ગુજરાત રાજ્યના શિક્ષણવિભાગના પત્ર-ક્રમાંક
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PHYSICS

Standard 12

(Semester III)

PLEDGE

India is my country.
All Indians are my brothers and sisters.
I love my country and I am proud of its rich and varied heritage.
I shall always strive to be worthy of it.
I shall respect my parents, teachers and all my elders and treat everyone with courtesy.
I pledge my devotion to my country and its people.
My happiness lies in their well-being and prosperity.

રાજ્ય સરકારની વિનામૂલ્યે યોજના હેઠળનું પુસ્તક

Gujarat State Board of School Textbooks
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Authors

Prof. P. N. Gajjar (Convenor)
Dr. V. P. Patel
Prof. M. S. Rami
Dr. Arun P. Patel
Dr. Deepak H. Gadani
Dr. Nisharg K. Bhatt
Prof. Mahesh C. Patel

Reviewers

Dr. P. B. Thakor
Dr. Mukesh M. Jotani
Prof. Manish V. Acharya
Prof. V. A. Bheda
Prof. Hiren H. Rathod
Prof. Sanjay D. Zala
Shri Jayesh D. Darji
Shri Shailesh S. Patel
Shri Anand N. Thakkar
Shri Mukesh N. Gandhi

Language Correction

V. Balakrishnan

Artist

Shri G. V. Mevada

Co-Ordinator

Shri Chirag H. Patel
(Subject Co-ordinator : Physics)

Preparation and Planning

Shri Haresh S. Limbachiya
(Dy. Director : Academic)

Lay-out and Planning

Shri Haresh S. Limbachiya
(Dy. Director : Production)

PREFACE

The Gujarat State Secondary and Higher Secondary Education Board has prepared new syllabi in accordance with the new national syllabi prepared by N.C.E.R.T. based on N.C.F. 2005 and core-curriculam. These syllabi are sanctioned by the Government of Gujarat.

It is a pleasure for the Gujarat State Board of School Textbooks, to place before the students this textbook of **Physics, Standard 12, (Semester III)** prepared according to the new syllabus.

Before publishing the textbook, its manuscript has been fully reviewed by experts and teachers teaching at this level. Following suggestions given by teachers and experts, we have made necessary changes in the manuscript before publishing the textbook.

The Board has taken special care to ensure that this textbook is interesting, useful and free from errors. However, we welcome any suggestions, from people interested in education, to improve the quality of the textbook.

Dr. Bharat Pandit	**Dr. Nitin Pethani**
Director	Executive President
Date : 3-3-2015	Gandhinagar

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FUNDAMENTAL DUTIES

It shall be the duty of every citizen of India

(A) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(B) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(C) to uphold and protect the sovereignty, unity and integrity of India;

(D) to defend the country and render national service when called upon to do so;

(E) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(F) to value and preserve the rich heritage of our composite culture;

(G) to protect and improve the natural environment including forests, lakes, rivers and wild life, and to have compassion for living creatures;

(H) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(I) to safeguard public property and to abjure violence;

(J) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

(K) to provide opportunities for education by the parent or the guardian, to his child or a ward between the age of 6-14 years as the case may be.

* Constitution of India : Section 51 A

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About This Textbook...

We have pleasure in presenting this textbook of physics of Standard 12 to you. This book is on the syllabi based on the courses of National Curriculum Framework (NCF), Core-Curriculum and National Council of Educational Research and Training (NCERT) and has been sanctioned by the State Government keeping in view the National Education Policy.

The State Government has implemented the semester system in science stream. The semester system will reduce the educational load of the students and increase the interest towards study.

In this Textbook of Physics for Standard-12, Seven chapters are included, looking into the depth of the topics, time which will be available for classroom teaching, etc...

The real understanding of the theories of physics is obtained only through solving related problems. Hence, for the new concept, solved problems are given. One of the positive sides of the book is that at the end of each chapter extended summary is given. On the basis of this one can see the whole contents of the chapter at a glance.

Keeping in view the formats of various entrance test conducted on all India basis, we have included MCQs, Short questions, objective questions and problems in this book. At the end of the book, Hints for solving the problems are also included so that students themselves can solve the problems.

This book is published in quite a new look in four-colour printing so that the figures included in the book are much clear. It has been observed, generally, that students do not preserve old text-books, once they go to the higher standard. In the semester system, each semester has its own importance and the look of the book is also very nice so the students would like to preserve this book and it will become a reference book in future.

The previous textbook got excellent support from students, teachers and experts. So a substantial portion from that book is taken in this book either in its original form or with some changes. We are thankful to that team of authors. We are also thankful to the teachers who remained present in the Review workshop and gave their inputs to make this textbook error-free.

Proper care has been taken by authors, subject advisors and reviewers while preparing this book to see that it becomes error-free and concepts are properly developed. We welcome suggestions and comments for the importance of the textbook in future.

Authors/Editors

1 Electric Charge and Electric Field

1.1 Introduction

Whatever facilities an individual is enjoying in this modern age is due to technological development. From all kinds of energy, electric energy holds an important role for human comfort. Electric energy can be easily stored and can be transferred to another form of energy. There is no exaggeration in calling the electricity is the mother of technology. Electric charges are the foundation stones of electricity.

In this chapter we will study about static charges, their properties and interaction between them. Such a study is called static electricity. Static electricity is used in copier machine, laser printer, television etc. Natural phenomenon such as lightning can be understood through static electricity. Here, we will study about electric fields due to different system of charges and its characteristics.

1.2 Electric Charge

Any matter consists of certain fundamental particles. Fundamental particles are more than 100. Out of them three particles are most important namely electron, proton and neutron. Because of their masses these particles exert gravitational force on each other. For example two electrons 1cm apart exert 5.5×10^{-67} N gravitational force on each other, which is attractive. However, an electron is found to repel another electron at the same distance (1 cm) with a force of 2.3×10^{-24} N. This additional force other than gravitational force is an electric force. **The fundamental intrinsic property due to which such a force acts is called the electric charge.**

Just as masses of two particles are responsible for the gravitational force, charges are responsible for the electric force.

Two protons placed at a distance of 1 cm also repel each other with a force of 2.3×10^{-24} N, which shows that proton has the electric charge. The magnitude of this charge is same as the charge of an electron. Now if a proton and electron are placed 1 cm apart, they exert a force of 2.3×10^{-24} N on each other but this force is attractive.

Thus, we conclude that magnitude of charge on electron and proton is same but they are of opposite type.

Electric charges are of two types : Positive charge and Negative charge. Traditionally, charge of a proton considered positive and that of an electron negative. Though it makes no difference whatsoever to Physics if this sign convention is reversed.

The force acting between two like charges is repulsive and it is attractive between two unlike charges.

All material bodies contain equal number of electrons and equal number of protons in their normal state. So they are electrically neutral. In any substance, electrons are comparatively weakly bound than the force with which the protons are bound inside the nucleus. Hence, whenever there is an exchange of charge between two bodies due to some process (e.g. friction), it is the electrons are transferred from one body to the other. The body that receives the extra electrons, becomes negatively charged. The body that loses the electrons, becomes positively charged because it has more number of protons than electrons. Thus, when a glass rod is rubbed with a silk cloth, some electrons are transferred from the glass rod to the silk cloth. The glass rod becomes positively charged and the cloth becomes negatively charged because it receives extra electrons. To detect these charges a simple device is used, known as **electroscope.**

Electic charge is a fundamental property like mass. It is difficult to define. The SI unit of the quantity of charge is coulomb and abbreviated as C.

One coulomb is the charge flowing through any section of the conductor in one second when the electric current in it is 1 ampere. The charge on a proton is $e = +1.6 \times 10^{-19}$ C. The charge on the electron is $e = -1.6 \times 10^{-19}$ C.

Quantization of Electric Charge

All the experiments carried out so far show that the **magnitude of all charges found in nature are in integral multiple of a fundamental charge.**

$Q = ne$

This fact is known as quantization of charges. The fundamental charge is the charge of an electron or proton. It is denoted by e and it is called the fundamental unit of charge.

Out of all the fundamental particles, the building blocks of all matters, the particles having possesed charge equal to e. For example, charge on proton and positron (positive electron) is $+e$, while charge on electron is $-e$. Thus, charge on any object can be increased or decreased only in step of e. The quantization of charge was first suggested by English scientist Faraday. It was experimentally demonstrated by Millikan in 1912.

No theory, so far, has been able to explain satisfactorily, the quantization of charges.

According to new research, the proton and neutron consists of another fundamental particles called **quarks.**

A proton and neutron consist of three quarks each. These quarks are of two types : the quark possessing $+\frac{2}{3}e$ charge is called an up quark (u) and another having $-\frac{1}{3}e$ charge is called a down quark (d). (The composition of proton is indicated as uud and composition of neutron is indicated as udd). Thus, **matter is formed of such quarks and electrons.** The independent existence of quark is not detected so far.

Conservation of Electric Charge

The algebraic sum of electric charges in an electrically isolated system always remains constant irrespective of any process taking place. This statement represents the law of conservation of charge.

In an electrically isolated system, a charge can neither enter from outside nor escape from inside. Any chargeless thing can enter or leave such a system.

In the experiment of glass rod and silk cloth, before rubbing glass rod with silk the net charge on them is zero. After rubbing the glass rod with silk cloth, the glass rod becomes positively charged and same amount of negative charge is received by the silk cloth. Thus, after the process of friction the net charge of system (glass rod + silk cloth) is zero.

Now, to understand the conservation of electric charge we consider another illustration.

As shown in figure 1.1, the initial charge in a box having thin walls is zero. A highly energetic photon enters in the box. A photon is a chargeless particle. As the photon enters through a box it produces an electron-positron pair. After the pair production in the isolated system the net charge is zero because the charges on the electron and positron are equal and opposite type. The initial charge of the system was zero. Thus, in this event also charge is conserved.

In other words in an electrically isolated system, only those processes are possible in which charges of equal magnitude and opposite types are either produced or destroyed.

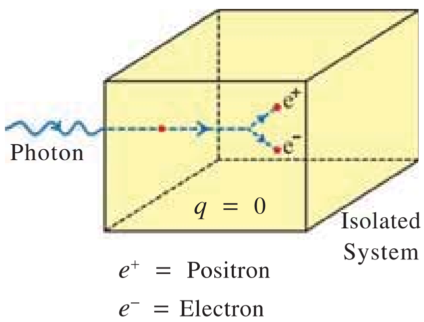

Figure 1.1 Conservation of Electric Charge

Charging by Induction

Consider two identical isolated sphere placed on an insulated stand, one carrying net charge +Q (i.e. positively charged) and other having no net charge. If they are brought directly in contact or brought in contact with conducting wire, some of the electrons from the chargeless sphere transferred to positively charged sphere. As a result, the positive charge on the positively charged sphere reduced and chargeless sphere becomes positive, because it loses the electrons. Now, both the spheres will have equal amount of charge$+\frac{Q}{2}$ after the separation because they are identical. Thus we have established $\frac{Q}{2}$ electric charge on the other sphere through contact or that the charging of the second sphere has taken place.

There is another method of charging the object. In that method the charged body does not looses its own charge and without coming in physical contact with other object it will induce opposite charge in that. This phenomenon is called **induction of electric charge.**

Figure 1.2(a) shows an isolated metal sphere. The net charge on the sphere is zero. As shown in figure 1.2(b), a negatively charged plastic rod is brought close to the sphere the free electrons of the sphere move away from the rod because of repulsion and go to the other part of the sphere. Consequently the part of the sphere close to the rod becomes positively charged due to deficiency of electron in that region.

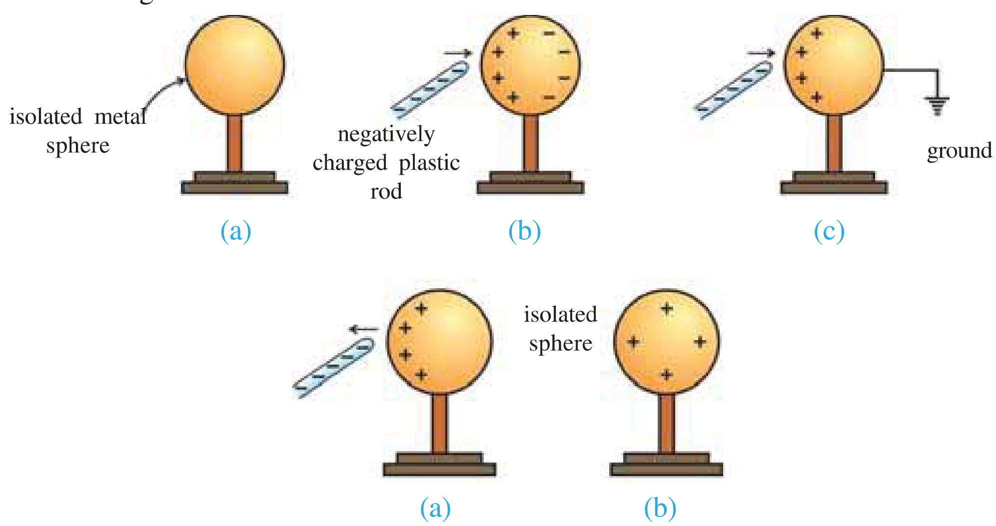

Figure 1.2 Induction of Electric Charge

As shown in figure 1.2(c) when the sphere is connected to the earth through a conducting wire, the some of the electrons of the sphere will flow to the ground. (The earth is a good conductor and it act as a practically infinite source of extra electrons or sink of electrons.)

As shown in figure 1.2(d), even if the connection with the earth is removed, the sphere retains the positive charge. When the plastic rod is moved away from the sphere, the electrons get redistributed on the sphere such that the same positive charge is spread all over the surface of the sphere. (Figure 1.2 (e))

1.3 Coulomb's Law

French scientist Charles Coulomb (1736-1806) measured electrical attraction and repulsion between two electric charges through a number of experiments and deduced the law that governs them, which is known as Coulomb's law. The law is as under :

'The electric force (Coulombian force) between two stationary point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.' This force is along the line joining the two charges.

According to Coulomb's law, the electric force between the two point charges q_1 and q_2 separated by a distance r can be given as,

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\therefore \ F = k\frac{q_1 q_2}{r^2} \qquad (1.3.1)$$

Where k is a Coulomb's constant. It's value depends on the unit of q_1, q_2 and r. Experimentally the value of k in vacuum in SI unit is 8.9875×10^9 Nm^2C^{-2}. For practical purposes, $k = 9 \times 10^9$ Nm^2C^{-2}. (In CGS unit value of k is 1).

For the simplification of formula in electrostatic k is expressed as $\frac{1}{4\pi\varepsilon_0}$.

$$k = \frac{1}{4\pi\varepsilon_0}$$

Where, ε_0 is the permittivity of free space. From the above equation,

$$\varepsilon_0 = \frac{1}{4\pi k} = \frac{1}{4\pi \times 8.9875 \times 10^9} \approx 8.854 \times 10^{-12} \ C^2N^{-1}m^{-2}$$

$$\text{Thus, } F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}. \qquad (1.3.2)$$

If the charges are in any other insulating medium and not in vacuum, the permittivity of vacuum ε_0 in equation (1.3.2) should be replaced by the permittivity ε of that medium. Hence force in that medium,

$$F_m = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2}. \qquad (1.3.3)$$

Thus, Coulombian force acting on two point charges is also depend on the medium between the two charges. By taking ratio of equation (1.3.2) and (1.3.3),

$$\frac{F}{F_m} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r = K$$

$$\therefore \ F_m = \frac{F}{K} \qquad (1.3.4)$$

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Where, ε_r is known as relative permittivity of the medium or dielectric constant (K). A detailed study about this we will learn in Chapter 2. From equation (1.3.4) it is clear that the force between given charges held at a given distance apart in insulating medium is only $\frac{1}{K}$ times (i.e.k-th part) of the force between them in vacuum.

Remember that Coulomb's law holds only for stationary point charges. Generally, this law is also applicable for charged objects whose sizes are much smaller than the distance between them.

Coulomb's law resembles inverse square law of gravitation. The charge q plays the same role in Coulomb's law that the mass m plays in gravitational law. The gravitational forces are always attractive, whereas electrostatic forces can be repulsive or attractive, because electric charges are of two types.

Illustration 1 : The repulsive force between two particles of same mass and charge, separated by a certain distance is equal to the weight of one of them. Find the distance between them.

Mass of particle $= 1.6 \times 10^{-27}$kg

Charge of particle $= 1.6 \times 10^{-19}$C, $k = 9 \times 10^9$ MKS, $g = 10\text{ms}^{-2}$.

Solution : Here,

Repulsive force between two particles = Weight of one of the particles

$$\therefore \; k\frac{q_1q_2}{r^2} = mg$$

$$\therefore \; r^2 = \frac{kq_1q_2}{mg} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{(1.6\times10^{-27})(10)} = 1.44 \times 10^{-2}$$

$\therefore \; r = 0.12$ m.

Illustration 2 : Two spheres of copper, having mass 1g each, are kept 1 m apart. The number of electrons in them are 1% less than the number of protons. Find the electrical force between them. Atomic weight of copper is 63.54 g/mol, atomic number is 29, Avogadro's number $N_A = 6.023 \times 10^{23}$ mol^{-1}. $k = 9 \times 10^9$ SI.

Solution : In a neutral atom of copper the number of electrons and protons are 29 each. Here, the number of electrons are less than that of protons by 1%.

$$\therefore \text{ Net charge on each atom } q' = \begin{pmatrix}\text{Total Charge}\\ \text{of Protons}\end{pmatrix} + \begin{pmatrix}\text{Total Charge}\\ \text{of Electrons}\end{pmatrix}$$

$$= (+29e) + (-29e) - (-0.29e)$$

$$= +\, 0.29e$$

$\therefore$ Net positive charge of 1 g copper,

$$q = \begin{pmatrix}\text{No. of Atoms}\\ \text{in 1g Copper}\end{pmatrix} \times 0.29e = \frac{6.023\times10^{23}}{63.54} \times 0.29e$$

$\therefore$ Electric force between two copper spheres,

$$F = k\frac{q\,q}{r^2} = k\frac{q^2}{r^2} = \frac{9\times10^9}{1^2} \times \left(\frac{6.023\times10^{23}\times0.29\times1.6\times10^{-19}}{63.54}\right)^2$$

$$= 1.74 \times 10^{15}\text{N}$$

It can be seen in above example that even a difference of 1% between positive and negative charges in any substance can give rise to a very large force. Most of the matters are electrically neutral so that there is a dominance of weak gravitational force on them.

Illustration 3 : Charge Q is uniformly distributed over a body. How should the body be divided into two parts, so that force acting between the two parts of body is maximum for a given separation between them ?

Solution : Suppose the body is broken into two parts such that the charge on one part of body is q and on the other is $Q - q$. The force existing between the two parts separated by distance r will be,

$$F = k\frac{q(Q-q)}{r^2}$$

The force F to be maximum, the quantity $y = q(Q - q) = Qq - q^2$

should be maximum. For this $\frac{dy}{dq}$ should be zero. $\therefore \frac{dy}{dq} = Q - 2q = 0$

$$\therefore q = \frac{Q}{2}$$

Thus, the body should be divided into two parts such equal charges are present on each part.

Coulomb's Law in Vector from :

Force is a vector quantity, so the Coulomb's law can be represented in vector form as follows :

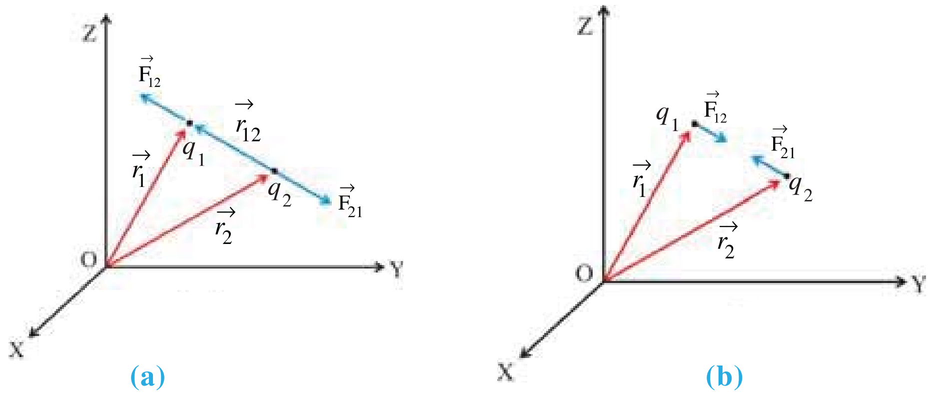


Figure 1.3 Coulomb's Law in Vector Form

As shwon in figure 1.3(a), let $\vec{r}_1$ and $\vec{r}_2$ be the position vectors of the charges q_1 and q_2 respectively in a Cartesian co-ordinate system. Let $\vec{r}_{12}$ be the unit vector pointing from q_2 to q_1, $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$.

According to Coulomb's Law, force acting on charge q_1 due to charge q_2 is,

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} \tag{1.3.5}$$

Where, $r_{12} = |\vec{r}_1 - \vec{r}_2|$ is the distance between the two charges and $\hat{r}_{12}$ is a unit vector of $\vec{r}_{12}$ in the direction from q_2 to q_1.

$$\hat{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}$$

$$\therefore \vec{F}_{12} = k\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \cdot \frac{(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}$$

$$\vec{F}_{12} = k\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) \qquad (1.3.6)$$

Above equation is valid for any sign of the charges whether positive or negative. If q_1 and q_2 are of the same sign (either both positive or both negative)

$\vec{F}_{12}$ is along $\hat{r}_{12}$, which denotes repulsive force. If q_1 and q_2 are of opposite sign, $\vec{F}_{21}$ is along $-\hat{r}_{12}$, which denotes the attraction between the opposite charges. (See Figure 1.3(b)).

The coulombian force on charge q_1 due to charge q_2 can be given by replacing 1 and 2 in equation (1.3.6)

$$\vec{F}_{21} = k\frac{q_1 q_2}{r_{21}^{\ 2}}\vec{r}_{21} \qquad (1.3.7)$$

$$= k\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1) \qquad (1.3.8)$$

Where, $\vec{r}_{21}$ is a unit vector directed from q_1 to q_2.

Here, $\vec{r}_2 - \vec{r}_1 = -(\vec{r}_1 - \vec{r}_2)$

Thus, from equation (1.3.8),

$$\vec{F}_{21} = -k\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) = -\vec{F}_{12}$$

Thus, Coulomb's Law agrees with the Newton's Third Law.

1.4 Forces between more than two charges : The Superposition Principle

We can use Coulomb's Law to find the force acting between two electric charges. When more than two charges (Suppose they are q_1, q_2, ..., q_n) are present and to calculate the net force acting on any one charge, we have to use superposition principle in addition to Coulomb's Law.

Superposition Principle : When more than one coulombian forces are acting on a charge, the resultant coulombian force acting on it is equal to the vector sum of the individual force.

Thus, the coulombian force acting between two charges is not influenced by the presence of a third charge. Hence, the coulombian force is called a two body force.

Consider a system of charges q_1, q_2, q_3 and q_4 as shown in figure 1.4. Let $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$ and $\vec{r}_4$ are their respective position vectors in a given co–ordinate system. Here, we will find the resultant force $\vec{F}_2$ acting on charge q_2 due to the other charges.

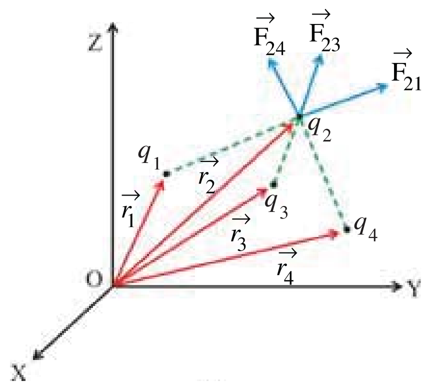


Figure 1.4 Superposition Principle

The force on charge q_2 due to charge q_1 is,

$$\vec{F}_{21} = k\frac{q_1 q_2}{r_{21}^{\ 2}}\hat{r}_{21}$$

The force on charge q_2 due to q_3 is,

$$\vec{F}_{23} = k\frac{q_2 q_3}{r_{23}^{\ 2}}\,\hat{r}_{23}$$

The force on charge q_2 due to q_4 is,

$$\vec{F}_{24} = k\frac{q_2 q_4}{r_{24}^{\ 2}}\,\hat{r}_{24}$$

According to superposition principle,

$$\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24} = k\frac{q_1 q_2}{r_{21}^{\ 2}}\,\hat{r}_{21} + k\frac{q_2 q_3}{r_{23}^{\ 2}}\,\hat{r}_{23} + k\frac{q_2 q_4}{r_{24}^{\ 2}}\,\hat{r}_{24}$$

$$= kq_2\left[\frac{q_1}{r_{21}^{\ 2}}\hat{r}_{21} + \frac{q_3}{r_{23}^{\ 2}}\hat{r}_{23} + \frac{q_4}{r_{24}^{\ 2}}\hat{r}_{24}\right]$$

$$= kq_2 \sum_{\substack{j=1 \\ j\neq 2}}^{4} \frac{q_j}{r_{2j}^{\ 2}}\,\hat{r}_{2j} \qquad (1.4.1)$$

or

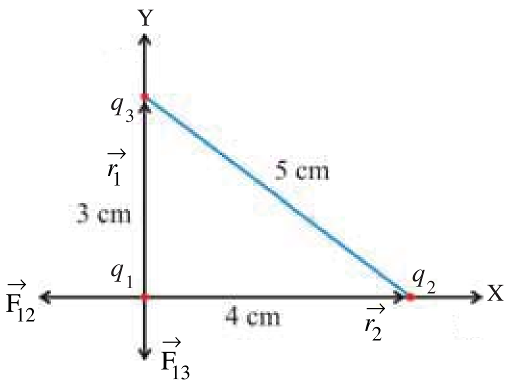
$$\vec{F}_2 = kq_2 \sum_{\substack{j=1 \\ j\neq 2}}^{4} \frac{q_j}{|\vec{r}_2 - \vec{r}_j|^3}(\vec{r}_2 - \vec{r}_j) \qquad (1.4.2)$$

In general, the force acting on charge q_i due to system of n electric charges will be,

$$\vec{F}_i = kq_i \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{q_j}{r_{ij}^{\ 2}}\,\hat{r}_{ij}$$

$$\vec{F}_i = kq_i \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{q_j}{|\vec{r}_i - \vec{r}_j|^3}(\vec{r}_i - \vec{r}_j) \qquad (1.4.3)$$

Illustration 4 : Three equal charges each having a magnitude of 2.0×10^{-6}C are placed at the three corners of a right angled triangle of sides 3cm, 4cm and 5cm. Find the force on the charge at the right angle corner.



Solution :

The situation is as shown in the figure.

$q_1 = q_2 = q_3 = q = 2 \times 10^{-6}$C

The position vectors of q_1, q_2 and q_3 are respectively $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$.

$\vec{r}_1 = (0,\ 0)$

$\vec{r}_2 = (4,\ 0)\text{cm} = (0.04,\ 0)\text{m}$

$\vec{r}_3 = (0,\ 3)\text{cm} = (0,\ 0.03)\text{m}.$

q_1 is placed at the right angle of right angled triangle. Net force acting on q_1 is,

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}$$

$$= k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + k\frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} = kq^2\left[\frac{\hat{r}_{12}}{r_{12}^2}+\frac{\hat{r}_{13}}{r_{13}^2}\right] \quad (\because\ q_1 = q_2 = q) \qquad (1)$$

Now, $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2 = (0,\ 0) - (0.04,\ 0) = (-0.04,\ 0)\text{m}.$

$\therefore\ r_{12} = \sqrt{(-0.04)^2 + (0)^2} = 0.04\text{m}.$

$$\hat{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_{12}|} = \frac{(-0.04, 0)}{0.04} = (-1,\ 0)\text{m}.$$

$\vec{r}_{13} = \vec{r}_1 - \vec{r}_3 = (0,\ 0) - (0,\ 0.03) = (0,\ -0.03)\text{m}$

$r_{13} = \sqrt{(0)^2 + (-0.03)^2} = 0.03\text{m}$

$$\hat{r}_{13} = \frac{\vec{r}_1 - \vec{r}_3}{|\vec{r}_{13}|} = \frac{(0, -0.03)}{0.03} = (0,\ -1)\text{m}$$

Put all these values in equation (1),

$$\vec{F}_1 = (9 \times 10^9)\ (2 \times 10^{-6})^2 \left[\frac{(-1, 0)}{(0.04)^2} + \frac{(0, -1)}{(0.03)^2}\right]$$

$= 36 \times 10^{-3}[625\ (-1,\ 0) + 1111.1\ (0,\ -1)]$

$= (-22.5,\ -40)\text{N}$

$\therefore\ |\vec{F}_1| = \sqrt{(-22.5)^2 + (-40)^2} = 45.88\text{N}$

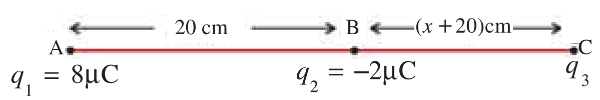
Direction of force,

$$\theta = tan^{-1}\left(\frac{F_y}{F_x}\right) = tan^{-1}\left(\frac{-40}{-22.5}\right) = tan^{-1}(1.777)$$

$= 60.6^{\circ}$

θ is the angle with respect to negative X-axis.

Illustration 5 : Two electric charges having magnitude 8.0μC and −2.0μC are separated by 20cm. Where should a third charge be placed so that the resultant force acting on it is zero ?



Solution : Let the two charges q_1 = 8μC and q_2 = −2μC be placed at points A and B respectively as shown in figure. The resultant force on the third charge q_3 will be zero only if the forces due to two charges are equal in magnitude and opposite in direction. This is possible only if the third charge is placed at a point on the line joining the two charges. Third charge q_3 cannot be placed anywhere between points A and B since q_1 and q_2 have opposite sign. As the magnitude of charge on A is greater than that on B the third charge has to be nearer to B.

Suppose the third charge is placed at point C and BC = x cm.

According to superposition principle, the net force on charge q_3,

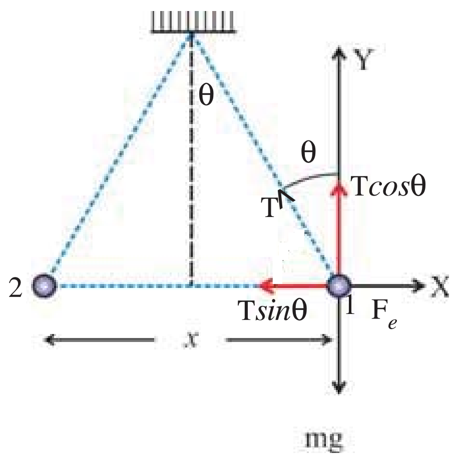
$F_3 = F_{31} + F_{32}$

$$0 = k\frac{q_1 q_3}{(r+x)^2} + k\frac{q_2 q_3}{x^2} = \frac{q_1}{(r+x)^2} + \frac{q_2}{x^2} = \frac{8\times10^{-16}}{(20+x)^2} - \frac{2\times10^{-16}}{x^2}$$

$\therefore \frac{20+x}{x} = 2 \qquad \therefore x = 20\text{cm}$

Illustration 6 : Two spheres having same radius and mass are suspended by two strings of equal length from the same point, in such a way that their surfaces touch each other. On depositing 4×10^{-7}C charge on them, they repel each other in such a way that in equilibrium the angle between their strings becomes 60°. If the distance from the point of suspension to the center of the sphere is 20cm, find the mass of each sphere. $k = 9 \times 10^9$ SI and $g = 10\text{ms}^{-2}$.

Solution : If the spheres are identical in all respects then 4×10^{-7}C charge will be distributed equally between them. Hence charge on each sphere is 2×10^{-7}C. The force acting on sphere 1 in equilibrium will be :



(1) Weight mg in the vertically downward direction.

(2) F_e, the repulsive force between the spheres,

(3) The tension T produced in the string.

Under the balanced condition, if we consider the X and Y components in the Cartesian co-ordinate system as shown in the figure,

$F_e = T\sin\theta$

$$\therefore k\frac{q^2}{x^2} = T\sin\theta \qquad (1)$$

and $mg = T\cos\theta \qquad (2)$

$$\frac{kq^2}{x^2 mg} = \tan\theta \Rightarrow m = \frac{kq^2}{x^2 g\tan\theta}$$

From figure, $\sin\theta = \frac{\frac{x}{2}}{l} = \frac{x}{2l}$

$\therefore x = 2l \sin\theta$

$$\therefore m = \frac{kq^2}{g\, 4l^2 \sin^2\theta \tan\theta}$$

$$\therefore m = \frac{(9\times10^9)(2\times10^{-7})^2}{10\times4(20\times10^{-2})^2\times(\sin30^\circ)^2\times(\tan30^\circ)} = 1.56 \times 10^{-3}\text{kg}$$

1.5 Electric Field

When we place a point charge q_0 in the region around another point charge q in the space, it will exert the electric force on q_0. We may ask the question. If charge q_0 is removed then what is left in the surrounding ? Is there nothing ? If there is nothing in the surrounding, then how does a force act on q_0 ? In order to answer these questions, the concept of electric field is very useful.

A charge produces some effect in the space around it. The region around the charge in which the effect of electric charge is prevailing is called the electric field of the charge. This electric field can interact with another charge q_0 placed in it and exerts the force on it. (It does not exert the force which produce the electric field). Thus, electric field acts as an agency between q and q_0.

Suppose a charge Q is placed at an origin of a co-ordinate system. Now bring a charge q_0 at the given point in the electric field without disturbing the position of charge Q. If the position vector of that point is $\vec{r}$, then electric field at that point can be defined as follows :

$$\vec{E}(\vec{r}) = \frac{\vec{F}(\vec{r})}{q_0} \qquad (1.5.1)$$

Here, $\vec{E}$ is called the electric field or electric field intensity of charge Q at a position vector $\vec{r}$. The quantity $\vec{E}$ is independent of q_0. It is dependent solely on the magnitude of electric charges of the system, their arrangement and the position vector $\vec{r}$ of q_0.

The charge q_0 used to define or to measure intensity of electric field is called a test charge. The charges producing electric field are called the source charges.

In SI system, the unit of electric field is NC^{-1} or Vm^{-1}.

In equation (1.5.1), if $q_0 = 1C$ then $\vec{E} = \vec{F}$ and definition of electric field can be given as follows :

'The force acting on a unit positive charge at a given point in an electric field of a point charge of a system at charges is called the electric field or intensity of electric field $\overset{\circledR}{E}$ at that point.

Electric field is a vector quantity and it is in the direction of force acting on unit positive charge at a given point.

If the system of charges consists of more than one charge, then electric field at a given point can be obtained by using Coulomb's Law and superposition principle.

Consider a system of charges q_1, q_2, ..., q_n with position vectors $\vec{r_1}$, $\vec{r_2}$, ..., $\vec{r_n}$ relative to origin. The electric field is produced in the region surrounding the system due to the system of charges. We want to determine the electric field at a point P(x, y, z) having position vector $\vec{r}$. For this purpose place a very small test charge q_0 at that point and use the superposition principle.

Electric field at point P due to charge q_1 is given by,

$$\vec{E_1} = \frac{\vec{F_1}}{q_0} = k\frac{q_1}{|\vec{r} - \vec{r_1}|^3}(\vec{r} - \vec{r_1})$$

Electric field at point P due to charge q_2 is.

$$\vec{E_2} = \frac{\vec{F_2}}{q_0} = k\frac{q_2}{|\vec{r} - \vec{r_2}|^3}(\vec{r} - \vec{r_2})$$

Same way, electric field at point P due to charge q_n is

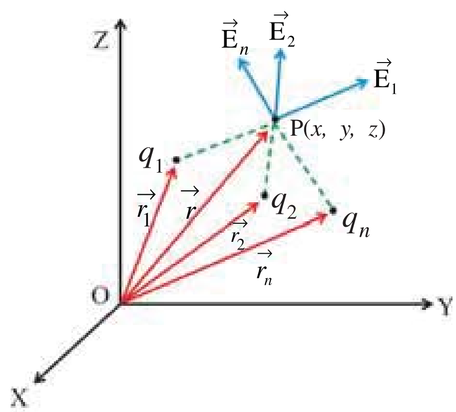


Figure 1.5 Superposition Principle for Electric Field

$$\vec{E}_n = \frac{\vec{F}_n}{q_0} = k\frac{q_n}{|\vec{r}-\vec{r}_n|^3}(\vec{r}-\vec{r}_n)$$

According to superposition principle, net electric field at a point P is.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + ... + \vec{E}_n$$

$$= k\frac{q_1}{|\vec{r}-\vec{r}_1|^3}(\vec{r}-\vec{r}_1) + k\frac{q_2}{|\vec{r}-\vec{r}_2|^3}(\vec{r}-\vec{r}_2) + ... + k\frac{q_n}{|\vec{r}-\vec{r}_n|^3}(\vec{r}-\vec{r}_n)$$

$$\vec{E} = k\sum_{j=1}^{n}\frac{q_j}{|\vec{r}-\vec{r}_j|^3}(\vec{r}-\vec{r}_j) \quad (1.5.2)$$

Here, q_1, q_2, ..., q_n are the sources of electric field.

The following points are noteworthy for an electric field :

(1) To determine the electric field there should not be any change in the original system of charges due to the presence of a test charge. So it is necessary that the test charge should be very small. To define electric field more precisely $q_0 \rightarrow 0$. But minimum value of q_0 is 1.6×10^{-19}C.

(2) Equation 1.5.2 indicates the force acting on unit positive charge at point $\vec{r}(x, y, z)$. Once $\vec{E}(\vec{r})$ is known, we do not have to worry about the source of electric field. In this sense, the electric field itself is a special representation of the system of charges producing electric field, as far as the effect on other charge is concerned. Once such a representation is done, the force acting on charge q kept at that point in the field can be determined using following equation.

$$\vec{F}(\vec{r}) = q\vec{E}(\vec{r}) \quad (1.5.3)$$

(3) The direction of force acting on unit positive charge at a given point is the direction of electric field at that point.

(4) Faraday was the first person to introduce the concept of electric field. Electric field is not an imaginary concept but a physical reality.

1.6 Electric Field Due to a Point Charge

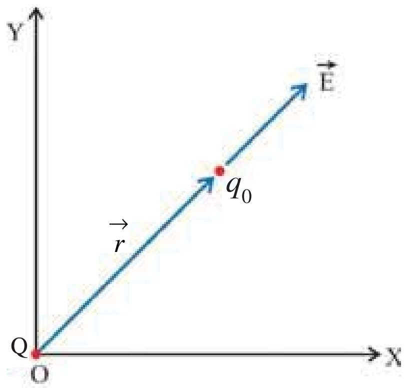


Figure 1.6 Electric Field Due to a Point Charge

As shown in figure 1.6, consider a point charge Q on the origin of a cartesian co-ordinate system.

In order to calculate electric field due to charge Q, consider a test charge q_0 at a distance r from the charge Q. Force acting on charge q due to Q is,

$$\vec{F} = k\frac{Qq_0}{r^2}\hat{r}$$

Therefore, electric field intensity at r due to Q will be,

$$\vec{E} = \frac{\vec{F}}{q_0} = k\frac{Q}{r^2}\hat{r} \quad (1.6.1)$$

Figure 1.7 shows the electric field due to point charge in two dimensions using field vectors.

From figure 1.7 it is clear that for positive charge (Q > 0), direction of field vectors are radially outward while those of a negative charge (Q < 0) are radially inward. The length of the arrow decreases, indicating the decreasing strength of the electric field, as we go away from the charge.

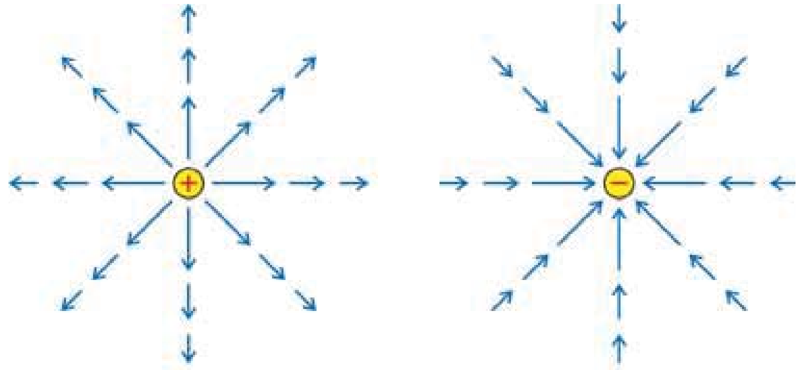
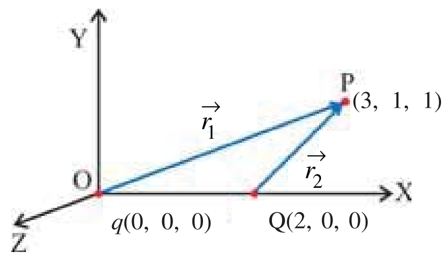


Figure 1.7 Electric Field of a Point Charge

Illustration 7 : A charge $+10^{-9}$C is located at the origin of cartesian co-ordinate system and another charge Q at (2, 0, 0)m. If X-component of electric field at (3, 1, 1)m is zero, calculate the value of Q.

Solution : As shown in the figure, Position vector of $q = 10^{-9}$C is (0, 0, 0) and position vector of Q is (2, 0, 0)m.



The co-ordinates of point P is (3, 1, 1) m.

$$\therefore \vec{r_1} = (3, 1, 1) - (0, 0, 0) = (3, 1, 1)$$

$$= 3\hat{i} + \hat{j} + \hat{k}$$

$$|\vec{r_1}| = \sqrt{(3)^2 + (1)^2 + (1)^2} = \sqrt{11} \text{ m.}$$

$$\vec{r_2} = (3, 1, 1) - (2, 0, 0) = (1, 1, 1) \text{ m}$$

$$= \hat{i} + \hat{j} + \hat{k}$$

$$|\vec{r_2}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3} \text{ m.}$$

Electric field at point P, $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$= k\frac{q}{r_1^3}\vec{r_1} + k\frac{Q}{r_2^3}\vec{r_2} = k\left[\frac{10^{-9}(3\hat{i} + \hat{j} + \hat{k})}{(\sqrt{11})^3} + \frac{Q(\hat{i} + \hat{j} + \hat{k})}{(\sqrt{3})^3}\right]$$

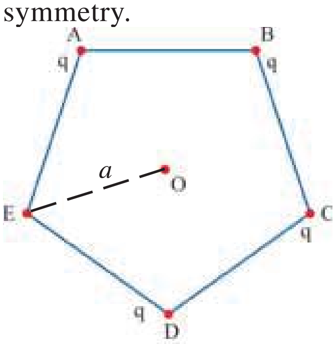
Now, x component of electric field is zero.

$$\therefore E_x = k\left[\frac{10^{-9} \times 3}{(11)^{\frac{3}{2}}} + \frac{Q}{(3)^{\frac{3}{2}}}\right] = 0$$

$$\therefore Q = -\left(\frac{3}{11}\right)^{\frac{3}{2}} \times 3 \times 10^{-9} = -0.43 \times 10^{-9} \text{ C.}$$

Illustration 8 : Four particles, each having a charge q, are placed on the four vertices of a regular pentagon. The distance of each corner from the centre is a. Find the electric field at the centre of the pentagon.

Solution : Let the charges be placed at the vertices A, B, C and D of the pentagon as shown in figure. If we put a charge q at the corner E also, the field at O will be zero by symmetry.



Therefore, $\vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_D + \vec{E}_E = 0$

$\therefore \vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_D = -\vec{E}_E$

Thus, the field at the centre due to charges at A, B, C and D is equal and opposite to the field due to the charges q at E alone.

The field at the centre due to the charge q at E is.

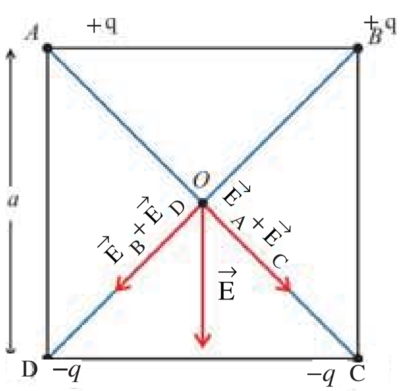
$\vec{E}_E = k\dfrac{q}{a^2}$ (along EO).

Thus, the field at O due to the charges on A, B, C and D is

$\vec{E} = k\dfrac{q}{a^2}$ (along OE direction).

Illustration 9 : Four electric charges $+q$, $+q$, $-q$ and $-q$ are respectively placed on the vertices A, B, C and D of a square. The length of the square is a, calculate the intensity of the resultant electric field at the centre.

Solution : All the electric charges are equidistant from the centre O of the square, hence the magnitude of intensity of electric field due to all the charges will be the same at point O. If r is the distance of vertices from the centre, we have,



$E_A = E_B = E_C = E_D = \dfrac{kq}{r^2}$

The directions of these electric field are as shown in figure.

If E' is the resultant field of E_A and E_C, then

$$E' = E_A + E_C = 2\frac{kq}{r^2} \tag{1}$$

In a similar way E'' is the resultant field of E_B and E_D.

$$E'' = E_B + E_D = 2\frac{kq}{r^2} \tag{2}$$

Resultant electric field, $\vec{E} = \vec{E}' + \vec{E}''$

$\therefore E = \sqrt{(E')^2 + (E'')^2}$ ($\because$ Angle between $\vec{E}'$ and $\vec{E}''$ is 90°)

$= \sqrt{\left(2\dfrac{kq}{r^2}\right)^2 + \left(2\dfrac{kq}{r^2}\right)^2}$ (from equation (1) and (2))

$$= \sqrt{\left(\frac{8k^2q^2}{r^4}\right)} = \left(\frac{2\sqrt{2}kq}{r^2}\right) \tag{3}$$

From the figure, $(2r)^2 = a^2 + a^2$

$\therefore\ 2r = \sqrt{2a^2} \quad \therefore\ r = \frac{a}{\sqrt{2}}$

Putting the value of r in equation (3),

$$E = \frac{2\sqrt{2}kq}{\left(\frac{a}{\sqrt{2}}\right)^2} = 4\sqrt{2}\,\frac{kq}{a^2}$$

The direction of E is parallel to AD (or BC).

Illustration 10 : An electron falls through a distance of 1.5 cm in a space, devoid of gravity, having uniform electric field of intensity 2.0×10^4 N C^{-1}. (Figure (a)). The direction of electric field intensity is then reversed keeping its magnitude same, in which a proton falls through the same distance. (Figure (b)). Calculate the time taken by both of them. $m_e = 9.1 \times 10^{-31}$ kg, $m_p = 1.7 \times 10^{-27}$ kg and $e = 1.6 \times 10^{-19}$ C.

Solution : As shown in the Figure (a), the direction of the electric field is vertically upward because of which the electron experiences a force eE in the vertically downward direction.

The acceleration of electron,

$$a_e = \frac{eE}{m_e}$$

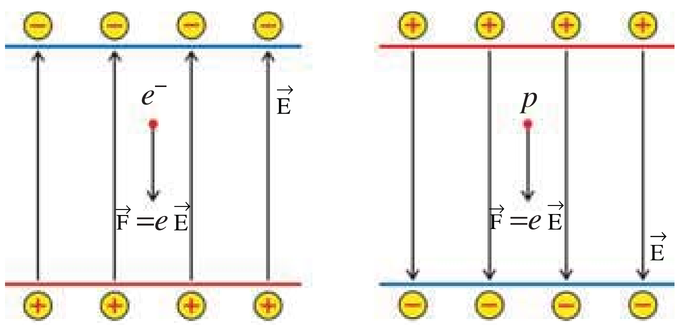


Figure (a) Figure (b)

From the equation of motion $d = v_0 t + \frac{1}{2}at^2$ (considering $v_0 = 0$) the time taken by electron to travel distance h.

$$t_e = \sqrt{\frac{2h}{a_e}} = \sqrt{\frac{2hm_e}{eE}}$$

Substituting the given data, we have $t_e = 2.9 \times 10^{-9}$s = 2.9 ns.

As shown in the figure (b) electric field is now in the vertically downward direction, the proton experiences the electric force eE in the vertically downward direction.

Therefore, acceleration of proton $a_p = \frac{eE}{m_p}$.

Therefore, the time taken by the proton to travel distance h is $t_p = \sqrt{\frac{2hm_p}{eE}}$

Substituting the given data $t_p = 1.3 \times 10^{-7}$s = 0.13μs (microsecond).

Hence, we can see that the time taken by a heavier particle is more than the time taken by a lighter particle having the same magnitude of charge in a uniform electric field.

(On the contrary, as we have studied in Standard 11, the time taken for free fall in gravitational field is independent of the mass.)

1.7 Electric Dipole

A system of two equal and opposite charges, separated by a finite distance is called an electric dipole.

As shown in figure 1.8, the two electric charges of electric dipoles are $+q$ and $-q$ and distance between them is $2a$. Electric dipole moment ($\vec{p}$) of the system can be defined as follows :

$$\vec{p} = q(2\vec{a}) \qquad (1.7.1)$$

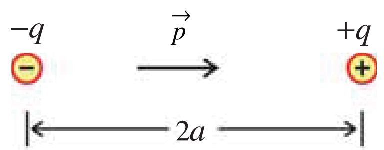


Figure 1.8 Electric Dipole

The SI unit of electric dipole is coulomb-meter (Cm). Electric dipole is a vector quantity and its direction is from the negative charge ($-q$) to positive charge ($+q$).

The net electric charge on an electric dipole is zero but its electric field is not zero, since the position of the two charges is different.

If lim $q \to \infty$ and $2a \to 0$ in $\vec{p} = 2\vec{a}q$, then the electric dipole is called a point dipole.

Electric field of a Dipole :

To find the electric field due to an electric dipole, placed the co-ordinate system such that its Z-axis coincides with the dipole and origin of system coincides with the centre of dipole. The separation between the charges of the dipole $+q$ and $-q$ is $2a$.

Here, we will determine the electric field at the point on the axis as well as point on the equator of a dipole.

Electric field at the point on the axis of a Dipole :

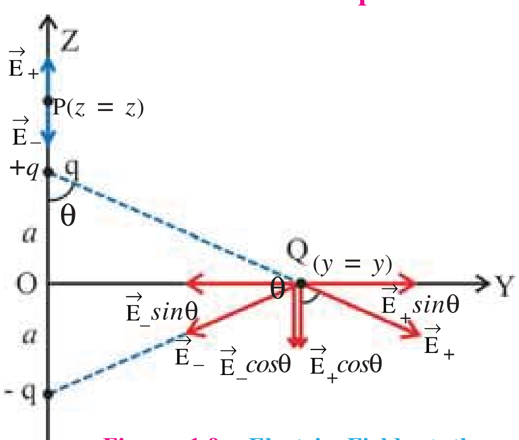


Figure 1.9 Electric Field at the Point on the Axis of a Dipole

As shown in figure 1.9, we want to determine the electric field at a point P on the axis of a dipole. Let the point P be a distance z from the origin. Hence, the distance of point P will be $z - a$ and $z + a$ from charges $+q$ and $-q$ respectively.

Electric field at point P due to charge $+q$ is,

$$\vec{E}_+ = k\frac{q}{(z-a)^2}\hat{p} \qquad (1.7.2)$$

Where, $\hat{p}$ is the unit vector along the dipole axis from $-q$ to $+q$.

Now, Electric field at point P due to charge $-q$ is, $\vec{E}_- = -k\frac{q}{(z+a)^2}\hat{p}$ (1.7.3)

According to superposition principle, the net electric field at point P is,

$$\vec{E}(z) = \vec{E}_+ + \vec{E}_- = kq\left[\frac{1}{(z-a)^2} - \frac{1}{(z+a)^2}\right]\hat{p} = kq\frac{4za}{(z^2-a^2)^2}\hat{p}$$

$$\therefore \quad \vec{E}(z) = \frac{2kpz}{(z^2-a^2)^2}\,\hat{p} \quad (\because\ 2aq = p) \qquad (1.7.4)$$

If $z >> a$, then a^2 can be neglected in comparison with z^2.

$$\vec{E}(z) = \frac{2kp}{z^3}\,\hat{p} \quad (z >> a) \qquad (1.7.5)$$

The direction of this resultant electric field is from O to P.

Electric Field at a Point on the Equator of a Dipole

The perpendicular bisector to the line joining the two electric charges of the dipole is called the equator of a dipole. Here, we want to determine the electric field at a point Q on the equator. Point Q is at a distance y from the centre of a dipole. The magnitude of electric field due to the two charges $+q$ and $-q$ will be same since they are at equal distance from point Q.

Magnitude of Electric field due to $+q$ is,

$$E_+ = k\frac{q}{(y^2+a^2)} \qquad (1.7.6)$$

Magnitude of Electric field due to $-q$ is,

$$E_- = k\frac{q}{(y^2+a^2)} \qquad (1.7.7)$$

The direction of $\vec{E}_+$ and $\vec{E}_-$ at point Q are shown in Figure 1.9.

The components of $\vec{E}_+$ and $\vec{E}_-$ normal to the dipole axis are $E_+ \sin\theta$ and $E_- \sin\theta$ respectively. These components cancelled each other, since they are of equal magnitude with opposite directions.

The components of $\vec{E}_+$ and $\vec{E}_-$ along the dipole axis are $E_+ \cos\theta$ and $E_- \cos\theta$ respectively. They will be added up since they are in the same direction.

The net electric field at point Q is opposite to $\hat{p}$ we have,

$$\vec{E}(y) = -(E_+ + E_-)\cos\theta\,\hat{p}$$

$$= -\left(\frac{kq}{(y^2+a^2)} + \frac{kq}{(y^2+a^2)}\right)\left(\frac{a}{(y^2+a^2)^{\frac{1}{2}}}\right)\hat{p} = -k\frac{(2aq)}{(y^2+a^2)^{\frac{3}{2}}}\,\hat{p}$$

$$\therefore \quad \vec{E}(y) = -\frac{kp}{(y^2+a^2)^{\frac{3}{2}}}\,\hat{p} \qquad (1.7.8)$$

If $y >> a$ then, $\vec{E}(y) = -\frac{kp}{y^3}\,\hat{p} \qquad (y >> a)$ (1.7.9)

From the equations (1.7.5) and (1.7.9) it is clear that electric field of dipole at large distance falls of not as $\frac{1}{r^2}$ but as $\frac{1}{r^3}$.

Behaviour of an Electric Dipole in a Uniform Electric Field :

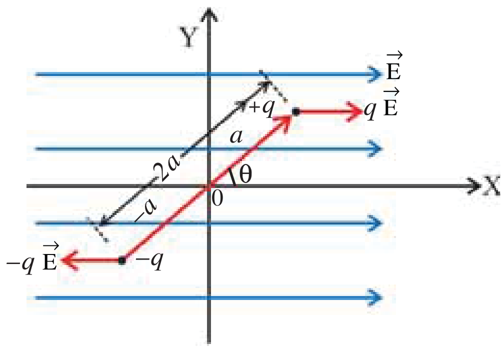


Figure 1.10 Electric Dipole in Uniform Electric Field

As shown in figure 1.10, an electric dipole of $|\vec{p}| = q|2\vec{a}|$ is kept in a uniform electric field $\vec{E}$. The origin O of co-ordinate system coincides with the centre of a dipole and electric field $\vec{E}$ is in positive X-axis. Suppose, at any instant, the angle between $\vec{p}$ and $\vec{E}$ is θ.

The forces $q\vec{E}$ and $-q\vec{E}$ are acting on $+q$ and $-q$ charges respectively. These forces are equal and in opposite direction. The resultant force being zero, keeps the dipole in translational equilibrium.

But, the two forces have different lines of action, hence the dipole will experience a torque. A torque acting on $+q$ with respect to point O, due to force $q\vec{E}$ is,

$$\vec{\tau}_1 = (\vec{a} \times q\vec{E}) \qquad (1.7.10)$$

In a similar way, the torque acting on $-q$ with respect to point O due to force $-q\vec{E}$ is,

$$\vec{\tau}_2 = (-\vec{a}) \times (-q\vec{E}) = (\vec{a} \times q\vec{E}) \qquad (1.7.11)$$

Here, $\vec{a}$ and $-\vec{a}$ are the position vectors of $+q$ and $-q$ respectively.

From the equation (1.7.10) and (1.7.11) the resultant torque acting on a dipole,

$$\vec{\tau} = (\vec{a} \times q\vec{E}) + (\vec{a} \times q\vec{E}) = 2\vec{a} \times q\vec{E} = 2\vec{a}q \times \vec{E}$$

$$\therefore \vec{\tau} = \vec{p} \times \vec{E} \qquad (1.7.12)$$

Magnitude of torque, $|\vec{\tau}| = pE\sin\theta$ (1.7.13)

The direction of torque is perpendicular to the paper, going inside of it.

The torque rotates the dipole in such a way that the angle θ reduces (In this case dipole rotates in clockwise direction), when the dipole align itself along the direction of the electric field ($\theta = 0$), the torque becomes zero. This is the normal position of dipole in electric field. If the dipole is to be rotated by an angle θ from this position, work is required to be done against the torque. This work is equal to the change in the potential energy of the dipole.

Behaviour of electric dipole in non-uniform electric field :

If the electric field is non-uniform the intensity of electric field will be different at different points as a result the electric force acting on the positive charge and negative charge of the dipoles will also be different. In this situation both the net force and torque are acting on the dipole. As a result dipole experiences a linear displacement in addition to rotation. This rotation of dipole continues only till the dipole aligns in the direction of the electric field. But linear motion of the dipole will continue.

Our common experience is that when a dry comb is rubbed against dry hair, it attracts the small pieces of papers.

Here, the comb acquires negative charge through friction. But the paper is not charged, then why does paper attract by comb ?

The non-uniform electric field is produced by the charge on the comb. Electric dipole is induced along the direction of non-uniform electric field in the small pieces of papers. When charged comb is brought near to the small pieces of paper, this non-uniform electric field exerts a net force on the small pieces of paper and paper move in the direction of comb.

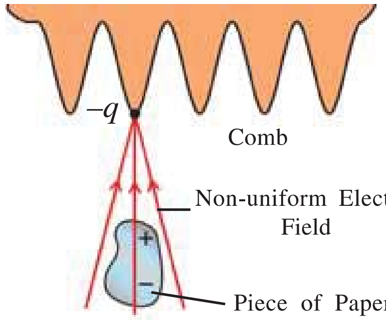


Figure 1.11 Electric Field of Comb

Illustration 11 : Calculate the magnitude of the torque on an electric dipole having dipole moment of 4×10^{-9} Cm placed in a uniform electric field of intensity of 5×10^{4} NC^{-1} making an angle 30^{o} with the field.

Solution : $p = 4 \times 10^{-9}$ Cm, $E = 5 \times 10^{4}$ NC^{-1}, $\theta = 30^{o}$, $\tau = ?$

$\tau = pEsin\theta = (4 \times 10^{-9})(5 \times 10^{4})sin30^{o} = 10^{-4}$ Nm.

1.8 Continuous Distribution of Charges

We can determine the net electric force acting on a point charge due to the discrete charges in the space using Coulomb's law and superposition principle. But, in practical situation we need to work with the continuous charge distribution. For example, a continuous charge distribution on a surface. In this situation, it is difficult to describe the effect of these charges using superposition principle. Therefore, we use the concept of charge density to describe the system of continuous charge distribution. It is not necessary that charge density will be uniform in the system.

The continuous charge distribution of electric charge can be of three types :

(1) Linear charge distribution, (2) Surface charge distribution and (3) Volume charge distribution.

(1) Linear Charge Distribution : Consider a continuous charge distribution over a line as shown in figure 1.12. We want to determine the force acting on a charge q situated at point P due to this charge distribution.

Let the amount of charge per unit length of line be λ. It is called the linear charge density.

$$\lambda = \frac{\text{Total Charge on a Line}}{\text{Length of a Line}} = \frac{Q}{l}, \text{ Unit of } \lambda \text{ is } Cm^{-1}$$

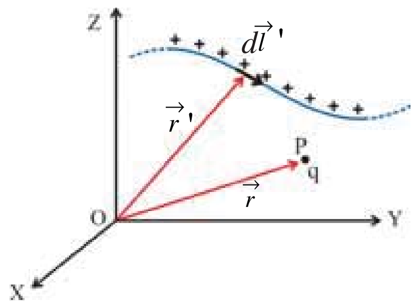


Figure 1.12 Linear Charge Distribution

If the charge distribution is not uniform, then λ will be different at different points on the line. In that case linear charge density is represented as $\lambda(\vec{r}\,')$ at a point on a line having position vector $\vec{r}\,'$.

Imagine the line to be divided into a large number of small segments of length $\vec{dl}\,'$. Such a line element $\vec{dl}\,'$ having position vector $\vec{r}\,'$ with respect to O is shown in figure 1.12. Hence, the charge in line element $\vec{dl}\,'$ will be,

$$dq = \lambda(\vec{r}\,')\,|\vec{dl}\,'| \qquad (1.8.1)$$

The force acting on charge q having position vector $\vec{r}$ will be,

$$d\vec{F} = k\frac{(q)(dq)}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,') \tag{1.8.2}$$

In order to calculate total force acting on charge q we have to add the forces like $d\vec{F}$ due to all the line elements of entire linear charge distribution. If the line elements of the charges are distributed continuously then the sum results into integration.

Total force,

$$\vec{F} = \int_l d\vec{F} = \int_l \frac{k(q)(dq)}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,')$$

$$\therefore \vec{F} = kq\int_l \frac{\lambda(\vec{r}\,')|d\vec{l}\,'|}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,') \text{ (from equation 1.8.2)} \tag{1.8.3}$$

If the charge q situated at point P is very small ($q \to 0$), then the intensity of electric field at that point due to linear charge distribution will be,

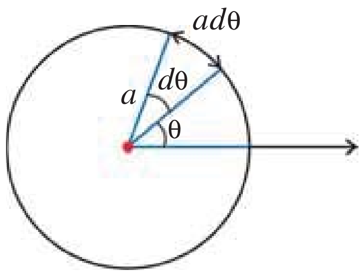
$$\vec{E} = \frac{\vec{F}}{q} = k\int_l \frac{\lambda(\vec{r}\,')|d\vec{l}\,'|}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,') \tag{1.8.4}$$

Illustration 12 : A circle, as shown in the figure, having radius 'a' has line charge distribution over its circumference having linear charge density $\lambda = \lambda_0 cos^2\theta$. Calculate the total electric charge residing on the circumference of the circle. [**Hint :** $\int_0^{2\pi} cos^2\theta\, d\theta = \pi$]

Solution : The length of an infinitesimally small line element shown in the figure is $a d\theta$, then the charge on the line element is

$dq = \lambda a d\theta = \lambda_0 cos^2\theta \; a d\theta$

In order to calculate the total charge Q residing on the surface, we have to integrate dq over the entire surface.



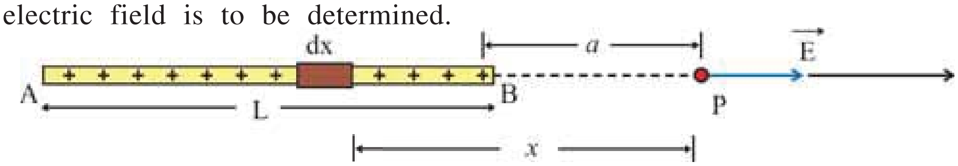
$$\therefore Q = \oint dq$$

Here the symbol $\oint$ indicates the integration over the entire closed path (circumference of the circle)

$$\therefore Q = \int_0^{2\pi} \lambda_0 cos^2\theta\, a d\theta = a\lambda_0 \int_0^{2\pi} cos^2\theta d\theta = \pi a \lambda_0$$

Illustration 13 : A conducting wire of length L carries a total charge q which is uniformly distributed on it. Find the electric field at a point located on the axis of the wire at a distance 'a' from the nearer end. (Neglect the thickness of a wire).

Solution : Consider a small element dx of the rod located at a distance x from point P where the electric field is to be determined.



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The charge in this element will be, $dq = \frac{q}{L} dx$

Hence, magnitude of electric field at point P will be,

$$dE = k\frac{dq}{x^2} = k\frac{q}{L}\frac{dx}{x^2}$$

Now, electric field at P due to entire wire,

$$E = \int_a^{L+a} dE = \frac{kq}{L}\int_a^{L+a}\frac{dx}{x^2} = \frac{kq}{L}\left[-\frac{1}{x}\right]_a^{L+a}$$

$$= \frac{kq}{L}\left[-\frac{1}{L+a}+\frac{1}{a}\right] = k\frac{q}{a(L+a)} = \frac{1}{4\pi\varepsilon_0}\frac{q}{a(L+a)}$$

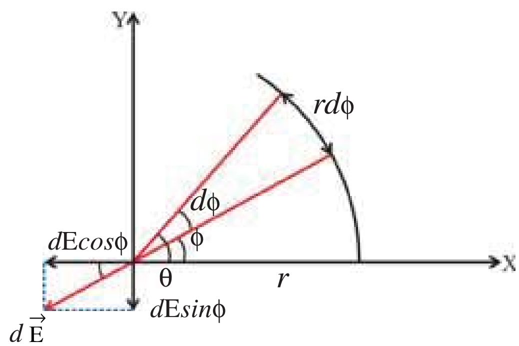
Note : If L << a, then $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$ which is same as electric field due to a point charge. If the charge q is positive, the direction of the field will be along the AP.

Illustration 14 : An arc of raidus r subtends an angle θ at the centre with the X-axis in a cartesian co-ordinate system. A charge is distributed over the arc such that the linear charge density is λ. Calculate the electric field at the origin.

Solution : The electric charge distributed on the portion of arc making an angle $d\phi$ is $d_q = \lambda r d\phi$.

The electric field at the origin due to this charge

$$dE = \frac{k\lambda r d\phi}{r^2}$$



The electric field vector $d\vec{E}$ is shown in the figure. Taking two components of $d\vec{E}$:

$$d\vec{E}_x = -\frac{k\cdot\lambda r d\phi}{r^2}\cos\phi\ \hat{i} \quad \text{and}$$

$$d\vec{E}_y = -\frac{k\cdot\lambda r d\phi}{r^2}\sin\phi\ \hat{j}$$

Now, $$\vec{E}_x = \int_0^\theta dE_x = -\frac{k\lambda}{r}\int_0^\theta \cos\phi\, d\phi\,\hat{i} = -\frac{k\lambda}{r}\left[\sin\phi\right]_0^\theta \hat{i}$$

$$\therefore \vec{E}_x = -\frac{k\lambda}{r}\sin\theta\,\hat{i} \qquad (1)$$

Now, $$\vec{E}_y = -\frac{k\lambda}{r}\int_0^\theta \sin\phi\, d\phi\,\hat{j} = -\frac{k\lambda}{r}\left[-\cos\phi\right]_0^\theta \hat{j}$$

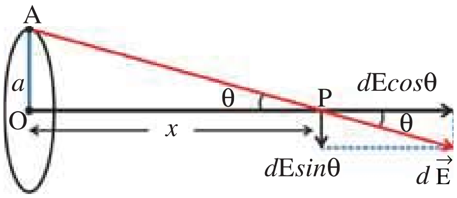
$$\therefore \vec{E}_y = -\frac{k\lambda}{r}\left[\cos\theta - 1\right]\hat{j} \qquad (2)$$

$$\vec{E} = \vec{E}_x + \vec{E}_y = -\frac{k\lambda}{r}\sin\theta\,\hat{i} + \frac{k\lambda}{r}(\cos\theta - 1)\,\hat{j} \qquad \text{(From equations (1) and (2))}$$

$$\therefore \vec{E} = \frac{k\lambda}{r}\left[(-\sin\theta)\,\hat{i} + (\cos\theta - 1)\,\hat{j}\right]$$

Illustration 15 : A charge Q is uniformly distributed on the circumference of a circular ring of radius a. Find the intensity of electric field at a point at a distance x from the centre on the axis of the ring.

Solution : Given situation is depicted in the figure. Consider an infinitesimal element at point A on the circumference of the ring. Let charge on this element be dq. The magnitude of the intensity of electric field $d\vec{E}$, at a point P situated at a distance x from the centre on its axis is,



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{AP^2} = k\frac{dq}{(a^2+x^2)} \qquad (1)$$

Its direction is from A to P. Now consider two components of $d\vec{E}$, (i) $dE\sin\theta$, perpendicular to the axis of the ring and (ii) $dE\cos\theta$, parallel to the axis.

Here it is clear that in the vector sum of intensities due to all such elements taken all over the circumference, the $dE\sin\theta$ components of the diametrically opposite elements will meet each other as they are mutually opposite. Hence only $dE\cos\theta$ components only should be considered for integration.

∴ The total intensity of electric field at point P.

$$E = \int dE\cos\theta = \int k\frac{dq}{(a^2+x^2)}\frac{OP}{AP} \qquad \left(\because \cos\theta = \frac{OP}{AP}\right)$$

$$E = k\int \frac{dq}{(a^2+x^2)}\frac{x}{(a^2+x^2)^{\frac{1}{2}}} \qquad \text{(from equation 1)}$$

$$\therefore E = k\frac{x}{(a^2+x^2)^{\frac{3}{2}}}\int_{surface} dq = \frac{kxQ}{(a^2+x^2)^{\frac{3}{2}}} = \frac{1}{4\pi\varepsilon_0}\frac{xQ}{(a^2+x^2)^{\frac{3}{2}}}$$

(2) Surface Charge Distribution :

As shown in figure 1.13 suppose the charge is distributed continuously over a surface. We want to determine the force on the charge q placed at point P having position vector $\vec{r}$ due to these charge distribution.

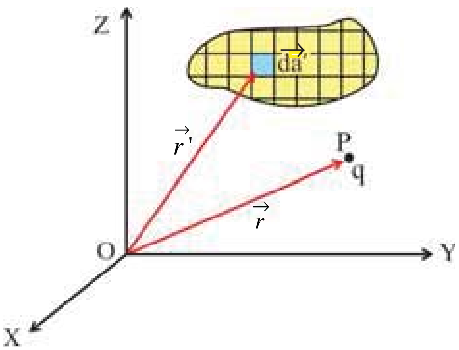


Figure 1.13
Surface Charge Distribution

Here, charge is distributed continuously over a surface having surface charge density $\sigma(\vec{r}\,')$.

Surface charge density is the charge per unit area.

$$\sigma = \frac{\text{Total Charge on the Surface}}{\text{Surface Area}} = \frac{Q}{A}, \text{ Unit of } \sigma \text{ is } Cm^{-2}.$$

Imagine the entire surface to be divided into large number of small surface element of $\vec{da}\,'$. The charge in area element $\vec{da}\,'$ will be,

$$dq = \sigma(\vec{r}\,')\,|\vec{da}\,'| \qquad (1.8.5)$$

Force acting on charge q due to this charge (dq) will be,

$$d\vec{F} = k\frac{(q)(dq)}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,') \qquad (1.8.6)$$

The total force on q due to charge on the surface can be determined by taking surface integration of above equation, From equation 1.8.5 and 1.8.6,

$$\vec{F} = \int_s d\vec{F} = kq\int_s \frac{\sigma(\vec{r}\,')|d\vec{a}\,'|}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,') \tag{1.8.7}$$

If the charge at point P is very small, then electric field at that point,

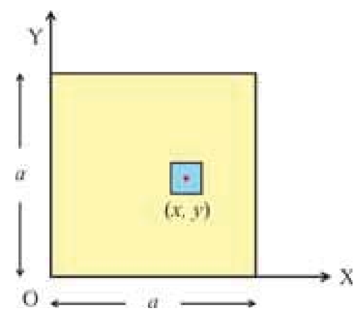
$$\vec{E} = \frac{\vec{F}}{q} = k\int_s \frac{\sigma(\vec{r}\,')|d\vec{a}\,'|}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,')$$

Illustration 16 : As shown in the figure, a square having length a has electric charge distribution of surface charge density $\sigma = \sigma_0 xy$. Calculate the total electric charge on the square. The Cartesian co-ordinate system is shown in the figure.

Solution : As shown in the figure, consider an element of area $dxdy$ at a point (x, y). The charge on the area element is,

$dq = \sigma_0 xy\ dx\ dy$

∴ Therefore, the total electric charge on the surface,



$$Q = \sigma_0 \int_0^a x\,dx \cdot \int_0^a y\,dy = \sigma_0\left[\frac{x^2}{2}\right]_0^a\left[\frac{y^2}{2}\right]_0^a = \sigma_0\left(\frac{a^2}{2}\right)\left(\frac{a^2}{2}\right)$$

$$\therefore\ Q = \frac{\sigma_0 a^4}{4}$$

(3) Volume Charge Distribution :

As shown in figure 1.14, suppose electric charge is distributed continuously over some volume and volume charge density is $\rho(\vec{r}\,')$.

Volume charge density is charge per unit volume.

$$\rho = \frac{\text{Total Charge}}{\text{Total Volume}} = \frac{Q}{V}, \text{ Unit of } \rho \text{ is } Cm^{-3}.$$

Imagine the entire volume divided into small volume elements dV'. The charge in this volume element will be,

$dq = \rho(\vec{r}\,')dV'$

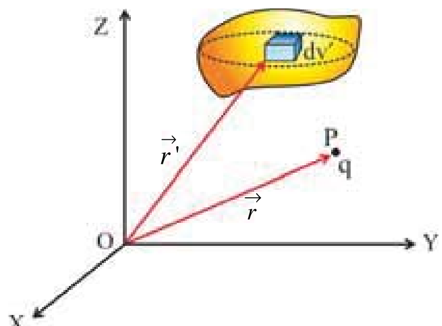


Figure 1.14 Volume Charge Distribution

Force acting on the charge q at point P having position vector $\vec{r}$ due to charge dq will be $d\vec{F} = k\frac{k(q)(dq)}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,')$

As explained earlier, the total force acting on charge q can be determined by taking volume integration of above equation.

$$\vec{F} = \int_V d\vec{F} = kq\int_V \frac{\rho(\vec{r}\,')dV'}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,')$$

If charge q is very small, then electric field at point P will be,

$$\vec{E} = \frac{\vec{F}}{q} = k\int_V \frac{\rho(\vec{r}\,')dV'}{|\vec{r}-\vec{r}\,'|^3}(\vec{r}-\vec{r}\,')$$

1.9 Electric Field Lines

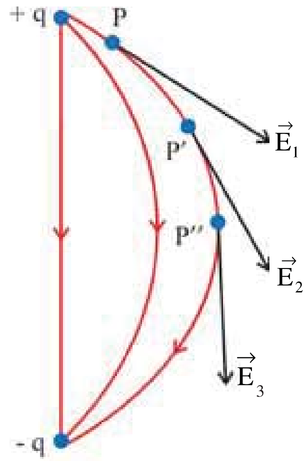


Figure 1.15 To Draw Electric Field Line of an Electric Dipole

Electric field lines are the pictorial representation of the electric field produced by the electric charge. Scientist Michael Faraday introduced the concept of electric field lines and obtained important results of electric field. (Faraday called these electric field of lines as lines of electric force.)

An electric field line is a curve drawn in an electric field in such a way that the tangent to the curve at any point is in the direction of net electric field at that point.

In fact, an electric line of field is the path along which a positive charge would move if free to do so.

Now we consider an example of electric dipole to understand the method of drawing electric field lines.

We can use the equation of electric field to determine the intensity of electric field at any point. As shown in figure 1.15 draw a vector of electric field ($\vec{E}_1$) at a point P, according to magnitude and direction at electric field at that point. Now consider another point P' close to P and draw a vector of electric field $\vec{E}_2$ at that point according to its magnitude and direction. Similarly draw a vector $\vec{E}_3$ at point P'', very close to P'. Same way other vectors of electric field can be drawn.

P, P', P'' all these points are so close to each other that a continuous curve passes through the tails of these vectors can be drawn. This curve represents the **electric field line.** Thus the curve on which the tangents drawn at different points like P, P', P'' ... and so on, represent the direction of the electric field at the respective points, is called the **electric field line.**

Characteristics of Electric Field Lines :

(1) Electric field lines start from positive charges and end at negative charges.

(2) The tangent drawn at any points on the electric field lines shows the direction of electric field at that point.

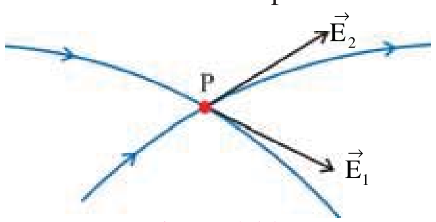


Figure 1.16

(3) Two field lines never cross each other.

If two lines intersect at a point, two tangents can be drawn at that point indicating two directions of electric field at that point which is not possible.

(4) Electric field lines of stationary electric charge distribution do not form closed loops.

(5) The separation of neighbouring field lines in a region at electric field indicates the strength of electric field in that region.

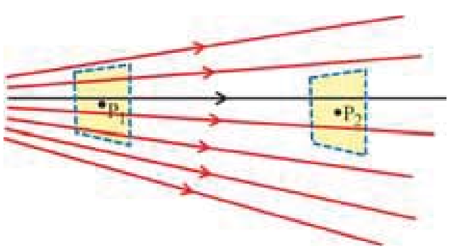


Figure 1.17 Intensity of Electric Field

In practice, the number of field lines are so restricted that the number of field lines passing through unit cross sectional area about a point, kept perpendicular to electric field lines is proportional to the intensity of electric field at that point. If the field lines are close to each other, the electric field in that region is relatively strong, if the field lines are far apart, the field is weak in that region.

From the figure 1.17, it is clear that at point P_1 electric field is relatively strong than electric field at point P_2.

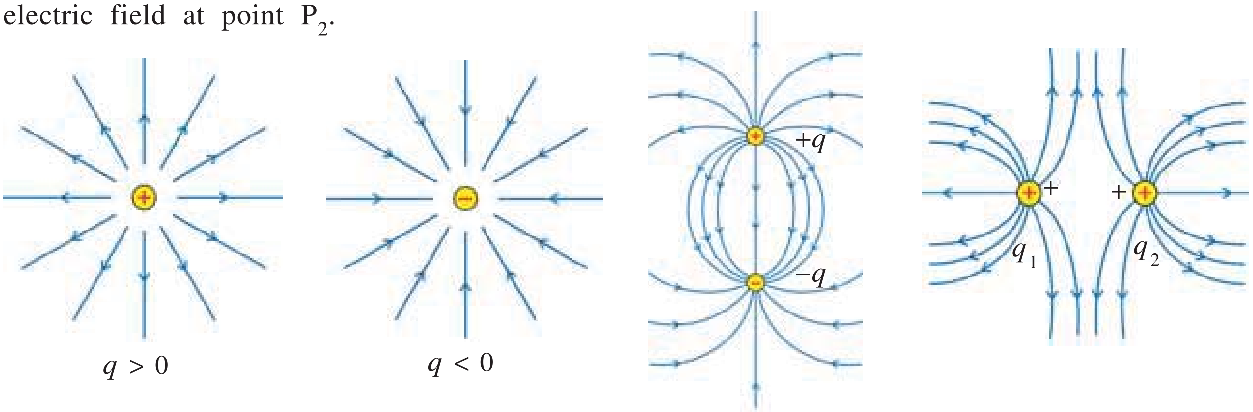


Figure 1.18 Electric Field Lines of Some Systems of Charges

(6) Field lines of uniform electric field are mutually parallel and equidistant.

Note : The electric field lines are geometrical representation of electric field and are not real. But electric field is a reality.

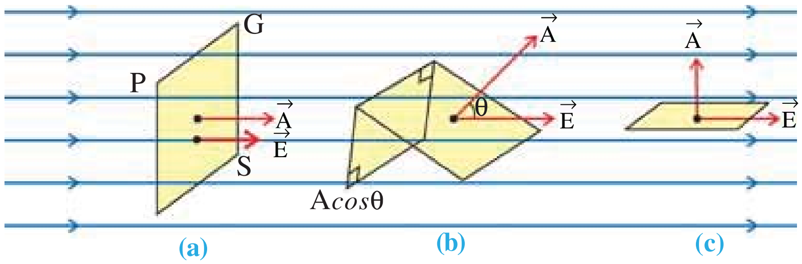
Figure 1.18 shows the electric field lines of some systems of charges.

