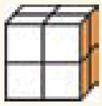
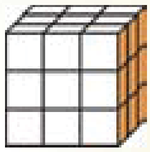
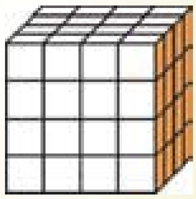
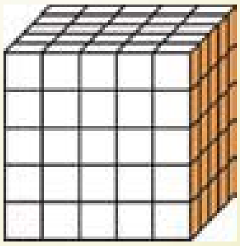
1 Cube and Cube Root

Let us remember :

You have learnt about cube and its volume in Standard VII. All sides of a cube are equal in measure. Aren't they ? Now fill in the blanks according to example :

Question	Figure	Volume (cm^3)
What will be the volume of cube having side of measure 1 cm ?		$1 \times 1 \times 1 = 1^3 = 1$
What will be the volume of cube having side of measure 2 cm ?		$2 \times 2 \times 2 = 2^3 = 8$
What will be the volume of cube having side of measure 3 cm ?		$3 \times 3 \times 3 = 3^3 = 27$
What will be the volume of cube having side of measure 4 cm ?		
What will be the volume of cube having side of measure 5 cm ?		

Let's learn new :

On the basis of Table the numbers 1, 8, 27, 64, 125 are obtained if same number is multiplied three times. Such obtained number is called cube of that number.

Cube of a number is obtained by multiplying that number with its square. Therefore, $x^2 = x \times x$.

Here, $1 \times 1 \times 1 = 1^3 = 1$. Therefore cube of 1 is '1' and cube root of '1' is '1'.

The symbol for cube root is '$\sqrt[3]{\ }$'.

Symbolically $\sqrt[3]{1} = 1$ (Read as : Cuberoot of 1 is equal to 1)

$2 \times 2 \times 2 = 2^3 = 8$. Therefore cube of 2 is '8' and cuberoot of 8 is '2'.

Symbolically $\sqrt[3]{8} = 2$ (Read as : Cuberoot of 8 is 2)

$3 \times 3 \times 3 = 3^3 = 27$. $\therefore$ Cube of 3 is '27' and $\sqrt[3]{27} = 3$

$4 \times 4 \times 4 = 4^3 = 64$ $\therefore$ Cube of 4 is '64' and $\sqrt[3]{64} = 4$

$5 \times 5 \times 5 = 5^3 = 125$ $\therefore$ Cube of 5 is '125' and $\sqrt[3]{125} = 5$

The numbers 1, 8, 27, 64, 125,... etc obtained above are called perfect cube numbers.

- **Now complete the following Table :**

Table 1

Number	Cube	Number	Cube
1	$1^3 = 1$	11	....
2	$2^3 =$	12	$12^3 = 1728$
3	$3^3 =$	13	$13^3 = 2197$
4	$4^3 = 64$	14	....
5	$5^3 =$	15	....
6	$6^3 = 216$	16	$16^3 = 4096$
7	....	17	$17^3 = 4913$
8	....	18	....
9	$9^3 =$	19	....
10	....	20	$20^3 = 8000$

- **Answer the following questions on the basis of Table 1 :**

(1) Is your roll number is a perfect cube number ? Why ?

....

(2) Which are the perfect cube numbers from 1 to 100 ?

....

(3) How many perfect cube numbers are there from 1 to 1000 ?

....

- **Fill in the blank in the given Table 2 on the basis of Table 1 :**

Table 2

Digit at unit place of number	Digit at unit place of a number obtained by cubing number	To get more information of the numbers on the basis of Table 1 and Table 2, cube the following numbers : 21 to 30	Digit at unit place of number	Digit at unit place of a number obtained by cubing number
1	1		2	8
4	....		8	....
5	5		3	7
6	....		7	....
9	....		0	....

It is clear from the Table 2,

- If 1, 4, 5, 6, 9 and 0 are the digit at unit place of numbers then the digit at unit place of the cube of numbers will be 1, 4, 5, 6, 9 and 0 respectively.
- If digit at unit place of a number is 2 then 8 will be the digit at unit place of cube of that number and if 8 lies at unit place then cube of that number has 2 at its unit place. If 3 lies at unit place then its cube has 7 will be at its unit place and if 7 lies at unit place then its cube has 3 at its unit place.

Examine the number of zero of the number obtained by cube of 10, 20, 30.

$10^3 = 10 \times 10 \times 10 = 1000$ $\qquad$ $20^3 = 20 \times 20 \times 20 =$ ....

$30^3 = 30 \times 30 \times 30 =$ .... $\qquad$ $40^3 = 40 \times 40 \times 40 =$ ....

On the basis of this we can say that the number having zero at its unit's place, the number obtained by cube of them have zero at unit's ten's and hundred's place.

Example 1 : What will be the digit at unit place obtained by cubing the given numbers :

(1) 11 (2) 58 (3) 104 (4) 407 (5) 596 (6) 840

Number	Digit at unit place obtained by cube of number	Number	Digit at unit place obtained by cube of number
11	1	407	3
58	2	596	6
104	4	840	0

Example 2 : What will be the digit at unit place of cube-root of given perfect cube number ?

(1) 6859, (2) 3375, (3) 17576, (4) 39304, (5) 35397, (6) 64000

(1) 6859 : 9 is the digit at unit place of cube root of perfect cube number 6859.
(2) 3375 : 5 is the digit at unit place of cube root of perfect cube number 3375.
(3) 17576 : 6 is the digit at unit place of cube root of perfect cube number 17576.
(4) 39304 : 4 is the digit at unit place of cube root of perfect cube number 39304.
(5) 35397 : 7 is the digit at unit place of cube root of perfect cube number 35397.
(6) 64000 : 0 is the digit at unit place of cube root of perfect cube number 64000.

Practice 1

1. What will be the 'digit at unit place' obtained by cubing the following numbers :

(1) 401 (2) 258 (3) 344 (4) 47 (5) 66 (6) 25 (7) 79 (8) 10

2. What will be the 'digit at unit place' in the cube root of the following numbers :

(1) 729 (2) 4096 (3) 15625 (4) 13824
(5) 12167 (6) 8000 (7) 5832 (8) 1331

Examine that the given number is perfect cube or not :

Example 3 : Verify 64 is perfect cube or not.

2	64
2	32
2	16
2	8
2	4
2	2
	1

$$64 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$$
$$= 2^3 \times 2^3$$
$$= (2 \times 2)^3$$
$$= 4^3$$

$\therefore$ 64 is a perfect cube number.

Example 4 : Verify 2312 is perfect cube or not.

2	2312
2	1156
2	578
17	289
17	17
	1

$$2312 = \underline{2 \times 2 \times 2} \times \underline{17 \times 17}$$

Here 2 is three times but 17 is two times only.

There 2312 is not a perfect cube number.

Practice 2

- **Which of the following numbers are perfect cube ? For what ?**

  (1) 729 (2) 100 (3) 243 (4) 400 (5) 3375 (6) 127000 (7) 4913 (8) 4096

*

Example 5 : By which smallest number 2312 must be multiplied to get a perfect cube.

According to Example 4, $2312 = \underline{2 \times 2 \times 2} \times \underline{17 \times 17}$

Here, 2 is three times but 17 is only two times. If here 17 will be three times, then the new number obtained will be perfect cube.

Therefore, $2312 \times 17 = \underline{2 \times 2 \times 2} \times \underline{17 \times 17 \times 17}$

Hence the smallest natural number by which 2312 should be multiplied to make it perfect cube is 17.

Example 6 : By which smallest number 1029 should be multiplied to get a perfect cube ?

Example 7 : By which smallest number 704 should be divided to get a perfect cube ?

2	704
2	352
2	176
2	88
2	44
2	22
11	11
	1

$704 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 11$

Here 11 is only one time. If 11 is removed, therefore, if given number is divided by 11, then new number is a perfect cube.

Example 8 : By which smallest number 1600 should be divided to get a perfect cube number ?

Practice 3

1. **Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube ?**

   (1) 256 (2) 100 (3) 576 (4) 81 (5) 1715

2. **Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube :**

   (1) 88 (2) 875 (3) 1512 (4) 625 (5) 13500

To find cube root through factorisation method :

We have learnt to find square root through factorisation method. Now let's learn to find cube root through factorisation method.

Example 9 : Find cube root of 15625 by factorisation method.

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$$15625 = \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}$$
$$= 5^3 \times 5^3$$
$$= (5 \times 5)^3$$
$$= (25)^3$$
$$\therefore \sqrt[3]{15625} = 25$$

Example 10 : Find the cube root of 10648 by factorisation method.

2	10648
2	5324
2	2662
11	1331
11	121
11	11
	1

$$10648 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11}$$
$$= 2^3 \times 11^3$$
$$= (2 \times 11)^3$$
$$= (22)^3$$
$$\therefore \sqrt[3]{10648} = 22$$

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To find cube-root of fractional and decimal-fractional numbers :

As we have learnt to find out the cube of perfect cube integers, similarly let us learn to find the cube-root of perfect cube fractions.

Example 11 : Find cube root of $\sqrt[3]{\frac{27000}{8000}}$ or $\frac{27000}{8000}$ by factorisation method

2	27000
2	13500
2	6750
5	3375
5	675
5	135
3	27
3	9
3	3
	1

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$\frac{27000}{8000} = \frac{2\times2\times2\times5\times5\times5\times3\times3\times3}{2\times2\times2\times2\times2\times2\times5\times5\times5}$$
$$= \frac{2^3\times5^3\times3^3}{2^3\times2^3\times5^3}$$
$$= \frac{(2\times5\times3)^3}{(2\times2\times5)^3}$$
$$= \frac{(30)^3}{(20)^3}$$
$$\sqrt[3]{\frac{27000}{8000}} = \sqrt[3]{\frac{(30)^3}{(20)^3}} = \frac{30}{20}$$

Example 12 : Find cube root of $\sqrt[3]{\frac{19683}{15625}}$ or $\frac{19683}{15625}$ by factorisation method.

3	19683
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$$\frac{19683}{15625} = \frac{3\times3\times3\times3\times3\times3\times3\times3\times3}{5\times5\times5\times5\times5\times5}$$

$$= \frac{3^3\times3^3\times3^3}{5^3\times5^3}$$

$$= \frac{(3\times3\times3)^3}{(5\times5)^3}$$

$$= \frac{(27)^3}{(25)^3}$$

$$\sqrt[3]{\frac{19683}{15625}} = \sqrt{\frac{(27)^3}{(25)^3}} = \frac{27}{25}$$

Example 13 : Find cube root of $\sqrt[3]{1.331}$ or 1.331 by factorisation method.

$1.331 = \frac{1331}{1000}$

11	1331
11	121
11	11
	1

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$\frac{1331}{1000} = \frac{11\times11\times11}{2\times2\times2\times5\times5\times5}$$

$$= \frac{11^3}{(2^3\times5^3)}$$

$$= \frac{11^3}{(2\times5)^3}$$

$$= \frac{11^3}{10^3}$$

$$\sqrt[3]{1.331} = \sqrt{\frac{11^3}{10^3}} = \frac{11}{10} = 1.1$$

Example 14 : Find cube root of $\sqrt[3]{12.167}$ or 12.167 by factorisation method.

$12.167 = \frac{12167}{1000}$

23	12167
23	529
23	23
	1

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$$\frac{12167}{1000} = \frac{23\times23\times23}{2\times2\times2\times5\times5\times5}$$

$$= \frac{23^3}{2^3\times5^3}$$

$$= \frac{23^3}{(2\times5)^3}$$

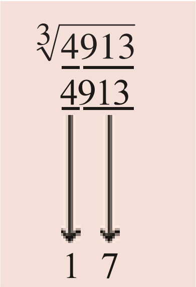
$$= \frac{23^3}{10^3}$$

$$\sqrt[3]{12.167} = \sqrt{\frac{23^3}{10^3}} = \frac{23}{10} = 2.3$$

- **To find cube root through estimation method :**

If given number is perfect cube then the cube root of same number can be found by estimation method.

Example 15 : Find the cube root of 4913 through estimate.



Explanation :

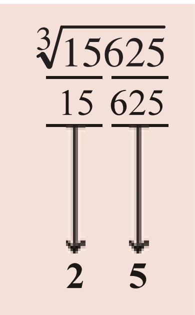
Step 1 : Divide the given number into two parts. One part of number having hundred's, ten's and unit's digit and other part of number. Here, in 4913, 913 is first part and 4 is the second part.

Step 2 : Here, first part is 913 whose unit place digit is 3. Here 3 comes at the unit's place of a number between 1 and 9 when it's cube root ends in 3 ? This number is 7.

Step 3 : Here, the number of second part is the smallest number out of the numbers between the cube of two numbers. Here, the second part is 4, which lies between $1^3 = 1$ and $2^3 = 8$. 1 is smaller between 1 and 2. Therefore, 1 will be taken at ten's place of cube root of a perfect cube number.

$\therefore \sqrt[3]{4913} = 17$

Example 16 : Find the cube root of 15625 through estimation.



Explanation :

Step 1 : Divide the given number into two parts. One part of number having handred's, ten's and unit's digit and other part of number having remaining digits. Here, dividing 15625 into two parts, 625 is first part and 15 is second part.

Step 2 : Here, first part is 625 whose digit at unit's place is 5. Therefore, 5 will be at unit place of the cube root of this perfect cube number.

Step 3 : Here, the number second part is the smallest number out of the numbers between cube of two numbers. Here, the second part is 15, which lies between $2^3 = 8$ and $3^3 = 27$. 2 is smaller between 2 and 3. Therefore, 2 will be taken at ten's place of cube root of a perfect cube number.

$\therefore \sqrt[3]{15625} = 25$

Note : If the second part is perfect cube then write the digit of its cube root.

Practice 4

1. Find the cube root of the following numbers by factorisation method :

(1) $\frac{8}{27}$ (2) $\frac{27}{125}$ (3) $\frac{125}{729}$ (4) $\frac{2744}{2197}$ (5) $\frac{3375}{4096}$

(6) 0.8 (7) 0.125 (8) 0.216 (9) 4.913 (10) 5.832

2. Find the cube root of the following numbers by estimation method :

(1) 8000 (2) 9261 (3) 13824 (4) 15625 (5) 19683

Exercise

1. **What will be the 'digit at unit place' in the perfect cube of the following numbers on the basis of digit at unit's place of the given numbers :**
   (1) 711 (2) 408 (3) 544 (4) 57 (5) 26 (6) 70
2. **What will be the 'digit at unit place' in the cube root of the following numbers on the basis of digit at unit's place of the given numbers :**
   (1) 216 (2) 2197 (3) 2744 (4) 6859 (5) 42875 (6) 125000
3. **Which numbers of the following numbers are perfect cube ? For which ?**
   (1) 400 (2) 9000 (3) 343 (4) 17576
4. **Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube :**
   (1) 675 (2) 392 (3) 968 (4) 875
5. **Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube :**
   (1) 1536 (2) 8019 (3) 7000 (4) 5400
6. **Find the cube root of the following numbers through factorisation method :**
   (1) 512 (2) 3375 (3) 17576 (4) 35937 (5) 32768 (6) 29791
7. **Find the cube root of the following numbers through estimation :**
   (1) 4096 (2) 42875 (3) 85184 (4) 54872 (5) 74088 (6) 140608

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Answers

Practice 1

1. (1) 1 (2) 2 (3) 4 (4) 3 (5) 6 (6) 5 (7) 9 (8) 0
2. (1) 9 (2) 6 (3) 5 (4) 4 (5) 3 (6) 0 (7) 8 (8) 1

Practice 2

1. (1) a perfect cube number (2) not a perfect cube number (3) not a perfect cube number (4) not a perfect cube number (5) a perfect cube number (6) a perfect cube number (7) a perfect cube number (8) a perfect cube number

Practice 3

1. (1) 2 (2) 10 (3) 3 (4) 9 (5) 25 **2.** (1) 11 (2) 7 (3) 7 (4) 5 (5) 4

Practice 4

1. (1) $\frac{2}{3}$ (2) $\frac{3}{5}$ (3) $\frac{5}{9}$ (4) $\frac{14}{13}$ (5) $\frac{15}{16}$ (6) 0.2 (7) 0.5 (8) 0.6 (9) 1.7 (10) 1.8
2. (1) 20 (2) 21 (3) 24 (4) 25 (5) 27

Exercise

1. (1) 1 (2) 2 (3) 4 (4) 3 (5) 6 (6) 0 **2.** (1) 6 (2) 3 (3) 4 (4) 9 (5) 5 (6) 0
3. (1) not a perfect cube number (2) not a perfect cube number
   (3) a perfect cube number (4) a perfect cube number
4. (1) 5 (2) 7 (3) 11 (4) 49 **5.** (1) 3 (2) 11 (3) 7 (4) 25
6. (1) 8 (2) 15 (3) 26 (4) 33 (5) 32 (6) 31
7. (1) 16 (2) 35 (3) 44 (4) 38 (5) 42 (6) 52

2 Rational Numbers

- **Let's learn :**
  - **Natural numbers :**
    - 1, 2, 3, ... are natural numbers.
    - 1 is the smallest natural number.
    - Natural numbers are also called calculative numbers. These are also called positive integers.
    - They are infinite.
  - **Whole numbers :**
    - 0, 1, 2, 3, ... are whole numbers.
    - 0 (zero) is the smallest whole number.
    - They are infinite.
  - **Negative integers :**
    - (–1), (–2), (–3), ... are negative integers.
    - They are infinite.
  - **Integers :**
    - The group of positive integers, negative integers and zero are called integers.
    - They are infinite.
  - **Rational numbers :**
    - $\frac{1}{2}, \frac{2}{5}, \frac{3}{7}, \frac{4}{8}, \ldots$ etc. are rational numbers.
    - $\left(-\frac{1}{5}\right), \left(-\frac{3}{5}\right), \left(-\frac{7}{8}\right), \left(-\frac{5}{7}\right), \ldots$ are negative rational numbers.
    - In $\frac{1}{2}$, 1 is numerator and 2 is denominator. Similarly in $\left(-\frac{3}{5}\right)$, (–3) is numerator and 5 is denominator.
  - **Decimal rational numbers :**
    - 0.5, –0.2, 1.3, 2.25 are decimal rational numbers.
    - Decimal rational numbers can be shown as below

$0.5 = \frac{5}{10}$ | $1.3 = \frac{13}{10}$

$-0.2 = \frac{-2}{10}$ | $2.25 = \frac{225}{100}$

Let's learn new :

We know that $\frac{2}{5}$, $-\frac{4}{7}$ etc. are non-integers, -0.3, 0.7 etc. are decimal non-integers which are denoted as $-\frac{3}{10}$ and $\frac{7}{10}$ respectively as simple fractions. Similarly, number 5, 9, 0 etc are also can be denoted by writing 1 as denominators.

For example, $5 = \frac{5}{1}$, $9 = \frac{9}{1}$, $0 = \frac{0}{1}$ etc.

Therefore, any number can be denoted in the $\frac{p}{q}$ form.

- The numbers which can be in the form $\frac{p}{q}$ (where p is a zero, positive and negative integer and q is a positive integer) is called a rational number.

Therefore, group of integers and non-integers are called rational numbers.

In given rational number $\frac{p}{q}$,

- If p is a positive integer, then it is a positive rational number.

  For example, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{2}{5}$, $\frac{5}{7}$, $\frac{2}{10}$, 5, ...

- If p is a negative integer, then it is a negative rational number.

  For example, $-\frac{2}{3}$, $-\frac{5}{7}$, $-\frac{5}{8}$, $-\frac{3}{7}$, -11, -20, ...

- If p is a zero, then 0 (zero) is a rational number.

