9758	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9883	9888	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9