Here, the field lines are drawn in a plane but are actually they are in three dimensional space.

1.10 Electric Flux

Coulomb's law is the fundamental law in electrostatics, we can apply Coulomb's law to find electric field at any point. Another equivalent of Coulomb's law is Gauss's law. Gauss's law is useful to determine the electric field of the system of charges having symmetry. Before we discuss Gauss's law we discuss the concept of electric flux.

The concepts of electric flux relates the electric field with its source. Flux is simply a mathematical concept which can be interpreted physically. Flux is a characteristics of all types of the vector fields.



$\theta = 0,\ \phi = EA$ $\phi = EA\cos\theta$ $\theta = 90^{\circ},\ \phi = 0$

Figure 1.19 Electric Flux for Uniform Electric Field

Electric flux is quantity proportional to the number of electric field lines passing through surface. (Here we use the word proportional because the number of lines we choose to draw arbitrary.) Consider a surface of area $\vec{A}$ placed in a uniform electric field $\vec{E}$. Surface $\vec{A}$ is perpendicular to $\vec{E}$ as shown in figure 1.19. Area $\vec{A}$ is vector quantity and its direction is along the outward drawn normal to the area. Here, area vector $\vec{A}$ and $\vec{E}$ both are in the same direction.

Electric field can also be defined in terms of the electric field lines. Electric field at any point is the number of electric field lines are passing through a surface of unit area placed perpendicular to the electric field at that point. Therefore, the number of lines passing through surface of area A will be EA. This is an electric flux ϕ associated with the given surface.

Thus, electric flux is the number of lines passing through the surface. It is represented as ϕ.

$$\therefore\ \phi = EA \tag{1.10.1}$$

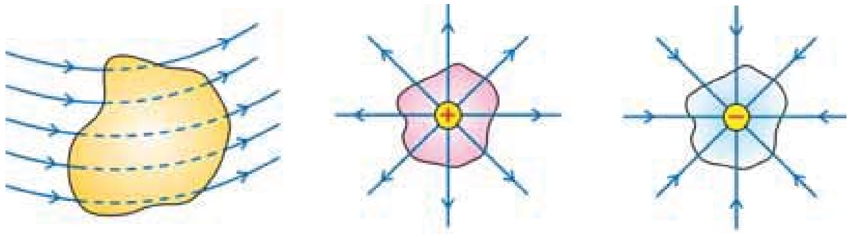
If the surface under the consideration is not perpendicular to the field, the number of lines passing through it must be less. As shown in figure 1.19(b), if the surface of area $\vec{A}$ is making an angle θ with the direction of electric field $\vec{E}$, then to determine the electric flux linked with the surface, we have to consider the $A\cos\theta$ component of the $\vec{A}$ parallel (or anti-parallel) to the electric field. Hence, Electric flux linked with the surface is,

$$\phi = EA\cos\theta \tag{1.10.2}$$

In the vector form,

$$\phi = \vec{E}\cdot\vec{A} \tag{1.10.3}$$

Electric flux is a scalar quantity. Its SI unit is $Nm^2\ C^{-1}$ or V m. From equation 1.10.2, it is clear that flux can be positive, negative or zero. If the surface is parallel to the electric field then, $\vec{A} \perp \vec{E}$. Hence, the flux linked with surface will be $\phi = EA\cos 90^\circ = 0$. For $\theta < 90^\circ$, flux is positive and for $\theta > 90^\circ$, it is negative. If the field lines are entering in the close surface, then flux linked with this surface is considered to be negative and if the field lines are leaving the surface, the flux is considered to be positive. (See the figure 1.20).



$\phi = 0$ $\quad$ $\phi > 0$ (Positive Flux) $\quad$ $\phi < 0$ (Negative Flux)

Figure 1.20 Electric flux

Now, we will discuss the general definition of electric flux.

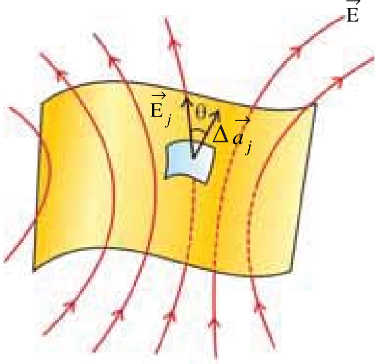


Figure 1.21 Electric Flux Linked with the Surface Placed in Non-uniform Electric Field

As shown in figure 1.21, consider an arbitrary surface in the electric field. Divide this imaginary surface into small surface elements. If the element is infinitesimally small and surface is not highly irregular, each surface element can be considered as a plane. In such a small element we can consider an electric field to be uniform. Each of these small surface element can be represented by an area vector. The magnitude of this vector should be equal to the area of surface element and direction is along the normal to the surface. If the surface is closed, i.e. surface enclosed the volume, then such vectors are drawn in outward direction of closed surface.

Suppose the vector $\Delta\vec{a}_j$ is an area vector of j^{th} element and the electric field at this element is $\vec{E}_j$. As the area of this element is very small the electric field does not change appreciably at all the points over the element. Hence, the electric flux associated with the j^{th} surface element will be.

$$\phi_j = \vec{E}_j\cdot\Delta\vec{\phi}_j \tag{1.10.4}$$

Total flux ϕ linked with the surface can be determined by adding the flux associated with all such elements.

$$\phi = \sum_j \vec{E}_j \cdot \Delta\vec{\phi}_j \qquad (1.10.5)$$

Taking $\lim |\Delta \vec{a}_j| \to 0$, i.e. considering each element as small as possible, the summation taken in equation 1.10.5 can be written as integration.

$$\phi = \lim_{|\Delta \vec{a}_j| \to 0} \sum_j \vec{E}_j \cdot \Delta\vec{\phi}_j$$

$$\phi = \int_{\text{surface}} \vec{E} \cdot d\vec{a} \qquad (1.10.6)$$

Equation 1.10.6 is known as the surface integration of $\vec{E}$ over surface a.

Thus, the general definition of electric flux can be given as follows :

'The flux lined with any surface is the surface integration of the electric field over the given surface.'

1.11 Gauss's Law

The integration of electric field over a closed surface which enclosed the charges, leads us towards the Gauss's law. Gauss's law is one of the fundamental laws of nature. To understand this law, consider the following example :

Consider a point charge $+q$ located at the centre O of the sphere of raidus r. See figure 1.22. Now, we will determine the total flux linked with the surface of a sphere.

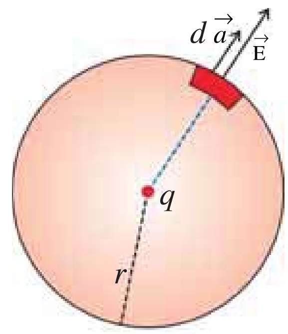


Figure 1.22 Flux Associated with the Sphere

According to definition of flux, total flux linked with the surface,

$$\phi = \int_s \vec{E} \cdot d\vec{a} = \int_s E\, da\, cos\theta \qquad (1.11.1)$$

All the points on the surface are at equidistant from the centre, hence magnitude of $\vec{E}$ will be same at every points at the surface. The electric field due to a point charge is radially outward.

Hence, area vector $\vec{da}$ of each surface element will be along the direction of $\vec{E}$ ($\theta = 0$). From equation 1.11.1

$$\phi = \int_s E\, da \quad (\because cos 0 = 1)$$

$$= E\int da = \frac{q}{4\pi\varepsilon_0 r^2} \times 4\pi r^2 \quad \text{(Area of the surface of sphere is } 4\pi r^2\text{)}$$

$$\phi = \frac{q}{\varepsilon_0} \qquad (1.11.2)$$

Here, the flux is independent of the radius of the sphere; hence it is true for any closed surface. Equation (1.11.2) is the general result of Gauss's law. Gauss's law statement is as follows, we will accept it without proof.

Guass's Law : The total electric flux associated with any closed surface is equal to the ratio of the net electric charge enclosed by the surface to ε_0.

Flux associated with any closed surface, $\phi = \int_s \vec{E} \cdot \vec{da} = \frac{\Sigma q}{\varepsilon_0}$ (1.11.3)

The law implies that the total electric flux through a closed surface is zero if no charge is enclosed by the surface.

Let us note some important points regarding this law :

(1) Gauss's Law is true for any closed surface, no matter what its shape or size.

(2) The term q on the right side of equation (1.11.3), includes the sum of all charges enclosed by the surface. The charges may be located anywhere inside the surface.

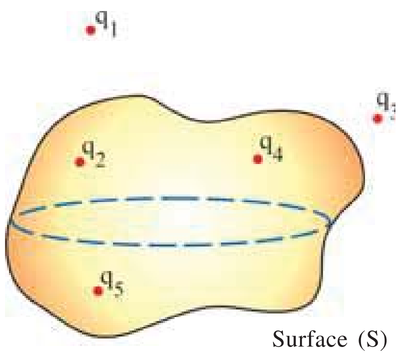


Figure 1.23

(3) The electric field appearing on the left hand side of equation 1.11.3 is the electric field produced due to a system of charges, whether enclosed by the surface or outside it.

As for example q_1, q_2, q_3, q_4 and q_5 as shown in figure 1.23. To calculate the electric flux passing through surface S, $\vec{E}$ is determined by taking the vector addition of the electric fields at the surface due to all the charges, which is used in the left side of the equation (1.11.3). But on the right side of the equation (1.11.3) we should consider the charges q_2, q_4 and q_5 to calculate net charge Σq.

Flux linked with the surface S,

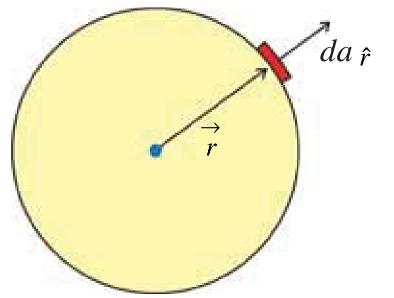
$$\phi = \frac{q_2 + q_4 + q_5}{\varepsilon_0}$$

(4) The surface that we choose for the application of Gauss's Law is called Gaussian surface.

(5) Gauss's Law is useful towards a much easier calculation of electric field when system has some symmetry.

Illustration 17 : An electric field prevailing in a region depends only on x and y co-ordinates according to an equation, $\vec{E} = b\frac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ where b is a constant. Find the flux passing through a sphere of radius r whose centre is on the origin of the co-ordinate system.

Solution : As shown in the figure, $\hat{r}$ is the unit vector in the direction of $\vec{r}$.



$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{r}$ Now, $\vec{E} = b\frac{x\hat{i}+y\hat{j}}{x^2+y^2}$

$\therefore \vec{E}\cdot d\vec{a} = b\left(\frac{x\hat{i}+y\hat{j}}{x^2+y^2}\right)\cdot\frac{x\hat{i}+y\hat{j}+z\hat{k}}{r}da = \frac{bda}{r}\frac{x^2+y^2}{x^2+y^2} = \frac{b}{r}da$

$\therefore \int\vec{E}\cdot d\vec{a} = \frac{b}{r}\int da = \frac{b}{r}.4\pi r^2 = 4\pi br$

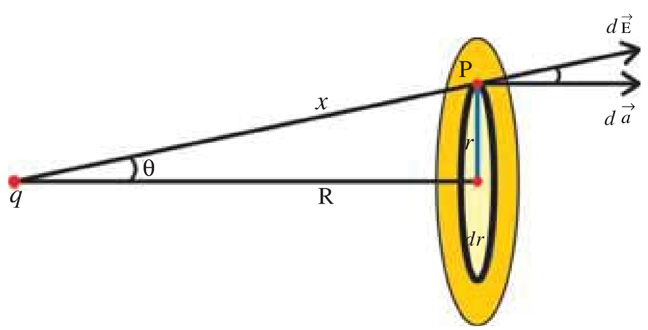
$\therefore \phi = 4\pi br$

Illustration 18 : Calculate the total electric flux linked with a circular disc of radius a, situated at a distance R from a point charge q.

[**Hint :** $\int\frac{rdr}{(R^2+r^2)^{\frac{3}{2}}} = -\frac{1}{\sqrt{(R^2+r^2)}}$]

Solution : Consider a thin circular ring of radius r and width dr as shown in figure. The electric field intensity at some point P on the ring is given by,

$|d\vec{E}| = \frac{kq}{x^2}$



The area of the ring is $|d\vec{a}| = 2\pi rdr$.

$d\vec{a}$ is perpendicular to the plane of the ring and makes an angle θ with $d\vec{E}$. The flux passing through the small area element of the disc is given by,

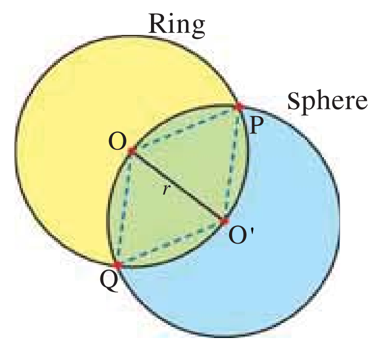
$d\phi = |d\vec{E}|\;|d\vec{a}|cos\theta$

$= \frac{kq}{x^2}\times 2\pi rdr\times\frac{R}{x} = 2\pi kqR\times\frac{rdr}{x^3} = 2\pi kqR\times\frac{rdr}{(R^2+r^2)^{\frac{3}{2}}}$ ($\because x^2 = R^2 + r^2$)

$\therefore$ Total flux $\phi = 2\pi kqR\int_0^a\frac{rdr}{(R^2+r^2)^{\frac{3}{2}}} = 2\pi kqR\left[-\frac{1}{\sqrt{(R^2+r^2)}}\right]_0^a = 2\pi kqR\left[\frac{1}{R}-\frac{1}{\sqrt{(R^2+a^2)}}\right]$

Illustration 19 : Q amount of electric charge is uniformly distributed on a ring of radius r. A sphere of radius r is drawn in such a way that the centre of the sphere lies on the surface of the ring. Calculate the electric flux associated with the surface of the sphere.

Solution : It is evident from the geometry of the sphere that OP = OO' and O'P = O'O. Hence, ΔOPO' is an equilateral triangle.



$\therefore$ ∠POO' = 60° or ∠POQ = 120°

Hence, the chord PO'Q of the ring will subtend an angle 120° at its centre. Hence, it is evident that the length of the chord will be equal to one third of the circumference of the ring. The total charge residing on this chord (enclosed by sphere) PO'Q will be equal to $\frac{Q}{3}$.

From Gauss's Law, the total flux passing through surface of the sphere is equal to $\frac{Q}{3\varepsilon_0}$.

1.12 Applications of Gauss's Law

The electric field of any symmetric charge distribution can be easily determined by using Gauss's Law. Let us consider some examples.

(1) Electric Field Due to an Infinitely Long Straight Uniformly Charged Wire :

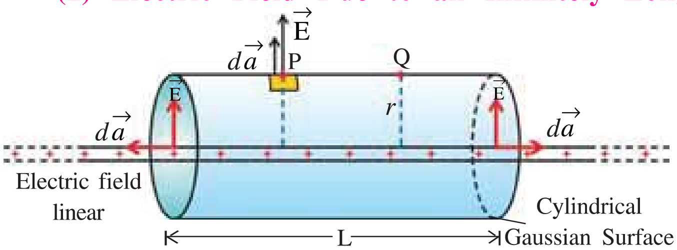


Figure 1.24 **Infinitely Long Wire having Linear Charge Distribution**

An infinitely long and linear charge distribution having uniform linear charge density λ is shown in figure 1.24. We want to find the intensity of electric field at point P situated at a perpendicular distance r from the linear charge distribution.

Since the wire is of infinite length, the electric field at all points line P, Q, ... situated at the same perpendicular distance from the wire will be same.

Now imagine a cylindrical Gaussian surface of radius r and length L, whose axis coincides with the line of linear charge distribution. At all the points at such a cylindrical surface the electric field is same and directed radially outward. The area of this curved surface of cylinder is $2\pi r$L and cross sectional area is πr^2. The charge enclosed by the cylinder of length L is $q = \lambda L$.

As shown in figure, electric flux associated with cylindrical surface of radius r and length L is,

$$\phi_1 = \int \vec{E} \cdot d\vec{a} = \int E\, da\, cos0 = E \int da$$

$$\therefore \phi_1 = E(2\pi r L) \qquad (1.12.1)$$

Now, the flux associated with the two end sides of cylinder perpendicular to axis is,

$$\phi_2 = \int \vec{E} \cdot d\vec{a} = \int E\, da\, cos90^{\circ} = 0$$

$$\therefore \text{Total flux, } \phi = \phi_1 + \phi_2 = (2\pi r L)E$$

According to Gauss's Law, $\phi = (2\pi r L)E = \frac{q}{\varepsilon_0} \Rightarrow (2\pi r L)E = \frac{\lambda L}{\varepsilon_0}$

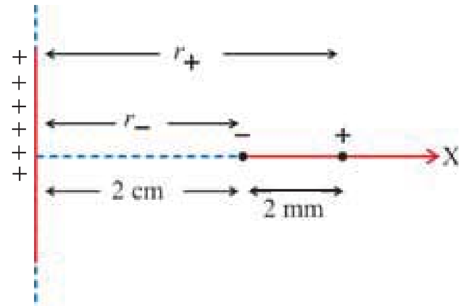
$$\therefore E = \frac{\lambda}{2\pi\varepsilon_0} \frac{1}{r} \qquad (1.12.3)$$

Electric field is in the direction of raidus, hence taking $\hat{r}$ as the unit vector in the direction of radius.

$$\therefore \vec{E} = \frac{\lambda}{2\pi\varepsilon_0} \frac{1}{r} \hat{r} \qquad (1.12.4)$$

Illustration 20 : An electric dipole is prepared by taking two electric charges of 2×10^{-8}C separated by distance 2 mm. This dipole is kept near a line charge distribution having density 4×10^{-4}C/m in such a way that the negative electric charge of the dipole is at a distance 2 cm from the wire as shown in the figure. Calculate the force acting on the dipole. Take $k = 9 \times 10^9$ Nm^{2}C^{-2}.

Solution : The electric field intensity at some point r from continuous line charge distribution having density λ is given by the following formula.



From $E = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r} = \frac{2k\lambda}{r}$, $\vec{F}_- = \frac{-2k\lambda q}{r_-}\hat{i}$ and $\vec{F}_+ = \frac{2k\lambda q}{r_+}\hat{i}$

$\therefore$ Resultant force, $\vec{F} = \vec{F}_+ + \vec{F}_- = 2k\lambda q\left[\frac{1}{r_+} - \frac{1}{r_-}\right]\hat{i}$

$$= 2 \times 9 \times 10^9 \times 4 \times 10^{-4} \times 2 \times 10^{-8}\left[\frac{1}{2.2\times10^{-2}} - \frac{1}{2.0\times10^{-2}}\right]\hat{i}$$

$$= -0.65\,\hat{i}\text{ N}$$

(2) Electric Field Due to a Uniformly Charged Infinite Plane Sheet :

As shown in Figure 1.25, we want to find the electric field at point P situated at a perpendicular distance r from the infinite plane sheet of uniform surface charge density σ.

(The figure shows only a small part at the infinite plane sheet.)

It can be inferred from the symmetry that on either side and equidistant from the plane, points like P and P' the magnitude of electric field will be same. But the direction of electric field at these two points are perpendicular to the plane and mutually opposite. (If the charge on the plane is positive / negative, the direction of the field will be away / towards the plane).

As shown in the figure 1.25 consider a close cylindrical Gaussian surface having cross-sectional area A and equal length on either side of the cylinder. The charge enclosed by the close cylinder is $q = \sigma A$ since the surface charge density of plane is σ.

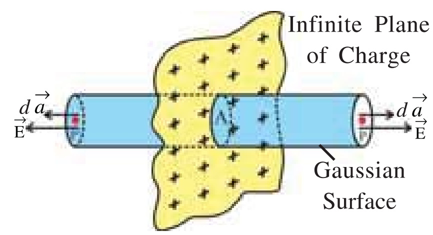


Figure 1.25 Electric Field due to Infinite Plane Sheet of Uniform Surface Charge Density

The flux linked with the curved surface of cylinder is

$$\phi_1 = \int \vec{E}\cdot d\vec{a} = \int E\,da\,cos90^\circ = 0 \qquad (1.12.5)$$

because for curved surface, $\vec{E}$ and $\vec{da}$ both are perpendicular to each other.

The flux linked with the surface of area A at the end of cylinder at point P is,

$$\phi_p = \int \vec{E}\cdot d\vec{a} = \int E\,da\,cos0 = \int E\,da = EA \qquad (1.12.6)$$

Same way, flux linked with surface area A at point P' is, $\phi_{p'} = EA$ (1.12.7)

Thus, total flux, $\phi = \phi_1 + \phi_p + \phi_{p'} = 0 + EA + EA = 2EA$

According to Gauss's Law, $\phi = 2EA = \frac{q}{\varepsilon_0}$

$\Rightarrow 2EA = \frac{\sigma A}{\varepsilon_0}$ $(\because\ q = \sigma A)$

$$\therefore\ E = \frac{\sigma}{2\varepsilon_0} \qquad (1.12.8)$$

This equation shows that electric field at a point is independent of the distance of the point from the plane.

Electric field in a vector form is represented as,

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n} \qquad (1.12.9)$$

Where, $\hat{n}$ is a unit vector normal to the plane and going away from it. If the charge on a plane is negative, then $\vec{E}$ is towards the plane and perpendicular to it.

Equation (1.12.8) is used to calculate the electric field intensity and direction of the electric field between two planes having surface charge density σ_1 and σ_2.

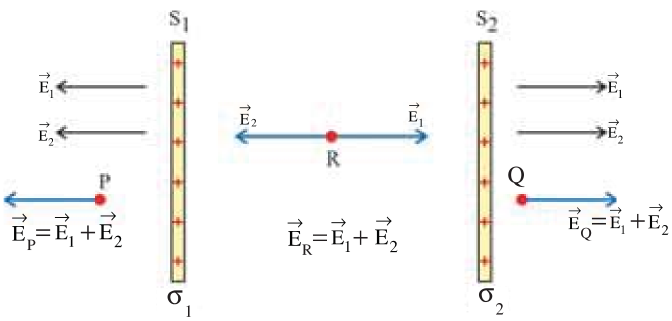


Figure 1.26

Two parallel planes S_1 and S_2 having surface charge density σ_1 and σ_2 respectively are shown in figure 1.26. $\vec{E}_1$ and $\vec{E}_2$ are the electric field produced due to the charge on the S_1 and S_2 respectively.

From the figure, electric field at point P,

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} = \frac{\sigma_1+\sigma_2}{2\varepsilon_0} \qquad \text{(in } S_2S_1 \text{ direction)}$$

Electric field at point Q,

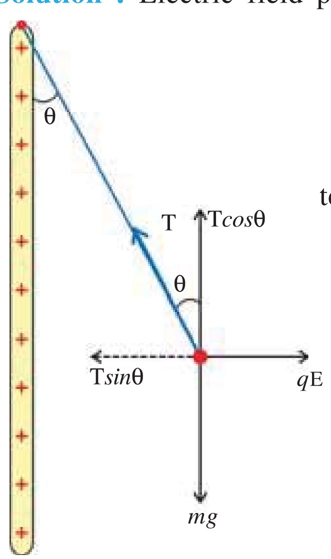
$$\vec{E}_Q = \vec{E}_1 + \vec{E}_2 = \frac{\sigma_1+\sigma_2}{2\varepsilon_0} \qquad \text{(in } S_1S_2 \text{ direction)}$$

Electric field at point R, (for $\sigma_1 > \sigma_2$)

$$\vec{E}_R = \vec{E}_1 + \vec{E}_2 = \frac{\sigma_1-\sigma_2}{2\varepsilon_0} \quad \text{(in } S_1S_2 \text{ direction)} \qquad (1.12.10)$$

Illustration 21 : A particle of mass *m* and charge *q* is attached to one end of a thread. The other end of the thread is attached to a large, vertical positively charged plate, having surface charge density σ. Find the angle the thread makes with the plate vertical in equilibrium.

Solution : Electric field produced due to positively charged plane is,



$$E = \frac{\sigma}{2\varepsilon_0}$$

The forces acting on the charge and components of the tension (T) produced in string are shown in the figure.

In the equilibrium,

$$T\,cos\theta = mg \text{ and } T\,sin\theta = qE$$

$$\therefore\ tan\theta = \frac{qE}{mg} = \frac{q\sigma}{2mg\varepsilon_0}$$

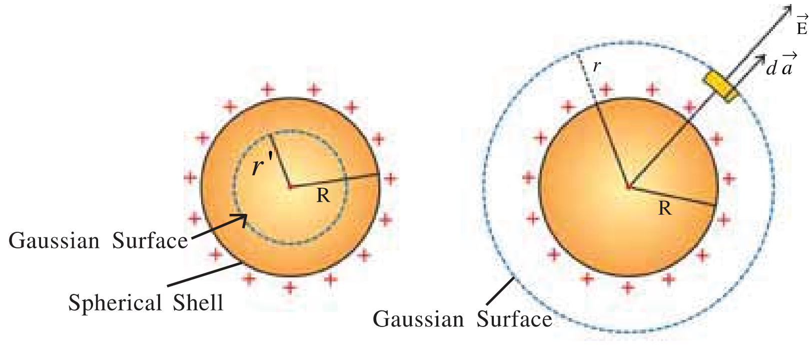
$$\therefore\ \theta = tan^{-1}\left(\frac{q\sigma}{2mg\varepsilon_0}\right)$$

(3) Electric Field Due to a Uniformly Charged Thin Spherical Shell :

Let σ be the surface charge density on a spherical shell having radius R, as shown in figure 1.27. Therefore, total charge on the shell,

$$q = \sigma A = \sigma(4\pi R^2) \qquad (1.12.11)$$

The electric field produced from such a spherical shell is radial. We want to determine the electric field at points inside and outside the spherical shell.



(a) (b)

Figure 1.27 Electric Field of a Spherical Shell

(1) For a Point Lying Inside a Shell : Consider a spherical Gaussian surface of radius $r'(r' < R)$, concentric with the shell (See figure 1.27)

Since the charge enclosed by such a surface is zero then according to Gauss's Law,

$$\int \vec{E}\cdot d\vec{a} = \frac{q}{\varepsilon_0} = 0 \qquad (\because q = 0)$$

$$\therefore \vec{E} = 0 \qquad (1.12.12)$$

Thus, electric field inside the charged spherical shell is zero.

(2) For a Point Lying Outside the Shell :

To determine electric field outside the shell, consider a spherical Gaussian surface of radius $r(r > R)$. (See figure 1.27 (b))

According to Gauss's Law, flux linked with this surface,

$$\int \vec{E}\cdot d\vec{a} = \frac{q}{\varepsilon_0}$$

$$\int E\, da \cos 0 = \frac{q}{\varepsilon_0} \quad (\because \vec{E} \text{ and } d\vec{a} \text{ are in the same direction.})$$

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad (1.12.3)$$

For an electric field on the surface of a shell, put $r = R$.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} \qquad (1.12.4)$$

Equations 1.12.3 and 1.12.4 shows that for an electric field outside the sphere the entire charge on a shell can be treated as concentrated at its centre.

Putting, $q = (4\pi R^2)\sigma$ in equation 1.12.4

$$E = \frac{1}{4\pi\varepsilon_0}\frac{(4\pi R^2)\sigma}{r^2}$$

$$\therefore E = \frac{\sigma}{\varepsilon_0}\cdot\frac{R^2}{r^2} \qquad (1.12.5)$$

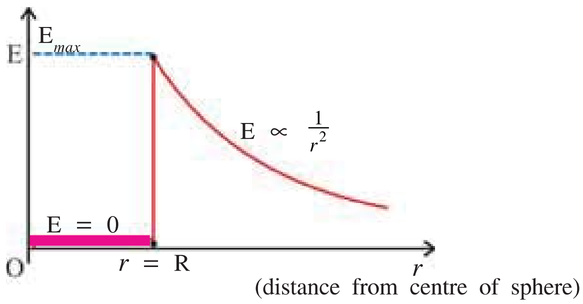


Figure 1.28 Electric Field of a Spherical Shell

Figure 1.28 shows the variation of electric field intensity with distance from the centre O to the region outside the uniformly charged spherical shell. Inside the shell $\vec{E} = 0$. The magnitude of E is maximum on the surface (r = R). However, outside the shell electric field decreases) according to $\frac{1}{r^2}$.

(4) Electric Field Intensity Due to Uniformly Charged Sphere :

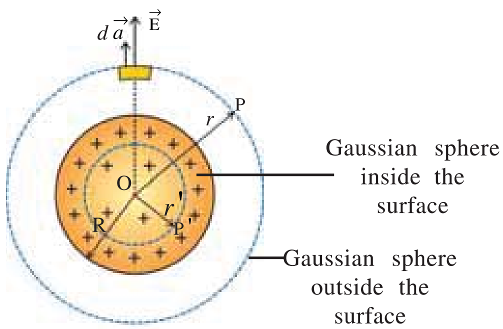


Figure 1.29 Electric Field due to Uniformly Charged Sphere

Let ρ be the volume charge density of a charged sphere having radius R as shwon in figure 1.29. The charge inside the sphere is

$$q = (\tfrac{4}{3}\pi R^3)\rho. \qquad (1.12.6)$$

The electric field due to such a sphere is radial. We want to determine the electric field at points inside and outside for such a charge sphere.

(1) For Point Lying Inside the Sphere : Imagine spherical Gaussian surface of radius r' (where $r' <$ R) concentric with sphere to determine the electric field at a distance r' (point P') from the centre of sphere. The charge enclosed by such a sphere is.

$$q' = (\tfrac{4}{3}\pi r'^3)\rho \qquad (1.12.7)$$

$$= \frac{4}{3}\pi r'^3 \times \frac{q}{\frac{4}{3}\pi R^3} \qquad \text{(From equation 1.12.6)}$$

$$\therefore q' = q\frac{r'^3}{R^3} \qquad (1.12.8)$$

The flux linked with the Gaussian surface.

$$\int \vec{E}\cdot d\vec{a} = \frac{q'}{\varepsilon_0}$$

$$E(4\pi r'^2) = \frac{q\,r'^3}{\varepsilon_0 R^3} \qquad \text{(From equation 1.12.8)}$$

$$\therefore \ E = \frac{q}{4\pi\varepsilon_0}\frac{r'}{R^3} \qquad (\text{for } r' \le R) \qquad (1.12.9)$$

i.e. Inside the sphere, $E \propto r'$

By putting the value of q from equation 1.12.6, we can represent electric field in terms of charge density.

$$E = \frac{\rho r'}{3\varepsilon_0} \qquad (\text{for } r' \le R) \qquad (1.12.10)$$

(2) For Point Lying Outside the Sphere : Now, consider a Gaussian surface of radius r (where $r > R$). The centres of two spheres coincide with each other. The charge enclosed by this surface is q. According to Gaussian's Law.

$$\int \vec{E} \cdot d\vec{a} = \frac{q}{\varepsilon_0}$$

$$\int E\, da = \frac{q}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\therefore \ E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad (\text{for } r \ge R) \qquad (1.12.11)$$

This shows that a point outside the sphere the entire charge of the sphere can be considered as concentrated at its centre. Thus, for a point outside the sphere, $E \propto \frac{1}{r^2}$.

In above equation put $q = (\frac{4}{3}\pi R^3)\rho$,

We can have electric field in terms of ρ.

$$E = \frac{R^3\rho}{3r^2\varepsilon_0} \qquad (1.12.12)$$

Figure 1.30 shows the variation of electric field intensity with distance r from the centre O to the region outside the charged sphere. Note that electric field on a surface of sphere is maximum $(E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2})$

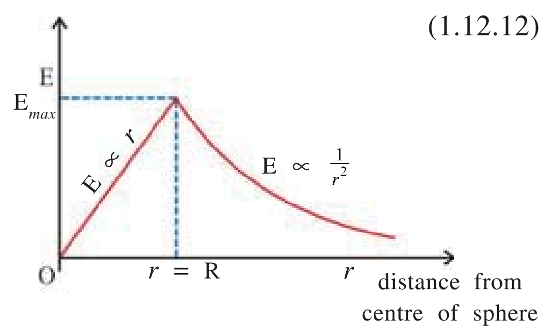


Figure 1.30 Electric Field of a Charged Sphere

SUMMARY

1. **Electric Charge :** Just as masses of two particles are responsible for the gravitational force, charges are responsible for the electric force. Electric charge is an intrinsic property of a particle.

   Charges are of two types : (1) Positive charge (2) Negative charge.

   The force acting between two like charges is repulsive and it is attractive between two unlike charges.

   The SI unit of charge is coulomb (C).

2. **Quantization of Electric Charge :** The magnitude of all charges found in nature are in integral multiple of a fundamental charge. Q = ne where, e is the fundamental unit of charge.
3. **Conservation of Electric Charge :** Irrespective of any process taking place, the algebraic sum of electric charges in an electrically isolated system always remains constant.
4. **Coulomb's Law :** The electric force between two stationary point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.

   $$F = k\frac{q_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

   If $q_1q_2 > 0$, then there is a repulsion between the two charges and for $q_2q_2 < 0$ there is an attractive force between the charges.
5. **Electric Field Intensity :** The force acting on a unit positive charges at a given point in an electric field of a system of charges is called the electric field or the intensity of electric field ($\vec{E}$) at that point.

   $$\vec{E} = \frac{\vec{F}}{q}$$

   The SI unit of $\vec{E}$ is NC^{-1} or Vm^{-1}.

   If $\vec{r_1}$, $\vec{r_2}$, ... $\vec{r_n}$ are the position vectors of the charge q_1, q_2, ... q_n respectively then neot electric field at a point of position vector $\vec{r}$ is,

   $$\vec{E} = k\sum_{j=1}^{n}\frac{q_j}{|\vec{r}-\vec{r_j}|^3}(\vec{r}-\vec{r_j})$$

6. **Electric Dipole :** A system of two equal and opposite charges, separated by a finite distance is called an electric dipole.

   Electric dipole moment $\vec{p} = (2\vec{a})q$

   The direction of $\vec{p}$ is from the negative electric charge to the positive electric charge.
7. Electric field of a dipole on the axis of the dipole at point $z = z$,

   $$\vec{E}(z) = \frac{2kp}{z^3}\hat{p} \quad (\text{for } z >> a)$$

   Electric field of a dipole on the equator of the dipole at point y = y.

   $$\vec{E}(y) = -\frac{kp}{y^3}\hat{p} \quad (\text{for } y >> a)$$

8. The torque acting on the dipole place in the electric field at an angle θ,

   $$\vec{\tau} = \vec{p} \times \vec{E},\ |\vec{\tau}| = pE\sin\theta$$

9. **Electric Flux :** Electric flux associated with surface of area $\vec{A}$, placed in the uniform electric field.

$\phi = \vec{E} \cdot \vec{A} = EA\cos\theta$

where θ is the angle between $\vec{E}$ and $\vec{A}$.

Its unit is Nm^2C^{-1} or Vm.

10. **Gauss's Law :** The total electric flux associated with the closed surface,

$$\phi = \int_s \vec{E} \cdot d\vec{a} = \frac{\Sigma q}{\varepsilon_0}$$

where Σq is the net charge enclosed by the surface.

11. Electric field due to an infinitely long straight charged wire,

$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0} \frac{1}{r} \hat{r}$, where, r is the perpendicular distance from the wire.

12. Electric field due to uniformly charged infinite plane, $E = \frac{\sigma}{2\varepsilon_0}$

13. Electric field due to uniformly charged thin spherical shell,

(1) Electric field inside the shell $\vec{E} = 0$.

(2) Electric field at a distance r from the centre outside the shell,

$E = k\frac{q}{r^2} = \frac{\sigma}{\varepsilon_0} \frac{R^2}{r^2}$, where R = radius of spherical shell.

14. Electric field due to a uniformly charged sphere of radius R,

(1) Electric field inside the region of the sphere :

$$E = \frac{Q}{4\pi\varepsilon_0} \frac{r}{R^3} = \frac{\rho r}{3\varepsilon_0}$$

(2) Electric field outside the sphere,

$$E(r) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2} = \frac{R^3\rho}{3r^2\varepsilon_0}$$

where, Q is the total charge inside the sphere.

EXERCISE

For the following statements choose the correct option from the given options :

1. The force acting between two point charges kept at a certain distance is ϕ. Now magnitudes of charges are doubled and distance between them is halved, the force acting between them is ....

(A) ϕ (B) 4ϕ (C) 8ϕ (D) 16ϕ

2. An electric dipole is placed in a uniform field. The resultant force acting on it ....

(A) always be zero (B) depends on its relative position

(C) never be zero (D) depends on its dipole moment.

3. An electric dipole is placed in an electric field of a point charge, then ....

(A) the resultant force acting on the dipole is always zero

(B) the resultant force acting on the dipole may be zero

(C) torque acting on it may be zero

(D) torque acting on it is always zero.

4. When an electron and a proton are both placed in an electric field ....

(A) the electric forces acting on them are equal in magnitude as well as direction.

(B) only the magnitudes of forces are same

(C) accelerations produced in them are same

(D) magnitudes of accelerations produced in them are same.

5. The electric force acting between two point charges kept at a certain distance in vacuum is α. If the same two charges are kept at the same distance in a medium of dielectric constant K. The electric force acting between them is ....

(A) α (B) $K\alpha$ (C) $K^2\alpha$ (D) α/K

6. The distance between two point charges $4q$ and $-q$ is r. A third charge Q is placed at their midpoint. The resultant force acting on $-q$ is zero then Q = ...

(A) $-q$ (B) q (C) $-4q$ (D) $4q$

7. The linear charge density on the circumference of a circle of radius 'a' varies as $\lambda = \lambda_0 cos\theta$. The total charge on it is ....

(A) zero (B) infinite (C) $\pi a\lambda_0$ (D) $2\pi a$

8. Two identical metal spheres A and B carry same charge q. When the two spheres are at distance r from each other, the force acting between them is F. Another identical sphere C is first brought in contact with A, then it is touched to sphere B and then separated from it. Now the force acting between A and B at the same distance is ....

(A) F (B) 2F (C) $\frac{3F}{8}$ (D) $\frac{F}{4}$

9. Two point charges of q and $4q$ are kept 30 cm apart. At a distance ...., on the straight line joining them, the intensity of electric field is zero.

(A) 20 cm from $4q$ (B) 7.5 cm from q

(C) 15 cm from $4q$ (D) 5 cm from q

10. The dimensions of permittivity $[\varepsilon_0]$ are .... .Take Q as the dimension of charge.

(A) $M^1L^{-2}T^{-2}Q^{-2}$ (B) $M^{-1}L^2T^{-3}Q^{-1}$ (C) $M^{-1}L^{-3}T^2Q^2$ (D) $M^{-1}L^3T^{-2}Q^{-2}$

11. The electric dipole moment of an HCL atom is 3.4×10^{-30} Cm. The charges on both atoms are unlike and of same magnitude. Magnitude of this charge is .... The distance between the charges is 1 $\overset{o}{A}$

(A) 1.7×10^{-20} C (B) 3.4×10^{-20} C (C) 6.8×10^{-20} C (D) 3.4×10^{-10} C

12. There exists an electric field of 100 N/C along Z–direction. The flux passing through a square of 10 cm sides placed on XY plane inside the electric field is ....

(A) 1.0 Nm^2/C (B) 2.0 Vm (C) 10 Vm (D) 4.0 Nm^2/C

13. The radius of a conducting spherical shell is 10 mm and a 100 μC charge is spread on it. The force acting on a 10 μC charged placed at its centre is .... $k = 9 \times 10^9$ MKS.

(A) 10^3N (B) 10^2N (C) zero (D) 10^5N

14. When a 10 μC charge is enclosed by a closed surface, the flux passing through the surface is ϕ. Now another -10 μC charge is placed inside the closed surface, then the flux passing through the surface is ....

(A) 2ϕ (B) ϕ (C) 4ϕ (D) zero

15. An electric dipole is placed at the centre of a sphere. The flux passing through the surface of the sphere is ....

(A) Infinity (B) zero (C) cannot be found (D) $\frac{2q}{\varepsilon_0}$

16. Two spheres carrying charge q are hanging from a same point of suspension with the help of threads of length 1 m, in a space free from gravity. The distance between them will be ....

(A) 0 (B) 0.5

(C) 2 m (D) cannot be determined.

17. One point electric charge Q is placed at P. A closed surface is placed near the point P. The electrical total flux passing through a surface of the sphere will be

(A) Q ε_0 (B) $\frac{\varepsilon_0}{Q}$ (C) $\frac{Q}{\varepsilon_0}$ (D) zero

18. Charge Q each is placed on $(n - 1)$ corners of a polygon of sides n. The distance of each corner from the centre of the polygon is r. The electric field at its centre is ....

(A) $k\frac{Q}{r^2}$ (B) $(n - 1)\ k\frac{Q}{r^2}$ (C) $\frac{n}{n-1}k\frac{Q}{r^2}$ (D) $\frac{n-1}{n}k\frac{Q}{r^2}$

19. When two spheres having 2Q and $-$Q charge are placed at a certain distance, the force acting between them is F. Now they are connected by a conducting wire and again separated from each other. How much force will act between them if the separation now is the same as before ?

(A) F (B) $\frac{F}{2}$ (C) $\frac{F}{4}$ (D) $\frac{F}{8}$

20. The number of electric field of lines emerged out from 1 C charge is ....

($\varepsilon_0 = 8.85 \times 10^{-12}$ MKS)

(A) 9×10^9 (B) 8.85×10^2 (C) 1.13×10^{11} (D) infinite

21. When 10^{19} electrons are removed from a neutral metal plate through some process, the charge on it becomes ....

(A) -1.6 C (B) $+ 1.6$ C (C) 10^9 C (D) 10^{-19} C

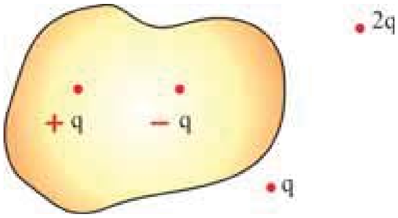
22. A charge Q is placed at the centre of a cube. The electric flux emerging from any one surface of the cube is ....

(A) $\frac{Q}{\varepsilon_0}$ (B) $\frac{Q}{2\varepsilon_0}$ (C) $\frac{Q}{4\varepsilon_0}$ (D) $\frac{Q}{6\varepsilon_0}$

23. The liquid drop of mass m has a charge q. What should be the magnitude of electric field E to balance this drop ?

(A) $\frac{mg}{q}$ (B) $\frac{E}{m}$ (C) mgq (D) $\frac{mq}{g}$

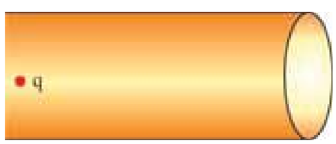
24. As shown in figure the electric flux associated with close surface is ....



(A) $\frac{3q}{\varepsilon_0}$ (B) $\frac{2q}{\varepsilon_0}$

(C) $\frac{q}{\varepsilon_0}$ (D) zero

25. As shown in the figure, q charge is placed at the open end of the cylinder with one end open. The total flux emerging from the surface of cylinder is ....



(A) $\frac{q}{\varepsilon_0}$ (B) $\frac{2q}{\varepsilon_0}$

(C) $\frac{q}{2\varepsilon_0}$ (D) zero

ANSWERS

1. (D)	2. (A)	3. (C)	4. (B)	5. (D)	6. (A)
7. (A)	8. (C)	9. (A)	10. (C)	11. (B)	12. (A)
13. (C)	14. (D)	15. (B)	16. (C)	17. (D)	18. (A)
19. (D)	20. (C)	21. (B)	22. (D)	23. (A)	24. (D)
25. (C)					

Answer the following questions in brief :

1. How many number of protons of the charge is equivalent to a 1 μC ?
2. Two identical metal spheres of equal radius are taken. One of the spheres has charge of 1000 electrons and another has charge of 600 protons. When the two spheres are brought in contact with copper wire and removed, what will be the charges on each sphere ?
3. If $q_1q_2 > 0$, which type of the force acting between two charges ?
4. What is a test charge ? What should be its magnitude ?
5. Define the electric dipole moment and give its SI unit.
6. What will be the torque acting on the dipole if it is placed parallel to the electric field.
7. Explain the behaviour of electric dipole placed in the non-uniform electric field.
8. Give the statement of Gauss's Law.
9. Why does the two electric field lines not intersecting each other ?
10. Draw the electric field lines of electric dipole.
11. A charge enclosed by the spherical Gaussian surface is 8.85×10^{-8}C. What is the electric flux linked with this surface ? If the radius of sphere is doubled, what is the electric flux ?

12. Write the conservation law of electric charge.
13. An electric dipole is placed at the centre of the cube. What is the total electric flux linked with the surfaces of the cube ?

Answer the following questions :

1. Write the Coulomb's Law and represent the forces between the two charges in vector form.
2. Explain the linear charge density, surface charge density and volume charge density. Also give their units.
3. What is electric field ? Explain, giving the characteristics of the electric field.
4. Obtain the expression of the electric field at a point on the axis of the electric dipole.
5. Obtain the expression for the torque acting on the electric dipole place in the uniform electric field.
6. Write the characteristics of the lines of the electric field.
7. Write and explain the Gauss's Law.
8. Obtain the expression of the electric field due to an infinitely long linear charged wire along the perpendicular distance from the wire.
9. Derive the expression of the electric field produced due to uniformly charged infinite plane.
10. Using Gauss's Law, find the intensity of the electric field inside and outside the charged sphere having uniform volume charge density.

Solve the following examples :

1. A metal sphere is suspended through a nylon thread. When another charged sphere (identical to A) is brought near to A and kept at a distance d, a force of repulsion F acts between them. Now A is brought in contact with an identical uncharged spheres C and B also brought in contact with an identical uncharged sphere D and then they are separated from each other. What will be the force between the spheres A and B when they are at a distance $\frac{d}{2}$? [**Ans. :** F]
2. Two identically charged spheres are suspended by strings of equal length. When they are immersed in kerosene, the angle between their strings remains the same as it was in the air. Find the density of the spheres. The dieletric constant of kerosene is 2 and its density is 800 kg m^{-3}. [**Ans. :** 1600 kg m^{-3}]
3. Three point charges 0.5 μC, −0.25 μC and 0.1 μC are placed at the vertices A, B and C of an equilateral triangle ABC. The length of the side of triangle is 5.0 cm. Calculate resultant force acting on the charge placed at point C. $k = 9 \times 10^9$ SI.

   [**Ans. :** $\vec{F}_3 = 0.045\ (3, \sqrt{3})$N]
4. Three identical charges q are placed on the vertices of an equilateral triangle. Find the resultant force acting on the charge $2q$ kept at its centroid. (The distance of the centroid from vertices is 1 m). [**Ans. :** Zero]
5. An electric dipole of momentum $\vec{p}$ is placed in a uniform electric field. The dipole is rotated through a very small angle θ from equilibrium and is released. Prove that it executes simple harmonic motion with frequency $f = \frac{1}{2\pi}\sqrt{\frac{pE}{I}}$. Where, I = moment of inertia of the dipole.

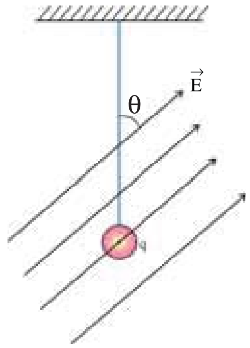
6. The surface charge density of a very large surface is $-3.0 \times 10^{-6} Cm^{-2}$. From what distance should an electron of 150 eV energy be projected towards the plane so that its velocity becomes zero on reaching the plane ? Charge of an electron = 1.6×10^{-19}C, 1 eV = 1.6×10^{-19}J, $\varepsilon_0 = 9 \times 10^{-12}$ SI. [**Ans. :** 9×10^{-4}m]

7. Two small, identical spheres, one positively charged and another negatively charged are placed 0.5m apart attract each other with a force of 0.108N. If they are brought in contact for some time and again separated by 0.5m, they repelled each other with force of 0.036N. What were the initial charges on the spheres ?

[**Ans. :** $q_1 = \pm 3.0 \times 10^{-6}$C, $q_2 = \mp 1.0 \times 10^{-6}$C]

8. Two charged particles of mass m and 2m have charges $+2q$ and $+q$ respectively. They are kept in a uniform electric field far away from each other and then allowed to move for some time t. Find the ratio of their kinetic energy. [**Ans. :** 8 : 1]

9.

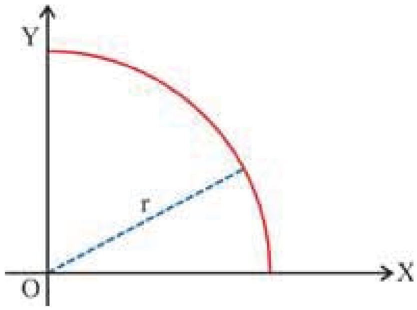


A simple pendulum is suspended in a uniform electric field $\vec{E}$ as shown in the figure. What will be its period if its length is l ? Charge on the bob of pendulum is q and mass is m.

[**Ans. :** $T = 2\pi\sqrt{\dfrac{l}{\left(g^2 + \dfrac{q^2E^2}{m^2} - \dfrac{2gqE}{m}\cos\theta\right)^{\frac{1}{2}}}}$]

10. A charge of 4×10^{-8}C is uniformly distributed over the surface of sphere of radius 1cm. Another hollow sphere of radius 5cm is concentric with the smaller sphere. Find the intensity of the electric field at a distance 2cm from the centre. $k = 9 \times 10^{-9}$ SI. [**Ans. :** 9×10^5 NC^{-1}]

11.



An arc of radius r, lying in the first quadrant is shown in the figure. The linear charge density on the arc is λ. Calculate the magnitude and direction of electric field intensity at the point of origin.

[**Ans. :** $E = \dfrac{\sqrt{2}k\lambda}{r}$, making an angle 45° with X-axis in the third quadrant]

12. A particle of mass 5×10^{-9} kg is held at some distance from very large uniformly charged plane. The surface charge density on the plane is $4 \times 10^{-6} C/m^2$. What should be the charge on the particle so that the particle remains stationary even after releasing it ? $\varepsilon_0 = 8.85 \times 10^{-12} C^{-2}N^{-1}m^{-2}$, $g = 9.8$ ms^{-2}

[**Ans. :** $q = 2.17 \times 10^{-13}$C)

13. In the hydrogen atom, an electron revolves around a proton in a circular orbit of radius 0.53 Å. Calculate the radial acceleration and the angular velocity of the electron. $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$C.

[**Ans. :** $a_r = 9.01 \times 10^{22}$ m/s^2, ω = 3 9 × ^{16} rad/s)

•

2 Electrostatic Potential and Capacitance

2.1 Introduction

In Chapter 1, we learned about the types of electric charge, the forces acting between the charges, the electric fields produced by a point charge and by different charge distributions and Gauss' theorem. The **force** acting on a given **charge** q can be found by knowing the electric field. Now, if the electric charge is able to move due to this force, it will start moving and in such a motion work will be done. So, now in this chapter we shall study in detail, the physical quantities like electrostatic energy, electrostatic potential that give information about the work done on the charge. Moreover electric potential and electric field, both the quantities can be obtained from each other. We will also know the relation between them.

A simple device which stores the electric charge and electrical energy is a **capacitor**. We shall also study about the capacitance of a capacitor, the series and parallel combinations of capacitors, the electrical energy stored in it, etc. The capacitors are used in different electrical and electronic circuits e.g. electric motor, flashgun of a camera, pulsed lasers, radio, TV etc. At the end of the chapter we shall see about a device–with the help of which we can get a very large potential difference–Van de Graaff generator.

2.2 Work done during the Motion of an Electric Charge in the Electric Field

We had seen in Chapter-1 that when an electric charge q is placed at a point in an electric field $\vec{E}$, a force $\vec{F} = q\vec{E}$, acts on it. Now, if this charge is able to move, it starts moving. To discuss the work done during such a motion, initially we will consider a unit positive charge.

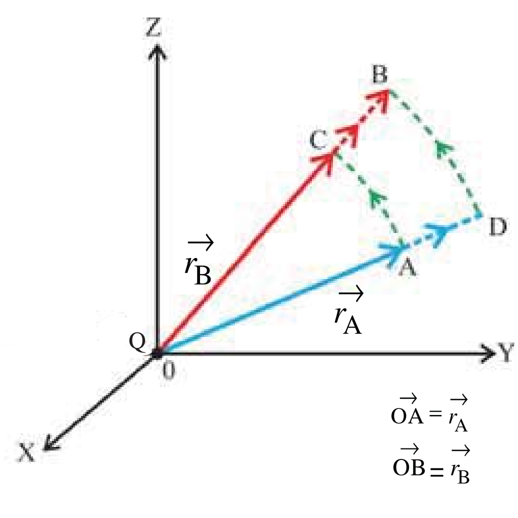


Figure 2.1 Work during the Motion of a Charge

As shown in the figure 2.1, we want to take a unit positive charge (q = +1 C charge) from point A to point B, in the electric field produced by a point charge (Q), and also want to find the **work done by the electric field** during this motion. Many different paths can be thought of to go from A to B. In the figure 2.1 ACB and ADB paths are shown as illustrations.

According to the definition, the force on the unit positive charge at a given point, is the electric field $\vec{E}$ at that point. According to the formula $E = \frac{kQ(1)}{r^2}$ this force varies continuously with distance. Hence the work done by the electric field on unit positive charge in a small displacement is given by $dW = \vec{E} \cdot \vec{dr}$ and the work done during A to B by $W_{AB} = \int_A^B \vec{E} \cdot \vec{dr}$ (2.2.1)

Here, $\int_A^B \vec{E} \cdot \vec{dr}$ is called the **line integral of electric field** between the points A and B.

ACB Path : (1) First, we go from A to C on the circular arc AC having radius OA and then we go from C to B in $\vec{OC}$ direction. The electric field produced by Q, is normal to the arc AC at every point on it (the angle between $\vec{E}$ and $\vec{dr}$ = 90°). Hence $W_{AC} = \int_A^C \vec{E} \cdot \vec{dr} = 0$.

The work done by the electric field on the path CB, is

$$W_{CB} = \int_C^B \vec{E} \cdot \vec{dr} \tag{2.2.2}$$

$$= \int_C^B \frac{kQ}{r^2} \hat{r}_B \cdot dr \hat{r}_B = kQ \int_C^B \frac{1}{r^2} dr = kQ \left[-\frac{1}{r} \right]_{r_C}^{r_B}$$

$$W_{CB} = kQ \left[\frac{1}{r_C} - \frac{1}{r_B} \right] \tag{2.2.3}$$

Thus, on the path ACB, the work done by the electric field

$$W_{ACB} = W_{AC} + W_{CB} = kQ \left[\frac{1}{r_C} - \frac{1}{r_B} \right] \tag{2.2.4}$$

Here, since $r_C < r_B$, it is self-evident that this work is positive.

(2) Path ADB : From A to D, just like the above, the work done by the electric field is obtained as $W_{AD} = kQ \left[\frac{1}{r_A} - \frac{1}{r_D} \right]$. Moreover, since the electric field is normal to the arc DB, the work done in this motion = 0.

Hence the work done by the electric field on ADB path is

$$W_{ADB} = W_{AD} + W_{DB} = kQ \left[\frac{1}{r_A} - \frac{1}{r_D} \right] \tag{2.2.5}$$

Here $|\vec{r_D}| = |\vec{r_B}|$ and $|\vec{r_A}| = |\vec{r_C}|$. Hence from equations 2.2.4 and 2.2.5,

$$W_{ACB} = W_{ADB} = W_{AB} = kQ \left[\frac{1}{r_A} - \frac{1}{r_B} \right] \tag{2.2.6}$$

Thus, in an electric field, the work done by the **electric field** in moving a unit positive charge from one point to the other, **depends only on the positions of those two points and does not depend on the path joining them.**

Now, if we move the unit positive charge from point B to A, on any path, the work done by the electric field, will be given by (according to equation 2.2.6)

$$W_{BA} = kQ\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \quad (2.2.7)$$

If a unit positive charge is taken from point A to B on any path and then is brought back to A on any path, a closed loop is formed (e.g. ACBDA or ADBCA) and on this closed loop the total work done by the electric field ($\oint \vec{E} \cdot \vec{dr}$); will be $W_{AB} + W_{BA} = 0$ (using equations 2.2.6 and 2.2.7). You are aware of the fact that a field with this property is known as a **conservative field.** Thus electric field is also a conservative field. [In Standard 11 you had also seen that the gravitational field is also a conservative field.]

Although we have considered the work done on unit positive charge, all these aspects are also applicable to the work done on any charge q, but for that, the right hand side of the above equations for the work, should be multiplied by q. e.g., Work for A to B will be $W_{AB} = \int_A^B q\vec{E} \cdot \vec{dr}$. Moreover, you will be able to understand that instead of the work done **by the electric field,** if we want to find the **work required to be done by the external force against the electric field** (for the motion without acceleration), then the **negative sign** will have to be put on the right hand side of the above equation (2.2.1) for the work. Hence for unit positive charge, such a work will be given by $W'_{AB} = -\int_A^B \vec{E} \cdot \vec{dr}$, which is the same in magnitude as work given by equation 2.2.1 but has the opposite sign to it. For charge q such a work will be given by $W''_{AB} = -\int_A^B q\vec{E} \cdot \vec{dr}$.

From this discussion we should remember that $\int_A^B \vec{E} \cdot \vec{dr}$, that is the line integral of electric field between A to B – is the work done by the electric field in moving a unit positive charge from A to B and it does not depend on the path. Moreover, $\oint \vec{E} \cdot \vec{dr} = 0$. $\vec{E} \cdot \vec{dr}$ is also sometimes written as $\vec{E} \cdot \vec{dl}$ where $\vec{dl}$ is also a small displacement vector

2.3 Electrostatic Potential

We know that the work done by the electric field in moving a unit positive (+1 C) charge from one point to the other, in the electric field, depends only on the positions of those two points and not on the path joining them.

If we take a reference point A, and take the unit positive charge from point A to B; A to C; A to D; ..... etc in the electric field, then the work done by the electric field is obtained as $W_{AB} = \int_A^B \vec{E} \cdot \vec{dr}$, $W_{AC} = \int_A^C \vec{E} \cdot \vec{dr}$, $W_{AD} = \int_A^D \vec{E} \cdot \vec{dr}$,.... respectively. But the reference point A is already fixed, hence the above mentioned work depends on the position of the other points (B, C, D, ...) only. Conventionally the reference point is taken as a point at infinite distance from the source of electric field. Hence to bring a unit positive charge from that point to a point P in the field, the work done by the electric field is given by the formula $W_{\infty P} = \int_\infty^P \vec{E} \cdot \vec{dr}$

and it becomes the function only of the position of point P. But, if we want to find the work required to be done **against** the electric field; in order that the motion becomes **"motion without acceleration,"**

the formula $W'_{\infty P} = -\int_{\infty}^{P} \vec{E} \cdot \vec{dr}$ has to be used.

An important characteristic of an electric field is called **electrostatic potential** and with reference to the work done on unit positive charge, it is defined as under :

"Work required to be done against the electric field in bringing a unit positive charge from infinite distance to the given point in the electric field is called the electrostatic potential (V) at that point."

Here the meaning of "against the electric field" is actually **"against the force by the electric field".** We will call the electrostatic potential as electric potential in short.

According to the above definition, the electric potential at a point P is given by the formula :

$$V_P = -\int_{\infty}^{P} \vec{E} \cdot \vec{dr} \tag{2.3.1}$$

In other words this formula represents the definition of electric potential.

From this formula the potential difference betwen points Q and P is given by

$$V_Q - V_P = \left(-\int_{\infty}^{Q} \vec{E} \cdot \vec{dr}\right) - \left(-\int_{\infty}^{P} \vec{E} \cdot \vec{dr}\right) \tag{2.3.2}$$

$$= \int_{Q}^{\infty} \vec{E} \cdot \vec{dr} + \int_{\infty}^{P} \vec{E} \cdot \vec{dr} = \int_{Q}^{P} \vec{E} \cdot \vec{dr} \tag{2.3.3}$$

$$= -\int_{P}^{Q} \vec{E} \cdot \vec{dr} \tag{2.3.4}$$

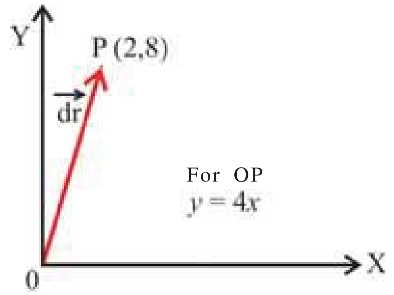
This potential difference shows the **work required to be done to take a unit positive charge from P to Q, against the electric field** and in that sense it also shows the potential of Q with respect to P. Very often the potential difference is in short written as p.d. also. The unit of electric potential (and hence that of the potential difference also) is joule / coulomb which is called volt (symbol V) in memory of the scientist Volta. i.e., volt $= \frac{\text{joule}}{\text{coulomb}}$ or $V = \frac{J}{C}$. It's dimensional formula is $M^1L^2T^{-3}A^{-1}$.

Electric potential is a scalar quantity. Moreover, we have obtained electric potential from the vector quantity-electric field $\vec{E}$ (See equation 2.3.1). In future we will also obtain electric field from the electric potential. In the calculations involving electric field $\vec{E}$, its three components E_x, E_y, E_z have to be considered and the calculations become longer, while in the calculations involving the electric potential, **only one scalar** appears and hence the calculations become shorter and easier. Hence the concept of electric potential is widely used. Absolute value of electric potential has no importance, only the difference in potential is important.

[For Information Only : Galvani (1737–1798) produced electricity by placing two different metallic electrodes in the tissue of frog. He called it Animal Electricity. Volta explained that the above process had nothing to do with the characteristics of the frog, but one can generate electricity by placing two dissimilar metallic electrodes on any wet body. He was the one who designed the electro chemical cell, which we studied earlier as voltaic cell.

The importance of electric potential in electricity is similar to the importance of temperature in thermodynamics and the height of fluid in hydrostatics. The electricity flows (i.e. the electric current flows) from an electrically charged material having higher electric potential to an electrically charged material having lower electric potential. Quite similar to water, which flows from a higher level to a lower level or like the flow of heat which flows from a region having higher temperature to a region having lower temperature. Thus, the direction of the flow of electric current between two materials depends on their electric potentials.]

Illustration 1 : Suppose an electric field due to a stationary charge distribution is given by $\vec{E} = ky\hat{i} + kx\hat{j}$, where k is a constant. (a) Find the line integral of electric field on the linear path joining the origin O with point P(2, 8), in the Figure. (b) Obtain the formula for the electric potential at any point on the line OP, with respect to (0, 0)



Solution : (a) The displacement vector $\vec{dr}$ on the line OP is $\vec{dr} = dx\hat{i} + dy\hat{j}$

$$\therefore \vec{E}\cdot\vec{dr} = (ky\hat{i} + kx\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$= kydx + kxdy = k(ydx + xdy)$$

Moreover, on the entire OP line $y = 4x$ ($\because$ the slope of a straight line is constant)

$\therefore\ dy = 4dx$

$\therefore$ The line integral of electric field from O to P, is

$$\int_O^P \vec{E}\cdot\vec{dl} = k\int_O^P (ydx + xdy) = k\int_{(0,0)}^{(2,8)} [4xdx + x(4dx)] = k\int_0^2 8x\,dx \qquad (A)$$

$$= 8k\left[\frac{x^2}{2}\right]_0^2 = 16k$$

(b) In order to obtain the potential at any point Q(x, y) on the line OP with respect to (0, 0) we can use $V(Q) = -\int_O^Q \vec{E}\cdot\vec{dl}$

$$\therefore\ V(Q) = -\int_{(0)}^{(x)} 8kx\,dx \ldots \text{ (from equation A)}$$

$$= -\ 8k\left[\frac{x^2}{2}\right]_0^x = -4kx^2$$

Illustration 2 : The electric field at distance r perpendicularly from the length of an infinitely long wire is $E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}$, where λ is the linear charge density of the wire. Find the potential at a point having distance b from the wire with respect to a point having distance a from the wire ($a > b$). [**Hint :** $\int \frac{1}{r} dr = ln\ r$].

Solution : $V_b - V_a = -\int_a^b \vec{E} \cdot \vec{dr}$

$$= -\int_a^b \frac{\lambda}{2\pi \epsilon_0 r} dr \quad (\because \vec{E} \parallel \vec{dr})$$

$$= -\frac{\lambda}{2\pi\varepsilon_0} \int_a^b \frac{1}{r} dr = -\frac{\lambda}{2\pi\varepsilon_0} [ln\ r]_a^b = -\frac{\lambda}{2\pi\varepsilon_0 r} [ln\, b - ln\, a]$$

$$= \frac{\lambda}{2\pi\varepsilon_0} [ln\, a - ln\, b]$$

$$= \frac{\lambda}{2\pi\varepsilon_0} ln \left(\frac{a}{b}\right)$$

For reference point a, taking $V_a = 0$

$$\therefore V_b = \frac{\lambda}{2\pi\varepsilon_0} ln \left(\frac{a}{b}\right)$$

Illustration 3 : An electric field is represented by $\vec{E} = Ax\hat{i}$, where $A = 10 \frac{V}{m^2}$. Find the potential of the origin with respect to the point (10, 20)m.

Solution : $\vec{E} = Ax\hat{i} = 10x\hat{i}$

$$V(0, 0) - V(10, 20) = -\int_{(10, 20)}^{(0, 0)} \vec{E} \cdot \vec{dr}$$

$$= -\int_{(10, 20)}^{(0, 0)} (10x\hat{i}) \cdot (dx\hat{i} + dy\hat{j}) = -\int_{10}^{0} 10x\, dx$$

$$= -10 \left[\frac{x^2}{2}\right]_{10}^{0} = [0 - (-500)] = 500 \text{ volt}$$

Since V(10, 20) is to be taken as zero,

V(0, 0) = 500 volt.

2.4 Electrostatic Potential Energy

In the previous article (2.2), we had discussed the work done by the electric field on a unit positive charge and then also on the charge q, during the motion in the electric field. Moreover we had also talked about the work required to be done by the external force against the electric field, in which the **motion of charge is without acceleration only.** Hence its velocity remains constant and its kinetic energy does not change. But the work done by this external force is stored in the form of potential energy of that charge. From this, the electric potential energy is defined as under :

"The work required to be done against the electric field in bringing a given charge (*q*), from infinite distance to the given point in the electric field is called the electric potential energy of that charge at that point." Here "motion without acceleration" is implied when we mentioned" "work required to be done."

From the definitions of electric potential energy and the electric potential, we can write the electric potential energy of charge q at point P, as

$$U_P = -\int_{\infty}^{P} q\vec{E}\cdot\vec{dr} = -q\int_{\infty}^{P}\vec{E}\cdot\vec{dr} \quad (2.4.1)$$

$$= qV_P \quad (2.4.2)$$

Moreover, we can also call the electric potential at point P as the electric potential energy of unit positive charge (q = +1 C) at that point. That is,

{electric potential at a given point} = {electric potential energy of unit positive charge at that point}

For more clarity in this discussion, we note a few important points as under :

(1) When we bring charge q (or a unit positive charge) from infinite distance to the given point or when we move it from one point to the other in the field, the **positions of the sources (charges) producing the field are not changed**. (We will imagine these sources as being clamped on their positions by some invisible force !!)

(2) The absolute value of the electric potential energy is not at all important, only the difference in its value is important. Here, in moving a charge q, from point P to Q, **without acceleration, the work required to be done by the external force,** shows the difference in the electric potential energies ($U_Q - U_P$) of this charge q, at those two points.

$$\therefore\ U_Q - U_P = -q\int_{P}^{Q}\vec{E}\cdot\vec{dr} \quad (2.4.3)$$

(3) Here, electric potential energy is **of the entire system** of the sources producing the field and the charge that is moved, for **some one configuration,** and when the configuration changes the electric potential energy of the system also changes. e.g., when the distance between them is r, it is one configuration and if distance r changes, the configuration is also said to be changed and hence the electric potential energy of the system is also said to be changed. But as the conditions of the sources producing the field are not changed, the entire change in the electric potential energy is **experienced by this charge *q* only which we have moved.** Hence we are able to write $U_Q - U_P$ as the difference in potential energy **of this charge *q* only.** Because of this reason we have mentioned "potential energy of charge q" for equation 2.4.1 and "potential energy of unit positive charge" in the discussion that followed it.

2.5 Electric Potential due to a Point Charge

We want to find the electric potential V(P), due to a point charge q, at some point P, at a distance r from it.

For this we will put the origin of co-ordinate axes 0, at the position of that charge. See figure 2.2. Here $\vec{OP} = \vec{r}$. According to the definition of electric potential we can use the equation.

$$V(P) = -\int_{\infty}^{P}\vec{E}\cdot\vec{dr} \quad (2.5.1)$$

Moreover, we can also write this equation in another form as

$$V(P) = \int_{P}^{\infty}\vec{E}\cdot\vec{dr} \quad (2.5.2)$$

because, $\int_{\infty}^{P}\vec{E}\cdot\vec{dr} = -\int_{P}^{\infty}\vec{E}\cdot\vec{dr}$.

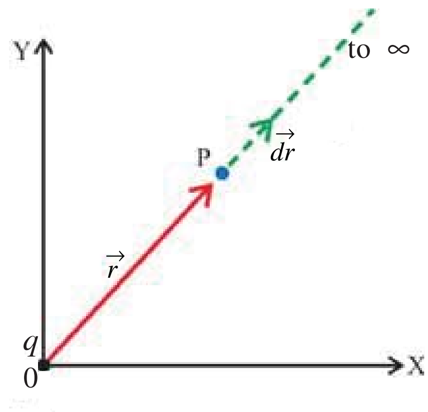


Figure 2.2 Potential due to Point Charge

At this point P, $\vec{E} = \frac{kq}{r^2}\hat{r}$ (2.5.3)

∴ From equation 2.5.2

$$V(P) = \int_P^{\infty} \frac{kq}{r^2}\hat{r}\cdot dr\hat{r} = \int_r^{\infty} \frac{kq}{r^2} dr$$

$$= kq \int_r^{\infty} \frac{1}{r^2} dr = kq \left[-\frac{1}{r}\right]_r^{\infty}$$

$$V(P) = \frac{kq}{r} \qquad (2.5.6)$$

or $$V(P) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad (2.5.7)$$

This equation is true for any charge, positive or negative. The potential due to a positive charge is positive and that due to a negative charge is negative (as q is to be put with negative sign in the above equation.)

It is self evident from equation 2.5.6 that as the distance r increases, the electric potential decreases as $\frac{1}{r}$. In case of potential also superposition principle is applicable. To find the electric potential due to many point charges we should find the potential due to every charge according to equation 2.5.7 and they should be added algebraically.

Illustration 4 : A point P is 20 m away from a 2 μC point charge and 40 m away from a 4 μC point charge. Find the electric potential at P.

(1) Find the work required to be done to bring 0.2 C charge from infinite distance to the point P.

(2) Find the work required to be done to bring −0.4 C charge from infinite distance to the point P. [$k = 9 \times 10^9$ N m^2 C^{-2}]

Solution : $$V_P = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = k\left[\frac{q_1}{r_1}+\frac{q_2}{r_2}\right]$$

$$= 9 \times 10^9 \left[\frac{2\times10^{-6}}{20}+\frac{4\times10^{-6}}{40}\right] = 1800 \text{ volt}$$

(1) $W_1 = V_P\, q_1' = (1800)(0.2) = 360$ J.

(2) $W_2 = V_P\, q_2' = (1800)(-0.4) = -720$ J

2.6 Electric Potential due to an Electric Dipole

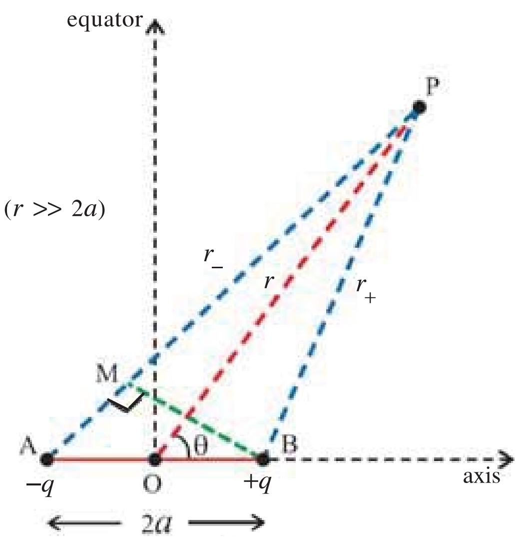


Figure 2.3 Potential due to an Electric Dipole

We have seen in Chapter-1 that two equal and opposite charges (+ q and − q) separated by a finite distance (= $2a$) constitute an electric dipole.

Such a dipole is shown in the figure 2.3, with the origin of co-ordinate system O at its mid-point. The magnitude of the dipole moment of the dipole is $p = q(2a)$ and its direction is from negative to the positive charge that is, in AB direction.

We want to find the electric potential at point P far away from the mid-point O of dipole and in the direction making an angle θ with the axis of the dipole. Let OP = r, AP = r_-, and BP = r_+. At P, the electric potential is equal to the sum of the potentials produced by each of the charges.

$$\therefore \quad V(P) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{r_-} \qquad (2.6.1)$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_+} - \frac{1}{r_-}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r_- - r_+}{r_- r_+}\right] \qquad (2.6.2)$$

Since P is a far distant point, $r >> 2a$ and hence we can take AP || OP || BP. In this condition the figure 2.3 shows that

$$\left\{\begin{array}{l}\text{for numerator of equation (2.6.2), } r_- - r_+ = AM = 2a\ cos\theta \\ \text{and for denominator, } r_- \approx r_+ \approx r\end{array}\right\} \qquad (2.6.3)$$

We have considered a very far distant point as compared to the length ($2a$) of the dipole. The molecular dipoles are very small and such an approximation is very well applicable to them. From equations (2.6.2) and (2.6.3), we get

$$V(r) = \frac{q}{4\pi\varepsilon_0}\left(\frac{2a\ cos\theta}{r^2}\right) \qquad (2.6.4)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p\ cos\theta}{r^2} \qquad (2.6.5)$$

Writing the unit vector in the direction $\overrightarrow{OP}$ as $\hat{r}$, we can wirte $\vec{p}\cdot\hat{r} = p\ cos\theta$.

$$\therefore \quad V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2} \text{ (for } r >> 2a) \qquad (2.6.6)$$

Note : The dipole obtained in the limits $q \to \infty$ and $a \to 0$, is called the point dipole. For such a point dipole the above equation is more accurate, while for the physical dipole - found in practice - this equation gives an approximate value of the electric potential. Let us note a few points evident from equation (2.6.4), as under :

(1) Potential on the Axis : For a point on the axis of the dipole

$$\theta = 0 \text{ or } \pi. \therefore V = \pm\frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

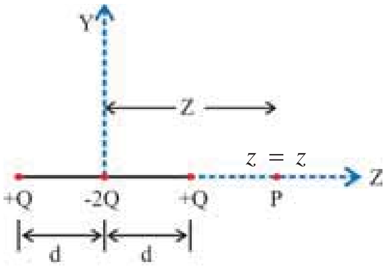
From the given point, if the nearer charge is $+q$, then we get V as positive and if it is $-q$, then we get V as negative.

(2) Potential on the Equator : For a point on the equator $\theta = \frac{\pi}{2}$ $\therefore$ V = 0

(3) The potential at any point depends on the angle between its position vector $\vec{r}$ and $\vec{p}$.

(4) The potential due to a dipole decreases as $\frac{1}{r^2}$ with distance (while the potential due to a point charge decreases as $\frac{1}{r}$ with distance). We have seen in Chapter 1 that the electric field due to a dipole decreases as $\frac{1}{r^3}$)

Ilustration 5 : When two dipoles are lined up in opposite direction, the arrangement is known as a quadruple (as shown in the Figure). (1) Calculate the electric potential at a point $z = z$ along the axis of the quadruple and (2) If $z >> d$, then show that,



$$V(z) = \frac{Q}{4\pi\varepsilon_0}\frac{2d^2}{z^3}$$

Note : $2|Q|d^2$ is called the quadruple moment.

Solution : (1) Let z be the Z co-ordinate of point P.

The electric potential at point P, due to +Q charge (which is at the left hand side of the origin) is,

$$V_1 = \frac{kQ}{z+d} \qquad (1)$$

The electric potential at point P due to the +Q charge which is at the right hand side of the origin is,

$$V_2 = \frac{kQ}{z-d} \qquad (2)$$

The electric potential at point P, due to −2Q charge present at the origin is,

$$V_3 = -\frac{k(2Q)}{z} \qquad (3)$$

∴ The total potential at point P,

$V(z) = V_1 + V_2 + V_3$

$$= kQ\left[\frac{1}{z+d}+\frac{1}{z-d}-\frac{2}{z}\right] = kQ\left[\frac{2z}{z^2-d^2}-\frac{2}{z}\right] = kQ\left[\frac{2d^2}{z(z^2-d^2)}\right]$$

(2) If $z >> d$, we can neglect d^2 in comparison with z^2 in the denominator of right hand side of the above equation.

$$\therefore\ V(z) = \frac{kQ(2d^2)}{z^3} = \frac{Q}{4\pi\varepsilon_0}\frac{2d^2}{z^3}$$

Illustration 6 : Charge Q is distributed uniformly over a non-conducting sphere of radius R. Find the electric potential at distance r from the centre of the sphere (r < R). The electric field at a distance r from the centre of the sphere is given as $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\ r\hat{r}$. Also find the potential at the centre of the sphere.

Solution : The electric potential on the surface of such a sphere is,

$$V(R) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

As a result, we can use the equation $V(r) - V(R) = -\int_R^r \vec{E}\cdot\vec{dr}$

$$\therefore\ V(r) - V(R) = -\int_R^r \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\, rdr\ \hat{r}\cdot\hat{r} \qquad (\because \vec{dr} = dr\,\hat{r})$$

$$= -\frac{Q}{4\pi\varepsilon_0 R^3}\int_R^r rdr = -\frac{Q}{4\pi\varepsilon_0 R^3}\left[\frac{r^2}{2}\right]_R^r$$

$$= -\frac{Q}{4\pi\varepsilon_0 R^3}\left[\frac{r^2}{2}-\frac{R^2}{2}\right]$$

$$\therefore \ V(r) = V(R) + \frac{Q}{4\pi\varepsilon_0 R^3}\left[\frac{R^2}{2} - \frac{r^2}{2}\right]$$

$$\therefore \ V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} + \frac{Q}{8\pi\varepsilon_0 R^3}(R^2 - r^2)$$

$$\therefore \ V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R}\left(3 - \frac{r^2}{R^2}\right), \ r < R$$

At the centre of the sphere $r = 0$, $\therefore$ V (centre) $= \frac{1}{4\pi\varepsilon_0}\left(\frac{3Q}{2R}\right)$.

2.7 Electric Potential due to a System of Charges

In a system of charges, point charges could have been distributed descretely (separated from each other) while in some system they could have been distributed continuously with each other. In some system of charges the distribution of charges could be a mixture of any type of these two distributions.

(a) Descrete Distribution of Charges :

In figure 2.4, point charges q_1, q_2, q_3 ...... q_n are shown as distributed descretely. The position vectors of these charges with respect to the origin of co-ordinate system are $\vec{r}_1$, $\vec{r}_2$,.... $\vec{r}_n$ respectively. We want to find the electric potential due to this system, at point P with position vector $\vec{r}$. For this we will find the electric potential due to every point charge and then will make summation.