  For example, 0

Write examples of required numbers in the following Table :

Negative rational number	Positive rational number	Zero rational number

- **Representation of rational numbers on the number line :**

You have learnt to represent integers on a number line. Let us revise them.

Represent the following numbers on the number line as required :

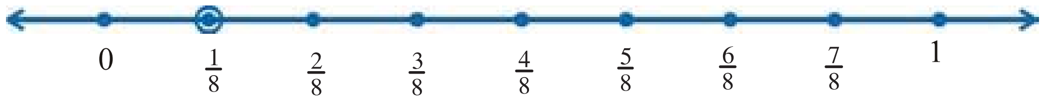
(1) Any two integers less than 3.

(2) Two integers more than (–4) and less than (–1).

(3) Integer more than 2 and less than 4.

Example 1 : Represent $\frac{1}{8}$ on the number line.

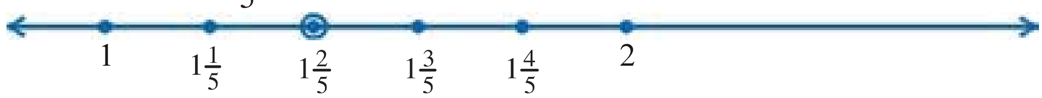
$\frac{1}{8} < 1$, therefore $\frac{1}{8}$ can be represented on the number line between 0 and 1. In the denominator of $\frac{1}{8}$, there is 8, so we will have to divide 0 to 1 in 8 equal parts on number line. His first part is $\frac{1}{8}$, second part is $\frac{2}{8}$ is represented respectively. Represent $\frac{1}{8}$ as ◉ :



Think : How many rational numbers can be represented between 0 and 1 on the number line.

Example 2 : Represent $1\frac{2}{5}$ on the number line.

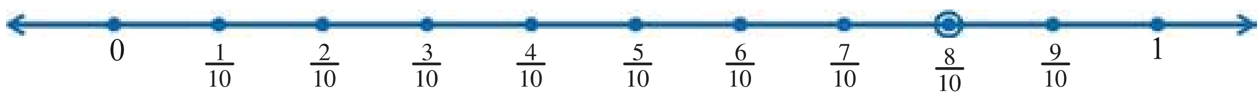
In $1\frac{2}{5}$, 1 is integer and $\frac{2}{5}$ is rational. Therefore, $1\frac{2}{5}$ will have to represent between 1 and 2. In the denominator of $\frac{2}{5}$, there is 5, so we will have to divide 1 to 2 in 5 equal parts. Here, 2 is in the numerator of $\frac{2}{5}$, therefore, on the second part we can represent $1\frac{2}{5}$.



Example 3 : Represent 0.8 on the number line.

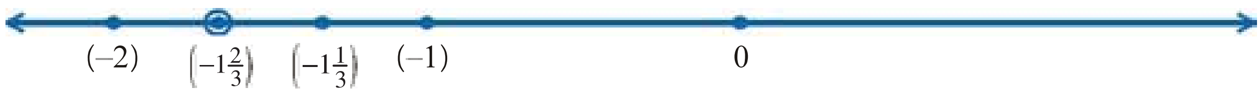
$0.8 = \frac{8}{10}$, here 8 is in numerator and 10 is in denominator.

See, by dividing 10 equal parts from 1 to 10, each part shows $\frac{1}{10}$. In the numerator of $\frac{8}{10}$ there is 8, therefore eighth point is represented as 0.8.



Example 4 : Represent $\left(-1\frac{2}{3}\right)$ on the number line.

$(-1) > \left(-1\frac{2}{3}\right) > (-2)$, therefore $\left(-1\frac{2}{3}\right)$ is to be represented between (-1) and (-2) on the number line. Because $\left(-1\frac{2}{3}\right)$ is greater than (-2) and smaller than (-1). The line segment from (-2) to (-1) is divided in three equal parts the $\left(-1\frac{2}{3}\right)$ is represented on L.H.S. of (-1) on the second point.



Think : Where to represent (-2.5) on the number line ?

Practice 1

- **Represent the given rational numbers on separate number lines :**

(1) $\left(-1\frac{4}{5}\right)$ (2) $2\frac{1}{4}$ (3) $\frac{4}{7}$ (4) $\left(-\frac{3}{5}\right)$ (5) 0.5 (6) (-1.5)

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- **The additive inverse negative and the multiplicative inverse (reciprocal) of rational number :**

Negative of a number : Which number should be added to $\frac{2}{5}$ to get zero result.

For every rational number always such a number exist whose addition to the given rational number results zero. These rational numbers are called negative numbers to each other.

For example, $\frac{2}{5} + \left(-\frac{2}{5}\right) = 0.$

Therefore, $\frac{2}{5}$ and $\left(-\frac{2}{5}\right)$ are negative numbers to each other.

$\frac{1}{3} + \left(-\frac{1}{3}\right) = 0$

Therefore, $\frac{1}{3}$ and $\left(-\frac{1}{3}\right)$ are negative number to each other.

$\left(-\frac{4}{7}\right) + \left(\frac{4}{7}\right) = 0$

Therefore, $\left(-\frac{4}{7}\right)$ and $\left(\frac{4}{7}\right)$ are negative number to each other.

- **Multiplicative inverse (Reciprocal) :** 3 is multiplied by such a number that the multiplication of both is 1 ?

Except zero for any number, we get such a number whose multiplication with given number results 1, then these both numbers are called reciprocal to each other.

For example, $2 \times \frac{1}{2} = 1$

Here, 2 and $\frac{1}{2}$ are reciprocal to each other.

Reciprocal of 2 is $\frac{1}{2}$ and reciprocal of $\frac{1}{2}$ is 2.

$\frac{1}{5} \times 5 = 1$

Here, $\frac{1}{5}$ and 5 are reciprocal to each other.

$\left(-\frac{4}{7}\right) \times \left(-\frac{7}{4}\right) = 1$

Here, $\left(-\frac{4}{7}\right)$ and $\left(-\frac{7}{4}\right)$ are reciprocal to each other.

If any number is multiplied with zero (0), the result is zero (0). Therefore, the reciprocal of zero (0) does not exist.

1. Write additive inverse of the given numbers :

(1) $\left(-\frac{3}{4}\right)$ (2) 0 (3) (-2) (4) $\frac{13}{20}$ (5) (-0.7)

(6) 0.8 (7) 0.01 (8) $1\frac{2}{3}$ (9) $\frac{2}{5}$ (10) $\frac{7}{8}$

2. Write multiplicative inverse (reciprocal) of the given numbers :

(1) (-1) (2) $\left(-\frac{5}{8}\right)$ (3) $\frac{1}{8}$ (4) $\frac{2}{5}$ (5) (-0.7)

(6) 0.8 (7) 0.01 (8) $1\frac{3}{2}$ (9) $\frac{2}{5}$ (10) $\frac{7}{8}$

*

- **Properties for addition and multiplicative of rational numbers :**

We have learnt about properties of addition and multiplication of integers. Let us now learn about properties of addition and multiplication of rational numbers.

- **Properties for addition of rational numbers :**

Note the result by adding the following rational numbers :

No.	Addition	Result	Properties
(1)	$\frac{4}{7}+\left(-\frac{2}{3}\right)=\dots$ $\left(-\frac{1}{4}\right)+\frac{3}{8}=\dots$	Is resulting number a rational number ?	**Closure property :** The addition of any two rational numbers is a rational number.
(2)	$\frac{4}{7}+\left(-\frac{2}{3}\right)=\dots$ $\left(-\frac{2}{3}\right)+\frac{4}{7}=\dots$	How result is obtained when order is changed.	**Commutative property :** Two rational numbers can be added in any order but the result is same.
(3)	$\left[\left(-\frac{3}{4}\right)+\frac{1}{2}\right]+\frac{2}{6}=\dots$ $\left(-\frac{3}{4}\right)+\left[\frac{1}{2}+\frac{2}{6}\right]=\dots$	How result is obtained when group is changed ?	**Associative property :** For any three rational numbers if in the group of any two numbers, third number is added, the result is same.
(4)	$\left(-\frac{3}{4}\right)+0=\dots$ $0+\frac{2}{3}=\dots$	How result is obtained when addition is done with zero (0) ?	**Existence of identity element :** For addition of a rational number and zero, we get the same rational number. Therefore, zero (0) is the identity element for addition.
(5)	$\left(-\frac{7}{17}\right)+\frac{7}{17}=\dots$ $\frac{3}{5}+\left(-\frac{3}{5}\right)=\dots$	What is the result when two opposite rational numbers are added ?	For any rational number there always exist an opposite number such that addition of both number is zero.

- **Properties for multiplication of rational numbers :**

Note the result by multiplication of following rational numbers :

No.	Addition	Result	Properties
(1)	$0 \times \frac{5}{9} = ...$ $\left(-\frac{3}{2}\right) \times \frac{2}{6} = ...$	Is resulting number a rational number ?	**Closure property :** The addition of any two rational numbers is a rational number.
(2)	$\left(-\frac{2}{5}\right) \times \frac{10}{3} = ...$ $\frac{10}{3} \times \left(-\frac{2}{5}\right) = ...$	How result is obtained when order of numbers are changed.	**Commutative property :** When two rational numbers are multiplied in any order then the result is same.
(3)	$\left[\left(-\frac{1}{3}\right) \times \frac{3}{4}\right] \times \frac{6}{7} = ...$ $\left(-\frac{1}{3}\right) \times \left[\frac{3}{4} \times \frac{6}{7}\right] = ...$	How result is obtained when group is changed ?	**Associative property :** For any three rational numbers if in the group of any two number, the third number is multiplied, the result is same.
(4)	$\left(-\frac{4}{9}\right) \times 1 = ...$ $\frac{3}{7} \times 1 = ...$	How result is obtained when any number is multiplied with one ?	**Existence of identity element :** Multiplication of any rational number and 1 is always the same rational number. Therefore, 1 is the identity element for multiplication.
(5)	$\frac{3}{5} \times \frac{5}{3} = ...$ $\left(-\frac{1}{2}\right) \times \left(-\frac{2}{1}\right) = ...$	What is the result when two inverse (reciprocal) rational numbers are multiplied ?	For any rational number there always exist a reciprocal number such that multiplication of both number is one.

No.	Addition	Result	Properties
(6)	$\left(-\frac{3}{5}\right) \times \left(-\frac{3}{4}\right) + \frac{2}{3} = \ldots$	Can the distribution of multiplication of rational numbers over addition is possible ?	**The distributive property :** The distribution of multiplication of rational numbers over addition is possible.

Exercise

1. Fill in the blanks :

(1) (–5) is an integer but it is not a .... number. (rational, natural)

(2) Zero is an integer but it is not considered in .... number. (rational, natural)

(3) $\frac{5}{7}$ is a/an .... (integer, rational number)

(4) In $\frac{3}{8}$, ... is numerator and ...... is denominator. (8, 3, 83)

(5) In $2\frac{3}{8}$, ... is an integer and ...... is a rational number. $\left(8, \frac{3}{8}, 2\right)$

2. Represent the following number in $\frac{p}{q}$ form :

(1) $2\frac{1}{7}$ (2) 0.6 (3) $\left(-3\frac{1}{4}\right)$ (4) 0 (5) 28

3. Write which properties are used in the following rational numbers :

(1) $\frac{2}{7} + \left(-\frac{7}{4}\right) = \left(-\frac{7}{4}\right) + \frac{2}{7}$

(2) $\left(-\frac{1}{8}\right) \times \frac{1}{7} = \frac{1}{7} \times \left(-\frac{1}{8}\right)$

(3) $\frac{1}{2} \times \left(\frac{1}{3} \times \frac{1}{4}\right) = \left(\frac{1}{2} \times \frac{1}{3}\right) \times \frac{1}{4}$

(4) $\frac{1}{5} \times \left(\frac{1}{2} + \frac{1}{3}\right) = \left(\frac{1}{5} \times \frac{1}{2}\right) + \left(\frac{1}{5} \times \frac{1}{3}\right)$

(5) $\left(-\frac{3}{5}\right) + 0 = \left(-\frac{3}{5}\right)$

(6) $\frac{1}{4} + \left(\frac{1}{6} + \frac{1}{3}\right) = \left(\frac{1}{4} + \frac{1}{6}\right) + \frac{1}{3}$

Answers

Practice 2

1. (1) $\frac{3}{4}$ (2) 0 (3) 2 (4) $\left(-\frac{13}{20}\right)$ (5) 0.7

(6) (–0.8) (7) (–0.01) (8) $\left(-1\frac{2}{3}\right)$ (9) $\left(-\frac{2}{5}\right)$ (10) $\left(-\frac{7}{8}\right)$

2. (1) (–1) (2) $\left(-\frac{8}{5}\right)$ (3) 8 (4) $\frac{5}{2}$ (5) $\frac{10}{7}$ (6) $\frac{10}{8}$ (7) $\frac{100}{1}$ (8) $\frac{2}{5}$ (9) $\frac{5}{2}$ (10) $\frac{8}{7}$

Exercise

1. (1) natural (2) rational (3) rational number (4) 3, 8 (5) 2, $\frac{3}{8}$

2. (1) $\frac{15}{7}$ (2) $\frac{6}{10}$ (3) $\left(-\frac{13}{4}\right)$ (4) $\frac{0}{1}$ (5) $\frac{28}{1}$

3. (1) Commutative property for addition (2) Commutative property for multiplication (3) Associative property for multiplication (4) Distributive property (5) Existence of identity element for addition (6) Associative property for addition

For information only

- **Irrational Numbers :**

$\sqrt{4} = 2$ is an integer, so it is also a rational.

$\sqrt{1.69} = 1.3$ is also a rational number.

$\sqrt[3]{8} = 2$ is also a rational number.

These each can be denoted in $\frac{p}{q}$ form. Additionally each rational number can be written as finite decimal or infinite repeating decimal.

For example, $\frac{1}{5} = 0.2$, $\frac{1}{8} = 0.125$, $\frac{1}{3} = 0.333...$, $\frac{1}{7} = 0.1428571428571$

Here, 0.333 and 0.1428571428571... can be written as $0.\dot{3}$ and $0.\dot{1}4285\dot{7}$ respectively.

Here, for 0.123456789101112... there is no possibility of finite and repeating. These numbers are called irrational numbers. Therefore, the numbers whose decimal form is infinite and non-repeating are called irrational numbers.

For example, $\sqrt{2}$, $\sqrt{3}$, π, $\frac{\sqrt{3}}{3}$

- **Real Numbers :**

The set which is formed by collection of rational and irrational numbers, is called the set of Real numbers. They are denoted by R.

3 Rational Indices

- **Let's remember :**

Let's remember the laws of rational indices which we have learnt in Std. 7.

- **Fill in the blanks :**

(1) $5^4 \times 5^3 = 5^{....}$ (2) $7^5 \div 7^2 = 7^{....}$

(3) $\frac{11^3}{11^5} = \frac{1}{11^{...}}$ (4) $(9^3)^2 = 9^{....}$

(5) $(...)^3 = 4^3 \times m^3$ (6) $\left(\frac{2}{....}\right)^5 = \frac{2^5}{3^5}$

- **Let's learn new :**
- **Laws of natural indices :**

(1) Law of multiplication : In the earlier class we have learnt that 'during multiplication of two rational indices with same base then their indices are added with same base.'

Multiplication of indices	Repeated multiplication	Result
$2^2 \times 2^3$	$\underline{2 \times 2} \times \underline{2 \times 2 \times 2}$	$2^5 = 2^{2+3}$
$(-8)^3 \times (-8)^4$	$\underline{(-8)\times(-8)\times(-8)} \times \underline{(-8)\times(-8)\times(-8)\times(-8)}$	$(-8)^7 = (-8)^{3+4}$
$x^2 \times x^4$	$\underline{x \times x} \times \underline{x \times x \times x \times x}$	$x^6 = x^{2+4}$
$\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^5$	$\underline{\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)}\times\underline{\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)}$	$\left(\frac{1}{2}\right)^7 = \left(\frac{1}{2}\right)^{2+5}$
$(\sqrt{3})^2\times(\sqrt{3})^3\times(\sqrt{3})^4$	$\underline{(\sqrt{3})\times(\sqrt{3})}\times\underline{(\sqrt{3})\times(\sqrt{3})\times(\sqrt{3})}\times$ $\underline{(\sqrt{3})\times(\sqrt{3})\times(\sqrt{3})\times(\sqrt{3})}$	$(\sqrt{3})^9 = (\sqrt{3})^{2+3+4}$

Therefore, $a^m \times a^n = a^{m+n}$; where $a \in$ R and $m, n \in$ N

Example 1 : Simplify :

(1) $(0.3)^2 \times (0.3)^4 \times (0.3)$ $= (0.3)^{2+4+1}$ $= (0.3)^7$

(2) $(\sqrt{7})^2 \times (\sqrt{7})^7 \times (\sqrt{7})^3$ $= (\sqrt{7})^{2+7+3}$ $= (\sqrt{7})^{12}$

(3) $\left(-\frac{4}{5}\right)^6 \times \left(-\frac{4}{5}\right)^{18} \times \left(-\frac{4}{5}\right)$ $= \left(-\frac{4}{5}\right)^{6+18+1}$ $= \left(-\frac{4}{5}\right)^{25}$

(2) **Law of division :** During division of two rational indices with same base their indices are subtracted at the same base.

Division of indices	Repeated multiplication form	Result
$(-3)^6 \div (-3)^4$	$\frac{(-3)^6}{(-3)^4} = \frac{(-3)\times(-3)\times(-3)\times(-3)\times(-3)\times(-3)}{(-3)\times(-3)\times(-3)\times(-3)}$	$(-3)^2 = (-3)^{6-4}$
$\left(\frac{1}{2}\right)^3 \div \left(\frac{1}{2}\right)^4$	$\frac{\left(\frac{1}{2}\right)^3}{\left(\frac{1}{2}\right)^4} = \frac{\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)}$	$\frac{1}{\left(\frac{1}{2}\right)^1} = \frac{1}{\left(\frac{1}{2}\right)^{4-3}}$
$(\sqrt{m})^4 \div (\sqrt{m})^4$ where $m \neq 0$	$\frac{(\sqrt{m})^4}{(\sqrt{m})^4} = \frac{(\sqrt{m})\times(\sqrt{m})\times(\sqrt{m})\times(\sqrt{m})}{(\sqrt{m})\times(\sqrt{m})\times(\sqrt{m})\times(\sqrt{m})} = 1$	$1 = (\sqrt{m})^{4-4} = (\sqrt{m})^0$

Therefore, (i) If $m > n$ and $a \neq 0$, then $a^m \div a^n = a^{m-n}$

(ii) If $m < n$ and $a \neq 0$, then $a^m \div a^n = \frac{1}{a^{n-m}}$

(iii) If $a \neq 0$, then $a^m \div a^m = a^{m-m} = a^0 = 1$

For any $a \neq 0$, $a^0 = 1$.

Example 2 : Simplify :

(1) $\left(\frac{1}{3}\right)^5 \div \left(\frac{1}{3}\right)^3$ $= \left(\frac{1}{3}\right)^{5-3}$ $= \left(\frac{1}{3}\right)^2$

(2) $\frac{(-7)^4}{(-7)^6}$ $= \frac{1}{(-7)^{6-4}}$ $= \frac{1}{(-7)^2}$

(3) $(-5)^7 \div (-5)^3$ $= (-5)^{7-3}$ $= (-5)^4$

(4) $\frac{a^7}{a^{14}} \times a^{12}$

First method : $\frac{a^7}{a^{14}} \times a^{12}, (a \neq 0)$

$= \frac{1}{a^{14-7}} \times a^{12}$

$= \frac{a^{12}}{a^7}$

$= a^{12-7}$

$= a^5$

Second method : $\frac{a^7}{a^{14}} \times a^{12}, (a \neq 0)$

$= \frac{a^7 \times a^{12}}{a^{14}}$

$= \frac{a^{7+12}}{a^{14}}$

$= \frac{a^{19}}{a^{14}}$

$= a^{19-14}$

$= a^5$

(5) $y^{19} \div y^{19}; (y \neq 0)$

$= y^{19-19}$

$= y^0$

$= 1$

(3) Law of power of a power :

For taking power of a power, the rational indices are multiplied on the same base.

Example : (1) $(2^2)^3 = 2^{2\times3} = 2^6$

(2) $[(-3)^4]^5 = (-3)^{4\times5} = (-3)^{20}$

(3) $\left[\left(\frac{1}{2}\right)^3\right]^4 = \left(\frac{1}{2}\right)^{3\times4} = \left(\frac{1}{2}\right)^{12}$

- **Try yourself :**

(1) $(7^3)^4 = 7^{....} = ...^{....}$ (2) $(a^5)^2 = a^{....} = ...^{....}$

Therefore, $(a^m)^n = a^{mn}$; where $a \in$ R and $m, n \in$ N

Example 3 : Simplify :

(1) $(a^{10})^3 \div (a^6)^5, (a \neq 0)$

$= a^{10 \times 3} \div a^{6 \times 5}$

$= a^{30} \div a^{30}$

$= a^{30 - 30}$

$= a^0 = 1$

(2) $\dfrac{\left[(-2)^3 \times (-2)^5\right]^2}{(-2)^7}$

$= \dfrac{\left[(-2)^{3+5}\right]^2}{(-2)^7}$

$= \dfrac{\left[(-2)^8\right]^2}{(-2)^7}$

$= \dfrac{(-2)^{16}}{(-2)^7}$

$= (-2)^{16-7} = (-2)^9$

(4) Law of power of product :

The power of multiplication of two numbers is multiplication of powers of two numbers.

Example : (1) $(3 \times 4)^5 = 3^5 \times 4^5$

(2) $[(-3) \times 2]^4 = (-3)^4 \times 2^4$

Try yourself : (1) $(8 \times 10)^3 = 8^{\cdots\cdots} \times 10^{\cdots\cdots}$ (2) $(a \times b)^5 = a^{\cdots\cdots} \times b^{\cdots\cdots}$

Therefore, $(ab)^n = a^n b^n$; where $a, b \in$ R and $n \in$ N

Note : If there is multiplication of more than two numbers, then this law can be applied.

Example 4 : Simplify :

(1) $(-2ab)^5$

$= (-2)^5 a^5 b^5$

(2) $(ab^2c)^3$

$= a^3(b^2)^3c^3$

$= a^3b^6c^3$

(3) $(6a^3)^2$

$= (6 \times a^3)^2$

$= 6^2 \times (a^3)^2$

$= 36 \times a^6$

$= 36a^6$

(5) Law of power of quotient :

The power of division of two numbers is the division of two numbers with same power, where divisor is not zero.

Example : (1) $\left(\frac{2}{5}\right)^{10} = \frac{2^{10}}{5^{10}}$, (2) $\left(\frac{-3}{4}\right)^{5} = \frac{(-3)^5}{4^5}$

- **Try yourself :** (1) $\left(\frac{7}{9}\right)^{8} = \frac{7^{\cdots\cdots}}{9^{\cdots\cdots}}$ (2) $\left(\frac{-2}{4}\right)^{6} = \frac{(-2)^{\cdots\cdots}}{4^{\cdots\cdots}}$

Therefore, $b \neq 0$ and $a, b \in$ R, then $\left(\frac{a}{b}\right)^{n} = \frac{a^n}{b^n}$.

Example 5 : Simplify :

(1) $[(a^6)^3 (b^4)^7] \div [(a^2)^9 (b^{14})^2]$; $(a \neq 0,\ b \neq 0)$

$= [a^{18}b^{28}] \div [a^{18}b^{28}]$

$= \frac{a^{18}b^{28}}{a^{18}b^{28}}$

$= a^{18-18}\ b^{28-28}$

$= a^0b^0$

$= 1 \times 1$

$= 1$

(2) $\frac{(3x^2)^2 \times (2x^3)^4}{(6x^2)^2}$

$= \frac{3^2 \times (x^2)^2 \times 2^4 \times (x^3)^4}{(2 \times 3 \times x^2)^2}$

$= \frac{3^2 \times x^4 \times 2^4 \times x^{12}}{2^2 \times 3^2 \times x^4}$

$= 2^{4-2} \times x^{12}$

$= 2^2 \times x^{12}$

$= 4x^{12}$

Practice 1

1. Fill in the blanks :

(1) $(-10)^6 \times (-10)^6 = (-10)^{\cdots\cdots}$

(2) $4^5 \div 4^3 = 4^{\cdots\cdots}$

(3) $\frac{x^3}{x^6} = \frac{1}{x^{\cdots\cdots}} = \cdots\cdots$

(4) $(21^3)^3 = 21^{\cdots\cdots}$

(5) $(\sqrt{3})^0 = \cdots\cdots$

(6) $[(-3) \times 4]^2 = (-3)^{\cdots\cdots} \times \cdots\cdots^2$

(7) $\left(\frac{5}{x}\right)^4 = \frac{5^{\cdots\cdots}}{x^{\cdots\cdots}}$

(8) $\left(\frac{1}{2}\right)^3 \div \left(\frac{1}{2}\right)^3 = \cdots\cdots$

(9) $12x^4 \div 3x^6 = \cdots\cdots$

(10) $a^{10} \div \cdots\cdots = a^8$

2. Simplify :

(1) $(2^7)^3 \times (2^4)^6$

(2) $(-3)^4 \times (-3)$

(3) $\frac{(a^{14})^4 \times (a^2)^3}{(a^7)^3}$; $(a \neq 0)$

(4) $\frac{(y^7)^3}{(y^6)^5}$; $(y \neq 0)$

(5) $\left(\frac{1}{3} \times x\right)^3$

(6) $\frac{(2a^2b^3)^5 \times (2a^2b^2)^3}{(5a^4b)^6}$; $(a, b \neq 0)$

- **Negative Rational indices :**

For number x except zero according to the law of division of power the indices are subtracted in the numerator only.

$$\frac{x^3}{x} = x^{3-1} = x^2$$

$$\frac{x^2}{x} = x^{2-1} = x^1$$

$$\frac{x}{x} = x^{1-1} = x^0 = 1$$

$$\frac{1}{x} = \frac{x^0}{x} = x^{0-1} = x^{-1} \quad (\because\ 1 = x^0)$$

$$\frac{1}{x} \div x = \frac{1}{x} \times \frac{1}{x} = \frac{1}{x \times x} = \frac{x^0}{x^2} = x^{0-2} = x^{-2} \quad (\because\ 1 = x^0)$$

Therefore, for any positive real number a, $\frac{1}{a} = a^{-1}$, $\frac{1}{a^2} = a^{-2}$, $\frac{1}{a^3} = a^{-3}$

$\because$ is read as 'because'

Similarly (i) $a^{-1} = \frac{1}{a}$, $a^{-5} = \frac{1}{a^5}$, $a^{-7} = \frac{1}{a^7}$ (ii) $5^{-1} = \frac{1}{5}$, $(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$

(iii) $\left(\frac{1}{4}\right)^{-3} = \frac{1^{-3}}{4^{-3}} = \frac{4^3}{1^3} = 4^3 = 64$ (iv) $\left(\frac{2}{7}\right)^{-5} = \frac{2^{-5}}{7^{-5}} = \frac{7^5}{2^5}$

Therefore, if $a \neq 0$ and $a \in R$, then for $n \in N$, $\boldsymbol{a^{-n} = \frac{1}{a^n}}$

Besides this, all the laws of rational indices for positive integers are also true for negative integers.

$a, b \neq 0$, $m, n \in Z$, then

(1) $a^m \times a^n = a^{m+n}$ (2) $(a^m)^n = a^{mn}$ (3) $\frac{a^m}{a^n} = a^{m-n}$

(4) $(ab)^n = a^n b^n$ (5) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Example 6 : Simplify :

(1) 4^{-3}

$= \frac{1}{4^3}$

$= \frac{1}{64}$

(2) $\left(\frac{1}{3}\right)^{-4}$

$= \frac{1}{\left(\frac{1}{3}\right)^4} = \frac{1}{\left(\frac{1^4}{3^4}\right)}$

$= \frac{1}{\frac{1}{81}} = \frac{81}{1} = 81$

(3) $\left(x^{-1}\right)^3$

$= x^{-3}$

$= \frac{1}{x^3}$

(4) $2^3 \times \left(\frac{1}{2}\right)^5 \times 2^{-6}$

First method : $2^3 \times \left(\frac{1}{2}\right)^5 \times 2^{-6}$

$= 2^3 \times (2^{-1})^5 \times 2^{-6}$

$= 2^3 \times 2^{-5} \times 2^{-6}$

$= 2^{3-5-6}$

$= 2^{-8}$

$= \frac{1}{2^8}$

$= \frac{1}{256}$

Second method : $2^3 \times \left(\frac{1}{2}\right)^5 \times 2^{-6}$

$= 2^3 \times \frac{1^5}{2^5} \times \frac{1}{2^6}$

$= \frac{2^3 \times 1 \times 1}{2^5 \times 2^6}$

$= \frac{2^3}{2^{5+6}}$

$= \frac{2^3}{2^{11}}$

$= \frac{1}{2^{11-3}}$

$= \frac{1}{2^8} = \frac{1}{256}$

Example 7 : Simplify :

(1) $(5x^{-2})^3 \times (3x^3)^4 \div (15x^2)^2$

$= \dfrac{5^3 \times x^{(-2)\times 3} \times 3^4 \times x^{3\times 4}}{(3\times 5\times x^2)^2}$

$= \dfrac{5^3 \times x^{-6} \times 3^4 \times x^{12}}{3^2 \times 5^2 \times x^{2\times 2}}$

$= \dfrac{3^4 \times 5^3 \times x^{-6+12}}{3^2 \times 5^2 \times x^4}$

$= \dfrac{3^4 \times 5^3 \times x^6}{3^2 \times 5^2 \times x^4}$

$= 3^{4-2} \times 5^{3-2} \times x^{6-4}$

$= 3^2 \times 5^1 \times x^2$

$= 9 \times 5 \times x^2$

$= 45x^2$

(2) $\left(\dfrac{x}{y}\right)^a (xy)^b \div \left(\dfrac{x}{y}\right)^b$

$= \dfrac{x^a}{y^a} \times x^b \times y^b \div \dfrac{x^b}{y^b}$

$= \dfrac{x^a \times x^b \times y^b \times y^b}{y^a \times x^b}$

$= \dfrac{x^a \times x^b \times y^{b+b}}{x^b \times y^a}$

$= x^a \times \dfrac{x^b}{x^b} \times y^{2b-a}$

$= x^a \cdot y^{2b-a}$

Practice 2

1. Fill in the blanks by choosing proper options :

(1) $4^{-3} =$ .... [(a) 64, (b) −12, (c) $\frac{1}{64}$]

(2) $\frac{1}{5} =$ .... [(a) 5^{-1}, (b) 5^{-2}, (c) 5^{-3}]

(3) $(8^{-1})^{-2} =$ .... [(a) $\frac{1}{8}$, (b) $\frac{1}{64}$, (c) 64]

(4) $a^{-2} \times \dfrac{1}{a^{-2}} =$ .... [(a) a^0, (b) a^2, (c) a^{-4}]

(5) $\dfrac{1}{(2^{-1})^2} =$ .... [(a) $\frac{1}{4}$, (b) 2^2, (c) $\frac{1}{2}$]

(6) $(\sqrt{5})^3 \div (\sqrt{5})^3 =$ .... [(a) $\sqrt{5}$, (b) 1, (c) $(\sqrt{5})^6$]

2. Simplify :

(1) $2^{-3} \times \left(\frac{1}{4}\right)^5 \times 8^{-3}$ (2) $\left(\frac{a}{b}\right)^{m+n} \left(\frac{a}{b}\right)^{m-n} (ab)^m$ (3) $4a^{-2} \times \left(\frac{a}{2}\right)^5 \div (2a)^2$

3. Evaluate :

(1) $2^2 \times 2^{-3} \times 2^{-1}$ (2) $\left(\frac{1}{3}\right)^3 \times 3^{-2} \times 3^5$ (3) $(8^{-2} \times 12^4) \div 27^2$

*

● Fractional (rational) indices :

Number	Square	Square root of square
1	1	$\sqrt{1} = \sqrt{1^2}$
2	4	$\sqrt{4} = \sqrt{2^2}$
3	9	$\sqrt{9} = \sqrt{3^2}$
4	16	$\sqrt{16} = \sqrt{4^2}$
5	25	$\sqrt{25} = \sqrt{5^2}$

Now, from the table comparing number and square root of square of the number we get,

$1 = \sqrt{1^2}$

$2 = \sqrt{2^2}$

$3 = \sqrt{3^2}$

$4 = \sqrt{4^2}$

$5 = \sqrt{5^2}$

Now, $\sqrt{3^2} = 3$ **Result (1)**

Here, 3 is square root of 3^2.

Now, let's find such indices of 3^2 such that value is 3.

If, we take such rational indices of 3^2 which when multiplied with 3, then rational indices is 1. (Law of power of power)

The reciprocal of two inverse numbers is 1 and so reciprocal of $2 = \frac{1}{2}$.

Thus, $(3^2)^{\frac{1}{2}} = 3^{2\times\frac{1}{2}}$

$= 3^1 = 3$ **Result (2)**

Comparing result (1) and result (2) we get,

$$3 = \sqrt{3^2} = (3^2)^{\frac{1}{2}}$$

Here, the symbol of square root ($\sqrt{\ }$) is denoted by index $\frac{1}{2}$.

Therefore, $\sqrt{2} = 2^{\frac{1}{2}}$, $\sqrt{3} = 3^{\frac{1}{2}}$, $\sqrt{5} = 5^{\frac{1}{2}}$ and $\sqrt{x} = x^{\frac{1}{2}}$; where x is a real positive number.

Similarly, for symbol of cube root $\sqrt[3]{\ }$, $\sqrt[3]{x} = x^{\frac{1}{3}}$, for symbol of fourth root $\sqrt[4]{\ }$, $\sqrt[4]{x} = x^{\frac{1}{4}}$, for symbol of fifth root $\sqrt[5]{\ }$, $\sqrt[5]{x} = x^{\frac{1}{5}}$, for symbol of nth root $\sqrt[n]{\ }$, $\sqrt[n]{x} = x^{\frac{1}{n}}$, where $n \in$ N.

Therefore, if x is a positive real number and $n \in$ N, then $\sqrt[n]{x} = x^{\frac{1}{n}}$ can be written.

Example 8 : For $a = 64$, $m = \frac{3}{2}$, $n = \frac{1}{3}$ verify $a^m \times a^n = a^{m+n}$.

L.H.S. $= a^m \times a^n$

$= (64)^{\frac{3}{2}} \times (64)^{\frac{1}{3}}$

$= (2^6)^{\frac{3}{2}} \times (2^6)^{\frac{1}{3}}$

$= 2^{6 \times \frac{3}{2}} \times 2^{6 \times \frac{1}{3}}$

$= 2^9 \times 2^2$

$= 2^{9+2}$

$= 2^{11}$

R.H.S. $= a^{m+n}$

$= (64)^{\frac{3}{2}+\frac{1}{3}}$

$= (64)^{\frac{11}{6}}$

$= (2^6)^{\frac{11}{6}}$

$= 2^{11}$

Therefore, $a^m \times a^n = a^{m+n}$

Note : To show exponent (indices) form of any number, write that number as exponent form of prime number.

Example 9 : For $a = 64$, $m = \frac{2}{3}$, $n = \frac{1}{2}$, verify $(a^m)^n = a^{mn}$.

L.H.S. $= (a^m)^n$

$= (64^{\frac{2}{3}})^{\frac{1}{2}}$

$= ((2^6)^{\frac{2}{3}})^{\frac{1}{2}}$

$= (2^4)^{\frac{1}{2}}$

$= 2^2$

$= 4$

R.H.S. $= a^{mn}$

$= (64)^{\frac{2}{3} \times \frac{1}{2}}$

$= (2^6)^{\frac{1}{3}}$

$= 2^{6 \times \frac{1}{3}}$

$= 2^2$

$= 4$

Therefore, $(a^m)^n = a^{mn}$

Example 10 : For $a = 27$, $b = 8$, $m = \frac{1}{3}$ verify $(ab)^m = a^m b^m$

L.H.S. $= (ab)^m$

$= (27 \times 8)^{\frac{1}{3}}$

$= (3^3 \times 2^3)^{\frac{1}{3}}$

$= (3^3)^{\frac{1}{3}} (2^3)^{\frac{1}{3}}$

$= 3^{3\times\frac{1}{3}} 2^{3\times\frac{1}{3}}$

$= 3 \times 2 = 6$

R.H.S. $= a^m b^m$

$= 27^{\frac{1}{3}} \times 8^{\frac{1}{3}}$

$= (3^3)^{\frac{1}{3}} (2^3)^{\frac{1}{3}}$

$= 3 \times 2$

$= 6$

$\therefore$ L.H.S. = R.H.S. Therefore, $(ab)^m = a^m b^m$

Example 11 : Simplify :

(1) $(343)^{\frac{1}{3}}$

$= (7^3)^{\frac{1}{3}}$

$= 7^{3\times\frac{1}{3}}$

$= 7$

(2) $\left(\frac{81}{625}\right)^{\frac{3}{4}}$

$= \left(\frac{3^4}{5^4}\right)^{\frac{3}{4}}$

$= \frac{3^{4\times\frac{3}{4}}}{5^{4\times\frac{3}{4}}}$

$= \frac{3^3}{5^3}$

$= \frac{27}{125}$

(3) $\left(\frac{32}{243}\right)^{\frac{-2}{5}}$

$= \left(\frac{2^5}{3^5}\right)^{\frac{-2}{5}}$

$= \frac{2^{5\times\frac{-2}{5}}}{3^{5\times\frac{-2}{5}}}$

$= \frac{2^{-2}}{3^{-2}}$

$= \frac{3^2}{2^2}$

$= \frac{9}{4}$

$= 2\frac{1}{4}$

Second method :

$\left(\frac{32}{243}\right)^{\frac{-2}{5}}$

$= \left(\frac{243}{32}\right)^{\frac{2}{5}}$

$= \left(\frac{3^5}{2^5}\right)^{\frac{2}{5}}$

$= \frac{3^{5\times\frac{2}{5}}}{2^{5\times\frac{2}{5}}}$

$= \frac{3^2}{2^2}$

$= \frac{9}{4}$

$= 2\frac{1}{4}$

Example 12 : Simplify : $\left(\dfrac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}}\right)^{\frac{1}{5}}\left(\dfrac{x^{\frac{1}{5}}}{x^{\frac{1}{3}}}\right)^{\frac{1}{2}}\left(\dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{5}}}\right)^{\frac{1}{3}}$, where $(x > 0)$.

$$= \frac{(x^{\frac{1}{3}})^{\frac{1}{5}}}{(x^{\frac{1}{2}})^{\frac{1}{5}}} \times \frac{(x^{\frac{1}{5}})^{\frac{1}{2}}}{(x^{\frac{1}{3}})^{\frac{1}{2}}} \times \frac{(x^{\frac{1}{2}})^{\frac{1}{3}}}{(x^{\frac{1}{5}})^{\frac{1}{3}}}$$

$$= \frac{x^{\frac{1}{3}\times\frac{1}{5}}}{x^{\frac{1}{2}\times\frac{1}{5}}} \times \frac{x^{\frac{1}{5}\times\frac{1}{2}}}{x^{\frac{1}{3}\times\frac{1}{2}}} \times \frac{x^{\frac{1}{2}\times\frac{1}{3}}}{x^{\frac{1}{5}\times\frac{1}{3}}}$$

$$= \frac{x^{\frac{1}{15}}}{x^{\frac{1}{10}}} \times \frac{x^{\frac{1}{10}}}{x^{\frac{1}{6}}} \times \frac{x^{\frac{1}{6}}}{x^{\frac{1}{15}}}$$

$$= \frac{x^{\frac{1}{15}}}{x^{\frac{1}{15}}} \times \frac{x^{\frac{1}{10}}}{x^{\frac{1}{10}}} \times \frac{x^{\frac{1}{6}}}{x^{\frac{1}{6}}}$$

$$= 1$$

Example 13 : If $x = 64$, then find the value of $x^{\frac{1}{6}} + x^{-\frac{1}{6}}$.

$$x^{\frac{1}{6}} + x^{-\frac{1}{6}} = 64^{\frac{1}{6}} + 64^{-\frac{1}{6}}$$

$$= (2^6)^{\frac{1}{6}} + (2^6)^{\frac{1}{6}}$$

$$= 2^{6\times\frac{1}{6}} + 2^{6\times\frac{-1}{6}}$$

$$= 2^1 + 2^{-1}$$

$$= 2 + \frac{1}{2}$$

$$= 2\frac{1}{2}$$

Example 14 : If $x = \frac{9}{4}$, then prove $\left(x\sqrt{x}\right)^x = x^{x\sqrt{x}}$.

$$\left(x\sqrt{x}\right)^x = \left(\frac{9}{4}\sqrt{\frac{9}{4}}\right)^{\frac{9}{4}} = \left(\frac{3^2}{2^2}\times\frac{3}{2}\right)^{\frac{9}{4}} = \left[\left(\frac{3}{2}\right)^3\right]^{\frac{9}{4}} = \left(\frac{3}{2}\right)^{\frac{27}{4}}$$

$$x^{x\sqrt{x}} = \left(\frac{9}{4}\right)^{\frac{9}{4}\sqrt{\frac{9}{4}}} = \left(\frac{9}{4}\right)^{\frac{9}{4}\times\frac{3}{2}} = \left(\frac{9}{4}\right)^{\frac{27}{8}} = \left[\left(\frac{3}{2}\right)^2\right]^{\frac{27}{8}} = \left(\frac{3}{2}\right)^{\frac{27}{4}}$$

Therefore, $\left(x\sqrt{x}\right)^x = x^{x\sqrt{x}}$.