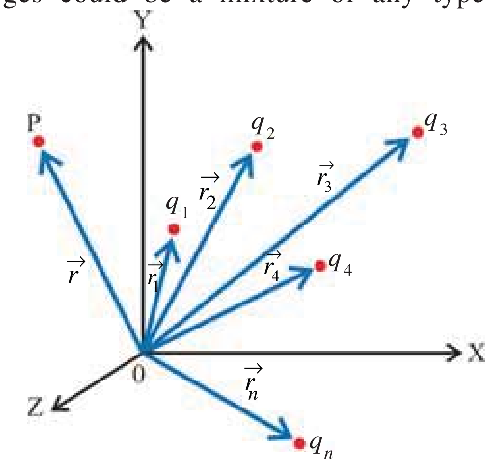


Figure 2.4 Potential Due to Descrete Charges

That is, $V = V_1 + V_2 + ... + V_n$ (2.7.1)

$$\therefore \ V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1p}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2p}} + ... + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_{np}} \qquad (2.7.2)$$

Where r_{1p} = distance of P from $q_1 = |\vec{r} - \vec{r}_1|$.

Similarly r_{2p}, ..., r_{np} are the corresponding distances.

$$\therefore \ V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{|\vec{r} - \vec{r}_1|} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{|\vec{r} - \vec{r}_2|} + ... + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{|\vec{r} - \vec{r}_n|} \qquad (2.7.3)$$

$$\therefore \ V = \sum_{i=1}^{n}\frac{kq_i}{|\vec{r} - \vec{r}_i|} \qquad (2.7.4)$$

(b) Electric Potential due to a Continuous Distribution of Charges :

Suppose in a certain region electric charge is distributed continuously. Imagine this region to be divided in a large number of volume-elements, each one with extremely small volume. If the volume of such an element having position vector $\vec{r}\,'$ is $d\tau'$ and at this position the volume-density of charge is $\rho(\vec{r}\,')$, then the charge in this element is $\rho(\vec{r}\,')\,d\tau'$, and it can be treated as a point charge. The electric potential due to this small, volume element at point P having the position vector $\vec{r}$, is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\rho(\vec{r}')d\tau'}{|\vec{r}-\vec{r}'|}. \quad (2.7.5)$$

By integrating this equation over the entire volume of this distribution, we get the total potential at point P, which can be written as under :

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{volume}\frac{\rho(\vec{r}')d\tau'}{|\vec{r}-\vec{r}'|} \quad (2.7.6)$$

If the charge distribution is uniform, $\rho(\vec{r}\,')$ can be taken as canstant (= ρ).

(c) A Spherical Shell with Uniform Charge Distribution :

In Chapter 1, we had seen that the electric field **at a point outside** and **at a point on the surface** of spherical shell with uniform charge distribution is equal to the electric field obtained by considering the entire charge of the shell as cencentrated.at the centre of the shell. We have obtained the electric potential from the electric field ($V = -\int_{\infty}^{r}\vec{E}\cdot\vec{dr}$). For electric potential also the entire charge can be considered as concentrated at the centre of the shell. Hence the potential at a point outside and at a point on the surface of the shell having charge q and radius R, is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad (\text{for } r \geq R) \quad (2.7.7)$$

where r = distance of the given point from centre of shell.

Moreover, we also know that the electric field inside the shell is zero. Hence during the **motion** of **unit positive charge inside the shell** no work is required to be done. Hence the potentials at **all points** inside the shell are equal having the value equal to the potential on the surface of that shell. i.e. $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$ (for $r \leq R$) (2.7.8)

(Note that here, only that work is accounted for which is done during the motion of unit positive charge from ∞ to the surface of the shell.)

2.8 Equipotential Surfaces

An equipotential surface is that surface **on which the electric potentials at all points are equal.**

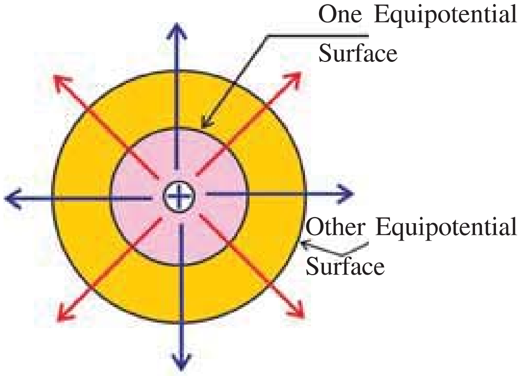


Figure 2.5 Equipotential Surfaces

The electric potential due to a point charge is given by $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$. Hence if r is constant, V also becomes constant. From this we can say that for a single point charge q, the equipotential surfaces are the surfaces of the spheres drawn by taking this charge as the centre. (See figure 2.5). The potentials on two such different surfaces are different but for all the points on the **same surface** the potentials **are equal.** The electric field produced by a point charge is along the radial directions drawn from it. [For $+q$ they are in radial directions going away from it and for $-q$ coming towards it.]. These radial lines are normal to those equipotential surfaces at every point. Hence at a given point the direction of electric field is normal to an equipotential surface passing through that point. We shall now prove that this is true not only for a point charge but in general for any charge configuration.

Suppose a unit positive charge is given a small displacement $d\vec{l}$ **on the** equipotential surface **(along this surface),** from a given point. In this process the work required to be done against the electric field (by the external force) is $dW = -\vec{E}\cdot d\vec{l}$ = potential difference between those two points.

But the potential difference on the equipotential surface = 0.

$\therefore \; \vec{E}\cdot d\vec{l} = 0 \Rightarrow E\,dl\,\cos\theta = 0$, where θ = angle between $\vec{E}$ and $d\vec{l}$.

But $E \neq 0$ and $dl \neq 0 \;\therefore\; \cos\theta = 0 \;\therefore\; \theta = \frac{\pi}{2} \;\therefore\; \vec{E} \perp d\vec{l}$.

But $d\vec{l}$ is along this surface. Hence the electric field $\vec{E}$ is normal to the equipotential surface at that point.

Like the field lines, the equipotential surface is also a useful concept to represent an electric field. For a uniform electric field prevailing in X-direction, the field lines are parallel to X-axis and equispaced, while the equipotential surfaces are normal to X-axis (i.e. parallel to YZ plane.) See figure 2.6.

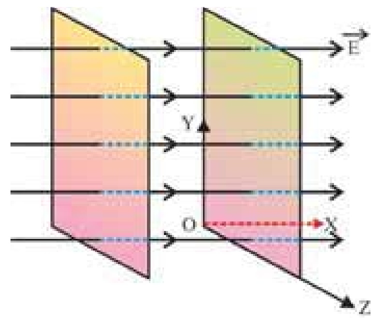


Figure 2.6 Equipotential Surface for a Uniform Electric Field

(a) Equipotential Surfaces of a Dipole (Only For Information)

(b) Equipotential Surfaces of a System of Two Positive and Equal Charge (Only for Information)

Figure 2.7

The equipotential surfaces of an electric dipole are shown in figure 2.7(a).

The equipotential surfaces of a system of two positive charges of equal magnitude are shown in figure 2.7(b).

2.9 Relation between the Electric Field and the Electric Potential

In article 2.3, we have obtained the electric potential $V = \left(-\int_{\infty}^{P} \vec{E}\cdot d\vec{r}\right)$ from the electric field.

Now, if we know about the electric potential in a certain region, we can get the electric field from it as well.

We have seen in article 2.3, that from the line integral of electric field between points P and Q, we can get the potential difference between those two points. (Equation 2.3.4) as

$$V_Q - V_P = \Delta V = -\int_P^Q \vec{E}\cdot\vec{dr} \quad (2.9.1)$$

Now, if these points P and Q are very close to each other, then for such a small displacement $d\vec{l}$, integration is not required and only one term $\vec{E}\cdot d\vec{l}$ can be kept.

$$\therefore\ dV = -\vec{E}\cdot d\vec{l} \quad (2.9.2)$$

If $d\vec{l}$ is in the direction of $\vec{E}$, $\vec{E}\cdot d\vec{l} = E\ dl\ cos 0^o = E\ dl$

$$\therefore\ dV = -E\ dl$$

$$\therefore\ E = \frac{-dV}{dl} \quad (2.9.3)$$

This equation gives the magnitude of electric field in the direction of displacement $d\vec{l}$. Here $\frac{dV}{dl}$ = potential difference per unit distance. It is called the **potential gradient.** Its unit is $\frac{V}{m}$. From equation (2.9.3) the unit of electric field is also written as $\frac{V}{m}$, which is equivalent to $\frac{N}{C}$.

If we had taken the displacement $d\vec{l}$ **not** in the direction of electric field, but in some other direction, then $\frac{-dV}{dl}$ would give us the **component of electric field in the direction of that displacement.** e.g. If the electric field is in X-direction only and the displacement is in any direction (in three dimensions), then

$$\vec{E} = E_x\hat{i} \text{ and } d\vec{l} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\therefore\ dV = -(E_x\hat{i})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= -E_x\ dx \quad (2.9.4)$$

$$\therefore\ E_x = \frac{-dV}{dx} \quad (2.9.5)$$

Similarly, if the electric field was only in Y and only in Z direction respectively, we would get,

$$E_y = \frac{-dV}{dy} \quad (2.9.6)$$

$$E_z = \frac{-dV}{dz} \quad (2.9.7)$$

Now, if the electric field also has all the three ($x-$, $y-$, $z-$) components then from equations (2.9.5) (2.9.6) and (2.9.7) we can write as under.

$$E_x = \frac{-\partial V}{\partial x},\ E_y = \frac{-\partial V}{\partial y},\ E_z = \frac{-\partial V}{\partial z} \quad (2.9.8)$$

$$\text{and } \vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) \quad (2.9.9)$$

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Here $\frac{\partial V}{\partial x}$, $\frac{\partial V}{\partial y}$, $\frac{\partial V}{\partial z}$ show the partial differentiation of V(x, y, z) with respect to x, y, z respectively. Moreover, the partial differentiation of V(x, y, z) with respect to x means the differentiation of V with respect to **only x** (i.e. $\frac{\partial V}{\partial x}$) by taking y and z in the formula of V, as constants.

In equation (2.9.1), the values of $\vec{E}$ at all points between P and Q come in the calculation, while equations (2.9.3) and (2.9.8) give relation between the potential difference near a given point and the electric field at that point.

The direction of electric field is that in which the rate of decrease of electric potential with distance $\left(\frac{-dV}{dr}\right)$ is maximum and this direction is always normal to the equipotential surface.

This entire discussion is based on the property that electric field is a conservative field.

2.10 Potential Energy of a System of Point Charges

As shown in the figure 2.8, in a system of charges three point charges q_1, q_2 and q_3 are lying stationary at points A, B and C respectively. Their position vectors from the origin of a co-ordinate system are $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$ respectively. We want to find the potential energy of this system.

$\vec{OA} = \vec{r_1}$, $\vec{OB} = \vec{r_2}$, $\vec{OC} = \vec{r_3}$

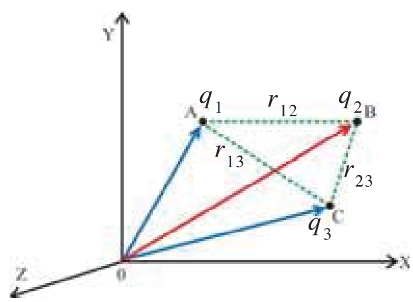


Figure 2.8 System of Point Charges

In the beginning we shall imagine that these charges are lying at infinite distances from the origin and also from each other. In this condition the electric force between them is zero, and their potential energy is also zero.

Moreover, the electric fields at A, B and C are also zero. From such a condition the work required to be done by the external forces (against the electric fields) to arrange them in the above mentioned configuration is stored in the form of potential energy of this system.

First, we bring the charge q_1 from infinite distance to point A. In this process since no electric field is present, the work done by the external force against the electric field is W_1 = zero. (You know that here the field produced by this charge itself is not to be considered.)

Now the charge set on q_1, produces an electric field and electric potential around it. The potential due to this charge q_1 at point B separated by distances r_{12} from it is (from equation 2.5.7) is

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{12}} \qquad (2.10.1)$$

Where $r_{12} = |\vec{r_2} - \vec{r_1}|$

Hence the work required to be done by the external force to bring charge q_2 from infinite distance to point B, is $W_2 = q_2 V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$ (2.10.2) (from equation 2.4.2).

(If we want to consider a system of these **two charges only,** then the total work $W_1 + W_2 = \frac{1}{4\pi\varepsilon_0}\frac{q\,q_2}{r_{12}}$ is the electric potential energy of this system.)

Now q_1 and q_2 both will produce electric fields and electric potentials around them. The electric potential produced due to them at point C is $V_C = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{23}}$ (2.10.3)

Therefore, the work required to be done to bring charge q_3 from infinite distance to point C is

$$W_3 = (V_C)q_3$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}} \qquad (2.10.4)$$

Hence the total work to be done to set these three charges in the above arrangement (= $W_1 + W_2 + W_3$) is the electric potential energy U of this system.

$$\therefore\ U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}} \qquad (2.10.5)$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right] \qquad (2.10.6)$$

$$= k\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right] \qquad (2.10.7)$$

From this, in general, the potential energy of a system of n−charges can be wirtten as

$$U = \sum_{\substack{i=1 \\ i<j}}^{n} \frac{k q_i q_j}{q_{ij}} \qquad (2.10.8)$$

As the electric field is conservative; it does not matter, which charge comes earlier or later. In that case the electric potential energy does not change (and given by equation 2.10.8 only)

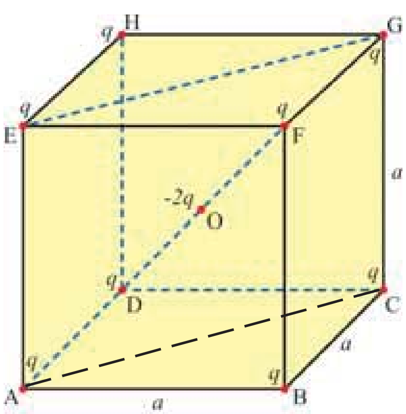


Illustration 7 : Calculate the potential energy of the system of charges, shown in the Figure.

Solution : The total potential energy of the system of charges is equal to the sum of the potential energy of all the pairs of charges.

(1) There are 12 pairs of charges like the AB pair. The distance between the electric charges in such pairs is equal to a.

The potential energy of all such pairs is

$$U_1 = \frac{kq^2}{a} \times 12 \qquad (1)$$

(2) There are 12 pairs of charges like the AC pair. The distance between charges in such a pair is $a\sqrt{2}$. ($\because$ AC $= \sqrt{AB^2 + AC^2} = \sqrt{a^2 + a^2} = a\sqrt{2}$). Their potential energy is,

$$U_2 = \frac{kq^2}{a\sqrt{2}} \times 12 \qquad (2)$$

(3) There are 4 pairs of charges like the AG pair. The distance between charges in these pairs is equal to $a\sqrt{3}$. ($\because$ AG = $\sqrt{AC^2+CG^2}$ = $\sqrt{2a^2+a^2} = a\sqrt{3}$)

Their potential energy is $U_3 = \frac{kq^2}{a\sqrt{3}} \times 4$

(4) There are eight pairs of electric charges similar to AO in which distance between charges is $\frac{a\sqrt{3}}{2}$. (AO = $\frac{AG}{2}$ = $\frac{a\sqrt{3}}{2}$)

Their potential energy is $U_4 = -\frac{kq\,2q}{\left(a\frac{\sqrt{3}}{2}\right)} \times 8$ (4)

$\therefore$ total potential energy $U = U_1 + U_2 + U_3 + U_4$

$$\therefore \ U = \frac{12kq^2}{a} + \frac{12kq^2}{a\sqrt{2}} + \frac{4\,kq^2}{a\sqrt{3}} - \frac{32kq^2}{a\sqrt{3}}$$

$$= \frac{kq^2}{a}\left[12+\frac{12}{\sqrt{2}}+\frac{4}{\sqrt{3}}-\frac{32}{\sqrt{3}}\right] = \frac{kq^2}{a}\left[12(1+\frac{1}{\sqrt{2}})-\frac{28}{\sqrt{3}}\right]$$

2.11 The Potential Energy of an Electric Dipole in an External Electric Field

As shown in figure 2.9, an electric dipole AB is placed in a uniform electric field $\vec{E}$ in X–direction such that the axis of the dipole makes an angle θ with the field $\vec{E}$. Its dipole moment is $q(2a)$ in AB direction. The electric potential energy of this dipole means the algebraic sum of the electric potential energies of both of its charges ($+q$ and $-q$). We arbitrarily take the potential at the position of $-q$ charge as zero. Hence its potential energy becomes zero. Now we will find the potential enegy of $+q$ charge with respect to it and it will become the potential energy of the entire dipole.

AB = $2a$, AC = AB$cos\theta$

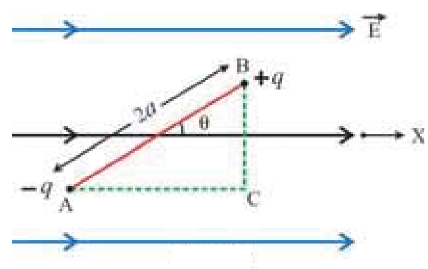


Figure 2.9 Potential Energy of Dipole

As the electric field is only in X-direction,

$$E = \frac{-\Delta V}{\Delta x} = \frac{-(V_B - V_A)}{AC}$$

$$= \frac{-V_B}{2a\,cos\theta} \quad (\because\ V_A = 0) \qquad (2.11.1)$$

$$\therefore\ V_B = -E\ (2a\ cos\theta) \qquad (2.11.2)$$

$\therefore$ Potential energy of $+q$ at B, is

$$U = qV_B = q[-E\ 2a\ cos\theta] \qquad (2.11.3)$$

$$= -E(q\ 2a\ cos\theta)$$

$$= -E\ p\ cos\theta\ [\because\ q(2a) = p]$$

$$= -\vec{E}\cdot\vec{p} \qquad (2.11.4)$$

$\therefore$ The potential energy of the entire dipole $U = -\vec{E} \cdot \vec{p} = -\vec{P} \cdot \vec{p}$ (2.11.5)

We note a few points :

(i) If the axis of the dipole is normal to the electric field, then $\theta = \frac{\pi}{2}$ and

$$U = Ep \cos\frac{\pi}{2} = 0$$

(ii) If the axis of the dipole is parallel to the field. ($\vec{AB} \parallel \vec{E}$)

Then $\theta = 0$ $\therefore$ $U = -pE$. This is the **minimum value** of potential energy. Hence the dipole tries to arrange its axis parallel to the electric field, so that $\vec{p}$ becomes parallel to $\vec{E}$. In this condition dipole remains in stable equilibrium. (A system always tries to remain in such a state that its potential energy becomes minimum.) (For $\theta = \pi$, the dipole is in an unstable equilibrium.)

2.12 Electrostatics of Conductors

It is interesting to know the effects produced when metallic conductors are placed in the electric field or when electric charges are placed on such conductors.

(a) Effect of External Electric Field on Conductors :

In a metallic conductor there are positive ions situated at the lattice points and the free electrons are moving randomly between these ions. They are free to move within the metal but not free to come out of the metal. When such a conductor is placed in an external electric field $\vec{E}'$, the free electrons move under the effect of the force in the direction opposite to the field and get deposited on the surface of one end of conductor. And an equal amount of **positive charge** can be considered as deposited on the other end. Thus electric charges are **induced.** These induced charges produce an electric field $\vec{E}''$ inside the conductor, in the direction **opposite** to the external electric field $\vec{E}'$. When these two electric fields become equal in magnitude, the resultant (net) electric field ($\vec{E}$) inside the conductor becomes zero. (See figure 2.10). Now the motion of charges in the conductor stops, and the charges become steady (stationary) on the end-surfaces.

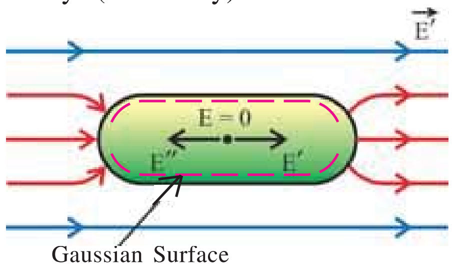


Figure 2.10 Conductor in Electric Field

Now let us consider a Gaussian Surface shown by dotted line, inside the conductor and close to the surface, as shown in figure 2.10. Every point on this surface is a point inside the conductor; the electric field $\vec{E}$ on this entire surface is zero. Hence the electric charge enclosed by it is also zero. ($\because \int \vec{E} \cdot \vec{dr} = \frac{q}{\varepsilon_0}$).

Thus in the case of a metallic conductor, placed in an external electric field,

(1) A steady electric charge distribution is induced on the surface of the conductor.

(2) The net electric field inside the conductor is zero.

(3) The net electric charge inside the conductor is zero.

(4) On the outer surface of the conductor, the electric field at every point is locally normal (perpendicular) to the surface. If the electric field were not normal (perpendicular) a component of electric field parallel to the surface would exist and due to it the charge would move on the surface. But now the motion is stoppd and the charges have become steady. Thus the component of electric field parallel to the surface would be zero, and hence the electric field would be normal to the surface.

(5) Since $\vec{E} = 0$ at every point inside the conductor, the electric potential everywhere inside the conductor is constant and equal to the value of potential on the surface.

(6) If there is a cavity inside the conductor then even when the conductor is placed in an external electric field ($\vec{E}'$), the net electric field inside the coductor is zero and also inside the cavity it is zero. Consider a Gaussian Surface around the cavity as shown in the figure 2.11. Since every point on this surface is a point inside the conductor, the electric field on this entire surface is zero.

Hence the total charge on the surface of the cavity is zero, ($\int \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$). And there is no charge inside the cavity. Hence the electric field everywhere inside the cavity is zero.)

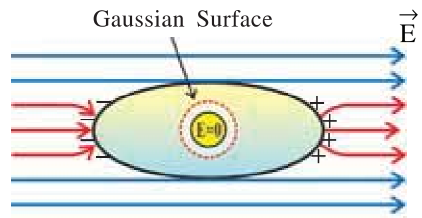


Figure 2.11
Cavity in a Conductor

This fact is called electrostatic shielding. If we are sitting in a car and suppose lightning strickes, we should close the doors of the car. (we suppose the car is fully made of metal !) By doing so, we happen to be in the cavity of car and we are protected due to electrostatic shielding.

(b) Effects Produced by Putting Charge on the Conductor :

In the above discussion we considered the effects produced when a metallic conductor is placed in an external electric field. Now we note the effects produced when a charge is placed on a metallic body, in the absence of an external electric field.

(1) Whether a metallic conductor is put in an external electric field or not and whether a charge is put or not, on it, in all such (but stable) conditions the **electric field everywhere inside** the conductor is always **zero.** This is a very important and a general fact. (This can be taken as a property to define a conductor).

(2) The charge placed on a coductor is always **distributed only on the outer surface** of the conductor. We can understand this by the fact that the electric field inside a coductor is zero. Consider a Gaussian Surface shown by the dots inside the surface and very close to it, (figure 2.12). Every point on it is inside the surface and not on the surface of conductor Hence the electric field at every point on this surface is zero. Hence according to Gauss's theorem the charge enclosed by that surface is also zero.

(3) In a stable condition these charges are steady on the surface. This shows that the electric field is locally normal to the surface. (See figure 2.12).

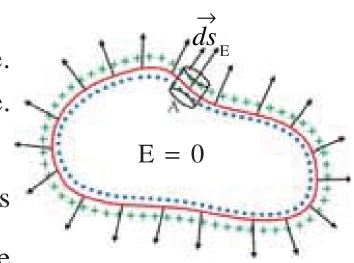


Figure 2.12

(4) The electric field at any point on the charged conductor is $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$, where $\hat{n}$ = unit vector coming out from the surface normally. To prove this, we consider a Gaussian surface of a pill-box (a cylinder) of extremely small length and extremely small cross-section ds. A fraction of it is inside the surface and the remaining part is outside the surface. The total charge enclosed by this pill-box is $q = \sigma ds$; where σ = surface density of charge on the conductor. At every point on the surface of the conductor $\vec{E}$ is perpendicular to the local surface element. Hence it is parallel to surface vector ($\vec{E} \parallel \vec{ds}$).

But inside the surface $\vec{E} = 0$. Hence the flux coming out from the cross-section of pill-box inside the surface = 0. For its side the area vector (surface vector) is normal to $\vec{E}$. Hence flux through it is zero. The flux coming out from the cross-section of pill-box outside the surface is $\vec{E} \cdot \vec{ds} = E ds$.

$\therefore$ Total flux = E ds

According to Gauss's theorem, $E\, ds = \frac{q}{\varepsilon_0} = \frac{\sigma\, ds}{\varepsilon_0}$ (2.12.1)

$$\therefore E = \frac{\sigma}{\varepsilon_0} \quad (2.12.2)$$

In the vector from $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$ (2.12.3)

If σ is positive, $\vec{E}$ is in the direction of normal coming out from the surface. If σ is negative $\vec{E}$ is in the direction of normal entering into the surface.

(5) If some charge is placed inside a cavity in the conductor, then the charges are so induced on the surface of the cavity and on the outer surface of conductor that the electric field in the region which is inside the conductor but outside the cavity becomes zero. The electric field inside the cavity is non-zero and the electric field outside the conductor due to that charge is also non-zero.

[**Note (For information only) :** In the above discussion we have considered the coductors to be insulated.

The sharp ends of the conductor have a large electric charge density. The electric field near such a region is very strong. This strong electric field can strip the electrons from the surface of the metal. This event is known as Corona discharge. In general, this event is called dielectric breakdown.

The electrons escaping the surface of a metal perform an accelerated motion, colliding with the air particles coming in their way. The excited atoms of the energetic particles emit electromagnetic waves and a greenish glow is observed. Apart from the above process, the ionization of the air molecules also takes palce, during collision

Sailors long ago saw these glows at the pointed tops of their masts and spars and dubbed the phenomenon St. Elmo's fire.]

2.13 Capacitors and Capacitance

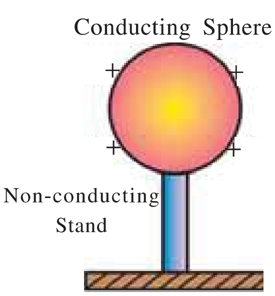


Figure 2.13

Consider an insulated conducting sphere as shown in the figure 2.13. Suppose we go on gradually adding positive charge on this sphere. As the charge on the sphere is gradually increased, the potential (V) on the surface of the sphere and the electric field around the sphere also go on gradually increasing. In this process at some one stage the electric field becomes sufficiently strong to ionize the air particles around the sphere. Hence the charge on the sphere is conducted through air and insulating property of air gets distroyed (i.e. it is not sustained.). This effect is called dielectric breckdown.

Thus the charge on the sphere is leaked and now the sphere is not able to store any additional charge. During this entire process the ratio of the charge (Q) on the sphere and the potential (V) on the sphere remains constant. This ratio is called the capacitance of the sphere. [$C = \frac{Q}{V}$]

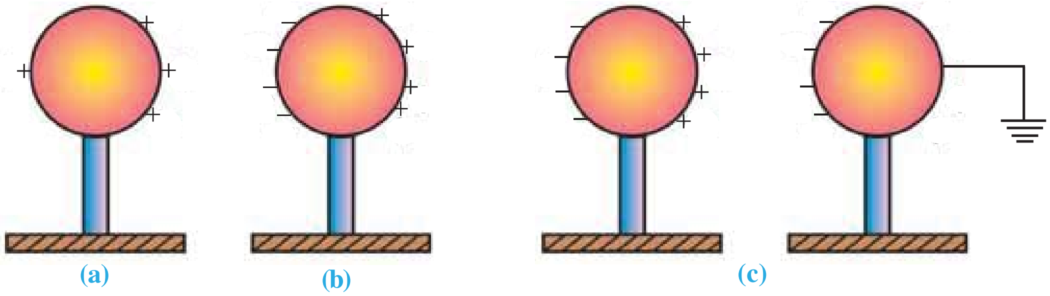


Figure 2.14

The maximum electric field upto which an insulating (non-conducting) medium can maintain its insulating property is called the **dielectric strength** of that medium (or the minimum electric field which starts ionization in a given non-conducting-medium is called its **dielectric strength.**). For air the dielectric strength is nearly 3000 $\frac{V}{mm}$.

Now, if we want to increase the capacity of the above mentioned sphere to store charge (capacitance C), then place another, insulated conducting sphere near the first one. So, electric charge is induced in this second sphere. See figure 2.14(b). If the second sphere is connected to Earth, as in figure 2.14(c) electrons from Earth will flow to it and neutralize the positive charge in it. Now due to negative charge on the second sphere the potential on the surface of the first sphere and the electric field near it are decreased. Now the capacity to store charge on the first sphere increases, as compared to earlier. In this condition also the ratio of the electric charge Q and the p.d. (V) between two spheres at every stage is found to be constant. This ratio is called the capacitance C of this system of two spheres. The value of this capacitance depends on the dimensions of the spheres, their relative arrangement and the medium between them.

"A device formed by two conductors insulated from each other is called a capacitor." These conductors are called the plates of the capacitor. The conductor with positive charge is called the positive plate and the one with negative charge is called the negative plate. The charge on the positive plate is called the charge on the capacitor and the potential difference between the two conductors is called the potential difference (V) between the two plates of the capacitor. Here the capacitance of the capacitor is $C = \frac{Q}{V}$.

The SI unit of capacitance is coulomb / volt and in memory of the great sceintist Michael Faraday it is known as Farad. Its symbol is F. Farad is a large unit for practical purposes and hence smaller units microfarad ($1\ \mu F = 10^{-6} F$) nanofarad ($1\ nF = 10^{-9} F$) and picofarad ($1\ pF = 10^{-12} F$) are used in practice.

A capacitor having a definite capacitance is shown by the symbol ⊣⊢ and the one having a variable capacitance is shown by the symbol .

Moreover, a single conducting sphere of radius R and having charge Q can also be considered as a capacitor, because it also has 'some' capacity to store charge. For such a capacitor other conductor (with $-Q$ charge) is considered to be at infinite distance (separation). Taking the potential at infinite distance from the sphere as zero, the potential on the surface of this sphere is $V = \frac{kQ}{R}$. Hence the potential difference between this sphere and the other one imagined at infinite distance is also $V = \frac{kQ}{R}$.

$\therefore$ The capacitance of this sphere is $C = \frac{Q}{V} = \frac{QR}{kQ} = \frac{R}{k} = 4\pi\varepsilon_0 R$ ($\because K = \frac{1}{4\pi\varepsilon_0}$). Earth can also be considered as a capacitor. You may calculate its capacitance.

2.14 Parallel Plate Capacitor

In such a capacitor, two conducting parallel plates of equal area (A) are insulated from each other and kept at a separation of (d). (See figure 2.15)

Considering vacuum (or air) as the non-conducting medium between them, we shall obtain the formula for its capacitance.

Suppose, the electric charge on this capacitor is Q. Therefore, the value of the surface density of charge on its plates is $\sigma = \frac{Q}{A}$. The value of d is kept very small as compared to the dimension of each plate. Due to this, the non-uniformity of the electric field near the ends of the plates can be neglected and in the entire region between the plates the electric field $\vec{E}$ can be taken as constant.

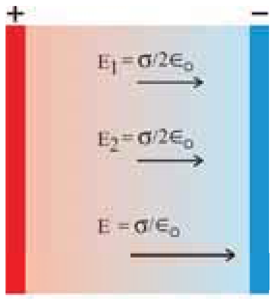


Figure 2.15 Parallel Plate Capacitor

The uniform electric field in the region between two plates due to the positive plate is $E_1 = \frac{\sigma}{2\varepsilon_0}$ in the direction from positive to negative plate. (2.14.1)

Similarly the uniform electric field in the same region due to the negative plate, is $E_2 = \frac{\sigma}{2\varepsilon_0}$ (2.14.2)

(Also in the direction from positive to negative plate.)

Since these two fields are in the same direction, the resultant uniform electric field is

$$E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} \quad (2.14.3)$$

It is in the direction from positive to negative plate.

$$\therefore E = \frac{Q}{\varepsilon_0 A} \quad (2.14.4)$$

In the regions on the other sides of the plates, E_1 and E_2 being equal but in opposite direction, the resultant electric field becomes zero.

If the potential difference between these two plates is V, then $V = Ed$ (2.14.5)

$\therefore$ From equations (2.14.4) and (2.14.5),

$$V = \frac{Q}{\varepsilon_0 A} d \quad (2.14.6)$$

$\therefore$ From the formula $C = \frac{Q}{V}$, we get the capacitance of parallel plate capacitor as

$$C = \frac{\varepsilon_0 A}{d} \quad (2.14.7)$$

From equation (2.4.7), it is clear that if the distance between two plates each of 1 m × 1 m is 1 mm, its capacitance is $C = \frac{(8.85\times10^{-12})(1)}{10^{-3}} = 8.85 \times 10^{-9}$F.

If we want 1F capacitance, then the area of each plate kept at a separation of 1 mm should be $A = \frac{Cd}{\varepsilon_0} = \frac{(1\times10^{-3})}{8.85\times10^{-12}} = 1.13 \times 10^8$ m^2. Thus each of the length and the breadth of each plate should be nearly 1×10^4 m = 10 km.

2.15 Combinations of Capacitors

The system, formed by the combination of capacitors having capacitances C_1, C_2, ..., C_n has some equivalent (effective) capacitance C. We shall discuss two types of combinations.

(a) Series Combination of Capacitors

The arrangement formed by joining the capacitors having capacitances C_1, C_2, C_3, ..., C_n by conducting wires as shown in figure 2.16 is called the series combination of capacitors.

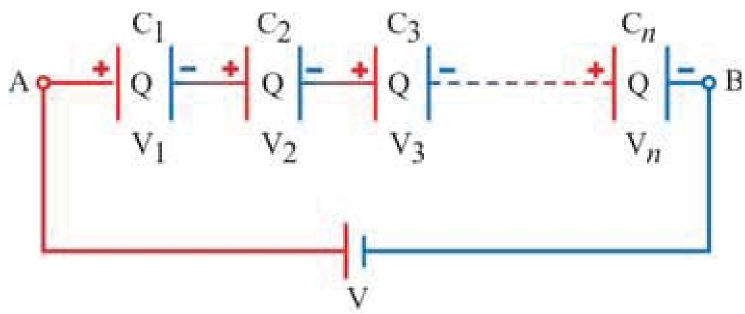


Figure 2.16 Series Combination of Capacitors

In such a condition the charge on every capacitor has the same value Q. As (−Q) charge is deposited by the battery on one plate, it induces (+Q) charge on the other plate. For this (−Q) charge from the second plate will be deposited on the near plate of the next capacitor. This induces +Q charge on the other plate. This continues further. Thus all capacitors have equal charge. but the potential difference between the two plates of different capacitors is different. From the figrue it is clear that

$$V = V_1 + V_2 + V_3 + ... + V_n \qquad (2.15.1)$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + ... + \frac{Q}{C_n} \qquad (2.15.2)$$

$(\because C_1 = \frac{Q}{V_1}, ... \text{ etc.})$

$$\therefore \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n} \qquad (2.15.3)$$

If the effective capacitance of this combination is C,

$$\frac{V}{Q} = \frac{1}{C} \qquad (2.15.4)$$

$$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n} \qquad (2.15.5)$$

Thus the value of effective capacitance is even smaller than the smallest value of capacitance in the combination.

[Note that here the formula obtained for series combination is similar to the formula for effective (equivalent) resistance obtained for the parallel combination of the resistances.]

(b) Parallel Combination of Capacitors

The arrangement formed by joining the capacitors having capacitances C_1, C_2, C_3 by conducting wires as shown in figure 2.17 is called the parallel combination of capacitors.

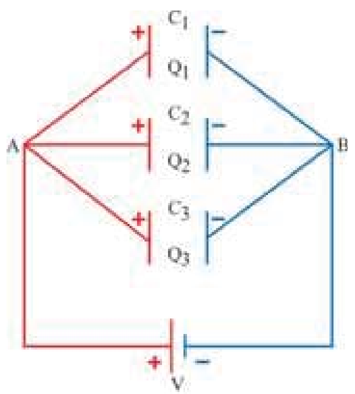


Figure 2.17 Parallel Combination of Capacitors

In such a combination the potential difference (V) between the plates of every capacitor is the same and is equal to the potential difference between their common points A and B. But the charge Q on every capacitor is different.

Here, $\left.\begin{aligned} Q_1 &= C_1 V \\ Q_2 &= C_2 V \\ Q_3 &= C_3 V \end{aligned}\right\}$ (2.15.6)

And the total electric charge

$$Q = Q_1 + Q_2 + Q_3$$
$$= C_1V + C_2V + C_3V$$
$$= (C_1 + C_2 + C_3)V \quad (2.15.7)$$

If the effective capacitance of this parallel combination is C, then

$$C = \frac{Q}{V} = C_1 + C_2 + C_3 \quad (2.15.8)$$

If such n-capacitors are joined in parallel connection, the effective capacitance is

$$C = C_1 + C_2 + C_3 + ... + C_n \quad (2.15.9)$$

Here, as the values of capacitances are added the value of effective capacitance is even greater than the largest value of capacitance in the connection.

[Note that the formula obtained here for parallel combination is similar to the formula for effective (equivalent) resistance obtained for the series combination of resistances.]

Illustration 8 : Prove that the force acting on one plate due to the other in a parallel plate capacitor is $F = \frac{1}{2}\ \frac{CV^2}{d}$.

Solution : The electric field due to one plate is $E_1 = \frac{\sigma}{2\varepsilon_0}$ (1)

A second plate having charge σA is present in the above electric field.

$\therefore$ The force acting on the second plate is

$F = (\sigma A)E_1$

Substituting the value of E_1 from (1), we have,

$$F = \frac{\sigma^2 A}{2\varepsilon_0}$$

But $\sigma = \frac{Q}{A}$

$$\therefore\ F = \frac{\frac{Q^2}{A^2}.A}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A} = \frac{Q^2/d}{2\varepsilon_0 A/d} = \frac{Q^2}{2dC} \qquad \left(\because \frac{\varepsilon_0 A}{d} = C\right)$$

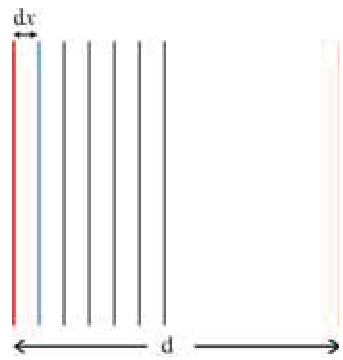
$$\therefore\ F = \frac{1}{2}\frac{CV^2}{d} \qquad (\because\ Q = CV)$$

Illustration 9 : Figure shows an infinite number of conducting plates of infinitesimal thickness such that consecutive plates are sparated by a small distance dx spread over a distance d to form a capacitor. Calculate the value of the capacitance of such an arrangement.

Solution : The capacitance of each of the capacitors in the above arrangement, $dC = \frac{\varepsilon_0 A}{dx}$

All these capacitors are in series combination with each other.

Therefore the total capacitance C is obtained from



$$\frac{1}{C} = \frac{1}{dC} + \frac{1}{dC} + ...$$

$$= \frac{dx}{\varepsilon_0 A} + \frac{dx}{\varepsilon_0 A} + ...$$

$$= \frac{1}{\varepsilon_0 A}(dx + dx + ... + dx)$$

$$\therefore \frac{1}{C} = \frac{d}{\varepsilon_0 A}$$

$$\therefore C = \frac{\varepsilon_0 A}{d}$$

This is equivalent to the capacitance of the capacitor formed by the first and the last plate of the above arrangement.

2.16 Energy Stored in a Charged Capacitor

In order to establish a charge on the capacitor, work has to be done on the charge. This work is stored in the form of the potential energy of the charge. Such a potential energy is called the energy of capacitor.

Suppose the charge on a parallel plate capacitor is Q. In this condition each plate of the capacitor is said to be lying in the electric field of the other plate.

The magnitude of the uniform electric field produced by one plate of capacitor is

$$= \frac{\sigma}{2\varepsilon_0} .... \quad (2.16.1)$$

where $\sigma = \frac{Q}{A}$ and A = area of each plate.

Hence by taking arbitrarily the potential on this plate as zero, that of the other plate at distance d from it will be $= \left(\frac{\sigma}{2\varepsilon_0}\right)d$ (2.16.2)

From this, the potential energy of the first plate is zero and that of the second plate will be = (potential) (charge Q on it)

$$= \left(\frac{\sigma d}{2\varepsilon_0}\right)Q \quad (2.16.3)$$

$\therefore$ Energy stored in the capacitor

$$U_E = \frac{\sigma d Q}{2\varepsilon_0} = \left(\frac{Q}{A}\right)\frac{dQ}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A/d} \quad (2.16.4)$$

$$= \frac{Q^2}{2C} \quad (2.16.5)$$

where, $C = \frac{\varepsilon_0 A}{d}$ = capacitance of capacitor.

Moreover, $C = \frac{Q}{V}$. From equation (2.16.5) and this formula we can write

$$U_E = \frac{VQ}{2} \quad (2.16.6)$$

and $U_E = \frac{1}{2}CV^2$ (2.16.7)

We have derived these equations (2.16.5), (2.16.6) and (2.16.7) for the parallel plate capacitor, but in general they are true for all types of capacitor.

To show energy stored in the capacitor in the form of energy density :

The energy stored in the capacitor is $U_E = \frac{1}{2}CV^2$. This energy is stored in the region between the two plates, that is, in the volume Ad, where A = area of each plate and d = separation between them. Hence, if we write the energy stored **per unit volume** in the region between the plates – that is energy density – as ρ_E, then

$$\rho_E = \frac{U_E}{\text{Volume}} = \frac{\frac{1}{2}CV^2}{Ad} \qquad (2.16.8)$$

$$= \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)\frac{V^2}{Ad} \qquad (2.16.9)$$

$$= \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)\left(\frac{V}{d}\right) \qquad (2.16.10)$$

$$= \frac{1}{2}\varepsilon_0 E^2 \qquad (\because \frac{V}{d} = E) \qquad (2.16.11)$$

Where $\frac{V}{d}$ = E = electric field between the two plates. Thus the energy stored in the capacitor can be considered as the energy stored in the electric field between its plates.

We have obtained this equation for a parallel plate capacitor but it is a result in general and can be used for the electric field of any arbitrary charge distribution.

Illustration 10 : A capacitor of 4 μF value is charged to 50 V. The above capacitor is then connected in parallel to a 2 μF capacitor. Calculate the total energy of the above system. The second capacitor is not charged prior to its connection with the 4 μF capacitor.

Solution : The energy stored in the capacitor of 4 μF will be

$$W_1 = \frac{1}{2}C_1V^2$$

$$= \frac{1}{2} \times 4 \times (50)^2 = 2 \times 2500 = 5000 \ \mu J$$

The two capacitors are connected in parallel. Let q_1 and q_2 be the electrical charges on capacitors C_1 and C_2 respectively after connection. If V' is their common potential difference across the capacitors. $(V' = \frac{q_1}{C_1} = \frac{q_2}{C_2})$

$$\frac{q_1}{q_2} = \frac{C_1}{C_2}$$

$$\therefore \frac{q_1+q_2}{q_2} = \frac{C_1+C_2}{C_2} \qquad (1)$$

By the law of conservation of charge.

$$q_1 + q_2 = Q \qquad (2)$$

Where Q is the initial charge

Now, $Q = C_1V = (4)(50)$

$= 200 \ \mu C$

Putting equation (2) in equation (1) and substituting the value of Q, we have,

$$\frac{200}{q_2} = \frac{(4+2)}{2}$$

$$\therefore\ q_2 = \frac{200\times 2}{6} = \frac{200}{3}\ \mu C$$

From Equation (2)

$$q_1 = 200 - \frac{200}{3}$$

$$= \frac{400}{3}\ \mu C$$

Calculation of energy : The energy of the first capacitor

$$\frac{q_1^2}{2C_1} = \left(\frac{400}{3}\right)^2 \times \frac{1}{2\times 4} = 2222\ \mu J$$

The energy of the second capacitor

$$\frac{q_2^2}{2C_2} = \left(\frac{200}{3}\right)^2 \times \frac{1}{2\times 2} = 1111\ \mu J$$

The total energy of the system, after combination = 2222 + 1111 = 3333 = μJ

Thus the energy decreased by 5000 − 3333 = 1667 μJ. This energy is dissipated in the from of heat.

2.17 Dielectric Substances and their Polarisation

Non-conducting materials are called **dielectric**. Faraday found that when a dielectric is introduced between the plates of a capacitor, the capacitance of the capacitor is increased. In order to understand how does this happen, we should know about the effects produced when a dielectric is placed in an electric field. Dielectric materials are of two types (1) polar and (2) non-polar.

A dielectric is called a polar dielectric if its molecules possess a permanent dipole moment (e.g. HC*l*, H_2O, ... etc.) If the molecules of the dielectric do not possess a permanent dipole moment, then that dielectric is called a non-polar dielectric (e.g. H_2, O_2, CO_2, ... etc.)

(a) Non-polar Molecule : In a non-polar molecule, the centre of the positive charge and the centre of the negative charge coincide with each other. Hence they do not possess a permanent dipole moment. Now, when it is placed in a uniform electric field ($\vec{E}_0$), these centres are displaced in mutually opposite directions. Hence they now, possess a dipole moment $p = qd$, where d = the distance between centres of positive and negative charges after being displaced, q = the value of positive or negative charge (See figure 2.18).

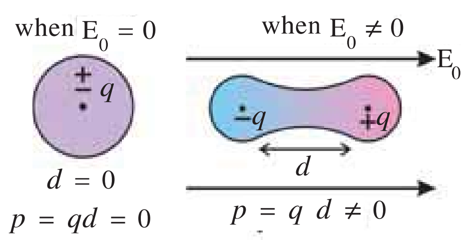


Figure 2.18 Polarisation of a Non-polar Molecule

Thus an electric dipole is induced in it. In other words due to an external electric field a dielectric made of such molecules is said to be polarised. If the external electric field ($\vec{E}_0$) is not very strong, it is found that this dipole moment of molecule is proportional to $\vec{E}_0$.

$$\therefore\ \vec{p} = \alpha \vec{E}_0 \qquad (2.17.1)$$

where α is called the polarisability of the molecule.

From units of $\vec{p}$ and $\vec{E}_0$, the unit of α is $C^2\ m\ N^{-1}$

(b) Polar Molecule : A polar molecule possesses a permanent dipole moment $\vec{p}$, but such dipole moments of different molecules of the substance are randomly oriented in all possible directions and hence the resultant dipole moment of the substance becomes zero.

Now, on applying an external electric field a torque acts on every molecular dipole. Therefore, it rotates and tries to become parallel to the electric field. Thus a resultant dipole moment is produced. In this way the dielectric made up of such molecules is said to be polarised. Moreover, due to thermal oscillations the dipole moment also gets deviated from being parallel to electric field. If the temperature is T, the dipoles will be arranged in such an equilibrium condition that the average thermal energy per molecule ($\frac{3}{2}k_B T$) balances the potential energy of dipole ($U = -\vec{p}\cdot\vec{E}_0$) in the electric field. At 0 K temperature since the thermal energy is zero, the dipoles become parallel to the electric field. We shall only discuss such an ideal situation.

(c) When there is air (or vacuum) between the charged plates of a capacitor, the electric field between the plates is $E_0 = \dfrac{\sigma_f}{\varepsilon_0}$. (2.17.2)

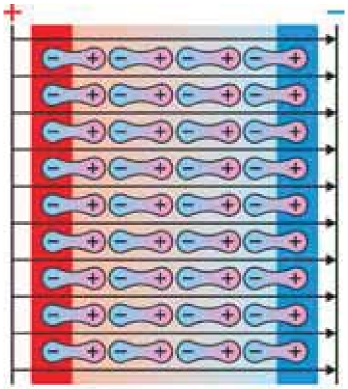


Figure 2.19 Polarisation in Dielectric

where σ_f = value of surface charge density on each plate.

The charge on these plates is called the free charge, because its value can be adjusted at our will (by joining proper battery).

Here, the area of each plate is = A. Now on placing a slab of dielectric material (polar or non-polar) in the region between the plates, the polarisation produced by the electric field $\vec{E}_0$ is shown in the figure 2.19. We want to find the electric field inside the dielectric.

It is clear from the Figure that the opposite charges in the successive dipoles inside the slab cancel the effect of each other, as they are very close to each other and a net (resultant) charge resides only on the faces of the slab, close to the plates. These charges are called **induced charges** or the **bound charges** or the **polarisation charges.** The charge induced on the surface of the slab close to the positive plate is $-\sigma_b A$ and that on the surface close to the negative plate is $+\sigma_b A$, where $-\sigma_b$ and $+\sigma_b$ are, the surface densities of the **bound charges** on **the** respective **surfaces.** This induced charges form a dipole. Its dipole moment is $P_{\text{total}} = (\sigma_b A)d$ (2.17.3)

where, d = thickness of the slab = distance between two plates. (if sides of slab touch the plates)

Here, Ad = volume of slab = V (2.17.4)

The dipole moment produced per unit volume is called the **intensity of polarisation** or in short **polarisation (P).**

$$\therefore\ P = \frac{P_{\text{total}}}{\text{volume}} = \frac{(\sigma_b A)d}{Ad} = \sigma_b \qquad (2.17.5)$$

Thus the magnitude of polarisation(P) in a dielectric is equal to the **surface density** of **bound charges (σ_b),** induced on its surface. The electric field produced by these induced charges is in the direction opposite to the external electric field $\vec{E}_0$. Hence, now the resultant electric field E inside the dielectric can be considered as produced due to ($\sigma_f - \sigma_b$).

$\therefore$ $E = \dfrac{\sigma_f - \sigma_b}{\varepsilon_0}$ (2.17.6)

Thus the net electric field inside the dielectric is less than the applied (external) electric field. (But recall that net electric field in the conductor was zero.)

Moreover, it is found that if the external electric field (E_0) is not very strong, then the polarisation (P), is proportional to the net electric field (E) inside the dielectric.

i.e. $P \propto E$

$\therefore$ $P = \varepsilon_0 x_e E$ (2.17.7)

where x_e = constant, which is called the electric susceptibility of the dielectric medium. It depends on the nature of dielectric and the temperature. The dielectric obeying $P \propto E$ is called a linear dielectric.

From equation (2.17.7) $x_e = \dfrac{P}{\varepsilon_0 E}$ (2.17.8)

Using $E_0 = \dfrac{\sigma_f}{\varepsilon_0}$ and $P = \sigma_b$ in equation (2.17.6), we get,

$E = \dfrac{\varepsilon_0 E_0 - P}{\varepsilon_0}$ (2.17.9)

$\therefore$ $\varepsilon_0 E = \varepsilon_0 E_0 - \varepsilon_0 x_e E$ ($\because$ From equation 2.17.8 $P = \varepsilon_0 x_e E$) (2.17.10)

$\therefore$ $\varepsilon_0 E + \varepsilon_0 x_e E = \varepsilon_0 E_0$ (2.17.11)

$E\varepsilon_0(1 + x_e) = E_0\varepsilon_0$ (2.17.12)

$\varepsilon_0(1 + x_e)$ is called the permittivity(ε) of the dielectric medium; i.e. $\varepsilon = \varepsilon_0(1 + x_e)$ (2.17.13)

$\therefore$ $E\varepsilon = E_0\varepsilon_0$ (2.17.14)

$\therefore$ $E = \dfrac{E_0}{\varepsilon / \varepsilon_0}$ (2.17.15)

Here, $\dfrac{\varepsilon}{\varepsilon_0}$ is called the relative permittivity ε_r of the medium and is also called the dielectric constant of the medium K. Value of K is always greater than 1.

Thus, $\dfrac{\varepsilon}{\varepsilon_0} = \varepsilon_r = K$ (2.17.16)

From equations (2.17.13) and (2.17.16), $\dfrac{\varepsilon_0(1+x_e)}{\varepsilon_0} = K$

$\therefore$ $K = 1 + x_e$ (2.17.17)

This equation shows the relation between two electrical properties x_e and K of the dielectric.

Now, equation (2.17.15) can be written as

$E = \dfrac{E_0}{K}$ (2.17.18)

Thus if the electric field in certain region in the free space is E_0, when a dielectrcic is placed in that region, the electric field in the dielectric becomes K^{th} part (i.e., $\frac{1}{K}$ times), the value in free space.

Electric Displacement : When a dielectric is placed between the plates of a capacitor, the net electric field produced in the dielectric is given by $E = \frac{\sigma_f - \sigma_b}{\varepsilon_0}$, where σ_f = value of surface density of free charges, σ_b = Value of surface density of bound charges.

Since σ_b = P, we get $E = \frac{\sigma_f - P}{\varepsilon_0}$ (2.17.19)

$\therefore \varepsilon_\circ E + P = \sigma_f$ (2.17.20)

The directions of $\vec{E}$ and $\vec{p}$ are the same. $\varepsilon_0 \vec{E} + \vec{P}$ is called electric displacement $\vec{D}$.

$\therefore \vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ (2.17.20)

It is a vector field. Using the definition of $\vec{D}$, many equations regarding electric field become simpler in form. Gauss' theorem in the presence of a dielectric is written as

$\oint \vec{D} \cdot d\vec{s} = q$ (2.17.22)

where q is **only the free charge** (it does not include the bound charge). Thus in case of dielectric the field related to the free charges is not $\vec{E}$, but it is $\vec{D}$, that is $\varepsilon_0 \vec{E} + \vec{P}$.

2.18 Capacitor with a Dielectric

When there is air (or vacuum) between the plates of the parallel plate capacitor, its capacitance is given by $C = \frac{\varepsilon_0 A}{d}$. (2.18.1)

where ε_0 = permittivity of vacuum, A = area of each plate and d = separation between two plates. Now if the entire region between these plates is filled with a dielectric medium having permittivity ε, then to obtain the formula for its capacitance C', ε should be placed in place of ε_0 in the above formula.

$\therefore C' = \frac{\varepsilon A}{d}$ (2.18.2)

$\therefore \frac{C'}{C} = \frac{\varepsilon}{\varepsilon_0} = K$ (2.18.3)

where K = dielectric constant of that medium.

$\therefore C' = KC$ (2.18.4)

Thus, putting a medium of dielectric constant K between the plates of the capacitor, its capacitance becomes K times and thus its capacity to store electric charge also becomes K times.

Illustration 11 : A capacitor consists of three parallel plates of equal area A. The distance between them is d_1 and d_2. Dielectric material having pemittivity ε_1 and ε_2 is present between the plates. (i) Calculate the capacitance of such a system. (ii) Express this capacitance in terms of K_1 and K_2.

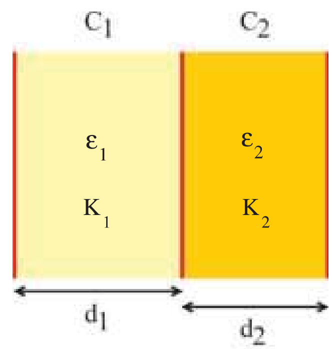
Solution : As shown in the figure, the two capacitors are connected in series. If C is the total capacitance, then

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{But } C_1 = \frac{\varepsilon_1 A}{d_1} \text{ and } C_2 = \frac{\varepsilon_2 A}{d_2}$$

$$\therefore \frac{1}{C} = \frac{d_1}{\varepsilon_1 A} + \frac{d_2}{\varepsilon_2 A}$$

$$= \frac{d_1\varepsilon_2 A + \varepsilon_1 A d_2}{\varepsilon_1\varepsilon_2 A^2} = \frac{d_1\varepsilon_2 + d_2\varepsilon_1}{\varepsilon_1\varepsilon_2 A}$$

$$\therefore C = \frac{\varepsilon_1\varepsilon_2 A}{d_1\varepsilon_2 + d_2\varepsilon_1} \text{ or } C = \frac{A}{\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}}$$



From $K_1 = \frac{\varepsilon_1}{\varepsilon_0}$, we get $\varepsilon_1 = \varepsilon_0 K_1$. Similarly $\varepsilon_2 = \varepsilon_0 K_2$, where ε_0 = permittivity of vacuum.

$$\therefore C = \frac{A}{\frac{d_1}{\varepsilon_0 K_1} + \frac{d_2}{\varepsilon_0 K_2}} = \frac{A\varepsilon_0}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

2.19 Van-De-Graaff Generator

With the help of this machine, a potential difference of a few million (1 million = 10^6 = ten lac) volt can be established. By suitably passing a charged particle through such a high potential difference it is accelerated (to very high velocity) and hence acquires a very high energy ($\frac{1}{2}mv^2$). Because of such a high energy they are able to penetrate deeper into the matter. Therefore, fine structure of the matter can be studied with the help of them. The principle of this machine is as under.

Suppose there is a positive charge Q, on an insulated conducting spherical shell of radius R, as shown in the figure 2.20. At the centre of this shell, there is a conducting sphere of radius $r (r < R)$, having a charge q.

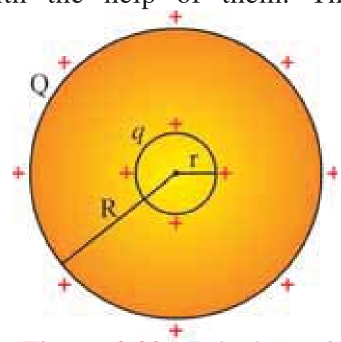


Figure 2.20 Principle of Van–de–Graaff Generator

Here the electric potential on the shell of radius R, is

$$V_R = \frac{kQ}{R} + \frac{kq}{R} \qquad (2.19.1)$$

and the potential on the surface of the sphere of radius r, is

$$V_r = \frac{kQ}{R} + \frac{kq}{r} \qquad (2.19.2)$$

It is clear from these two equations that the potential on the smaller sphere is more and the potential difference (p.d.) between them is

$$V_r - V_R = \left(\frac{kQ}{R} + \frac{kq}{r}\right) - \left(\frac{kQ}{R} + \frac{kq}{R}\right)$$

$$= kq\left[\frac{1}{r} - \frac{1}{R}\right] \qquad (2.19.3)$$

Hence, if the smaller sphere is brought in electrical contact with the bigger sphere then the charge goes from smaller on to the bigger sphere. Thus charge can be accumulated to a very large amount on the bigger sphere and thereby its potential can be largely increased.

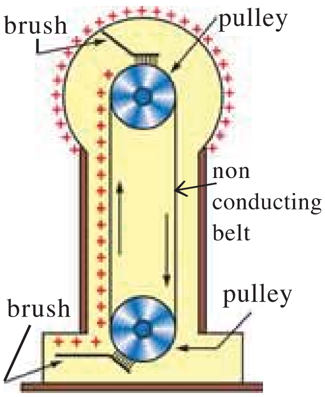


Figure 2.21
Van-de-Graaff Generator

The machine based on this principle made by Van-De-Graaff, is called the Van-De-Graaff generator.

As shown in the figure 2.21 a spherical shell of a few meter radius, is kept on an insulated support, at a height of a few meters from the ground.

A pully is kept at the centre of the big sphere and another pully is kept on the ground. An arrangement is made such that a non-conducting belt moves across two pulleys. Positive charges are obtained from a discharge tube and are continuously sprayed on the belt using a metallic brush (with sharp edges) near the lower pulley. This positive charge goes with the belt towards the upper pulley.

There it is removed from the belt with the help of another brush and is deposited on the shell (because the potential on the shell is less than that of the belt on the pulley.) Thus a large potential difference (nearly 6 to 8 million volt) is obtained on the big spherical shell.

Illustration 12 : Q amount of electric charge is residing on a conducting sphere having radius equal to R_1. This sphere is connected to another conducting sphere of radius R_2 by a conducting wire. Calculate the electric charge on each of the spheres. The two spheres are separated by a large distance.

Solution : Let q_1 and q_2 be the electric charge present on the two conducting spheres after being connected with each other.

$$\therefore\ Q = q_1 + q_2 \qquad (1)$$

The electric potentials of the two spheres have to be the same, since they are connected by a conducting wire

$$\therefore\ \frac{kq_1}{R_1} = \frac{kq_2}{R_2}\ \therefore\ \frac{q_1}{q_2} = \frac{R_1}{R_2} \qquad (2)$$

$$\therefore\ \frac{q_2 + q_1}{q_2} = \frac{R_1 + R_2}{R_2}$$

$$\therefore\ \frac{Q}{q_2} = \frac{R_1 + R_2}{R_2}$$

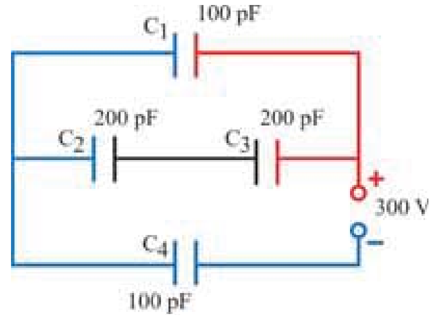
$$\therefore\ q_2 = \frac{R_2}{R_1 + R_2}Q \qquad (3)$$

Substituting the value of q_2 in the equation (1), we have

$$Q = q_1 + \frac{R_2}{R_1 + R_2}Q$$

This gives $q_1 = \frac{R_1}{R_1 + R_2}Q.$

Illustration 13 : Find the effective capacitance of the network shown in the figure and find the charge on each capacitor.



Solution : Here, C_2 and C_3 are in series. Their equivalent (effective) capacitance is $C' = \frac{C_2C_3}{C_2+C_3} = \frac{200\times200}{200+200}$ $= 100pF$.

This C' and C_1 are in parallel connection. Their equivalent capacitance is $C'' = C' + C_1 = 100 + 100 = 200\ pF$.

This C'' and C_4 are in series. Their Equivalent capacitance is $C''' = \frac{C''C_4}{C''+C_4} = \frac{200\times100}{200+100}$ $= \frac{200}{3}pF$.

Now the charge supplied by the battery is $Q = C'''V = \left(\frac{200\times10^{-12}}{3}\right)(300) = 2 \times 10^{-8}C$.

$\therefore$ charge on C_4 and C'' are equal and each is equal to $2 \times 10^{-8}C$.

$\therefore$ Charge on C_4 is $Q_4 = 2 \times 10^{-8}C = Q''$ (on C'')

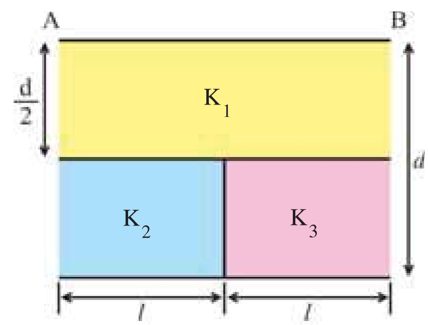
Charge on C'' is divided on C' and C_1. Since C' and C_1 are of equal values, that charge is divided equally on them.

$\therefore$ Charge on C_1 is $Q_1 = \frac{1}{2}Q_4 = 1 \times 10^{-8}C = Q'$... (on C')

The charge on C' has the same value as on C_2 and C_3.

$\therefore Q_2 = Q_3 = 1 \times 10^{-8}C$.

Illustration 14 : Find the capacitance of the capacitor shown in the figure. Area of AB is A. K_1, K_2, K_3 are dielectric constants of respective materials.



Solution : We shall use the formula $C = \frac{\varepsilon A}{d} = \frac{K\varepsilon_0 A}{d}$ and also use the formulae for series and parallel connections. Here, the capacitors formed by K_2 and K_3 are in parallel and hence their equivalent capacitance is

$$C_{23} = C_2 + C_3 = \frac{K_2\varepsilon_0(A/2)}{(d/2)} + \frac{K_3\varepsilon_0(A/2)}{(d/2)}$$

$$= \frac{\varepsilon_0 A}{d}(K_2 + K_3)$$

The capacitor formed by k_1 is in series connection with this C_{23}. $\therefore$ The equivalent capacitance of the entire system is

$$C = \frac{C_1C_{23}}{C_1+C_{23}} = \frac{\left(\frac{K_1\varepsilon_0 A}{d/2}\right)\left[\frac{\varepsilon_0 A}{d}(K_2+K_3)\right]}{\left(\frac{K_1\varepsilon_0 A}{d/2}\right)+\left[\frac{\varepsilon_0 A}{d}(K_2+K_3)\right]} = \frac{2\varepsilon_0 A}{d}\cdot\frac{K_1(K_2+K_3)}{(2K_1+K_2+K_3)}$$

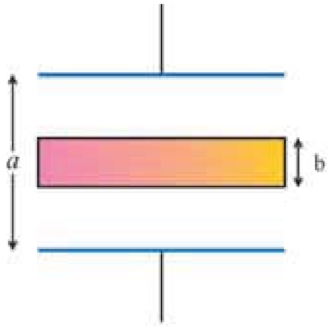
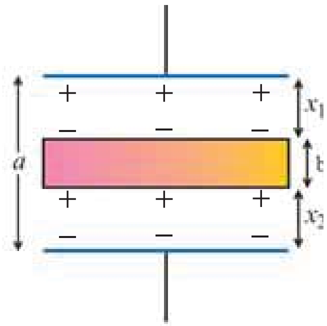


Illustration 15 : A capacitor has air between its plates having separation a. Now a metallic slab of thickness b is placed between its plates as shown in the figure. Show that now its capacitance is $C = \frac{\varepsilon_0 A}{a-b}$. Does this value of capacitance depend on the position of the metallic slab between the plates ?



Solution : One capacitor is formed in the upper region with thickness x_1. Let its capacitance = C_1. Other capacitor is formed in the lower region with thickness x_2. Let its capacitance = C_2. In the thickness b, there is metallic slab, hence no capacitor is formed (because its two surfaces cannot be considered as isolated from each other.) Here, C_1 and C_2 are in series. If their equivalent capacitance is C; $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{x_1}{\varepsilon_0 A} + \frac{x_2}{\varepsilon_0 A} = \frac{x_1 + x_2}{\varepsilon_0 A}$

$$\therefore C = \frac{\varepsilon_0 A}{x_1 + x_2}.$$

But Figure shows that $x_1 + x_2 = a - b$ $\therefore$ $C = \frac{\varepsilon_0 A}{a-b}$. This value does not depend on the position of the metallic slab. Put it anywhere but $(x_1 + x_2)$ remains constant and only in this much region capacitor is formed.

Illustration 16 : The area of each plate of a parallel plate capacitor is 100 cm^2 and the separation between the plates is 1.0 cm. When there is air between the plates, the capacitor is charged with a battery of 100 V. Now the battery is removed and a dielectric slab of thickness 0.4 cm and dielectric constant 4.0 is placed between the plates. (a) Find the capacitance before the dielectric is introduced. (b) Find the free charge on the plate and the surface charge density on it. (c) What is the electric field E_0 in the region between the plate and the dielectric ? (d) What is the electric field in the dielectric ? (e) What is the potential difference between two plates after the dielectric is introduced ?

Solution : $A = 100 \times 10^{-4} m^2$; $d = 1 \times 10^{-2} m$, $V_0 = 100$ V

$d' = 0.4 \times 10^{-2} m$, $K = 4.0$

(a) When there is air between the plates, the capacitance

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12})(100 \times 10^{-4})}{1 \times 10^{-2}} = 8.85 \times 10^{-12} \text{ F} = 8.85 \ p\text{F}.$$

(b) $q_0 = C_0 V_0 = (8.85 \times 10^{-12})(100) = 8.85 \times 10^{-10}$ C

This is free charge. The surface density of charge is

$$\sigma = \frac{q_0}{A} = \frac{8.85 \times 10^{-10}}{100 \times 10^{-4}} = 8.85 \times 10^{-8} \text{ C/m}^2$$

(c) The electric field between the plate and the dielectric is produced by the charge on the plate, that is by the free charge.

$$\therefore E_0 = \frac{\sigma}{\varepsilon_0} = \frac{8.85 \times 10^{-8}}{8.85 \times 10^{-12}} = 10000 \text{ V/m}$$

(d) In the absence of dielectric the electric field at that place would be E_0. Now on putting the dielectric there the electric field is $E = \frac{E_0}{K} = \frac{10000}{4} = 2500$ V/m.

(e) Now, the potential difference (p.d.) between the plates (from V = Ed) is

$$V' = E_0(1 - 0.4) \times 10^{-2} + E(0.4 \times 10^{-2})$$

$$= 10000\ (0.6 \times 10^{-2}) + 2500 \times 0.4 \times 10^{-2}$$

$$= 60 + 10 = 70 \text{ V}.$$

Illustration 17 : A substance has a dielectric constant 2.0 and its dielectric strength is 20×10^6 V/m. It is taken as a dielectric material in a parallel plate copacitor. What should be the minimum area of its each plate such that its capacitance becomes 8.85×10^{-2} μF and it can withstand a potential difference of 2000 V ?

Solution : K = 2; $E = 20 \times 10^6$ V/m , $C = (8.85 \times 10^{-2}) \times 10^{-6}$ F

V = 2000 V, A = ?

Charge on capacitor $Q = CV = (8.85 \times 10^{-8})\ (2000) = 17.7 \times 10^{-5}$ C

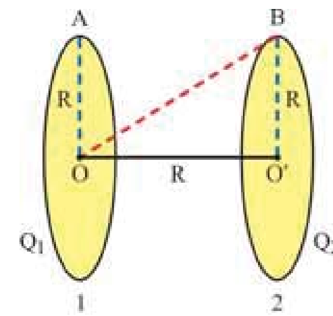
surface density of charge on the plate $\sigma = \frac{Q}{A} = \frac{17.7\times10^{-5}}{A}$ C/m^2.

If there were air between the plates, the electric field would be $E_0 = \frac{\sigma}{\varepsilon_0}$, but here a dielectric is placed. Hence the electric field is $E = \frac{E_0}{K} = \frac{\sigma}{K\varepsilon_0}$.

$$\therefore\ 20 \times 10^6 = \frac{17.7\times10^{-5}}{(A)(2)(8.85\times10^{-12})} \Rightarrow A = 0.5 \text{ m}^2$$

If the value of A is smaller than this, E becomes greater than 20×10^6 and dielectric breackdown occurs.

Illustration 18 : Two identical thin rings each of radius R m are kept on the same axis at a distance of R m apart. If charges on them are Q_1 C and Q_2 C respectively, calculate the work done in moving a charge Q C from the centre of one ring to that of the other.



Solution :

It is clear from the figure that $AO' = BO = \sqrt{R^2+R^2} = (\sqrt{2})R$

Centre of each ring is at equal distance = $(\sqrt{2})R$ from the circumference of the other ring.

$$\therefore \text{ Potential at O is } V_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{R\sqrt{2}}$$

$$\text{and potential at O' is } V_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R\sqrt{2}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{R}$$

$\therefore$ Potential difference $\Delta V = V_1 - V_2 = \frac{1}{4\pi\varepsilon_0 R}(Q_1 - Q_2) + \frac{1}{4\pi\varepsilon_0 R\sqrt{2}}[Q_2 - Q_1]$

$$= \frac{1}{4\pi\varepsilon_0 R}(Q_1 - Q_2) - \frac{1}{4\pi\varepsilon_0 R\sqrt{2}}[Q_1 - Q_2]$$

$$= \frac{1}{4\pi\varepsilon_0 R}(Q_1 - Q_2)\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0 R}(Q_1 - Q_2)\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right] \text{ V}$$

$\therefore$ Work $W = q(\Delta V) = \frac{q(Q_1 - Q_2)}{4\pi\varepsilon_0 R}\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$ J

SUMMARY

1. The information about the work done in taking an electric charge from one point to the other in the electric field is obtained from the quantities called electric potential and electric potential energy.

2. $\int_A^B \vec{E}\cdot\vec{dr}$ is the line-integral of electric field between points A to B and it shows the work done by the electric field in taking a unit positive charge from A and B. Moreover, it does not depend on the path and $\oint \vec{E}\cdot\vec{dr} = 0$.

3. "The work required to be done against the electric field to bring a unit positive charge from infinite distance to the given point in the electric field, is called the electric potential (V) at that point".

   Electric potential at point P is $V_p = -\int_\infty^P \vec{E}\cdot\vec{dr}$

   Its unit is $\frac{\text{joule}}{\text{coulomb}}$ = volt. Symbolically $V = \frac{J}{C}$

   Its dimensional formula is $M^1L^2T^{-3}A^{-1}$

   Absolute value of electric potential has no importance but only the change in it is important.

4. "The work required to be done against the electric field to bring a given change (q) from infinite distance to the given point in the electric field is called the electric potential energy of that charge at that point."

   $U_p = -q\int_\infty^P \vec{E}\cdot\vec{dr} = qV_p$

   The absolute value of electric potential energy has no importance, only the change in it is important.

5. Electric potential at point P lying at a distance r from a point charge q is $V_p = \frac{kq}{r}$

6. The electric potential at a point at distance r from an electric dipole is $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$

   ... (for $r >> 2a$)

   Potential on its axis is $V = \pm\frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$. Potential on its equator is $V = 0$

7. Electric potential at a point $\vec{r}$ due to a system of point charges q_1, q_2, ... q_n situated at positions r_1, r_2, ... q_n is $V = \sum_{i=1}^{n} \frac{kq_i}{|\vec{r} - \vec{r}_i|}$.

The electric potential at point $\vec{r}$, due to a continuous charge distribution is

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_{\text{volume}} \frac{\rho(\vec{r}\,')d\tau'}{|\vec{r} - \vec{r}\,'|}$$

The electric potential due to a spherical shell is

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ ... (for $r \geq R$) and $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$ ... (for $r \leq R$)

8. A surface on which electric **potential is equal at all points** is called an equipotential surface. The direction of electric field is normal to the equipotential surface.

9. $E = \frac{-dV}{dl}$ gives the magnitude of electric field in the direction of $\vec{dl}$. To find E from V, in general, we can use the equation

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

The direction of electric field is that in which the rate of decrease of electric potential with distance $\left(\frac{-dV}{dl}\right)$ is maximum, and this direction is always normal to the equipotential surface.

10. The electrostatic potential energy of a system of point charges q_1, q_2, ... q_n situated respectively at r_1, r_2, ... r_n is

$$U = \sum_{\substack{i=1 \\ i<j}}^{n} \frac{kq_iq_j}{r_{ij}} \quad \text{where } r_{ij} = r_j - r_i$$

11. The electrostatic potential energy of an electric dipole in an external electric field $\vec{E}$, is $U = -\vec{E}\cdot\vec{p} = -E\,p\,cos\theta$.

12. When a metallic conductor is placed in an external electric field,
   (i) A stationary charge distribution is induced on the surface of the conductor.
   (ii) The resultant electric field inside the conductor is zero.
   (iii) The net electric charge inside the conductor is zero.
   (iv) The electric field at every point on the outer surface of conductor is locally normal to the surface.
   (v) The electric potential everywhere inside the conductor is the same constant.
   (vi) If there is a cavity in the conductor then, even when the conductor is placed in an external electric field, the electric field inside the conductor and also inside the cavity is always zero. This fact is called the electrostatic shielding.

   When electric charge is placed on the metallic conductor :
   (i) The electric field everywhere inside the conductor is zero.
   (ii) That charge is distributed only on the outer surface of the conductor.

(iii) The electric field on the surface is locally normal, and is given by $\vec{E} = \left(\frac{\sigma}{\varepsilon_0}\right)\hat{n}$.

(iv) If a charge is placed inside the cavity in the conductor, the electric field in region which is outside the cavity but in the conductor remains zero.

13. "A device formed by two conductors insulated from each other is called a capacitor". Its capacitance is $C = \frac{Q}{V}$ = constant. The unit of C is coulomb/volt, which is also called **farad.**

$1\ \mu F = 10^{-6}$ F; $1\ pF = 10^{-12}$ F

14. The capacitance of the parallel plate capacitor is $C = \frac{\varepsilon_0 A}{d}$.

15. If the effective capacitance in the series combination of capacitors is C,

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$

If the effective capacitance in the parallel combination of capacitors is C,

$C = C_1 + C_2 + C_3 + ...$

16. The energy stored in the capacitor is $U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{VQ}{2}$ and the energy density = energy per unit volume = $\frac{1}{2}\varepsilon_0 E^2$, where E = electric field.

17. When a dielectric is placed in an external electric field E_0, polarisation of dielectric occurs due to electical induction. The electric field produced by these induced charges is in the direction opposite to the direction of external electric field. Hence the resultant electric field E, inside the dielectric is less than the external electric field E_0. The dipole moment produced per unit volume is called the intensity of polarisation or in short polarisation P.P = σ_b.

Since $P \propto E$, $P = \varepsilon_0 x_e E$. x_e is called the electric susceptibility of the dielectric medium. $\varepsilon_0(1 + x_e)$ is called the permittivity ε of the dielectric medium. $\frac{\varepsilon}{\varepsilon_0}$ is called the relative permittivity of that medium and it is also called the dielectric constant K.

i.e. $\frac{\varepsilon}{\varepsilon_0} = \varepsilon_r = K$.

$K = 1 + x_e$, $E = \frac{E_0}{K}$. Thus in the dielectric the electric field reduces to the K^{th} part.

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ is called the electric displacement. Gauss Law in the presence of dielectric is written as $\oint \vec{D}.\vec{ds} = q$, where q is only the net free charge.

18. When there is air (or vacuum) between the plates of a parallel plate capacitor, the capacitance is $C = \frac{\varepsilon_0 A}{d}$.On placing a medium of dielectric constant K, the capacitance is $C' = KC$. Thus the capacitance becomes K times, due to the presence of the dielectric.

19. With the help of Van-De-Graf generator a p.d. of a few million volt can be established.

EXERCISE

For the following statements choose the correct option from the given options :

1. For a uniform electric field $\vec{E} = E_0(\hat{i})$, if the electric potential at $x = 0$ is zero, then the value of electric potential at $x = +x$ will be ....

(A) $x\ E_0$ (B) $-x\ E_0$ (C) $x^2\ E_0$ (D) $-x^2\ E_0$

2. The line integral of an electric field along the circumference of a circle of radius r, drawn with a point charge Q at the centre will be ....

(A) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ (B) $\frac{Q}{2\varepsilon_0 r}$ (C) zero (D) $2\pi Q r$

3. A particle having mass 1 g and electric charge 10^{-8} C travels from a point A having electric potential 600 V to the point B having zero potential. What would be the change in its kinetic energy ?

(A) -6×10^{-6} erg (B) -6×10^{-6} J

(C) 6×10^{-6} J (D) 6×10^{-6} erg

4. The area of every plate shown in the Figure is A and the separation between the successive plates is d. What is the capacitance between points a and b ?

(A) $\varepsilon_0\ A/d$ (B) $2\varepsilon_0\ A/d$

(C) $3\varepsilon_0\ A/d$ (D) $4\varepsilon_0\ A/d$

5. A particle having mass m and charge q is at rest. On applying a uniform electric field E on it, it starts moving. What is its kinetic energy when it travels a distance y in the direction of force ?

(A) qE^2y (B) qEy^2 (C) qEy (D) q^2Ey

6. A parallel plate capacitor is charged and then isolated. Now a dielectric slab is introduced in it. Which of the following quantities will remain constant ?

(A) Electric charge Q (B) Potential difference V

(C) Capacitance C (D) Energy U.

7. A moving electron approaches another electron. What will happen to the potential energy of this system ?

(A) will remain constant (B) will increase

(C) will decrease (D) may increase or decrease

8. Energy of a charged capacitor is U. Now it is removed from a battery and then is connected to another identical uncharged capacitor in parallel. What will be the energy of each capacitor now ?

(A) $\frac{3U}{2}$ (B) U (C) $\frac{U}{4}$ (D) $\frac{U}{2}$

9. A uniform electric field is prevailing in Y-direction in a certain region. The co-ordinates of points A, B and C are (0, 0), (2, 0) and (0, 2) respectively. Which of the following alternatives is true for the potentials at these points ?

(A) $V_A = V_B$, $V_A > V_C$ (B) $V_A > V_B$, $V_A = V_C$

(C) $V_A < V_C$, $V_B = V_C$ (D) $V_A = V_B$, $V_A < V_C$

10. The capacitance of a parallel plate capacitor formed by the circular plates of diameter 4.0 cm is equal to the capacitance of a sphere of diameter 200 cm. Find the distance between two plates.