Practice 3

1. Evaluate :

(1) $\sqrt[5]{243}$ (2) $729^{\frac{1}{3}}$ (3) $\left(\frac{125}{343}\right)^{\frac{1}{3}}$ (4) $64^{\frac{1}{6}}$

(5) $625^{\frac{1}{4}}$ (6) $\left(\frac{81}{144}\right)^{\frac{1}{2}}$ (7) $\left(\frac{81}{625}\right)^{\frac{-3}{4}}$ (8) $\frac{32^{\frac{1}{5}}}{81^{\frac{1}{4}}}$

2. Prove that, $5^{\frac{1}{3}}\times\left(\frac{2}{5}\right)^{\frac{1}{3}}\times\frac{64^{\frac{1}{3}}}{3^{\frac{1}{3}}}\times\frac{9^{\frac{1}{6}}}{2^{\frac{1}{3}}} = 4$.

3. If $x = 243$, then find the value of $x^{\frac{1}{5}}\times x^{-\frac{1}{5}}$.

4. Verify : (1) $\sqrt{625} - 5\sqrt[3]{27} - (100)^{\frac{1}{2}} = 0$

(2) $\left[(81)^{\frac{1}{2}}+1\right]\left[(81)^{\frac{1}{4}}-1\right] = 20$

What did you learn ?

- **a, $b \neq 0$ and m, $n \in$ Q**

  (1) $a^m \times a^n = a^{m+n}$ (2) $(a^m)^n = a^{mn}$ (3) $\frac{a^m}{a^n} = a^{m-n}$

  (4) $(ab)^n = a^n b^n$ (5) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

- If $a \neq 0$, then $a^0 = 1$
- If $a \neq 0$ and $a \in$ R, then for $n \in$ N, $a^{-n} = \frac{1}{a^n}$.
- If x is a positive real number and $n \in$ N, then $\sqrt[n]{x} = x^{\frac{1}{n}}$.

Exercise

1. Fill in the blanks :

(1) $(-51)^0 =$ ....

(2) $x^5 \times x^{-4} \div x^2 =$ ....

(3) $(a^3)^{-4} =$ ....

(4) $(\sqrt{y}\,)^5 =$ ....

(5) $4^{-2} \times \frac{1}{4^{-2}} =$ ....

(6) $\frac{1}{(3\times4)^{-1}} =$ ....

(7) $\left[\left(\frac{2}{3}\right)^2\right]^{-2} =$ ....

(8) $\left[\frac{16}{81}\right]^{\frac{1}{4}} =$ ....

2. Simplify : (1) $\left(\frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}}\right)^2 \left(\frac{x^{\frac{1}{3}}}{x^{\frac{1}{4}}}\right)^3 \left(\frac{x^{\frac{1}{4}}}{x^{\frac{1}{2}}}\right)^4$, $(x > 0)$ (2) $\frac{(x^5)^{\frac{1}{6}} \times x^{\frac{1}{7}} \times (x^{\frac{2}{3}})^2}{(x^2)^{\frac{2}{3}} \times (x^{\frac{1}{6}})^5 \times x^{\frac{1}{7}}}$

3. Evaluate :

(1) $\left[\frac{49}{16}\right]^{\frac{1}{2}} \times \left(\frac{4^2}{7^2}\right) \times 4^{\frac{1}{2}}$

(2) $\left(\frac{25}{16}\right)^{\frac{1}{4}} \times \left(\frac{27}{8}\right)^{\frac{1}{6}} \times \left(\frac{2}{15}\right)^{\frac{1}{2}}$

(3) $\frac{\sqrt[3]{108}\times\sqrt[6]{4}}{\sqrt[4]{81}}$

(4) $\left(\frac{8}{27}\right)^{\frac{1}{3}} \times \left(\frac{9}{25}\right)^{\frac{1}{2}} \times \left(\frac{2}{5}\right)^{-1}$

4. Prove that : $\left(\dfrac{2^{\frac{1}{3}}}{2^{\frac{-1}{3}}}\right)^3 + \dfrac{3^{\frac{1}{2}}}{3^{\frac{-1}{2}}} = 7$

5. Prove that $\dfrac{(16)^{\frac{1}{4}}}{(27)^{\frac{1}{3}}} + \dfrac{(625)^{\frac{1}{4}}}{(81)^{\frac{1}{4}}} - \dfrac{1}{(243)^{\frac{1}{5}}} = 2$

6. Prove : $[(a^x)^y(a^y)^x]^z = a^{2xyz}$ $(x, y, z \in Q)$

7. If $x > 0$ and $x, y, z \in$ Q and a, b, c are non-zero integers, then prove that

$$\left[\left(\frac{x^a}{x^b}\right)^{\frac{1}{a}}\right]^{\frac{1}{b}}\left[\left(\frac{x^b}{x^c}\right)^{\frac{1}{b}}\right]^{\frac{1}{c}}\left[\left(\frac{x^c}{x^a}\right)^{\frac{1}{c}}\right]^{\frac{1}{a}} = 1.$$

Answers

Practice 1

1. (1) 12 (2) 2 (3) 3 (4) 9 (5) 1 (6) 2, 4 (7) 4, 4 (8) $\left(\frac{1}{2}\right)^0$ or 1 (9) $\frac{4}{x^2}$ (10) a^2

2. (1) 2^{45} (2) $(-3)^5$ (3) a^{41} (4) $\frac{1}{y^9}$ (5) $\frac{x^3}{3^3}$ (6) $\frac{2^8 b^{15}}{5^6 a^8}$

Practice 2

1. (1) c (2) a (3) c (4) a (5) b (6) b

2. (1) $\frac{1}{2^{22}}$ (2) $a^{3m} \div b^m$ (3) $\frac{a}{32}$

3. (1) $\frac{1}{4}$ (2) 1 (3) $\frac{4}{9}$

Practice 3

1. (1) 3 (2) 9 (3) $\frac{5}{7}$ (4) 2 (5) 5 (6) $\frac{3}{4}$ (7) $\frac{125}{27}$ (8) $\frac{2}{3}$ **3.** 1

Exercise

1. (1) 1 (2) $\frac{1}{x}$ (3) $\frac{1}{a^{12}}$ (4) $y^{\frac{5}{2}}$ (5) 1 (6) 12 (7) $5\frac{1}{16}$ (8) $\frac{2}{3}$

2. (1) $\frac{1}{x^{\frac{5}{12}}}$ (2) 1 **3.** (1) $\frac{8}{7}$ or $1\frac{1}{7}$ (2) $\frac{1}{2}$ (3) 2 (4) 1

4 Introduction to Set

- **Let's learn new :**
- **Identification of set :**

Dear students, have you ever seen herd of cattle ?
have you seen flock of sheep ?
have you ever seen caravan of camel ?
have you even seen such type of group anywhere ?

- In all these groups, the number of members is more than one or two. But number of members are definite. For example, 10, 15, 20, 28,... etc. Therefore, group of definite elements (members) i.e. **A set is a collection of well specified object. Set is an undefined term.**
- **The brackets { } are used to express a set**

(1) The group of birds like sparrow, parrot, crow are written in set form as {Sparrow, Parrot, Crow}.

(2) The group of flowers like rose, marigold, hibiscus, bougainvillea are denoted in set forms {rose, marigold, hibiscus, bougainvillea}.

(3) The group of numbers line 1, 2, 3, 4, 5 are denoted in set form as {1, 2, 3, 4, 5}.

- Each of the well specified objects forming a set is called a member or an element of the set.
- Each member of a set is separated by commas (,). In the last comma and fullstop is not denoted. No element or member of a set is repeated.
- The symbol $\in$ (Belongs to) is used to denote that an object is a member of set and the symbol $\notin$ (Does not Belong to) is used to denote that an object is not a member of set.

Methods of describing a set : (1) The listing method (2) property method

Sets are denoted by capital letters A, B, C, ..., X, Y, Z, etc.

For example, A = {1, 2, 3, 4, 5}, X = {Mango, Asopalav, Pipal, Gulmohor}

Here, the natural numbers from 1 to 5 are written in the set form. This method of denoting the set is called listing method.

If this list is to show briefly then first five natural number can be written as : A = {x / x is first five natural numbers}. This method of denoting (writing) a set is called the property method. If all the members of a set have some common properties then that property is denoted by P(x) and the set is denoted by {x/p(x)}.

- **Some special sets :**

Complete the following table :

No.	The listing method	The property method
1		B = {x / x is an even prime number}
2.	X = {2, 4, 6, 8, 10}	
3.	Y = {–2, –1, 0, 1, 2}	
4.		C = {x / $x < 6$, $x \in$ N}
5.		Z = {x / x is tools of geometry box}
6.	M = {a, b, c, d, e, f}	

(1) Which is the set of having one and only one member ?

(2) Which is the set of having members more than 5 ?

(3) Which two sets have equal numbers of members ?

- **Some special sets :**

Empty Set :

The set without any member is called the empty set. It is denoted by symbol ϕ (phi) or "{ }".

For example : A = {x / x is a prime number less than 2, $x \in$ N}, then A = ϕ or A = { }

B = {x / x is a prime of female chief minister of Gujarat}, then B = ϕ or B = { }

Singleton Set :

A set having only one element (member) is called the singleton set.

For example : P = {x / x is an odd prime number less than 5}

P = {3}

Finite Set :

If the number of members in a set is a definite non-negative integer, then the set is called a finite set. For example, A = {1, 2, 3, ... 10}

Here, 1 to 10 natural numbers are included in set A, which can be counted. Therefore, the number of members of set A is 10 which is definite. Therefore, the set A is a finite set.

The number of members of a set A is denoted by n(A).

Here, the number of members of set A is 10. Therefore, n(A) = 10

- The empty set is also a finite set.

Infinite Set :

A set which is not finite is an infinite set.

For example : A = {x/x is a natural number} ∴ A = {1, 2, 3, 4, ...}

Here, there is no end of the list of members in set A. Such set is called an infinite set. To an infinite set after writing some members, generally three points are denoted.

- The set of natural numbers is denoted by a special symbol N.
  N = {1, 2, 3, ...}
- The set of whole numbers is denoted by a special symbol W.
  W = {0, 1, 2, 3, ...}
- The set of integers is denoted by a special symbol Z.
  Z = {..., −3, −2, −1, 0, 1, 2, 3, ...}
- The set of rational numbers (quotients) is denoted by a special symbol Q.

  $$Q = \left\{\frac{p}{q} \mid p \in Z, q \in N\right\}$$

- N, W, Z and Q, all these are infinite sets.
- Why is the set of leaves of a tree is called an infinite set ? Why ? Think !

1. Fill in the blanks with appropriate symbol ∈ or ∉ :

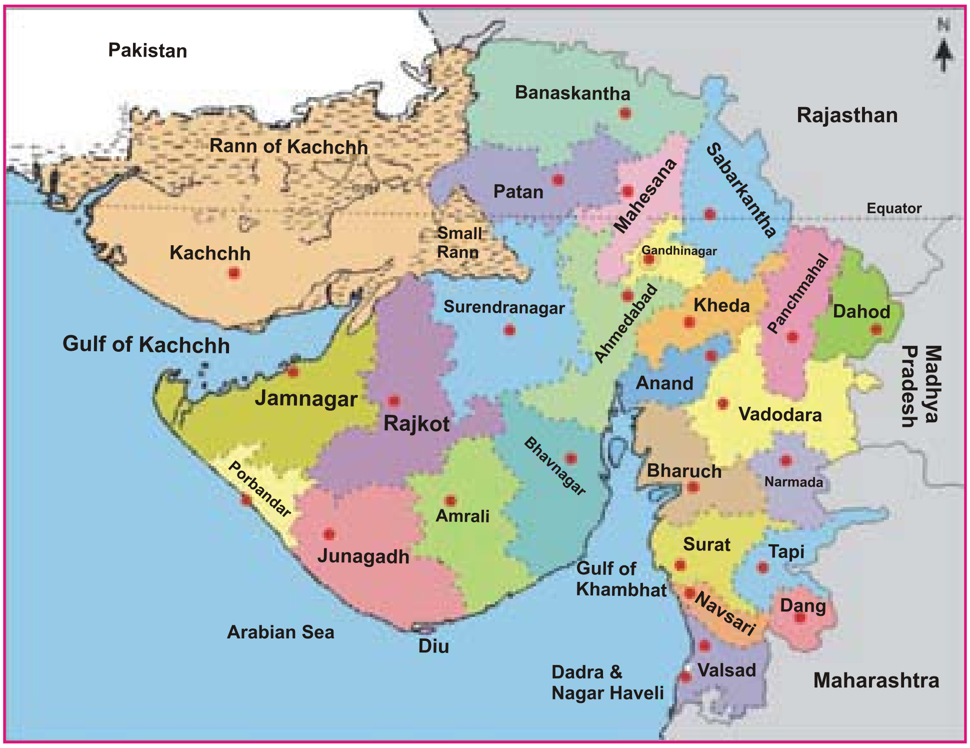
(1) 3 .... {1, 2, 3, 4}

(2) 100 .... {1, 2, 3, ..., 99}

(3) 5 .... $\{x / x$ is a multiple of $10\}$ (5) 0 .... $\{x / x$ is a natural number$\}$

(4) 2 .... $\{x / x$ is a prime factor of $15\}$

2. On the basis of given map write the given groups by the listing method :



No.	Groups	The listing method
1.	The district of Gujarat starting with 'A'	
2.	Such a district of Gujarat whose boundary touches Uttar Pradesh	
3.	The bay lying in Gujarat	
4.	The smallest district	
5.	The largest district	

3. Answer the following questions on the basis of groups given in the above Table :

(1) Which are the singleton sets ?

(2) Which are the finite sets ?

(3) Which one is the empty set ?

- **Subset :**

If every member of the set B is a member of the set A, then B is called a subset of A.

For example, A = $\{x / x$ is a positive integer less than $10\}$
$= \{1, 2, 3, ..., 9\}$
B = $\{1, 2, 3, 6\}$

Here, each and every member of set B lies in the set A. Therefore, set B is called subset of set A. Symbolically it is denoted as $B \subset A$.

Think : In every set, each and element of that set is member of itself, then can we say every set is a subset of itself.

A = $\{a, b, c\}$ and B = $\{x, b, c\}$, then can we say that set A is subset of set B ? Why ?

- **Keep in mind :** Every set is a subset of itself, i.e. $A \subset A$.
  - The empty set is a subset of each and every set, i.e. $\phi \subset A$.
  - Every set has at least two subsets, except the empty set.
  - The empty set has only one subset, i.e., itself only.
  - If A is not a subset of set B, then it is written as $A \not\subset B$.

Number of subsets of a given set : Look and understand

No.	Set	Number of member in set	Subsets	Number subsets	Number of subsets in exponent form
1.	A = { }	0	{ }	1	2^0
2.	B = $\{x\}$	1	{ }, $\{x\}$	2	2^1
3.	C = $\{p, q\}$	2	{ }, $\{p\}$ $\{q\}$, $\{p, q\}$	4	2^2
4.	D = $\{a, b, c\}$	3	{ }, $\{a\}$, $\{b\}$ $\{c\}$, $\{a, b\}$ $\{b, c\}$, $\{c, a\}$ $\{a, b, c\}$	8	2^3

On the basis of above Table it can be said that number subsets of set having number of members n in itself = 2^n.

Think :

(1) How many subsets are there in a set having four members in it ? ....

(2) How many subsets are there in a set having five members in it ? ....

Equal Set :

If set A and B contain the same elements, then set A and set B are said to be equal sets. Symbolically it is written as A = B.

For example : A = $\{x \,/\, x$ is a natural number less than 5$\}$, A = $\{1, 2, 3, 4\}$

B = $\{x \,/\, x$ is a factor of 12 less than 5$\}$, B = $\{1, 2, 3, 4\}$

Here, set A and set B have the same elements, therefore set A and set B are equal sets.

$\therefore$ A = B

Additionally here $A \subset B$ and $B \subset A$.

$\therefore$ $A \subset B$ and $B \subset A$, then A = B

One to One Correspondence :

Suppose there are 10 students in a class. A unique (one and only one) roll number is given to each student.

(1) Utsav or $1 \leftrightarrow$ Utsav

(2) Vijay or $2 \leftrightarrow$ Vijay

(3) Chahana or $3 \leftrightarrow$ Chahana

⋮ ⋮ ⋮

(10) Rehana or $10 \leftrightarrow$ Rehana

Therefore, there is only one number from 1 to 10 corresponding to each student, which is his roll number and no student have two roll numbers because there will not be two students with same roll number. Such correspondence is called one to one correspondence.

Now, A = $\{1, 2, 3\}$ and B = $\{a, b, c\}$, then there can be 6 correspondence between them.

(i)	(ii)	(iii)	(iv)	(v)	(vi)
$1 \leftrightarrow a$	$1 \leftrightarrow a$	$1 \leftrightarrow b$	$1 \leftrightarrow b$	$1 \leftrightarrow c$	$1 \leftrightarrow c$
$2 \leftrightarrow b$	$2 \leftrightarrow c$	$2 \leftrightarrow a$	$2 \leftrightarrow c$	$2 \leftrightarrow a$	$2 \leftrightarrow b$
$3 \leftrightarrow c$	$3 \leftrightarrow b$	$3 \leftrightarrow c$	$3 \leftrightarrow a$	$3 \leftrightarrow b$	$3 \leftrightarrow a$

- **Equivalent Set :**

If the number of the elements of two finite sets is the same, then they are called equivalent sets. Its symbol is '$\sim$'.

$A = \{1, 4, 6\}$ $\qquad$ $B = \{x, y, z\}$

$n(A) = 3$ $\qquad$ $n(B) = 3$

Here, $n(A) = n(B)$. Therefore set A and set B are equivalent sets.

Symbolically they are written as $A \sim B$.

- **Universal Set :**

Generally, in the discussion about sets, all sets are assumed to be the subsets of a definite set. This definite set with respect to its subsets is called the universal set. The symbol U is used for universal set.

For example, with reference to the set of all students of the school, the set of the player of Kho-kho team, the set of players of Kabaddi team, the set of members of the prayer committee, the set of students of VIII Standard etc are the subsets of the set of the students of school. Therefore, in this reference the set of the students of the school is the universal set.

Practice 2

1. Fill in the blanks using symbol $\subset$ or $\not\subset$:

(1) N .... Z $\qquad$ (2) $\{3, 1, -1\}$ .... N

(3) Z .... Q $\qquad$ (4) $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}\right\}$ .... Q

2. Write the following with symbols :

(1) 3 lies in the set A.

(2) $\frac{1}{2}$ does not lie in the set A.

(3) The set A is an empty set.

(4) Set A and set B are equal sets.

(5) Set C is subset of set D.

(6) Set B and set C are equivalent sets.

(7) Set A is not subset of set B.

(8) $\{0\}$ is a subset of set B.

3. A = {x / x is an even natural number less than 10} and

B = {−2, −3, −4, −5}, then whether they are equal sets or equivalent set ? Write with symbol.

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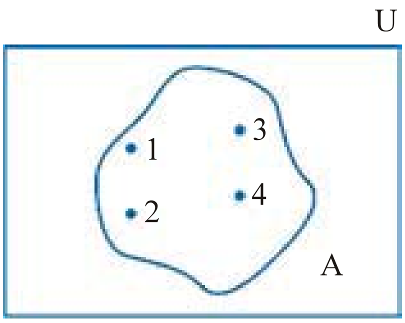
Venn Diagram :

According to the method of British logician John Venn, a universal set is expressed by the interior part of a rectangle and other sets (subsets of the universal set) are expressed by freehand interior part of the rectangle. This pictorial representation of the sets are known as Venn Diagram.

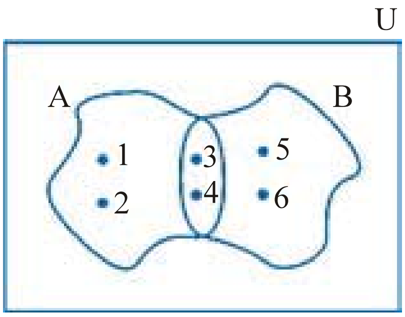


Venn diagram of a single set :

A = {1, 2, 3, 4}

Venn diagrams of two sets :

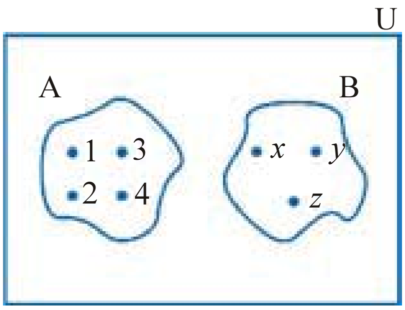
There are four possibilities for Venn diagrams of two sets.



(1) When one or more than one members between the two sets are common.

A = {1, 2, 3, 4}

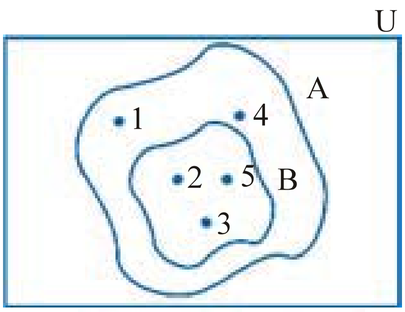
B = {3, 4, 5, 6}



(2) When both sets are disjoint sets

A = {1, 2, 3, 4}

B = {x, y, z}



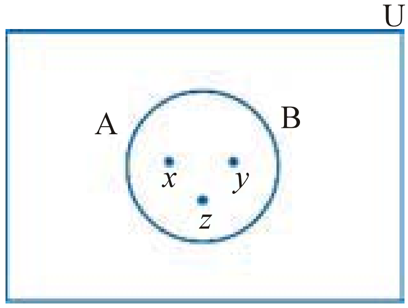
(3) When out of two sets one set is subset of the other.

A = {1, 2, 3, 4, 5}

B = {2, 3, 5}

(4) When both are equal sets.

A = {x, y, z} and B = {x, y, z}



Think : How many possibilities are there for Venn diagram of three sets ?

- **Complement of a set :**

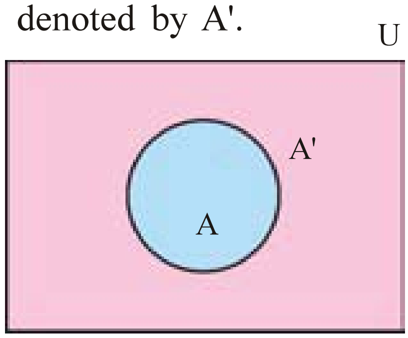
The set of all those elements which are present in the universal set U but not present in the set A, is called complement of the set A. It is denoted by A'.

Therefore A' = {x/x ∈ U and x ∉ A}

In short A' = U − A

U = {1, 2, 3, 4, 5} and A = {1, 2}

A' = {3, 4, 5}



Think : (1) U' = ... (2) (A')' = ...

Set operations : There are two types of set opeations : (1) Union (2) Intersection.

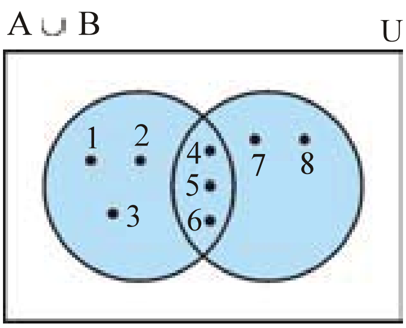
- **Union :**

The set consisting of all elements of the set A and the set B is called the union of the sets A and B and it is denoted by A ∪ B.

A ∪ B = {x / x ∈ A or x ∈ B}

For example, A = {1, 2, 3, 4, 5, 6} and set B = {4, 5, 6, 7, 8}

A ∪ B = {1, 2, 3, 4, 5, 6, 7, 8}

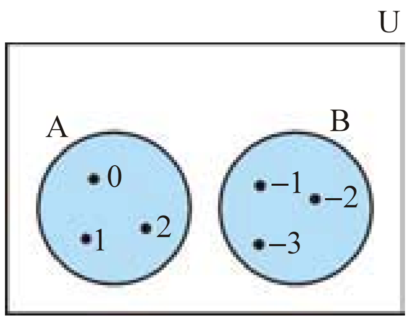


In the diagram, A ∪ B is denoted by shaded portion. The elements lying in both set A and set B are not written two times but they are shown in the intersection part of two circles.

For example, : A = {1, 0, 2}

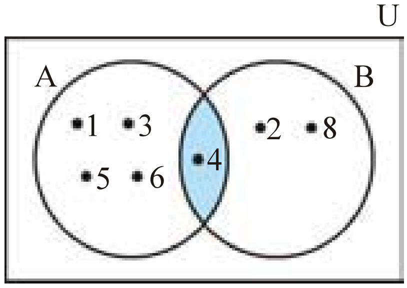
B = {−2, −3, −1}, then

A ∪ B = {1, 0, 2, −1, −2, −3}



Intersection :

The set consisting of elements which are in both A and B is called the intersection of A and B. It is denoted by $A \cap B$.



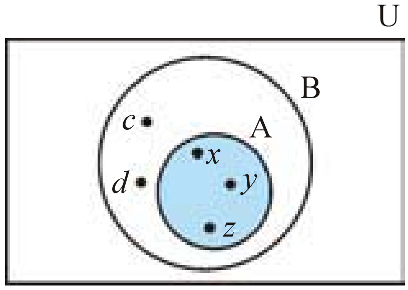
$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

For example, : $A = \{1, 3, 4, 5, 6\}$ and

$B = \{2, 4, 8\}$, then

$A \cap B = \{4\}$

In this figure the shaded region shows $A \cap B$.



For example, $A = \{x, y, z\}$

$B = \{x, y, z, c, d\}$, then

$A \cap B = \{x, y, z\} = A$

If $A \subset B$, then $A \cap B = A$

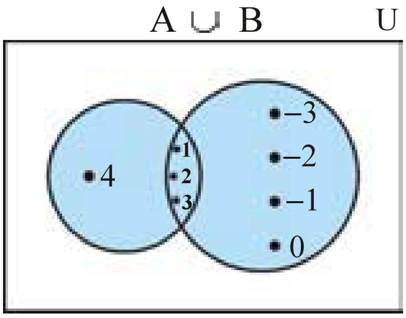
Example 1 : $A = \{1, 2, 3, 4\}$

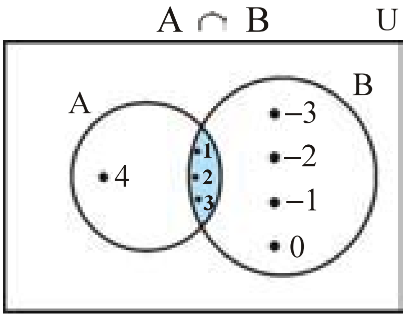
$B = \{x / x$ is an integer from -3 to $3\}$, then represent $A \cup B$ and $A \cap B$ by Venn diagram.

Solution : $A \cup B = \{1, 2, 3, 4\} \cup \{-3, -2, -1, 0, 1, 2, 3\}$

$= \{-3, -2, -1, 0, 1, 2, 3, 4\}$

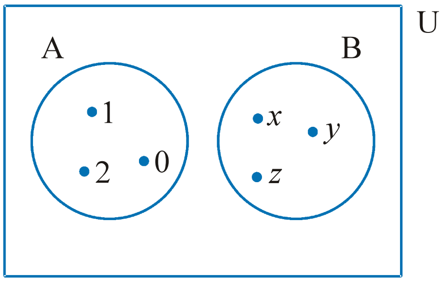
$A \cap B = \{1, 2, 3, 4\} \cap \{-3, -2, -1, 0, 1, 2, 3\} = \{1, 2, 3\}$





Example 2 : If $A = \{1, 0, 2\}$ and $B = \{x, y, z\}$, then show by Venn diagram such that $A \cap B = \{\} = \phi$.

Solution :



> **Keep in mind :** For two non-empty sets A and if $A \cap B = \phi$, then A and B are called **Disjoint Sets**.

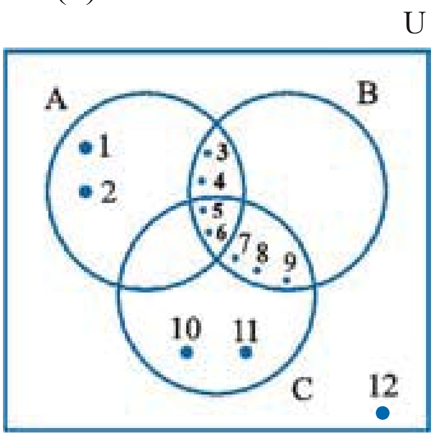
Example 3 : U = {x / $x \in$ N, $1 \le x \le 12$}, A = {1, 2, 3, 4, 5, 6}

B = {3, 4, 5, 6, 7, 8, 9}, C = {5, 6, 7, 8, 9, 10, 11}

Draw a Venn diagram representing these sets. From that, find the following sets :

(1) $B \cup C$ (2) $A \cap C$ (3) $A \cap B$

(4) $(A \cup B) \cap C$ (5) $(A \cap C) \cup (B \cap C)$ (6) $A \cap B \cap C$



(1) $B \cup C$ = {3, 4, 5, 6, 7, 8, 9, 10, 11}

(2) $A \cap C$ = {5, 6}

(3) $A \cap B$ = {3, 4, 5, 6}

(4) $A \cup B$ = {1, 2, 3, 4, 5, 6, 7, 8, 9}

C = {5, 6, 7, 8, 9, 10, 11}

$(A \cup B) \cap C$ = {5, 6, 7, 8, 9}

(5) $A \cap C$ = {5, 6}, $B \cap C$ = {5, 6, 7, 8, 9}

$(A \cap C) \cup (B \cap C)$ = {5, 6, 7, 8, 9}

(6) For $A \cap B \cap C$, taking intersection $A \cap B$ as in (3) with C, we will obtain the result.

$A \cap B \cap C$ = {5, 6}

- Please do activity by game of BINGO under the guidance of the teacher.

Practice 3

1. Calculate as asked :

(1) U = {x / x is name of months of English calendar},

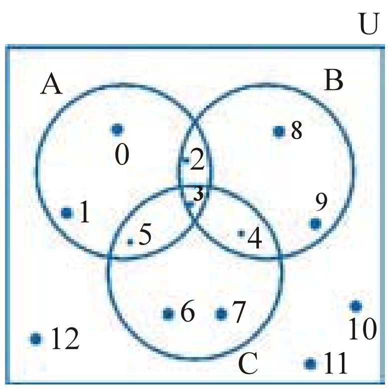
A = {March, May, July, June}, then find A'.

(2) U = {x / x is main colour of Rainbow}, R = {Violet, Red, Yellow}, then find R'.

(3) U = {$x \in$ N / $x \le 9$}, A = {2, 3, 5}

B = {4, 5, 7}, then find A', (A')', B' and (B')'.

2. Find the following results from the Venn diagram :



(1) $A \cup B$ = ...

(2) $A \cap B$ = ...

(3) $(A \cap C) \cup B$ = ...

(4) $(A \cup C) \cup B$ = ...

(5) U = ...

3. Find union and intersection of the given sets and also show them by Venn diagram.

A = $\{x / x$ is a natural number less than $5\}$, B = $\{x / 3 < x < 7; x \in N\}$

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Exercise

1. Fill in the blanks using proper symbol $\in$, $\notin$, $\subset$, $\not\subset$, $\sim$ or = :

(1) 6 .... {1, 2, 4, 6}

(2) {20} .... {20, 30, 40}

(3) 7 .... $\{x / x$ is a prime natural number$\}$

(4) 9 .... $\{x / x$ is a multiple of $18\}$

(5) {1, 2, 3} .... N.

(6) {−1, 1, 0} .... N.

(7) If A = $\{a, b, c\}$ and B = {1, 2, 3}, then A .... B.

2. Show the following groups by the listing method and by the property method :

No.	Groups	The listing method	The property method
1.	Multiplies of 5		
2.	Prime numbers between 21 and 30		
3.	Positive integers less than 6		
4.	Factors of 21		

3. State which of the following sets are empty sets and which are singleton sets :

(1) $\{x / x$ is a prime number less than $3\}$

(2) {5}

(3) $\{x / x + 1 = 1, x \in N\}$

(4) $\{x / x$ is the additive identity$\}$ or $\{x / x$ is a neutral demant for addition1$\}$

4. Which of the following are finite sets and infinite sets, write them :

(1) The set of citizens of India.

(2) The set of numbers of three digits more than 100.

(3) A = {x / x is a number whose unit digit is 7}

(4) {x / x is a prime number}

5. Classify the following sets are equal sets and equivalent set and write them in symbolic form :

(1) P = {a, b, c}, Q = {x, y, z}

(2) F = { }, G = {x / x is a four digit number less than 1000}

(3) A = {1, 4, 9, 16}, B = {x / x is a perfect square number less than 25}

(4) D = {p, q, r}, E = {r, q, p}

(5) A = {1, 2, 3}, B = {7, 8, 9}

6. If U = N and A = {1, 2, 3,..., 10}, then find A' and (A')'.

7. Write all one-to-one correspondence between the given two sets :

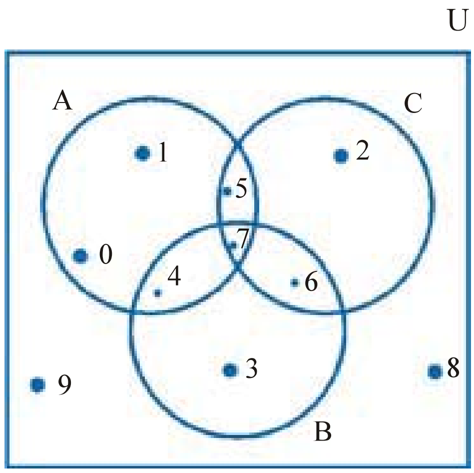
A = {x, y} and B = {a, b}

8. Draw Venn diagram for given sets and write union and intersection for each set :

(1) A = {x, y, z, w}, B = {a, b, c, x, y}

(2) S = {5, 10, 15}, R = {10, 15, 25, 20}

9. From the given Venn diagram find the following results :



(1) A ∩ B

(2) (A ∪ B) ∪ (B ∪ C)

(3) A ∩ (B ∪ C)

(4) (A ∪ C) ∩ B

(5) U

Answers

Practice 1

1. (1) ∈ (2) ∉ (3) ∉ (4) ∉ (5) ∉

Practice 2

1. (1) $\subset$ (2) $\not\subset$ (3) $\subset$ (4) $\subset$

2. (1) $3 \in A$ (2) $\frac{1}{2} \notin A$ (3) $A = \phi$ (4) $C = B$

(5) $C \subset D$ (6) $B \sim C$ (7) $A \not\subset B$ (8) $\{0\} \subset B$

3. (1) Equivalent sets, $A \sim B$

Practice 3

1. (1) A' = {January, February, March, April, May, June, July, August, September, October, November, December}

(2) R' = {Blue, Indigo, Green, Orange}

(3) A' = {1, 4, 6, 7, 8, 9}, (A')' = A, B' = {1, 2, 3, 6, 8, 9}, (B')' = B

2. (1) {0, 1, 2, 3, 4, 5, 8, 9} (2) {2, 3} (3) {2, 3, 4, 5, 8, 9}

(4) {0, 1, 2, ..., 9} (5) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

3. $A \cap B = \{4\}$, $A \cup B = \{1, 2, 3, 4, 5, 6\}$

Exercise

1. (1) $\in$ (2) $\subset$ (3) $\notin$ (4) $\notin$ (5) $\subset$ (6) $\not\subset$ (7) $\sim$

2. (1) **Listing method :** {5, 10, 15, ...}, **Property method :** {x / x is a multiple of 5}

(2) **Listing method :** {23, 29},

Property method : {x / x is a prime number between 21 and 30}

(3) **Listing method :** {1, 2, 3, 4, 5},

Property method : {x / x is positive integer less than 6}

(4) **Listing method :** {1, 3, 7, 21}, **Property method :** {x / x is a factor of 21}

3. (1) Singleton set (2) Singleton set (3) Empty set (4) Singleton set

4. (1) Finite set (2) Finite set (3) Infinite set (4) Infinite set

5. (1) $P \sim Q$ (2) $F = G$ (3) $A = B$ (4) $D = E$ (5) $A \sim B$

6. A' = {11, 12, 13,...}, (A')' = A

7. $x \leftrightarrow a$, $x \leftrightarrow b$, $y \leftrightarrow a$, $y \leftrightarrow b$

8. (1) $A \cup B = \{a, b, c, x, y, z, w\}$, $A \cap B = \{x, y\}$

(2) $S \cup R = \{5, 10, 15, 20, 25\}$, $S \cap R = \{10, 15\}$

9. (1) {4, 7} (2) {0, 1, 2, 3, 4, 5, 6, 7} (3) {4, 5, 7} (4) {4, 6, 7}

(5) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

5 Expansion

- **Let us remember :**
  - A polynomial having one term is called Monomial.
  - A polynomial having two terms is called Binomial.
  - A polynomial having three terms is called Trinomial.
- **Let us learn new :**
  - **Multiplication (product) of monomial with monomial :**

$2 \times 3 = 6$ similarly $a \times b = ab$

$2 \times 3x = 2 \times 3 \times x = 6x$

$5x \times 3y = 5 \times x \times 3 \times y = 5 \times 3 \times x \times y = 15xy$

$5x \times 3x^2 = 5 \times x \times 3 \times x^2 = 5 \times 3 \times x \times x^2 = 15x^3$

$3x \times (-5y) = 3 \times x \times (-5) \times y = 3 \times (-5) \times x \times y = (-15)xy$

$4x \times 6y^2 = 24xy^2$

Therefore, in multiplication (product) of two monomials their coefficients are multiplied with coefficients and variable is multiplied with variable. Hence here multiplication (product) of monomials with monomial is monomial only.