(A) 2×10^{-4} m (B) 1×10^{-4} m (C) 3×10^{-4} m (D) 4×10^{-4} m

11. The capacitance of a variable capacitor joined with a battery of 100 V is changed from 2 μF to 10 μF. What is the change in the energy stored in it ?

(A) 2×10^{-2} J (B) 2.5×10^{-2} J (C) 6.5×10^{-2} J (D) 4×10^{-2} J

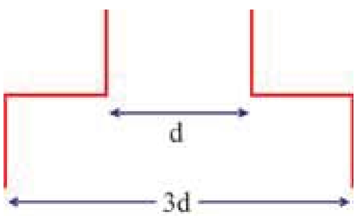
12. A parallel plate capacitor is charged with a battery, and then separated from it. Now if the distance between its two plates is increased, what will be the changes in electric charge, potential difference and capacitance respectively ?

(A) remains constant, decreases, decreases
(B) increases, decreases, decreases
(C) remains constant, decreases, increases
(D) remains constant, increases, decreases

13. 6 identical capacitors are joined in parallel and are charged with a battery of 10 V. Now the battery is removed and they are joined in series with each other. In this condition what would be the potential difference between the free plates in the combination ?

(A) 10 V (B) 30 V (C) 60 V (D) $\frac{10}{6}$ V

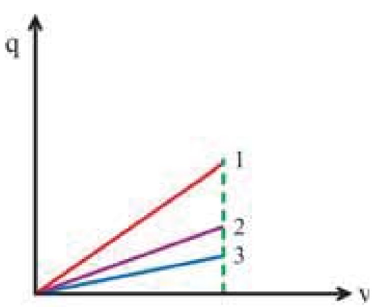
14. Six identical square metallic plates are arranged as in figure. Length of each plate is l. The capacitance of this arrangement would be ....



(A) $\frac{3\varepsilon_0 l^2}{d}$ (B) $\frac{4}{3}\frac{\varepsilon_0 l^2}{d}$

(C) $\frac{3}{2}\frac{\varepsilon_0 l^2}{d}$ (D) $\frac{4\varepsilon_0 l^2}{d}$

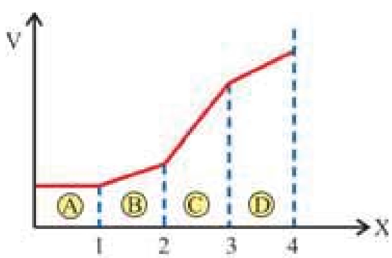
15. In the following table, the area of plates and separation between the plates are given. In the nearby Figure, $q - V$ graphs for them are shown. Determine which graph is for which capacitor.



Capacitor	area	separation
C_1	A	d
C_2	2A	d
C_3	A	$2d$

(A) $1 \rightarrow C_2$ $2 \rightarrow C_3$ $3 \rightarrow C_1$
(B) $1 \rightarrow C_1$ $2 \rightarrow C_2$ $3 \rightarrow C_3$
(C) $1 \rightarrow C_2$ $2 \rightarrow C_1$ $3 \rightarrow C_3$
(D) $1 \rightarrow C_3$ $2 \rightarrow C_1$ $3 \rightarrow C_2$

16. A V$-x$ graph for an electric field on X-axis is shown in the figure. In which region is the magnitude of electric field maximum ?



(A) A (B) B

(C) C (D) D

17. The distance between electric charges Q C and 9Q C is 4 m. What is the electric potential at a point on the line joining them where the electric field is zero ?

(A) 4 kQ V (B) 10 kQ V (C) 2 kQ V (D) 2.5 kQ V

18. If a capacitor having capacitance of 600 μF is charged at a uniform rate of 50 μC/s, what is the time required to increase its potential by 10 volts.

(A) 500 s (B) 6000 s (C) 12 s (D) 120 s

19. Two metallic spheres of radii R_1 and R_2 are charged. Now they are brought into contact with each other with a conducting wire and then are separated. If the electric fields on their surfaces are E_1 and E_2 respectively, E_1 / E_2 = ...

(A) R_2 / R_1 (B) R_1 / R_2 (C) R_2^2 / R_1^2 (D) R_1^2 / R_2^2

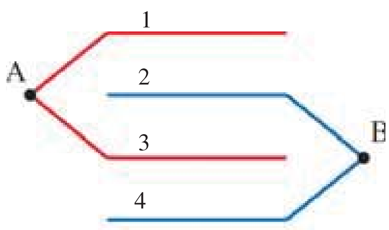
20. For a capacitor the distance between two plates is $5x$ and the electric field between them is E_0. Now a dielectric slab having dielectric constant 3 and thickness x is placed between them in contact with one plate. In this condition what is the p.d. between its two plates ?

(A) $\frac{13E_0x}{3}$ (B) 15 E_0 x (C) 7 E_0 x (D) $\frac{9E_0x}{2}$

21. In the figure area of each plate is A and the distance between consecutive plates is d. What is the effective capacitance between points A and B ?

(A) ε_0A/d (B) $2\varepsilon_0A/d$

(C) $3\varepsilon_0A/d$ (D) $4\varepsilon_0A/d$

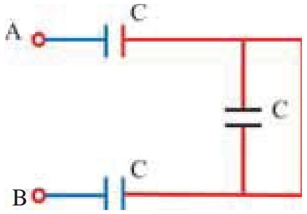


ANSWERS

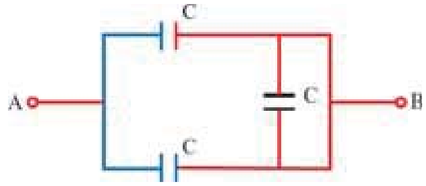
1. (B)	**2.** (C)	**3.** (C)	**4.** (B)	**5.** (C)	**6.** (A)
7. (B)	**8.** (C)	**9.** (A)	**10.** (B)	**11.** (D)	**12.** (D)
13. (C)	**14.** (B)	**15.** (C)	**16.** (C)	**17.** (A)	**18.** (D)
19. (A)	**20.** (A)	**21.** (C)			

Answer the following questions in brief :

1. What is line integral of electric field ? What does it indicate ?
2. If the electric potential at point P is V_P, what is the electric potential energy of charge q at this point ?
3. What is the electric potential at a point on the equator of an electric dipole ?
4. What is electic potential gradient ? Give its unit.
5. Electric field is always .... to the equipotential surface and in a direction in which the rate of decrease of potential is ....
6. Give the formulae for the equivalent (effective) capacitance of capacitors in series and parallel combinations.
7. How does the energy density associated with an electric field depend on the value of electric field ?
8. What is meant by a non-polar molecule ?

9. Define intensity of polarisation (or in short polarisation) P.
10. Write the formula showing the relation between x_e and P.
11. Electric field at a point in free space is 100 N/C. What would be the electric field in a medium with dielectric constant 5, placed at that place ?
12. What is the meaning of the relative permittivity ε_r of a dielectric medium ?
13. State the use of Van-de-Graf generator.
14. 

What is the equivalent capacitance between points A and B in the figure ? (**Hint :** The last capcitor on right side is short circuited. ∴ it is not effective)

[**Ans :** C/2]
15. 

What is the equivalent capacitance between points A and B shown in the figure ? (**Hint :** The last capcitor on right side is short circuited. ∴ it is not effective)

[**Ans :** 2C]

Answer the following questions :

1. Show that the work done by the electric field in moving a unit positive charge from one point to the other point in an electric field depends only on the positions of those two points and not on the path joining them.
2. Define electric potential and give the formula corresponding to it. Write its units and dimensions.
3. Define electric potential and obtain the formula for the electric potential due to a point charge.
4. Derive the formula for the electric potential due to an electric dipole at a far distant point from it.
5. What is an equipotential surface ? Show that the direction of the electric field at a given point is normal to the equipotential surface passing through that point.
6. Obtain the formula which can give electric field from the electric potential.
7. Derive the formula for the electric potential energy of an electric dipole in a uniform electric field.
8. Explain in short the effects produced inside a metallic conductor placed in an external electric field.
9. What is a capacitor ? Give the definition, and units of capacitance. On which factors does the value of capacitance depend ? Give the symbol of capacitor.
10. Obtain the formula for the equivalent (effective) capacitance in the series / parallel combination of capacitors.
11. Obtain the formula for the capacitance of a parallel plate capacitor.
12. Obtain the formula for the energy, stored in the capacitor and also for the energy density.
13. Explain the polarisation produced in the dielectric placed between the two plates of a parallel plate capacitor and obtain the formula $P = \sigma_b$.
14. The resultant electric field inside a dielectric placed between two plates of a capacitor is $E = \frac{\sigma_f - \sigma_b}{\varepsilon_0}$. Hence obtain $E = \frac{E_0}{K}$, where E_0 = external electric field on the dielectric.
15. Using $E = \frac{\sigma_f - \sigma_b}{\varepsilon_0}$ obtain the formula for the electric displacement $\vec{D}$. State the importance of $\vec{D}$.
16. Obtain the formula showing the principle of Van-De-Graaff generator.
17. Only draw the Figure and explain the working of Van-de-Graf generator.

Solve the following examples :

1. q_1 = 2C and q_2 = −3 C charges are placed at (0, 0) and (100, 0)m points respectively. At which point(s) on the X-axis is the electrical potential zero ?

   [**Ans. :** 40m, −200 m]

2. Two metallic spheres having radii a and b, are placed very far from each other and are joined by a conducting wire. The total charge on them is Q. Find (i) the charge on each sphere and (ii) potential on each sphere.

   [**Ans. :** $Q_a = \frac{aQ}{a+b}$, $Q_b = \frac{bQ}{a+b}$, $V_a = V_b = \frac{kQ}{a+b}$]

3. In a certain region the electric potential is given by the formula $V(x, y, z) = 2x^2y + 3y^3z - 4z^4x$. Find the components of electric field and the vector electric field at point (1, 1, 1) in this field.

   [**Ans. :** $E_x = 0$, $E_y = -11$ unit, $E_z = 13$ units, $\vec{E} = -11\hat{j} + 13\hat{k}$ unit]

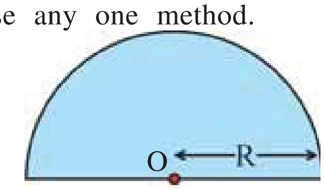
4. A spherical drop of water has 3×10^{-10} C amount of charge residing on it. 500 V electric potential exists on its surface. Calculate the radius of this drop. If eight such drops (Having identical charge and radii) combine to form a single drop, calculate the electric potential on the surface of the new drop. $k = 9 \times 10^9$ SI.

   [**Ans. :** Radius of the first drop = 0.54 cm,
   Electric Potential on the new drop = 2000 V]

5. Q amount of electric charge is present on the surface of a sphere having radius R. Calculate the total energy of the above system. [**Ans. :** $\frac{1}{2}\frac{kQ^2}{R}$]

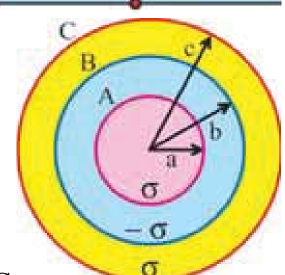
   Note : The above example can be calculated in three different ways, (1) By multiplying the electric charge with the average value of the initial and the final electric potential, (2) By considering the above system to be a capacitor and calculating the energy of the capacitor and (3) By considering an electric charge q and taking the integration of the work done to increase the above charge by an amount dq. Use any one method.

6. σ is the uniform charge density present on the surface of a semi-sphere of radius R. Derive the formula for the electric potential at the centre.



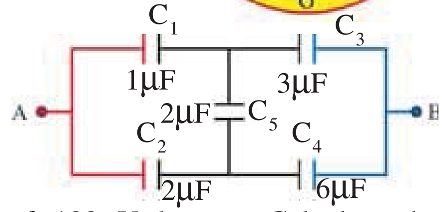
   [**Ans. :** $\frac{R\sigma}{2\varepsilon_0}$]

7. Consider A, B and C to be the co-centric shells of metal. Their radii are a, b and c respectively ($a < b < c$). Their surface charge densities are σ, −σ and σ respectively. Calculate the electric potential on the surface of shell A.



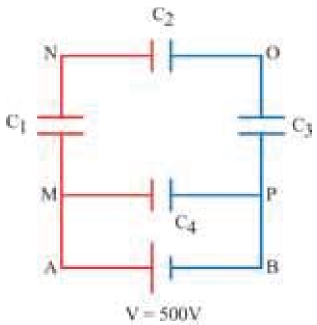
   [**Ans. :** $\frac{\sigma}{\varepsilon_0}[a - b + c]$]

8. Calculate the equivalent capacitance between points A and B of the connections of capacitors shown in the figure. [**Ans. :** $\frac{9}{4}$ μF]



9. (1) A capacitor of 900 pF is charged with the help of 100 V battery. Calculate the electric potential energy of this capacitor. (2) The above capacitor is disconnected from the battery and is connected to another identical uncharged capacitor. What will be the total energy of the system ? [**Ans. :** (1) 4.5×10^{-6} J (2) 2.25×10^{-6} J]

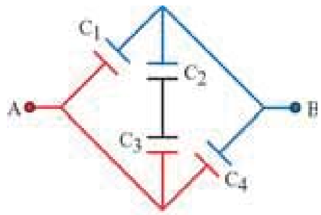
10.



Calculate the equivalent capacitance for the connection of capacitors shown in the figure and the electric charge present on each of the capacitors. The value of each capacitance is 10 μF.

[**Ans. :** Equivalent capacitance = 13.3 μF $Q_1 = Q_2 = Q_3 = 1.7 \times 10^{-3}$ C, $Q_4 = 5.0 \times 10^{-3}$ C]

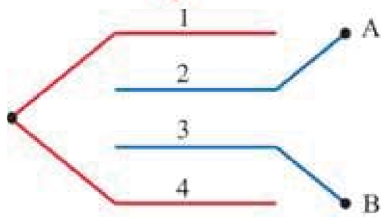
11.



Find the equivalent capacitance between A and B in the circuit shown in the Figure. $C_1 = C_4 = 1$ μF; $C_2 = C_3 = 2$ μF.

[**Ans. :** 3 μF]

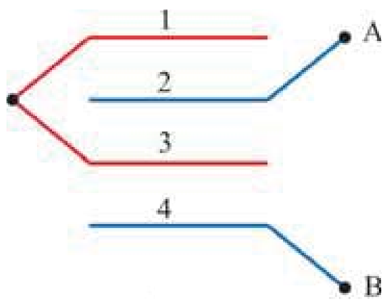
12.



The area of each plate shown in the figure is A and the distance between consecutive plates is *d*. What is the equivalent capacitance between points A and B ?

[**Ans. :** $\frac{3}{2}\frac{\varepsilon_0 A}{d}$]

13.



The area of each plate shown in the figure is A and the distance between consective plates is *d*. What is the equivalent capacitance between points A and B ?

[**Ans. :** $\frac{2}{3}\frac{\varepsilon_0 A}{d}$]

3

Current Electricity

3.1 Introduction

In the previous two chapters, all electric charges (whether free or bound) were considered to be stationary (at rest) and we mainly studied the interaction between them. The study of this branch of electricity is called electrostatics.

In the present chapter, we will bring the charges in motion by providing energy to them. Such charges in motion constitute an electric current.

Such currents occur naturally in many situations. When there is lightning in the sky, charges flow from the clouds to the earth through the atmosphere. Flow of charges in lightning is of short duration, resulting in current called transient current. This flow of charges in lightning is not steady.

In everyday life, we see many devices where charges flow in a steady manner, like water flowing smoothly in a river. A cell-driven clock, torch and a transistor radio are examples of such devices.

In the present chapter, we shall study some of the basic laws concerning steady electric current and the quantities associated with flow of charges like electric current density, drift velocity and mobility. Moreover, we shall study about resistors, cells and their different connections, Kirchhoff's rules for the analysis of network and conversion of electrical energy into heat energy during conduction of electricity through conductors. Further, we shall get the information about potentiometer for the measurement of emf of a cell and wheatstone bridge which is used for the measurement of resistor.

The study of this branch of electricity is called **current electricity.**

3.2 Electric Current

The flow of electric charges constitutes an electric current.

If net amout of charge Q is flowing through a cross-sectional area of the conductor in time t, then for a steady flow of charge,

$$I = \frac{Q}{t} \tag{3.2.1}$$

is defined as the current flowing through that cross-sectional area.

The amount of charge flowing per unit time across any cross-section of a conductor held perpendicular to the direction of flow of charge is called current (I).

Electric current (I) is considered as fundamental quantity in SI unit system. The SI unit of electric current is (a) which is equal to $\frac{\text{coulomb}}{\text{second}}$.

In the above equation (3.2.1), if we take,

t = 1 second

Q = 1 coulomb

then I = 1 ampere

If 1 coulomb of charge crosses any cross-section of a conductor. Perpendicular to the direction of current in 1 second, then the current through that cross-section is said to be 1 ampere.

For small currents, milliampere (mA = 10^{-3} A) and microampere (μA = 10^{-6}A) units are used.

In metallic conductors the current is due to the motion of negatively charged electrons. In electrolytes, the current is due to the motion of both positive and negative ions moving in opposite directions. While in semi-conductors, partly the electrons and partly the holes (hole is the deficiency of electron in the covalent bond) are responsible for the flow of the current.

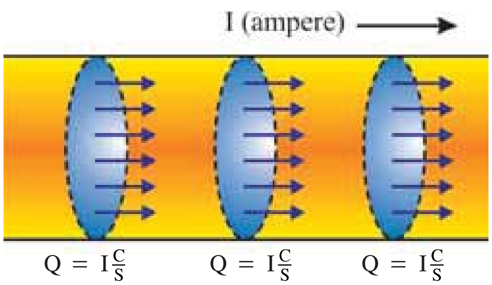


Figure 3.1 Conservation of Charge

Let I ampere current be flowing through any conductor as shown in figure 3.1. Hence, I coulomb electric charge is flowing through every cross-sectional area of the conductor per second.

In other words, the amount of electric charge entering any cross-section of the conductor from one side in a given time interval is equal to the amount of electric charge leaving that cross-section from the other side in the same interval of time. As a result of this **Electric charge is never accumulated at any point in the conductor. The electric charge is neither created nor destroyed at any point in the conductor. This means that electric charge is conserved.**

By convention, the direction of motion of positive charges is taken as the direction of electric current. It is called conventional current. However, in conductors the current is due to the motion of negatively charged electrons, so the direction of current is opposite to the electron current.

In some cases, current (rate of flow of charge) varies with time means the flow of charge is not steady. In this circumstances, let ΔQ be the net amount of electric charge flowing across any cross-sectional area of a conductor during the time interval Δt between times t and $(t + \Delta t)$, then the average electric current flowing during time interval Δt is given by,

$$< I > = \frac{\Delta Q}{\Delta t}$$

The electric current at time t will be,

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \qquad (3.2.2)$$

Illustration 1 : The current through a wire varies with time as $I = I_0 + \alpha t$, where $I_0 = 10$ A and $\alpha = 4$ As^{-1}. Find the charge that flows across a cross-section of the wire in first 10 seconds.

Solution : Current $I = \frac{dq}{dt} = I_0 + \alpha t$

$\therefore\ dq = (I_0 + \alpha t)dt$

Integrating on both sides,

$$\int dq = \int_{t=0}^{t=10} (I_0 + \alpha t)dt$$

$$\therefore\ q = \left[I_0 t + \frac{\alpha t^2}{2} \right]_{t=0}^{t=10} = 10\ I_0 + 50\ \alpha$$

Substituting $I_0 = 10$ and $\alpha = 4$,

$q = 10(10) + 50(4) = 300$ C.

3.3 Electric Current Density

It is possible that the rate of flow of the electric charge through different cross-sectional areas of the conductor may not be same. Apart from this, the flow of the electric charge may not be perpendicular to the cross-sectional area of the conductor. In such circumstances, to study the flow of charge through a cross-section of the conductor at a particular point, a vector quantity known as electric current density $\vec{J}$ is defined.

To define the current density at a point P, imagine a small cross-section of area Δa through P perpendicular to the flow of charges as shown in figure 3.2 (a).

(a) (b)

Figure 3.2 Cross–section of a Current Carrying Conductor

If ΔI be the current through the area Δa, the average current density is,

$$\langle J \rangle = \frac{\Delta I}{\Delta a}$$

The current density at the point P is,

$$J = \lim_{\Delta a \to 0} \frac{\Delta I}{\Delta a} = \frac{dI}{da} \qquad (3.3.1)$$

The direction of the current density is the same as the direction of the current.

If a current I is uniformly distributed over an area A and is perpendicular to it,

$$J = \frac{I}{A} \qquad (3.3.2)$$

Thus, **The electric current density at any point is defined as the amount of electric current flowing per unit cross-section perpendicular to the current at that point. (amount of electric charge flowing per unit time)**

The SI unit of the current density is Am^{-2}.

Now let us consider a cross-section Δa which is not perpendicular to the current, (Figure 3.2 (b)) then the component of cross-section in the direction of current $\Delta a \cos\theta$ should be considered.

Average current density at point P,

$$\langle J \rangle = \frac{\Delta I}{\Delta a \cos\theta} \qquad (3.3.3)$$

Where, ΔI = Current flowing through small area Δa near point P.

and θ = angle made by the normal to the cross-section with the direction of the current.

For very small area $\Delta \vec{a}$,

current density $J = \lim_{\Delta a \to 0} \frac{\Delta I}{\Delta a \cos\theta} = \frac{dI}{da \cos\theta}$ (3.3.4)

$\therefore\ dI = J da \cos\theta$

$\therefore\ dI = \vec{J} \cdot \vec{da}$ (3.3.5)

Taking the surface integration of equation (3.3.5) over the entire cross–sectional area, we have,

$$\int dI = \int \vec{J} \cdot \vec{da}$$

$$I = \int_a \vec{J} \cdot \vec{da} \quad (3.3.6)$$

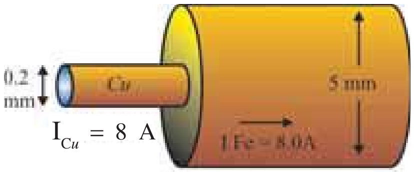
If the cross-sectional area is perpendicular to the current and if J is constant over the entire cross-section then,

$$I = \int \vec{J} \cdot \vec{da} = J \int da$$

$\therefore\ I = JA$ (3.3.7)

The concept of electric current density is very useful in the discussion of the flow of electric charges.

Illustration 2 : 0.2 mm diameter copper wire is connected to 5.0 mm diameter iron wire. The current flows through both the wires. If 8.0 A current flows through the copper wire, then calculate the following quantities.



(1) The current flowing through the iron wire and current density in it.

(2) The current density in the copper wire.

Solution : As per the conservation law of charges, equal amount of time is taken for a given quantity of charge to enter the copper wire and leave the iron wire.

(1) $\therefore\ I_{Cu} = 8.0 = I_{Fe}$

Let A_{Fe} be cross-sectional area of the iron wire and let d_{Fe} and r_{Fe} be the diameter and radius.

$$\therefore\ J_{Fe} = \frac{I_{Fe}}{A_{Fe}} = \frac{8.0}{\pi r^2_{Fe}} = \frac{8.0}{\pi\left(\frac{d_{Fe}}{2}\times 10^{-3}\right)^2} = \frac{8.0\times 4}{(3.14)(5\times 10^{-3})^2}$$

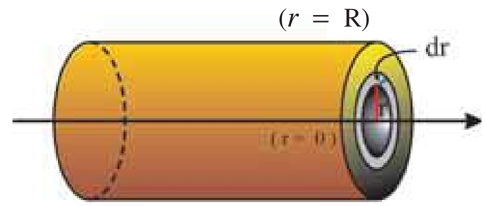
$$\therefore\ J_{Fe} = 407\ kA/m^2$$

(2) The current density in the copper wire $J_{Cu} = \frac{8.0}{(3.14)(0.1\times 10^{-3})^2} = 2.5 \times 10^8\ A/m^2$

Illustration 3 : The current density along the axis of a cylindrical conductor having radius equal to R is given by $J = J_0\left(1 - \frac{r^2}{R^2}\right)$. Find the current along the length of the conductor. The distance from the axis is given by r.

Solution : Consider a ring of thickness dr at a distance r from the axis on the cross-section perpendicular to the axis of a cylinder.

The current flowing through the ring,



$$dI = \vec{J} \cdot \vec{da} = J da \quad (\because \cos\theta = 1)$$

$$\therefore dI = J_0\left(1 - \frac{r^2}{R^2}\right)(2\pi r dr)$$

The current along the length of the conductor,

$$I = \int dI = \int_{r=0}^{r=R} J_0\left(1 - \frac{r^2}{R^2}\right)(2\pi r dr) = 2\pi J_0 \int_{r=0}^{r=R}\left(1 - \frac{r^2}{R^2}\right)(r)\ dr$$

$$I = 2\pi J_0 \int_0^R\left(r - \frac{r^3}{R^2}\right)dr = 2\pi J_0\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$$

$$I = 2\pi J_0\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right] = 2\pi J0\left[\frac{R^2}{4}\right]$$

$$I = \pi J_0 \frac{R^2}{2}$$

3.4 Ohm's Law

Why do we not experience a fatal shock on touching a 6V supply while on touching a 230 V source, one experiences a fatal shock ?

In these examples for different voltages, the electric current flowing through the body is different.

In 1828, a German physicist, George Simon Ohm was the first person to give a mathematical relationship between voltage and current, famously known as Ohm's law. Ohm experimentally proved that **"Under a definite physical condition, (e.g. constant temperature) the current (I) flowing through the conductor is directly proportional to the potential difference (V) applied across its ends."** This statement is called Ohm's law.

According to Ohm's law, $I \propto V$

$$\therefore \frac{V}{I} = \text{constant}$$

This constant ratio $\frac{V}{I}$ is called the resistance (R) of the conductor.

$$\therefore \frac{V}{I} = R \qquad (3.4.1)$$

$$\text{OR } V = IR \qquad (3.4.2)$$

The SI unit of resistance is $\frac{\text{volt}}{\text{ampere}}$ which is known as ohm, and is denoted by the symbol Ω. At a given temperature, the resistance R not only depends on the material of the conductor but also on the dimensions of the conductor.

The reciprocal of a resistance i.e. $\frac{1}{R}$ is called the conductance of the material of the given conductor. Its unit is Ω^{-1} or mho and is symbolised as ℧.

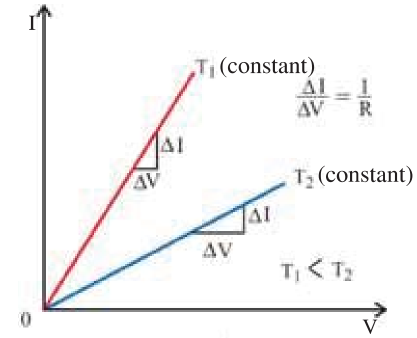


Figure 3.3 I – V Characteristics for a Conductor

Ohm's law is not a fundamental law of nature, like the gravitational law of Newton or the Coulomb's law for electrical charges. Ohm's law gives us the relationship between the potential difference across the conductor and the current flowing through it, under a given situation.

All the metals, some of the insulators and some of the electrical devices obey the Ohm's law. Such devices are called Ohmic devices.

The I – V graph for a conductor obeying Ohm's law at a constant temperature will be a straight line. i.e. such relation is linear.

3.4.1 Limitations of Ohm's Law

There do exist some materials and devices used in electric circuits where the proportionality of V and I does not hold. In such devices,

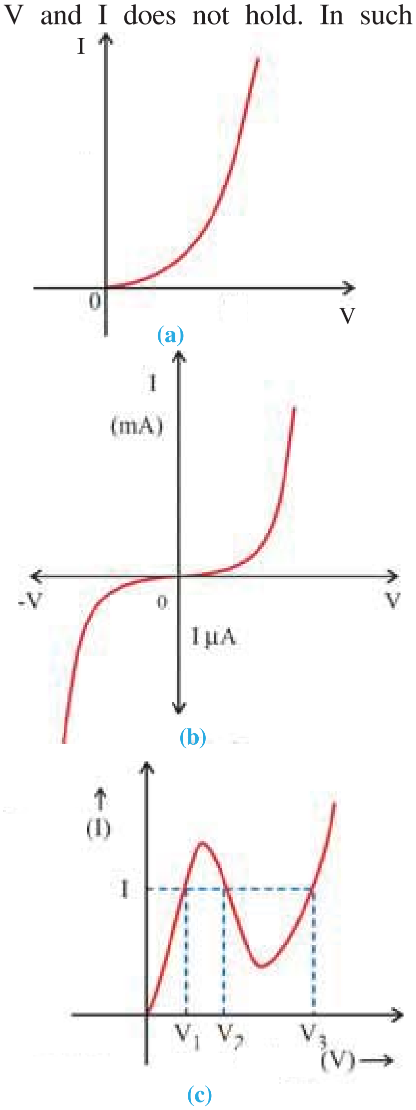


Figure 3.4 I-V Characteristics of Difference Devices

(1) V–I relations are non-linear. e.g. semi-conductor devices like diode and transistor. (figure 3.4 (a))

(2) The relation between V and I depends on the sign of V. In other words, if I is the current for a certain voltage V, then reversing the direction of V keeping its magnitude fixed, does not produce a current of the same magnitude as I in the opposite direction. This happens in a semi-conductor diode which we will study in future. (See figure 3.4 (b))

(3) The relation between V and I is not unique. i.e. there is more than one value of V for the same current I. A graph of device exhibiting such behaviour (e.g. tunnel diode) is shown in figure 3.4 (c).

Materials and devices not obeying Ohm's law are called non-ohmic devices. Such devices are widely used in electronic circuits.

3.5 Electrical Resistivity and Conductivity

Let us try to understand the dependence of the resistance (R) of a conductor on the dimensions of the conductor. Consider a conductor having cross−sectional area A and length l. Experimentally it is found that, at a given temperature, resistance (R) of a conductor is proportional to length of the conductor (l) and inversely proportional to the cross-sectional area (A).

$$R \propto l \text{ and } R \propto \frac{1}{A}$$

$$\therefore R \propto \frac{l}{A}$$

$$\therefore R = \rho . \frac{l}{A} \qquad (3.5.1)$$

Here the constant ρ is called the resistivity of the material. It depends on the material of the conductor, temperature and the pressure existing on the given conductor. It does not depend on the dimensions of the conductor.

The unit of resistivity ρ is ohm meter (Ωm).

(**Information :** The resistivity of a material changes at higher pressure, due to the changes in the composition of the crystals.)

Using equation (3.5.1), Ohm's law can be written as,

$$V = IR$$

$$V = \frac{I\rho l}{A} \qquad (3.5.2)$$

$$V = J\rho l \qquad (3.5.3)$$

where, $\frac{I}{A} = J$ is the current density.

Further, if E is the magnitude of uniform electric field in the conductor whose length is l, then the potential difference V across its ends is $V = El$

$$\therefore El = J\rho l$$

$$\therefore E = J\rho \qquad (3.5.4)$$

The current density $\vec{J}$ is a vector quantity and is directed along $\vec{E}$. Thus, the above equation can be written in the vector form as,

$$\vec{E} = \vec{J}\rho$$

$$\text{OR } \vec{J} = \frac{\vec{E}}{\rho} = \sigma\vec{E} \qquad (3.5.5)$$

where, $\sigma = \frac{1}{\rho}$ (reciprocal of resistivity) is called the conductivity of that material.

The unit of σ is $(\Omega \text{ m})^{-1}$ or mho m^{-1} (℧ m^{-1}) or siemen m^{-1} (Sm^{-1}).

Note that equation (3.5.5) is the vector form of Ohm's law.

3.6 Drift Velocity, Mobility and its Relations with Current

In atoms and molecules, the negatively charged electrons and the positively charged nuclei are bound to each other due to Coulombian electric force. Bulk matter is made up of many molecules.

We will focus only on solid conductors in which current is carried by the negatively charged free electrons.

In metallic conductors, the electrons in the outershells are less bound with the nucleus. Due to thermal energy at room temperature, such valence electrons are liberated from the atom leaving behind positively charged ions. These ions are arranged in a regular geometric arrangement on the lattice points. The electrons liberated from the atom are called free electrons and ions are oscillating about their mean position.

In the absence of electric field, free electrons in a solid conductor move like the molecules in a gas due to their thermal velocities. During their motion they collide with the ions. The directions of their velocities after the collision are completely random. At a given time, there is no preferential direction for the velocities of the electrons. Such random motion of an electron is shown by continuous line AB in figure 3.5.

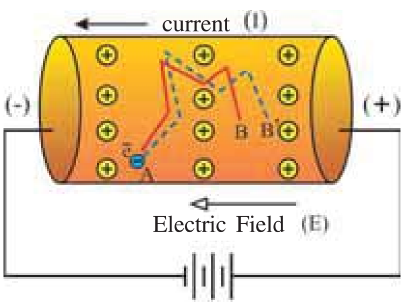


Figure 3.5 Drift Velocity

Thus on the average, the number of electrons travelling in any direction will be equal to the number of electrons travelling in the opposite direction in the absence of an external electric field. Therefore the net charge passing through any cross-section of the conductor is zero hence there will be no flow of electric current in the conductor.

Now, an electric field (E) is applied across the conductor by connecting a battery between two ends of a conductor as shown in figure 3.5. Due to electric field in the conductor, the electron will experience a force F = E*e* in the direction opposite to the electric field (towards the positive terminal of battery). The path of the electron will become AB' as shown by the dotted lines. This is because of the electron, executing the motion under the oscillatory electric field of the ions, constantly gets scattered from its path. This gives rise to the resistance in a conductor.

In the presence of electric field (E), the acceleration of the electron in the direction opposite to the electric field is $a = \frac{E \cdot e}{m}$. This acceleration of the electron is momentary, since the electrons are continuously colliding with the ions. (In the real sense, the electrons are scattered in the oscillating electric field of the ions.) As a result, the electrons are dragged in the direction opposite to the electric field. The velocity of the electron becomes zero after every such collision with the ions and after each collision the electron is accelerated once again due to electric field and collide with the ions. The above process keeps on repeating.

Thus, electron travels from A to B' in the presence of electric field rather than travelling from A to B in the absence of electric field. The effective displacement of the electron is equal to BB' in the presence of electric field. The velocity of electron corresponding to this displacement is known as the **drift velocity** (v_d). In this situation the average number of electrons passing through any corss-sectional area of the conductor is not zero in the presence of electric field. As a result, there will be a net flow of charge of current through the conductor.

The average time between two successive collisions of the electron with the ions is called relaxation time (τ).

The drift velocity achieved by the electron during the relaxation time (τ) is,

$$v_d = a\tau$$

$$v_d = \left(\frac{E \cdot e}{m}\right)\tau \qquad (3.6.1)$$

Relation between the Drift Velocity and Current Density :

To find the relation between the current density and the drift velocity, let us consider a cylindrical conductor of uniform cross-sectional area A. An electric field E exists in the conductor when its ends are connected to the battery.

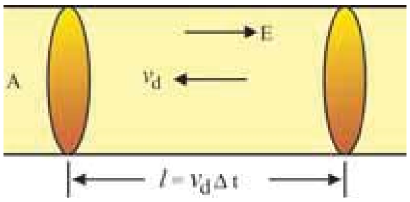


Figure 3.6

If the drift velocity of the electron is v_d, then distance travelled by the electron during time Δt is $l = v_d \Delta t$.

The volume of the portion of the conductor whose length is $v_d \Delta t = Al = Av_d \Delta t$.

If there are n free electrons per unit volume (number density) of the conductor, the number of free electrons in this portion is $= nAv_d\Delta t$.

All these electrons cross the area A in time Δt.

Thus, the charge crossing this area in time Δt is, $\Delta Q = nAv_d \Delta t e$ (3.6.2)

$$\therefore \text{ Current } I = \frac{\Delta Q}{\Delta t} = nAv_d e \qquad (3.6.3)$$

$$\text{and current density } J = \frac{I}{A} = nev_d \qquad (3.6.4)$$

In general, equation (3.6.4) can be written in the vector from as,

$$\vec{J} = nq\vec{V}_d$$

For negative charge q, $\vec{J}$ and $\vec{v}_d$ will be in opposite direction.

Comparing two equations (3.5.5) and (3.6.4) of current density,

$$\sigma E = nev_d$$

Substituting the value of v_d from equations (3.6.1),

$$\sigma E = ne\left(\frac{Ee}{m}\tau\right)$$

$$\therefore \sigma = \frac{ne^2\tau}{m} \qquad (3.6.5)$$

$$\text{since, } \sigma = \frac{1}{\rho}$$

$$\rho = \frac{1}{\sigma}$$

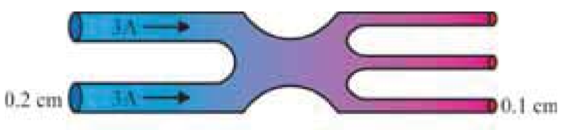
$$\therefore \rho = \frac{m}{ne^2\tau} \qquad (3.6.6)$$

In a metal, number density n is not dependent on temperature to any appreciable extent. The oscillations of the ions increases with the temperature and become more erratic. As a result, the relaxation time (τ) decreases. Thus, the resistivity of the conductor increases with temperature according to above formula.

For insulators and semi-conductors, the relaxation time τ and number density n of charge carriers varies with the temperature.

The number density (n) of charge-carriers increases with temperature in semi-conductors. Therefore, the conductivity of semiconductor increases with temperature i.e. its resistivity (ρ) decreases.

Illustration 4 : 3A current is flowing through two identical conducting wires having diameter equal to 0.2 cm. These conducting wires are then split into three identical conducting wires, each having 0.1 cm diameter (as shown in the Figure). Calculate the drift velocities in the thicker and the thinner conductors.



The electron density = 7×10^{28} m^{-3}. All the conductors are made of the same material. The electric charge on electron is equal to = 1.6×10^{-19} C.

Solution : The current density in the thicker wire $J = \frac{I}{A} = \frac{3}{\pi r^2} = \frac{3}{\pi(0.1\times10^{-2})^2}$

Current density $J = nev_d$

$$\therefore v_d = \frac{J}{ne} = \frac{3}{\pi(0.1\times10^{-2})^2\times7\times10^{28}\times1.6\times10^{-19}}$$

$\therefore v_d = 8.5 \times 10^{-5}$ m s^{-1}

6A total current is flowing through the three indentical conductors (As per Kirchoff's First Law).

$\therefore$ The current flowing through each of the wires = 2 A

$$\therefore v_d' = \frac{J'}{ne} = \frac{2}{\pi(\frac{0.1}{2}\times10^{-2})^2\times7\times10^{28}\times1.6\times10^{-19}}$$

$$= 2.3 \times 10^{-4} \text{ m s}^{-1}$$

Illustration 5 : A copper wire is stretched to make it 0.1% longer. What is the percentage change in its resistance ? [Assume that the volume of the wire remains constant.]

Solution : Suppose the length of the wire is l and area of cross-section is A.

The resistance of a wire, $R = \rho.\frac{l}{A}$

$$\therefore R = \frac{\rho l^2}{Al} = \frac{\rho l^2}{V} \quad (1)$$

$$\frac{dR}{dl} = \frac{\rho}{V}.2l$$

$$\therefore dR = \frac{\rho}{V}2l\cdot dl \quad (2)$$

Taking ratio of equations (2) and (1),

$$\frac{dR}{R} = \frac{\frac{\rho}{V}.2ldl}{\frac{\rho}{V}.l^2}$$

$$\therefore \frac{dR}{R} = 2\cdot\frac{dl}{l}$$

Percentage Change $\frac{dR}{R} \times 100\% = 2\left(\frac{dl}{l}\right) \times 100\% = 2\ (0.1\%)$

$= 0.2\ \%$

Thus, the resistance of the wire increases by 0.2%.

Note : If the change in the length of the wire is infinitesimally small, then the above method of differentiation can be used to calculate the change in the resistance. But if the change is very large, then we have to find the change in resistance according to the change in the length.

3.6.1 Mobility

We have seen that the conductivity of any material is due to the mobile charge-carriers. Mobile charge carriers in the conductor are free electrons. In the ionized gas they are electrons and positive ions. Positive and negative both types of ions are the mobile charge carriers in the electrolytes. In semi-conductors the flow of current is partly due to electrons and partly due to holes. (We shall study about semi-conductors and holes in the next chapters. At present, we will note that hole will behave like a positively charged particles.)

Comparing two equations (3.6.4) and (3.5.5) of current density,

$$nev_d = \sigma E$$

$$\therefore \frac{V_d}{E} = \frac{\sigma}{ne}$$

$\frac{V_d}{E}$ is the drift velocity of a charge carrier per unit electric field intensity. This quantity is known as mobility (μ) of a charge carrier.

$$\therefore \text{Mobility } \mu = \frac{V_d}{E} = \frac{\sigma}{ne} \qquad (3.6.7)$$

SI unit of mobility is $m^2V^{-1}s^{-1}$.

From equations (3.6.7)

$$\text{Conductivity } \sigma = ne\mu \qquad (3.6.8)$$

If charge carriers are electrons, then

$$\sigma_e = n_e e\mu_e \qquad (3.6.9)$$

$$\text{For holes, } \sigma_h = n_h e\mu_h \qquad (3.6.10)$$

In a semi-conductor, the holes and the electrons both constitute current in the same direction. The total conductivity,

$$\sigma = \sigma_e + \sigma_h$$

$$\sigma = n_e e\mu_e + n_h e\mu_h \qquad (3.6.11)$$

3.7 Temperature Dependence of Resistivity

The resistivity of a material is found to be dependent on the temperature. Different materials do not exhibit the same dependence on temperatures. Over a limited range of temperatures, that is not too large, the relationship between the resistivity of a metallic conductor and temperature is approximately given by,

$$\rho_\theta = \rho_{\theta_0} [1 + \alpha (\theta - \theta_0)] \qquad (3.7.1)$$

where, ρ_θ = resistivity at a temperature θ

ρ_{θ_0} = resistivity at a proper reference temperature θ_0.

and α is called the temperature co-efficient of resistivity and its unit is $(°C)^{-1}$ or K^{-1}.

The above equation can be written in the form of resistance as follows.

$$R_\theta = R_{\theta_0} [1 + \alpha (\theta - \theta_0)] \quad (3.7.2)$$

The resistivity (ρ) and temperature-coefficient (α) for some materials are given in Table 3.1.

Table 3.1 : The Value of ρ and α for various materials

(For information purpose only)

Material	At 0°C Temperature Resistivity (Ω m)	Temperature Co-efficient $(\alpha)(°C)^{-1}$
(A) Conductors		
Silver	1.6×10^{-8}	0.0041
Copper	1.7×10^{-8}	0.0068
Aluminium	2.7×10^{-8}	0.0043
Tungsten	5.6×10^{-8}	0.0045
Iron	10×10^{-8}	0.0065
Platinum	11×10^{-8}	0.0039
Mercury	98×10^{-8}	0.0009
Nichrome	$\sim 100 \times 10^{-8}$	0.0004
(B) Semi-conductors		
Carbon (graphite)	3.5×10^{-5}	-0.0005
Germanium	0.46	-0.05
Silicon	2300	-0.07
(C) Insulators		
Pure water	2.5×10^{5}	
Glass	$10^{10} - 10^{14}$	
Solid rubber	$10^{13} - 10^{16}$	
NaCl	$\sim 10^{14}$	
Fused Quartz	$\sim 10^{16}$	

From the above table, note that for metals, α is positive. Therefore the resistivity of the metal increases with temperature. For metallic conductors, the relation between resistivity (ρ) and temperature is non-linear at lower temperature (< 50 K) and the graph becomes linear near the room temperature. Finally at very higher temperature the graph again becomes non-linear, (Figure 3.7).

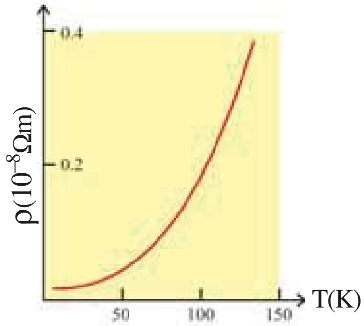


Figure 3.7 Graph of $\rho \rightarrow T$ for a Metal

Some materials like Nichrome (which is an alloy of nickel, iron and chromium) have very high value of resistivity, exhibit a very weak dependence of resistivity with temperature. (See figure 3.8). The resistivity of manganin (an alloy of copper, magnesene and nickel) is almost independent of temperature.

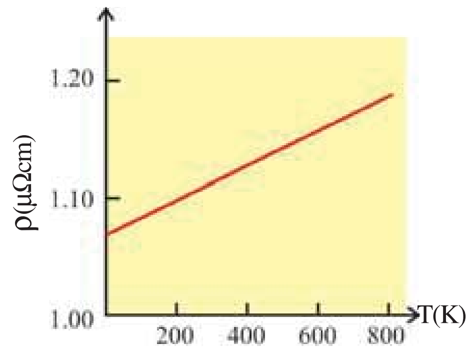


Figure 3.8 Graph of $\rho \rightarrow T$ for an Alloy

The resistivity of Nichrome does not become zero even at absolute zero temperature (0 K), while the resistivity of a pure metal becomes almost zero at absolute zero temperature. Using this fact the purity of the metal can be tested.

As shown in Table 3.1, semi-conductors like carbon, germanium and silicon have negative values of α. This means that the resistivity of such materials decreases with temperature (as shown in figure 3.9).

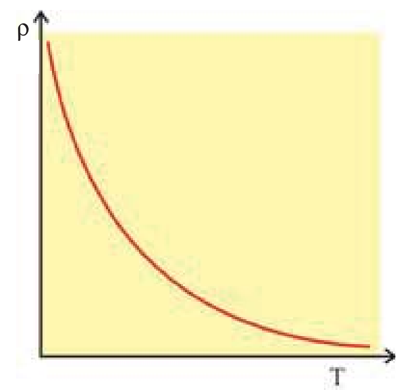


Figure 3.9 Graph of $\rho \rightarrow T$ for a Semiconductor

3.7.1 Classification of materials on the basis of resistivity

The materials are classified as conductors, semiconductors and insulators depending on their resistivities.

An ideal conductor has zero resistivity or infinite conductivity, while the ideal insulator has infinite resistivity (means zero conductivity).

Metals have low resistivities in the range of $10^{-8}\Omega$m to 10^{-6} Ωm. At the other end are insulators like ceramic, rubber and plastics having resisitivities 10^{18} times greater than metals or more.

Semiconductors lie between these two. They have resistivities characteristically decreasing with a rise in temperature. The resistivities of semiconductors are also affected by presence of small amount of impurities.

It is generally found that good conductors of electricity like the metals are also good conductors of heat (superconductors are an exception in this regard), while the bad conductors of electricity like ceramic, plastic etc. are also found to be bad conductors of heat.

Resistors used in laboratories are of two types.

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(1) Wire Wound Resistors : Wire wound resistors are made by winding the wires of an alloy, viz., manganin, constantan, nichrome or similarones on a proper base. The resistivity of such materials does not change appreciably with temperature.

(2) Carbon Resistors : Carbon resistors are widely used in electronic circuits (like radio, television, amplifier etc.). The carbon resistors have very small dimension and it is very inexpensive. (Now a days thin film resistors are used very extensively in the electronic circuits.)

To make a carbon resistor, pure graphite mixed with resin is moulded into a cylinder at high temperature and pressure. Wire leads are attached to two ends of a cylinder and the entire resistor is enclosed in an insulating jacket (ceramic or plastic). Carbon resistors are available in the range of 1 Ω to 100 MΩ.

Colour Code for Carbon Resistors :

The value of the carbon resistor can be found from the colour bands, marked on the surface of the cylinder of carbon resistor. Let us refer the resistor and colour code shown in figure 3.10 in order to understand this.

Colour Code for Resistors (ohm)

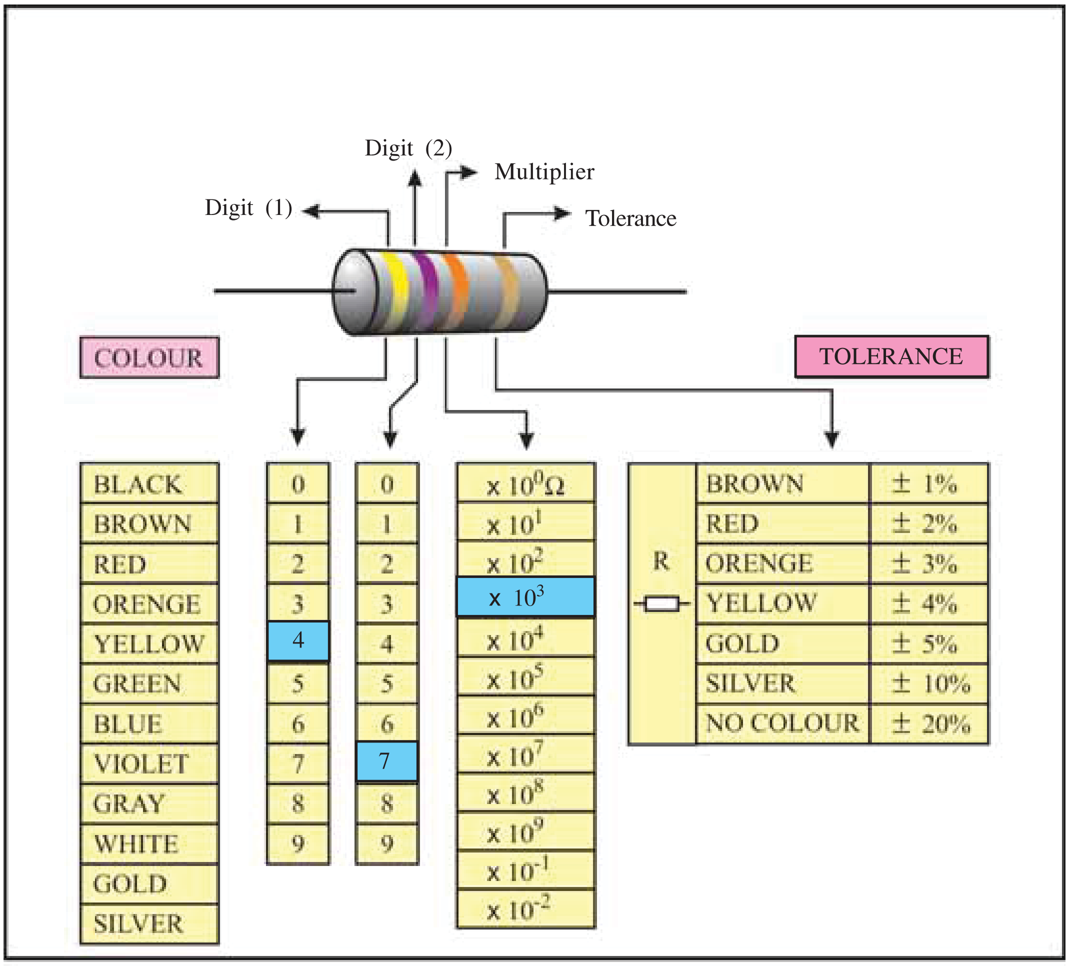


Figure 3.10

Colour of the first band on the resistor shows its value in **"tens"**. The colour of the second band shows its value in **"units"**. The digits for different colours are shown in the colourcode. (Figure 3.10)

The third band implies that the number formed by the first and second digit is to be multiplied by 10^n. The multiplier 10^n for different colours is given in the colourcode. The fourth colour band shows the possible deviation (tolerance) in the value of resistor.

Let us take the example of the resistor shown in figure 3.10. The first band on the resistor is yellow and the digit for this colour is 4. The second band is violet and its corresponding digit is 7. The number formed due to combination of this two digits is 47.

The third colourband is orange, which shows the multiplier 10^3. 47 multiplied by 10^3 gives the value of the resistance = $47 \times 10^3 = 47$ KΩ. The last colourband on the resistor is golden, which indicates that the value of the resistor calculated above can have a variation of 5%. Thus, the value of this resistor is (47 KΩ $\pm$ 5%)

Dear students, give the colourcode for the resistor 1KΩ $\pm$ 10% using the colourcode given in Figure 3.10.

3.7.2 Super Conductivity

In 1911, Dutch physicist Kamerlingh Onnes experimentally discovered that the resistivity of mercury absolutely disappears at temperatures below about 4.2 K. As per his observation at 4.3 K temperature, the resistance of mercury is about 0.084Ω and at 3 K temperature it becomes 3×10^{-6}Ω (Which is about 10^6th part of the resistance at 0°C). This showed that,

"The resistance of certain materials reduces to almost zero, when its temperature is lowered below a certain definite temperature (which is known as critical temperature T_C). The material in this state is known as superconductor and this phenomenon is known as superconductivity."

Superconductivity is a specific state of the material. Most of the metals and alloys can achieve the state of superconductivity. Some of the semi-conductors like Si, Ge, Se and Te exhibit the state of suprconductivity under high pressure and low temperatures.

The current flowing through a super-conductor can be sustained over a long interval of time. The reason is that in an ordinary conductor the electrical energy is dissipated as heat energy due to the resistance offered by the conductor, while in super-conductor there will be no loss of electrical energy since the resistance of super-conductor is almost zero.

From the above discussion it seems that using superconductors, the problem of energy loss during the transmission of electrical energy can be solved. One of the important fact we have ignored is that the temperature of the material should be lowered to its critical temperature. Liquid helium and liquid nitrogen are used to achieve the temperature of the material below its critical temperature T_C. This situation can be best described by the proverb 'Penny wise pound foolish.'

In fact, the best of the normal conductors have higher critical temperature (but very much lower than room temperature) than oxide alloys. It shows that, compared to normal conductors, insulators like ceramic can easily achieve the state of superconductivity. Thus, superconductivity is a specific state of a material.

According to latest research, the critical temperature (T_C) of the compound Hg–Ba–Ca–Cu–O can be raised upto 164K.

Such superconductors are known as high temperature superconductors (HTS). HTS has applications in the areas of thin film devices, electric transmission over long distances, levitating trains (maglev trains) which can achieve speed of 550 km/h.

Illustration 6 : The resistance of the platinum wire of a platinum resistance thermometer at the icepoint is 5Ω and at steampoint is 5.23Ω. When the thermometer is inserted in a hot bath, the resistance of the platinum wire is 5.795Ω. Calculate the temperature of the bath.

Solution : $R_0 = 5\Omega$, $R_{100} = 5.23\Omega$ and $R_\theta = 5.795\Omega$

From equation, $R_\theta = R_{\theta_0}\,[1 + \alpha\,(\theta - \theta_0)]$

$R_\theta = R_0[1 + \alpha\theta]$ ($\because\ \theta_0 = 0$)

$\therefore\ R_\theta - R_0 = R_0\alpha\theta$

For steam, $R_{100} - R_0 = R_0\alpha\ (100)$ (1)

For heat bath, $R_\theta - R_0 = R_0\ \alpha\ \theta$ (2)

Dividing equation (2) by (1),

$$\frac{R_\theta - R_0}{R_{100} - R_0} = \frac{\theta}{100}$$

$$\therefore \theta = \frac{R_\theta - R_0}{R_{100} - R_0} \times 100$$

$$= \frac{5.795 - 5}{5.23 - 5} \times 100$$

$\therefore \theta = 345.65\ ^\circ C$

Illustration 7 : Two materials have the value of α_1 and α_2 as $6 \times 10^{-4} (^\circ C)^{-1}$ and $-5 \times 10^{-4} (^\circ C)^{-1}$ respectively. The resistivity of the first material $\rho_{20} = 2 \times 10^{-8}$. A new material is made by combining the above two materials. The resisitivity does not change with temperature. What should be the resistivity ρ_{20} of the second material ? Considering the reference temperature as 20°C assume that the resistivity of the new material is equal to the sum of the resistivity of its component materials.

Solution :

Here the reference temperature is 20 °C.

Resistivity of a material at temperature θ is,

$\rho_\theta = \rho_{20}\ [1 + \alpha\ (\theta - 20)]$

$$\therefore\ \frac{d\rho_\theta}{d\theta} = \rho_{20}\alpha$$

For material 1, $\left(\dfrac{d\rho_\theta}{d\theta}\right)_1 = (\rho_{20})_1\ \alpha_1$

For material 2, $\left(\dfrac{d\rho_\theta}{d\theta}\right)_2 = (\rho_{20})_2\ \alpha_2$

The resistivity of the mixture $\rho_\theta = (\rho_\theta)_1 + (\rho_\theta)_2$ does not change with temperature. Therefore,

$$\left(\frac{d\rho_\theta}{d\theta}\right) = \left(\frac{d\rho_\theta}{d\theta}\right)_1 + \left(\frac{d\rho_\theta}{d\theta}\right)_2 = 0$$

$$\therefore\ \left(\frac{d\rho_\theta}{d\theta}\right)_1 = -\left(\frac{d\rho_\theta}{d\theta}\right)_2$$

$\therefore\ (\rho_{20})_1\ \alpha_1 = -(\rho_{20})_2\ \alpha_2$

$$\therefore\ (\rho_{20})_2 = -\frac{(\rho_{20})_1\alpha_1}{\alpha_2}$$

$$= \frac{-(2\times10^{-8})(6\times10^{-4})}{-(5\times10^{-4})}$$

$\therefore\ (\rho_{20})_2 = 2.4 \times 10^{-8}\ \Omega\ m$

Illustration 8 : The tungsten filament of bulb has resistance equal to 18 Ω at 20° C temperature. 0.185 A of current flows, when 30 V is connected to it. If $\alpha = 4.5 \times 10^{-3}\ K^{-1}$ for a tungsten, then find the temperature of the filament.

Solution : As per Ohm's Law,

$$I = \frac{V}{R} \quad \therefore R_\theta = \frac{V}{I} = \frac{30.0}{0.185} = 162\ \Omega$$

When the bulb is ON its resistance is 162 Ω,

Now, $R_\theta = R_{\theta_0}\ [1 + \alpha\ (\theta - \theta_0)]$

$\therefore\ 162 = 18[1 + 4.5 \times 10^{-3}\ (\theta - 293)]$

$$\therefore \frac{9-1}{4.5\times10^{-3}} = \theta - 293$$

$\therefore\ \theta = 2070.7\ K$

3.8 Electromotive Force and Terminal Voltage of a Cell

We have seen that current is constituted due to the motion of a charged particles. In order to bring them in motion, force must be exerted on them, in other words energy has to be supplied to the charged particles. The device, which serves the above purpose is called the source of electromotive force i.e. "emf". There are many ways in which force can be exerted on the charge. For example, the force exerted on the charge in an electric cell is due to the chemical processes, The force can be exerted on the charge due to the varying magnetic field and by temperature difference. All the above mentioned devices are the source of emf. A battery (cell) is also a source of emf. What does this emf mean ? We shall consider the example of an electric cell in order to understand the cell.

Figure 3.11 shows a schematic diagram of a battery. There are positive and negative charges present in the chemical of a battery. Due to certain chemical reactions occurring in the battery, force is exerted on these charges. Such a force is called chemical force or non-electrical force F_n. This force (F_n) drives positive charges towards one terminal (i.e. positive terminal) A and drives the negative charges towards the other terminal (i.e. negative terminal) B.

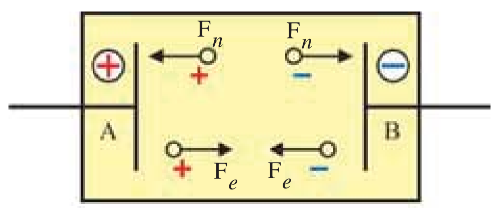


Figure 3.11 Schematic diagram of a battery

As the positive and negative charges build up on the positive and negative terminals A and B respectively, a potential difference (or electric field $\vec{E}$) is set up between them, which keep on increasing gradually. As a result of this, electric force $\vec{F_e}$ $(= q\vec{E})$ is exerted on the charge q in the direction opposite to $\vec{F}n$. In the steady state, the charges stop accumulating further at the terminals A and B and $F_n = F_e$.

The work done by the non-electrical force in taking a unit positive charge from a negative terminal to the positive terminal is equal to $W = \int_{line} \vec{F}_n \cdot \vec{dl}$, where the line intergral is from negative to positive terminal. As per the definition of an emf, this work done is equivalent to the emf. Therefore the definition of emf can be given as follows.

When unit positive charge is driven from negative to positive terminal due to non-electrical forces, the energy gained by the charge (or work done by the non-electrical forces) is called an emf (ε) of a battery. The unit of emf is $\frac{\text{joule}}{\text{coulomb}}$ = volt. (in the memory of great scientist Volta) Remember that emf is not a force but energy per unit charge.

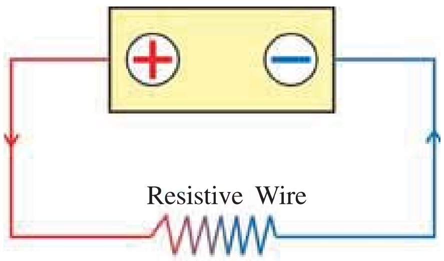


Figure 3.12 Terminal Voltage of a Battery

In the steady state of a battery (when $F_n = F_e$), the electric charge in the battery does not execute any motion i.e. no current is flowing through the battery. (I = 0). In this condition, battery is said to be in **open circuit condition**.

Let us consider a wire of resistance R connected across the two terminals of a battery as shown in figure 3.12. The electric field is thus established in the wire. As a result, positive charges which are at higher potential will move towards the negative terminal of a battery through the wire and constitute an electric current. The question then arises as to why didn't the positive charges move towards the negative terminal (inside the battery) rather than the longer route of the wire ? The reason for this is the non-electrical forces which oppose the motion of positive charges towards the negative terminal of the battery inside the battery.

The energy of the positive charge is consumed against the resistance of the wire. As it reaches the negative terminal, its energy becomes zero. This happens at every rotation of its motion.

During the flow of current, the positive charge is moving from negative terminal to the positive terminal because of non-electrical forces. During the motion, the charge has to pass through the chemical materials of a battery. In other words battery offers a resistance to the charge which is called the internal resistance (r) of a battery.

Due to this internal resistance (r), when a unit positive charge reaches to positive terminal, some part of its energy (which is gained due to the work done by the non-electrical forces.) is consumed against the internal resistance. If the current through the battery is I, then the energy consumed per unit charge against internal resistance = Ir.

Therefore, the energy of a unit positive charge at the positive terminal of battery is less by an amount Ir compared to the energy (ε) in the open circuit condition. Thus, the net energy per unit charge will be $(\varepsilon - Ir)$. Thus, during the flow of current this energy is called the potential difference between two terminals of a battery or the terminal voltage (V) of a battery.

$\therefore\ V = \varepsilon - Ir$

3.8.1 Secondary Cell : Lead Accumulator

Electrochemical cells are of two types.

(1) Primary Cell : The cells which get discharged only are called primary cells. e.g. Voltaic cell. Primary cells cannot be recharged.

(2) Secondary Cell : The cells which can be restored to original condition by reversing the chemical processes (i.e. by recharging) are called secondary cells.

In a secondary cell, one can pass current in both directions.

(i) When (conventional) current leaves the cell at the positive terminal and enters the cell at the negative terminal, the cell is said to be discharging. This is the normal working of the cell during which chemical energy is converted into electrical energy.

(ii) If the cell is connected to some other source of larger emf, current may enter the cell at the positive terminal and leave it at the negative terminal. The electrical energy is then converted into chemical energy and the cell gets charged.

The most commonly used secondary cell is a lead accumulator.

Lead Accumulator : A lead accumulator consists of electrodes made of PbO_2 and of Pb immersed in an electrolyte of dilute sulphuric acid (H_2SO_4). PbO_2 acts as the positive electrode and Pb as the negative electrode.

When the cell is in use, (i.e. when the cell is discharging) SO_4^{-2} ions move towards the Pb electrode, give up the negative charge and form $PbSO_4$ there. The H^+ ions move to the PbO_2 electrode, give up the positive charge and reduce PbO_2 to PbO.

The PbO so formed reacts with the H_2SO_4 to form $PbSO_4$ and water.

Thus, $PbSO_4$ is formed at both the electrodes and the concentration of the electrolyte decreases.

The concentration of the electrolyte can be measured by a device called hydrometer. When the cell is fully charged, the specific gravity of an electrolyte is 1.285 and emf of a cell is about 2.1 volt. In the discharged condition, the specific gravity falls to 1.15 and emf may fall to 1.8 V.

Charging : To charge a secondary cell of emf ε, direct current (d.c.) is passed through the cell as shown in figure 3.13. The positive terminal of the cell is connected to positive end of a d.c. source and the negative terminal is connected to negative of d.c. supply (opposing condition) for the charging of a cell. (Here V > E)

Due to the chemical reactions occurring in the cell during charging process, $PbSO_4$ deposited at the two electrodes is dissolved. Pb is deposited at the negative electrode and PbO_2 at the positive electrode, simultaneously H_2SO_4 is also formed. This restores the capacity of the cell to provide current.

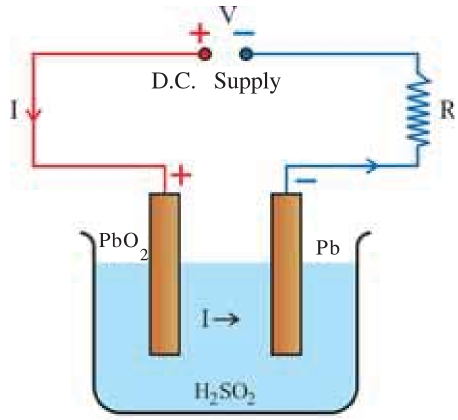


Figure 3.13 Charging of a Secondary Cell

Here, the elecrical energy V*It* consumed by a *d.c.* source provides εIt energy for the charging of a cell and $I^2Rt + I^2rt$ energy dissipated in the external (series) resistance (R) and internal resistance (r) of a cell.

$$\therefore \ VIt = \varepsilon It + I^2Rt + I^2rt \qquad (3.8.2)$$

$$\therefore \ V = \varepsilon + I(R + r)$$

$$\therefore \ I = \frac{V - \varepsilon}{R + r} \qquad (3.8.3)$$

Above equation gives the charging current. Here, the resistance R is connected to control the current.

Illustration 9 : 6 batteries, each of 2.0 volts are connected in series so that they are helping each other. Internal resistance of each is 0.5 Ω. They are being charged using a direct voltage supply of 110 volts. To control the current, a resistance of 46 Ω. is used in the series. Obtain (1) power drawn from the supply, and (2) power dissipated as heat. Why are the two different ?

Solution : $V = \varepsilon. + Ir + IR$ gives

$V = 6\varepsilon + 6\ Ir + IR$

$$V = 110\ V$$
$$\varepsilon = 2.0\ V$$
$$r = 0.50\ \Omega$$
$$R = 46\ \Omega$$

Now, $I = \frac{V - 6\varepsilon}{6r + R} = \frac{110 - 12}{6 \times 0.50 + 46} = \frac{98}{49}$ $\therefore$ I = 2 A

power drawn from the supply,

$W = V \times I = 110 \times 2 = 220$ W

power dissipated as heat $= 6I^2r + I^2R$

$= I^2(6r + R)$

$= 4 \times (6 \times 0.50 + 46)$

$= 4 \times (3 + 46)$

$= 196$ W

$\therefore$ Difference = (220 − 196) W = 24 W. This power is used to charge the batteries.

3.9 Kirchoff's Rules

In different electronic circuits, components like resistors, inductors, capacitors and batteries are connected with each other in a complicated way. Such circuits cannot be considered as a simple series or parallel connections. Generally, such complicated circuits are known as network.

Ohm's law alone is not sufficient to analyze a network. There are several rules for the analysis of a network. Kirchoff's two rules are amongst them.

Let us try to understand the two terms concerning circuits before the discussion of Kirchoff's rules.

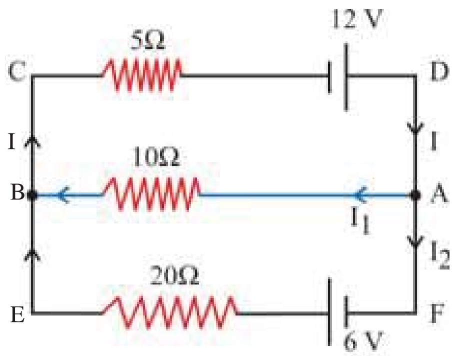


Figure 3.14 Network

Junction or Branch Point : The point in a network at which more than two conductors (minimum three) meet is called a junction or a branch point. (You will be aware that how many railway lines make a junction.) In figure 3.14 three conductors are meeting at points A and B. Therefore, points A and B are called junction or branch points.

Loop : A closed circuit formed by conductors is known as loop. As shown in figure 3.14, CDABC, AFEBA and CDAFEBC are some of the closed path of conductors known as loop.

In the analysis of a network, unknown quantities like V, I, R ...... in a given circuit can be determined from the known quantities.

Kirchhoff's Rules :

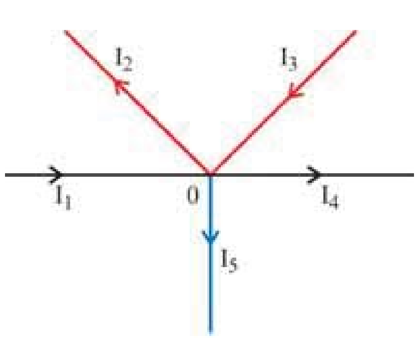


Figure 3.15

Kirchoff's First Rule : Kirchoff's first rule is the consequence of the law of conservation of charge.

Consider junction O of a network as shown in figure 3.15. The currents meeting at the junction point O are represented as I_1, I_2, ..., I_5. Their directions are represented by arrows in the Figure.

Let Q_1, Q_2, ..., Q_5 be electrical charges flowing through the cross-sectional area of the respective conductor in time interval t which constitutes current I_1, I_2, ..., I_5.

Hence, $I_1 = \frac{Q_1}{t} \rightarrow Q_1 = I_1 t$

$I_2 = \frac{Q_2}{t} \rightarrow Q_2 = I_2 t$

$I_5 = \frac{Q_5}{t} \rightarrow Q_5 = I_5 t$

It is evident from the Figure that the total electric charge entering the junction is $Q_1 + Q_3$, while $Q_2 + Q_4 + Q_5$ amount of electric charge is leaving the junction in the same interval of time.

As per the law of conservation of charge,

$Q_1 + Q_3 = Q_2 + Q_4 + Q_5$ (3.9.1)

$\therefore\ I_1 t + I_3 t = I_2 t + I_4 t + I_5 t$

$\therefore\ I_1 + I_3 + (-I_2) + (-I_4) + (-I_5) = 0$ (3.9.2)

$\therefore$ At the junction, $\Sigma\ I = 0$ (3.9.3)

Thus, **"The algebraic sum of all the electric currents meeting at the junctions is zero."** This statement is known as Kirchoff's first rule.

In the above sum I_1 and I_3 currents are positive while I_2, I_4 and I_5 are negative. Thus the electric currents entering the junction are considered as positive and the currents leaving the junction are considered as negative. One can also consider an opposite convention to arrive at the same result.

Kirchoff's Second Rule : Using law of conservation of energy and the concept of electric potential any closed circuit can be analyzed. Kirchoff's second rule is the essence of the above mentioned concepts. Let us consider a closed path ABCDEA as shown in figure 3.16.

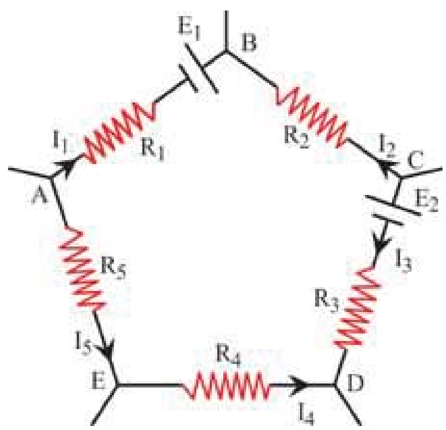


Figure 3.16

Here, resistors R_1, R_2, R_3, R_4, R_5 and batteris of emf's ε_1 and ε_2 form a closed loop ABCDEA. If the internal resistance of a battery is ignored, the rise in the electric potential while going from negative to positive terminal of a cell is equal to the emf (ε) of a battery. The potential difference across the ends of a resistor is equal to the product of the resistor and the current flowing through it ($V = IR$).

The electric potential at any point in a steady circuit does not change with time. If V_A is the electric potential at point A, and if we measure the changes in the electric potential while moving in clockwise or anticlockwise direction in a closed circuit and come back to point A, the potential V_A should remain unchaged. This is called the singlevaluedness of the electric potential. In fact, the singlevaluedness of the electric potential is a consequence of the law of conservation of energy.

The electric potential drops by an amount I_1R_1 when we move in a clockwise direction from A through the resistor R_1. Here, the direction of current is arbitrarily taken from A to B i.e. current flows through resistor R_1 from a point of higher potential (A) to lower potential. Hence there will be a drop in potential equal to I_1R_1 as we move from A to B. There is a rise in the potential ε_1 while going from the negative terminal to the positive terminal of a battery of emf ε_1. The potential rises by I_2R_2 when we go from B to C through resistor R_2. As the direction of current is assumed from C to B, the electric potential of point C is higher than B. Therefore, potential rises by an amount I_2R_2 while going from B to C.

In a similar way, there is a decrease in potential equal to ε_2 when we go from positive to negative terminal of a battery of emf ε_2. There is a potential drop I_3R_3 while passing through R_3, rise in potential I_4R_4 through R_4 and rise in potential I_5R_5 through R_5.

Taking the algebraic sum of all these changes, the potential at point A should remain V_A.

$\therefore\ V_A - I_1R_1 + \varepsilon_1 + I_2R_2 - \varepsilon_2 - I_3R_3 + I_4R_4 + I_5R_5 = V_A$

$$- I_1R_1 + \varepsilon_1 + I_2R_2 - \varepsilon_2 - I_3R_3 + I_4R_4 + I_5R_5 = 0 \quad (3.9.4)$$

Thus, the algebraic sum of all the changes in potential around any closed loop is zero.

$$\therefore\ (-I_1R_1) + I_2R_2 + (-I_3R_3) + I_4R_4 + I_5R_5 = (-\varepsilon_1) + \varepsilon_2 \quad (3.9.5)$$

$$\therefore\ \Sigma IR = \Sigma\varepsilon \quad (3.9.6)$$

This equation suggests that **"for any closed loop the algebraic sum of the products of resistances and the respective currents flowing through them is equal to the algebraic sum of the emf's applied along the loop."** This statement is known as Kirchhoff's second rule.

Sign convention for applying Kirchoff's rules :

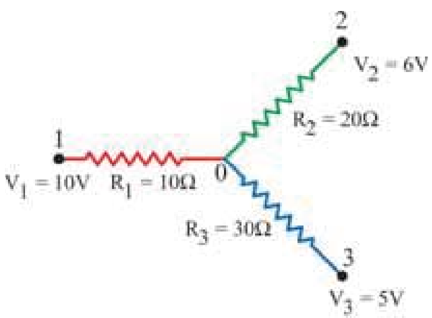
The following sign convention has to be followed in using equation (3.9.5).

(1) If our journey through the resistor is in the direction of flow of current which is arbitrarily chosen, IR should be considered negative and if the direction of journey and the direction of current is opposite to each other IR should be considered as positive.

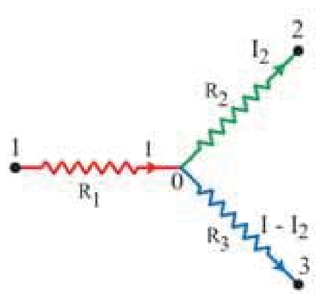
(2) The emf of a battery should be considered negative while moving from negative terminal of a battery to the positive terminal (while writing on the right hand side of the equation.) The emf of a battery is taken as positive while moving from positive to negative terminal of the battery.

The direction of the electric current can be arbitrarily chosen while using Kirchoff's rules to analyze any network. We shall get negative value of the current if the direction of current which is arbitrarily chosen is opposite to the actual direction of current.

Illustration 10 : Calculate the current flowing through the resistor R_1 in the given circuit. $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$ and $R_3 = 30\ \Omega$. The potentials of the points 1, 2 and 3 are respectively, $V_1 = 10$ V, $V_2 = 6$ V and $V_3 = 5$ V. Calculate the potential at the junction.



Solution : O is the junction point in the above circuit. The potential at point 1 is higher than the potential existing at point 2 and 3. Hence, the direction of the flow of the current is from point 1 to O, from O to 2 and from O to point 3. The Figure indicates the electric current and their direction.



Now for the 1O2 path, we have,

$V_1 - IR_1 - I_2R_2 = V_2$

$\therefore\ 10 - 10I - 20I_2 = 6$

$$\therefore\ 10I + 20I_2 = 4 \quad (1)$$

For the 1O3 path, we have,

$10I + 30\ (I - I_2) = 5$

$$\therefore\ 40I - 30I_2 = 5 \quad (2)$$

Solving equation (1) and (2), we have,

$I = 0.2$A

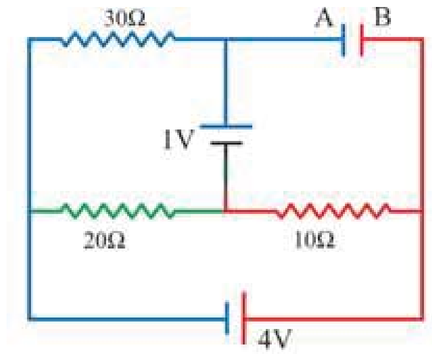
Let V_O, be the potential at point O, then

$10 - V_O = IR_1$

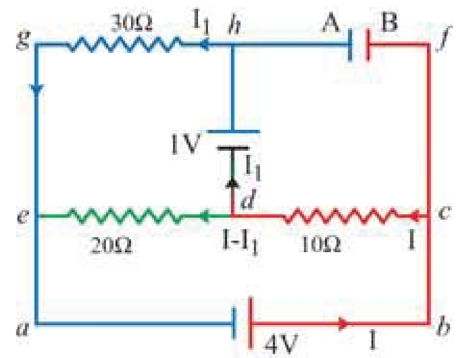
$\therefore\ 10 - V_O = 2$

$\therefore\ V_O = 8$ V

Illustration 11 : Calculate the potential difference between the plates A and B of the capacitor in the adjacent circuit.



Solution : The distributions of the current are shown in figure.



Applying Kirchhoff's Second Law to the closed loop abcdea, we have

$-10\ I - 20\ (I - I_1) + 4 = 0$

$\therefore\ 30\ I - 20\ I_1 = 4$ (1)

For the cdhge loop,

$20(I - I_1) + 1 - 30I_1 = 0$

$\therefore\ 20I - 50I_1 = -1$ (2)

Solving equation (1) and (2), we have,

$I_1 = 0.1$ A and $I = 0.2$ A.

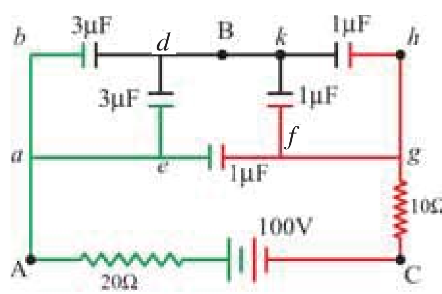
The p.d. between the two plates of the capacitor is equal to the p.d. between c and h point. Let V_c be the potential at point c and let V_h be the potential at point h. For the path cdh, we have

$\therefore\ V_c - 10 \times 0.2 + 1 = V_h$

$\therefore\ V_c - V_h = 2 - 1 = 1$

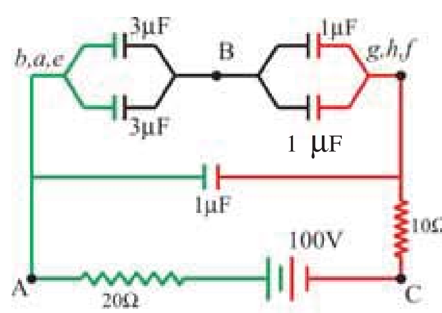
$\therefore$ The potential difference between the two capacitors = 1 V

Illustration 12 : Calculate the potential difference between points A and B as well as between, points C and B under a steady condition of the circuit shown in the figure.



Solution : e (or a or b) and d are the two ends of the capacitor 3 μF. 'k and g (or h or f) are the two ends of the capacitor 1 μF.

The equivalent circuit of the above circuit can be given as under :



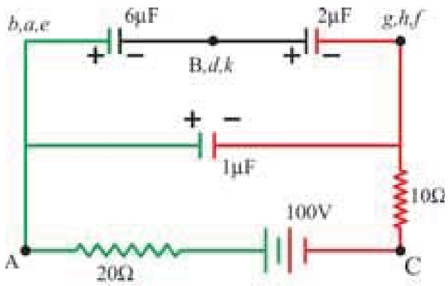
There are two 3 μF capacitors connected in parallel.

$\therefore$ Their equivalent capacitors = 6 μF

In a similar way equivalent capacitors of two 1 μF capacitors = 2 μF.

The above situation is represented in the figure below.

Since the circuit is in the steady state, no current flows through 20 Ω and 10 Ω resistances. It seems as if these resistors are not connected in the circuit. In this situation, the voltage of the battery (100 V) is applied between point a and h. The 6 μF and 2 μF capacitors are connected in a series combination between the two ends of the battery.



If the electrical charge on 6 μF and 2 μF capacitors is equal to q, then

$$V_1 + V_1 = V$$

$$\frac{q}{C_1} + \frac{q}{C_2} = V,\ \frac{q}{6} + \frac{q}{2} = 100$$

$$\therefore\ q = \frac{100\times12}{8} = 150\ \mu C$$

Now, the potential difference between points A and B are equal to voltage developed across 6 μF capacitors,

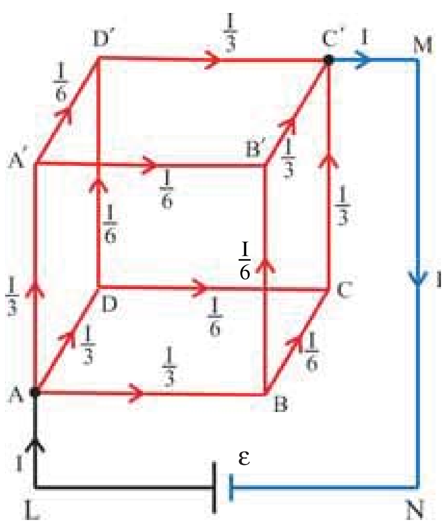
$$\therefore\ V_{AB} = \frac{150}{6} = 25\ V$$

Now, the voltage between B and C

$V_{BC} = 100 - 25 = 75$ V

Illustration 13 : A cube is made by connecting 12 wires of equal resistance R. Find the equivalent resistance between any two of its diagonally opposite points.

Solution : Let I be the current through the cell.



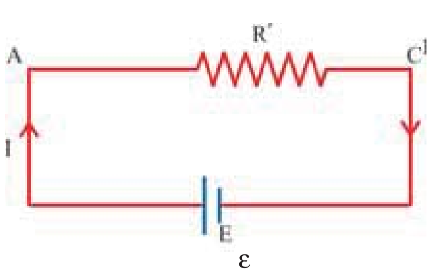
Since the paths AB, AD and AA' are symmetrical with respect to resistors, current through each of them is same (i.e. $\frac{I}{3}$). At the junctions B, D and A' the incoming current $\frac{I}{3}$ splits equally into the two outgoing branches, the current through each branch is $\frac{I}{6}$, as shown in Figure. At the junctions C, B' and D' these currents reunite and the currents along CC', B'C' and D'C' are $\frac{I}{3}$ each. These three currents reunite at the junction C' and the total current at junction C' is I again.

Applying Kirchoff's second rule to the closed loop AA'D'C'MNLA,

$$-\frac{I}{3}\cdot R - \frac{I}{6}R - \frac{I}{3}R = -\varepsilon$$

$$\therefore \varepsilon = \frac{5}{6}IR \qquad (1)$$

Let the equivalent resistance between two diagonally opposite points A and C' be R' this means that if R' is connected across the same battery (of emf ε) in place of the given network, the current I should remain same.



From the equivalent circuit shown in Figure,

$$\varepsilon = IR' \qquad (2)$$

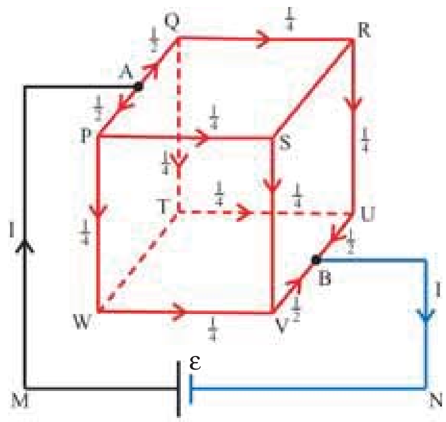
Comparing equation (1) and (2),

$$\frac{5}{6}IR = IR'$$

$$\therefore\ R' = \frac{5}{6}R$$

Illustration 14 : A cube is constructed by connecting 12 wires of equal resistance as shown in Figure. Find the equivalent resistance between the points A and B shown in the Figure. The resistance of each wire is of $r\ \Omega$. A and B are the midpoints of the sides PQ and VU respectively.

Solution : Note that with reference to the line joining A and B, the pairs AP and UB, AQ and VB, PW and RU, QT and SV, WV and QR are symmetric branches. Hence current flowing through each of this symmetric pair must be same. e.g. if the current flowing through PW is $\frac{I}{4}$, the same current (i.e. $\frac{I}{4}$) will flow through RU. With this consideration the proportional currents through the various circuit branches are as assigned their values in figure.



Points W and T being symmetric about A are at the same potential, so no current will flow through WT and similarly also through SR.

Appyling Kirchoff's second rule to the closed loop APWVBNMA, taking r as the resistance of each wire.

$$-\frac{I}{2}\left(\frac{r}{2}\right) - \frac{I}{4}r - \frac{I}{4}r - \frac{I}{2}\left(\frac{r}{2}\right) = -\varepsilon$$

$\therefore$ IR = ε (1)

If the equivalent resistance is r',

then $Ir' = \varepsilon$ (2)

Comparing equations (1) and (2),

$r' = r$

3.10 Series and Parallel Connections of Resistors

Resistors can be connected in series or parallel or a mixed combination of both the types between any two points. You have studied the series and parallel connections of resistors in Standard–X. Here we will make note of their results.

Series Combination of Resistors

Resistors are said to be connected in series between any two points, if the same current is flowing through each resistor or in other words, there is only one path available for the flow of current.

Figure 3.17 shows the series connection of n resistors R_1, R_2, R_3, ..., R_n between two points A and B.

If the equivalent resistance of the series connection is R_s, then,

$$R_s = R_1 + R_2 + R_3 + \ldots\ldots + R_n = \sum_{i=1}^{n} R_i \qquad (3.10.1)$$

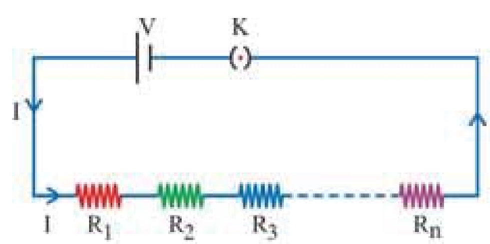


Figure 3.17 Series Connection of Resistors

Thus, the equivalent resistance of the series combination of the resistors is always greater than the greatest value of the resistors connected in series.

If n identical resistors each of value R are connected in series, the equivalent resistance is,

$$R_s = R + R + R + \ldots\ldots\ n \text{ times} = nR \qquad (3.10.2)$$

Parallel Connection of Resistors : The resistors are said to be connected in parallel between two points if there are more than one path available for the flow of current and potential difference (V) across each of them is same.

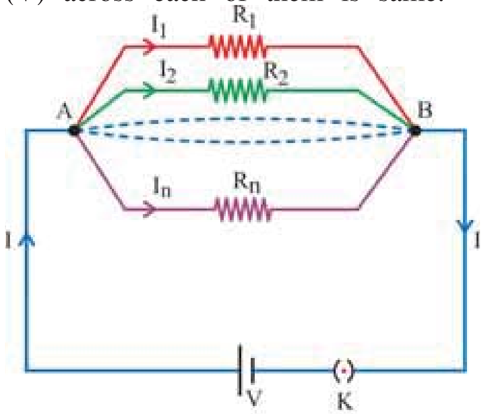


Figure 3.18 Parallel Connection of Resistors

In figure 3.18, n resistors R_1, R_2, R_3, ..., R_n are connected in parallel between two points A and B.

If the equivalent resistance of this parallel connection is R_P, then

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n} = \sum_{i=1}^{n}\frac{1}{R_i} \quad (3.10.3)$$

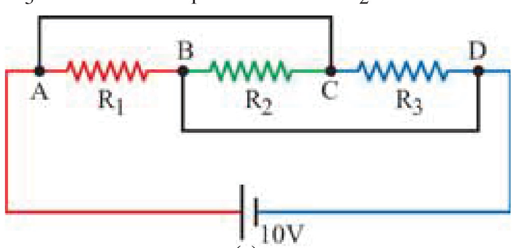
Thus, the equivalent resistance of a parallel combination of resistors is always smaller than the smallest value of resistors connected in parallel.

If n identical resistors having resistance R are connected in parallel, the equivalent resistance is,

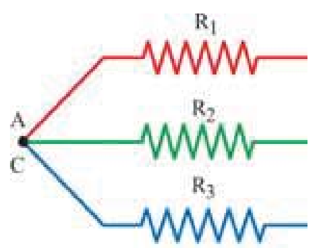
$$\frac{1}{R_P} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + ... n \text{ times} = \frac{n}{R}$$

$$\therefore R_P = \frac{R}{n} \quad (3.10.4)$$

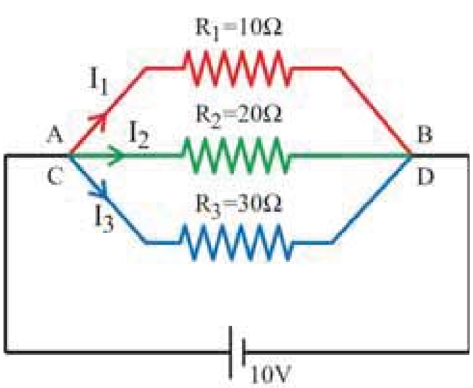
Illustration 15 : As shown in the figure (a), some current flows through resistors R_1, R_2 and R_3 resistors. $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$ and $R_3 = 30\ \Omega$ and the battery voltage is equal to 10 V.



(a)



(b)



(c)

Solution : Let us start from point A in order to obtain the equivalent circuit of the above given circuit. Here one end of the resistor R_1 is connected to point A. The common end of resistor R_2 and R_3 (point C) is connected to point A.

$\therefore$ The circuit (b) resembles partially the above circuit.

Similarly, the other end of the resistor R_1 and the common end of resistor R_2 and R_3 are connected to point B.

$\therefore$ The entire circuit can be represented by figure (c).

Hence, we have a situation in which 3 resistors are connected in parallel as shown in figure (c).

$\therefore$ The voltage developed across the two ends of each resistor will be equal to 10 V.

$\therefore$ Therefore, the current flowing through R_1, $I_1 = \frac{V}{R_1}$ $= \frac{10}{10} = 1A$, similarly the current flowing through R_2, $I_2 = \frac{V}{R_2} = \frac{10}{20} = 0.5A$ and current flowing through R_3 $I_3 = \frac{V}{R_3} = \frac{10}{30} = 0.33$ A.

Illustration 16 : An electric current of 5A is divided in three branches forming a parallel combination. The lengths of the wires in the three branches are in the proportion 2 : 3 : 4 and their radii are in the proportion 3 : 4 : 5. Find the currents in each branch if the wires are of the same material.

Solution : Let the lengths of the wires be $2l$, $3l$ and $4l$ and their radii be $3r$, $4r$ and $5r$ respectively. Their respective resistances are,

$$R_1 = \rho.\frac{2l}{\pi(3r)^2}$$

$$R_2 = \rho.\frac{3l}{\pi(4r)^2}$$

and $$R_3 = \rho.\frac{4l}{\pi(5r)^2}$$

or, $R_1 : R_2 : R_3 = \frac{2}{9} : \frac{3}{16} : \frac{4}{25}$

The currents must be in the inverse proportion of resistances.

$$\therefore I_1 : I_2 : I_3 = \frac{9}{2} : \frac{16}{3} : \frac{25}{4}$$

$$= 54 : 64 : 75$$

$\therefore$ Current in the first branch $I_1 = \frac{54 \times 5}{193} = 1.40$ A

Current in the second branch $I_2 = \frac{64 \times 5}{193} = 1.66$ A

Current in the third branch $I_3 = \frac{75 \times 5}{193} = 1.94$ A

3.11 Series and Parallel Connections of Cells

Like resistors, cells can also be connected in series, parallel and combination of both between two points.

Cells in Series :

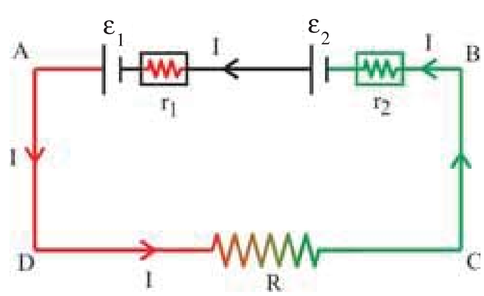


Figure 3.19 Series Connections of Cells

Suppose two cells having emfs ε_1 and ε_2 and internal resistances r_1 and r_2 are connected in series between two points A and B as shown in figure 3.19. An external resistance R is also connected across the connection.

Applying Kirchhoff's second rule for the closed loop ABCDA,

$$-\varepsilon_1 + Ir_1 - \varepsilon_2 + Ir_2 + IR = 0$$

$$\therefore Ir_1 + Ir_2 + IR = \varepsilon_1 + \varepsilon_2$$

$$\therefore I[R + (r_1 + r_2)] = \varepsilon_1 + \varepsilon_2$$

$$\therefore I = \frac{\varepsilon_1 + \varepsilon_2}{R + (r_1 + r_2)} = \frac{\varepsilon_{eq}}{R + r_{eq}} \qquad (3.11.1)$$

where, I is the current through the resistor R.

Thus, the series combination of two cells acts as a single cell of emf $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ and internal resistance $r_{eq} = r_1 + r_2$. In this sense ε_{eq} is an equivalent emf and r_{eq} is the equivalent internal resistance of the series connection of cells.

If the polarity of one of the cells is reversed, the equivalent emf will be $|\varepsilon_1 - \varepsilon_2|$ but the equivalent internal resistance will remain $r_1 + r_2$.

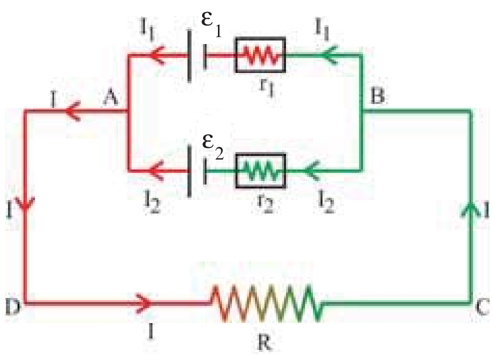


Figure 3.20 Parallel Connection of Cells

Cells in Parallel :

As shown in figure 3.20, suppose two cells of emfs ε_1 and ε_2 and internal resistances r_1 and r_2 are connected in parallel between two points A and B. The currents are also shown in the Figure.

We are interested in finding the current flowing through external resistor R.

At junction A, according to Kirchhoff's first rule,

$$I = I_1 + I_2 \tag{3.11.2}$$

Applying Kirchhoff's second rule to the closed loop ADRCBε_1A,

$$-IR - I_1 r_1 + \varepsilon_1 = 0$$

$$\therefore\ IR + I_1 r_1 = \varepsilon_1$$

$$\therefore\ I_1 = \frac{\varepsilon_1 - IR}{r_1} \tag{3.11.3}$$

Similarly for the closed loop ADRCBε_2A, we have

$$I_2 = \frac{\varepsilon_2 - IR}{r_2} \tag{3.11.4}$$

Substituting the values of I_1 and I_2 from equations (3.11.3) and (3.11.4) in equation (3.11.2), we have,

$$I = \left(\frac{\varepsilon_1 - IR}{r_1}\right) + \left(\frac{\varepsilon_2 - IR}{r_2}\right)$$

$$\therefore\ I = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} - IR\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\therefore\ I + IR\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}$$

$$\therefore\ I\left(1 + \frac{R}{r_1} + \frac{R}{r_2}\right) = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}$$

$$\therefore\ I = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{1 + \frac{R}{r_1} + \frac{R}{r_2}} \tag{3.11.5}$$

$$\text{or, } I = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{R(r_1 + r_2) + r_1 r_2} \tag{3.11.6}$$

Dividing numerator and denominator of equation (3.11.6) by $(r_1 + r_2)$,

$$I = \frac{\frac{(\varepsilon_1 r_2 + \varepsilon_2 r_1)}{(r_1 + r_2)}}{R + \frac{r_1 r_2}{(r_1 + r_2)}} = \frac{\varepsilon_{eq}}{R + r_{eq}} \quad (3.11.7)$$

Thus, the parallel combination of cells acts as a single cell whose emf is,

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} \quad (3.11.8)$$

and internal resistance is,

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \quad (3.11.9)$$

$$\therefore \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} \quad \text{(from equation 3.11.9)} \quad (3.11.10)$$

Taking the ratio of equations (3.11.8) and (3.11.9), we get

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \quad (3.11.11)$$

If emf's of two cells are $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and internal resistances are $r_1 = r_2 = r$, then, $\varepsilon_{eq} = \varepsilon$ and $r_{eq} = \frac{r}{2}$,

In figure 3.20, we had joined the positive terminals together (at point A) and similarly the two negative ones (at point B), so that the currents I_1 and I_2 flow out of positive terminals. If the negative terminal of the second is connected to positive terminal of the first, equations (3.11.10) and (3.11.11) would still be valid with $\varepsilon_2 \rightarrow -\varepsilon_2$

If there are n cells of emf ε_1, ε_2, ..., ε_n and of internal resistances r_1, r_2, ..., r_n respectively connected in parallel, the combination is equivalent to a single cell of emf ε_{eq} and internal resistance r_{eq}, such that

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots\ldots + \frac{1}{r_n} \quad (3.11.12)$$

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \ldots\ldots + \frac{\varepsilon_n}{r_n} \quad (3.11.13)$$

$$\text{and } I = \frac{\sum_{i=1}^{n} \frac{\varepsilon_i}{r_i}}{1 + R\sum_{i=1}^{n} \frac{1}{r_i}} \quad (3.11.14)$$

If n cells of emf ε_1, ε_2, ..., ε_n and internal resistances r_1, r_2, ..., r_n are connected in series to form a row and m such rows are connected in parallel, the current in such connection (which is called mixed connection) is given by following formula.

$$I = \frac{\sum_{i=1}^{n} \varepsilon_i}{R + \frac{1}{m}\sum_{i=1}^{n} r_i} \quad (3.11.15)$$

R = external resistance connected across the mixed connection

m = number of rows

n = number of cells in a row.

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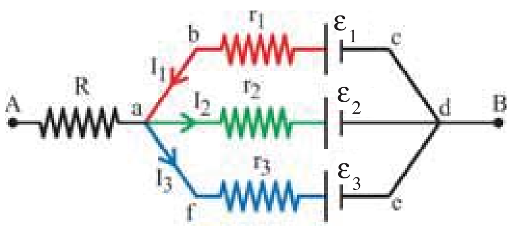
Illustration 17 : In the circuit shown in Figure, $\varepsilon_1 = 3V$, $\varepsilon_2 = 2V$, $\varepsilon_3 = 1V$ and $R = r_1 = r_2 = r_3 = 1\Omega$.. Find the current through each branch and potential difference between the points A and B.

Solution : Let I_1, I_2 and I_3 be the currents through the resistors r_1, r_2 and r_3 respectively as indicated in figure. Using Kirchhoff's second rule to loops abcda and abcdefa, we have,



$$+I_1r_1 - \varepsilon_1 + \varepsilon_2 + I_2r_2 = 0 \quad .....(1)$$

and $$I_1r_1 - \varepsilon_1 + \varepsilon_3 + I_3r_3 = 0 \quad .....(2)$$

From equation (1) and (2),

$$\varepsilon_1 - I_1r_1 = \varepsilon_2 + I_2r_2 = \varepsilon_3 + I_3r_3 \quad .....(3)$$

Appyling Kirchhoff's first rule to junction a, we have

$$I_1 = I_2 + I_3 \quad .....(4)$$

Using equation (4) in (3), we get

$\varepsilon_1 - (I_2 + I_3)r_1 = \varepsilon_3 + I_3r_3$

or, $2I_3 + I_2 = 2$ .....(5)

Also $\varepsilon_2 + I_2r_2 = \varepsilon_3 + I_3r_3$

or, $I_3 - I_2 = 1$ .....(6)

From equations (4), (5) and (6),

$I_1 = 1A$, $I_2 = 0$ A and $I_3 = 1A$

Potential difference between A and B

= Potential difference between a and d

$= \varepsilon_1 - I_1r_1$

$= 3 - 1 \times 1$

$= 2V$

3.12 Wheatstone Bridge

In 1843, Charles Wheatstone developed a circuit to measure unknown resistor with reference to standard known resistance. This circuit is known as Wheatstone bridge. Wheatstone bridge network is shown in figure 3.21. The bridge has four resistor arms R_1, R_2, R_3 and R_4 connected to form a closed loop. The source of emf (battery) is connected between A (common point of R_1 and R_3) and C (common point of R_2 and R_4) and sensitive galvanometer is connected between B (common point of R_1 and R_2) and D (common point of R_3 and R_4)

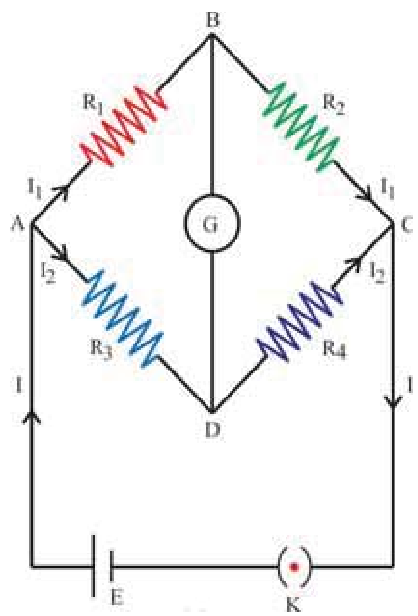


Figure 3.21 Wheatstone Bridge

Three resistors out of the four are known and the fourth one is unknown. The three resistors are chosen in such a way that galvanometer shows zero deflection. In this condition the potential at point B and D are same hence there will be no flow of current through the galvanometer. This condition of Wheatstone bridge is said to be balanced condition.

Applying Kirchhoff's second rule to loop ABDA in a balanced condition,

$$-I_1R_1 + I_2R_3 = 0$$

$$\therefore\ I_1R_1 = I_2R_3 \qquad (3.12.1)$$

Similarly, applying Kirchhoff's second rule to the loop BCDB,

$$-I_1R_2 + I_2R_4 = 0$$

$$\therefore\ I_1R_2 = I_2R_4 \qquad (3.12.2)$$

Dividing equation (3.12.1) by equation (3.12.2) we have,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad (3.12.3)$$

By knowing three resistors, the fourth unknown resistance can be found.

Meterbridge :

Meterbridge is the simplest practical device based on the principle of Wheatstone bridge. It is used to measure an unknown resistance experimentally. The Meterbridge used in the laboratory is shown in figure 3.22.

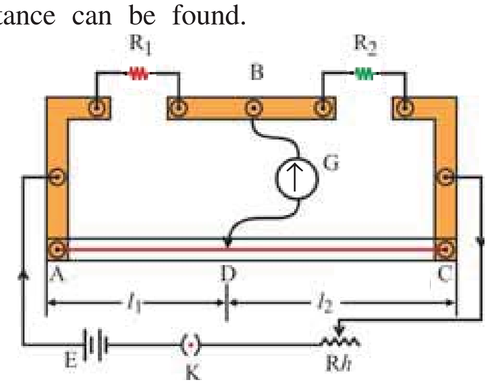


Figure 3.22 Meterbridge

Meterbridge consists of a constantan wire of length 1 m and of uniform cross sectional area which is used in place of resistors R_3 and R_4. This wire is stretched taut and clamped on a meterscale which is mounted on a wooden plateform. Two thick copper strips bent at right angles are connected at two ends A and C of the wire as shown in Figure. The connecting terminals are provided on this metallic strip (at the end points of a wire) where a battery can be connected. Another copper strip is fixed between two thick copperstrips in such a way that the metallic strip has two gaps across which resistors can be connected. One end of a sensitive galvanometer is connected to the copper strip midway (at point B) between the two gaps. The other end of the galvanometer is connected to a 'jockey' D which can slide over the wire to make electrical connection.

As shown in figure 3.22 unknown resistance R_1 is connected across one of the gaps and a standard known resistance R_2 is connected across the other gap.

For one value of known resistor R_2, the jockey is slided along the wire to get the position (say D) where the galvanometer will show no current. Point D is called balance (null) point.

Let the distance of the jockey from the end A at the balance point is AD = l_1 and the length of the wire DC is l_2, then from equation (3.12.3), we have,

$$\frac{R_1}{R_2} = \frac{\text{Resistance of Wire AD}}{\text{Resistance of Wire DC}}$$

$$\frac{R_1}{R_2} = \frac{l_1\rho}{l_2\rho} = \frac{l_1}{l_2} \qquad (3.12.4)$$

where, ρ = resistance per unit length of the wire

$$\frac{R_1}{R_2} = \frac{l_1}{(100-l_1)}$$

$$\therefore\ R_1 = R_2 \cdot \frac{l_1}{(100-l_1)} \qquad (3.12.5)$$

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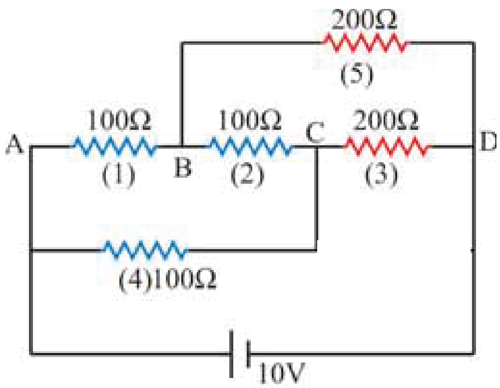
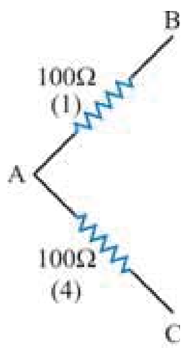
By choosing various values of known resistance R_2, the value of $\frac{l_1}{l_2}$ is calculated each time and the average value of unknown resistance R_1 can be found. The above method gives accurate valve of unknown resistance R_1 but this method is not useful for the measurement of small resistance.

Illustration 18 : Calculate the current flowing through the BC wire for the given circuit (a) shown here.

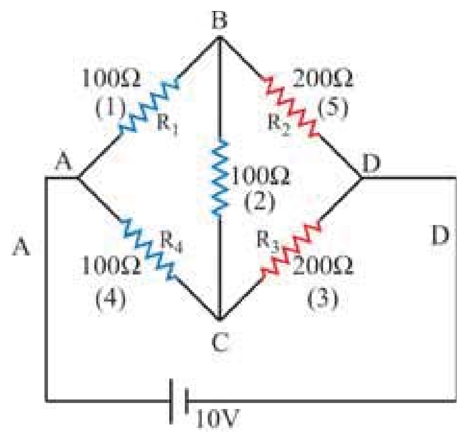


(a)

Solution : Kirchhoff's Law can be used to solve the above problem. We shall redraw the above circuit in a different way. The four points ABCD are common to two different resistances. Let us start from point A. At point A one end of resistances (1) and (4) is common. (Figure (b)). A 100 Ω resistance is connected between B and C. Resistance 200 Ω is connected between points C and D (resistance 3). A 200 Ω resistance present between B and D (resistance 5). The original circuit can be redrawn as shown in the figure (c).



(b)



(c)

In the above circuit a 10 V battery is connected between points A and D.

We form a close loop when we go from point A to B to D to C and back to A. A battery is connected between points A and D.

Under the balanced condition of Wheatstone bridge no current will flow from point B to C, since

$\frac{R_1}{R_2} = \frac{R_4}{R_3}$ condition is satisfied.

∴ No current flows through the resistor connected between the points B and C.

∴ $I_{BC} = 0$

Illustration 19 : 200 Ω resistor is connected in one of the gaps of the Meterbridge. Series combination of X Ω and 50 Ω resistors is connected in the second gap. Here unknown resistance X Ω is kept in a Heat bath at a certain temperature. Calculate the unknown resistance and its temperature if the balance point is obtained at 50 cm. The total length of the wire of the Meterbridge is equal to 1 meter. The resistance of the unknown resistance at 0 °C temperature is equal to 100 Ω $\alpha = 0.5 \times 10^{-3}$ °C^{-1} for the material of the X Ω resistors.

Solution : Here, we have $\frac{R_1}{R_2} = \frac{l_1}{l_2}$

$\therefore \frac{200}{X+50} = \frac{50}{50}$ $\qquad R_1 = 200\ \Omega$

$\therefore X = 150\ \Omega$ $\qquad R_2 = (X + 50)\Omega$

Now, $X = X_0[1 + \alpha(\theta - 0)]$ $\qquad l_1 = 50$ cm

$\therefore 150 = 100[1 + 5 \times 10^{-3}\theta]$ $\qquad l_2 = 100 - 50 = 50$ cm

$\therefore 1.5 = 1 + 5 \times 10^{-3}\theta$

$\therefore \theta = 100\ ^{\circ}C$

Note : We can understand from the above example that temperature of the resistor can be measured using wheatstone bridge. (The varying temperature of the resistor can also be measured.)

The thermometer can be constructed by knowing the relationship between resistance and temperature. Such a thermometer is known as **resistance thermometer**. The manufacturer of such a thermometer provides us with R $\rightarrow$ T graph. Presently thermometer with digital display is available. The resistance thermometer is an example of a transducer. In a transducer the physical quantity is converted into an electrical quantity or vice-versa.

3.13 Potentiometer

(A) The Requirement of Potentiometer : We have seen that the terminal voltage of a battery is given by,

$V = \varepsilon - Ir$ (3.13.1)

where ε = emf of battery

and r = internal resistance of a battery.

As shown in figure 3.23 if voltmeter used in the laboratory (table voltmeter) is connected across two terminals (between points a and b) of a battery, then it will measure the potential difference between two terminals of a battery or a terminal voltage (V).

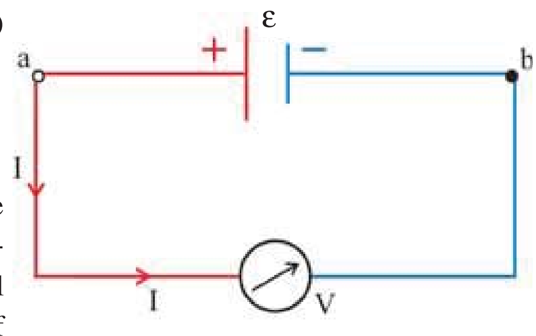


Figure 3.23

The equation (3.13.1) reduces to $V = \varepsilon$ when the internal resistance of a battery is zero (i.e. $r = 0$) or no current flows through the battery (i.e. $I = 0$). But the internal resistance (r) of a battery can never be zero. Therefore voltmeter can measure the emf (ε) of a battery only if no current is drawn from it. (i.e. $I = 0$ open circuit condition.)

The resistance of an ordinary voltmeter is approximately in the range of 5000 Ω to 6000 Ω. Hence a small amount of current flows through the battery when connected to voltmeter. **This means that the voltmeter measures only the terminal voltage (V) and not the emf (ε) of a battery.**

Thus, in order to measure the emf of a battery we have to design a new device in which open circuit condition ($I = 0$) is achieved. Such a device is called potentiometer.

Potentiometer is such a device in which one can obtain a continuously varying potential difference between any two points which can be measured simultaneously. This can be understood from the principle of potentiometer.

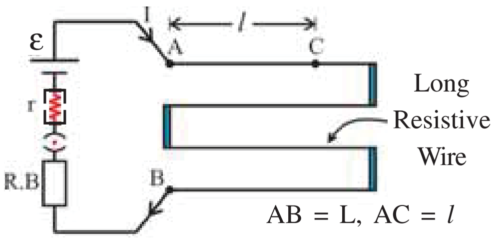


Figure 3.24 Principle of Potentiometer

(B) Principle of Potentiometer : As shown in figure 3.24 a battery of emf ε and having internal resistance r is connected in series with a resistance box R and a long resistive wire of uniform cross-sectional area (i.e. the resistance per unit length of the wire is same throughout the length of the wire.) It is to be noted that the resistance box R is not always necessary to connect.

(**Note :** In potentiometer a long piece of uniform wire few meters in length across which a battery is connected is clamped on a meterscale which is mounted on a wooden plateform.)

Let L be the length of the potentiometer wire AB and ρ be the resistance per unit length of the wire. Therefore the resistance of the wire AB = Lρ. If R is the resistance of the resistance box then the current flowing through wire AB can be given by Ohm's law as follows.

$$I = \frac{\varepsilon}{R + L\rho + r} \qquad (3.13.2)$$

If l = length of the wire from A to C then $l\rho$ = resistance of the AC part of wire,

Therefore, the potential difference between A and C is = $Il\rho$

This potential difference is denoted by V_l

$$\therefore \; V_l = Il\rho \qquad (3.13.3)$$

Substituting the value of I from equation (3.13.2) into equation (3.13.3),

$$V_l = \left(\frac{\varepsilon}{R + L\rho + r}\right) l\rho$$

$$\therefore \; V_l = \left(\frac{\varepsilon . \rho}{R + L\rho + r}\right) l \qquad (3.13.4)$$

$$\therefore \; V_l \propto l \qquad (3.13.5)$$

Principle : The potential difference between any two points of a potentiometer wire is directly proportional to the distance between that two points. By taking different values of l, different potential difference can be obtained. Points A and C of the wire behave as if they are positive and negative terminals of a battery. By changing the position of C (with the help of jockey) the emf of such a battery can be continuously varied.

From equation (3.13.4),

$$\sigma = \frac{V_l}{l} = \frac{\varepsilon . \rho}{R + L\rho + r} \qquad (3.13.6)$$

The potential difference per unit length of the wire $\frac{V_l}{l} = \sigma$ is called potential gradient.

Its unit is Vm^{-1}.

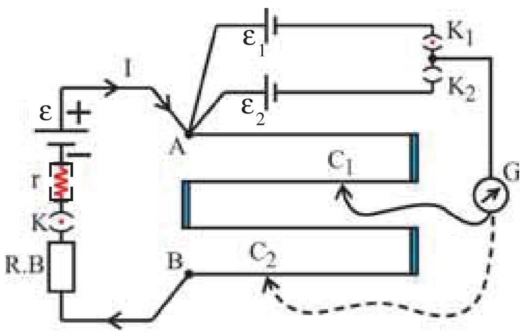
The sensitivity of the potentiometer depends on the potential gradient along the wire. Smaller the potential gradient, greater will be the sensitivity of potentiometer.

For a given V_{AB}, the sensitivity of a potentiometer can be increased by increasing the length of the potentiometer wire.

(C) Uses of Potentiometer :

(i) Comparison of emf's of two cells : Let ε_1 and ε_2 be the emf's of the two cells which are to be compared using potentiometer. For this purpose, the emf (ε) of the driver cell (main battery) in potentiometer should be greater than emf's of cells (ε_1 and ε_2) to be determined.

As shown in figure 3.25, firstly the positive terminal of cell ε_1 is connected to the end A of the potentiometer wire and the negative terminal of ε_1 is connected to jockey through a sensitive galvanometer. For this connection, plug key k_1 is inserted.



$AC_1 = l_1$, $AC_2 = l_2$

Figure 3.25 **Comparision of emfs of Two Cells**

The jockey is moved along the wire AB till the galvanometer shows no deflection. Let the position of the jockey be C_1. In this condition no current is flowing through cell ε_1 and hence its terminal voltage is equal to its emf (ε_1). Such a point on the wire is called **null point**. Suppose, null point C_1, is at a distance l_1 from point A of wire. In the balanced condition, potential difference between point A and C_1 of the wire should be equal to emf of cell ε_1.

From equation (3.13.4), we have

$$V_{AC_1} = \varepsilon_1 = \sigma l_1 \tag{3.13.7}$$

where, $\sigma = \left(\dfrac{\varepsilon.\rho}{R + L\rho + r}\right)$ respresents the potential gradient.

Now by inserting plug key K_2 battery ε_2 is connected in place of ε_1. The null point (C_2) is again obtained for cell ε_2 by sliding jockey on the wire. Let the balancing length be $AC_2 = l_2$, then,

$$V_{AC_2} = \varepsilon_2 = \sigma l_2 \tag{3.13.8}$$

Taking the ratio of equation (3.13.7) and (3.13.8), we get

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2} \tag{3.13.9}$$

Using the above equation the emfs of two cells can be compared.

In practice, the emf of a cell is determined by comparing it with the emf of a standard cell and equation (3.13.9) is employed.

One can obtain desired value of potential difference between any two points of the wire by choosing appropriate value of R from the resistance box. The potential difference of the order of 10^{-6}V (=1 μV) or of the order of 10^{-3}V(= 1mV) can be obtained. Thus, potentiometer can also be used to measure a very small emf.

Note : If two cells of emfs ε_1 and ε_2 are first connected in helping condition and then in opposing condition, the lengths of the null point is respectively l_3 and l_4, then,

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_3 + l_4}{l_3 - l_4} \tag{3.13.10}$$

(ii) To Determine the Internal Resistance of a Cell :

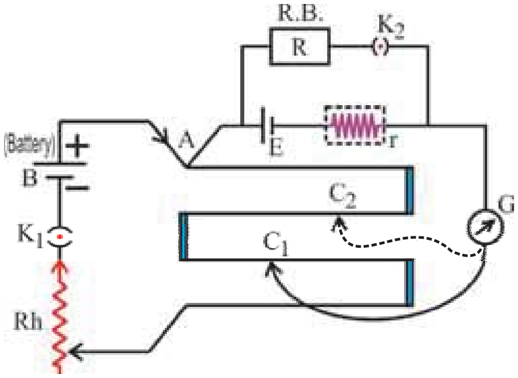


Figure 3.26 Internal Resistance of a Cell

We can also use a potentiometer to measure the internal resistance of a cell. For this as shown in figure 3.26 the cell (emf ε) whose internal resistance (r) is to be determined is connected across a small resistance box R through a key K_2.

The null point C_1 is obtained on the potentiometer wire when key K_2 is open. (i.e. when resistance box is not connected.) At this time there will be no flow of current through a cell (ε) which is called open circuit condition. If the null point C_1 is obtained at a distance l_1 from point A of the wire, then

$$V_{AC_1} = \varepsilon = \sigma l_1 \quad (3.13.11)$$

When key K_2 is closed, resistance box comes in the circuit. Null point C_2 is again obtained on the wire for an appropriate value of R. If the terminal voltage of a cell is V and null point is obtained at $AC_2 = l_2$,

$$V_{AC_2} = V = \sigma l_2 \quad (3.13.12)$$

$$\therefore \frac{\varepsilon}{V} = \frac{l_1}{l_2} \quad (3.13.13)$$

From Ohm's law $\varepsilon = I(R + r)$

and $V = IR$

$$\text{This gives, } \frac{\varepsilon}{V} = \frac{R+r}{R} \quad (3.13.14)$$

Using equation (3.13.14) into equation (3.13.13),

$$\frac{R+r}{R} = \frac{l_1}{l_2}$$

$$\therefore r = R\left(\frac{l_1}{l_2} - 1\right) \quad (3.13.15)$$

Using equation (3.13.15) we can find the internal resistance of a given cell.

3.14 Electrical Energy, Power : Joule's Law

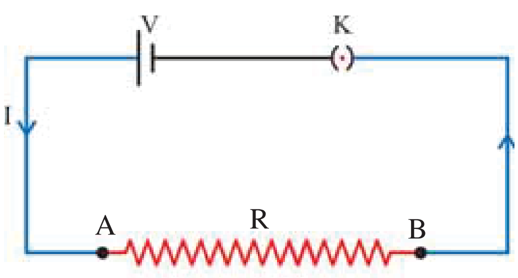


Figure 3.27

Consider figure 3.27 A battery having terminal voltage of V volt is connected to a resistance R and the circuit is completed. As explained above at end A of the resistor, energy of 1 C positive charge is V Joule. This energy per unit positive charge represents the electric potential at point A.

If the electric current is considered due to the motion of electrons (which is true in reality), it can be said that a unit negative charge possesses an electrical energy of V Joule at end B of the resistor.

We have also studied that when the electrons acquire drift velocity they experience collisions with the positive ions oscillating about their mean position, and the energy acquired by the electrons is partly transferred to the ions making their oscillations faster and more random. In Standard 11 we have already seen that heat energy is the kinetic energy associated with the random motion of constituent particles of a substance. Accordingly, this increase in the energy of oscillations of the ions due to collisions with electrons manifests as heat energy.

The heat energy released in a conductor on passing of an electric current is called the "Joule heat" and the effect is called the "Joule effect".

The p.d. of V volt applied between two ends of a conductor means that V joule energy is utilized when a unit charge passes through the conductor.

If Q coulomb charge passes through the conductor in t seconds, the electrical energy consumed in t second = heat energy produced during this time,

$$W = V\ Q \tag{3.14.1}$$

This is the heat energy produced in time t.

Let a steady current of I ampere be produced due to this charge then,

$$I = \frac{Q}{t}$$

$$\therefore\ Q = It$$

$$\therefore\ W = V\ I\ t \tag{3.14.2}$$

Now according to Ohm's law, V = IR

$$\therefore\ W = I^2Rt \tag{3.14.3}$$

$\therefore$ Electrical energy converted into the heat energy per unit time (power) is given by

$$P = I^2R \tag{3.14.4}$$

Here, R is the Ohmic resistance of the conductor, value of which does not depend upon V or I. Considering R as a constant, the heat energy produced per second.

$$P \propto I^2 \tag{3.14.5}$$

This equation is known as Joule's law.

Joule's Law : "The heat produced per unit time, on passing electric current through a conductor at a given temperature, is directly proportional to the square of the electric current."

The heat energy produced here is in joules.

We must know the relation between joule and calorie if we want to express heat energy in calories. Such a relation was given by Joule (James Prescott Joule, 1818–1889) according to which W = JH. Where W is in joule and H is in calorie. Here J is called joule's constant or mechanical equivalent of heat and its value is J = 4.2 J cal^{-1}.

$$\therefore\ H = \frac{I^2Rt(\text{joule})}{J(\text{Joule/cal})} = \frac{I^2Rt}{J}\ \text{cal} \tag{3.14.6}$$

3.15 Practical Applications of Joule Heating

Generation of heat on passing electric current through a conductor is an inevitable phenomenon. In most cases it is unwanted, as electrical energy gained by charges is wasted in the form of heat energy. This is known as **'Ohmic dissipation'** or **'Ohmic loss'**. For example, a considerable part of electrical power supplied to an electric motor used to pump water to overhead tank in our houses, is wasted in the form of heat. Moreover, when current is passed through a circuit, properties of some components in the circuit change due to heat produced. Long distance electric transmission is done at very high voltage to reduce this Ohmic loss.

Joule heat is useful in case of some domestic applications also. Usefulness of Joule heat will be immediately clear if you think of electrical appliances used such as electric iron, electric toaster, electric oven, electric kettle, room heater etc. Joule heat is also used in electric bulbs to produce light. When electric current is passed through the filament of a bulb, its temperature rises considerably due to the heat produced, and hence it emits light. The filament should consist of a metal of very high melting point (e.g. tungsten's melting point is 3380^{o} C). As far as possible this filament should be thermally isolated from the surrounding.

Note that only a very small fraction of electrical power supplied converts into light. Normally bulbs emit 1 Candela of light energy per 1 W electrical power consumed.

A very common application of Joule heat is fuse wires used in circuits (and in our houses.) A fuse consists of a piece of wire of metals having low melting point (such as aluminium, iron, lead etc.) and is connected in series with an appliance. If a current larger than a Pre-decided value flows, the fuse wire melts and breaks the circuit and thus protects the appliance.

Illustration 20 : Electric current divides among two resistors connected in parallel in such a way that the joule heat developed becomes minimum. Using this fact, obtain the equation of division of currents.

Solution : Suppose total current I divides into two parts among two resistances R_1 and R_2 connected in parallel. Let current passing through R_1 be I_1, then the current passing through R_2 will be $I_2 = I - I_1$. Joule heat produced in unit time in this case will be,

$$H = I_1^2R_1 + (I - I_1)^2R_2$$

For this heat to be minimum, we should have $\frac{dH}{dI_1} = 0$

$$\therefore \frac{dH}{dI_1} = 2I_1R_1 + 2(I - I_1)(-1)R_2 = 0$$

On simplifying, we get $I_1 = \frac{IR_2}{R_1+R_2}$

Also, $I_2 = I - I_1 = I - \frac{IR_2}{R_1+R_2}$ $\therefore I_2 = \frac{IR_1}{R_1+R_2}$

Note : How does electric current know that some resistance is low so that more of it should pass through it ! This is due to a fundamental principle of physics which you will study in future. That fundamental principle is reflected here.

Illustration 21 : When two resistors are connected with voltage V individually, the powers obtained are P_1 and P_2 respectively. Then,

(i) they are connected in series

(ii) they are connected in parallel

Shown that the product of powers in (i) and (ii) is P_1P_2.

Solution : Here suppose that the given resistances are R_1 and R_2. When they are connected separately,

$$P_1 = \frac{V^2}{R_1} \text{ and } P_2 = \frac{V^2}{R_2} \quad (1)$$

$$\therefore R_1 = \frac{V^2}{P_1} \text{ and } R_2 = \frac{V^2}{P_2} \quad (2)$$

Their combined resistance in series connection is, $R_1 + R_2$. This combination is connected with voltage V.

$\therefore$ The power in case of series connection, $P_s = \frac{V^2}{R_1+R_2}$.

Substituting values of R_1 and R_2 from equations (2),

$$P_s = \frac{V^2}{\frac{V^2}{P_1}+\frac{V^2}{P_2}} = \frac{P_1P_2}{P_1+P_2} \quad (3)$$

Equivalent resistance for their parallel connection $= \frac{R_1R_2}{R_1+R_2}$

$\therefore$ The power for this combination, $P_p = \frac{V^2}{\left(\frac{R_1R_2}{R_1+R_2}\right)} = \frac{V^2}{R_1R_2}(R_1 + R_2)$

Using equation (2) in above equation,

$$P_p = \frac{V^2\left(\frac{V^2}{P_1}+\frac{V^2}{P_2}\right)}{V^4\left(\frac{1}{P_1}\times\frac{1}{P_2}\right)}$$

$$\therefore P_p = \frac{P_1P_2\times(P_1+P_2)}{P_1P_2}$$

$$\therefore P_p = P_1 + P_2 \quad (4)$$

Note : As the voltage across both the resistors is the same in parallel connection, we could have obtained equation (4) directly also !

From equations (3) and (4),

$P_s \times P_p = P_1 \times P_2$

Illustration 22 : A battery having an emf ε and an internal resistance r is connected with a resistance R. Prove that the power in the external resistance R is maximum when $R = r$.

Solution : Power in the external resistance $P = I^2R$

$$\therefore P = \left(\frac{\varepsilon}{R+r}\right)^2 R$$

$$\therefore \frac{dP}{dR} = -\frac{2\varepsilon^2 R}{(R+r)^3} + \frac{\varepsilon^2}{(R+r)^2} = 0$$

(being the condition for maximum or minimum P)

$\therefore R = r$

(It can readily be shown by the second differentiation of P with respect to R, that for $r = R$ it is negative. This shows that for $r = R$, Power P is maximum.)

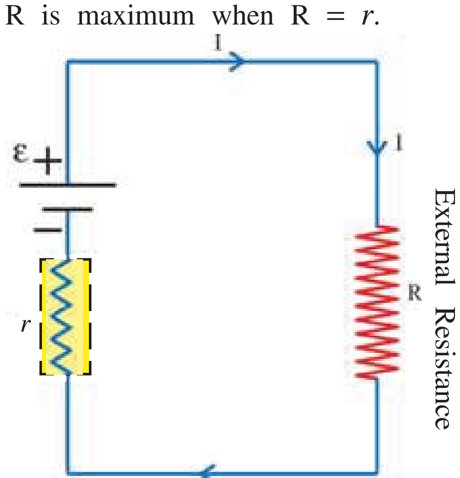


Figure 3.28

SUMMARY

1. **Electric Current :** Charges in motion constitute an electric current.
The amount of charge flowing per unit time across any cross-sectional area of a conductor held perpendicular to the direction of flow of charge is called current (I).

For a steady flow of charge, $I = \frac{Q}{t}$

If the rate of flow of charge varies with time,

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

2. **Electric Current Density :** It is the amount of electric current flowing (electric charge flowing per unit time) per unit cross–sectional area perpendicular to the current at that point. If a cross-sectional area is not perpendicular to the current, then the current density at any point,

$$J = \frac{dI}{da\cos\theta}$$

$$\therefore\ dI = J da\ cos\theta = \vec{J} \cdot d\vec{a}$$

If the cross-sectional area is perpendicular to the current and if J is constant over the entire cross-section then,

$$I = \int_a \vec{J} \cdot d\vec{a} = J\int da$$

$$\therefore\ I = JA$$

$$\therefore\ J = \frac{I}{A}$$

3. **Ohm's Law :** "Under a definite physical condition (e.g. constant temperature) the current (I) flowing through the conductor is directly proportional to the potential difference (V) applied across its ends."

From this, $\frac{V}{I} = R$ or $V = IR$

The reciprocal of a resistance i.e. $\frac{1}{R}$ is called the conductance of the material.

4. **Resistivity :** The resistance of a conductor,

$$R = \rho . \frac{l}{A}$$

$$\therefore\ \text{resistivity } \rho = \frac{RA}{l}$$

The reciprocal of a resistivity is called conductivity of the material.

$$\therefore\ \text{Conductivity } \sigma = \frac{1}{\rho}$$

5. **Drift Velocity and Relaxation Time :** The velocity of electron corresponding to the effective (drift) displacement of the electron in the presence of electric field is known as the drift velocity.
Relaxation Time : The average time between two successive collisions of the electron with the ions is called relaxation time (τ).
The drift velocity achieved by the electron during the relaxation time (τ) is,

$$v_d = a\tau = \left(\frac{E \cdot e}{m}\right)\tau$$

The relation between the drift velocity and current is, $I = nAv_d e$.

The relation between the drift velocity and current density is, $J = \frac{I}{A} = nev_d$.

6. The Relation between the Resistivity (ρ) and Conductivity (σ)

$$\sigma = \frac{ne^2\tau}{m} \text{ and } \rho = \frac{m}{ne^2\tau}$$

7. **Mobility :** It is the drift velocity of a charge carrier per unit electric field intensity.

$$\mu = \frac{v_d}{E} = \frac{\sigma}{ne}$$

$\therefore\ \sigma = ne\mu$

The conductivity of a semiconductor,

$$\sigma = n_e e\mu_e + n_h e\mu_h$$

8. **Temperature dependence of resistivity :**

The relation between the resistivity of a metallic conductor and temperature is given by the following empirical formula.

$$\rho_\theta = \rho_{\theta_0}[1 + \propto(\theta - \theta_0)]$$

where, θ_0 = reference temperature

For a resistance,

$$R_\theta = R_{\theta_0}[1 + \propto(\theta - \theta_0)]$$

$\propto$ = temperature coefficient of resistance

For metals $\propto$ is positive i.e., resistivity of metals increases with the increase in temperature.

For semiconductors $\propto$ is negative i.e., their resistivity decreases with the increase in temperature.

9. **Super Conductivity :** "The resistance of certain materials reduces to almost zero, when its temperature is lowered below a certain definite temperature (which is known as critical temperature T_C). The material in this state is known as superconductor and this phenomena is known as a superconductivity.

10. **The emf of a Cell and Terminal Voltage :** When unit positive charge is driven from negative terminal to the positive terminal due to non-electrical forces, the energy gained by the charge (or work done by the non-electrical forces) is called an emf (ε) of a battery.

The potential difference between the two terminals of a battery is called the terminal voltage (V).

The terminal voltage of a battery is, $V = \varepsilon - Ir$

11. **Secondary Cell :** The cells which can be restored to original condition by reversing chemical processes (i.e. by recharging) are called secondary cells. e.g. lead accumulator.

12. **Charging :** If the secondary cell is connected to some other source of larger emf, current may enter the cell at the positive terminal and leave it at the negative terminal. The electrical energy is then converted into chemical energy. This is called charging of the cell.

For the charging of a lead storage cell (lead accumulator),

$$VIt = \varepsilon It + I^2Rt + I^2rt \text{ and } I = \frac{V - \varepsilon}{r + R}$$

13. **Junction or Branch Point :** It is the point in a network at which more than two conductors (minimum three) meet.

14. **Loop :** A closed circuit formed by conductors is known as loop.

15. **Kirchoff's Rules :**

First rule : "The algebraic sum of all the electric currents meeting at the junction is zero."

$\therefore \ \Sigma I = 0$

Second rule : "For any closed loop the algebraic sum of the products of resistances and the respective currents flowing through them is equal to the algebraic sum of the emfs applied along the loop."

$\Sigma IR = \Sigma \varepsilon$

16. **Connections of Resistors**

Series Connection :

$R_s = R_1 + R_2 + R_3 + \ldots\ldots + R_n$

where, R_s = Equivalent resistance of n resistors connected in series.

Parallel connection :

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_n}$$

where, R_P = Equivalent resistance of n resistors connected in parallel.

17. **Series Connection of Cells :** For the series connections of two cells of emfs ε_1 and ε_2 and internal resistances r_1 and r_2,

$$I = \frac{\varepsilon_1 + \varepsilon_2}{R + (r_1 + r_2)} = \frac{\varepsilon_{eq}}{R + r_{eq}}$$

where, I = Current flowing through the external resistance R connected across the series connection.

Equivalent emf $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$

Equivalent internal resistance $r_{eq} = r_1 + r_2$

18. **Parallel Connection of Cells :** If two cells of emfs ε_1 and ε_2 and internal resistances r_1 and r_2 are connected in parallel, then we have,

$$I = \frac{\dfrac{\varepsilon_1}{r_1} + \dfrac{\varepsilon_2}{r_2}}{1 + \dfrac{R}{r_1} + \dfrac{R}{r_2}} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{R(r_1 + r_2) + r_1 r_2}$$

$$\therefore \ I = \frac{\dfrac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{(r_1 + r_2)}}{R + \dfrac{r_1 r_2}{(r_1 + r_2)}} = \frac{\varepsilon_{eq}}{R + r_{eq}}$$

Equivalent emf $\varepsilon_{eq} = \dfrac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$

Equivalent internal resistance $r_{eq} = \dfrac{r_1 r_2}{r_1 + r_2}$

19. Wheatstone Bridge : In the balanced condition of Wheatstone bridge,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

20. Potentiometer : It is a device in which one can obtain a continuously varying potential difference between any two points which can be measured simultaneously.

Principle : The potential difference between any two points of a potentiometer wire is directly proportional to the distance between that two points.

$\therefore\ V_l \propto l$

$$V_l = \left(\frac{\varepsilon.\rho}{R + L\rho + r}\right) \cdot l$$

where, $\sigma = \frac{V_l}{l} = \left(\frac{\varepsilon.\rho}{R + L\rho + r}\right)$ = Potential gradient

21. Joule Effect : "The heat energy released in a conductor on passing an electric current is called the 'Joule heat' and this effect is called the 'Joule effect' ".

Joule heat $W = I^2Rt$ (Joule)

$$H = \frac{I^2Rt}{J} \text{ (cal)}$$

Electrical energy consumed per unit time or heat energy produced per unit time i.e. electric power

$P = I^2R$

$P \propto I^2$

Joule's Law : "The heat energy produced per unit time, on passing electric current through a conductor at a given temperature, is directly proportional to the square of the electric current.

22. Ohmic loss : On passing electric current through a conductor, an electrical energy gained by charges is wasted in the form of heat energy. This is known as "Ohmic loss".

EXERCISE

For the following statements choose the correct option from the given options :

1. In a hydrogen atom, the electron is moving in a circular orbit of radius 5.3×10^{-11} m with a constant speed of 2.2×10^6 ms^{-1}. The electric current formed due to the motion of electron is ....

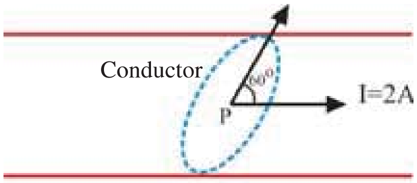
   (A) 1.12 A (B) 1.06 mA (C) 1.06 A (D) 1.12 mA

2. A ring of radius R and linear charge density λ on its surface is performing rotational motion about an axis perpendicualr to its plane. If the angular velocity of the ring is ω, how much current is constituted by the ring ?

   (A) $R\omega\lambda$ (B) $R^2\omega\lambda$ (C) $R\omega^2\lambda$ (D) $R\omega\lambda^2$

3. A cell supplies a current of 0.9 A through a 2 Ω resistor and current of 0.3 A through a 7 Ω resistor. What is the internal resistance of the cell ?

   (A) 0.5 Ω (B) 1.0 Ω (C) 1.2 Ω (D) 2.0 Ω

4. 

The cross-sectional area of the plane shown in the figure is equal to 1 cm^2. 2A current flows through a conductor. The current density at point P in the conductor will be ....

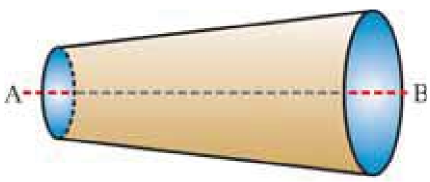
(A) $\frac{4}{\sqrt{3}} \times 10^4 Am^{-2}$ (B) $\frac{\sqrt{3}}{2} \times 10^4 Am^{-2}$

(C) $\frac{\sqrt{3}}{2} \times 10^{-4} Am^{-2}$ (D) $\frac{\sqrt{3}}{4} \times 10^{-4} Am^{-2}$

5. A current density of 2.5 Am^{-2} is found to exist in a conductor when an electric field of $5 \times 10^{-8} Vm^{-1}$ is applied across it. The resistivity of a conductor is ....

(A) 1×10^{-8} Ωm (B) 2×10^{-8} Ωm

(C) 0.5×10^{-8} Ωm (D) 12.5×10^{-8} Ωm

6. 

A wire has a non-uniform cross-section as shown in figure. A steady current is flowing through it. Then the drift speed of the electrons while going from A to B ....

(A) is constant throughout the wire (B) decreases

(C) increases (D) varies randomly

7. A resistive wire is stretched till its length is increased by 100 %. Due to the consequent decrease in diameter, the change in the resistance of a stretched wire will be ....

(A) 300 % (B) 200 % (C) 100 % (D) 50 %

8. At what temperature would the resistance of a copper conductor be double its resistance at 0°C ? Given ∝ for copper = 3.9×10^{-3} $°C^{-1}$

(A) 256.4°C (B) 512.8°C (C) 100°C (D) 256.4 K

9. You are given n identical resistors each of resistance $r\Omega$. First they are connected in such a way that the possible minimum value of resistance is obtained. Then they are connected in a way to get maximum possible resistance. The ratio of minimum and maximum resistance obtained in these ways is ....

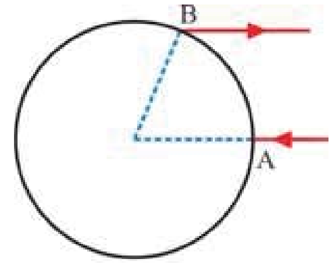
(A) $\frac{1}{n}$ (B) n (C) n^2 (D) $\frac{1}{n^2}$

10. P and Q are two points on a uniform ring of resistance R. O is the centre of the ring. If the part PQ of the ring subtends an angle θ at the centre O of the ring (i.e. ∠POQ = θ), the equivalent resistance of the ring between the points P and Q will be ....

[radius of the ring = r and resistance per unit length of the ring = ρ]

(A) $\frac{R\theta}{4\pi^2}(2\pi - \theta)$ (B) $R\left(1-\frac{\theta}{2\pi}\right)$ (C) $\frac{R\theta}{2\pi}$ (D) $R\left(\frac{2\pi-\theta}{4\pi}\right)$

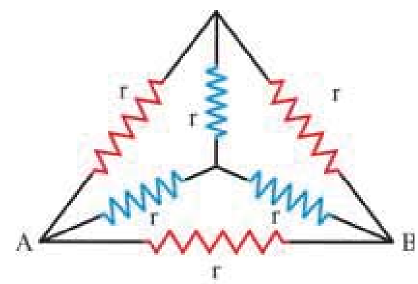
11. A wire in a circular shape has 10 Ω resistance. The resistance of wire per 1 m length is 1 Ω. If the equivalent resistance betwen A and B is 2.4 Ω, then the length of the chord AB will be equal to .... meter.



(A) 2.4 (B) 4

(C) 4.8 (D) 6

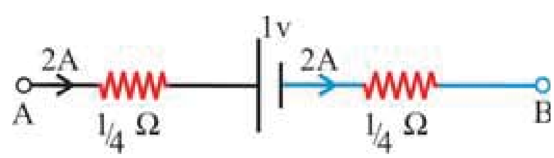
12. In the circuit shown in figure, what will be the effective resistance between points A and B ?



(A) r (B) $\frac{r}{2}$

(C) $\frac{r}{3}$ (D) $2r$

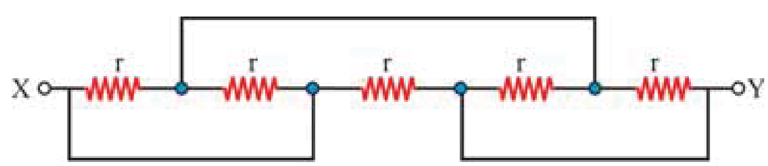
13. Figure shows a part of a closed circuit. If the current flowing through it is 2A, what will be the potential difference between points A and B ?



(A) +2 V (B) +1 V

(C) −2 V (D) −1 V

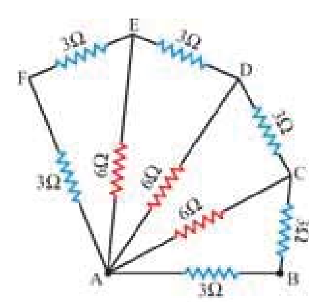
14. In the network shown in figure, the equivalent resistance between points X and Y will be ....



(A) r (B) $\frac{r}{2}$

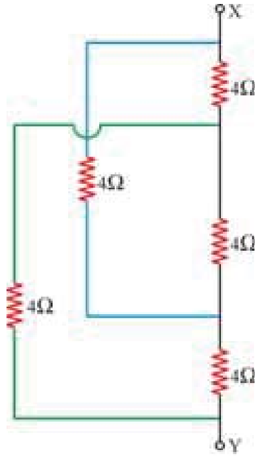
(C) $2r$ (D) $\frac{r}{3}$

15. The effective resistance between points A and B of the network shown in figure ....



(A) 2 Ω (B) 3 Ω

(C) 6 Ω (D) 12 Ω

16. 

The equivalent resistance between points X and Y in the following figure is ....

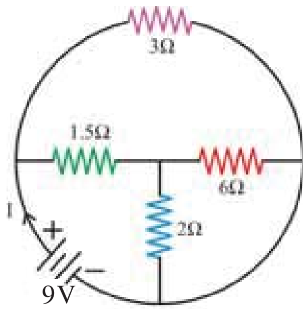
(A) 4 Ω

(B) 2 Ω

(C) 1 Ω

(D) 3 Ω.

17. What is the total current supplied by the battery to the circuit shown in the adjoing figure ?



(A) 2 A

(B) 4 A

(C) 6 A

(D) 9 A

18. A uniform conductor of resistance R is cut into 20 equal pieces. Half of them are joined in series and the remaining half of them are connected in parallel. If the two combinations are joined in series, the effective resistance of all the pieces is :

(A) R (B) $\frac{R}{2}$ (C) $\frac{101R}{200}$ (D) $\frac{201R}{200}$

19. What will be the time taken by electron to move with drift velocity from one end to the other end of copper conductor 3 metre long and carrying a current of 3 A ?

[The cross-sectional area of the conductor = 2×10^{-6} m^2 and electron density for copper $n = 8.5 \times 10^{28}$ m^{-3}]

(A) 2.72×10^3s (B) 2.72×10^4s (C) 2.72s (D) 2.72×10^{-4}s

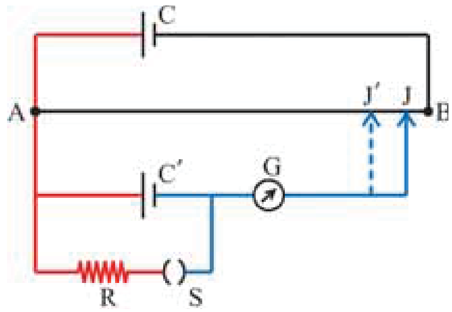
20. Masses of three wires of copper are in the ratio 5 : 3 : 1 and their lengths are in the ratio 1 : 3 : 5. The ratio of their electrical resistances are ....

(A) 5 : 3 : 1 (B) $\sqrt{125}$: 15 : 1 (C) 1 : 15 : 125 (D) 1 : 3 : 5

21. The resistance of a 10 m long potentiometer wire is 20 Ω. It is connected in series with a 3 V battery and 10 Ω resistor. The potential difference between two points separated by distance 30 cm is equal to ....

(A) 0.02 V (B) 0.06 V (C) 0.1 V (D) 1.2 V

22. In the potentiometer circuit shown in figure, the balance length AJ = 60 cm when switch S is open. When switch S is closed and the value of R = 5 Ω, the balance length AJ' = 50 cm. What is the internal resistance of cell C' ?



(A) 0.5 Ω (B) 1 Ω

(C) 1.5 Ω (D) 0.1 Ω

23. n identical cells each of emf ε and internal resistance r are connected in parallel with resistor R. The current flowing through resistor R is, ...

(A) $\frac{n\varepsilon}{R+nr}$ (B) $\frac{n\varepsilon}{nR+r}$ (C) $\frac{\varepsilon}{R+r}$ (D) $\frac{\varepsilon}{nR+r}$

24. A wire is uniformly stretched to make its area of cross-section $\frac{1}{n}$ th times (n > 0). What will be its new resistance ?

(A) $\frac{1}{n^2}$ times (B) n^2 times (C) $\frac{1}{n}$ times (D) n times

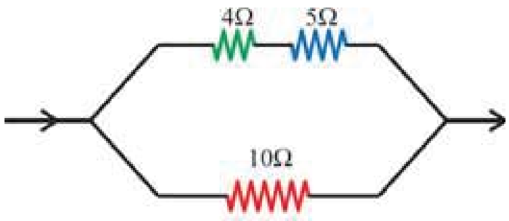
25. If the current in an electric bulb increases by 1 %, what will be the change in the power of a bulb ?

[Assume that the resistance of the filament of a bulb remains constant.]

(A) increases by 1 % (B) decreases by 1 %

(C) increases by 2 % (D) decreases by 2 %

26. In the following circuit if the heat evolved in the 10 Ohm resistor is 10 cal/s. The heat evolved in the 4 Ω resistor is approximately .... cal/s.



(A) 4 (B) 5

(C) 10 (D) 20

27. Two bulbs of 220 V and 100 W are first connected in series and then in parallel with a supply of 220 V. Total power in both the cases will be ....

(A) 50 W, 100 W (B) 100 W, 50 W

(C) 200 W, 150 W (D) 50 W, 200 W

ANSWERS

1. (B)	2. (A)	3. (A)	4. (A)	5. (B)	6. (B)
7. (A)	8. (A)	9. (D)	10. (A)	11. (B)	12. (B)
13. (A)	14. (B)	15. (A)	16. (A)	17. (C)	18. (C)
19. (B)	20. (C)	21. (B)	22. (B)	23. (B)	24. (B)
25. (C)	26. (B)	27. (D)			

Answer the following questions in brief :

1. Why is an electric current density defined ?
2. How many electrons are present in 1 nenocoulomb (1 *nc*) charge ?
3. The internal resistance of a battery of 2 V terminal voltage is 0.2 Ω. If the current flowing through the battery is 0.5 A, what will be the emf of battery ?
4. Define the mobility of a charge carrier.
5. Give the relation between the drift velocity and current flowing through the conductor.
6. The drift velocity of the electron is v when current I is flowing through a conductor of radius r. What will be the drift velocity of electron in a similar conductor of radius 2 r if the same current (I) is flowing through it ?
7. Give the empirical formula showing the relation between the resistivity of a metallic conductor and temperature.
8. Resistance of a wire is 10 Ω. What will be the required change in the length of it to increase its resistance to 1000 Ω ?
9. State the law of conservation of charge.
10. Kirchoff's second law is the consequence of which law ?
11. Why the current in a superconductor can be sustained over a long interval of time ?
12. Why the emf (ε) of a battery cannot be measured using table voltmeter ?
13. State the principle of potentiometer.
14. Write Joule's law.
15. Give the examples of "Ohmic loss".
16. What are the changes made in the temperature of a semiconductor in order to reduce its conductivity.

Answer the following questions :

1. Define an electric current density. Clarify the differences between the electric current and electric current density.
2. Explain the emf of a battery. When the battery is said to be in "open circuit condition" ?
3. Write Ohm's law. Explain the I–V characteristics for a conductor obeying Ohm's law.
4. Using necessary diagram explain the drift velocity of electron in a conductor in the presence of external electric field.
5. Explain the mobility of a charge carrier and obtain the formula for the conductivity of a semiconductor.
6. Obtain the relation between the drift velocity and current density.
7. Accepting the single valuedness of electric potential in an appropriate closed circuit, derive Kirchoff's second rule by drawing necessary circuit diagram.
8. Deduce the principle of potentiometer with the help of necessary circuit diagram.
9. Explain the method of finding the internal resistance of a cell using potentiometer.
10. Giving appropriate circuit diagram, describe the charging process of a lead storage cell (accumulator). Obtain the formula for the charging current.
11. Derive the expression to find the unknown resistance in the balanced condition of Wheatstone bridge.
12. Obtain an expression for the equivalent emf and equivalent internal resistance in the parallel connection of two cells.
13. State the limitations of Ohm's law.
14. Write notes on superconductivity.
15. What is Joule heat and Joule effect ? Obtain Joule's law for Joule heating.

Solve the following examples :

1. A stream of electron moves from the electron gun to a screen of a television. The electric current of the 10 μ A is constituted. Calculate the number of electrons striking the screen at every second. Also calculate magnitude of the charges striking the screen in one minute.

[**Ans. :** $n = 6.25 \times 10^{13}$ electron/Sec., Q = − 600 μC]

2. An electron in the hydrogen atom is revolving around a proton with a speed of $\frac{e^2}{\hbar}$. The radius of the electron orbit is equal to $\frac{\hbar^2}{me^2}$. Obtain the formula for the electric current in the above case. Mass of the electron = m, charge on electron = e. (**Hint :** $\hbar = \frac{h}{2\pi}$)

[**Ans. :** $I = \frac{4\pi^2 me^5}{h^3}$]

3. A current of 1.0 A is flowing through a copper wire of length 0.1 m and corss-section 1.0×10^{-6} m^2.

(i) If the resistivity of copper be 1.7×10^{-8} Ω m, calculate the potential difference across the ends of a wire.

(ii) Determine the drift velocity of electrons.

[Density of copper = 8.9×10^3 kg m^{-3}, valancy of Cu = 1, atomic weight of copper = 63.5 g mol^{-1}, $N_A = 6.02 \times 10^{23}$ mol^{-1}]

[**Ans. :** V = 1.7×10^{-3} V and $v_d = 7.4 \times 10^{-5}$ ms^{-1}]

4. An n-type semi-conductor has 4×10^{-3} meter width, 25×10^{-5} metre thickness and 6×10^{-2} metre length. 4.8 mA current is flowing through it. Here voltage is applied parallel to the length of the semi-conductor. Calculate the current density. The density of the free electron is equal to 10^{22} m^{-3}. What will be the time taken by the electron across the length of the semi-conductor ?

[**Ans. :** 4.8×10^3 Am^{-2}, 2×10^{-2} s]

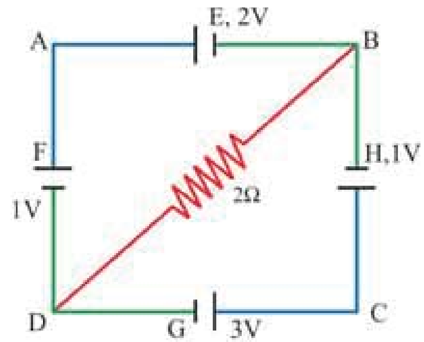
5. A cylindrical wire is stretched to increase its length by 10%. Calculate the percentage increase in resistance. [**Ans. :** 21%]

6. One conducting wire of length 1 m is cut into two unequal part P and Q respectively. Now, part P is stretched to double its length. Let the modified wire be R. If the resistance of the R and Q wires are same, then calculate the length of P and Q wires.

[**Ans. :** Length of the P wire is $\frac{1}{5}$ meter, Length of Q wire is $\frac{4}{5}$ meter]

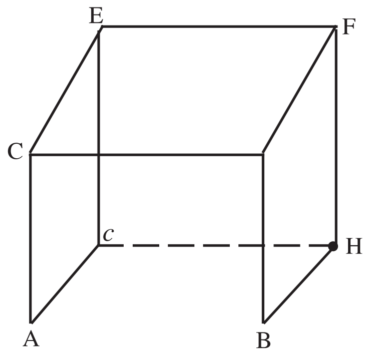
7. The resistance of one aluminium and one copper wires, having identical lengths is equal. Which of the two wires will be lighter ? $\rho_{Al} = 2.63 \times 10^{-8}$ Ω m, $\rho_{Cu} = 1.72 \times 10^{-8}$ Ω m the density of the aluminium is 2.7×10^3 kg m^{-3} and density of cooper is 8.9×10^3 kg m^{-3}.

[**Ans. :** aluminium]

8. 

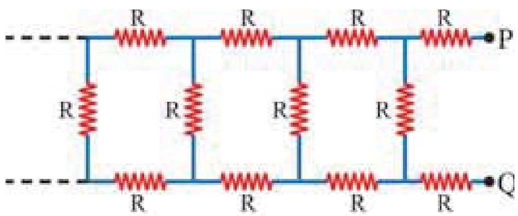
The emf of the batteries E, F, G and H are 2V, 1V, 3V and 1V respectively. Their internal resistance are respectively 2 Ω, 1 Ω, 3 Ω respectively. Calculate p.d. between B and D.

[**Ans. :** $\frac{2}{13}$ V]

9. 

Find the effective resistance between the points A and B in the network given below. All wires in the network have the same resistance '*r*' Ohm.

[**Ans. :** $\frac{7r}{5}$ Ohm]

10. 

Consider an infinite network as shown in the figure. The resistance of each of the wires of the network is equal to R. Calculate the resultant resistance between points P and Q.

[**Ans. : R (1 + $\sqrt{3}$)**]

11. The length of a potentiometer wire is 200 cm. For a given cell, the null point is obtained at 80 cm. What will be the length of wire required for balancing the cell if the length of the same wire is made 300 cm ? [**Ans. :** 120 cm]

12. A battery having an emf of 12 volt and an internal resistance of 2 Ω is connected to another battery having an emf of 18 volt and an internal resistance of 2 Ω in such a way that they are opposing each other and the circuit is closed. Calculate the following :

(1) current flowing in the circuit.
(2) electrical power in the two batteries.
(3) terminal voltage of the two batteries.
(4) electric power consumed in the batteries.

[**Ans. :** (1) 15 A (2) 18 W, 27 W (3) 15 V, 15 V (4) 4.5 W, 4.5 W]

13. An electric kettle has two heating coils. When one of the coils is switched on, a given quantity of water in the kettle starts boiling in 6 minutes. When the other coil only is switched on, then the same amount of water starts boiling in 8 minutes. If the two coils are switched on in parallel how much time will the same amount of water take to boil ? Each time the voltage applied is the same. [**Ans. :** 3.43 min]

14. Two wires which are made of the same material have the same cross-sectional area, but different lengths l_1 and l_2. Prove that if they are used as fuse wires, they will melt for the same value of the current flowing through them, in the same time.

15. A and B are two electric bulbs with their ratings respectively 40 W, 110 V and 100 W and 110 V. Find their respective filament resistances. If the bulbs are connected in series with a supply of 220 V, which bulb will fuse ? [**Ans. :** R_A = 302.5 Ω, R_B = 121 Ω, bulb A]

4

Magnetic Effects of Electric Current

4.1 Introduction

Branches of electricity and magnetism have been known for more than 2000 years. Danish physicist Oersted's observation and contributions given by Rowland, Faraday, Maxwell and Lorentz, unified these two branches, initially developed independently.

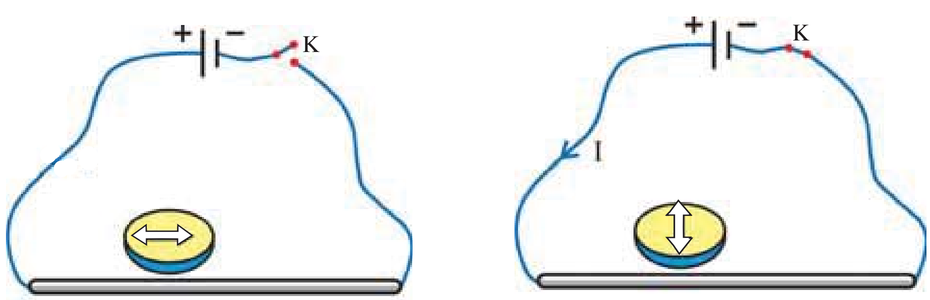
New concept was developed when the Laws obtained from the experimental studies of electricity and magnetism were presented mathematically and led to fundamental unification of these two branches. This helped in understanding nature of light and production of electromagnetic waves and its propagation become possible. As a result of this revolution is created in communication.

The branch of physics which envelops a comprehensive study of electricity and magnetism is called **electrodynamics**. In the modern technology of communication electrodynamics is of prime importance.

In the present chapter we will study, magnetic field produced due to electric current, force on electric charge moving in the magnetic field, force on current carrying conductor placed in magnetic field, cyclotron, galvanometer etc.

4.2 Oersted'd Observation

Some experimental observations are involved in the development process of the study of electricity and magnetism. One of theses observations is the Danish physicist Oerted's observation. In the year 1819 A.D. he made (Hans Christian Oersted 1771–1851) the following observation. he was a school teacher in Denmark.



(a) (b)

Figure 4.1 Oersted's Observation

Arrange the conductor (wire) parrallel to the magnetic needle such that it remains below the wire as shown in figure 4.1(a). On completing electric circuit shown in figure 4.1(a) current passes through the conducting wire and magnetic needle gets deflected and becomes perpendicular to the conducting wire see figure 4.1(b).

Thus in this observation of the experiment, he noted that when electric current passes through the conducting wire magnetic field is produced arround it.

This observation was presented to the French Academy by scientist Arago on 11th September 1820.

4.3 Biot–Savart's Law

When Biot and Savart, in Paris, came to know about Oested's above mentioned discovery, they, from the analysis of experimental studies, presented a Law for magnetic field produced due to electric current element in the following form.