- **Multiplication (product) of monomial with binomial :**

In the earlier class we have learnt about distribution of multiplication over addition.

$a \times (b + c) = (a \times b) + (a \times c)$ (Distribution of multiplication over addition)

$a \times (b + c) = ab + ac$

Now, if $a = 2x,\ b = 3y$ and $c = 5z$, then

$2x \times (3y + 5z) = (2x \times 3y) + (2x \times 5z)$

$= 6xy + 10xz$

There by using distributive law, the product on left hand side is expressed in the form of addition on right hand side. This process is called expansion. i.e. The process of getting $ab + ac$ from $a(b + c)$ is expansion. **"To express the product of polynomials as a single polynomial means expansion."** The product of monomial with binomial is a binomial.

Example 1 : $a \times (5a - 6b)$

$= (a \times 5a) - (a \times 6b)$

$= 5a^2 - 6ab$

or

$= (a \times 5a) + [(a)(-6b)]$

$= 5a^2 + (-6ab)$

$= 5a^2 - 6ab$

Example 2 : $(2x - 4) \times (-3x)$

$= (-3x) \times (2x - 4)$

$= [(-3x) \times (2x)] - [(-3x) \times (4)]$

$= (-6x^2) - (-12x)$

$= (-6x^2) + 12x$

Example 3 : $2m(3m^2 + 5)$

$= (2m \times 3m^2) + (2m \times 5)$

$= 6m^3 + 10m$

- **Product (multiplication of binomial with binomial :**

To get product of binomial $(a + b)(c + d)$, both the terms of any one binomial is multiplied with other nonomial respectively.

$(a + b)(c + d)$

$= a(c + d) + b(c + d)$

$= ac + ad + bc + bd$

or

$(a + b)(c + d)$

$= (a + b)\ c + (a + b)d$

$= ac + bc + ad + bd$

- **Expand :**

Example 4 : $(x + y)(2y + 5)$

$= x(2y + 5) + y(2y + 5)$

$= 2xy + 5x + 2y^2 + 5y$ (No two terms are like terms)

Example 5 : $(2a + 3b)(5x - 3y)$

$= 2a(5x - 3y) + 3b(5x - 3y)$

$= 10ax - 6ay + 15bx - 9by$ (No two terms are like terms)

Example 6 : $(5a - 7b)(3a - 2b)$

$= 5a(3a - 2b) - 7b(3a - 2b)$

$= 15a^2 - \underline{10ab - 21ab} + 14b^2$ (Two terms are like terms)

$= 15a^2 - 31ab + 14b^2$

Example 7 : $(x^2 - 5)(x^2 + 3)$

$= x^2(x^2 + 3) - 5(x^2 + 3)$

$= x^4 + \underline{3x^2 - 5x^2} - 15$ (Two terms are like terms)

$= x^4 - 2x^2 - 15$

Therefore, product of a binomial with a binomial is a polynomial.

Example 8 : $(2x + 3y)(2x - 3y)$

$= 2x(2x - 3y) + 3y(2x - 3y)$

$= 4x^2 - \underline{6xy + 6xy} - 9y^2$ (Two terms are like terms)

$= 4x^2 - 9y^2$

We can see from the examples in expansion that, the product of a binomial with a binomial, get (1) Four terms **or** (2) Three terms **or** (3) Two terms.

Practice 1

1. Complete the given Table by multiplying monomial with monomial :

1st monomial → 2nd monomial ↓	$2x$	$-5y$	$3a^2$	$-4xy$	mn
$2x$	$4x^2$	$-10xy$			
$3y^2$			$9a^2y^2$		
$-2a$				$8axy$	
$3mn$		$-15mny$			
$5xy$				$-20x^2y^2$	

2. Expand :

(1) $2a(3x + 5y)$ (2) $3x^2(5x - 4y)$ (3) $5a(6a + 3b)$

(4) $(-6y)(5x - 7y^2)$ (5) $(-10a)(5a^2 + b)$ (6) $(-3ab)(2a + 3b)$

(7) $(6x^3y^3 - 1)(-2x^2)$ (8) $2ab(3ab - 1)$

3. Expand :

(1) $(2x + y)(a + 3b)$ (2) $(5x - 3)(2a + 5)$ (3) $(x - 2)(x^2 + 3)$

(4) $(5x^2 + 3)(2x^2 + 5)$ (5) $(x - 3)(x - 7)$ (6) $(2m^2 + 5)(3m + 1)$

(7) $(3x + 5)(2x - 4)$ (8) $(x - 3a)(4x + 5a)$

- **Expansion of square of binomial :**

$(a + b)^2 = (a + b) \times (a + b)$

$= a(a + b) + b(a + b)$

$= a^2 + ab + ba + b^2$

$= a^2 + ab + ab + b^2$ **($\because$ $ba = ab$)**

$\boldsymbol{(a + b)^2 = a^2 + 2ab + b^2}$

(F.T. = First Term, S.T. = Second Term)

In words,

(F.T. + S.T.)^2 = (F.T.)^2 + 2(F.T.)(S.T.) + (S.T.)^2

$(a - b)^2 = (a - b) \times (a - b)$

$= a(a - b) - b(a - b)$

$= a^2 - ab - ba + b^2$

$= a^2 - ab - ab + b^2$ **($\because$ $ba = ab$)**

$\boldsymbol{(a - b)^2 = a^2 - 2ab + b^2}$

In words (F.T. – S.T.)^2 = (F.T.)^2 – 2(F.T.)(S.T.) + (S.T.)^2

- **Expand :**

Example 8 : $(x + 3)^2$

$= (x)^2 + 2(x)(3) + (3)^2$

$= x^2 + 6x + 9$

Example 9 : $(a - 5)^2$

$= (a)^2 - 2(a)(5) + (5)^2$

$= a^2 - 10a + 25$

Example 10 : $(2x + 3y)^2$

$= (2x)^2 + 2(2x)(3y) + (3y)^2$

$= 4x^2 + 12xy + 9y^2$

Example 11 : $(2ab - y)^2$

$= (2ab)^2 - 2(2ab)(y) + (y)^2$

$= 4a^2b^2 - 4aby + y^2$

- **Find the values using expansion formula :**

Example 12 : $(12)^2$

$= (10 + 2)^2$

$= (10)^2 + 2(10)(2) + (2)^2$

$= 100 + 40 + 4$

$= 144$

Example 13 : $(34)^2$

$= (30 + 4)^2$

$= (30)^2 + 2(30)(4) + (4)^2$

$= 900 + 240 + 16$

$= 1156$

Example 14 : $(26)^2$

$= (30 - 4)^2$

$= (30)^2 - 2(30)(4) + (4)^2$

$= 900 - 240 + 16$

$= 676$

or $(26)^2$

$= (20 + 6)^2$

$= (20)^2 + 2(20)(6) + (6)^2$

$= 400 + 240 + 36$

$= 676$

Practice 2

1. Fill in the blanks :

(1) $(x - y)^2 = x^2 - 2xy +$ ....

(2) $(a + 7)^2 =$ .... $+ 14a + 49$

(3) $(2m - n)^2 = 4m^2 -$ .... $+ n^2$

(4) $(x + 1)^2 =$ ....

(5) $(2a - 3)^2 =$ ....

(6) $(m + 2)^2 =$ ....

2. Expand :

(1) $(x + 7)^2$

(2) $(m - 4)^2$

(3) $(x + 3y)^2$

(4) $(8x + 5y)^2$

(5) $(2a - 3b)^2$

(6) $(4ab - 3xy)^2$

(7) $(3xy - 7)^2$

(8) $(2xy - 3z)^2$

3. Find the value of the following using expansion formulae of $(a + b)^2$ or $(a - b)^2$:

(1) $(43)^2$ (2) $(82)^2$ (3) $(67)^2$ (4) $(48)^2$

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- Expansion of $(a + b)(a - b)$: $(a + b)(a - b)$

$= a(a - b) + b(a - b)$

$= a^2 - ab + ab - b^2$

$= a^2 - b^2$

$$(a + b)(a - b) = a^2 - b^2$$

In words : (F.T. + S.T.) (F.T. − S.T.) = $(\text{F.T.})^2 - (\text{S.T.})^2$

- **Expand :**

Example 15 : $(x + 5)(x - 5)$

$= (x)^2 - (5)^2$

$= x^2 - 25$

Example 16 : $(2a - 3b)(2a + 3b)$

$= (2a)^2 - (3b)^2$

$= 4a^2 - 9b^2$

Example 17 : Find the value of 22×18 using expansion formula.

$22 \times 18 = (20 + 2)(20 - 2)$

$= (20)^2 - (2)^2$

$= 400 - 4$

$= 396$

Practice 3

1. Make pairs :

	Section 'A'	Section 'B'
(1)	$(m + n)(m - n)$	(a) $m^2 - 49$
(2)	$(m + 7)(m - 7)$	(b) $25 - m^2$
(3)	$(5 + m)(5 - m)$	(c) $9m^2 - 1$
(4)	$(3m - 1)(3m + 1)$	(d) $m^2 - n^2$

2. Expand :

(1) $(4x + 1)(4x - 1)$ (2) $(3x - 7y)(3x + 7y)$

(3) $(6 - x)(6 + x)$ (4) $(a + 8b)(a - 8b)$

(5) $(11 + 3xy)(11 - 3xy)$ (6) $(2mn + 5)(2mn - 5)$

3. Find values using expansion formula :

(1) 41×39 (2) 56×64 (3) 73×67 (4) 33×27

*

- **Expansion of $(x + a)(x + b)$:**

$$(x + a)(x + b) = x(x + b) + a(x + b)$$
$$= x^2 + xb + ax + ab$$
$$= x^2 + ax + bx + ab$$
$$\therefore \quad (x + a)(x + b) = x^2 + (a + b)x + ab$$

- **Expand :**

Example 18 : $(x + 3)(x + 2)$

$= (x)^2 + (3 + 2)(x) + (3)(2)$

$= x^2 + 5x + 6$

Example 19 : $(x + 3)(x - 5)$

$= (x)^2 + (3 - 5)(x) + (3)(-5)$

$= x^2 - 2x - 15$

Example 20 : $(x + 8)(x - 3)$

$= (x)^2 + (8 - 3)(x) + (8)(-3)$

$= x^2 + 5x - 24$

Example 21 : $(x - 4)(x - 3)$

$= (x)^2 + (-4 - 3)(x) + (-4)(-3)$

$= x^2 - 7x + 12$

Example 22 : $(3x + 5y)(3x - 2y)$

$= (3x)^2 + (5y - 2y)(3x) + (5y)(-2y)$

$= 9x^2 + (3y)(3x) + (-10y^2)$

$= 9x^2 + 9xy - 10y^2$

- **Find the values using expansion formula :**

Example 23 : 26×32

$= (30 - 4)(30 + 2)$

$= (30)^2 + (-4 + 2)(30) + (-4)(2)$

$= 900 + (-2)(30) + (-8)$

$= 900 - 60 - 8$

$= 832$

Example 24 : 35×33

$= (30 + 5)(30 + 3)$

$= (30)^2 + (5 + 3)(30) + (5)(3)$

$= 900 + (8)(30) + (15)$

$= 900 + 240 + 15$

$= 1155$

Practice 4

1. Expand :

(1) $(y + 2)(y + 4)$ (2) $(m + 6)(m - 2)$

(3) $(2a - 5)(2a + 3)$ (4) $(4x - 2y)(4x + y)$

(5) $(a - 3b)(a + 2b)$ (6) $(5ab - 3)(5ab + 2)$

(7) $(6x + 3)(6x + 5)$ (8) $(7a + 4)(7a + 3)$

2. Find the values using expansion formula :

(1) 43×42 (2) 68×73 (2) 52×51 (2) 24×19 (2) 23×18 (2) 27×32

Exercise

1. Multiply (find product) :

(1) $2a(-3a^2)$ (2) $(-4ab)(6a^2b)$

(3) $(2m)(3m + n)$ (4) $(-4n)(6n + 5m)$

(5) $(5a + 3b)(6a - 2b)$ (6) $(2x + 3y)(6x - 2y)$

(7) $(6xy + 1)(2xy - 3)$ (8) $(a - 2b)(2a - b)$

2. Expand :

(1) $(a + 5)^2$ (2) $(m - 7)^2$

(3) $(3m + 2n)^2$ (4) $(4xy - 3)^2$

(5) $(5ab + 3c)^2$ (6) $(4b^2 + 3)^2$

(7) $(2x - 7)^2$ (8) $(5 - 3mn)^2$

3. Expand :

(1) $(x - 7)(x + 7)$ (2) $(2a + 3b)(2a - 3b)$

(3) $(2m + 5)(2m - 5)$ (4) $(2mn + 3)(2mn - 3)$

4. Expand :

(1) $(a + 3)(a + 2)$ (2) $(m - 2)(m - 5)$

(3) $(x - 9)(x + 2)$ (4) $(x + 6y)(x - 2y)$

(5) $(5x - 2y)(5x - 4y)$ (6) $(2m + 3n)(2m + 5n)$

(7) $(xy - 7)(xy + 4)$ (8) $(x^2 - 5)(x^2 + 3)$

5. Find the values using expansion formula :

(1) 62^2 (2) 57^2 (3) 43×37

(4) 97×103 (5) 16×22

What did we learn ?

- Product of monomial with monomial.
- Product of monomial with binomial.
- Product of binomial with binomial.
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$
- $(x + a)(x + b) = x^2 + (a + b)(x) + ab$

Answers

Practice 1

1. (1) $6ax + 10ay$ (2) $15x^3 - 12x^2y$ (3) $30a^2 + 15ab$

(4) $-30xy + 42y^3$ (5) $-50a^3 - 10ab$ (6) $-6a^2b - 9ab^2$

(7) $-12x^5y^3 + 2x^2$ (8) $6a^2b^2 - 2ab$

2.

1st Monomial → / 2nd Monomial ↓	$2x$	$-5y$	$3a^2$	$-4xy$	mn
$2x$	$4x^2$	$-10xy$	$6xa^2$	$-8x^2y$	$2xmn$
$3y^2$	$6xy^2$	$-15y^3$	$9a^2y^2$	$-12xy^3$	$3y^2mn$
$-2a$	$-4ax$	$10ay$	$-6a^3$	$8axy$	$-2amn$
$3mn$	$6mnx$	$-15mny$	$9a^2mn$	$-12xymn$	$3m^2n^2$
$5xy$	$10x^2y$	$-25xy^2$	$15a^2xy$	$-20x^2y^2$	$5xymn$

3. (1) $2xa + 6xb + ya + 3yb$ (2) $10xa + 25x - 6a - 15$ (3) $x^3 + 3x - 2x^2 - 6$
(4) $10x^4 + 31x^2 + 15$ (5) $x^2 - 10x + 21$ (6) $6m^3 + 2m^2 + 15m + 5$
(7) $6x^2 - 2x - 20$ (8) $4x^2 - 7xa - 15a^2$

Practice 2

1. (1) y^2 (2) a^2 (3) $4mn$
(4) $x^2 + 2x + 1$ (5) $4a^2 - 12a + 9$ (6) $m^2 + 4m + 4$

2. (1) $x^2 + 14x + 49$ (2) $m^2 - 8m + 16$
(3) $x^2 + 6xy + 9y^2$ (4) $64x^2 + 80xy + 25y^2$
(5) $4a^2 - 12ab + 9b^2$ (6) $16a^2b^2 - 24abxy + 9x^2y^2$
(7) $9x^2y^2 - 42xy + 49$ (8) $4x^2y^2 - 12xyz + 9z^2$

3. (1) 1849 (2) 6724 (3) 4489 (4) 2304

Practice 3

1. (1) d (2) a (3) b (4) c

2. (1) $16x^2 - 1$ (2) $9x^2 - 49y^2$ (3) $36 - x^2$
(4) $a^2 - 64b^2$ (5) $121 - 9x^2y^2$ (6) $4m^2n^2 - 25$

3. (1) 1599 (2) 3584 (3) 4891 (4) 891

Practice 4

1. (1) $y^2 + 6y + 8$ (2) $m^2 + 4m - 12$ (3) $4a^2 - 4a - 15$
(4) $16x^2 - 4xy - 2y^2$ (5) $a^2 - ab - 6b^2$ (6) $25a^2b^2 - 5ab - 6$
(7) $36x^2 + 48x + 15$ (8) $49a^2 + 49a + 12$

2. (1) 1806 (2) 4964 (3) 2652 (4) 456 (5) 414 (6) 864

5 : Expansion

Exercise

1. (1) $-6a^3$ (2) $-24a^3b^2$ (3) $6m^2 + 2mn$
(4) $-24n^2 - 20mn$ (5) $30a^2 + 8ab - 6b^2$ (6) $12x^2 + 14xy - 6y^2$
(7) $12x^2y^2 - 16xy - 3$ (8) $2a^2 - 5ab + 2b^2$

2. (1) $a^2 + 10a + 25$ (2) $m^2 - 14m + 49$ (3) $9m^2 + 12mn + 4n^2$
(4) $16x^2y^2 - 24xy + 9$ (5) $25a^2b^2 + 30abc + 9c^2$
(6) $16b^4 + 24b^2 + 9$ (7) $4x^2 - 28x + 49$ (8) $25 - 30mn + 9m^2n^2$

3. (1) $x^2 - 49$ (2) $4a^2 - 9b^2$ (3) $4m^2 - 25$ (4) $4m^2n^2 - 9$

4. (1) $a^2 + 5a + 6$ (2) $m^2 - 7m + 10$ (3) $x^2 - 7x - 18$
(4) $x^2 + 4xy - 12y^2$ (5) $25x^2 - 30xy + 8y^2$ (6) $4m^2 + 16mn + 15n^2$
(7) $x^2y^2 - 3xy - 28$ (8) $x^4 - 2x^2 - 15$

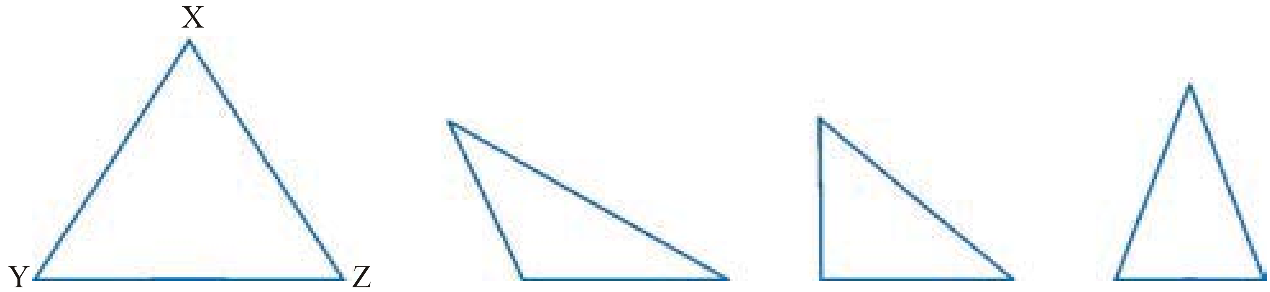
5. (1) 3844 (2) 3249 (3) 1591 (4) 9991 (5) 352

Relations between units :

- 1 inch = 2.54 cm
  1 foot = 30.48 cm
  1 foot = 12 inch
  3 feet = 1 bar = 36 inch
  1 metre = 39.37 inch = 3.280 feet = 1.0936 bar
- 100 m^2 = 1 acre
  100 acre = 1 hectare
  100 hectore = 1 sq km
- 9 sq feet = 1 sq bar
  121 sq. bar = 1 Guntha
  40 guntha = 1 Acre = 4840 sq bar
- 1 cubic foot = 1728 cubic inch
  1 cubic bar = 27 cubic feet
  100 cubic feet = 1 brass
- 1 gallon = 4.546 litre
  1 litre = 0.22 gallon
- 1 qusek = 1 cubic foot/second
  = 6.25 cubic gallon/second

6 Quadrilateral

◆ **Let us remember :**



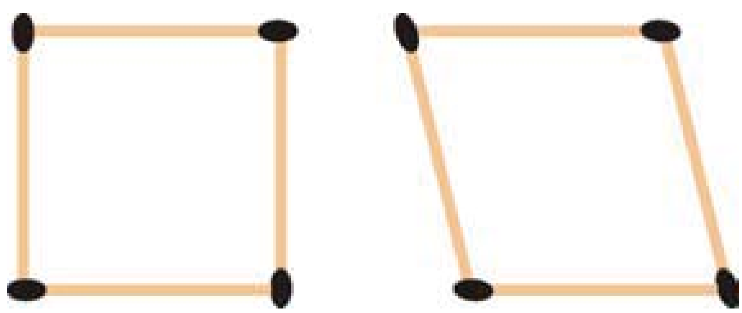
(1) (2) (3) (4)

- **Complete the Table by naming the each triangle with different names shown in the figure :**

No.	Vertices of Triangle	Name of Triangle	Angles	Sides
(1)	X, Y and Z	ΔXYZ	$\angle XYZ$ $\angle YZX$ $\angle ZXY$	$\overline{XY}$, $\overline{YZ}$, $\overline{ZX}$
(2)				
(3)				
(4)				

Activity 1 : Make such shapes that by taking four match-sticks closed figures are obtained.