The intensity of magnetic field due to an electric current element $I\vec{dl}$ at a point having position vector $\vec{r}$ with respect to the electric current element is given by the formula.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\hat{r}}{r^2} \qquad (4.3.1)$$

Here, $I\vec{dl}$ = Current element i.e. the product of electric current and length of line element $\vec{dl}$ of a conductor of very small length

μ_0 = magnetic permeability of vacuum

$= 4\pi \times 10^{-7}$ tesla meter ampere^{-1} (T m A^{-1})

$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$ unit vector along the direction of $\vec{r}$

Equation (4.3.1) can also be written as

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\vec{r}}{r^3} \qquad (4.3.2)$$

From equation (4.3.1) $|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{Idl\,sin\theta}{r^2}$ (4.3.3.)

Where θ is the angle between $\vec{dl}$ and $\vec{r}$.

Explanation : Consider a current currying conducting wire of any arbitrary shape as shown in figure 4.2. Suppose we wish to find magnetic field produced due to this current carrying conductor at any point P.

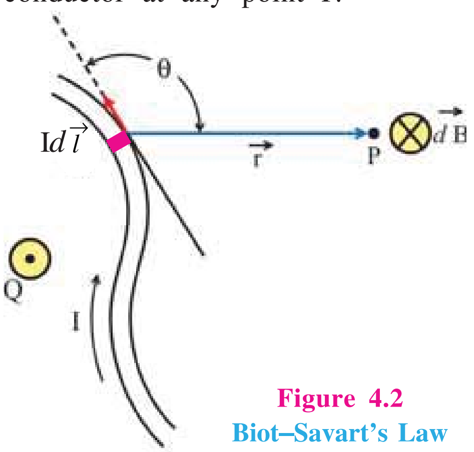


Figure 4.2
Biot–Savart's Law

We can think of the wire to be consisting of line elements dl_1, dl_2 ..... dl_n of infinitesimal lengths. Here, each element is so small that it can be locally considered straight and parallel to the direction of electric current. One such line element is shown in Figure 4.2 by $\vec{dl}$. $\vec{r}$ is the position vector of point P with respect to the current element $I\vec{dl}$. Intensity of magnetic field ($d\vec{B}$) at point P, due to this current element, can be found using equation (4.3.1).

Direction of $d\vec{B}$ is perpendicular to the plane formed by $\vec{dl}$ and $\vec{r}$ given by right hand screw rule. As $\vec{dl}$ and $\vec{r}$ taken in the plane of page of the book, the direction of $d\vec{B}$ at point

P is perpendicular to the plane of page of the book and going inside it shown by symbol ⊗ (As shown in the Figure, the direction of the magnetic field at Q is perpendicular to the plane of the page of the book towards the observer, and is shown by the symbol ⊙.)

To find the total magnetic field at point P, we will have to take the vector sum of magnetic field at P due to various current elements. As the current elements are continuous, the vector addition can be written as a line integral, as under.

$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \vec{r}}{r^2} \quad \text{or} \tag{4.3.4}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \vec{r}}{r^3} \tag{4.3.5}$$

Here, the line integral is taken over the entire path formed by the conducting wire.

Note that Biot-Savart's Law is an inverse square Law like Coulomb's Law and Newton's universal Law of gravitation.

The use of Biot-Savart's Law becomes simple in case of current carrying conductor of a simple geometrical shape.

Here, it is clear for the straight current currying conductor kept perpendicular to a plane, magnetic field at the equidistance point in this plane from the conductor will be same. That is as shown in the figure 4.3 magnetic field is equal of every point on the circumference of circle at radius OP and is along the tangent. For finding the direction of the magnetic field right hand thumb rule is as follows.

Hold the wire in such a way that the thumb is in the direction of electric current, the fingers encircling the wire indicate the direction of magnetic field as shown in figure 4.3.

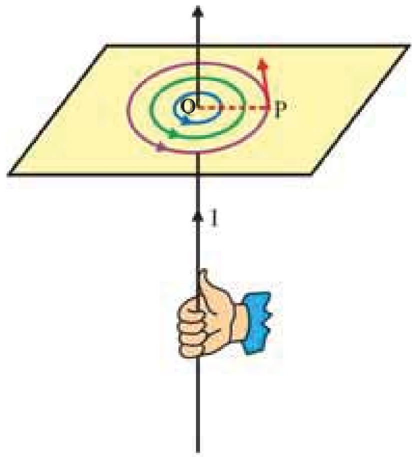


Figure 4.3 Right Hand Thumb Rule

4.4 Magnetic Field at a Point on the Axis of a Circular Ring Carrying Current

Consider a ring of thin wire carrying current I as shown in figure 4.4. Its radius is a. X-axis is taken along the axis of the ring. Suppose a point P is at a distance x from the centre of the ring on the axis of the ring.

Let the position vector of point P with respect to an element $\vec{dl}$ of wire be $\vec{r}$. The magnetic field $d\vec{B}$ at point P due to the current element $I\vec{dl}$ is in a direction perpendicular to the plane formed by $\vec{dl}$ and $\vec{r}$.

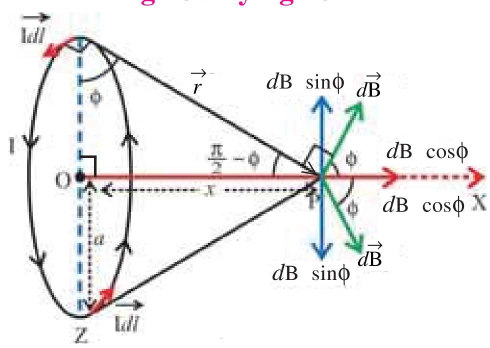


Figure 4.4 Magnetic Field Produced due to Circular Ring

Two mutually perpendicular components of this field $d\vec{B}$ are (1) a component $dB\cos\phi$ parallel to the X-axis and (2) a component $dB\sin\phi$ perpendicular to the X-axis. One thing is clear from the Figure 4.4 that when vector sum of magnetic field due to all such elements are considered, component $dB\sin\phi$ due to the diametrically opposite elements, which are in mutually opposite directions, will nullify each other.

Hence all axial components $dB\cos\phi$ will be in the X-direction and can be added together. Using Biot-Savart's Law

$$|d\vec{B}| = \left|\frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\vec{r}}{r^3}\right| = \frac{\mu_0}{4\pi}\frac{I\,dl\,r\sin\theta}{r^3} = \frac{\mu_0 I\,dl\,\sin\theta}{r^2}$$

where θ is angle between $\vec{dl}$ and $\vec{r}$.

But $\vec{dl} \perp \vec{r}$ $\therefore$ $\sin\theta = \sin\frac{\pi}{2} = 1$

$$\therefore\ |d\vec{B}| = \frac{\mu_0}{4\pi}\cdot\frac{I\,dl}{r^2} \tag{4.4.1}$$

Now, point P is at a distance x from centre of the circular ring.

$$\text{Hence } dB(x) = |d\vec{B}|\cos\phi \tag{4.4.2}$$

Using equation (4.4.1) in (4.4.2)

$$dB(x) = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\cos\phi = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\frac{a}{r} \quad \left(\because \text{ from Figure } \cos\phi = \frac{a}{r}\right)$$

Line integration should be taken over the circumference of the ring to find resultant magnetic field $B(x)$ at point P.

$$\therefore\ B(x) = \oint dB(x) = \frac{\mu_0 I a}{4\pi r^3}\oint dl$$

Here $\oint dl$ is the line integral taken over the whole ring. $\therefore$ $\oint dl = 2\pi a$.

$$\therefore\ B(x) = \frac{\mu_0 I a}{4\pi r^3}.2\pi a \text{ ring.}$$

But from the geometry of the Figure.

$$r^2 = a^2 + x^2 \Rightarrow r^3 = (a^2+x^2)^{\frac{3}{2}}$$

$$B(x) = \frac{\mu_0 I a^2}{2(a^2+x^2)^{\frac{3}{2}}}$$

The magnetic field is along the X-axis.

If the ring consists of N closely wound turns, we can write.

$$B(x) = \frac{\mu_0 N I a^2}{2(a^2+x^2)^{\frac{3}{2}}} \tag{4.4.3}$$

Magnitude of the field at the centre of the ring is obtained by sustituting $x = 0$ in the equation (4.4.4). Thus the magnetic field B (centre) at centre of the ring.

$$B_{(centre)} = \frac{\mu_0 N I}{2a} \tag{4.4.4}$$

For a point far away from the centre of the ring compared to its radius, we have $x >> a$. Neglecting a^2 in comparison to x^2 in equation (4.4.4)

$$B(x) = \frac{\mu_0 NIa^2}{2(x^2)^{\frac{3}{2}}} = \frac{\mu_0 NIa^2}{2x^3} \quad (\text{where } x >> a) \tag{4.4.5}$$

Illustration 1 : Electron is rotating in circular orbit with radius 5.2×10^{-11}m and with linear speed 2×10^6 m s^{-1} in an Hydrogen atom arround the proton. Find the magnetic field produced at the centre of the orbit.

Solution : Here $v = 2 \times 10^6$ m s^{-1}

$r = 5.2 \times 10^{-11}$ m

$e = 1.6 \times 10^{-19}$ C

Frequency of electron in the orbit f (No. of rotations completed in 1 second).

$$f = \frac{v}{2\pi r}$$

Electric current $I = f.e$

$$= \frac{v}{2\pi r} \times e$$

$$= \frac{2\times10^6}{2\times3.14\times5.2\times10^{-11}} \times 1.6 \times 10^{-19} = 9.8 \times 10^{-4}\text{A}$$

Magnetic field produced at the centre of the circular orbit.

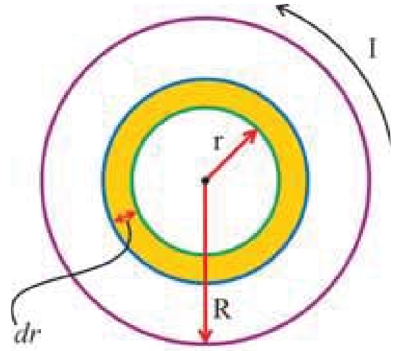
$$B = \frac{\mu_0 I}{2r}$$

$$= \frac{4\times3.14\times10^{-7}\times9.8\times10^{-4}}{2\times5.2\times10^{-11}}$$

$$= 11.8 \text{ T}$$

Illustration 2 : A charge Q is uniformly spread over a disc of radius R made from non-conducting material. This disc is rotated about its geometrical axis with frequency f. Find the magnetic field produced at the centre of the disc.

Solution : Suppose the disc with radius R is devided into the concentric rings with various radii. Consider one of these rings with radius r and width dr. Total charge on the disc is Q.



Hence charge per unit area $= \frac{Q}{\pi R^2}$

$\therefore$ The charge on the ring with radius r.

= (area of the ring) (charge per unit area)

$$= (2\pi r dr)\left(\frac{Q}{\pi R^2}\right)$$

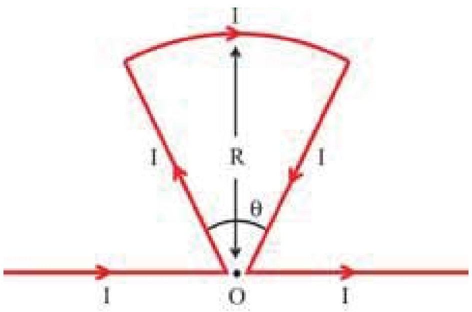
If the ring is rotating with frequency f, then current produced $I = \frac{Q}{\pi R^2} 2\pi r dr f$ and magnetic field produced at the centre due to this current $dB = \frac{\mu_0 I}{2r} = \frac{\mu_0 Q 2\pi}{\pi r^2}\frac{dr}{2r} f = \frac{\mu_0 Q f}{R^2} dr$

$\therefore$ Magnetic field B produced at the centre due to the whole disc.

$$B = \int dB = \int_0^R \frac{\mu_0 Qf}{R^2} dr = \frac{\mu_0 Qf}{R^2} \int_0^R dr$$

$$\therefore \ B = \frac{\mu_0 Qf}{R}$$

Illustration 3 : Find the intensity of magnetic field at point P shown in the figure. At point O, the wires do not touch each other. Corners of the two wires are very close to point O.



Solution : Here point O is on the line of horizontal currents, hence the magnetic field is not developed due to them. It also lies on the directions of radial currents hence magnetic fields due to them is also zero. So the magnetic field is produced only due to the arc. To find this, the formula of magnetic field at the center of a ring having n turns and radius R can be used. According to this equation,

$$B = \frac{\mu_0 nI}{2R} \text{ (in a direction going in to the plane of paper)} \qquad (1)$$

In the present case, the length of the arc is $= R\theta$

For one complete turn, the length of the arc is $2\pi R$, then the number of turns for length $R\theta$ will be,

$2\pi R$: 1 turn

$R\ \theta$: ? $\Rightarrow$ number of turns, $n = \frac{R\theta}{2\pi R} = \frac{\theta}{2\pi}$

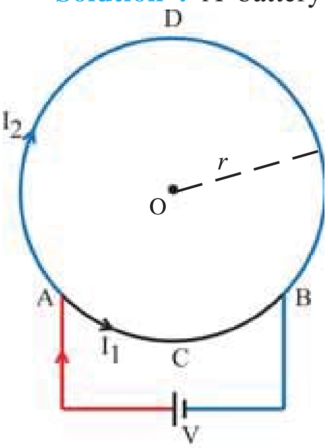
Using this in equation (1),

$$B = \frac{\mu_0 I\theta}{2R \times 2\pi}$$

$$\therefore \ B = \frac{\mu_0 I\theta}{4\pi R} \text{ (going in to the plane of figure)}$$

Illustration 4 : A circular loop is prepared from a wire of uniform cross section. A battery is connected between any two points on its circumference. Show that the magnetic induction at the centre of the loop is zero.

Solution : A battery is joined between points A and B of the loop as shown in the figure.



Since the cross-section of the wire is uniform, the resistance of the part of wire is proportional to the length of that part ($\because$ $R = \rho \frac{l}{A}$).

Let the resistance **per unit length** be R'.

Length of wire ACB $= l_1$

Length of wire ADB $= l_2$

$\therefore$ Resistance of wire ACB $= R_1 = R' l_1$

Resistance of wire ADB $= R_2 = R' l_2$

Current in wire ACB = I_1

Current in wire ADB = I_2

These two parts ACB and ADB are in parallel between A and B points.

$V = I_1R_1 = I_2R_2$

$I_1(R'l_1) = I_2(R'l_2)$

$\therefore I_1l_1 = I_2l_2$

Every small current element of this wire is perpendicular to the position vector of O, with respect to it.

$\therefore$ Biot-Savart's Law gives, magnetic field at O due to ACB, as

$$B_1 = \frac{\mu_0}{4\pi}\frac{I_1 l_1 sin90^o}{r^2}$$

and that due to ADB,

$$B_2 = \frac{\mu_0}{4\pi}\frac{I_2 l_2 sin90^o}{r^2}$$

Since, $I_1l_1 = I_2l_2$

we get, $B_1 = B_2$

According to right hand rule the directions of B_1 and B_2 are opposite to each other. Hence the resultant magnetic field at O will be zero.

4.5 Ampere's Circuital Law

We have obtained line integration in the case of electric field. Same can be done for magnetic field. Consider electric currents I_1, I_2, I_3, I_4, I_5 and I_6 as shown in figure 4.5. All these currents produce magnetic field in the region arround electric currents. A plane which is not necessarily horizontal is shown in the Figure. An arbitrary closed curve is also shown on it. Now let us take a line integration of magnetic field on this loop.

You must be remembering that we have taken a sign convention for electric charges (±) while considering surface integral in case of Gauss' theorem for electric field. In the same way we will have to decide a sign convention for the electric currents enclosed by the loop. One of the methods used in practice is as under.

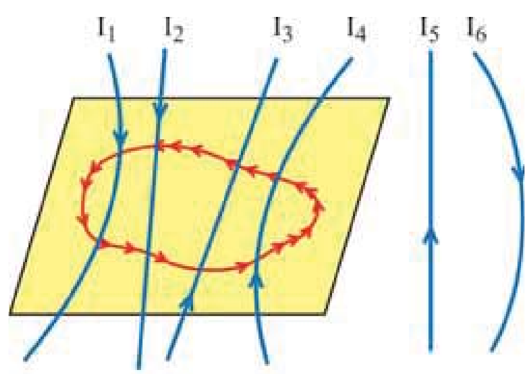


Figure 4.5 Ampere's Circuital Law

Arrange a right hand screw perpendicular to the plane containing closed loop and rotate it in direction of vector line elements taken for line integration. Electric currents in the direction of advancement of the screw are considered positive and the currents in the opposite direction are considered negative.

Now, using the above mentioned sign convention in figure 4.5, we have I_1 and I_2 negative and I_3 and I_4 positive.

Hence the algebraic sum of all these current will be

$I_3 + I_4 - I_1 - I_2 = \Sigma I$

Here do not worry about the currents which are not enclosed by the closed loop selected.

The statement of Ampere's circuital Law is as under :

"The line integral of magnetic induction over a closed loop in a magnetic field is equal to the product of algebraic sum of electric currents enclosed by the loop and the magnetic permeability."

The Law can be represented mathematically as

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \Sigma I \quad (4.5.1)$$

The magnetic induction in the above equation is due to all the currents (I_1, I_2, I_3, I_4, I_5, I_6 in our case). Whereas the algebraic sum of currents on the right hand side is only of those currents which are enclosed by the closed loop. It is important to note that Ampere's Law is true only for steady currents.

Just as in case of static electricity, the electric field due to a symmetric charge distribution can be determined using Gauss' Law, the intensity of magnetic field due to symmetric current distributions can be determined in the same manner using Ampere's Law.

Gauss' Law for the electric field and Ampere's Law for the magnetic field have their own importance in physics. Gauss' Law and Ampere's Law form two basic pillars out of four pillars of Maxwell's electromagnetic theory. Third pillar is the fact that magnetic field lines form closed loops and the fourth is the concept of displacement current.

Here note that Ampere's Law is the integral form of Biot–Savart's Law and Gauss' Law is the integral form of Coulomb's Law. These representations have become very fruitful in physics.

4.5.1 Uses of Ampere's Circuital Law

(1) To Find Magnetic Field Due to a Very Long Straight Conductor Carrying Electric Current, Using Ampere's Law :

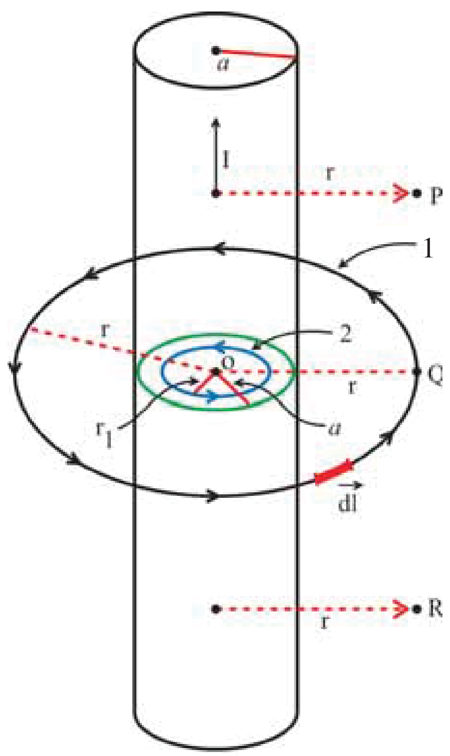


Figure 4.6 Magnetic Field Produced by Straight Conductor Carrying Electric Current

We have seen that magnetic field produced due to symmetric distribution of electric currents can easily be determined by Ampere's Law. Consider a very long (in principle infinitely long) straight conductor carrying electric current I as shown in figure 4.6.

Where is the symmetry in this case ? This can be understood as follows.

First of all see that uniform electric current I is flowing through the whole conductor. Now keep the wire between your two palms and rotate like a churn. This does not make any change in the magnetic field produced by the wire (electric current).

Now consider points like P, Q and R located at same perpendicular distance r from the wire. Both the ends of the wire are at infinite distance. Since the two ends of the wire are at infinite distance, these points P, Q and R can be considered at equal distance from the ends of the wire and in this sense they are equivalent.

This discussion of symmetry shows that the magnetic field at points like P, Q and R must be same. Moreover it is also clear from the fact of rotating the wire like churn that the magnetic field at all the points on the circumference of a circle of radius OQ = r with O at centre must also be the same. In this case we have to find magnetic field at point Q using

Ampere's Law. For this consider circle of radius OQ = r (amperean loop 1) as shown figure 4.6 which is perpendicular to the wire as a closed loop. Such a circle and line elements ($\vec{dl}$) over its circumference are shown in Figure 4.6.

Suppose the magnetic field of all such element is $\vec{B}$. Using this fact in the equation representing Ampere's Law.

$\oint \vec{B} \cdot \vec{dl} = \mu_0 \Sigma I$, we get

$\oint B.dl \cos\theta = \mu_0 I$

As $\vec{B}$ and $\vec{dl}$ are in the same direction at every element,
$\cos\theta = \cos 0 = 1$

$\therefore \oint B.dl = \mu_0 I$

As B is constant

$B \oint dl = \mu_0 I$

Here $\oint dl = dl$ circumference of the circle with radius $r = 2r$

$\therefore B(2\pi r) = \mu_0 I$

$$\therefore B = \frac{\mu_0}{2\pi} \frac{I}{R} \quad (4.5.2)$$

Here current is positive as per our sign convention.
from equation (4.5.2)

$B \propto \frac{1}{r}$ (outside the conductor)

Magnetic Field Inside the conductor : Now as shown in the figure 4.6 radius of the wire is a and we want to find magnetic field at a perpendicular distance r_1 from its axis inside the wire that is $r_1 < a$. Consider circle with radius r_1 as amperean loop 2 as shown in figure 4.6 (which is around the axis inside the wire). If current enclosed by this loop is I_e then

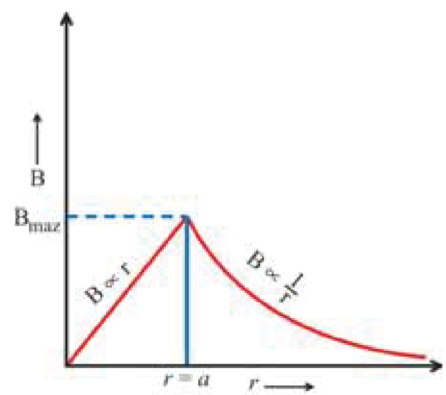


Figure 4.6 Magnetic Field B at distance *r* from the Centre of the Wire

$$I_e = \left(\frac{I}{\pi a^2}\right)\pi r_1^2 = I\frac{r_1^2}{a^2}$$

Using Ampere's Law

$$B(2\pi r_1) = \mu_0 \frac{r_1^2}{a^2} I$$

$$\therefore B = \left(\frac{\mu_0 I}{2\pi a^2}\right) r_1 \quad (4.5.3)$$

Now representing r_1 by r that is for $r < a$ (for magnetic field inside the conductor)
$B \propto r$

Hence in the form of common symbol r the above facts can be represented as follows

(i) If $r > a$, then $B \propto \frac{1}{r}$

(ii) If $r < a$, then $B \propto r$

(iii) At $r = a$ B is maximum.

These facts are shown in the from of plot of $B \rightarrow r$ in figure 4.7.

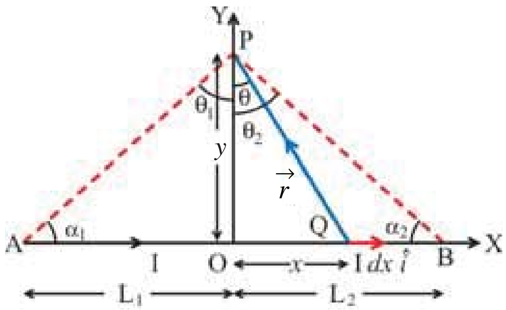


Figure 4.8 Magnetic Field Due to Current Carrying Conductor of Finite Length

For Conductor of Finite Length : To find the magnetic field $\vec{B}$ produced due to the conductor with finite length carrying current consider figure 4.8.

In this case following formula for $\vec{B}$ can be obtained using Biot–Savart Law.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{y}[sin\theta_1 + sin\theta_2]\hat{k} \qquad (4.5.4)$$

Where y is perpendicular distance of the given point P from the wire, θ_1 and θ_2 are the angles subtended with the perpendicular drawn on the wire from the given point by the lines joining given point and the ends of the wire (See Figure 4.8)

(2) Solenoid : As shown in the Figure 4.9 two identical rings carrying same current are placed closed to each other co-axially.

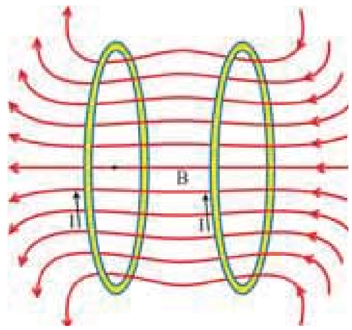


Figure 4.9

It is obvious from the Figure that the magnetic field produce due to the rings are in the same direction on their common axes. Moreover the lines close to the axis are almost parallel to the axis and in the same direction. Thus if a number of such rings (in principle of infinite number) are kept very close to each other and current is passed in the same direction, it is found that inside the region covered by the rings, the field lines are arranged at equal distance from each other obout the axis i.e. magnetic field is uniform. But the magnetic field due to two consecutive rings are in mutually opposite directions outside the rings, so they multiply each other. Hence, magnetic field in the outer region near the rings is zero. Solenoid is a device in which this situation is realized.

A helical coil consisting of closely wound turns of insulated conducting wire is called a solenoid

In practice long and short solenoids are used. **When length of a solenoid is very large as compared to its radius, the solenoid is called long solenoid.**

To find magnetic field inside a long solenoid using Ampere's Circuital Law.

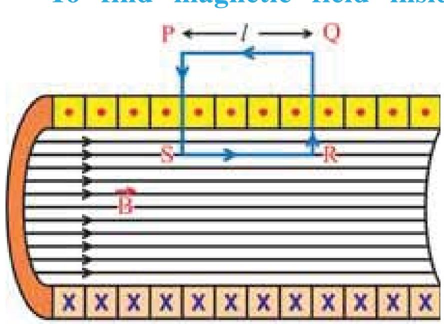


Figure 4.10 Solenoid

Figure 4.10 shows a cross-section of a long solenoid taken with plane of the page of the book. Symbol (X) shows the direction of currents going inside the plane of the page and symbol (•) shows the directions of the current coming out of the plane of the page.

Suppose we want to find the magnetic field at point S lying inside the solenoid. Considering a rectangular loop of length l, PQRS as shown in the Figure 4.10 as Amperean loop, we will take line integral $\vec{B}$ over the loop.

$$\therefore \oint \vec{B}\cdot\vec{dl} = \int_P^S \vec{B}\cdot\vec{dl} + \int_S^R \vec{B}\cdot\vec{dl} + \int_R^Q \vec{B}\cdot\vec{dl} + \int_Q^P \vec{B}\cdot\vec{dl}$$

From the figure 4.10 it is clear that the magnetic field on part PQ of the loop will be zero as it is lying outside the solenoid and hence $\int_Q^P \vec{B}\cdot\vec{dl} = 0$

Moreover, some part of sides QR and SP is outside the solenoid and the part which is inside is perpendicular to the magnetic field, therefore $\int_R^Q \vec{B}\cdot\vec{dl} = \int_P^S \vec{B}\cdot\vec{dl} = 0.$

$$\therefore \oint \vec{B}.\vec{dl} = \int_S^R B\,dl\,cos0^o = B\int_S^R dl = Bl \quad (4.5.5)$$

Now suppose that the number turns per unit length of the solenoid is n. Therefore, the number of turns passing through the Amperean loop is nl. Current passing through each turn is I, so total current passing through the loop is $\Sigma I = nlI$.

From Ampere's Circuital Law

$$\oint \vec{B}.\vec{dl} = \mu_0 nlI$$

$\therefore Bl = \mu_0 nlI$ (from equation 4.5.5)

$$\therefore B = \mu_0 nI \quad (4.5.6)$$

This method can be used only for a long solenoid because only in case of a long solenoid, all the points inside the solenoid can be considered equivalent and magnetic field inside the solenoid as uniform. In the region outside the solenoid in the vicinity of it is zero. This method should not be used for a solenoid of finite length.

For Solenoid of Finite Length : For solenoid of finite length magnetic field inside of it can be determined using Biot–Savart's Law. For this consider figure 4.11. Formula for the magnetic field inside the solenoid of finite length is as under.

$$B = \frac{\mu_0 nI}{2}(sin\alpha_1 + sin\alpha_2) \quad (4.5.7)$$

Here α_1 and α_2 are the angles subtended by two ends of the solenoid with normal drawn at point P respectively.

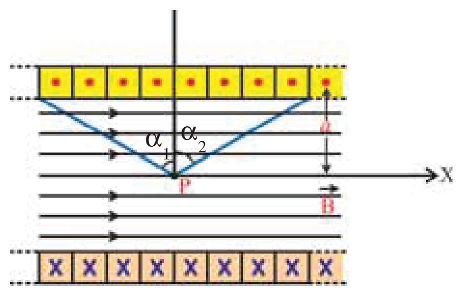


Figure 4.11 Solenoid of Finite Length

Toroid : If a solenoid is bent in the form of a circle and its two ends are joined with each other then the device is called a toroid.

A toroid can also be prepared by closely winding an insulated conducting wire arround non-conducting hollow ring. (In short, the shape of a toroid is the same as that of an inflated tube, also called doughnut shape.) The magnetic field produced inside the toroid carrying electric current can be obtained using Ampere's Circuital Law.

Suppose we want to find the magnetic field at a point P inside the toroid which is at a distance r from its centre as shown in the figure 4.12. If we consider a circle of radius r with its centre at O as an Amperean loop from the symmetry it is clear that the magnitude of the magnetic field at every point on the loop is same and directed towards the tangent to the circle. Therefore,

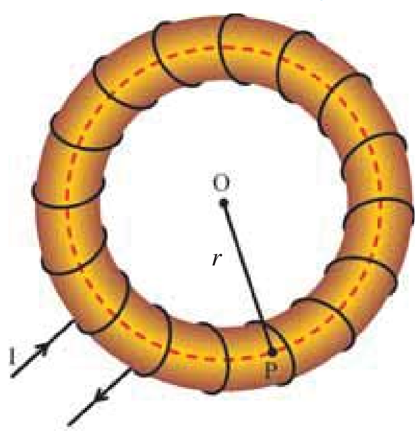


Figure 4.12 Toroid

$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl = B\oint dl = B(2\pi r) \qquad (4.5.8)$$

If the total number of turns is N and current passing is I, the total current passing through the said loop must be NI. From Ampere's Circuital Law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 NI \qquad (4.5.9)$$

Comparing equations (4.5.8) and (4.5.9)

$$B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 nI}{2\pi r} = \mu_0 NI \qquad (4.5.10)$$

Here, $n = \frac{N}{2\pi r}$ the number of turns per unit length of the toroid. This is the equation of magnetic field produced inside the toroid. This magnetic field is uniform at each point inside the toroid.

In an ideal toroid, the turns are completely circular. In such a toroid magnetic field the inside the toroid is uniform and outside the toroid is zero. But in the toroid used in practice, the will is helical and hence, a small magnetic field also exist outside the toroid.

For nuclear fusion, the device Tokamak is used for the confinement of plasma. Toroid is an important component of Tokamak.

4.6 Force on a Current Carrying Wire Placed in a Magnetic Field

Within week of the publicity of the news of Oersted's observation scientist Ampere made another observation. In this observation he showed that **"Two parallel wires placed near each other exert an attractive force if they are carrying currents in the same direction, and exert a repulsive force if they are carrying currents in the opposite directions."**

We have seen that magnetic field is created around the wire carrying electric current. Now, if another wire carrying current is placed in its neighbourhood (i.e. second wire carrying current is placed in the magnetic field produced by the current in the first wire) then the force acts on the other wire due to magnetic field produced by current in the first wire. In the same manner the first wire is lying in the magnetic field produced by the current in the other wire. Hence the force acts on the first wire due to the magnetic field produced by the current in the other wire. This is the magnetic force between two wires.

This interaction can schematically be represented as follows.

$$\left.\begin{matrix}\text{Current in the}\\ \text{first wire}\end{matrix}\right\} \rightleftarrows \text{Magneticfield} \left\{\begin{matrix}\text{Current in the}\\ \text{second wire}\end{matrix}\right.$$

Thus in other words the force acts between the two wires (carrying current) is due to magnetic field.

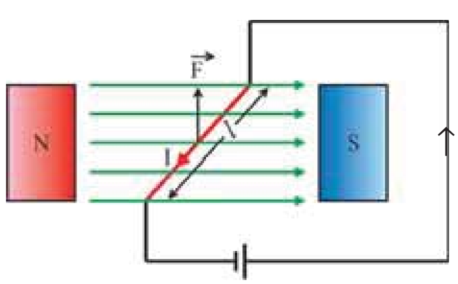


Figure 4.13

To find this force acting between two wires, one must know, the force acting on a wire currying a current placed in magnetic field. The Law giving this force was established by Ampere through the experimental studies is as under :

The force acting on a current element $I\vec{dl}$ due to the magnetic induction $\vec{B}$ is given by

$$\vec{dF} = I\vec{dl} \times \vec{B} \qquad (4.6.1)$$

If a straight wire of length l carrying current I is placed in uniform magnetic field $\vec{B}$, the force acting on the wire can be given by

$$\vec{F} = I\vec{l} \times \vec{B} \quad (4.6.2)$$

Such an arrangement is shown in the Figure 4.13.

The direction of force can be determined using the right hand rule for vector product.

4.6.1 The Force between Two parallel Current Carrying Wires

Consider two very long conducting wires placed parallel to each other along X-axis, separated by a distance y and carrying currents I_1 and I_2 in the same direction (See figure 4.14)

Magnetic field at a distance y from first conductor carrying current I_1 is

$$\vec{B}_1 = \frac{\mu_0}{2\pi} \cdot \frac{I_1}{y} \hat{k} \quad (4.6.3)$$

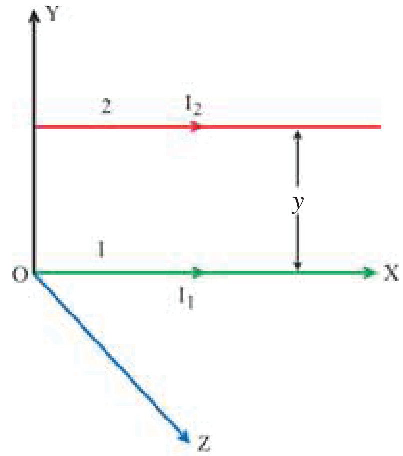


Figure 4.14

The strength of this field is same at all points on the second wire carrying current I_2 and directed along Z-axis. Therefore, the force acting on the second wire over its length l will be

$$\vec{F}_2 = I_2\vec{l} \times \vec{B}_1 \quad \text{(From equation 4.6.2)}$$

substituting value of B_1 from equation (4.4.3) in the above equation

$$\vec{F}_2 = I_1 I_2 \frac{\mu_0}{2\pi y} l\hat{i} \times \hat{k} \quad \text{(As current } I_2 \text{ being along the X-axis)}$$

$$\therefore \vec{F}_2 = -\frac{\mu_0}{2\pi}\frac{I_1 I_2 l}{y}\hat{j} \quad (4.6.4)$$

Above equation shows that the force $\vec{F}_2$ acts along negative Y-direction.

Now the force $\vec{F}_1$ acting on the first wire carrying current I_1 can be obtained in the same manner which is as under :

$$\vec{F}_1 = \frac{\mu_0}{2\pi}\frac{I_1 I_2 l}{y}\hat{j} \quad (4.6.5)$$

The above equation shows that the force F_1 acting on the first wire is in positive y direction.

$$\text{Thus } \vec{F}_1 = -\vec{F}_2 \quad (4.6.6)$$

This fact shows that force acting between indicates attraction takes place between them.

If the currents are flowing in the mutually opposite directions in the two wires then repulsion is produced between them.

From equation (4.6.6) it is obvious that here also Newton's third Law is obeyed.

Definition of Ampere :

In equation (4.6.4) if we take

$I_1 = I_2 = 1A, \quad y = 1$ m and $l = 1$m

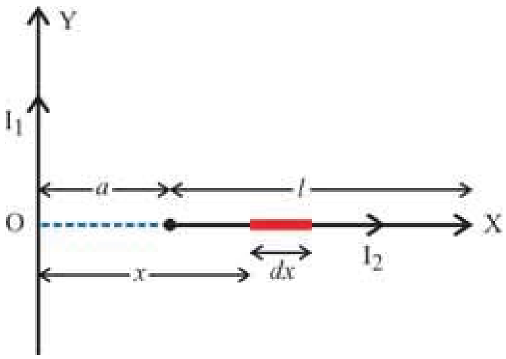
$$|\vec{F}_2| = \frac{\mu_0}{2\pi} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7} \text{ N} \quad (4.6.7)$$

Using this fact definition of SI unit of 1 ampere current is given as under :

"When the magnetic force acting per metre length in two infinitely long wires placed parallel to each other at a distance of 1 meter in vacuum, carrying identical current is 2×10^{-7} N, the current passing through each wire is 1 ampere."

Illustration 5 : As shown in the figure very long conducting wire carrying current I_1 is arranged in y direction. Another conducting wire of length l carrying current I_2 is placed on X-axis at a distance from this wire. Find the torque acting on this wire with respect to point O.

Solution : The force acting on a current element $I_2 dx$ located at a distance x from O is,



$$d\vec{F} = I_2 dx\,\hat{i} \times \vec{B}$$

where, $\vec{B} = \frac{\mu_0 I_1}{2\pi x}(-\hat{k})$

(the magnetic field due to a very long conductor)

$$\therefore\ d\vec{F} = I_2 dx\,\hat{i} \times \frac{\mu_0 I_1}{2\pi x}(-\hat{k})$$

$$= \frac{\mu_0 I_1 I_2 dx}{2\pi x}\hat{j}$$

$\therefore$ The torque acting on this element with respect to O is,

$$d\vec{\tau} = x\hat{i} \times d\vec{F} = x\hat{i} \times \frac{\mu_0 I_1 I_2 dx}{2\pi x}\hat{j} = \frac{\mu_0 I_1 I_2}{2\pi} dx\,\hat{k}$$

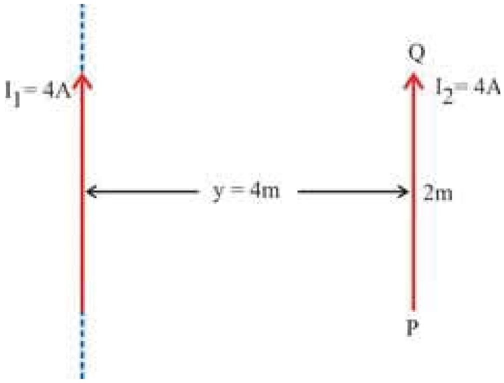
Total torque acting on this coil can be obtained by taking integration of this equation between $x = a$ to $x = a + l$,

$$\therefore\ \vec{\tau} = \frac{\mu_0 I_1 I_2}{2\pi}\int_a^{a+l} dx\,\hat{k} = \frac{\mu_0 I_1 I_2}{2\pi}[x]_a^{a+l}\,\hat{k} = \frac{\mu_0 I_1 I_2}{2\pi}[a + l - a]\hat{k}$$

$$\therefore\ \vec{\tau} = \frac{\mu_0 I_1 I_2 l}{2\pi}\hat{k}$$

Illustration 6 : As shown in the figure, a straight wire PQ of length 2 m carrying 4A current is placed parallel to a very long wire at a distance of 2m. Find the force acting on wire PQ if the current passing through the long wire is also 4A.

Solution : According to Newton's 3rd Law of motion, the force exerted by the smaller wire on the longer wire is the same as the force exerted by the long wire on the smaller one. Hence, we will find the force acting on the smaller wire.



Suppose magnetic field on the smaller wire due to the longer wire is $\vec{B}$

$$\therefore\ \vec{B} = \frac{\mu_0 I_1}{2\pi y}\hat{n} \qquad (1)$$

where $\hat{n}$ is the unit vector in the direction of $\vec{B}$. (1)

Now force on the longer wire is,

$$\vec{F} = I_2\vec{l} \times \vec{B}$$

$\therefore\ |\vec{F}| = I_4 l B\ (\because\ \vec{l}\ \perp\ \vec{B})$

Using the equation (1),

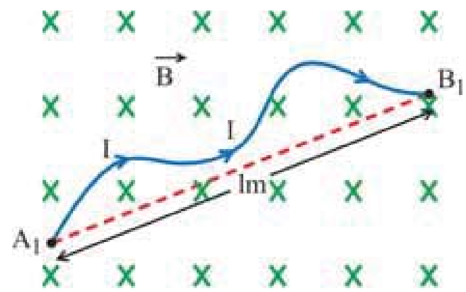
$$\therefore\ |\vec{F}| = \frac{I_2 l \mu_0 I_1}{2\pi y}$$

$$= \frac{4\times2\times4\times3.14\times10^{-7}\times4}{2\times3.14\times4}$$

$\therefore\ |\vec{F}| = 16 \times 10^{-7}$ N

This force produced here is attractive.

Illustration 7 : A wire carrying electric current I is placed on the plane of paper. A magnetic field of induction $\vec{B}$ is applied in a direction going into the plane of paper normally. Find the force acting on the wire.



A straight line joining A_1 and B_1, which is not a part of the wire, of length 1 m is shown in the figure.

Solution : The force acting on a current element $I\vec{dl}$ due to the magnetic field $\vec{B}$ is,

$$d\vec{F} = I\,\vec{dl} \times \vec{B}$$

$\therefore$ The total force acting on the wire is,

$\vec{F} = \int I\cdot\vec{dl} \times \vec{B}$ (Here integration is taken over the whole length of the wire.) Here, n is the number of (free) charge carriers per unit volume of the conductor.

$$\therefore\ \vec{F} = I[\int_l \vec{dl}\,] \times \vec{B}$$

But, $\int \vec{dl} = \vec{A_1B_1} = 1\hat{n}\ (\because\ A_1B_1 = 1\text{m})$

where, $\hat{n} = \vec{A_1B_1}$ the unit vector in the dirction of

$$\therefore\ \vec{F} = I\hat{n} \times \vec{B} \Rightarrow |\vec{F}| = IB$$

4.7 Force on an Electric Charge Moving in a Magnetic Field and Lorentz Force

In Chapter-3 we studied that the current I flowing through a cross section A of a conductor is

$I = nAv_d q$

Here q = Charge on the positively charged particle.

n = number of (free) charge carrier per unit volume of the conductor

v_d = drift velocity

$\therefore\ I\vec{dl} = qnAv_d\vec{dl} = qnA\vec{v_d}\,dl$ ($\because\ v_d$ and dl are in the same direction)

When this conductor is placed in a magnetic field of intensity $\vec{B}$, the force acting on current element $I\vec{dl}$ is given by

$$\vec{dF} = I\vec{dl} \times \vec{B}$$

$$\therefore \vec{dF} = qnAdl(\vec{v_d} \times \vec{B}) \qquad (4.7.1)$$

But $nAdl$ = total number of charged particle in current element

∴ the magnetic force acting on a single particle of charge q will be given by

$$\vec{F}_m = \frac{\vec{dF}}{nAdl} = \frac{qnAdl(\vec{v_d} \times \vec{B})}{nAdl}$$

$$\therefore \vec{F}_m = q(\vec{v_d} \times \vec{B}) \qquad (4.7.2)$$

$|\vec{F}_m| = Bqv_d \sin\theta$. This shows (i) if charge is stationary this force is zero (ii) moreover if charge is moving paralel or anti-parallel to the magnetic field then also this force is zero.

Now, if this electric charge q is moving in the electric field of intensity $\vec{E}$ over and above the magnetic field $\vec{B}$, the force $\vec{F}_e = \vec{E} \cdot q$ due to electric field acts on the charge q. In this circumstanes total force acting on the charge.

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

$$\therefore \vec{F} = q[\vec{E} + (\vec{v_d} \times \vec{B})] \qquad (4.7.3)$$

$$\therefore |\vec{F}_m| = Bqv_d \sin\theta$$

the force obtained by this equation is called Lorentz Force.

The magnetic force acting on a charge moving through the magnetic field is perpendicular to the velocity of the particle, work done by the force is zero and hence its kinetic energy remains constant. Only direction of velocity goes on changing at every instant.

The magnitude of the magnetic force depends on the velocity of the particle, hence such a force is called velocity dependent force.

Illustration 8 : A particle having 2 C charge passes through magnetic field of $4\hat{k}$ T and some uniform electric field with velocity $25\hat{j}$. If the Lorentz force acting on it is $400\hat{i}$ N find the electric field in this region.

Solution : Lorentz force

$$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$$

Here, $q = 2$ C, $\vec{v} = 25\hat{j}$ m s^{-1}, $B = 4\hat{k}$ T, $\vec{F} = 400\hat{i}$

$$\therefore 400\hat{i} = 2 \ [\vec{E} + (25)(4)(\hat{j} \times \hat{k})]$$

$$= 2\vec{E} + 200\hat{i}$$

$$\therefore 2\vec{E} = 200\hat{i}$$

$$\therefore \vec{E} = 100\hat{i} \text{ V m}^{-1}$$

Illustration 9 : In copper there are 8×10^{28} free (conducting) electrons per cubic meter. A current copper wire having length 1 m and cross-sectional area 8×10^{-6} m^2 is placed perpendicularly in the magnetic field of 4×10^{-3} T. The force acting on this wire is $8\text{-}0 \times 10^{-2}$ N. Find the drift velocity of the free electron.

Solution : Magnetic force acting on the wire is given by the formula $\vec{F} = I\vec{l} \times \vec{B}$. Here wire perpendicular to the magnetic field. $|\vec{F}| = IlB$ where $F = 8.0 \times 10^{-2}$, $B = 4.0 \times 10^{-3}$T and l = 1m

$$\therefore\ I = \frac{F}{Bl} = \frac{8\times10^{-2}}{4\times10^{-3}\times1} = 20\ \text{A.}$$

Now $I = Av_d n.e$

n = No. of electrons in the unit volume = 8×10^{28}

$A = 8 \times 10^{-6}$ m^2 and $e = 1.6 \times 10^{-19}$ C

$$\therefore\ v_d = \frac{I}{n.A.e}$$

$$= \frac{20}{8\times10^{28}\times8\times10^{-6}\times1.6\times10^{-19}} = 1.953 \times 10^{-4}$$

$$\approx 2 \times 10^{-4}\ \text{m s}^{-1}$$

Illustration 10 : Write the equation of magnetic force acting on a particle moving through a magnetic field. Using it obtain Newton's equation of motion and show that kinetic energy of the particle remains constant with time.

Solution : $\vec{F}_m = q(\vec{v} \times \vec{B})$

$$\therefore\ m\frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B})$$

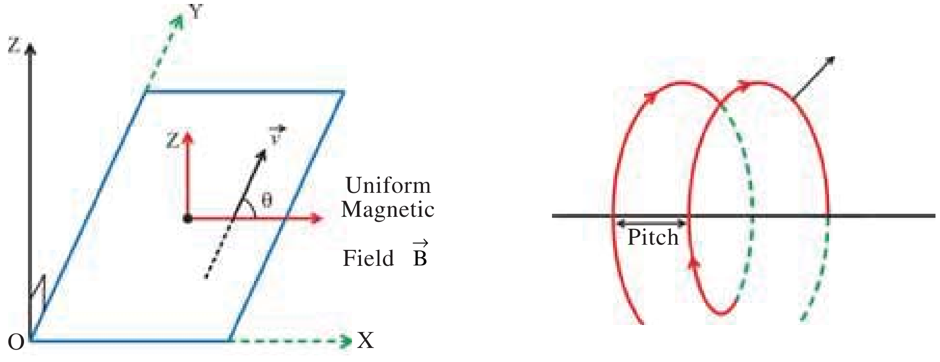
Taking dot product $\vec{v}$ with on both the sides,

$$m\vec{v}\cdot\frac{d\vec{v}}{dt} = q\vec{v}(\vec{v} \times \vec{B})$$

$$\therefore\ m\frac{1}{2}\frac{d}{dt}(\vec{v}\cdot\vec{v}) = 0\ (\because\ .\vec{v}\ \text{and}\ \vec{v} \times \vec{B}\ \text{are mutually perpendicular})$$

$$\therefore\ \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = 0 \Rightarrow \frac{1}{2}mv^2 = \text{constant}$$

Illustration 11 : Suppose a particle of mass m and charge q is incident on XZ plane with velocity v in a direction making angle θ with a uniform magnetic field applied along X-axis according to figure (a). Show that motion of this particle is helical and find the pitch of the path.



(a) (b)

Solution : Considering two components of velocity in XZ plane,

$v_z = v \ sin\theta$ and $v_x = v \ cos\theta$

As v_x component is in the direction of magnetic field, $qv_x\hat{i} \times B\hat{i} = 0$. Since this force is zero, the particle will continue to move with constant velocity $v_x = v \ cos\theta$ along X axis.

Now the force due to v_z component $= qv_z\hat{k} \times B\hat{i} = qv_zB\,\hat{j}$. This force acts perpendicularly to v_z hence the particle will perform circular motion on YZ plane with linear velocity v_z.

Now the centripetal force needed for circular motion is,

$$\frac{mv_z^2}{r} = qv_zB$$

$$\therefore \ r = \frac{mv_z}{qB} = \frac{mv \ sin\theta}{qB}$$

Radius of the circular path of the particle can be determined using above equation, period, $T = \dfrac{2\pi r}{v_z}$.

$$\therefore \ T = \frac{2\pi r}{v \ sin\theta} = \frac{2\pi m}{qB}$$

The particle covers a distance of v_xT during the time interval equal to its period along X axis.

$$\therefore \text{ distance travelled along X direction } = \frac{2\pi mv_x}{qB} = \frac{2\pi mv \ cos\theta}{qB}$$

It is clear from this discussion that the particle moves on a helical path whose axis is along X direction. Here, distance v_xT is called the pitch of the helix (See figure (b)).

4.8 Cyclotron

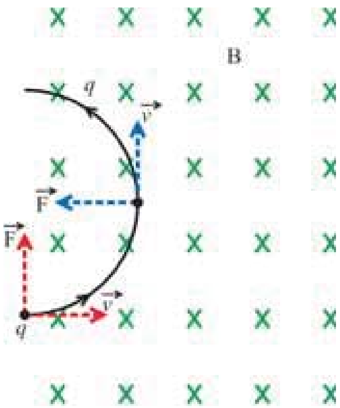


Figure 4.15
Motion of Charged Particle Entering Normally in the Magnetic Field

In the study of nuclear structure very high energy particles are required to be Bombarded on the Nucleus. For this purpose the charged particles are to be accelerated. To do so E.O. Lawrence and M. S. Livingston developed an instrument called cyclotron.

In this instrument the force on a charged particle moving perpendicularly inside a magnetic field is being used. Hence to understand its working we have to study the motion of a charged particle moving perpendicularly inside a magnetic field.

Consider a particle with charge q, moving with velocity $\vec{v}$ in the magnetic field of induction $\vec{B}$ as shown in the figure 4.15.

Here the magnetic field $\vec{B}$ perpendicularly entering into the plane of paper and the electron is maving in the plane of paper.

According to equation (4.7.2), the magnetic force on this particle is $\vec{F} = q(\vec{v} \times \vec{B})$

The value of this force is $qvBsin\theta$ and the direction is normal to the plane formed by $\vec{v}$ and $\vec{B}$. Here, since the particle is moving perpendicular to the magnetic field the value of this force is qvB. It is clear that in this condition the path of the particle will be circular. Since this force is normal to its velocity at every moment, the value of velocity will not change, only its direction will be continuously changing. As a result it will perform circular motion. The necessary centripetal force for this motion is the magnetic force Bqv.

$$\therefore \; qvB = \frac{mv^2}{r}$$

Where m = mass of particle and r = radius of circular path.

$$\therefore \; r = \frac{mv}{qB} = \frac{p}{qB} \qquad (4.8.1)$$

This equation shows that the radius of the circular path of the particle is proportional to the momentum of particle $p = mv$. If the momentum increases the radius of the circular path of the particle also increases.

Here for the circular motion we can write $v = rw_c$. w_c is the angular frequency of the particle which is called the cyclotron frequency. Substituting this value in equation (4.8.1), we get

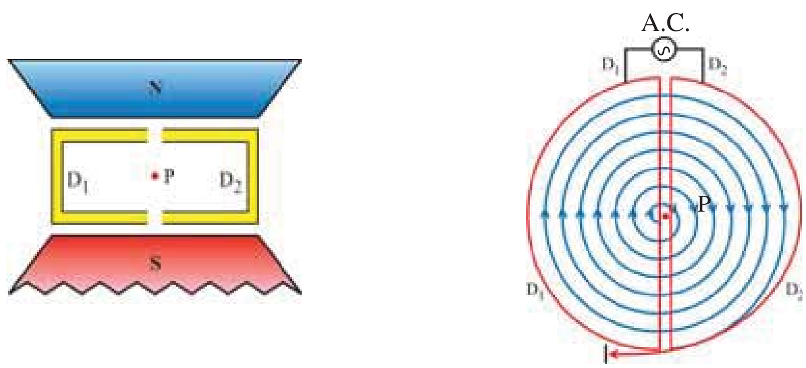
$$r = \frac{m(w_C r)}{qB}$$

$$\therefore \; w_C = \frac{qB}{m} \qquad (4.8.2)$$

$$\therefore \; f_C = \frac{qB}{2\pi m} \qquad (4.8.3)$$

This f_C is called cyclotron frequency.

Here, it is clear that the angular frequency of the particle w_C does not depend on its momentum. Hence on increasing the linear momentum of the particle, the radius of its cercular path definitely increases but the frequency w_C does not change. This fact is used in the design of a cyclotron.



(a) Side View (b) Top View

Figure 4.16 Schematic Diagram of Cyclotorn

Construction : Two hollow matallic boxes of D-shape are kept in front of each other with their diameters facing each other and with a small gap between them as shown in the figure 4.16. Two strong electromagnets are kept in such a way that a uniform magnetic field is developed in the space enveloped by the two boxes. These two boxes are called Dees as they are D-shaped. An A.C. of high frequency is applied between the two Dees. This device is then kept in an evacuated chamber in order to avoid the possible collision of charged particle with the air molecules.

Working : Suppose a charged particle is released from the centre P of the gap between the Dees of time $t = 0$. Exactly at the same time suppose one of the Dees is at negative potential. **If the particle is positively charged,** it gets attracted towards this Dee. Now as a uniform magnetic field is existing in the space between the Dees, the charged particle performs circular motion in the gap and enters the magnetic field in the Dees perpendicularly with a certain momentum. Now there is no electric field in the Dees, hence the particle moves on a circular path of radius depending on its momentum and comes out of the Dee after completing a half circle.

Now, if the opposite Dee becomes negative at the moment at which the particle emerges from one Dee the particle gains momentum due to electric field while passing through the gap before entering the other Dee. It moves in the other Dee on a circular path of larger radius. When this particle emerges out from the second Dee, if the opposit Dee acquires negative potential, the particle gets even more momentum and moves on a circular path of even greater radius in this Dee.

If this process is repeated the radius of circular path goes on increasing but the frequency w_c remains constant. To make this possible the frequency of A.C. voltage (f_{AC}) should be equal to the frequency of revolution f_C. (Here $w_C = 2\pi f_C$). This is nothing but resonance.

In this manner the charged particle goes on gaining energy which becomes maximum on reaching the circumference of the Dee.

For bambarding this charged particle on some target it should be brought out of the Dee. For this when the particle is on the edge, it is brought out of the Dee by deflecting with the help of another magnetic field and allowed to hit the nuclei of the atoms of target.

Here, we have discussed about accelerating positively charged particle (e.g. proton, positive ions), such accelerated particles are used in the study of nuclear reactions, preparation of artificial radioactive substances, treatment of cancer and ion implantation in solids.

Limitations : According to the theory of relativity as the velocity of particle approaches that of light, its mass goes on increasing. In this situation the condition of resonance ($f_{AC} = f_C$) is not satisfied.

To accelerate very light particles like electron, the frequency of A.C. is required to be very high (of the order of GHz)

Moreover, the size of Dees is also large. It is difficult to maintain a uniform magnetic field over a large region. Hence accelerators like synchrotron are developed.

4.9 Torque Acting on a Rectangular Current Carrying Coil Kept in Uniform Magnetic Field

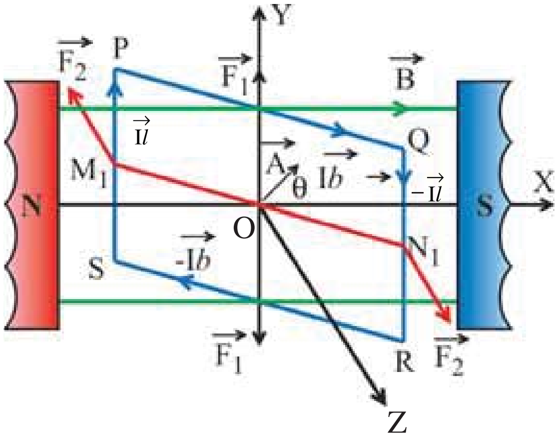


Figure 4.17

Consider a rectangular coil of length QR = l and width PQ = b carrying Current I as shown in figure 4.11. Here, direction of the magnetic field $\vec{B}$ is taken along X-axis.

$\therefore\ \vec{B} = B\hat{i}$

The force acting on the element constituted by side PQ of the coil = $I\vec{b}$.

Therefore force acting on this element $\vec{F_1} = I\vec{b} \times \vec{B}$. (Positive Y-direction). Similarly the force acting on the element formed by side RS is $\vec{F_1}' = I\vec{b} \times \vec{B}$ (negative Y-direction).

Here, forces $\vec{F_1}$ and $\vec{F_1}'$ are equal in magnitude, opposite in direction and collinear hence, they cancel each other.

Now consider the element (QR)I = $-Il\hat{j}$. The force acting on it

$$\vec{F_2} = -Il\hat{j} \times \vec{B}\hat{i} = -IlB(\hat{j} \times \hat{i}) = IlB\hat{k} \qquad (4.9.1)$$

is along positive Z-direction.

Similarly the force acting on the element (SP) I = $Il\hat{j}$ is

$$\vec{F_2}' = Il\hat{j} \times \vec{B}\hat{i} = -IlB\hat{k} \qquad (4.9.2)$$

is in negative Z-direction.

Equations (4.9.1) and (4.9.2) show that $|\vec{F_2}| = |\vec{F_2}'|$

It is also clear from the figure 4.17 that they are opposite in direction. But they are non-collinear. So they constitute a torque (couple)

Viewing the coil from above (in negative Y−direction), $\vec{F_2}$, $\vec{F_2}'$, X-axis and vector $\vec{A}$ appear as shown in figure 4.18. Here $\vec{A}$ is the vector representing the area of the plane of the coil which makes an angle θ with X-axis.

Thus,

Torque acting on coil = (magnitude of a force) (Perpendicular distance between two forces)

The perpendicular distance between two forces is (See Figure 4.18)

$$M'N' = 2\frac{b}{2}cos(\frac{\pi}{2} - \theta) = bsin\theta \quad (4.9.3)$$

$$\therefore \text{ Torque } |\vec{\tau}| = |\vec{F_2}|\ (M'N') = (IlB)(bsin\theta) \quad (4.9.4)$$

$$\therefore |\vec{\tau}| = IABsin\theta$$

Where $lb = A$ is the area of the coil.

For coil having N turns,

$$|\vec{\tau}| = NIAB\ sin\theta \quad (4.9.5)$$

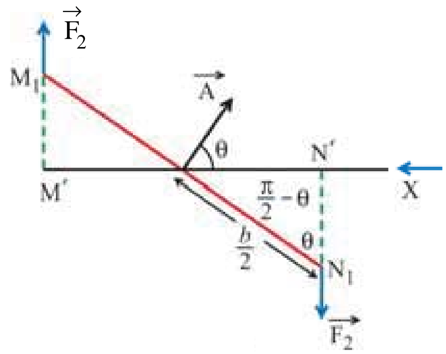


Figure 4.18 Torque Acting on Rectangular Coil

Taking area A of the coil in the vector form, equation (4.9.5) can be written in the vector form as

$$\vec{\tau} = NI\vec{A} \times \vec{B} \quad (4.9.6)$$

The vector quantity $NI\vec{A}$ is called "magnetic moment" linked with the coil and denoted by ($\vec{\mu}$)

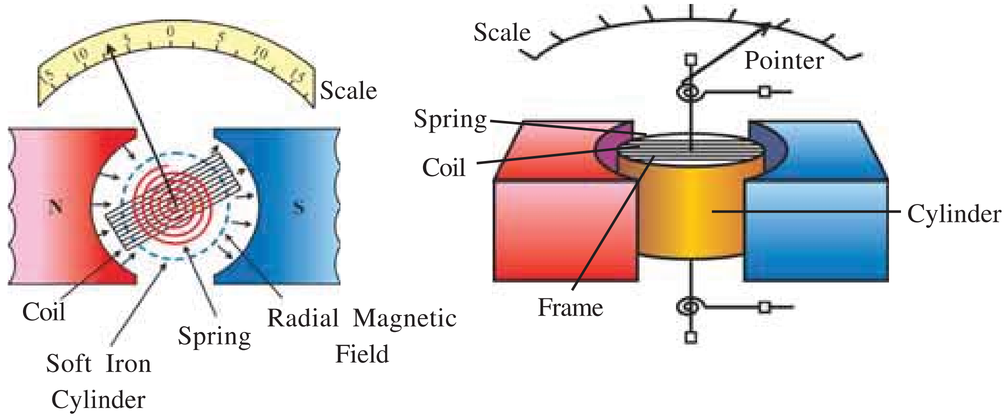
$$\therefore \vec{\tau} = \vec{\mu} \times \vec{B} \quad (4.9.7)$$

equation (4.9.7) is valid for any shape of the coil.

Direction of $\vec{\mu}$ can be determined using right hand screw rule. Keep a right hand screw perpendicular to the plane of the coil and rotate it in the direction of current, the direction in which screw advances shifts gives the direction of $\vec{\mu}$.

4.10 Galvanometer

Galvanometer is a device used to detect and measure small electric currents.



(a) (b) **[For Information Only]**

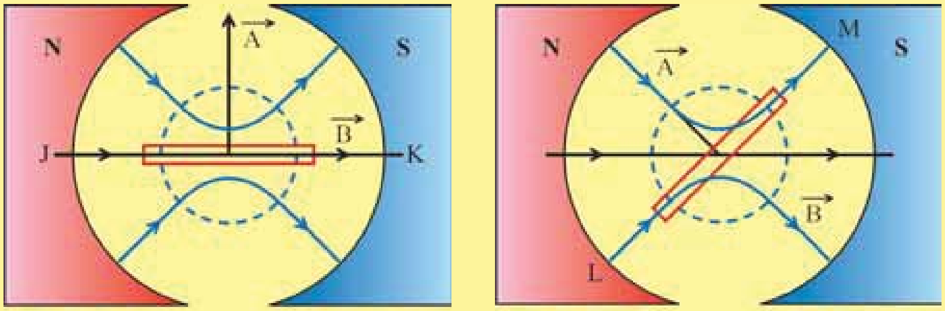
Figure 4.19 Construction of Galvanometer

In galvanometer, a coil of thin insulated copper wire is wound on a light rectangular (non-magnetic) frame. The frame is pivoted between two almost frictionless pivots and placed between two cylindrical poles of a permanent magnet so that it can freely move in the region between the poles. A small soft iron cylindrical core is placed at the axis of the coil (free from coil) so that uniform radial magnetic field is produced. When current is passed through the coil a torque acts on it and deflected. The steady deflection coil is indicated by a pointer attached with it. Knowing the position of the pointer on the scale current can be known.

Principle and Working : If the area vector of the coil marks an angle θ with the magnetic field, from equation (4.9.5) torque acting on the coil.

$\tau = NIAB\sin\theta$ (where N = number of turns in the coil) (4.10.1)

(For Information Only : In the present case magnetic field is radial)



Position (1) Position (2)

Figure 4.20

Figure 4.10 represent figure 4.20 the radially uniform magnetic field obtained in presence of a cylinder of soft iron. For convenience only a few magnetic field lines are shown here. When the coil is in position 1, the line JK is the only effective line. In this case the angle between $\vec{A}$ and $\vec{B}$ is 90°.

Similarly for position 2 of the coil, the line LM becomes effective. In this case also the angle between $\vec{A}$ and $\vec{B}$ is 90°. Thus for any position of the coil the angle between $\vec{A}$ and $\vec{B}$ is 90°.

Due to the raidal field, the angle between $\vec{A}$ and $\vec{B}$ will always be 90°.

$\therefore \ \tau = NIAB$ (4.10.2)

which is called deflecting torque. (The torque due to which the coil is deflected.)

Due to the deflection of the coil, the restoring torque is produced in the springs which is directly proportional to the deflection of the coil.

$\therefore \ \tau \text{ (restoring)} = k\phi$ (4.10.3)

Here k = effective torsional constant of the springs.

If the coil becomes steady after a deflection ϕ,

Deflecting torque = Restoring torque. $NIAB = k\phi$

$$\therefore \ I = \left[\frac{k}{NAB}\right]\phi \qquad (4.10.4)$$

$$\therefore \ I \propto \phi \qquad (4.10.5)$$

The scale of a galvanometer can be appropriately calibrated to measure I by knowing ϕ.

From equation (4.10.5)

$$\frac{\phi}{I} = \frac{NAB}{k} \qquad (4.10.6)$$

Where $\frac{\phi}{I}$ is called current sensitivety(s) of the galvanometer.

Thus, deflection produced per unit current is called current sensitivity of the galvanometer one of the ways to increase the current sensitivity of the galvanometer is to use stronger magnetic field $\vec{B}$.

To measure very weak currents of the order of 10^{-11} A, the galvanometer with coil suspended by an elastic fibre between magnetic poles are used.

4.10.1 Measurement of Electric Current and Potential Difference

We often need to measure the parameters related to a circuit component like the electric current passing through it and the potential difference across its two ends. The instruments to measure these quantities are called an ammeter and a voltmeter respectively. The basic instrument to measure electric current or the voltage is the galvanometer.

4.10.1 (a) Ammeter : A galvanometer has to be joined in series with the component through which the electric current is to be measured. If the potential difference between the two ends of a component is to be measured, the galvanometer has to be joined in parallel between these two ends.

In practice if a galvanometer is directly used as a current–meter, two difficulties arise.

(1) To measure the electric current passing through a component of a circuit, the current–meter is to be joined in series with that component. As for example, we want to measure current passing through the resistance R in a circuit shown in the figure 4.21(a). For this purpose, current meter is joined in series with resistance R, as shown in the figure 4.21(b). In such a connection the resistance G of the galvanometer is added in the circuit. As the total resistance of the circuit is changed the value of current to be measured itself is changed. Thus the true value of current is not obtained. This fact indicates that the resistance of current meter should be as small as possible (in principle zero)

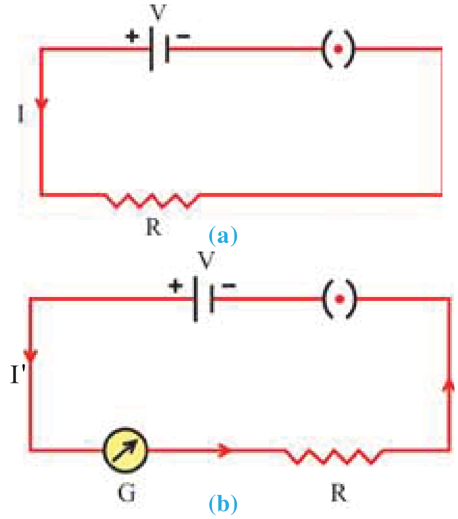


Figure 4.21

(2) Moreover, the moving coil galvanometers are very sensitive. Even when a small fraction of one ampere current (of the order of 10^{-6} A) passes through it, it shows full scale deflection.

The electric current, for which the galvanometer shows full scale deflection, is called the current capacity of galvanometer (I_G). If the galvanometer is used to measure a current greater than its range (current capacity), it is likely to be damaged.

Moreover due to larger current passing through thin copper wire of its coil, large quantity of heat is produced according to I^2Rt and hence it is likely to be burnt.

In order to remove the above mentioned difficulties a resistance of proper small value is joined in parallel to the coil of galvanometer. This resistance is called a **Shunt.** As the value of shunt is very much smaller than the resistance of galvanometer (G), most of the current passes through the shunt and the galvanometer is protected against the damage.

Moreover the shunt and the resistance of galvanometer being in parallel their equivalent resistance becomes even smaller than the value of shunt. Thus after joining the shunt the resistance of the current meter becomes very small. Hence both of the above mentioned difficulties are removed.

Known currents are passed through the instrument prepared after joining the shunt and its scale is calibrated in ampere, milliampere or microampere.

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The instrument thus prepared is called ammeter, milliammetre or microammeter respectively. For this purpose the proper value of shunt is obtained as follows :

Formula for shunt : Suppose a galvanometer having resistance G and current capacity I_G is to be converted into an ammeter which can measure a maximum current I. For this the value of required shunt is suppose S. Here the shunt should be so chosen that out of current I, only I_G current passes through the galvanometer and the remaining $I_S = I - I_G$ current passes through the shunt. This situation is shown in the figure 4.22.

Using Krichoff's first Law, at junction A,

$$I = I_G + I_S \tag{4.10.7}$$

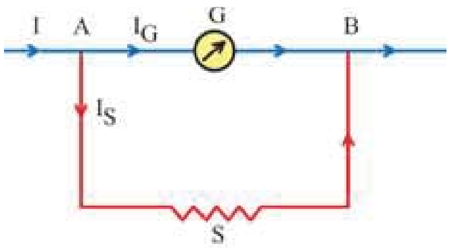


Figure 4.22

Using Kirchoff's second Law on ASBGA path,

$$- I_GG + I_SS = 0$$

$$\therefore\ S = \frac{GI_G}{I_S}$$

From equation 4.10.7, $I_S = I - I_G$

$$\therefore\ S = \frac{GI_G}{I - I_G} \tag{4.10.8}$$

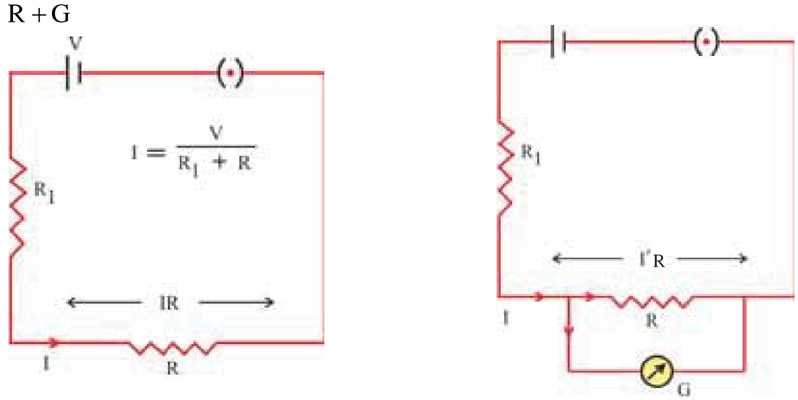
This is the formula for the required shunt. It is clear from this that in order to make the range of ammeter higher and higher the value of the required shunt is smaller and smaller.

To make the range of ammeter n times, the required shunt will be $S = \frac{G}{n-1}$, which you may varify for yourself.

4.10.1 (b) Voltmeter : The instrument to measure the potential difference (also called valtage) between the two ends of component in a circuit, is called voltmeter. For this purpose the voltmeter is joined in parallel to that component.

Suppose the voltage across the two ends of the resistance R shown in the figure 4.23(a) is to be measured. For this if a galvanometer with resistance G and current capacity I_G is used, we find the following difficulties. On joining the galvanometer as shown in the figure 4.23(b), the total resistance of circuit becomes

$$R' = R_1 + \frac{RG}{R+G} \tag{4.10.9}$$



(a) Figure 4.23 (b)

As a result, after joining the galvanometer, the resistance of circuit change and the current passing through R also changes. Thus value of potential difference = IR (which is to be measured), between two ends of the resistance R, also changes.

If the value of G is very high, then in R + G; neglecting R as compared to G,

$$R' = R_1 + \frac{RG}{R+G} \approx R_1 + R \tag{4.10.10}$$

In this condition the resistance of the circuit is not appreciably changed and since value of G is greater, most of the current passes through R and hence the value of IR is almost maintained.

The above discussion shows that the resistance of the instrument measuring the electric potential difference should be as great as possible (in principle infinite). Thus by joining a proper greater resistance in series with the galvanometer, it can be converted into a voltmeter. Here since the resistane is very large, the current passing through the galvanometer is very small and it is not likely to be damaged.

The maximum voltage that can be measured with a galvanometer ($I_G G$) is called its (voltage capacity).

Formula for Series Resistance : Suppose the resistance of a galvanometer is G and its current capacity is I_G. Hence its voltage capacity will be $I_G G$. This galvanometer is to be converted into a voltmeter which can measure a maximum potential difference of V volt. For this the required series resistance is suppose R_S. In figure 4.24 if the potential difference between A and B is V, then by joining the galvanometer and R_S between these points, the galvanometer shows full scale deflection that is the current passing through it will be I_G. From the Figure,

$$I_G G + I_G R_S = V$$

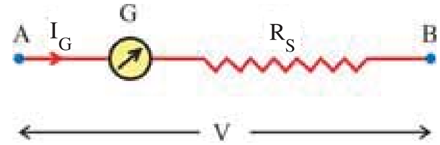


Figure 4.24

$$\therefore\ R_S = \frac{V}{I_G} - G \qquad (4.10.11)$$

By joining a resistance given by the above formula in series with the given galvanometer, and then by properly calibrating the scale of galvanometer, the voltmeter is prepared. From equation 4.10.11, it is clear that in order to make the range of voltmeter greater and greater the larger and larger value of series resistance (R_S) should be taken.

In order to make the voltage capacity of voltmeter, n times, the required series resistance will be $R_S = (n - 1)G$; which you may varify.

By dividing both the sides of equation 4.10.6 by the resistance of voltmeter R.

$$\frac{\phi}{IR} = \frac{NAB}{k}\frac{1}{R}$$

$$\therefore\ \frac{\phi}{V} = \frac{NAB}{kR} \qquad (4.10.12)$$

Here, $\frac{\phi}{V}$ is called the voltage sensitivity (S_V) of voltmeter.

Illustration 12 : There are 21 marks (zero to 20) on the dial of a galvanometer, that is there are 20 divisious. On passing 10 μA current through it, it shows a deflection of 1 division. Its resistance is 20 Ω (a). How can it be converted into an ammeter which can measure 1 A current ? (b) How can the original galvanometer be converted into a voltmeter which can measure a potential difference of 1 V ? Also find the effective resistance of both of the above mentioned meters.

Solution : (a) When a current of 10 μA passes through the galvanometer, its pointer shows a deflection of 1 division. There are 20 divisions in this galvanometer.

∴ The maximum current which can be measured by it (current capacity) $I_G = 10 \times 10^{-6} \times 20 = 200 \times 10^{-6}$A.

For ammeter, the required shunt to be joined in parallel to galvanometer is

$$S = \frac{GI_G}{I-I_G} \qquad I_G = 200 \times 10^{-6}A = 2 \times 10^{-4}A$$

$$= \frac{20\times200\times10^{-6}}{(10000\times10^{-4})-(2\times10^{-4})} \qquad G = 20\ \Omega$$

$$= \frac{20\times2\times10^{-4}}{(10000\times10^{-4})-(2\times10^{-4})} \qquad I = 1\ A = 10000 \times 10^{-4}A$$

$$= \frac{40}{9998} \approx 0.004\ \Omega$$

Thus to convert this galvanometer into an ammeter which can measure 1 A current, a shunt of 0.004 Ω should be joined.

The effective resistance of this ammeter will be $G' = \frac{GS}{G+S} = \frac{20\times0.004}{20+0.004} \approx 0.004\ \Omega$.

(b) For Voltmeter : In order to convert the galvanometer into a voltmeter, the required series resistance is

$$R_S = \frac{V}{I_G} - G \qquad \text{Here, } V = 1 \text{ volt}$$

$$I^G = 2 \times 10^{-4}A$$

$$G = 20\ \Omega$$

$$= \frac{1}{2\times10^{-4}} - 20$$

$$= 0.5 \times 10^4 - 20$$

$$= 5000 - 20$$

$$= 4980\ \Omega$$

In order to convert this galvanometer into a voltmeter which can measure 1 volt, a series resistance of 4920 should be joined with it.

The effective resistance of this voltmeter will be $R'_S = R_S + G = 4980 + 20 = 5000\ \Omega$. ($\therefore$ R_S and G are in series)

SUMMARY

1. **Oersted's Observation :** "When electric current is passed through a conducting wire kept parallel to and below the magnetic needle, the magnetic needle is deflected."

2. **Biot-Savart's Law :** The magnetic field due to a current element $I\vec{dl}$ at a point with position vector $\vec{r}$ with respect to it, is given by

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\hat{r}}{r^2}$$

Since such elements are continuously distributed in the entire conducting wire, the magnetic field due to such a wire can be written in the form of a line integral as

$$\vec{B} = \int\vec{dB} = \frac{\mu_0 I}{4\pi}\int\frac{\vec{dl}\times\hat{r}}{r^2}$$

or $$\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{\vec{dl}\times\vec{r}}{r^3}$$

Here, the line integral is on the entire circuit made up with the conducting wire.

3. The magnetic field due to a circular coil (ring) of N turns, radius a and carrying current I at a point on its axis at a distance x from its centre is

$$B(x) = \frac{\mu_0 N I a^2}{2(a^2 + x^2)^{\frac{3}{2}}}$$

For magnetic field at the centre of the coil (ring),

taking $x = 0$, $B(\text{centre}) = \frac{\mu_0 N I}{2a}$

For a point very much away from the centre,

taking $x >> a$;

$$B(x) = \frac{\mu_0 N I a^2}{2x^3}$$

4. **Ampere's Circuital Law :** "The line integral of magnetic field on a closed curve (loop) in a magnetic field, is equal to the product of the algebraic sum of the electric currents enclosed by that closed curve and the permeability of vacuum."

In the form of an equation this Law can be written as under :

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \Sigma I.$$

5. If current I is passed through a very long straight wire, the magnetic field at a point at normal distance r from the wire is,

$$B = \frac{\mu_0 I}{2\pi r}$$

6. The magnetic field at a point on the axis of a very long solenoid carrying current is $B = \mu_0 n I$

Where n = number of turns per unit length of solenoid.

7. The force on a conducting wire of length l and carrying current I placed in a magnetic field $\vec{B}$, is $\vec{F} = I\vec{l} \times \vec{B}$

The direction of this force can be found by the right hand screw rule for the vector product.

8. The force between two very long parallel current carrying conductors is $F = \frac{\mu_0}{2\pi}\frac{I_1 I_2 l}{y}$,

Where y = perpendicular distance between two wires. If the currents in the wires are in mutually opposite directions, the force is repulsive and if the currents are in the same direction, the force is attractive.

9. The magnetic force on a charge q, moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is $\vec{F}_m = q(\vec{v} \times \vec{B})$

The force on the charge q in an electric field $\vec{E}$ is $\vec{F}_e = q\vec{E}$

The force on the charge in the region where both the fields are present simultaneously, is $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$, which is called the Lorentz force.

10. Cyclotron is the instrument to accelerate the charged particles. The radius of the circular path of the charged particle moving in it, is

$r = \frac{mv}{Bq}$ which is dependent on its momentum.

The angular frequency w of this particle is called the cyclotron frequency (w_e)

$w_C = \frac{qB}{m}$ or $f_C = \frac{qB}{2\pi m}$ ... $(\because\ w_C = 2\pi f_C)$

11. The torque acting on a current carrying coil suspended in a uniform magnetic field is $\vec{\tau} = NI\vec{A} \times \vec{B}$

$\vec{\mu} = NI\vec{A}$ is called the magnetic moment of the coil.