- Keep it in mind that two black ends of match-sticks do not touch each other

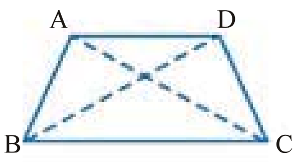


Now, think how many sides are formed ?

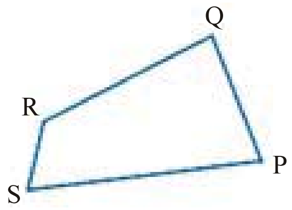
How many angles are formed ?

Let us learn new :

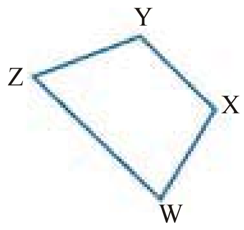
- **Examining the following figures, answer the given questions :**



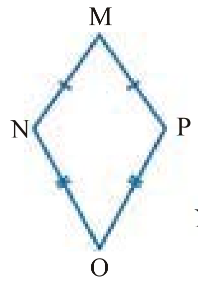
(5)



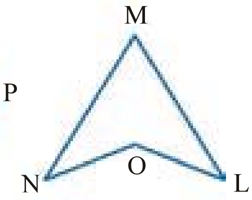
(6)



(7)



(8)



(9)

Join the opposite vertices in figures from (5) and (9) and write the line-segment so formed :

No.	(5)	(6)	(7)	(8)	(9)
Line-segment	$\overline{AC}$, $\overline{BD}$				

(1) How many vertices are there in above each figure ? ....

(2) In the above each figure are there any three vertices in only one line ? ....

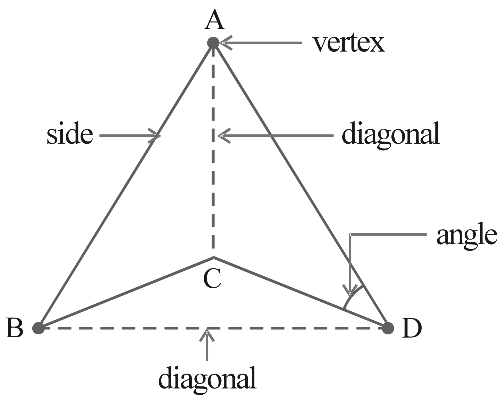
(3) How many sides are there in each figure ? ....

(4) How many angles are there in each figure ? ....

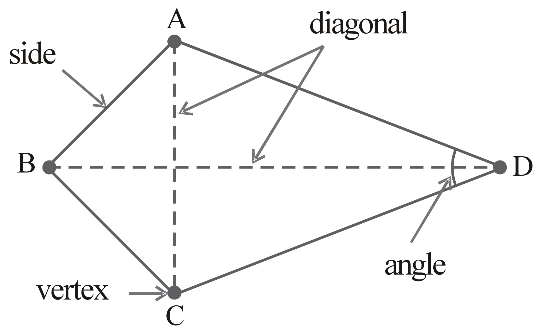
(5) .... is the figure in which line-segments intersect.

(6) .... is the figure in which line-segments do not intersect.

- **Examine the following figures and understand :**



(10)



(11)

- **Understand :**

In the above figure (10) and (11)

- A, B, C, D are the vertices of a quadrilateral.

- $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$ are the sides of quadrilateral.
- ∠ABC, ∠BCD, ∠CDA, ∠DAB are the angles.
- $\overline{AC}$ and $\overline{BD}$ are their diagonals.
- **A closed figure made by four line-segments having four angles and not intersecting each other at any point except the end points is a quadrilateral.**

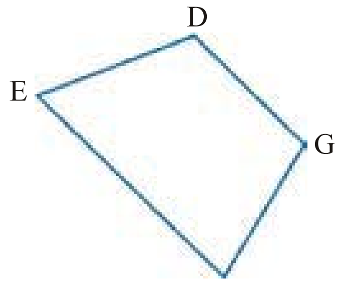
□ ABCD can be written as in set form as under :

□ ABCD = $\overline{AB} \cup \overline{BC} \cup \overline{CD} \cup \overline{DA}$

- Therefore, a quadrilateral is the union of four line-segments.
- Each line-segment which is formed by connecting opposite vertices of a quadrilateral is called a diagonal.
- If the diagonals of a quadrilateral intersect each other then that quadrilateral is called a convex quadrilateral.
- If the diagonals of a quadrilateral do not intersect each other then that quadrilateral is called a concave quadrilateral.
- Each quadrilateral contains four sides, four angles and two diagonals.

Here, we will discuss about the convex quadrilateral.

In the figure, D, E, F, G are the vertices of a quadrilateral, therefore, its name is given quadrilateral DEFG. Symbolically it is written as □ DEFG.

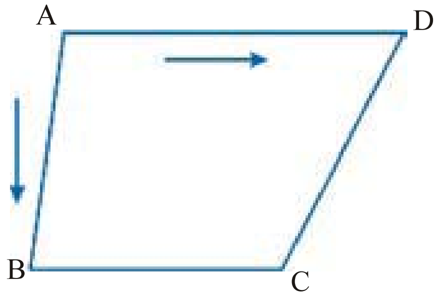


(12)

Read as : Quadrilateral DEFG

- **Naming of quadrilateral :**

□ ABCD
□ BCDA
□ CDAB
□ DABC



(13)

□ ADCB
□ DCBA
□ CBAD
□ BADC

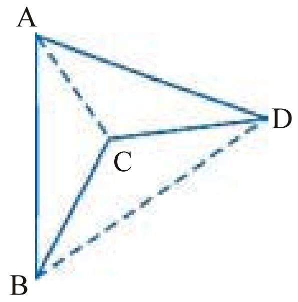
- A quadrilateral can be given name by two methods as clockwise and anticlockwise.
- Fill the details of the Table on the basis of above figures (12) and (13) :

Figures	Sides	Angles	Diagonals	Diagonals intersect ?
(12)				
(13)				

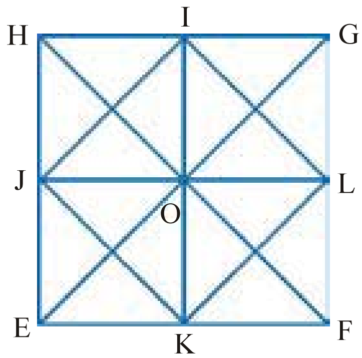
6 : Quadrilateral

Remember the following :

- A quadrilateral is formed with four coplanar points.
- Among four distinct coplanar points no three are colinear in a quadrilateral.
- A quadrilateral in which diagonals do not intersect is called a concave quadrilateral according to given figure.
- Here, $\overline{AC}$ and $\overline{BD}$ are diagonals.

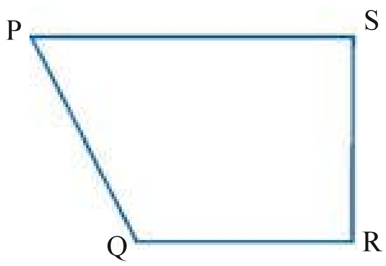


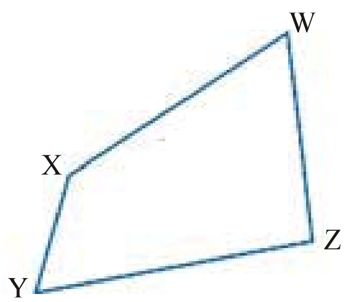
1. By studying the given figure, write all the possible quadrilaterals formed in your notebook. Out of these write informations of any three in the table :



No.	Name of quadrilateral	Sides	Angles	Diagonals
1.	□EFGH	$\overline{EF}$, $\overline{FG}$, $\overline{GH}$, $\overline{HE}$	∠EFG, ∠FGH, ∠GHE, ∠HEF	$\overline{FH}$, $\overline{EG}$

2. Write all the possible methods of writing names of the following quadrilaterals :





3. Draw and give names of such a quadrilateral whose vertices are S, T, U and V.

*

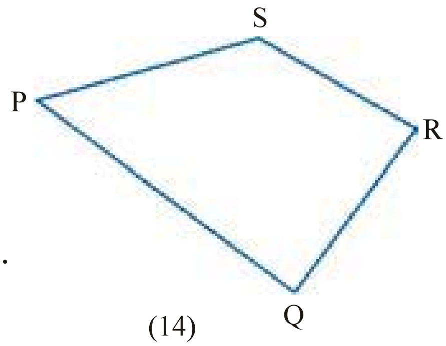
The opposite sides and opposite angles, adjacent sides and the adjacent angles of a quadrilateral :

In figure (14), $\overline{PQ}$ and $\overline{RS}$ are opposite sides.

$\overline{QR}$ and $\overline{PS}$ are opposite sides.

$\angle P$ and $\angle R$ are opposite angles.

$\angle Q$ and $\angle S$ are opposite angles.



(14)

Therefore, two pairs of opposite sides are obtained. Similarly two pairs of opposite angles are obtained.

Now, The adjacent sides of $\overline{PQ}$ are $\overline{PS}$ and $\overline{QR}$.
The adjacent sides of $\overline{QR}$ are $\overline{PQ}$ and $\overline{RS}$.
The adjacent sides of $\overline{RS}$ are $\overline{QR}$ and $\overline{SP}$.
The adjacent sides of $\overline{PS}$ are $\overline{PQ}$ and $\overline{RS}$.

The adjacent angles of $\angle P$ are $\angle Q$ and $\angle S$.
The adjacent angles of $\angle Q$ are $\angle P$ and $\angle R$.
The adjacent angles of $\angle R$ are $\angle Q$ and $\angle S$.
The adjacent angles of $\angle S$ are $\angle P$ and $\angle R$.

Therefore, four pairs of adjacent sides are obtained. Similarly four pairs of adjacent angles are obtained.

Practice 2

Fill in the following blanks on the basis of given figure :

(1) .... is the opposite side of $\overline{AB}$.

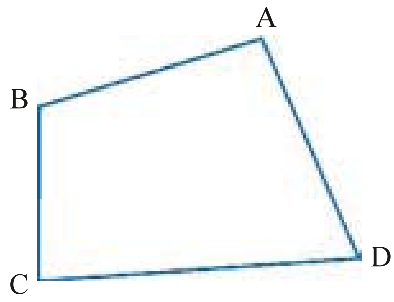
(2) $\overline{AD}$ is the opposite side of ....

(3) The adjacent sides of $\overline{AB}$ are .... and ....

(4) .... and .... are the adjacent sides of $\overline{BC}$.

(5) The adjacent sides of $\overline{CD}$ are .... and ....

(6) .... and .... are the adjacent sides of $\overline{DA}$.



(15)

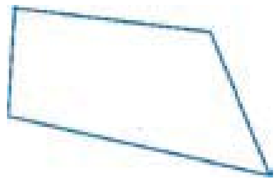
(7) Write opposite angle of $\angle A$.

(8) Which are the adjacent angles of $\angle D$?

(9) Which are the adjacent angles of $\angle A$?

(10) $\angle A$ and $\angle C$ are the adjacent angles of which angle ?

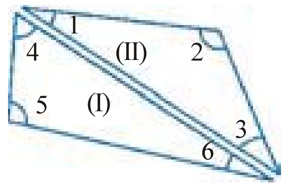
- **Activity 2 :**
  - First of all take a quadriangular paper of any measure.



  - Make the triangular from quadriangular shape by folding from the mid of dotted line.

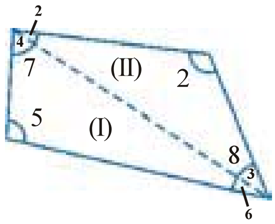
  - Measure all the three angles of triangular region respectively by protector.
  - After measuring the angles of both triangular regions, open the fold.

    Now, there will be four angles of quadrilateral instead of six angles of two triangles.



  - $m\angle 1 + m\angle 4 = m\angle 7$ and

    $m\angle 3 + m\angle 6 = m\angle 8$.



  - Now find the sum of measures of four angles of quadrilateral :

    $m\angle 7 + m\angle 2 + m\angle 8 + m\angle 5 =$ ....

Sum of measures of four angles of quadrilateral	=	Sum of measures of 1st triangular region	+	Sum of measures of angles of 2nd triangular region
	=	180°	+	180°
	=	360°		

- **Activity 3 :** Draw different types of quadrilaterals in chart paper. Cut with the help of scissors. Measure all four angles of each quadrilateral with the help of protractor and write in Table :

Name of quadrilateral	measure of $\angle 1$	measure of $\angle 2$	measure of $\angle 3$	measure of $\angle 4$	$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4$

Problems on the basis of measures of angles of quadrilateral :

Example 1 : The measures of three angles of a quadrilateral are 85°, 35° and 160° respectively. Find measure of its fourth angle.

The sum of measures of three angles of quadrilateral :

$85^\circ + 35^\circ + 160^\circ = 280^\circ$

The measures of all four angles of a quadrilateral is 360°.

Measure of fourth angle $= 360^\circ - 280^\circ = 80^\circ$

$\therefore$ The measure of fourth angle is 80°.

or

$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360^\circ$

$\therefore \quad 85^\circ + 35^\circ + 160^\circ + m\angle 4 = 360^\circ$

$\therefore \quad 280^\circ + m\angle 4 = 360^\circ$

$\therefore \quad m\angle 4 = 360^\circ - 280^\circ$

$\therefore \quad m\angle 4 = 80^\circ$

Example 2 : The measure of three angles of a quadrilateral are equal. If measure of each angle is 95°, then find measure of its fourth angle.

Suppose □ABCD is a quadrilateral.

$\therefore \quad m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$

$\therefore \quad 95^\circ + 95^\circ + 95^\circ + m\angle D = 360^\circ$

$\therefore \quad 285^\circ + m\angle D = 360^\circ$

$\therefore \quad m\angle D = 360^\circ - 285^\circ$

$\therefore \quad m\angle D = 75^\circ$

$\therefore$ The measure of fourth angle is 75°.

Example 3 : The measure of three angles of a quadrilateral are equal. If measure of fourth angle is 105°, then find the measure of each equal angle.

The sum measures of all three equal angles

$= 360^\circ$ − measure of fourth angle

$= 360^\circ - 105^\circ$

$= 255^\circ$

The measure of each equal angle $= \frac{255}{3} = 85$

The measure of each equal angle $= 85^\circ$

or

Suppose the measures of three equal angles is x°. Now,

$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360^\circ$

$\therefore \quad x^\circ + x^\circ + x^\circ + 105^\circ = 360^\circ$

$\therefore \quad 3x^\circ + 105^\circ = 360^\circ$

$\therefore \quad 3x^\circ = 360^\circ - 105^\circ = 255^\circ$

$\therefore \quad x^\circ = \frac{255}{3} = 85^\circ$

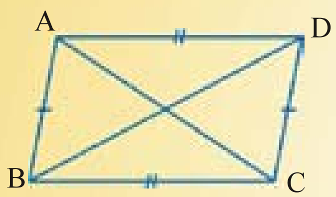
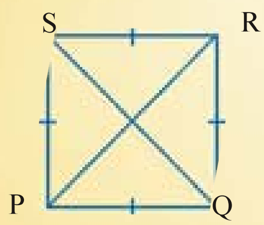
Practice 3

1. The measure of three angles of a quadrilateral are 75°, 65° and 120° respectively, then find the measure of fourth angle of the quadrilateral.

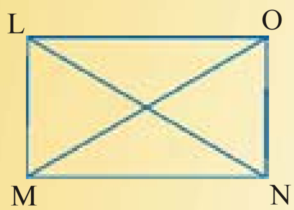
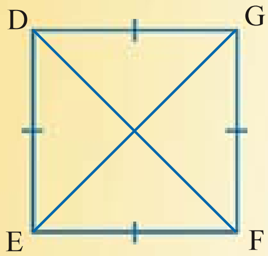
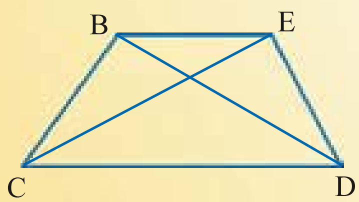
2. The measure of two adjacent angles of a quadrilateral are 80° and 100°. If remaining angles have equal measure then find the measures of equal angles.
3. In □ MNOP, $m\angle N$ is 10° more, $m\angle O$ is 20° more, and $m\angle P$ is 30° more than $m\angle M$, then find the measures of all four angles.
4. In □ DEFG, $m\angle D = 120^\circ$ and $m\angle F = 140^\circ$. If $m\angle E = m\angle G$, then find the measures of both angles.
5. If the measures of all four angles of a quadrilateral are equal, then find the measure of each angle.
6. One angle of a quadrilateral is a right angle and the measure of other angle is 110°. If remaining two angles are of equal measures, then find the measures of each equal angle.

- **Types of quadrilateral :**

See and understand :

No.	Figure and name	Definition	Characteristics
1.	**Parallelogram** 	If both the pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is called a parallelogram. $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{CD}$	• Diagonals bisect each other. • Diagonals are not of equal measures. • The measures of opposite sides are equal. • The measures of opposite angles are equal.
2.	**Rhombus** 	If all the sides of a parallelogram are equal, then it is called a rhombus. $\overline{QR} \parallel \overline{PS}$ and $\overline{PQ} \parallel \overline{SR}$ QR = RS = SP = PQ	• Diagonals are not of equal measurement. • Diagonals bisect each other at right angles.

6 : Quadrilateral

No.	Figure and Name	Definition	Characteristics
3.	**Rectangle** 	If all the angles of a parallelogram are right angles, then the parallelogram is called a rectangle. $\overline{LO} \parallel \overline{MN}$, $\overline{LM} \parallel \overline{ON}$ $m\angle L = m\angle M = m\angle N = m\angle O = 90^\circ$	• Diagonals are equal in measures. • Diagonals bisect each other.
4.	**Square** 	If the measures of four angles and four sides are equal, then that parallelogram is called a square. $\overline{DG} \parallel \overline{EF}$, $\overline{DE} \parallel \overline{GF}$ DG = GF = FE = ED $m\angle D = m\angle E = m\angle F = m\angle G = 90^\circ$	• The measures of diagonals are equal. • Diagonals bisect each other at right angles.
5.	**Trapezium** 	If in a quadrilateral one and only one pair of opposite sides are parallel then such quadrilateral is called a trapezium. $\overline{BE} \parallel \overline{CD}$	• The measures of diagonals are not equal. • Diagonals do not bisect each other.

Remember :

- The name of a quadrilateral can be given by two ways : clockwise and anti-clockwise.
- The quadrilateral name can be written by starting with any vertex.
- Each quadrilateral has two pairs of opposite sides and two pairs of opposite angles.
- Each quadrilateral has four pairs of adjacent sides and four pairs of adjacent angles.
- The sum of the measures of all our angles of any quadrilateral is 360°.

Therefore, for □ABCD, $m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$

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Exercise

1. Draw such a quadrilateral whose diagonals are $\overline{DG}$ and $\overline{EF}$. Name it. How many types can it be named ? Write all the names.
2. Make figure of □ XYZW and write its all pairs of adjacent sides and adjacent angles.
3. ∠S is a right angle in □ STUV. If $m\angle T = m\angle U$ and $m\angle V = 80^\circ$, then find $m\angle T$ and $m\angle U$.
4. In □ PQRS, $m\angle Q$ is 10° less, $m\angle R$ is 20° less and $m\angle S$ is 30° less, then $m\angle P$, then find measure of each angle.
5. The measure of an angle of a quadrilateral is 120°. The measures of remaining angles are equal, then find the measure of each angle.
6. Answer the following questions :
   (1) Write the number of sides, angles and diagonals of a quadrilateral.
   (2) What is the sum of measures of four angles of quadrilateral ?
   (3) What is the sum of measures of three angles of a square ?
   (4) Which quadrilaterals have equal measures of four sides ?

Answers

Practice 2

(1) $\overline{CD}$ (2) $\overline{BC}$ (3) $\overline{BC}$, $\overline{AD}$ (4) $\overline{AB}$, $\overline{CD}$ (5) $\overline{BC}$, $\overline{AD}$
(6) $\overline{AB}$, $\overline{CD}$ (7) ∠C (8) ∠A, ∠C (9) ∠B, ∠D (10) ∠B and ∠D

Practice 3

(1) 100° (2) 90°, 90° (3) 75°, 85°, 95°, 105°
(4) 50°, 50° (5) 90°, 90°, 90°, 90° (6) 80°, 80°

Exercise

1. □ DEGF, □ EGFD, □ GFDE, □ FDEG, □ DFGE, □ FGED, □ GEDF, □ EDFG
2. (1) $\overline{XY}$ and $\overline{ZW}$ (2) $\overline{YZ}$ and $\overline{XW}$
   (3) The adjacent angles of ∠X are ∠W and ∠Y. The adjacent angles of ∠Y are ∠X and ∠Z. The adjacent angles of ∠Z are ∠Y and ∠W. The adjacent angles of ∠W are ∠Z and ∠X.
3. $m\angle T = 95^\circ$, $m\angle U = 95^\circ$ **4.** 105°, 95°, 85°, 75° **5.** 80°, 80°, 80°
6. (1) 4, 4, 2 (2) 360° (3) 270° (4) Square, Rhombus

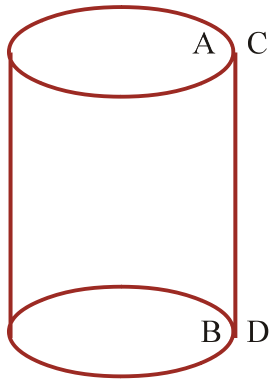
7 Area and Volume of Cylinder

- **Let us learn new :**

- **Activity 1 :**

Dear children, take a rectangular piece of paper as shown in the figure. Stick side $\overline{AB}$ of this rectangle with the side $\overline{CD}$ such that a figure shown below is obtained.

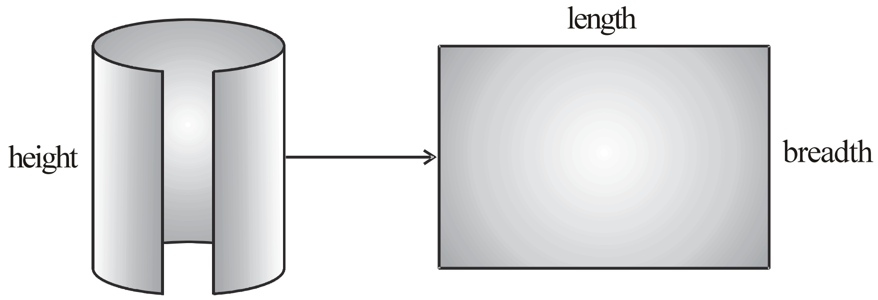
Above and below the surface of this figure are circular while the middle surface is curved surface. Water pipe, grainary tank, power box, drum, *Bhunglu*, etc. have the same type of shape. This shape is known as cylinder. Two ends of cylinder are circular, while other surfaces are curved which is known as curved (lateral) surface of cylinder.



Curved surface area of cylinder :

- **Activity 2 :**

  - Take a tin, metal or plastic, scissors and a paper.
  - Wrap the paper all around the tin.
  - Mark with pencil on the paper which is totally wrapped to cylinder.
  - Cut the extra paper with the help of scissors.
  - Now take the paper from cylinder and open it as shown in the figure.



The wrapped paper on cylinder is the curved surface area of cylinder. If you want to find the curved surface area of cylinder, then you will have to find the area of wrapped paper.

- Curved surface area of cylinder on the basis of figure = Area of rectangular paper.
- Now breadth of rectangular paper becomes the height of cylinder and length of rectangular paper becomes the circumference of circular part.
- Let's see the above reality in the formula :

Curved surface of cylinder = Area of rectangular paper

= Length × Breadth

= Circumference × Height

= $2\pi r \times h$

= $2\pi rh$

We know that circumference = $\pi \times$ diameter = πd

Curved surface area of cylinder = $\pi d \times h = \pi dh$

Curved surface area of cylinder = $2\pi rh = \pi dh$

- An open cylinderical thing with base which is closed from one side.

For example, water filling tin is such type of thing in which there are two surfaces : (1) circular base and (2) curved surface.

Total surface area of open cylinder with base

= Curved surface area + Area of base

= $2\pi rh + \pi r^2$

= $\pi r(2h + r)$

Therefore, total surface area of open cylinder with base = $\pi r(2h + r)$

- Such a cylinderical thing which is closed on both sides (base with cover)

  For example : A tin of grain have (1) circular base (2) curved surface and (3) circular top, such three surfaces.

- Total surface area of closed cylinder :

  = Curved surface area + Area of base + Area of top

  $= 2\pi rh + \pi r^2 + \pi r^2$

  $= 2\pi rh + 2\pi r^2$

  $= 2\pi r(h + r)$

Total surface area of closed cylinder = $2\pi r(h + r)$

- **Keep in mind :**

- 1 metre = 100 cm, 1 sq metre = 10000 sq cm

- We take value of π as $\frac{22}{7}$. The value of π up to two decimal places is 3.14 approximately. In this chapter if value of π is not given then take value of π as $\frac{22}{7}$.

Example 1 : The radius of base is 14 metre and height is 20 metre of a cylinder. Find curved surface area of this cylinder.

Radius of cylinder r = 14 m, Height of cylinder h = 20 m

Curved surface area of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 14 \times 20$

$= 1760 \text{ m}^2$

$\therefore$ **Curved surface area of cylinder = 1760 m^2**

Example 2 : The radius of base of a cylinder is 10 cm and its height is 40 cm. Find curved surface area of cylinder. (π = 3.14)

Radius of cylinder (r) = 10 cm, Height of cylinder (h) = 40 cm

Curved surface area of cylinder $= 2\pi rh$

$= 2 \times 3.14 \times 10 \times 40$

$= 2 \times \frac{314}{100} \times 10 \times 40$

$= 2512$ sq cm

∴ **Curved surface area of cylinder = 2512 m^2**

Practice 1

1. The radius of base of a cylinder is 7 cm and height is 10 cm, then find the curved surface area of this cylinder.
2. The radius of base of a cylinder is 3.5 cm and height is 40 cm, then find the curved surface area of cylinder.
3. The diameter of base of a cylinder is 50 cm and height is 20 cm, then find curved surface area of cylinder. (Take $\pi = 3.14$)
4. The radius of the base of cylinder is 20 cm and height is 30 cm, then find the curved surface area of cylinder. (Take $\pi = 3.14$)
5. The diameter of a cylinder is 28 cm and height is 10 cm. How much will be the curved surface area of this cylinder ?

*

Example 3 : The diameter of a cylinderical chinmey of iron is 2 m and its height is 7 m. How much will be the cost of painting the chimney from outside at the rate of ₹ 160 per m^2 ?

Radius of chimney $r = \frac{d}{2} = \frac{2}{2} = 1$ m, height $h = 7$ m

Curved surface area of cylindrical chimney $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 1 \times 7 = 44\ m^2$

Cost of painting 1 m^2 = ₹ 160

∴ Cost of painting 44 m^2 = 44 × 160 = ₹ 7040

The cost of painting the chimney from outside will be ₹ 7040.

Example 4 : The radius of cylinder having base is 7 cm and its height is 50 cm, then how much will be its area ?

Radius of base of cylinder $r = 7$ cm, Height of cylinder $h = 50$ cm

Here, base of cylinder is circular suface.

Area of base $= \pi r^2$

$= \frac{22}{7} \times 7 \times 7 = 154$ sq cm

Curved surface area of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 50$

$= 2200$ sq cm

Curved surface area of cylinder with base

= Area of base + Curved surface area

= 154 sq cm + 2200 sq cm

= 2354 sq cm

or

Second method :

Curved surface area of cylinder with base $= 2\pi rh + \pi r^2$

$= \pi r(2h + r)$

$= \frac{22}{7} \times 7(2 \times 50 + 7)$

$= \frac{22}{7} \times 7(100 + 7)$

$= 22 \times 107$

$= 2354\ cm^2$

$\therefore$ **Curved surface area of cylinder with base is 2354 cm^2.**

Example 5 : The diameter of a grain filling cylindrical tin with cap is 100 cm and its height is 2.5 m. Find the total surface area of this tin in sq cm. ($\pi = 3.14$)

Radius of cylinder $r = \frac{\text{diameter}}{2} = \frac{100}{2} = 50$ cm $= 0.50$ m

Height $h = 2.5$ m

Total surface area of cylinder with cap $= 2\pi r(h + r)$

$= 2 \times 3.14 \times 0.50\ (2.5 + 0.50)$

$= 3.14 \times 3$

$= 9.42$ sq m

Total surface area of tin = 9.42 sq m

Practice 2

1. There is a cylindrical platform of 2 m radius 50 cm height in school playground. What will be the cost of white washing the curved surface of this platform at the rate of ₹ 1.25 per 100 sq cm. ($\pi = 3.14$)
2. The radius of a cylindrical tank of oil without cap is 1.40 m and height is 2.3 m. What will be the cost of painting this tank from outside at the rate of ₹ 160 per m ?
3. The length of a roller of levelling the soil is 91 cm and the radius of its circular part is 30 cm. This roller rolls 100 revolutions on the soil, then how much sq metre of soil is levelled ?
4. The diameter of base of a chimney of kiln is 80 cm and height is 12.5 m. What will be the cost of painting this chimney from outside at the rate of ₹ 140 per sq m ? ($\pi = 3.14$)
5. The radius of cylindrical tank with cap is 2.1 m and height is 2.9 m. Find its total surface area.
6. How much sq metre of sheet is required to make 50 cylinders of 20 cm height and diameter of 14 cm ? If cost of 1 sq m cost of sheet is ₹ 200 then how much will be expenditure ?

*

- **Volume of cylinder :**

  - The surface of one rupee coin is circular.
  - By arranging coins on this rupee coin we get a pile of coin. If we see carefully the shape of this pile seems like a cylinder.
  - The pile of coins occupies (covers) space on the surface of table, additionally occupied (covered) suffice also covers space in the upper side. Therefore, the measure of space occupied by such cylinder in the space (universe) is called volume of cylinder.
  - To find volume of cylinder we will have to multiply area of base and height. Cylinder also have equal shape from base to top. The base is circular.

    Therefore, Volume of cylinder = Area of base × height

    = Area of circle × height

- **Volume of cylinder = $\pi r^2 h$**

Keep in mind :

1 cubic metre	= 10,00,000 cubic cm	1 litre	= 1000 cubic cm
1 litre	= 1000 ml	1 cubic metre	= 1000 litre = 1 kilolitre
		1 cubic cm	= 1 ml

Example 6 : The radius of the base of a cylinder is 7 cm and height is 10 cm. Find volume of this cylinder.

Radius of cylinder r = 7 cm, Height of cylinder h = 10 cm

Volume of cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 10$

$= 1540 \text{ cm}^3$

Volume of cylinder = 1540 cm^3

Example 7 : The diameter of a rod of iron is 2 cm and height is 100 cm, find its volume. ($\pi = 3.14$)

Radius of rod $r = \frac{\text{diameter}}{2} = \frac{2}{2} = 1$ cm and height h = 100 cm

Volume of rod = $\pi r^2 h$

$= 3.14 \times 1 \times 1 \times 100$

$= 314 \text{ cm}^3$

$\therefore$ **Volume of iron rod = 314 cm^3**

- **Practical problems based on volume of cylinder :**

Example 8 : The radius of a cylindrical tank of water is 35 cm and height is 1 m. How much maximum litres of water can be occupied by this tank ?

Radius of cylindrical tank r = 35 cm, height h = 1 m = 100 cm

Volume of cylindrical tank = $\pi r^2 h$

$= \frac{22}{7} \times 35 \times 35 \times 100$

$= 3{,}85{,}000 \text{ cm}^3$

$\therefore$ Volume of cylindrical tank = 3,85,000 cm^3

$1000 \text{ cm}^3 = 1$ litre

$\therefore$ $3{,}85{,}000 \text{ cm}^3 = \frac{385000}{1000}$ litre $= 385$ litre

Maximum 385 litres of water can be occupied by cylindrical tank.

Example 9 : The radius of base of a cylindrical tank is 50 cm and its height is 100 cm. This tank is fully filled with kerosene. How many cans of 5 litres can be filled from this tank ? ($\pi = 3.14$)

Solution : Radius of cylindrical tank $r = 50$ cm and height $h = 100$ cm

Radius of cylindrical tank $= \pi r^2 h$

$= 3.14 \times 50 \times 50 \times 100$

$= 7{,}85{,}000 \text{ cm}^3$

Volume of cylindrical tank $= 7{,}85{,}000 \text{ cm}^3$

$1000 \text{ cm}^3 = 1$ litre

$\therefore$ $7{,}85{,}000 \text{ cm}^3 = \frac{785000}{1000}$ litre $= 785$ litre

Number of cans filled in 5 litres $= 1$

$\therefore$ Number of cans filled in 785 litres $= \frac{785}{5} = 157$

$\therefore$ **157 cans of kerosene will be filled.**

Practice 3

1. The radius of base of cylinder is 20 cm and height is 21 cm, then find its volume.
2. The diameter of the base of a cylinder is 80 cm and height is 50 cm, then find its volume. ($\pi = 3.14$)
3. What will be the cost of digging a well of radius 3.5 m and height 4 m at the rate of ₹ 100 per cubic metre ?
4. A cylinder of diameter 70 cm and height 80 cm is fully filled with medicine. If 25 ml medicine is filled in one bottle, then how much bottles will be filled from the medicine of this cylinder ?
5. The radius of the base of a cylindrical tank is 25 cm. It is filled with milk upto 2 m height. How many bags of 500 ml can be filled from this milk ? ($\pi = 3.14$)

6. The radius of base of a metallic cylinder is 14 cm and height is 10 cm. If the weight of 1 cm^3 metal is 8 gm, then find total weight of the metal.

*

What did you learn ?

- Curved surface area of a cylinder = $2\pi rh$
- Total surface area of an open cylinder with base = $\pi r(2h + r)$
- Total surface area of closed cylinder = $2\pi r(h + r)$
- Volume of a cylinder = $\pi r^2 h$

Exercise

1. The diameter of a cylindrical tin is 80 cm and height is 1.5 m. How many square metre will be its curved surface area ? ($\pi = 3.14$)
2. The radius of a water filling cylindrical tank without cover is 1.4 m and height is 2 m. Find total surface area of this tin.
3. The base diameter of a closed cylinder is 3.6 cm and height is 8.2 cm, then find its total surface area. ($\pi = 3.14$)
4. How much sheet is required to prepare 50 open cylinders of height 15 cm and diameter 4 cm ? What Will be the total cost at the rate ₹ 20 for 100 sq cm sheet ? ($\pi = 3.14$)
5. What will be the labour cost of digging a well of radius 3.5 m and weight 10 at the rate of ₹ 60 per cubic metre ?
6. The radius of a cylindrical tank of Municipality is 7 m and height is 4 m. How much kilolitre of water can be occupied in the tank ?
7. The diameter of a cylinder is 20 cm. If its height is equal to its radius, then find its volume ? ($\pi = 3.14$)

7 : Area and Volume of Cylinder

Answers

Practice 1

1. 440 cm^2 **2.** 880 cm^2 **3.** 3140 cm^2

4. 3768 cm^2 **5.** 880 cm^2

Practice 2

1. ₹ 7850 **2.** ₹ 4224 **3.** 171.6 m^2

4. ₹ 4396 **5.** 66 m^2 **6.** 4.4 m^2, ₹ 880

Practice 3

1. 26400 cm^3 **2.** 251200 cm^3 **3.** ₹ 15400

4. 12320 bottles **5.** 785 bags **6.** 49.280 kilogram

Exercise

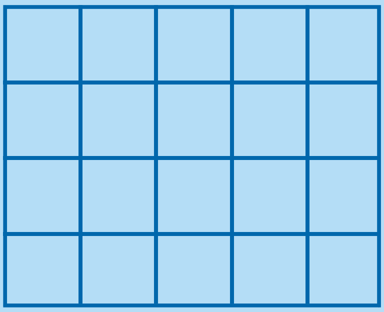
1. 3.768 m^2 **2.** 23.76 m^2 **3.** 113.04 cm^2

4. 9420 cm^2, ₹ 1884 **5.** ₹ 23,100 **6.** 616 kilolitre

7. 3140 cm^3

Learn something special :

- How many total squares are there in this rectangle ?



Here a rectangle of size 5×4 is given.

$\therefore$ Number of squares $= (5 \times 4) + (4 \times 3) + (3 \times 2) + (2 \times 1)$

$= 20 + 12 + 6 + 2$

$= 40$