$\therefore\ \vec{\tau} = \vec{\mu} \times \vec{B}$

12. For measuring very small electric currents galvanometer is used. In a moving and pivoted coil galvanometer, $\tau = NIAB$. Due to this the coil is diflected and springs attached with it are twisted. Hence restoring torque is produced. The restoring torque is $\tau = k\phi$.
In equilibrium condition.
$k\phi = NIBA$

$\therefore\ I = \frac{k}{NBA}\phi \qquad \therefore\ I \propto \phi$

13. The small resistance joined in parallel to a galvanometer to convert it into an ammeter is called a shunt. Its formula is $S = \frac{G\,I_G}{I - I_G}$.

To convert a galvanometer into a voltmeter a resistance of a high value is joined in series with it. The formula to find this series resistance Rs is $R_S = \frac{V}{I_G} - G$.

EXERCISE

For the following statements choose the correct option from the given options :

1. Two concentric rings are kept in the same plane. Number of turns in both the rings is 20. Their radii are 40 cm and 80 cm and they carry electric currents of 0.4 A and 0.6 A respectively, in mutually opposite directions. The magnitude of the magnetic field produced at their centre is ...... T.

(A) $4\mu_0$ (B) $2\mu_0$ (C) $\frac{10}{4}\mu_0$ (D) $\frac{5}{4}\mu_0$

2. A particle of mass m has an electric charge q. This particle is accelerated through a potential difference V and then entered normally in a uniform magnetic field B. It performs a circular motion of radius R. The ratio of its charge to the mass $\left(\frac{q}{m}\right)$ is = ... [$\left(\frac{q}{m}\right)$ is also called specific charge.]

(A) $\frac{2V}{B^2R^2}$ (B) $\frac{V}{2BR}$ (C) $\frac{VB}{2R}$ (D) $\frac{mV}{BR}$

3. A proton, a deuteron ion and an α-particle of equal kinetic energy perform circular motion normal to a uniform magnetic field B. If the radii of their paths are r_p, r_d and r_α respectively then..... [Here, $q_d = q_p$, $m_d = 2m_p$]

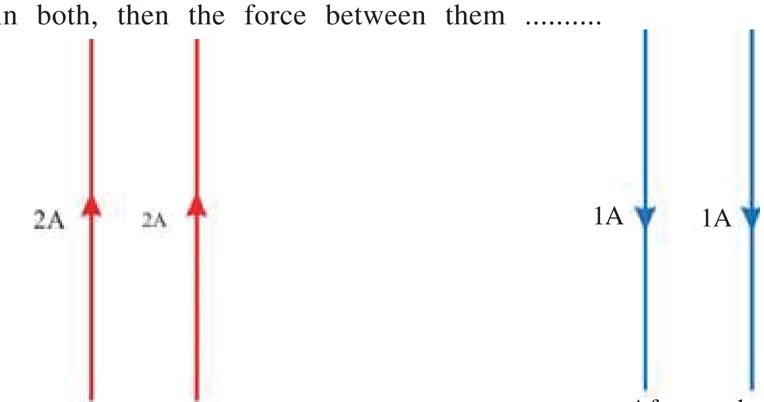
(A) $r_\alpha = r_p < r_d$ (B) $r_\alpha = r_d > r_p$

(C) $r_\alpha > r_d > r_p$ (D) $r_\alpha = r_d = r_p$

4. An electron performs circular motion of radius r, perpendicular to a uniform magnetic field B. The kinetic energy gained by this electron in half the revolution is ....

(A) $\frac{1}{2}mv^2$ (B) $\frac{1}{4}mv^2$ (C) zero (D) $\pi rBev$

5. As shown in the figure two very long straight wires are kept parallel to each other and 2A current is passed through them in the same direction. In this condition the force between them is F. Now if the current in both of them is made 1 A and directions are reversed in both, then the force between them ....



Initially
(a)

Afterwards
(b)

(A) will be $\frac{F}{4}$ and attractive (B) will be $\frac{F}{2}$ and repulsive

(C) will be $\frac{F}{2}$ and attractive (D) will be $\frac{F}{4}$ and repulsive.

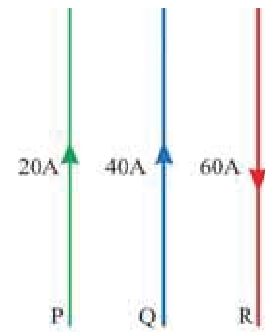
6. As shown in the figure 20A, 40A and 60A currents are passing through very long straight wires P, Q and R respectively in the directions shown by the arrows. In this condition the direction of the resultant force on wire Q is

(A) towards left of wire Q

(B) towards right of wire Q

(C) normal to the plane of paper

(D) in the direction of current passing through Q.



7. As shown in the figure a circular conducting wire carries current I. It lies in XY-plane with centre at O.

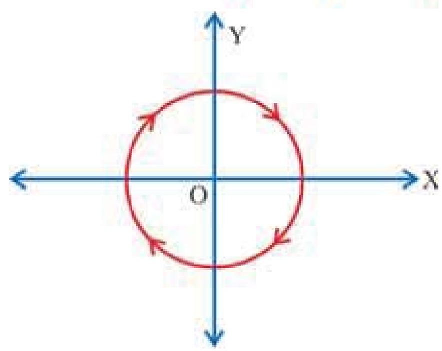
The tendency of this circular loop is to

(A) contract

(B) expand

(C) move towards positive X-direction

(D) move towards negative X-direction.

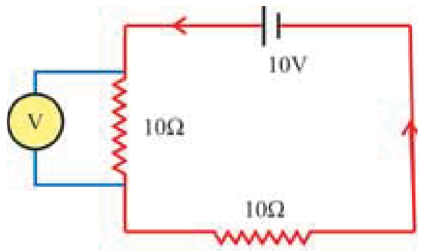


8. At a place an electric field and a magnetic field are in the downward direction. There an electron moves in the downward direction. Hence this electron ....
(A) will bend towards left (B) will bend towards right
(C) will gain velocity (D) will lose velocity.

9. Two parallel long thin wires, each carrying current I are kept at a separation r from each other. Hence the magnitude of force per unit length of one wire due to the other wire is ....

(A) $\frac{\mu_0 I^2}{r^2}$ (B) $\frac{\mu_0 I^2}{2\pi r}$ (C) $\frac{\mu_0 I}{2\pi r}$ (D) $\frac{\mu_0 I}{2\pi r^2}$

10. A voltmeter of a very high resistance is joined in the circuit as shown in the figure. The voltage shown by this voltmeter will be ....



(A) 5 V (B) 10 V
(C) 2.5 V (D) 7.5 V

11. A particle of charge q and mass m moves on a circular path of radius r in a plane inside and normal to a uniform magnetic field B. The time taken by this particle to complete one revolution is ....

(A) $\frac{2\pi mq}{B}$ (B) $\frac{2\pi q^2 B}{m}$ (C) $\frac{2\pi qB}{m}$ (D) $\frac{2\pi m}{Bq}$

12. A long wire carries a steady current. When it is bent in a circular form, the magnetic field at its centre is B. Now if this wire is bent in a circular loop of n turns, what is the magnetic field at its centre ?
(A) nB (B) n^2B (C) $2n$B (D) $2n^2$B

13. A conducting wire of 1 m length is used to form a circular loop. If it carries a current of 1 ampere, its magnetic moment will be .... Am^2.

(A) 2π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{1}{4\pi}$

14. When a charged particle moves in a magnetic field its kinetic energy ....
(A) remains constant (B) can increase
(C) can decrease (D) can increase or decrease

15. At each of the two ends of a rod of length $2r$, a particle of mass m and charge q is attached. If this rod is rotated about its centre with angular speed ω, the ratio of its magnetic dipole moment to the total angular momentum of this particle is ....

(A) $\frac{q}{2m}$ (B) $\frac{q}{m}$ (C) $\frac{2q}{m}$ (D) $\frac{q}{\pi m}$

16. There are 100 turns per cm length in a very long solenoid. It carries a current of 5 A. The magnetic field at its centre on the axis is .... T.
(A) 3.14×10^{-2} (B) 6.28×10^{-2} (C) 9.42×10^{-2} (D) 12.56×10^{-2}

17. Two very long conducting parallel wires are separated by a distance d from each other and equal currents are passed through them in mutually opposite directions. A particle of charge q passes through a point, at a distance $\frac{d}{2}$ from both wires, with velocity v perpendicularly to the plane formed by the wires. The resultant magnetic force acting on this particle is ....

(A) $\frac{\mu_0 Iqv}{2\pi d}$ (B) $\frac{\mu_0 Iqv}{\pi d}$ (C) $\frac{2\mu_0 Iqv}{\pi d}$ (D) zero

18. A very long solenoid of length L has n layers. There are N turns in each layer. Diameter of the solenoid is D and it carries current I. The magnetic field at the centre of the solenoid is ....

(A) directly propotional to D (B) inversely proportional to D.
(C) independent of D (D) directly proportional to L.

19. The angular speed of the charged particle is independent of ....

(A) its mass (B) its linear speed
(C) charge of particle (D) magnetic field.

20. A charged particle gains energy due to ....

(A) electric field (B) magnetic field
(C) both these fields (D) none of these fields.

21. A charged particle is moving with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. The magnetic force acting on it will be maximum when ....

(A) $\vec{v}$ and $\vec{B}$ are in same direction

(B) $\vec{v}$ and $\vec{B}$ are in opposite direction

(C) $\vec{v}$ and $\vec{B}$ are mutually perpendicular

(D) $\vec{v}$ and $\vec{B}$ moke an angle of 45^o with each other

22. Equal currents are passing through two very long and straight parallel wires in mutually opposite directions. They will ....

(A) attract each other (B) repel each other
(C) lean towards each other (D) neither attract nor repel each other.

23. A charged particle is moving in a uniform magnetic field. Then ....

(A) its momentum changes but kinetic energy does not change
(B) its momentum and kinetic energy both change
(C) neither the momentum nor kinetic energy changes.
(D) Kinetic energy charges but the momentum does not change.

24. If the speed of a changed particle moving through a magnetic field is increased, then the radius of curvature of its trajectory will ....

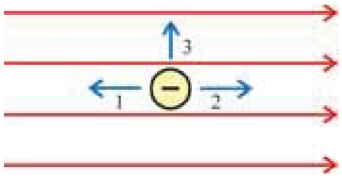
(A) decrease (B) increase (C) not change (D) become half

ANSWERS

1. (C)	2. (A)	3. (A)	4. (C)	5. (A)	6. (A)
7. (B)	8. (D)	9. (B)	10. (A)	11. (D)	12. (B)
13. (D)	14. (A)	15. (A)	16. (B)	17. (D)	18. (C)
19. (B)	20. (A)	21. (C)	22. (A)	23. (A)	24. (B)

Answer the following questions in brief :

1. State the observation made by Oersted.
2. Write the statement of Biot–Savart's Law.
3. Give the formula showing Ampere's Circuital Law.
4. State the Law giving the direction of magnetic field due to a straight conductor carrying current.
5. What is the magnitude of the magnetic field in the region near the outside of the solenoid.
6. State the direction of magnetic field due to current in a toroid.
7. State Ampere's observation after the observation made by Oersted.
8. Does the angular frequency of particle depend on its momentum in cyclotron ? Yes or No ?
9. Can a neutron be accelerated using cyclotron ? Why ?
10. State the functions of electric field and magnetic field in a cyclotron.
11. State two limitations of cyclotorn.
12. What should be the resistances of an ideal ammeter and an ideal voltmeter ?
13. What is meant by current sensitivity of a galvanometer ?
14. What should be done to increase the voltage capacity of a voltmeter.
15. If the radius of the ring and the current through it both are doubled, what change would occur in the magnetic field at its centre ?
16. Give the magnitude of the magnetic force on the electron for the three cases of its motion shown in the Figure.



Answer the following questions :

1. Write Biot–Savart's Law and explain it.
2. Write the formula for the magnetic field at a point on the axis of a current carrying circular ring and explain with a suitable diagram the right hand rule to find the direction of this magnetic field.
3. State and explain Ampere's Circuital Law.
4. Using Ampere's Circuital Law, obtain the magnitude of magnitic field at a perpendicular disance r due to very long staright conductor carrying current I.
5. Using Ampere's circuital Law obtain the formula for the magnitude of magnetic field due to current in a toroid.
6. Obtain the formula for the force of attraction between two parallal wires carrying currents in the same direction.
7. Obtain the formula for the Lorentz force on a moving electric charge
8. Explain the working of cyclotron and obtain the formula for the cyclotron frequency w_C.
9. With a suitable diagram explain the construction of galvanometer.
10. What should be d one to convert a galvanometer into an ammeter. Obtain the formula for the shunt.
11. Derive an expression for the magnetic field at a point on the axis of a current carrying circular ring.
12. Obtain the formula for the magnetic field produced inside a very long current carrying solenoid usiing Ampere's Circuital Law.
13. Obtain the formula for the torque acting on a rectangular coil carrying current, suspended in a uniform magnetic field.

Solve the following examples :

1. Distance between two very long parallel wires is 0.2 m. Electric currents of 4 A in one wire and 6A in the other wire are passing in the same direction. Find the position of a point on the perpendicular line joining the two wires at which the magnetic field intensity is zero.

   [**Ans :** 80 mm away from the wire with 4A current and between the two wires]

2. A very long wire is held vertical in a direction perpendicular to the horizontal component of Earth's magnetic field. Find the value of current to be passed through this wire so that the resultant magnetic field at a point 10 cm away from this wire becomes zero. What will be the magnetic induction at a point 10 cm away from the wire on the opposite side of this point ? Horizontal component of Earth's magnetic field H = 0.36×10^{-4}T, $\mu_0 = 4\pi \times 10^{-7}$T *m*/A. [**Ans. :** 18 A, 0.72×10^{-4} T]

3. When a galvanometer with a shunt is joined in an electrical circuit 2% of the total current passes through the galvanometer. Resistance of galvanometer is G. Find the value of shunt. [**Ans. :** $\frac{G}{49}$]

4. Two particles of masses M_1 and M_2 and having the equal electric charge are accelerated through equal potential difference and then move inside a uniform magnetic field, normal to it. If the radii of their circular paths are R_1 and R_2 respectively find the ratio of their masses. [**Ans :** $\frac{M_1}{M_2} = \left(\frac{R_1}{R_2}\right)^2$]

5. A circular coil having N turns is made from a wire L meter long. If a current of I ampere is passed through this coil suspended in a uniform magnetic field of B tesla, find the maximum torque that can act on this coil. [**Ans. :** $\frac{IL^2B}{4\pi N}$ N m]

6. A proton and a deuteronion having the same kinetic energies enter a region of uniform magnetic field perpendicularly. Deuteron's mass is twice that of proton. Calculate the ratio of the radii of their circular paths. [**Ans. :** $\frac{r_d}{r_p} = \sqrt{2}$]

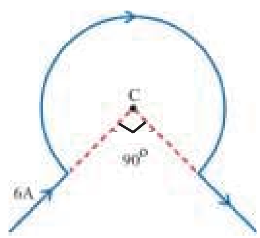
7. A rectangular coil of 120 turns and an area of 10×10^{-4} m^2 is suspended in a radial magnetic field of 45×10^{-4} T. If a current of 0.2 mA through the coil gives it a deflection of 36° find the effective torsional constant for the spring system holding the coil. [**Ans. :** 17.2×10^{-8} N m/rad]

8. Two rings X and Y are placed in such a way that their axes are along the X and the Y axes respectively and their centres are at the origin. Both the rings X and Y have the same radii of 3.14 cm. If the current through X and Y rings are 0.6 A and 0.8 A respecively, find the value of the resultant magnetic field at the origin. $\mu_0 = 4\pi \times 10^{-7}$SI. [**Ans. :** 2×10^{-5}T]

9. Two parallel very long straight wires carrying currents of 20 A and 30 A respectively are at a separation of 3 m between them. If the currents are in the same direction, find the attractive force between them per unit length. [**Ans. :** 4×10^{-5}N m^{-1}]

10. A very long straight wire carries a current of 5 A. An electron moves with a velocity of 10^6 m s^{-1} remaining parallel to the wire at a distance of 10 cm from wire in a direction opposite to that of electric current. Find the force on this electron. (Here the mass of electron is taken as constant) $e = -1.6 \times 10^{-19}$C, $\mu_0 = 4\pi \times 10^{-7}$SI. [**Ans. :** 16×10^{-19}N]

11. A current of 6 A passes through the wire shown in the Figure. Find the magnitude of magnetic field at point C. The radius is 0.02 m $\mu_0 = 4\pi \times 10^{-7}$T m A^{-1}. [**Ans. :** 1.41×10^{-4}T]



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5

MAGNETISM AND MATTER

5.1 Introduction

The word magnet is derived from the name of an island in Greece called Magnesia, where magnetic ore deposits were found as early as 800 BC. Shepherds on this island complained that the nails of their shoes were getting stuck to the ground. The tip of their staff were also getting stuck to chunks of magnetite while they pastured their flocks. Greeks observed that the stone of magnetite (Fe_3O_4) attracts the pieces of iron.

The chinese were the first to use magnetic needles for navigation on ships. Caravans used the magnetic needles to navigate across the Gobi desert. Magnetism is much older than the genesis of life and the subsequent evolution of human beings on earth. It exists everywhere in the entire universe. The earth's magnetism predates human evolution.

In 1269 a Frenchman named Pierre-de Maricourt mapped out the directions of magnetic lines on the surface of a spherical natural magnet by using magnetic needle. He observed that the directions of magnetic lines formed on the sphere were passing through two points diametrically opposite to each other, which he called the poles of the magnet. Afterwards other experiments also showed that every magnet, regardless of its shape and size, has two poles called north and south poles. Some commonly known facts regarding magnetism are as follows :

(1) The Earth behaves as a magnet with the magnetic field pointing approximately from geographic south to north direction.

(2) When a bar magnet is suspended from its mid–point such that it can rotate freely in horizontal plane, then it continues to rotate (oscillate) until it aligns in the north–south direction. The end of the magnet pointing towards the north is called the magnetic **North pole** of the magnet, and the end pointing towards the south pole is called the magnetic **South pole** of the magnet.

(3) Like magnetic poles repel each other, and the unlike poles attract each other.

(4) The positive and negative charges in **electric dipole** may be separated and can exist independently, called **electric monopoles.** The magnet with two poles may be regarded as a **magnetic dipole.** But the magnetic poles are always found in pairs. The north and south magnetic poles cannot be separated by splitting the magnet into two parts. Even if the bar magnet is broken into two or more parts, then also each fragment of the magnet behaves as an independent magnet with north and south magnetic poles with somewhat weaker magnetic field (See figure 5.1). Thus an independent magnetic monopole does not exist. The search for magnetic monopoles is going on.

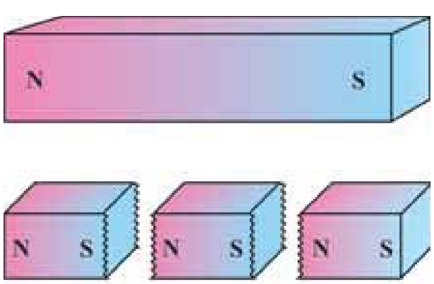


Figure 5.1 Magnet and its Fragments Behaving as Independent Magnets

(5) Magnets can be prepared from iron and its alloys.

In this chapter you will learn the equivalence between magnetic field of a bar magnet and a solenoid, the magnetic dipole moment of a current carrying loop and the dipole moment of orbiting electron in an atom.

The magnetic field strength produced by a magnetic dipole at a point on its equator and at a point along its axis is calculated. The magnetic field of the earth, geomagnetic elements, as well as, para, dia and ferro–magnetic materials are also discussed with suitable examples in this chapter. At the end of this chapter, the applications of permanent magnets and electromagnets are explained in brief.

5.2 The Bar Magnet

The great scientist Albert Einstein got a magnet as a gift when he was a child. He was much fascinated by it and used to play with it. When the magnet attracted iron nails, pins etc., he wondered how the magnet could attract the things without touching them.

Figure 5.2 shows the arrangement of iron filings sprinkled on a plane paper, which is kept on a bar magnet. When the paper is tapped twice or thrice, the iron filings rearrange in a systematic pattern representing the magnetic field lines. Similar picture of magnetic field lines can be formed if the bar magnet is replaced by a short solenoid, through which a DC current passes.

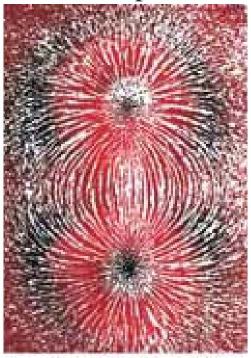
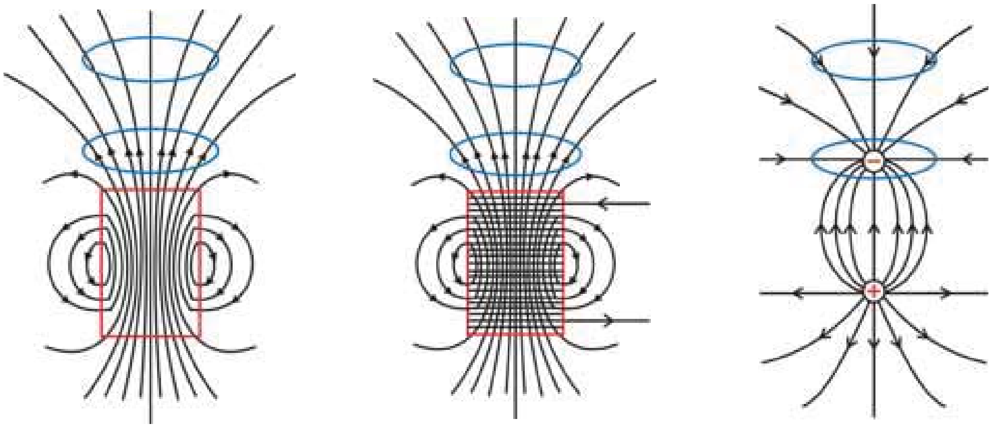


Figure 5.2 Systematic Arrangement of Iron Filings Representing Magnetic Field Lines of a Bar Magnet



(a) Bar Magnet **(b) Solenoid** **(c) Electric-Dipole**

Figure 5.3 Magnetic and Electric Field Lines (Only for Information)

Figure 5.3 shows the magnetic field lines due to a bar magnet and a short solenoid. Electric field lines due to an electric dipole are also shown for comparison.

Following conclusions can be made from the study of figure 5.3 :

(1) **The magnetic field lines of a magnet (or a solenoid) form continuous closed loops. The magnetic field lines emerge out from the magnetic north pole, reach the magnetic south pole and then passing through the magnet, reach the north pole to complete the loop.** In the electric dipole, these field lines begin from a positive charge and end on the negative charge or escape to infinity.

It is impossible to have a static arrangement of electric charges, whose electric field lines form closed loops. This is a typical property of the static electric field.

(2) The tangent to a magnetic field line at a point through which it passes, indicates the direction of magnetic field $\vec{B}$ at that point.

For example, a compass needle may be used to trace out the magnetic field lines of a bar magnet by putting it at different positions surrounding the bar magnet.

(3) The magnitude of magnetic field in the region surrounding a magnet can be represented by the number of magnetic field lines passing normally through a unit area in that region. In figures 5.3 (a) and 5.3 (b) the magnitude of magnetic field B is larger around region (i) than in region (ii).

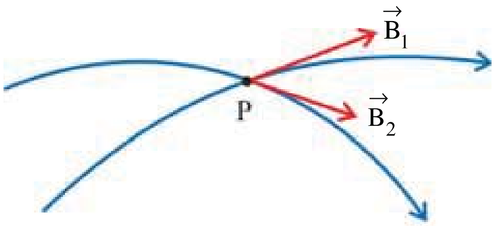


Figure 5.3 (d)

(4) **The magnetic field lines do not intersect with each other.** If they intersect at a point, then the tangents to the lines at the point of intersection would represent two different directions of the magnetic field at that point, which is impossible. (See figure 5.3(d))

If the magnetic field lines intersect at point P, the magnetic fields $\vec{B}_1$ and $\vec{B}_2$ point in different directions.

5.3 Current Loop as a Magnet and its Magnetic Moment

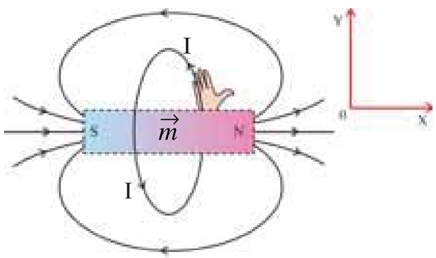


Figure 5.4 Magnetic Field Produced by a Current Loop Like that of a Bar Magnet of Magnetic Dipole Moment $\vec{m}$

In Chapter-4 you studied that, a loop of area A and carrying current I behaves as a magnet, with magnetic depole moment

$$m = IA \tag{5.3.1}$$

The direction of magnetic moment $\vec{m}$ of the loop can be found using right hand rule as shown in Figure (5.4)

$$\text{Thus, } \vec{m} = I\vec{A} \tag{5.3.2}$$

If there are N turns in the loop, then

$$\vec{m} = NI\vec{A} \tag{5.3.3}$$

For the points on the axis of the loop of radius a, far from its centre ($x >> a$), the magnetic field (Chapter-4) is given by

$$B(x) = \frac{\mu_0 I a^2}{2x^3} \tag{5.3.4}$$

$$= \frac{\mu_0}{2\pi}\frac{I\pi a^2}{x^3} = \frac{\mu_0}{2\pi}\frac{IA}{x^3} \qquad (A = \pi a^2 = \text{area of the loop})$$

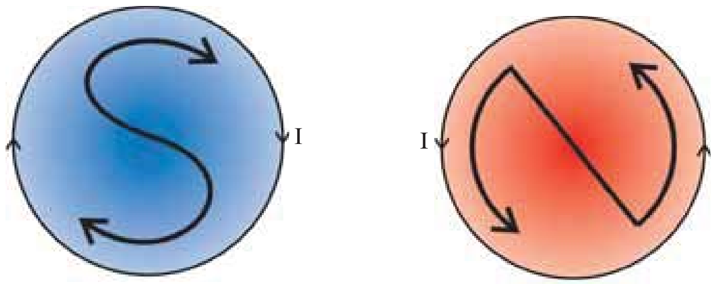
$$\therefore \ B(x) = \frac{\mu_0}{2\pi}\frac{m}{x^3} \tag{5.3.5}$$

Since B(x) and m have same direction,

$$\vec{B}(x) = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{x^3} \tag{5.3.6}$$

which is the axial magnetic field in terms of magnetic dipole moment $\vec{m}$ of the loop at $x >> a$. Equation (5.3.6) is equally applicable for a (short) bar magnet of magnetic dipole moment $\vec{m}$.

5.3.1 Direction of Magnetic Pole in a Current Carrying Loop :



(a) Magnetic South Pole (b) Magnetic North Pole

Figure 5.5

Figure 5.5(a) shows the current I flowing in clockwise direction in a circular loop lying in the plane of the page. According to right hand rule, the side of the loop towards us behaves as a magnetic south pole whereas the opposite side of the loop behave as a magnetic north pole. The symbolic notation S indicates magnetic south pole pointing towards us.

Simliarly, if the current flows in anticlockwise direction in the loop, the side of the loop towards us behaves as a magnetic north pole and opposite side as a magnetic south pole (See Figure 5.5(b)). The symbolic notation N indicates the magnetic north pole pointing outwards.

5.4 Magnetic moment of an electron rotating around the nucleus of an atom :

Dear students, now you know that a magnetic field is produced by the motion of charged particles or by an electric current. Any material is made up of atoms, and in these atoms definite number of electrons (depending on the nature of the element), move in various possible orbits. Such motion of electrons in orbits can be considered as an electric current around a closed path, with magnetic moment IA (I = electric current, and A = area enclosed by the orbit). The magnetic dipole moment of an atom of any given element, depends upon the distribution of electrons in various orbits and on their spins.

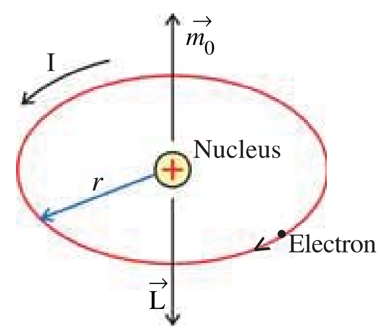


Figure 5.6 Non-zero Magnetic Moment of Atom

As shown in figure 5.6, consider an electron moving with constant speed v in a circular orbit of radius r about the nucleus. If the electron travels a distance $2\pi r$ (circumference of the circle) in time T, then its orbital speed is $v = \frac{2\pi r}{T}$. Thus the current I associated with this orbiting electron of charge e is, $I = \frac{e}{T}$.

Here, $T = \frac{2\pi}{\omega}$, and $\omega = \frac{v}{r}$

$\therefore I = \frac{e\omega}{2\pi} = \frac{ev}{2\pi r}$

The orbital magnetic moment associated with this orbital current loop is

$$m_0 = IA = \frac{ev}{2\pi r} \times \pi r^2 = \frac{1}{2} evr \quad (5.4.1)$$

where $A = \pi r^2$ = area enclosed by the circular orbit.

For this electron, the orbital angular momentum is $L = m_e vr$. Hence, the orbital magnetic moment of the electron can be represented as

$$m_0 = \left(\frac{e}{2m_e}\right)(m_e vr) = \left(\frac{e}{2m_e}\right)L \quad (5.4.2)$$

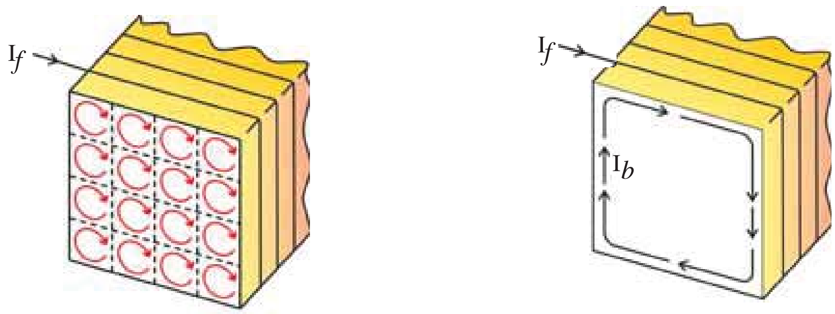
Equation (5.4.2) shows that the magnetic moment of the electron is proportional to its orbital angular momentum L. But since the charge of electron is negative, the vectors $\vec{m_0}$ and $\vec{L}$ point in opposite directions, perpendicular to the plane of the orbit.

$$\therefore \ \vec{m_0} = -\left(\frac{e}{2m_e}\right) \vec{L} \qquad (5.4.3)$$

The ratio $\frac{e}{2m_e}$ is a constant called the gyro–magnetic ratio, and its value is 8.8×10^{10} C kg^{-1}.

5.5 Magnetism in Matter

In general, the magnets are prepared from iron (*Fe*). The atoms of iron normally possess magnetic dipole moment, but an ordinary piece of iron does not behave as a magnet .



(a) Atomic Current Loops Due to Current I_f (Magnetized Iron)

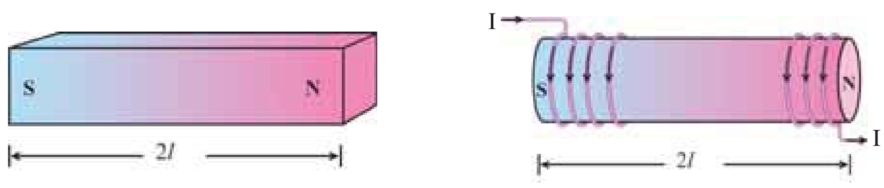
(b) Overall Bound Current Loop Equivalent to Loops in Part (a)

Figure 5.7

The same iron piece can be converted into a magnet, if it is kept in a strong magnetic field for some time and then the applied magnetic field is removed. As shown in figure 5.7 a wire is wound on a piece of iron. If $I_f = 0$, then the magnetic dipole moments of current loops of atoms are randomly oriented. Thus the resultant magnetic moment of the iron piece becomes zero and the iron piece does not behave as a magnet.

When sufficient current I_f passes through the wire, a strong magnetic field is generated in the iron piece, due to which the elemental atomic currents redistribute in the iron piece. Thus a resultant bound current I_b is generated in the iron piece (See Figure 5.7(b)). When the current I_f is slowly reduced to zero, all of the elemental atomic currents do not return to original state even though the external magnetic field becomes zero. This way the iron piece sustains magnetic field.

5.6 Equivalence between a Bar Magnet and a Solenoid



(a)

(b)

Figure 5.8 A Bar Magnet and a Solenoid

Figure 5.8 shows a bar magnet and a solenoid. If the pole strength of bar magnet is p_b (even though such individual poles do not exist), and the distance between two poles is $2l$ then according to definition, the magnetic dipole moment of bar magent is

$$m_b = 2lp_b \qquad (5.6.1)$$

$$\therefore \; p_b = \frac{m_b}{2l} \qquad (5.6.2)$$

The suffix b here indicates that the magnetic moment is due to bar magnet.

Note : Only for infomation : The poles (p_b) of the bar magnet are not on the end faces of the bar magnet, but are situated inside, in such a way that the distance between the two poles (magnetic length) is $2l_m$, which is slightly less than the geometric length $2l$ of the bar magnet. For practical purposes the magnetic length $2l_m = \frac{5}{6} \times 2l$, is taken as geometric length $2l$, in this book.

In a solenoid of cross sectional area A, carrying current I, each turn can be treated as a closed current loop, and hence a magnetic dipole moment IA can be associated with each turn. As the magnetic dipole moment of every turn is in the same direction, the magnetic dipole moment of the solenoid is a vector sum of dipole moments of all turns. If there are total N turns in length $2l$ of the solenoid, then its magnetic moment is

$$m_s = \text{NIA} \qquad (5.6.3)$$

From equations (5.6.1) and (5.6.3), we can define equivalent pole strength of solenoid as

$$p_s = \frac{m_s}{2l} = \frac{\text{NIA}}{2l} = n\text{IA} \qquad (5.6.4)$$

where $n = \frac{\text{N}}{2l}$ = number of turns per unit length of solenoid.

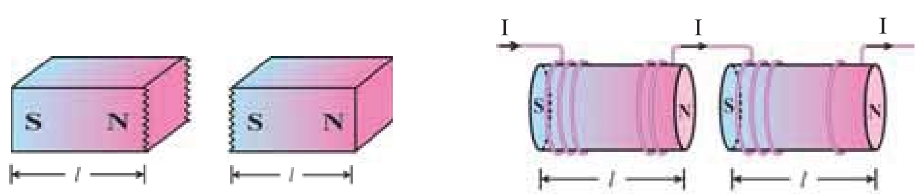
From equation (5.6.4), the unit of pole strength is A m.

As mentioned in the article (5.3) the magnetic field along the axis of dipole moment $\vec{m}$ is

$$\vec{\text{B}}(x) = \frac{\mu_0}{4\pi} \cdot \frac{2\vec{m}}{x^3} \qquad (5.6.5)$$

Hence, the magnetic field produced by a bar magnet or a solenoid can be calculated by replacing $\vec{m}$ by $\vec{m}_b$ or $\vec{m}_s$, respectively, in equation (5.6.5).

What happens if bar magnet is broken ?



(a) (b)

Figure 5.9 Broken Bar Magnet and a Solenoid

If the solenoid of figure 5.8 is broken into two equal pieces as shown in figure 5.9.(b), then the pole strength of each piece of solenoid remains same as nIA, since the number of turns per unit length (n) remains same. By analogy we can say that the pole strength of each piece of bar magnet also remains same.

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In both cases, the magnetic length becomes half of the original length. Hence the magnetic dipole moment also becomes half.

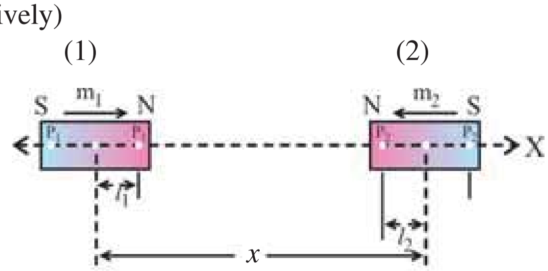
5.6.1 The Electrostatic Analogue : Comparing equatians (5.6.1) and (5.6.5) with corresponding equations for electric charge (chapter 1), it can be observed that the magnetic field at large distances due to a bar magnet or current loop of magnetic moment $\vec{m}$ can be obtained directly from the equations of electric field due to an electric dipole of dipole moment $p = 2aq$, by making following replacements.

$$\vec{E} \rightarrow \vec{B}, \ \vec{p} \rightarrow \vec{m}, \ \frac{1}{4\pi\varepsilon_0} \rightarrow \frac{\mu_0}{4\pi}$$

Table 5.1 Analogy between Electric and Magnetic Dipoles

Quantity	Electrostatics	Magnetics
Constant	$\frac{1}{4\pi\varepsilon_0}$ q (charge)	$\frac{\mu_0}{4\pi}$ p (pole strength)
Dipole moment	$\vec{p} = q(2\vec{a})$	$\vec{m} = p(2\vec{l})$
Equatorial Field $y >> a$ $y >> l$	$\vec{E}(y) = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{(y^2+a^2)^{\frac{3}{2}}}$ $= -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{y^3}$	$\vec{B}(y) = -\frac{\mu_0}{4\pi}\frac{\vec{m}}{(y^2+l^2)^{\frac{3}{2}}}$ $= -\frac{\mu_0}{4\pi}\frac{\vec{m}}{y^3}$
Axial Field $z >> a$ $z >> l$	$\vec{E}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}z}{(z^2-a^2)^2}$ $= \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{z^3}$	$\vec{B}(z) = \frac{\mu_0}{4\pi}\frac{2\vec{m}z}{(z^2-l^2)^2}$ $= \frac{\mu_0}{4\pi}\frac{2\vec{m}}{z^3}$
Force	$\vec{F} = q\vec{E}$	$\vec{F} = p\vec{B}$
Torque (in External Field)	$\vec{\tau} = \vec{p} \times \vec{E}$	$\vec{\tau} = \vec{m} \times \vec{B}$
Energy (in External Field)	$U = -\vec{p}\cdot\vec{E}$	$U = -\vec{m}\cdot\vec{B}$

Illustration 1 : Find the force between two small bar magnets of magnetic moments $\vec{m}_1$ and $\vec{m}_2$ lying on the same axis, as shown in the Figure. (p_1 and p_2 are the pole strength of magnets (1) and (2) respectively)



$x >> l_1$, $x >> l_2$

Solution : To find the force on magnet (2) due to magnet (1), calculate the magnetic field due to magnet (1) at the poles of magnet (2). The axial magnetic field at the north pole of magnet (2) due to magnetic moment m_1 is (from the geometry of Figure)

$$B_N = \frac{\mu_0}{4\pi} \frac{2m_1}{(x-l_2)^3} \qquad (1)$$

Similarly, the axial magnetic field at the south pole of magnet (2) is

$$B_S = \frac{\mu_0}{4\pi} \cdot \frac{2m_1}{(x+l_2)^3} \qquad (2)$$

The repulsive force F_N acting on the north pole of magnet (2) having pole strength p_2 is (like $F = qE$ in electrostatics)

$$F_N = p_2 B_N = \frac{\mu_0}{4\pi} \frac{2p_2 m_1}{(x-l_2)^3} \qquad (3)$$

which is acting away from magnet (1)

Similarly, the attractive force F_S acting on the south pole of magnet (2) is

$$F_S = p_2 B_S = \frac{\mu_0}{4\pi} \frac{2p_2 m_1}{(x+l_2)^3} \qquad (4)$$

which is acting towards magnet (1)

Hence the resultant force on magnet (2) is

$$F = F_N - F_S$$

$$= \frac{\mu_0}{4\pi} \cdot 2p_2 m_1 \left[\frac{1}{(x-l_2)^3} - \frac{1}{(x+l_2)^3} \right] = \frac{\mu_0}{2\pi} \; p_2 m_1 \left[\frac{(x+l_2)^3 - (x-l_2)^3}{\{(x-l_2)(x+l_2)\}^3} \right]$$

$$= \frac{\mu_0}{2\pi} \; p_2 m_1 \left[\frac{6x^2 l_2}{(x^2 - l_2{}^2)^3} \right]$$

[Because $(a \pm b)^3 = a^3 \pm b^3 \pm 3ab(a \pm b)$ and $l_2^3 << x^2 l_2$ in numerator]

$\therefore \; F = \dfrac{\mu_0 m_1}{2\pi} \cdot \dfrac{2l_2 P_2 \cdot 3x^2}{x^6}$ (Since $l_2^2 << x^2$, and hence l_2^2 can be neglected)

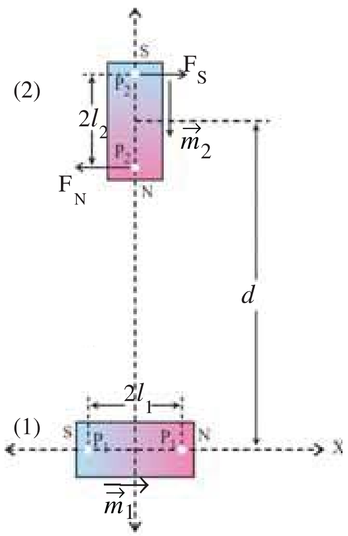
$$\therefore \; F = \frac{3\mu_0 m_1 m_2}{2\pi x^4} \qquad (5)$$

Where $m_2 = 2l_2 p_2$ = magnetic moment of magnet (2)

This resultant force is repulsive for the magnet positions shown in Figure, and acts on magnet (2) in a direction away from magnet (1).

[What will be the resultant force between the two bar magnets, if the direction of one of the magnets is reversed ? Think !]

Illustration 2 : Find the torque on small bar magnet (2) due to small bar magnet (1), when they are placed perpendicular to each other as shown in Figure. ($l_1 << d$, $l_2 << d$)



Solution : From the geometry of Figure, it is seen that both the poles of magnet (2) are lying on the equatorial line of magnet (1).

The magnetic field B_N produced by the small bar magnet (1) at distance $(d - l_2)$ on its equatorial plane is

$$B_N = \frac{\mu_0}{4\pi} \frac{m_1}{(d-l_2)^3} \tag{1}$$

Similarly the magnetic field B_S produced by the magnet (1) at south pole of magnet (2), lying at a distance $(d + l_2)$ on its equatorial plane is

$$B_S = \frac{\mu_0}{4\pi} \frac{m_1}{(d+l_2)^3} \tag{2}$$

Thus as shown in figure the forces F_N and F_S acting on the north and south poles of magnet (2) having pole strength p_2 are

$$F_N = p_2 B_N = \frac{\mu_0}{4\pi} \frac{m_1 p_2}{(d-l_2)^3} \tag{3}$$

$$F_S = p_2 B_S = \frac{\mu_0}{4\pi} \frac{m_1 p_2}{(d+l_2)^3} \tag{4}$$

As $l_1 << d$ and $l_2 << d$, l_1 and l_2 can be neglected in comparison with d in equations (3) and (4).

$$\therefore\ F_S = F_N = \frac{\mu_0}{4\pi} \frac{m_1 p_2}{d^3} \tag{5}$$

As the non-colinear forces F_S and F_N are acting on magnet (2) in opposite direction, they form a couple. Hence the torque due to these forces is

$$\vec{\tau} = 2\vec{l_2} \times \vec{F_N} = 2\vec{l_2} \times \vec{F_S} \quad (\because \vec{\tau} = \vec{r} \times \vec{F})$$

Since $\vec{F_N} \perp \vec{l_2}$ and $\vec{F_S} \perp \vec{l_2}$, the magnitude of the torque with respect to centre of magnet (2)

$$\tau = 2F_N l_2 = \frac{\mu_0}{4\pi} \frac{m_1 2 l_2 p_2}{d^3} = \frac{\mu_0}{4\pi} \frac{m_1 m_2}{d^3} \tag{6}$$

where $2l_2 p_2 = m_2$ = magnetic moment of magnet (2).

5.7 Torque Acting on a Magnetic Dipole (Bar Magnet) in a Uniform Magnetic Field

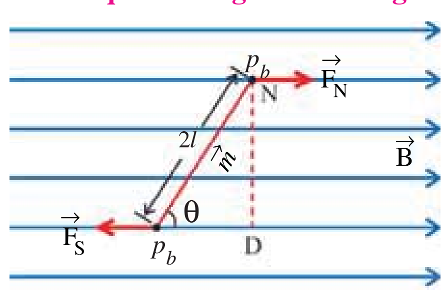


Figure 5.10 Torque Acting on a Magnetic Dipole of Magnetic Moment $\vec{m}$ in Uniform Magnetic Field $\vec{B}$

In Chapter-4 we have studied that the torque acting on a rectangular coil of magnetic moment $\vec{m}$, placed in a uniform magnetic field $\vec{B}$ is

$$\left.\begin{aligned} \vec{\tau} &= \vec{m} \times \vec{B} \\ \therefore\ \tau &= mB\sin\theta \end{aligned}\right\} \tag{5.7.1}$$

Where θ is the angle between $\vec{m}$ and $\vec{B}$ (sometimes magnetic moment is also represented by symbol $\vec{\mu}$).

This fact can be observed by placing a bar magnet or magnetic needle of magnetic dipole moment $\vec{m}$ in a uniform magnetic field $\vec{B}$ (See figure 5.10). In terms of pole strength, the magnetic field $\vec{B}$ can be considered equivalent to the force acting on unit pole strength. The magnetic field exerts equal and opposite forces $\vec{F}_N$ and $\vec{F}_S$ on the north and south poles. But since these forces do not lie on a straight line, they form a couple. Perpendicular distance between these two forces is ND. Under the influence of this couple, the magnetic dipole rotates to a new position making angle θ with the direction of magnetic field $\vec{B}$.

If the angle θ (in radian) in equation (5.7.1) is small, then $\sin\theta \approx \theta$.

$$\therefore \ \tau = m\text{B}\theta \qquad (5.7.2)$$

This torque, in the figure, is trying to rotate the dipole in a clockwise direction. Now if we try to rotate the dipole in anticlockwise direction further by a small angle θ with respect to this equilibrium position, then the torque represented by equation (5.7.1) will act in opposite direction. Thus we may write this restoring torque with negative sign as

$$\tau = -m\text{B}\theta \qquad (5.7.3)$$

According to Newton's second law of motion (for rotational motion)

$$\text{I}_\text{m}\frac{d^2\theta}{dt^2} = -m\text{B}\theta \qquad (5.7.4)$$

Where I_m is the moment of inertia of the magnetic dipole with respect to an axis perpendicular to the plane of figure and passing through the centre of the dipole.

$$\therefore \frac{d^2\theta}{dt^2} = -\frac{m\text{B}}{\text{I}_\text{m}}\theta = -\omega^2\theta \qquad (5.7.5)$$

Equation (5.7.5) is similar to the differential equation for angular simple harmonic motion. Hence the angular frequency

$$\omega = \sqrt{\frac{m\text{B}}{\text{I}_\text{m}}} \qquad (5.7.6)$$

$$\therefore \ \text{T} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\text{I}_\text{m}}{\text{mB}}} \qquad (5.7.7)$$

$$\text{which gives B} = \frac{4\pi^2\text{I}_\text{m}}{\text{mT}^2} \qquad (5.7.8)$$

The potential energy of the magnetic dipole in the external field $\vec{B}$ is given by

$$\text{U}_\text{B} = \int \tau d\theta = \int m\text{B}\sin\theta d\theta = m\text{B}\int \sin\theta d\theta$$

$$\therefore \ \text{U}_\text{B} = -m\text{B}\cos\theta = -\vec{m}\cdot\vec{\text{B}} \qquad (5.7.9)$$

In equation (5.7.9) we have taken the constant of integration to be zero by considering the potential energy to be zero at $\theta = 90°$, i.e. when the magnetic dipole is perpendicular to the field.

At $\theta = 0°$, $\text{U}_\text{B} = -m\text{B}\cos 0° = -m\text{B}$,

which is the minimum value of potential energy representing most stable position of the magnetic dipole.

At $\theta = 180°$, $\text{U}_\text{B} = -m\text{B}\cos 180° = m\text{B}$,

which is the maximum value of potential energy representing most unstable position of the magnetic dipole.

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Illustration 3 : A magnetic needle placed in uniform magnetic field has magnetic moment 6.7×10^{-2} A m^2, and moment of inertia of 15×10^{-6} kg m^2. It performs 10 complete oscillations in 6.70 s. What is the magnitude of the magnetic field ?

Solution : The periodic time of oscillation is, $T = \frac{6.70}{10} = 0.67$ s, and

$$B = \frac{4\pi^2 I_m}{mT^2} = \frac{4\times(3.14)^2\times 15\times 10^{-6}}{6.7\times 10^{-2}\times(0.67)^2} = 0.02 \text{ T}$$

Illustration 4 : A short bar magnet is placed in an external magnetic field of 600 G. When its axis makes an angle of 30° with the external field, it experiences a torque of 0.012 N *m*.

(a) What is the magnetic moment of the magnet ?

(b) What is the work done in moving it from its most stable to most unstable position ?

(c) The bar magnet is replaced by a solenoid of cross-sectional area 2×10^{-4} m^2 and 1000 turns, but having the same magnetic moment. Determine the current flowing through the solenoid.

Solution : $B = 600$ G $= 600 \times 10^{-4}$T, $\theta = 30°$, $\tau = 0.012$ N *m*, N = 1000,

$A = 2 \times 10^{-4}$ m^2

(a) From equation (5.7.1)

$\tau = mB\sin\theta$

$\therefore\ 0.012 = m \times 600 \times 10^{-4} \times \sin 30°$

$\therefore\ m = 0.40$ A m^2 (since $\sin 30° = \frac{1}{2}$)

(b) From equation (5.7.9), the most stable position is at $\theta = 0°$ and the most unstable position is at $\theta = 180°$. Hence the work done,

$W = U_B(\theta = 180°) - U_B(\theta = 0°) = mB - (-mB) = 2mB$

$= 2 \times 0.40 \times 600 \times 10^{-4} = 0.048$ J

(c) From equation (5.6.3)

$m_S = NIA$

But $m_S = m = 0.40$ A m^2, from part (a).

$\therefore\ 0.40 = 1000 \times I \times 2 \times 10^{-4}$

$\therefore\ I = 2$ A

5.8. Gauss's Law for Magnetic Field

From Figure (5.3-*a*) and (5.3-*b*) we can see that, for any closed surface like (i) or (ii), the number of magnetic field lines entering the closed surface is equal to the number of field lines leaving the surface. Since the magnetic field lines always form closed loops, the magnetic flux, associated with any closed surface is always zero.

$$\therefore \oint_{\text{closed surface}} \vec{B}\cdot\vec{da} = 0 \qquad (5.8.1)$$

where $\vec{B}$ is the magnetic field and $d\vec{a}$ is an infinitesimal area vector of the closed surface. **"The net magnetic flux passing through any closed surface is zero."** This statement is called Gauss's law for magnetic field.

According to the Gauss's law for electric field

$$\oint \vec{E}\cdot \vec{da} = 0 = \frac{\Sigma q}{\varepsilon_0} \qquad (5.8.2)$$

In equation (5.8.2) if $\sum q = 0$, then

$$\oint \vec{E}\cdot \vec{da} = 0 \qquad (5.8.3)$$

Comparing this equation with equation (5.8.1), we can write that the Gauss law for magnetic fields indicate that there does not exist any net magnetic monopole (magnetic charge ?) that is enclosed by the closed surface. The unit of magnetic flux is Weber (Wb).

$1 \text{ Wb} = 1 \text{ T m}^2 = 1 \text{ NmA}^{-1}$

5.9. The Magnetism of Earth and Magnetic Elements

We all are aware of the fact that the Earth has its own magnetic field. The magnetic field on the surface of Earth is of the order of 10^{-5} T (T = tesla).

The magnetic field on the Earth resembles that of a (hypothetical) magnetic dipole as shown in figure 5.11.

The magnitude of magnetic moment $\vec{m}$ of this (hypothetical) dipole is of the order of 8.0×10^{22} J T^{-1}. The axis MM of the dipole moment $\vec{m}$ does not coincide with the axis of rotation RR of the Earth, but is tilted by about 11.5°. The dipole axis MM intersects the Earth's geomagnetic north pole somewhere in north Canada, and the geomagnetic south pole in Antarctica. The magnetic field lines emerge out in the southern hemisphere and enter in the northern hemisphere. The actual south pole of earth's magnetic dipole is lying in the direction in which the north pole of magnetic needle, capable of rotating freely in the horizontal plane, remains stationary. Generally, we call this direction on earth as "Earth's magnetic north." The geomagnetic poles of Earth are located approximately 2000 km away from the geographic poles.

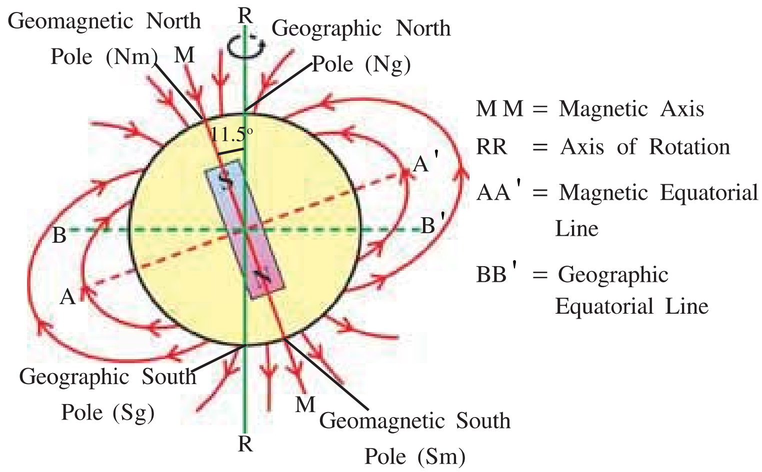


Figure 5.11 Magnetic Field of Earth

The geographic and geomagnetic equators intersect each other at longitude 6° west and 174° east. In India, Thumba near Trivandrum is on the magnetic equator, and hence it has been selected as the rocket launching station.

Each place on earth has a particular latitude and longitude which can be obtained from a good book of horoscope or map. The longitude circle passing through any place determines its geographic North-South direction. An imaginary vertical plane at a place on the Earth containing the longitude circle and the geographic axis of the Earth is called the **geographic meridian** (See figure 5.12).

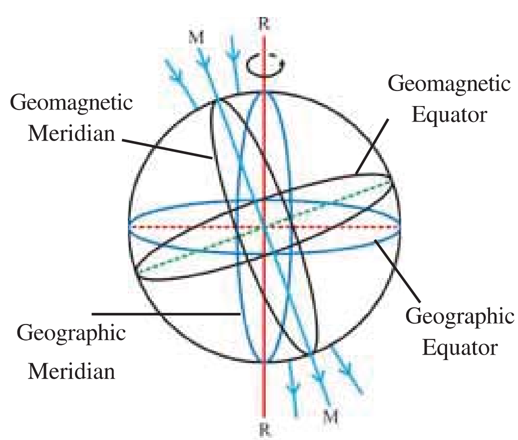


Figure 5.12 Geographic and Geomagnetic, Equator and Meridian of the Earth

Further, the magnetic field lines of geomagnetic dipole are also passing through every place on Earth. **Hence an imaginary vertical plane at a place on the Earth, passing through the magnetic axis and containing magnetic field lines is called magnetic meridian at that place.**

Illustration 5 : The Earth's magnetic field at some place on magnetic equator of Earth is 0.4 G. Estimate the magnetic dipole moment of the Earth. Consider the radius of Earth at that place to be 6.4×10^6 *m*. ($\frac{\mu_0}{4\pi} = 10^{-7}$T m A^{-1}, and 1 G = 10^{-4} T)

Solution : The magnitude of equatorial magnetic field, according to equation (5.6.6) is

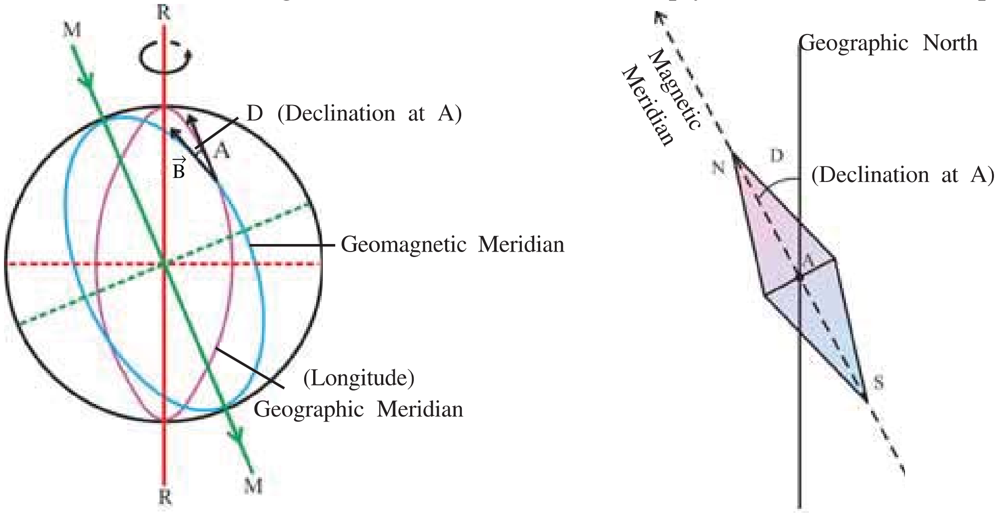
$$B_E = \frac{\mu_0 m}{4\pi y^3}$$

But $B_E = 0.4$ G $= 4 \times 10^{-5}$ T

$$\therefore \ m = \frac{4\pi y^3 B_E}{\mu_0} = \frac{B_E y^3}{\left(\mu_0 / 4\pi\right)} = \frac{4\times10^{-5}\times(6.4\times10^6)^3}{10^{-7}} = 1.05 \times 10^{23} \text{ A m}^2$$

5.9.1. Geomagnetic Elements : In order to describe the magnetic field of Earth scientifically, certain magnetic parameters are defined, called geo-magnetic elements.

Magnetic Declination : The angle between the magnetic meridian and the geographic meridian at a place on surface of Earth is called magnetic declination at that place. Thus, the angle between the true geographic north and the magnetic north at any place on the surface of Earth is the magnetic declination (D) or simply declination at that place.



(a) Declination at Point A on the Surface of Earth

(b) Magnetic Needle Placed in Horizontal Plane at Point A

Figure 5.13

As shown in figure (5.13-*a*) consider point A on the surface of Earth. At this point, the direction of true geographic north is determined from tangent to A the longitude circle of geographic meridian. A magnetic needle free to rotate in horizontal plane aligns along the magnetic meridian at point A. The north pole of the needle points towards the geomagnetic north pole (tangent to the magnetic meridian at A). The angle between the geographic meridian and magnetic meridian at point A indicates the declination at the point A.

The declination is larger at higher latitudes and smaller near the equator. The declination is small in India, it being 0°58' west at Bombay, and 0°41' east at Delhi. Thus, at both these places the magnetic needle shows true north quite accurately.

Magnetic dip angle or inclination : Magnetic dip angle or inclination is the angle ϕ (up or down) that the magnetic field of Earth makes with the horizontal at a place in magnetic meridian.

Magnetic field lines are not locally horizontal at all places on Earth. At a place near north Canada, magnetic field lines point vertically downwards, whereas at a place on the magnetic equator, these field lines are horizontal. At the magnetic equator, dip angle is zero. As we move towards magnetic pole, the dip angle increases and becomes 90° at magnetic poles.

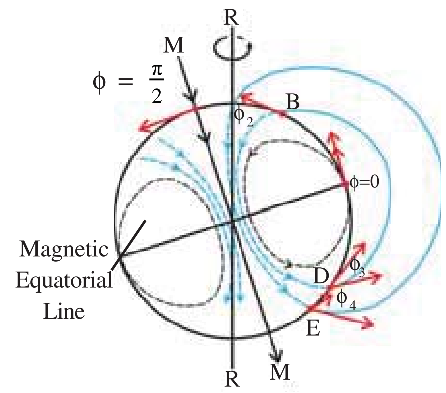


Figure 5.14 Magnetic Dip Angle at Different Places on the Earth's Surface in Magnetic Meridian [Only for Information]

Horizontal Component and Veritical Component of Earth's Magnetic field

Figure 5.15 shows the Earth's magnetic field ($\vec{B}$), angle of declination (D) and the angle of dip (ϕ) at a place (P).

The magnetic field $\vec{B}$ at point P is resolved into horizontal component $\vec{B}_H$ pointing towards geomagnetic north pole, and vertical component $\vec{B}_V$ pointing towards the centre of Earth. The angle made by $\vec{B}_H$ with geographic meridian is the angle of declination (D), whereas the angle between $\vec{B}_H$ and $\vec{B}$ is the angle of dip or inclination (ϕ).

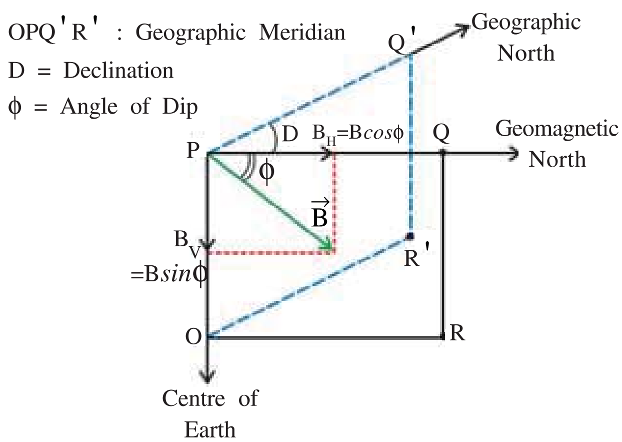


Figure 5.15 Components of Earth's Magnetic Field $\vec{B}$

The declination D, the angle of dip ϕ, and the horizontal component of Earth's field $\vec{B}_H$ are known as geomagnetic elements or the elements of Earth's magnetic field.

For the magnetic meridian OPQR of figure (5.15), we have

$$B_V = B\sin\phi \qquad (5.9.1)$$

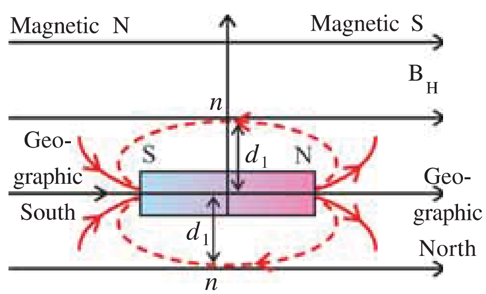
$$B_H = B\cos\phi \qquad (5.9.2)$$

$$\therefore \tan\phi = \frac{B_V}{B_H} \qquad (5.9.3)$$

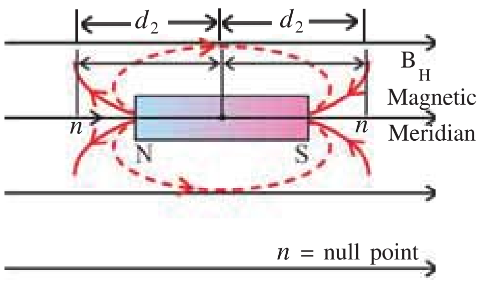
$$\text{and } B = \sqrt{{B_V}^2 + {B_H}^2} \qquad (5.9.4)$$

Illustration 6 : A short bar magnet with magnetic dipole moment 1.6 A m^2 is kept in magnetic meridian in such a way that its north pole is in north direction. In this case, the null (neutral) point is found at a distance of 20 cm from the centre of the magnet. Find the horizontal component of the Earth's magnetic field.

Next, the magnet is kept in such a way that its magnetic north pole is in south direction. Find the positions of neutral (null) points in this case.



(a)



(b)

Solution : From the figure (a) one can observe that on the magnetic equator of the magnet, horizontal field lines of the earth's magnetic field and the magnetic field lines due to the magnet are in mutually opposite directions. Hence in this case, one finds two points on magnetic equator of the magnet at equal distance from the magnet (one above and one below) in such a way that at these points the above mentioned two magnetic fields are equal in magnitude and opposite in directions. At such points the resultant magnetic field is zero. Such points are called **neutral** or **null points.**

Here, $m = 1.6 \text{ A } m^2$

Let, the distance of neutral points from the centre of the magnet is

$d_1 = 20 \text{ cm} = 0.2 \ m$

Now the magnetic field due to a short bar magnet on its equatorial plane B_1 must equal B_H.

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{m}{d_1^3} = B_H$$

$$\therefore \ B_H = \frac{10^{-7} \times 1.6}{(0.2)^3} = 2 \times 10^{-5} \text{ T}$$

However if the bar magnet is kept as in part (b) of the Figure, then it is clear that on the magnetic axis, B_H and the magnetic field due to the magnet are in mutually oposite directions. In this case the neutral points are on the axis. Let d_2 be the distance of such points from the centre of magnet, then B_2, the magnetic field on axis, must be equal to B_H,

$$\therefore \ B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2m}{d_2^3} = B_H$$

$$\therefore \ d_2^3 = \frac{10^{-7} \cdot 2m}{B_H} = \frac{10^{-7} \times 2 \times 1.6}{2 \times 10^{-5}} = 16 \times 10^{-3}$$

$$\therefore \ d_2 = 2.52 \times 10^{-1} \ m = 2.52 \text{ cm}$$

Illustration 7 : A magnet is hung horizontally in the magnetic meridian by a wire without any twist. If the supporting wire is given a twist of 180° at the top, the magnet rotates by 30°. Now if another magnet is used, then a twist of 270° at the supporting end of wire also produces a rotation of the magnet by 30°. Compare the magnetic dipole moments of the two magnets.

Solution : If resultant twist in the wire = δ,

$$\delta_1 = 180° - 30° = 150° = 150 \times \frac{\pi}{180} \text{ rad}$$

and $\delta_2 = 270° - 30° = 240° = 240 \times \frac{\pi}{180}$ rad

If the twist–constant for the wire is k then

Rotating torque, $\tau_1 = k\delta_1$ and $\tau_2 = k\delta_2$

Here α is the angle made by the magnetic dipole moment with the magnetic meridian.

$$\tau_1' = m_1 B_H \sin\alpha$$

Since the second magnet is also rotated by the same angle.

$$\tau_2' = m_2 B_H \sin\alpha$$

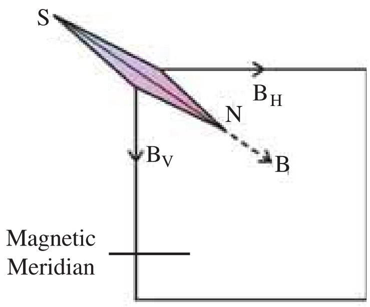
At equilibrium $\tau_1 = \tau_1'$ and $\tau_2 = \tau_2'$

$\therefore \frac{\tau_1'}{\tau_2'} = \frac{\tau_1}{\tau_2}$

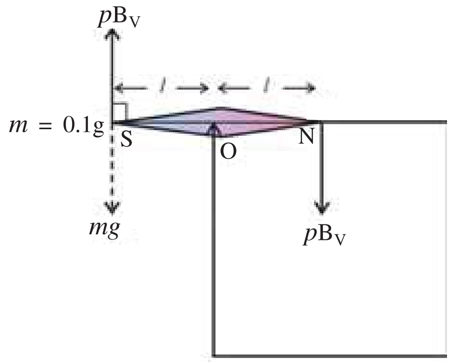
$\therefore \frac{m_1}{m_2} = \frac{\delta_1}{\delta_2} = \frac{150}{240} = \frac{5}{8}$

Illustration 8 : A magnetic needle is hung by an untwisted wire, so that it can rotate freely in the magnetic meridian. In order to keep it in the horizontal position, a weight of 0.1g is kept on one end of the needle. If the magnetic pole strength of this needle is 10 A *m*, find the value of the vertical component of the earth's magnetic field. (g = 9.8 m s^{-2})

Solution :



(a) Normal Position



(b) Position after inserting weight

Figure (a) shows the position of the magnetic needle in the magnetic meridian without any weight. In figure (b), a mass m is kept on the S–pole of the needle.

The vector sum of torques due to all forces must be zero for the equilibrium of the needle in horizontal direction.

$\therefore -pB_v(l) - pB_v(l) + m.g(l) = 0$

[The torque producing rotations in clockwise direction is taken as negative.]

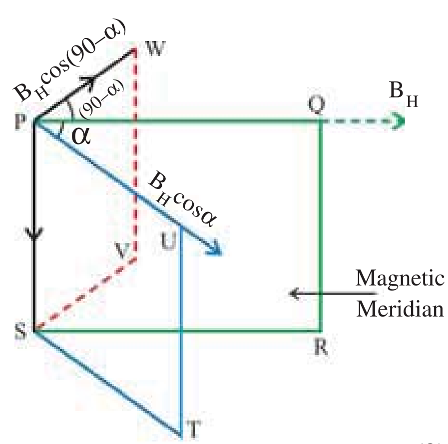
$\therefore 2pB_v = mg$

$\therefore B_v = \frac{mg}{2P} = \frac{10^{-4} \times 9.8}{2 \times 10}$ | $m = 0.1 \text{ g} = 10^{-4}$ kg,

$\therefore B_v = 4.9 \times 10^{-5}$ T | $p = 10$ A m

Illustration 9 : As shown in figure, plane PSTU forms an angle of α and plane PSVW makes an angle of $(90° - \alpha)$ with the magnetic meridian, respectively. The value of magnetic dip angle in plane PSTU is ϕ_1 and its value in plane PSVW is ϕ_2. If the actual dip angle at the place is ϕ, show that,

$cot^2\phi = cot^2\phi_1 + cot^2\phi_2$



Solution : $tan\phi = \frac{B_V}{B_H}$ (1)

In plane PSTU horizontal component is $B_H cos\alpha$

$\therefore tan\phi_1 = \frac{B_V}{B_H cos\alpha} \Rightarrow cos\alpha = \frac{tan\phi}{tan\phi_1} = tan\phi \cdot cot\phi_1$

(from equation (1)) (2)

Similarly for plane PSVW

$sin\alpha = tan\phi \cdot cot\phi_2$ (3)

Squaring and summing the equations (2) and (3)

$cos^2\alpha + sin^2\alpha = 1 = tan^2\phi\ (cot^2\phi_1 + cot^2\phi_2)$

$\therefore cot^2\phi = cot^2\phi_1 + cot^2\phi_2$

5.10 Magnetization and Magnetic Intensity

Consider a solenoid of N turns having length l. When a current I_f is passed through it, the magnetic field produced inside the solenoid (with air or vacuum) is

$$B_0 = \mu_0 n I_f \tag{5.10.1}$$

Where $n = \frac{N}{l}$ = number of turns per unit length of solenoid

This current I_f is called **free current**. If we denote the free current per unit length by i_f then

$$i_f = nI_f \tag{5.10.2}$$

$$\therefore\ B_0 = \mu_0 i_f \tag{5.10.3}$$

Now a material whose magnetic properties are to be studied is placed inside the solenoid. Let l be the length of the material, and A be its cross-sectional area. The magnetic field B_0, present inside the solenoid due to magnetizing current i_f, magnitizes the material such that it acquires some magnetic moment, say $\vec{m}$. This magnetic moment $\vec{m}$ of the material can be considered to be produced due to an equivalent surface current loop carrying current I_b. This current is called **bound current.** The dipole moment of this current loop is

$$\vec{m} = I_b \vec{A} \tag{5.10.4}$$

where A = area of cross-section of the material = area of current loop.

The net magnetic moment per unit volume of the material is called magnetization M of the material. Thus

$$M = \frac{m}{V} = \frac{I_b A}{lA} = \frac{I_b}{l} = i_b \tag{5.10.5}$$

Here $i_b = \frac{I_b}{l}$ = bound current per unit length of the core material.

The unit of M is A m^2 m^{-3} = A m^{-1}. Here M is a vector quantity. Its direction is along $\vec{m}$.

Thus the total magnetic field inside the magnetic core material placed inside the solenoid is due to both currents i_f and i_b.

$$\therefore\ B = \mu_0 (i_f + i_b) \tag{5.10.6}$$

Using equation (5.10.5) in (5.10.6)

$$B = \mu_0 (i_f + M) \tag{5.10.7}$$

$$\therefore\ \frac{B}{\mu_0} - M = i_f \tag{5.10.8}$$

Here, $\frac{B}{\mu_0} - M$ is defined as magnetic intensity H, and its value is equal to magnetizing current, i_f. Hence

$$\frac{B}{\mu_0} - M = H = i_f \tag{5.10.9}$$

$$B = \mu_0 (H + M) \tag{5.10.10}$$

Thus, the magnetic field B induced in a substance, depends on H and M. Further, it is observed that, if H is not too much strong, then the magnetization M induced in the substance is proportional to magnetic intensity H.

$\therefore$ $M = \chi_m H$ (5.10.11)

Here χ_m is a constant, called magnetic susceptibility of the material of the substance. It is a dimensionless quantity. Its value depends on the type of material and its temperature. It is a measure of how a magnetic material responds to external magnetic field. The magnetic susceptibility of some of the substances is listed in table (5.2) for information only.

Table 5.2 Magnetic Susceptibility of some Elements at 300 K (for information only)

Dimagnetic Substance	χ_m	Paramagnetic Substance	χ_m
Bismuth	-1.66×10^{-5}	Aluminium	2.3×10^{-5}
Copper	-9.8×10^{-6}	Calcium	1.9×10^{-5}
Dimond	-2.2×10^{-5}	Choromium	2.7×10^{-4}
Gold	-3.6×10^{-5}	Lithium	2.1×10^{-5}
Lead	-1.7×10^{-5}	Magnesium	1.2×10^{-5}
Mercury	-2.9×10^{-5}	Niobim	2.6×10^{-5}
Nitrogen (STP)	-5.0×10^{-9}	Oxygen (STP)	2.1×10^{-6}
Silver	-2.6×10^{-5}	Platinum	2.9×10^{-4}
Silicon	-4.2×10^{-6}	Tungsten	6.8×10^{-5}

The interpretation of equation (5.10.6) shows that, without putting magnetic material in solenoid, if the same magnetic field $[B = \mu_0 (i_f + i_b)]$ is required to be produced, then over and above the current I_f, an additional current I_m must be passed through the solenoid, such that the additional magnetizing current per unit length $nI_m = i_b$ is produced.

The substances for which χ_m is positive are called paramagnetic, for which $\vec{M}$ and $\vec{H}$ are in the same direction. The substances for which χ_m is negative are called diamagnetic, for which $\vec{M}$ and $\vec{H}$ are in opposite direction.

Substituting (5.10.11) in (5.10.10),

$B = \mu_0 [H + \chi_m H] = \mu_0 (1 + \chi_m) H = \mu H$ (5.10.12)

Where $\mu = \mu_0 (1 + \chi_m)$ is called permeability (magnetic permeability) of the material. $\frac{\mu}{\mu_0}$ is called relative permeability of the material, denoted by μ_r.

$\therefore$ $\mu_r = \frac{\mu}{\mu_0} = 1 + \chi_m$ (5.10.13) which gives,

$$B = \mu_0\mu_r H \quad (5.10.14)$$

Note : The vacuum cannot be magnetized. Hence for vacuum M = 0. Thus from equation (5.10.10), for vacuum $B = \mu_0 H$.

Illustration 10 : A solenoid has a core of material with relative permeability of 400. The current passing through the wire of solenoid is 2A. If the number of turns per cm are 10, calculate the magnitude of

(a) H, (b) B, (c) χ_m, (d) M, and (e) the additional magnetizing current I_m. (Take $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}).

Solution : Here $\mu_r = 400$, I = 2 A, $n = 10 \frac{\text{turns}}{\text{cm}} = 1000 \frac{\text{turns}}{\text{m}}$

(a) Magnetic intensity $H = i_f = nI = 1000 \times 2 = 2000$ A m^{-1}

(b) Magnetic field $B = \mu_0\mu_r H = 4\pi \times 10^{-7} \times 400 \times 2000 = 1.0$ T

(c) Magnetic susceptibility of the core material is

$$\chi_m = \mu_r - 1 = 400 - 1 = 399$$

(d) Magnetization

$$M = \chi_m H = 399 \times 2000 = 7.98 \times 10^5 \approx 8 \times 10^5 \text{ A m}^{-1}$$

(e) The additional magnetizing current I_m is obtained from $M = nI_m = i_b$ as

$$I_m = \frac{M}{n} = \frac{8\times10^5}{1000} = 800 \text{ A}$$

Illustration 11 : The region inside a current carrying torodial winding is filled with tungsten of susceptibility 6.8×10^{-5}. What is the percentage increase in the magnetic field in the presence of the material with respect to the magnetic field without it ?

Solution : The magnetic field in the current carrying torodial winding without tungsten is

$$B_0 = \mu_0 H$$

The magnetic field in the same current carrying torodial winding with tungsten is

$$B = \mu H$$

$$\therefore \quad \frac{B - B_0}{B_0} = \frac{\mu - \mu_0}{\mu_0}$$

But $\mu = \mu_0 (1 + \chi_m) \Rightarrow \frac{\mu}{\mu_0} = 1 + \chi_m \Rightarrow \frac{\mu}{\mu_0} - 1 = \chi_m \Rightarrow \frac{\mu - \mu_0}{\mu_0} = \chi_m$

Hence, $\frac{B - B_0}{B_0} = \chi_m$

$\therefore$ Percentage increase in the magnetic field in presence of tungsten is

$$\frac{B - B_0}{B_0} \times 100 = (6.8 \times 10^{-5}) \times 100 = 6.8 \times 10^{-3} \ \%$$

5.11 Magnetic Properties of Materials : Dia, Para and Ferro Magnetism

We know that each electron in an atom possess an orbital magnetic dipole moment and a spin magnetic dipole moment, that add vectorially. This type of resultant magnetic moment of each electron in an atom add vectorially, and the resultant dipole moment of each atom in the sample of a material add vectorially. If the resultant of all these dipole moments produces a magnetic field, then the material is said to be magnetic material.

The behaviour of a material in presence of an external magnetic field classifies the material as diamagnetic, paramagnetic or ferromagnetic. The classification of dia, para and ferro magnetic materials in terms of their susceptibility, relative permeability, and a small positive number ε (this 'ε' should be not be taken as permitivity of the medium) used to quantity paramagnetic material are briefly represented in Table 5.3.

Table 5.3

Diamagnetic	Parmagnetic	Ferromagnetic
$-1 \le \chi_m < 0$	$0 < \chi_m < \varepsilon$	$\chi_m >> 1$
$0 \le \mu_r < 1$	$1 < \mu_r < 1 + \varepsilon$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$

5.11.1 Diamagnetic Materials : The atoms/molecules of gold, silver, copper, silicon, water and bismuth etc. do not possess permanent magnetic dipole moments. The orbital motion of the electrons and their spins are such that their total magnetic dipole moment is zero. Such materials are called diamagnetic materials.

When the diamagnetic material is placed in an external magnetic field, a net magnetic moment in a direction opposite to that of the external magnetic field is induced in each atom. Due to this, each atom of diamagnetic material experiences repulsion.

Figure 5.16 shows a bar of diamagnetic material placed in an external magnetic field $\vec{B}$. The field lines are repelled by induced magnetic field (weak) in the material, and the resultant field inside the material is reduced.

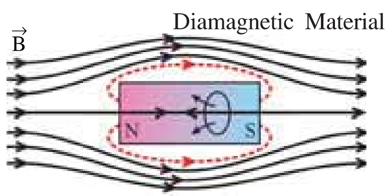
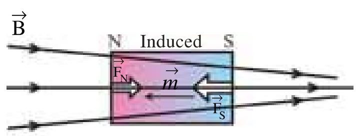


Figure 5.16 Diamagnetic Material in External Magnetic Field

As shown in figure 5.17, when the bar of diamagnetic material is placed in a non-uniform magneic field, the induced magnetic south pole is in the strong magnetic field, and the induced north pole is in the weak magnetic field. Hence, the magnetic force on the induced S-pole ($\vec{F}_S$ acting towards left) is more than the force on induced N-pole ($\vec{F}_N$ towards right). As a result the bar of diamagnetic material experiences a resultant force towards the region of weaker magnetic field. The magnetic susceptibility χ_m of diamagnetic materials is negative.



$\vec{F} = \vec{F}_S - \vec{F}_N$ (Towards Left)

Figure 5.17 Force Acting on Diamagnetic Material Placed in Non-uniform Magnetic Field

For superconductors $\chi_m = -1$ and $\mu_r = 0$. When superconductors are placed in an external magnetic field, the field lines are completely expelled. The phenomenon of perfect diamagnetism in superconductors is called the **Meissner effect,** after the name of its discoverer. Superconducting magnets can be used for running magnetically leviated superfast trains.

5.11.2 Paramagnetism : In paramagnetic material, the atoms/molecules possess permanent magnetic dipole moments. Normally, the molecules are arranged such that, their magnetic dipole moments are randomly oriented. Hence the resultant magnetic moment of the material is zero (See figure 5.18).

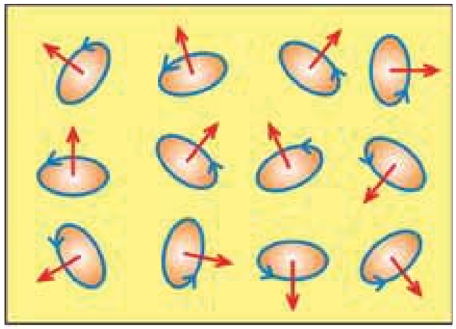


Figure 5.18 **Normal Dipole Distribution in Paramagnetic Material**

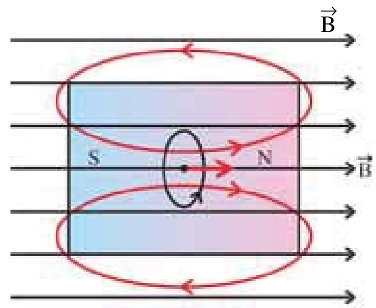


Figure 5.19 **Magnetic Dipole Moment of One Dipole Shown Aligned with $\vec{B}$**

When the paramagnetic material is placed in external magnetic field $\vec{B}$, these tiny dipoles try to align in the direction of $\vec{B}$. However, due to thermal agitation, all dipoles could not attain 100% alignment in the direction of $\vec{B}$.

Figure 5.19 shows the magnetic field due to the magnetic dipole aligned with $\vec{B}$. The field lines get concentrated inside the material (see figure 5.20)

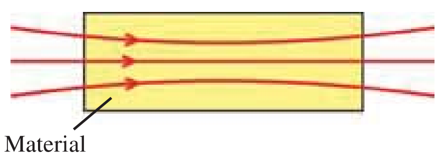


Figure 5.20 **Magnetic Field Lines in Paramagnetic Material**

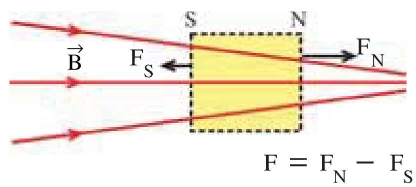


Figure 5.21 **Paramagnetic Material in Non-uniform Magnetic Field**

When a bar of paramagnetic material is placed in non-uniform magnetic field (See figure 5.21) the resultant north pole of the magnetized material feels strong magnetic field, whereas the south pole experiences comparatively weak magnetic field. As a result of which the resultant force ($F_N - F_S$) acts towards the stronger magnetic field (towards right) on the bar of paramagnetic material. In practice, this force is very weak.

Aluminium, sodium, calcium, oxygen at STP and copper chloride are few examples of paramagnetic materials. The magnetic susceptibility χ_m of paramagnetic materials is positive.

In 1895 Pierre Curie observed that the magnetization M of a paramagnetic material is directly proportional to the external magnetic field $\vec{B}$ and inversely proportional to its absolute temperature T, called Curie's law,

$$M = C\frac{B}{T} \qquad (5.11.1)$$

Where C = Curie's constant

From equation (5.11.1)

$$M = C\frac{B}{\mu_0}\frac{\mu_0}{T} = CH\frac{\mu_0}{T}$$

$$\therefore \frac{M}{H} = \chi_m = C\frac{\mu_0}{T} \qquad (5.11.2)$$

$$\therefore\ \mu_r - 1 = C\frac{\mu_0}{T} \qquad (5.11.3)$$

As we increase the applied external magnetic field or decrease the temperature of the paramagnetic material, or both, then alignment of atomic magnetic moments increase. Thus magnetization M increases. When magnetic moments of all atoms are aligned parallel to the external magnetic field, M, μ_r and χ_m become maximum. This situation is called **satuaration magnetization**. Curie's law is not obeyed after this state. If there are N atoms in volume V of the sample, each with magnetic moment $\vec{m}$, then at saturation magnetization

$$\vec{M}_{max} = \frac{N\vec{m}}{V} \qquad (5.11.4)$$

5.11.3 Ferromagnetism : The atoms of iron, cobalt, nickel and their alloys possess permanent magnetic dipole moments due to spin of electrons in outermost orbits. The atoms of such materials are arranged in such a way that over a region called **domain**, the magnetic moment of the atoms are aligned in the same direction. In unmagnetized sample, such domains having a net magnetization are randomly oriented so that the effective magnetic moment is zero (See figure 5.22).

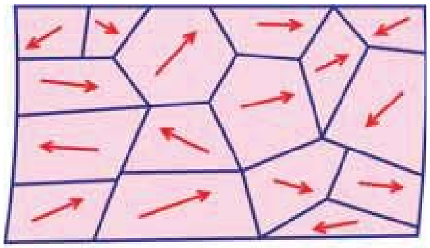


Figure 5.22 Random Arrangement of Domains

The explanation about the formation of such domains requires quantum mechanics which is beyond the scope of this book. The typical domain size is about 1 mm and the domain contains about 10^{11} atoms. The boundaries between the adjacent domains, having different orientations of magnetic moment, are called **domain walls**.

Hysteresis : The effect of an external magnetic field on ferromagnetic material is quite interesting. To understand this, consider an unmagnetized ferromagnetic material having initial magnetic field B = 0. Suppose this material is placed in a solenoid of n turns per unit length as shown in figure 5.8 (b). On passing a current through the solenoid, the magnetic field is generated, which induces magnetic moment inside the rod. Knowing the volume of the rod, we can evaluate M, the magnetic moment per unit volume. We already know that

$$\frac{B}{\mu_0} - M = i_f = H \text{ (See Equation (5.10.9))}$$

where, i_f = current passing through unit length of the solenoid.

From the values of H and M, we can evaluate B and study its variation with i_f (hence the variation of H). The graph of B versus. H can be drawn as shown in figure 5.23.

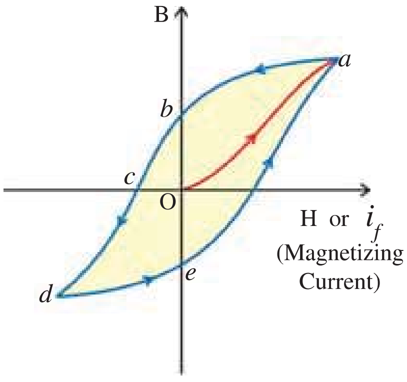


Figure 5.23 Hysteresis Loop

At the point 0 in the graph, the substance is in its normal condition, without any resultant magnetic field. As H (or i_f) is increased, B increases, but this increase is not linear. Near point *a*, B is maximized, which is the saturation magnetization condition of the rod.

One can explain the curve O*a* as follows : Starting from O, as long as the value of H is small, most of the atoms, due to strong bonding with their neighbours, do not respond to the external magnetic field. But the atoms near the domain boundary are in precarious situation. Hence the domain boundaries, instead of remaining sharp, start shifting. In this situation one domain of the two adjacent domains, increases in size and the other one reduces in size. If we still keep on increasing the value of H, ultimately only one domain survives in the substance and the **saturation magnetization** is acquired near point *a* on the graph.

This process is not reversible. At this stage, if the current in the solenoid is reduced, we do not get back the earlier domain constitution, and when H = 0, we do not get B = 0. This means that when H is made zero, the substance retains certain magnetic moment, hence the curve *ab* represents the effect of reducing H.

The value of B, when H = 0, is called **retentivity** or **remanence**. Now, if the current is increased in reverse direction, then we reach at point *c* in the graph, the value of H for which B = 0 is called **coercivity**. At this point, the magnetic moments of the domains are again in random directions but according to some different domain structure.

If we keep on increasing the current in the reverse direction, B goes on increasing in the reverse direction and saturation magnetization is again acquired, but in opposite direction. After reaching *d*, if the current is reduced, the substance follows the curve *de* and again by reversing the current direction and increasing its value, we obtain the curve *ea*. This process is called **hysteresis cycle.** The area enclosed by the B-H curve represents the heat energy (in joules) lost in the sample per unit volume per cycle.

Hard ferromagnetic substances : The substances with large retentivity are called hard ferromagnetic substances. These are used in producing permanent magnets. Obviously, the hysteresis cycle for such substances is broad (See figure 5.24 (a)). Alnico (an alloy of Al, Ni, Co and Cu) is a hard ferromagnetic meterial. Hence permanent magnets are made using Alnico.

Soft Ferromagnetic Substances : The substances with small retentivity, which means the materials with narrow hysteresis cycle (See figure 5.24 (b)), are called soft ferromagnetic substances. For example soft iron; such materials are used for making electromagnets.

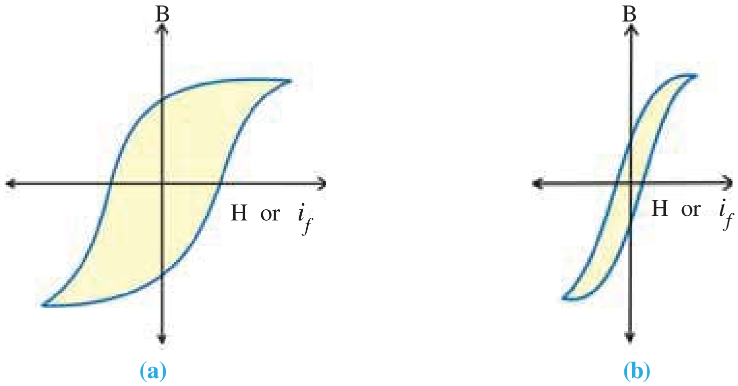


Figure 5.24 Hysteresis Loops for (a) Hard and (b) Soft Ferromagnetic Materials

Effect of Temperature : As the temperature of ferromagnetic substance is increased, the domain structure starts getting distorted. At a certain temperature depending upon the material, it is totally broken up. Each and every atomic magnetic moment attains independence from one another and the substance gets converted to a paramagnetic material.

The temperature at which a ferromagnetic substance is converted into a paramagnetic substance is called **Curie temperature T_C** of that substance. The relation between the magnetic susceptibility of the substance in the acquired paramagnetic form and the temperature T is given by

$$\chi_m = \frac{C_1}{T - T_C}, \ (T > T_C) \tag{5.11.1}$$

where, C_1 is a constant.

Finally, note that the ferromagnetic material is attracted towards the strong field region whenever it is kept in a non-uniform magnetic field.

The hysteresis loop shows that the magnetization of a ferromagnetic material depends on the history (the previous state) of the material as well as on the magnitude of applied field H. The shape and size of the hysteresis loop depends on the properties of ferromagnetic material as well as on the maximum value of applied magnetic field H.

5.12 Permanent Magnets and Electromagnets

The ferromagnetic materials which retain magnetism for a longer period of time at room temperature, are called permanent magnets. These materials have higher retentivity.

Before 400 years, the iron rods were fixed in north-south direction and hammered repeatedly to prepare magnets. Further, if one end of a magnet is continuously rubbed on a fixed steel rod only in one direction, then it acquires permanent magnetism.When a current is passed through a solenoid containing a steel rod, then the rod gets magnetized. Due to hysteresis, the rod retains magnetism even after the current is switched off. The materials like steel, hard alloys, and alnico have high retentivity and high coercivity, and hence are used to prepare permanent magnets.

Soft iron has large permeability and small retentivity, and hence is used to prepare electromagnets. For this purpose, a rod of soft iron is placed in a solenoid as a core, as shown in Figure 5.7(b). On passing a current through the solenoid, the magnetic field associated with the solenoid increases by a thousand fold. When the current through the solenoid is switched off, the associated magnetic field effectively becomes zero.

Electromagnets are used in electric bells and loudspeakers. Giant electromagnets are used in cranes to lift heavy loads made of iron or loads packed in iron containers (boggies).

In certain applications, an AC current is passed through the solenoid containing ferromagnetic material, for example in transformer cores and telephone diaphragms. The hysteresis loop of such materials must be narrow to reduce dissipation of energy in the form of heat.

Illustration 12 : A magnet has coercivity of 3×10^3 A m^{-1}. It is kept in a 10 cm long solenoid with a total of 50 turns. How much current has to be passed through the solenoid to demagnetize it ?

Solution : The value of H for which magnetization is zero is called coercivity.

For a solenoid $H = nI$

Here, $H = 3 \times 10^3$, $n = \frac{N}{l} = \frac{50}{0.1} = 500$

$$\therefore I = \frac{H}{n} = \frac{3 \times 10^3}{5 \times 10^2} = 6 \text{ A}$$

Illustration 13 : There are 2.0×10^{24} molecular dipoles in a paramagnetic salt. Each has dipole moment 1.5×10^{-23} A m^2 (or J T^{-1}). This salt kept in a uniform magnetic field 0.84 T is cooled to a temperature of 4.2 K. In this case the magnetization acquired is 15% of the saturation magnetization. What must be the dipole moment of this sample in magnetic field 0.98 T and at temperature of 2.8 K ? (Assume the applicability of the Curie's law).

Solution : Dipole moment of every molecular dipole = 1.5×10^{-23} A m^2

There are 2.0×10^{24} dipoles in the sample.

$\therefore$ Maximum (saturation) magnetization $= 1.5 \times 10^{-23} \times 2.0 \times 10^{24} = 30$ A m^2

But at 4.2 K, sample has 15 % of saturation magnetization

$\therefore m_1 = 30 \times 0.15 = 4.5$ A m^2

Now according to Curie's law, if m_1 is the dipole moment at T_1 and m_2 the dipole moment at T_2 then

$$\frac{m_1}{m_2} = \frac{B_1}{T_1} \times \frac{T_2}{B_2} \quad (\text{from } m \propto \frac{B}{T})$$

Here B_1 and B_2 are applied magnetic fields

$$\therefore m_2 = m_1 \times \frac{T_1}{T_2} \times \frac{B_2}{B_1}$$

Here, $m_1 = 4.5$ A m^2, $T_1 = 4.2$ K, $T_2 = 2.8$ K, $B_1 = 0.84$ T and $B_2 = 0.98$ T

$$\therefore m_2 = \frac{4.5 \times 4.2 \times 0.98}{2.8 \times 0.84} = 7.87 \text{ A m}^2$$

SUMMARY

1. The north and south magnetic poles cannot be separated by splitting the magnet into two or more pieces. The independent magnetic monopoles does not exist.
2. The magnetic field lines do not intersect at a point.
3. The magnetic field lines of a magnet form continuous closed loops. The magnetic field lines emerge out from the magnetic north pole, reach the magnetic south pole and then passing through the magnet, reach the north pole to complete the loop.
4. The magnetic moment of a current loop of area A, carrying current I is given by $m = IA$. If there are N turns of a loop, then $m = NIA$

   If there are N turns of a loop, then $m = NIA$
5. The axial magnetic field of a current loop is given by $\vec{B}(x) = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{x^3}$
6. The orbital magnetic moment of an electron in an atom is given by $m_0 = \frac{1}{2}evr$
7. When a bar magnet is divided into two equal pieces, the pole strength p_b of each piece remains the same, but the magnetic dipole moment of each piece becomes half of the original value.
8. When a magnet of magnetic moment $\vec{m}$ is placed in external magnetic field $\vec{B}$, the torque acting on it is given by $\vec{\tau} = \vec{m} \times \vec{B}$ or $\tau = mB\sin\theta$ and has potential energy $U_B = -\vec{m}\cdot\vec{B}$
9. The Gauss's law for magnetic field is $\oint_{\text{closed surface}} \vec{B}\cdot\vec{da} = 0$ which states that "the net magnetic flux passing through any closed surface is zero.
10. **Magnetic Meridian :** An imaginary vertical plane at a place on the Earth, passing through the magnetic axis is called magnetic meridian at that place.

11. The angle between the magnetic meridian and the geographic meridian at a place on the surface of Earth is called the magnetic declination (D) at that place.
12. **Magnetic dip or inclination(ϕ) :** It is the angle (up or down) that the magnetic field of Earth makes with the horizontal at a place in magnetic meridian.
$\phi = 0^o$ at magnetic equator and $\phi = 90^o$ at geomagnetic poles.
13. The net magnetic moment per unit volume of the material is called magnetization of the material, represented by $\vec{M} = \frac{\vec{m}}{V}$.
14. The magnetic susceptibility χ_m of a material is a measure of how a magnetic material responds to external magnetic field. It is dimensionless quantity.
15. When a diamagnetic material is placed in non-uniform magnetic field, it experiences a resultant force towards the region of weak magnetic field. The magnetic susceptibility χ_m of diamagnetic material is negative.
16. When a paramagnetic material is placed in non-uniform magnetic field, it experiences a (weak) force towards strong magnetic field. The magnetic susceptibility χ_m of paramagnetic material is positive.
17. According to Curie's law, the magnetization M of a paramagnetic material is given by $M = C\frac{B}{T}$.
When magnetic moments of all atoms are aligned with external magnetic field M, χ_m and μ_r become maximum, called saturation magnetization. Curie's law is not obeyed after saturation magnetization.
18. The atoms of ferromagnetic material possess permanent magnetic dipole moment due to spin of electrons in outermost orbits. These atoms are arranged in such a way that over a region called domain, the magnetic moments of such atoms are aligned in the same direction. In unmagnetized sample, such domains having a net magnetization are randomly oriented so that the effective magnetic moment is zero.
19. The temperature at which a ferromagnetic substance is converted into a paramagnetic substance is called Curie temperature T_C of that substance. The relation between the magnetic susceptibility of the substance in the acquired form and the temperature T is
$\chi_m = \frac{C_1}{T-T_C}$, $(T > T_C)$, where C_1 = constant
20. Permanent magnets have higher retentivity and high coercivity.
21. Soft iron used to prepare electromagnets have large permeability and small retentivity.

EXERCISE

For the following statements choose the correct option from the given options :

1. A magnet of magnetic dipole moment 5.0 A m^2 is lying in a uniform magnetic field of 7×10^{-4} T such that its dipole moment vector makes an angle of 30^o with the field. The work done in increasing this angle from 30^o to 45^o is about .... J.
(A) 5.56×10^{-4} (B) 24.74×10^{-4} (C) 30.3×10^{-4} (D) 5.50×10^{-3}
2. A bar magnet is oscillating in Earth's magnetic field with periodic time T. If a similar magnet with the same mass and dimensions has magnetic dipole moment, which is 4 times that of this magnet, then its periodic time will be ....
(A) $\frac{T}{2}$ (B) 2T (C) T (D) 4T

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3. A circular loop carrying current I is replaced by a bar magnet of equivalent magnetic dipole moment. The point on the loop is lying ....

(A) on equatorial plane of magnet

(B) on axis of the magnet

(C) A and B both

(D) except equatorial plane or axis of bar magnet

4. When a current carrying loop is replaced by an equivalent magnetic dipole

(A) the distance l between the poles is fixed.

(B) the pole strength p of each pole is fixed.

(C) the dipole moment is reversed.

(D) the product pl is fixed.

5. Let r be the distance of a point on the axis of a bar magnet from its centre. The magnetic field at r is always proportional to

(A) $\frac{1}{r^2}$ (B) $\frac{1}{r^3}$

(C) $\frac{1}{r}$ (D) not necessarily $\frac{1}{r^3}$ at all points

6. Magnetic meridian is a plane ....

(A) perpendicular to magnetic axis of Earth.

(B) perpendicular to geographic axis of Earth.

(C) passing through the magnetic axis of Earth.

(D) passing through the geographic axis.

7. At geomagnetic pole, a magnetic needle allowed to rotate in horizontal plane will

(A) stay in north-south direction only (B) stay in any position

(C) stay in east-west direction only (D) become rigid showing no movement

8. The horizontal and vertical components of magnetic field of Earth are same at some place on the surface of Earth. The magnetic dip angle at this place will be ....

(A) 30^o (B) 45^o (C) 0^o (D) 90^o

9. Inside a bar magnet, the magnetic field lines

(A) are not present

(B) are parallel to the cross-sectional area of the magnet

(C) are in the direction from N-pole to S-pole

(D) are in the direction from S-pole to N-pole

10. In non-uniform magnetic field, a diamagnetic substance experiences a resultant force

(A) from the region of strong magnetic field to the region of weak magnetic field.

(B) perpendicular to the magnetic field.

(C) from the region of weak magnetic field to the region of strong magnetic field.

(D) which is zero.

11. A straight steel wire of length l has magnetic moment m. If the wire is bent in the form of a semicircle, the new value of the magnetic dipole moment is

(A) m (B) $\frac{2m}{\pi}$ (C) $\frac{m}{2}$ (D) $\frac{m}{\pi}$

12. At a place on Earth, the horizontal component of Earth's magnetic field is $\sqrt{3}$ times its vertical component. The angle of dip at this place is ....

(A) 0 (B) $\frac{\pi}{2}$ rad (C) $\frac{\pi}{3}$ rad (D) $\frac{\pi}{6}$ rad

13. A place, where the vertical component of Earth's magnetic field is zero has the angle of dip equal to

(A) 0^o (B) 45^o (C) 60^o (D) 90^o

14. A place where the horizontal component of Earth's magnetic field is zero lies at

(A) geographic equator (B) geomagnetic equator

(C) one of the geographic poles (D) one of the geomagnetic poles

15. When a paramagnetic substance is brought near a north pole or a south pole of a bar magnet, it

(A) experiences repulsion (B) experiences attraction

(C) does not experience attraction or repulsion

(D) experiences attraction or repulsion depending upon which pole is brought near to it.

16. A magnetic needle kept on horizontal surface oscillates in Earth's magnetic field. If the temperature of this needle is raised beyond the Curie temperature of the material of the needle, then ....

(A) the periodic time of oscillation will decrease.

(B) the periodic time of oscillation will increase.

(C) the periodic time of the oscillation will not change.

(D) the needle will stop oscillating.

17. A bar magnet of length l, pole strength 'p' and magnetic moment '$\vec{m}$' is split $\frac{l}{2}$ into two equal pieces each of length. The magnetic moment and pole strength of each piece is respectively .... and ....

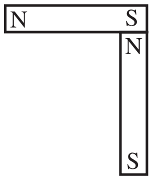
(A) $\vec{m}$, $\frac{p}{2}$ (B) $\frac{\vec{m}}{2}$, p (C) $\frac{\vec{m}}{2}$, $\frac{p}{2}$ (D) $\vec{m}$, p

18. Magnetization for vacuum is ....
(A) negative (B) positive (C) infinite (D) zero

19. A bar magnet of magnetic moment $\vec{m}$ is placed in uniform magnetic field $\vec{B}$ such that $\vec{m} \parallel \vec{B}$. In this position, the torque and force acting on it are .... and .... respectively.
(A) 0, 0 (B) $\vec{m} \times \vec{B}$, mB (C) $\vec{m} \cdot \vec{B}$, mB (D) $\vec{m} \cdot \vec{B}$, $\vec{m} \times \vec{B}$

20. Relative permeability of a substance is 0.075. Its magnetic susceptibility is
(A) 0.925 (B) −0.925 (C) 1.075 (D) −1.075

21. Two similar magnets of magnetic moment m are arranged as shown in figure. The magnetic dipole moment of this combination is



(A) $2m$ (B) $\sqrt{2}\ m$ (C) $\frac{m}{\sqrt{2}}$ (D) $\frac{m}{2}$

22. A magnetic needle kept non-parallel to the magnetic field in a non-uniform magnetic field experiences
(A) a force but not a torque. (B) a torque but not a force
(C) both a force and a torque. (D) neither a force nor a torque

23. A steamer would like to move in the direction making an angle of 10^{o} south with the west. The magnetic declination at that place is 17^{o} west from the north. The steamer should move in a direction ....
(A) making an angle of 83^{o} west with the north pole of Earth.
(B) making an angle of 83^{o} east with the north pole of Earth.
(C) making an angle of 27^{o} west with the south pole of Earth.
(D) making an angle of 27^{o} east with the south pole of Earth.

24. A toroid wound with 100 turns/m of wire carries a current of 3 A. The core of toroid is made of iron having relative magnetic permeability of $\mu_r = 5000$ under given conditions. The magnetic field inside the iron is .... (Take $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})
(A) 0.15 T (B) 0.47 T (C) 1.5×10^{-2} T (D) 1.88 T

ANSWERS

1. (A)	2. (A)	3. (A)	4. (D)	5. (D)	6. (C)
7. (B)	8. (B)	9. (D)	10. (A)	11. (B)	12. (D)
13. (A)	14. (D)	15. (B)	16. (D)	17. (B)	18. (D)
19. (A)	20. (B)	21. (B)	22. (C)	23. (A)	24. (D)

Answer the following questions in brief :

1. What happens if a bar magnet is cut into two pieces transverse to its length/along its length ?
2. Does a current carrying toroid have a north pole and a south pole ?
3. Which phase / phases of matter cannot be ferromagnetic in character ?
4. Magnetic properties of which materials are affected by temperature ?
5. What should be retentivity and coercivity of permanent magnet ?
6. What happens to a ferromagnetic material when its temperature increases above Curie temperature ?
7. What is the unit of magnetic intensity ?
8. What does the hysteresis loop represent ?

9. What are the applications of electromagnet ?
10. What could be the equation for Gauss's law of magnetism, if a monopole of polestrength p is enclosed by a surface ?
11. What happens when a paramagnetic material is placed in a non-uniform magnetic field ?
12. What is the unit of magnetic susceptibility ?
13. What is the declination for Delhi ?
14. Mention the names of diamagnetic materials.
15. Which property of soft iron makes it useful for preparing electromagnet ?

Answer the following questions :

1. Obtain an expression for axial magnetic field of a current loop in terms of its magnetic moment.
2. Explain symbolic notation for detecting north and south pole of magnetic field in a current carrying loop.
3. Obtain an expression for orbital magnetic moment of an electron rotating about the nucleus in an atom.
4. Explain in brief, the Gauss's law for magnetic fields.
5. What is a geographic meridian and a geomagnetic meridian ? What is the angle between them ?
6. Give definition of magnetic declination. How does the declination vary with latitude ? Where is it minimum ?
7. Give definition of magnetic dip. What is the dip angle at magnetic equator ? What happens to dip angle as we move towards magnetic pole from the magnetic equator ?
8. What happens when a diamagnetic material is placed in non-uniform magnetic field ? Explain with necessary Figure.
9. Discuss Curie's law for paramagnetic materials.
10. Discuss why the soft iron is suitable for preparing electromagnets.

Solve the following examples :

1. A toroidal core with 3000 turns has inner and outer radii of 11 cm and 12 cm, respectively. When a current of 0.70 A is passed, the magnetic field produced in the core is 2.5 T. Find the relative permeability of the core. ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})

   [**Ans. :** 685]

2. A paramagnetic gas has 2.0×10^{26} atoms/m^3 with the atomic magnetic dipole moment of 1.5×10^{-23} A m^2 each. The gas is at 27° C. (i) Find the maximum magnetization intensity of this sample. (ii) If the gas in this problem is kept in a uniform magnetic field of 3 T, is it possible to achieve saturation magnetization ? Why ?

   [**Hint :** Thermal energy of an atom of gas is $\frac{3}{2}k_B T$, and

   Maximum potential energy of the atom = mB.

   Find the ratio of thermal energy to the maximum potential energy and give answer.]

   ($k_B = 1.38 \times 10^{-23}$ J K^{-1}) [**Ans. :** 3.0×10^3 A m^{-1}, No]

3. Two small and similar bar magnets have magnetic dipole moment of 1.0 A m^2 each. They are kept in a plane in such a way that their axes are perpendicular to each other. A line drawn through the axis of one magnet passes through the centre of other magnet. If the distance between their centers is 2 m, find the magnitude of magnetic field at the mid point of the line joining their centers.

   [**Ans. :** $\sqrt{5} \times 10^{-7}$ T]

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4. A magnetic pole of bar magnet with pole-strength of 100 A m is 20 cm away from the centre of a bar magnet. Bar magnet has pole-strength of 200 A m and has a length of 5 cm. If the magnetic pole is on the axis of the bar magnet, find the force on the magnetic pole.

[**Ans. :** 2.5×10^{-2} N]

5. The work done for rotating a magnet with magnetic dipole moment *m*, by 90^o from its magnetic meridian is *n* times the work done to rotate it by 60^o. Find the value of *n*.

[**Ans. :** 2]

6. A magnet makes an angle of 45^o with the horizontal in a plane making an angle of 30^o with the magnetic meridian. Find the true value of the dip angle at the place.

[**Ans. :** tan^{-1} (0.866)]

7. An electron in an atom is revolving round the nucleus in a circular orbit of radius 5.3×10^{-11} m, with a speed of 2×10^6 m s^{-1}. Find the resultant orbital magnetic moment and angular momentum of the electron. Take charge of electron = 1.6×10^{-19} C, mass of electron = 9.1×10^{-31}kg. [**Ans. :** 8.48×10^{-24} Am^2, and 9.65×10^{-35} N m s]

8. The magnetic field from a current carrying loop of diameter 1 cm is 10^{-4} T at 10 cm from the centre, along the axis of the loop.

(a) Find the magnetic moment of the loop.

(b) Find the magnetic field at 10 cm from the centre, along the equator of the loop.

Take $\frac{\mu_0}{4\pi} = 10^{-7}$ T m A^{-1} [**Ans. :** (a) 0.5 A m^2, (b) 5×10^{-5} T]

9. A magnet in the form of a cylindrical rod has a length of 5 cm and a diameter of 2 cm. It has a uniform magnetization of 5×10^3 A m^{-1}. Find its net magnetic dipole moment.

[**Ans. :** 7.85×10^{-2} J T^{-1}]

10. An ionized gas consists of 5×10^{21} electrons/m^3 and the same number of ions/m^3. If the average electron kinetic energy is 6×10^{-20} J, and an average ion kinetic energy is 8×10^{-21} J, calculate the magnetization of the gas when a magnetic field of 1.0 T is applied to the gas. [**Ans. :** 340 J T^{-1} m^{-3}]

11. A closely wound solenoid of 6 cm, having 10 turns/cm and area of cross-section 3×10^{-4} m^2 carries a current of 1.0 A. Find the magnetic moment and the pole strength of the solenoid.

[**Ans. :** Magnetic moment of solenoid along its axis = 1.8×10^{-2} A m^2, pole strength of the solenoid = 0.3 A m]

•

6

RAY OPTICS

6.1 Introduction

Light is the agency which stimulates our sense of vision or sight. All curious questions regarding light: its nature, its generation, its interaction with matter, its speed and propagation through medium, etc., are described and explained in a branch of physics called optics. Developments in optics can be classified into three branches:

(1) Ray (Geometric) optics, (2) Wave optics and (3) Quantum optics

Since the wavelength of visible electromagnetic waves (400 nm to 800 nm) is too small compared to objects around us, light can be considered to travel from one point to another along a straight line. This is called rectilinear propagation of light. The path of the light propagation is called a ray, which is never diverging or converging. A bundle of such rays is called deam of light.

The optical phenomena like reflection, refraction and dispersion can be explained by the ray optics. The ray optics is based mainly on the following three assumptions.

(1) Rectilinear propagation of light

(2) Independence of light rays (i.e., they do not disturb one another when they intersect).

(3) Reversibility of path (i.e., they retrace exactly the same path on reversing their direction of propagation).

In the present chapter, we shall study reflection, refraction and dispersion phenomena using ray optics. Optical instruments like microscope and telescope are also studied at the end of the chapter.

6.2 Reflection by Spherical Mirrors

For studying reflection of light by spherical mirrors, we shall revise certain points as under :

The laws of reflection

(1) In the case of reflection of light, the angle of incidence and angle of reflection are equal.

(2) Incident ray, reflected ray and normal drawn at the point of incidence lie in the same plane. While the incident ray and the reflected ray are on either side of the normal.

These laws are valid at every point on any reflecting surface, whether plane or curved.

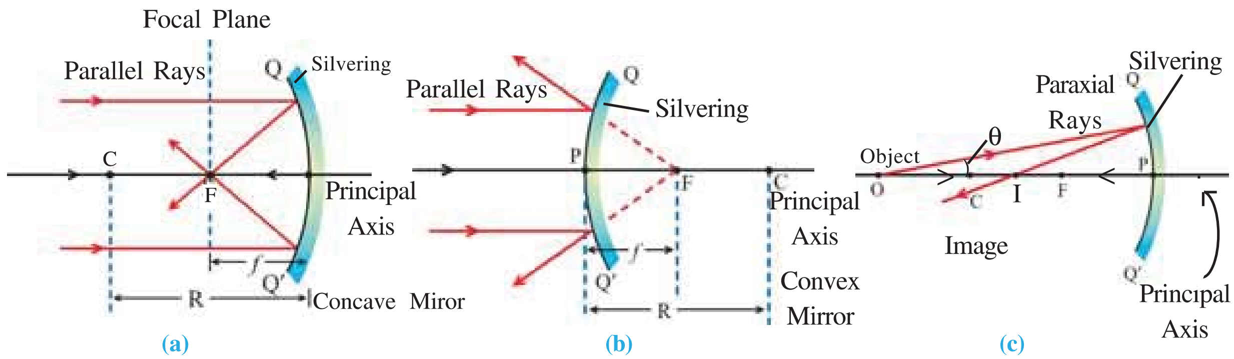

Figure 6.1 Image Formation by Curved Mirrors

Some useful terms used to study reflection of light by curved mirrors are as follows :

Pole : The centre of the reflecting surface of a curved mirror is called its pole (P)

Principal Axis : The imaginary line passing through the pole and the centre of curvature ($\overleftrightarrow{CP}$) is called the principal axis of the mirror.

Radius of Curvature : The radius of the spherical shell from which mirrors are made is called the radius of curvature (R) of the curved mirrors. It is the distance between C and P.

Centre of Curvature : The centre of the spherical shell from which mirrors are made is called the centre of curvature (C) of the mirror.

Aperture : The diameter of the reflecting surface (QQ') is called the aperture of the mirror.

Principal Focus : The point where the rays parallel to the principal axis meet for concave mirror or appear to meet for convex mirror on reflection is called the principal focus of the mirror.

Focal Plane : A plane passing through the principal focus and normal to the principal axis is called the focal plane of the mirror.

Focal Length : The distance between the pole and the principal focus of a mirror is called its focal length (f).

Paraxial Rays : Rays close to the principal axis are called Paraxial Rays. We shall study lens and mirrors in reference to Paraxial Rays only.

Sign Convention : In order to specify the position of the object and the image, we require a reference point and sign convention. We adopt Cartesian sign convention as follows.

(1) All the distances are measured from the pole of the mirror on the principal axis.

(2) Distances measured in the direction of the incident ray are taken positive, while those measured in the opposite direction are taken negative.

(3) Height above the principal axis is taken positive, while that below the principal axis is taken negative.

6.3 Relation between Focal Length and Radius of Curvature

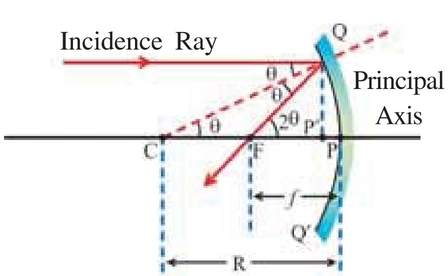

Figure 6.2 Relation between Focal Length and Radius of Curvature

In figure 6.2, a ray paraxial and parallel to the principal axis is shown to incident at point Q of a concave mirror of small aperture. The reflected ray passes through the principal focus. Normal drawn to the surface at point Q passes through centre of curvature. $\therefore$ CQ = CP. If the angle of incidence is θ, then the angle of reflection $\angle CQF = \theta = \angle QCF$.

From the geometry of the figure, exterior angle,

$\angle QFP = \theta + \theta = 2\theta$

Since the incident ray is paraxial and the aperture of the mirror is small, points P and P' are very close to each other. i.e., $CP' \approx CP = R$

and $FP' \approx FP = f$

In ΔQFP, $\sin 2\theta \approx 2\theta = \frac{QP'}{FP'} = \frac{QP}{FP}$

$$\therefore\ 2\theta = \frac{QP}{f} \Rightarrow \theta = \frac{QP}{2f} \qquad (6.3.1)$$

Similarly, from $\Delta CQP'$, $\sin\theta \approx \theta = \frac{QP'}{CP'} \approx \frac{QP}{CP}$

$$\therefore\ \theta = \frac{QP}{R} \qquad (6.3.2)$$

From equations (6.3.1) and (6.3.2) $R = 2f$ or $f = \frac{R}{2}$ (6.3.3)

Equation (6.3.3) is also true for a convex mirror. In the case of plane mirror, R is infinite, and therefore its focal length is also infinite.

6.4 Spherical Mirror Formula

Now we shall derive the relation between the object distance (u) image distance (v) and focal length (f) for a concave mirror. As shown in figure 6.3, consider a point object O on the principal axis at a distance u from the pole. Let the aperture of the mirror be small. Let the incident ray OQ makes a small angle (α) with the principal axis and gets reflected as QI. Another ray from object O moving along the axis is incident at P, and gets reflected in the direction PC. Both reflected rays meet at point I and forms the point like image.

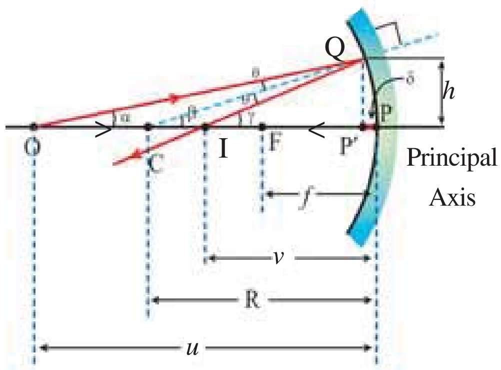

Figure 6.3 Image of a Point Object Due to Concave Mirror

Since the aperture of the mirror is small, distance $PP' = \delta$ is very small and can be neglected. Hence regions OPQ and IQP can be approximated by $\Delta OQP'$ and $\Delta IQP'$, respectively.

According to the laws of reflection, angle of incidence, $\angle OQC$ = angle of reflection, $\angle CQI = \theta$. Let CQ and IQ make angle β and γ, respectively, with the principal axis.

In ΔOCQ, exterior angle $\beta = \alpha + \theta$

In ΔCQI, exterior angle $\gamma = \beta + \theta$

Eliminating θ from above equations,

$$\alpha + \gamma = 2\beta \qquad (6.4.1)$$

Using the figure, in $\Delta OQP'$,

$$\alpha \text{ (rad)} = \frac{\text{arc QP}}{\text{OP}},$$

$$\beta \text{ (rad)} = \frac{\text{arc QP}}{\text{CP}} \quad \text{and}$$

$$\gamma \text{ (rad)} = \frac{\text{arc QP}}{\text{IP}}$$

Using these values in equation (6.4.1) we have,

$$\frac{\text{arc QP}}{\text{OP}} + \frac{\text{arc QP}}{\text{IP}} = 2\frac{\text{arc QP}}{\text{CP}}$$

$$\frac{1}{\text{OP}} + \frac{1}{\text{IP}} = \frac{2}{\text{CP}}$$

$$\therefore \frac{1}{u} + \frac{1}{v} = \frac{2}{R} \text{ or } \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \qquad (6.4.2)$$

Equation (6.4.2) represents the numerical relationship between object distance, image distance and focal length (or radius of curvature). While using it for calculating any of these physical quantities, we must apply sign convention. In the present case, $u \to -u$, $v \to -v$ and f (or R) $\to -f$ (or $-$R)

Equation (6.4.2) is the **Gauss' equation** for curved mirrors. It is also valid for convex mirror.

6.5 Lateral Magnification

The ratio of the height of the image (h') to the height of the object (h) is called the **transverse** or **lateral** magnification (m).

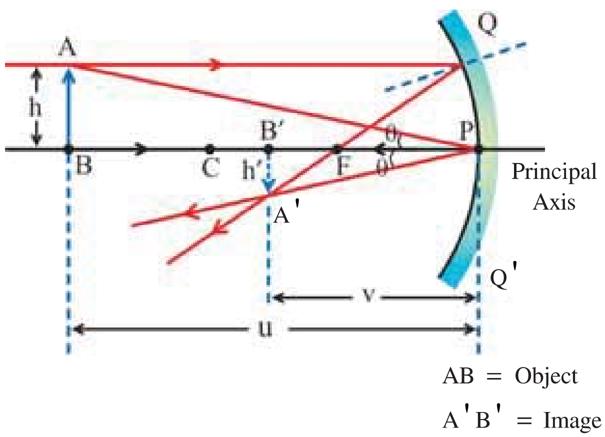

Figure 6.4 Image of an Extended Object

i.e., $m = \frac{h'}{h}$ (6.5.1)

For right angled triangles ABP and A'B'P,

$$\tan\theta = \frac{AB}{BP} = \frac{A'B'}{B'P} \qquad (6.5.2)$$

But AB = h, A'B' = $-h'$, PB = $-u$ and B'P = $-v$ (using sign convention), equation (6.5.2) becomes,

$$\frac{h}{-u} = \frac{-h'}{-v}$$

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$$\therefore \quad \frac{h'}{h} = \frac{-v}{u} \qquad (6.5.3)$$

Combining equations (6.5.1) and (6.5.3)

$$m = \frac{-v}{u} \qquad (6.5.4)$$

Equation (6.5.4) is also true for convex mirror.

Illustration 1 : An object lies on the principal axis of a concave mirror with radius of curvature 160 cm. Its image appears erect at a distance 70 cm from it. Determine the position of the object and also the magnification.

Solution : The mirror equation is

$$\frac{2}{R} = \frac{1}{u} + \frac{1}{v}$$

$$\therefore \quad \frac{1}{u} = \frac{2}{R} - \frac{1}{v} = \frac{2}{-160} - \frac{1}{70} \text{ (using sign convention)}$$

$$\therefore \quad u = -37 \text{ cm} = \frac{-15}{560}$$

i.e., The object is at a distance 37 cm in front of the mirror.

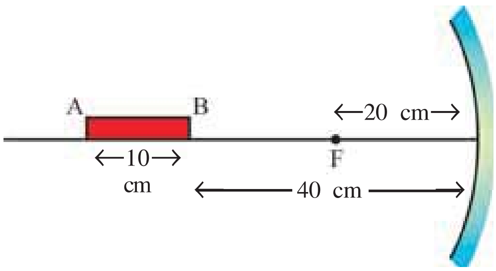
$$\text{Lateral magnification, } m = -\frac{v}{u} = -\frac{70}{-37} = 1.89$$

Illustration 2 : As shown in the figure, a thin rod AB of length 10 cm is placed on the principal axis of a concave mirror such that it's end B is at a distance of 40 cm from the mirror. If the focal length of the mirror is 20 cm, find the length of the image of the rod.

Solution : f = 20 cm and the end B is at distance 40 cm = $2f$ = R. Thus the image of B is formed at B only.

Now for end A,

u = –50 cm, f = – 20 cm, v = ?



In $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, putting these values

$$-\frac{1}{50} + \frac{1}{v} = -\frac{1}{20}$$

$$\therefore \quad \frac{1}{v} = \frac{1}{50} - \frac{1}{20} = \frac{20-50}{20\times50} = -\frac{30}{1000}$$

$$\therefore \quad v = -\frac{100}{3} = -33.3 \text{ cm}$$

This image A' is on the same side as the object.

Now, length of the image = 40 – 33.3 = 6.70 cm

Illustration 3 : Derive the formula for lateral magnification, $m = \frac{f}{f-u}$ for spherical mirrors; where f = focal length and u = object distance.

Solution : $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad \therefore \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{u-f}{uf}$

$$\therefore \quad v = \frac{fu}{u-f} \Rightarrow \frac{v}{u} = \frac{f}{u-f}$$

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and $m = -\frac{v}{u} = \frac{f}{f-u}$

Note : For a plane mirror $f \to \infty \therefore m = 1$ (Magnitude)

6.6 Refraction of Light

When a ray of light enters obliquely from one transparent medium to another transparent medium its direction changes at the surface separating two media. This phenomenon is known as refraction.

For information only :

• When the characteristics of a medium are same at all points, it is said to be homogeneous. When the characteristics are same in all directions it is said to be isotropic.

• If a medium is not homogenous then a light ray continuously gets refracted and its path is curved.

• If the medium is not isotropic light ray refracts by different amount in different directions.

Laws of Refraction :

(1) The incident ray, refracted ray and the normal drawn to the point of incidence are in the same plane.

(2) "The ratio of the sine of the angle of incidence to the sine of the angle of refraction for the given two media is constant." This constant is called **relative refractive index** of the two media. This statement is known as the Snell's law.

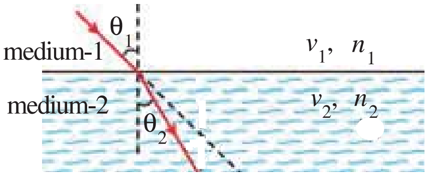

Figure 6.4 (a)

If θ_1 is the angle of incidence (in medium-1) and θ_2 is the angle of refraction (in medium-2) then,

$$\frac{\sin\theta_1}{\sin\theta_2} = n_{21}, \qquad (6.6.1)$$

where n_{21} is known as the refractive index of medium-2 with respect to medium-1.

n_{21} depends on the type of media, their temperature and the wavelength of light.

Relative refractive index may also be defined in terms of speed of light in two media.

$$n_{21} = \frac{v_1}{v_2}, \qquad (6.6.2)$$

where v_1 = speed of light in medium-1

and v_2 = speed of light in medium-2.

Similarly, refractive index of a medium with respect to vacuum (or in practice air) is

$$n = \frac{c}{v}. \qquad (6.6.3)$$

Here, n is known as absolute refractive index. Now,

$$n_{21} = \frac{v_1}{v_2} = \frac{c}{v_2} \times \frac{v_1}{c} = \frac{n_2}{n_1} \qquad (6.6.4)$$

$\therefore$ equation 6.6.1 becomes,

$$n_{21} = \frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

or $n_1\sin\theta_1 = n_2\sin\theta_2 \qquad (6.6.5)$

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This equation (6.6.5) is known as general form of Snell's law.

For given media, if $n_2 > n_1 \Rightarrow \sin\theta_1 > \sin\theta_2$

$\therefore \theta_1 > \theta_2$

When a light ray enters from optically rarer medium to optically denser medium, angle of refraction is smaller than the angle of incidence, and the ray bends towards the normal.

If $n_2 < n_1 \Rightarrow \sin\theta_1 < \sin\theta_2$

$\therefore \theta_1 < \theta_2$

When a light ray enters from optically denser medium to optically rarer medium, angle of refraction is greater than the angle of incidence, and the ray bends away from the normal.

The medium with greater refractive index is called **optically denser** medium and the one with smaller refractive index is called optically rarer medium. This optical density is different from the mass density.

Refraction Through Compound Slab :

As shown in figure 6.5, if light passes through a compound slab, refractive index of medium-3 with respect to medium-1 can be written as

$$n_{31} = \frac{v_1}{v_3}$$

$$= \frac{v_2}{v_3} \times \frac{v_1}{v_2} = n_{32} \times n_{21} \qquad (6.6.6)$$

Also, $n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 \qquad (6.6.7)$

and $n_{21} = \frac{v_1}{v_2} = \frac{1}{\left(\frac{v_2}{v_1}\right)} = \frac{1}{n_{12}}$

$\therefore n_{21} \times n_{12} = 1 \qquad (6.6.8)$

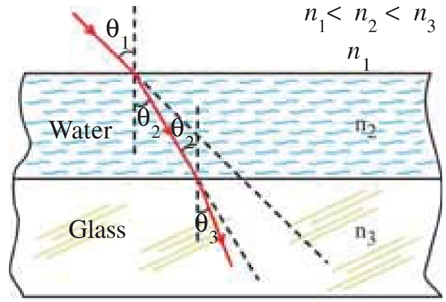


Figure 6.5 Refraction Through Compound Slab

For information only : The visibility of a transparent medium is due to the difference in its refractive index from that of the surrounding medium.

6.6.1 Lateral Shift : As shown in the figure 6.6, light rays undergo refraction twice, once from top (AB) and then from bottom (CD) surfaces of a given homogeneous medium. The emergent ray is parallel to PQR'S' ray. Here, PQR'S' is the path of light ray in absense of the other medium.

Since the emergent ray is parallel to the incident ray but shifted sidways by distance RN. This RN distance is called **lateral shift** (x). We can now calculate this lateral shift as follows :

Let n_1 and n_2 be the refractive indices of the rarer and denser medium, respectively. Also, $n_1 < n_2$. From the figure, $\angle RQN = \theta_1 - \theta_2$, $RN = x$.

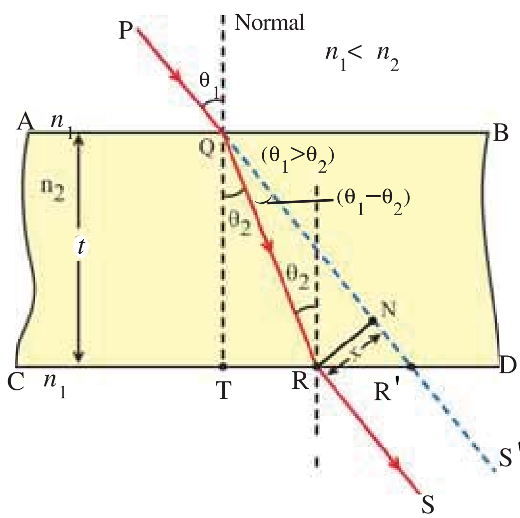


Figure 6.6 Lateral Shift Due to Rectangular Slab

From ΔQRN, $\sin(\theta_1 - \theta_2) = \frac{RN}{QR} = \frac{x}{QR}$ (6.6.9)

In ΔQTR, $\cos\theta_2 = \frac{QT}{QR}$

$\therefore QR = \frac{QT}{\cos\theta_1} = \frac{t}{\cos\theta_2}$

$\therefore$ from equation (6.6.9), $\sin(\theta_1 - \theta_2) = \frac{x}{\left(\frac{t}{\cos\theta_2}\right)}$

$$\therefore x = \frac{t \cdot \sin(\theta_1 - \theta_2)}{\cos\theta_2} \quad (6.6.10)$$

Since angle of incidence θ_1 is very small, θ_2 will also be small.

$\therefore \sin(\theta_1 - \theta_2) \approx (\theta_1 - \theta_2)$ & $\cos\theta_2 \approx 1$

$$\therefore x = \frac{t \cdot (\theta_1 - \theta_2)}{1}$$

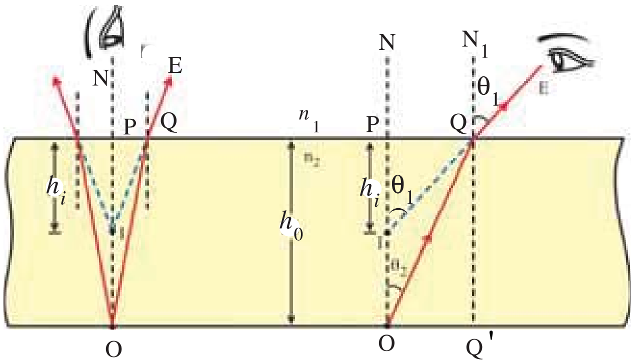
$$x = t \cdot \theta_1 \left(1 - \frac{\theta_2}{\theta_1}\right) \quad (6.6.11)$$

But according to Snell's law, $\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} \approx \frac{\theta_1}{\theta_2}$

$\therefore$ From equation (6.6.11),

$$x = t \cdot \theta_1 \left(1 - \frac{n_1}{n_2}\right)$$

6.6.2 Real Depth and Virtual Depth : Another manifestation of lateral shift is the apparant depth or hight seen through transparent medium.



(a) Normal Incidence (b) Oblique Incidence

Figure 6.7

As shown in figure 6.7, an object O is kept at depth h_0 in a denser medium (e.g. water) with refractive index n_2. In figure 6.7(b) Ray OQ on refraction moves along QE at the interface. If EQ is extended in the denser medium it meets the normal PN at point I.

So the observer sees the image of object O at position I. Here, PO = h_0 = real depth of an object.

PI = h_i = virtual depth of an image.

From figure 6.7(a) even when $\theta_1 = 0$, $h_0 \neq h_i$ (You will see this as a case of equation (6.8.10)).

But as θ_1 increases h_i becomes smaller compared to h_0. Also, the object appears curved when viewed obliquely through the refracting medium.

6.6.3 Real Height and Virtual Height :

Suppose an observer (e.g., fish) is inside a denser medium (e.g., water). It sees the eye (E) of a person at point E' instead of E. i.e., object is appeared lifted up (figure 6.8).

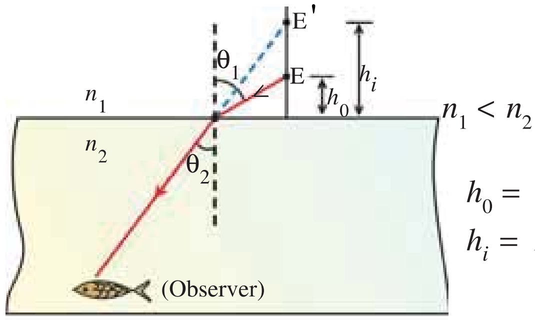


Figure 6.8 Virtual Height

Illustration 4 : Assuming that the angle of incidence at a refractive surface is sufficiently small, derive the relation between real depth, apparent depth and refractive index.

Solution : In figure 6.7, refractive index of denser medium = n_2 and refractive index of rarer medium = n_1. Real depth of the object O is PO = h_O. Depth of the image, i.e., apparent depth of the object = PI = h_i

Applying Snell's law at point Q,

$n_2 \sin\theta_2 = n_1 \sin\theta_1$

For nearly normal incidence, θ_1 and θ_2 are very small.

$\therefore \sin\theta \approx \theta \approx \tan\theta$

$n_2 \tan\theta_2 = n_1 \tan\theta_1$

But, $\tan\theta_2 = \frac{PQ}{PO} = \frac{PQ}{h_O}$ and $\tan\theta_1 = \frac{PQ}{PI} = \frac{PQ}{h_i}$

Using this in equation (1) $n_2\left(\frac{PQ}{h_O}\right) = n_1\left(\frac{PQ}{h_i}\right)$

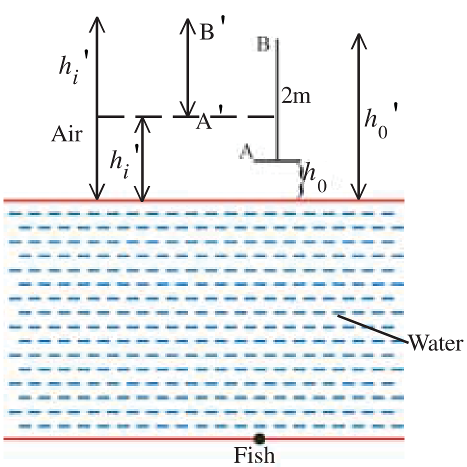
$$\therefore \frac{n_2}{n_1} = \frac{h_O}{h_i} \Rightarrow \frac{h_i}{h_O} = \frac{n_1}{n_2} = \frac{n(\text{rarer})}{n(\text{denser})}$$

Note : It can be proved that if an object kept in a rarer medium, at height h_O from the interface, is viewed normally from the denser medium and it appears to be at height h_i ($h_i > h_O$), then

$$\frac{h_i}{h_O} = \frac{n(\text{denser})}{n(\text{rarer})}$$

Illustration 5 : A swimmer is diving in a swimming pool vertically down with a velocity of 2 m s^{-1}. What will be the velocity as seen by a stationary fish at the bottom of the pool, right below the diver ? Refractive index of water is 1.33.

Solution : In the figure, vertical distance 2m is shown by AB. The height of A from the surface of water is h_O. Suppose it's apparent height is h_i ($h_i > h_O$).



$$\therefore \frac{h_i}{h_O} = \frac{n(\text{water})}{n(\text{air})}$$

$$\therefore h_i = h_O \times 1.33 \qquad (1)$$

Now the real height of B, $h_O' = (h_O + 2)$m

$\therefore$ if it's apparent height is h'_i, then

$$\frac{h_i'}{h_O'} = \frac{n(\text{water})}{n(\text{air})} = 1.33$$

$\therefore\ h'_i = h_O' \times 1.33$

$= (h_O + 2) \times 1.33$ (2)

From equations (1) and (2), the apparent distance, seen by the fish

$= h_i' - h_i = (h_O + 2) \times 1.33 - h_O \times 1.33$

$= 2 \times 1.33 = 2.66$ m

So the fish will see the swimmer falling with a speed of 2.66 m s^{-1}.

6.7 Total Internal Reflection

When light ray enters from one transparent medium to another, it is partially reflected and partially transmitted at an interface. This is true even if light is incident normally to a surface separating two media. In this case, intensity of reflected light is given by

$$I_r = I_0\left(\frac{n_2 - n_1}{n_1 + n_2}\right)^2 \tag{6.7.1}$$

where, I_O = intensity of incident light

I_r = intensity of reflected light.

n_1 = refractive index of the medium–1

n_2 = refractive index of the medium–2

For air (n_2 = 1.0) and glass (n_1 = 1.5), I_r is 4% of the incident intensity. It is to be noted that equation 6.7.1 is true for normal incidence only. For other cases, I_r also depends on the angle of incidence.

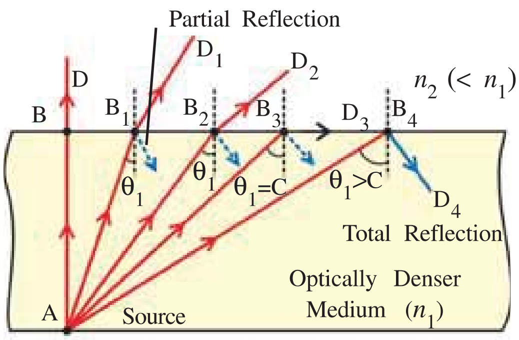


Figure 6.9 Total Internal Reflection

Here, A is a point object (or a light source) in a denser medium. Ray AB, AB_1, AB_2, ... get partially reflected and partially transmitted at points B, B_1, B_2 ... at the interface. It is observed that as the angle of incidence increases (going from B $\rightarrow$ B_1 $\rightarrow$ B_2 $\rightarrow$ ....) the angle of reflection ray also increases. It happens that at particular angle of incidence, refracted ray moves parallel to the surface separating two media. For this particular case, angle of refraction is 90°.

The angle of incidence for which the angle of refraction is 90° is called the critical angle (C) of the denser medium with respect to the rarer medium.

In this situation the interface appears bright. Using Snell's law for the critical angle of incidence,

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

when $\theta_1 = C$, $\theta_2 = 90°$

$\therefore\ n_1 \sin C = n_2$

$\therefore\ \sin C = \frac{n_2}{n_1}$

If rarer medium is air, i.e. $n_2 = 1$

$\therefore\ \sin C = \frac{1}{n_1} = \frac{1}{n}$ (Let $n_1 = n$)

$$\text{or } C = \sin^{-1}\left(\frac{1}{n}\right) \tag{6.7.2}$$

At the critical angle, the reflected ray is known as the **critical ray**.

Now if the angle of incidence is increased slightly more than the critical angle, the intensity of reflected light immediately increases, and the incident ray gets completely (i.e. 100%) reflected back into the denser medium. This is called total internal reflection. It is true for any of incidence greater than the critical angle. In this situation, the surface separating the two media behaves like a perfect mirror. It is to be noted that the total internal reflection obeys the laws of reflection.

For Information Only :

When total internal reflection is studied with respect to electromagnetic waves, it is found that a very small portion of incident light enters into the rarer medium upto a distance equals few wavelengths. Though, its intensity is diminutive. This in quantum mechanics is called **tunneling** effect.

Illustration 6 : As shown in figure, a ray of light is incident at angle of 30^o on a medium at $y = 0$ and proceeds ahead in the medium. The refractive index of this medium varied with distance y as given by,

$n(y) = 1.6 + \frac{0.2}{(y+1)^2}$ where y is in cm. What is the angle formed by the ray with the normal at a very large depth ?

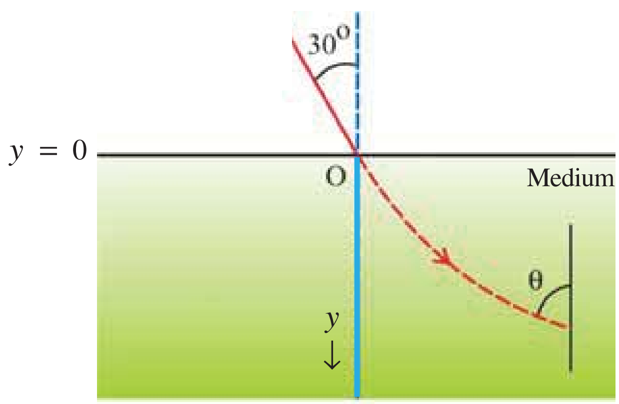
Solution : Suppose the angle is θ at distance y in the medium.

Appyling Snell's law at this point,

$n(y)\sin\theta = C$, where C = constant (1)

This formula is true for all the points.

Applying it to point O,



$n(0)\sin 30^o = C$

But, $n(0) = 1.6 + \frac{0.2}{(0+1)^2} = 1.8$

$\therefore\ 1.8 \times \frac{1}{2} = C$

$\therefore\ C = 0.9$

Putting this value in (1), $n(y)\sin\theta = 0.9$

$\therefore\ \left\{1.6+\frac{0.2}{(y+1)^2}\right\} sin\theta = 0.9 \ \therefore\ \sin\theta = \frac{0.9}{1.6+\frac{0.2}{(y+1)^2}}$.

When y is very large, taking $y \to \infty$; we get $\sin\theta = \frac{0.9}{1.6}$

$\therefore\ \theta = 34^o\ 14'$

6.7.1 Uses of Total Internal Reflection :

(1) The refractive index of diamond is 2.42 and its critical angle is 24.41^o. Thus, with proper cutting of its faces, whatever the angle at which light enters into the diamond, it undergoes many total internal reflections. Hence it looks bright from the inside, and we call the diamond is sparkling.

(2) For a glass with refractive index 1.50 has a critical angle for an air-glass interface,

$C = \sin^{-1}\left(\frac{1}{1.50}\right) \approx 42^o$

This angle is slightly less than 45°, which makes possible to use prisms with angles 45°–45°–90° as totally reflecting surface. (See figure 6.10).

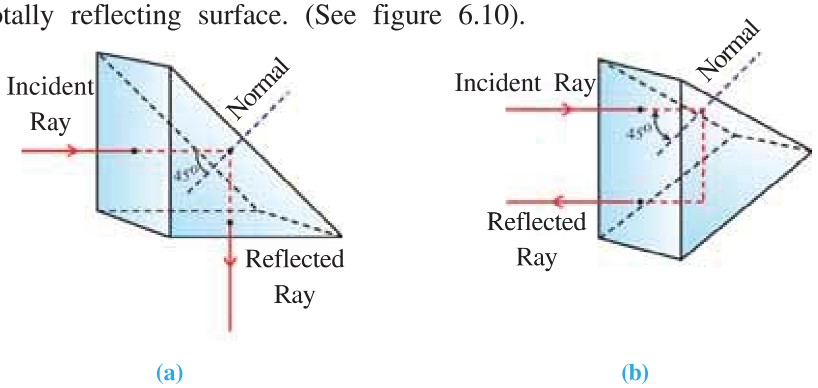


Figure 6.10 Totally Reflecting Prisms

The advantages of totally reflecting prisms over metallic reflectors are (1) superior reflection and (2) the reflecting properties are permanent and not affected by tarnishing.

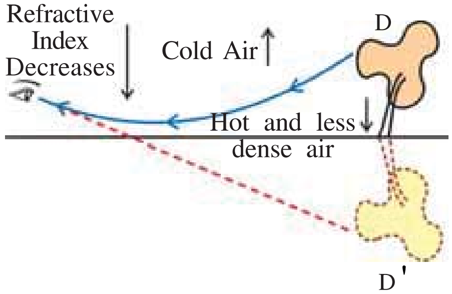


Figure 6.11 Mirage Formation

(3) Mirage : In summer, due to heat, the air in contact with the ground becomes hot while above it is cooler. Thus, air in contact with the ground is rarer and air above is denser. i.e., its refractive index increases as one moves upwards. As shown in the figure 6.11, a ray going from the top of the tree (D) to the ground is travelling continuously from a denser medium to a rarer medium. As it comes closer to the ground its angle of refraction increases and finally it undergoes total internal reflection, and enters into the eye of an observer. Thus, the image of D appears at D' to an observer, giving a feeling of image in a water surface. This phenomenon is called a mirage.

(4) Optical Fibres : The phenomenon of total internal reflection is used in optical fibres. They are made of glass or fused quartz of about 10 to 100 μm in diameter. They are in the form of long and thin fibres. The outer coating of the fibres **(cladding)** has a lower refractive index (n_1) than the core (material) of fibre (n_2). Here, $n_2 > n_1$.

In absense of the cladding layer, due to dust particles, oil or other impurities, some leakage of light may take place. In 1 m distance, in fact, light gets reflected thousands of times. Thus, if leakage occurs, light cannot travel far. Such leakage is prevented using cladding.

Fused quartz is usually used for making optical fibres because of its high transparency.

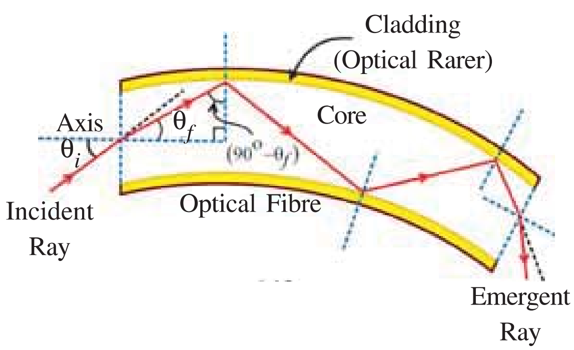


Figure 6.12 Schematic Diagram of Optical Fibre

In figure 6.12, a ray is incident at an angle θ_i to the axis of a fibre from air. θ_f is the angle of refraction. The refracted ray makes an angle θ_f with the axis of the fibre. As shown in the figure, this ray is incident on the wall of the fibre at an angle $(90° - \theta_f)$. It is clear that if angle $(90° - \theta_f)$ is greater than the critical angle for fibre cladding interface, the ray will undergo a total internal reflection. In short, the greater the value of $(90^o - \theta_f)$ the greater is the chance for total internal reflection. That is, a small value of θ_f is preferable. This also suggests that smaller the value of θ_i the greater are the chances of total internal reflection. Thus, for a given fibre the value of θ_i should not be greater than some particular value.

The above condition for total internal reflection can also be discussed in terms of the refractive index of the material of the fibre.

We have seen that the value $(90^o - \theta_f)$ should be greater than the critical angle. Thus, the smaller the value of critical angle, the more are the chances of total intermal reflection.

Now, $\sin C = \frac{1}{n}$ relation shows that n should be large in order to have small value of C. Thus, the material of an optical fiber should have value of n more than some minimum value. In our discussion we have taken the medium outside the optical fiber as air.

6.8 Refraction at a Spherically Curved Surface

Images can be formed by reflection as well as by refraction. Here we study the refraction at a spherical surface, i.e., at a spherical interface between two transparent media having different refractive indices. In the following discussion, we shall study refraction of paraxial rays at a spherically curved surface. This will help us to understand the image formation by lenses, though lens has two refracting surfaces. We will follow cartesian sign convention in our discussion, and the spherical surface as a very small part of the sphere.

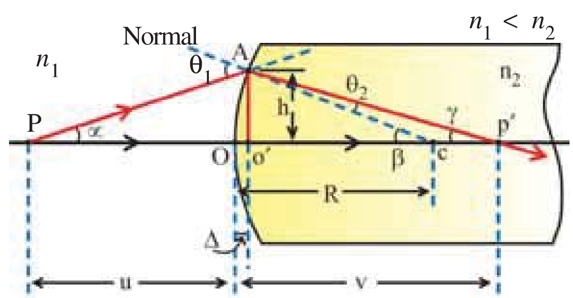


Figure 6.14 Refraction Due to Conve Curved Surface

As shown in the figure, O is the centre of the refracting surface, C is the centre of curvature, OC is the radius of curvature. A point object P is kept at a distance u from O on the principal axis.

To form image after refraction, consider two rays PO and PA from point object P.

For ray PO, angle of incidence is zero. Therefore, according to Snell's law this ray will move along OCP' without bending.

Ray PA is incident at point A on the surface. AC is the normal to the surface at point A. θ_1 is the angle of incidence. Suppose the refractive index (n_1) of the medium-1 is less than the refractive index (n_2) of the medium-2. As a result, the refracted ray bends towards the normal and moves along AP'. Let α, β and γ be the angles made by PA, CA and P'A respectively with principal axis.

Both refracted rays OP' and AP' meet at point P', and forms point like image of an object P. Here, θ_2 is the angle of refraction.

Applying Snell's law at point A,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad (6.8.1)$$

Since we are considering paraxial rays, θ_1 and θ_2 are small (measured in radian)

$$\therefore\ n_1\theta_1 = n_2\theta_2 \quad (6.8.2)$$

From figure, θ_1 is the exterior angle of ΔPAC.

$$\therefore\ \theta_1 = \alpha + \beta \quad (6.8.3)$$

Similarly, angle β is exterior to $\Delta CP'A$.

$\therefore\ \beta = \theta_2 + \gamma$

$\therefore\ \theta_2 = \beta - \gamma$ (6.8.4)

Using (6.8.3) and (6.8.4) in equation (6.8.2),

$n_1(\alpha + \beta) = n_2(\beta - \gamma)$

$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta$ (6.8.5)

From right angled triangle O'P'A, $\tan\gamma \approx \gamma = \frac{h}{v-\Delta}$ (6.8.6)

where v = image distance

From right angled ΔO'CA, $\tan\beta \approx \beta = \frac{h}{R-\Delta}$ (6.8.7)

And from ΔPAO', $\tan\alpha \approx \alpha = \frac{h}{-u+\Delta}$, (6.8.8)

where $u \rightarrow -u$, object distance, as per the sign convention.

The curved surface considered here is a very small part of the sphere from which it is cut. Thus, Δ is negligible compared to R, u and v.

$\therefore\ \gamma \approx \frac{h}{v},\ \beta = \frac{h}{R}$ and $\alpha = \frac{h}{-u}$ (6.8.9)

Combining equations (6.8.5) and (6.8.9), $n_1\left(\frac{h}{-u}\right) + n_2\left(\frac{h}{v}\right) = (n_2 - n_1).\frac{h}{R}$

$\therefore\ \frac{-n_1}{u} + \frac{n_2}{v} = \frac{(n_2-n_1)}{R}$ (6.8.10)

Equation (6.8.10) is valid for concave surface also. Equation (6.8.10) is the general equation which relates object distance, image distance and radius of curvature of the curved surface. This equation is derived for the ray travelling **from rarer medium (with refrective index n_1) to the denser medium (with refrective index n_2)**. In the similar way when the ray travels **from denser medium (with refrective index n_2) to the rarer medium (with refrective index n_1)**, we can derive the following equation using Snell's law.

$\frac{-n_2}{u} + \frac{n_1}{v} = \frac{(n_1-n_2)}{R}$ (6.8.11)

Case : If surface is plane (plane glass slab).

i.e. R = ∞. Therefore equation (6.8.10) becomes $\frac{+n_1}{u} = \frac{n_2}{v}$

or $\frac{v}{u} = \frac{n_2}{n_1} = \frac{h'}{h}$ (see the topic of magnification).

Whether the image is real or virtual is decided by the sign convention. If image distance is positive, i.e., image is formed on the right of point O, it is real or otherwise.

6.9 Spherical Lenses

In general, a lens is an image forming device, having two bounded refracting surfaces. Of the two surfaces at least one surface is curved. For example, the following figure depicts different types of lenses.

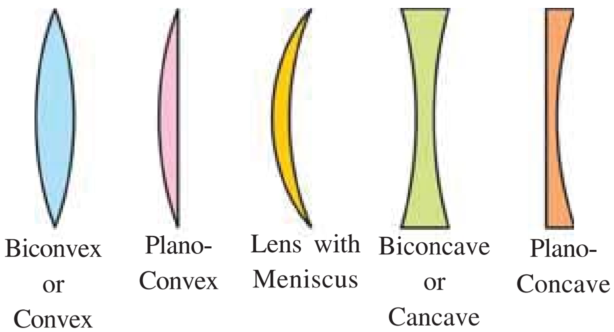
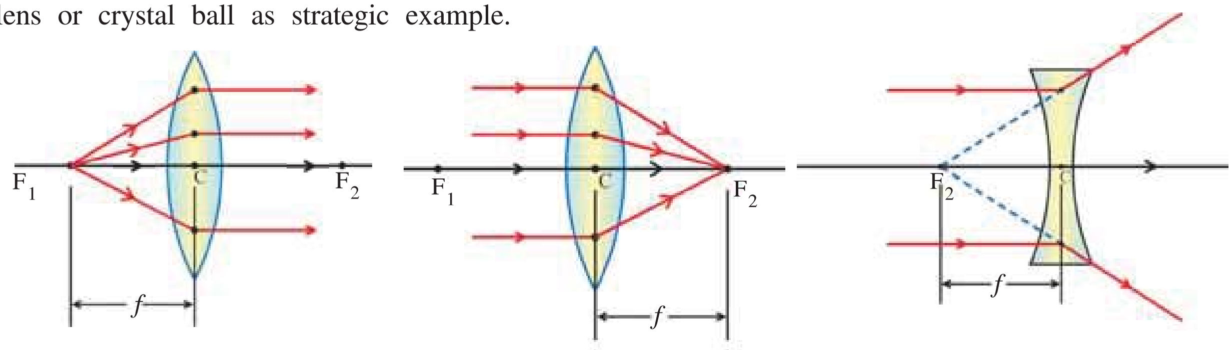


Figure 6.15 Different Type of Lenses

Since spherical surfaces are easy to construct, we first consider image formation by a spherical lens or crystal ball as strategic example.



(a) (b) (c)

Figure 6.16 Focus of Thin Lens

If a point object is placed on the principal axis of a convex lens such that the rays refracted are parallel to the axis figure (a), then the position of the point object is called the **first principal focus** (F_1) of the lens.

If the object is situated at infinite (figures (b) and (c)), refracted rays meet (or appear to meet) for convex (or concave) lens to a point (F_2), then the position of this point is known as **second principal focus** (F_2).

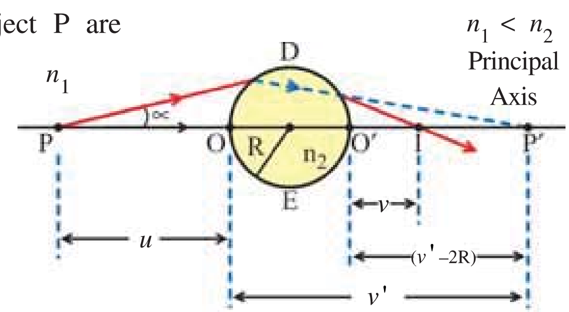
The geometrical centre of the medium of the lens is called its **optical centre** (C).

Distance of principal focus from the optical centre (C) is known as focal length (f) of the lens.

As per the sign convention, f is positive for convex lens and negative for concave lens.

Illustration 8 : Obtain the expression for image distance in terms of the radius of curvature for crystal.

Solution : Here, the rays coming from point object P are refracted twice at surfaces DOE and DO'E, respectively, before forming the final image. But for the sake of understanding, we consider both the refraction separately. Using the formula for spherical surface (equation 6.7.10) at both the surfaces we can determine the position of the (final) image.



At the surface DOE,

$$\frac{-n_1}{(-u)} + \frac{n_2}{v'} = \frac{(n_2 - n_1)}{R} \qquad (1)$$

(We have used Cartesian sign convention.)

Let $u > R$. In this case, v' will be large and positive. That is, image of P due to spherical surface DOE will form at point P' on the right and far from the ball.

Now, for surface DO'E image P' will behave as virtual object. Therefore at the surface DO'E,

$$-\frac{n_2}{(v' - 2R)} + \frac{n_1}{v} = \left(\frac{n_1 - n_2}{R}\right) \qquad (2)$$

Since v' is very large, $(v' - 2R)$ is positive. This gives v to be positive, i.e., the final point image will form on the right of the surface DO'E.

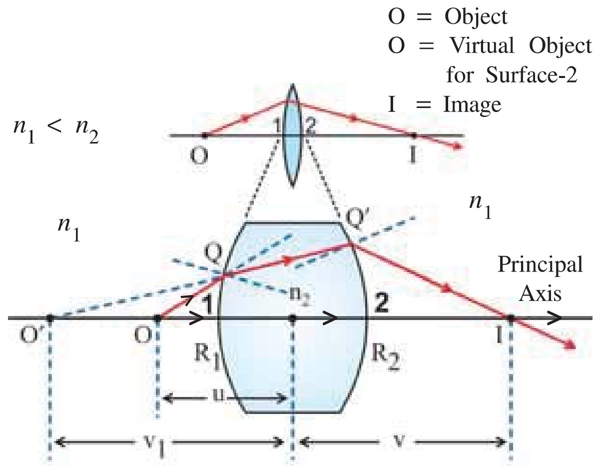


Figure 6.17 Image Formation Due to Thin Lens

6.9.1 Thin Lens : The lens for which the distance between the two refracting surfaces is negligible as compared to the object distance, the image distance and radius of curvature is called a thin lens. In general, radii of curvatures of the two refracting surfaces need not be equal. Being thin lens, the distance can be measured from either surface or even from the centre of the lens.

To obtain the relation between object distance, image distance and radii of curvatures for thin lens consider the following case as shown in the figure 6.17.

To understand, how final image due to thin lens is formed, assume that the two refracting surfaces are separated. Thus, the final image (I) is assumed to be formed due to two refractions at curved surface-1 and then due to surface-2, respectively.

The object O is in the medium having refractive index n_1. The incident ray OQ is refracted at surface-1 into the denser medium with refractive index n_2. (Here $n_2 > n_1$). The image is formed at O'. For the refraction at surface-1 using equation (6.8.10), we can write,

$$\frac{-n_1}{u} + \frac{n_2}{v_1} = \frac{(n_2 - n_1)}{R_1} \qquad (6.9.1)$$

Here, u = object distance and v_1 = image distance.

This image O' serves as virtual object for surface-2. For surface-2 the ray QQ' travelling from denser medium is refracted into rarer medium and meets the axial ray from O at I. Thus I is the final image. For refraction at surface-2, using equation (6.8.11), we can write,

$$\frac{-n_2}{v_1} + \frac{n_1}{v} = \frac{(n_1 - n_2)}{R_2} = \frac{(n_2 - n_1)}{-R_2} \qquad (6.9.2)$$

Here, v_1 = object distance for surface-2 and v = image distance.
Adding equations (6.9.1) and (6.9.2), we have

$$\frac{-n_1}{u} + \frac{n_1}{v} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore -\frac{1}{u} + \frac{1}{v} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (6.9.3)$$

Equation (6.9.3) is the disired equation. While using it in practice, proper sign convention should be employed.

6.9.2 Lens-Maker's Formula :

If medium on both sides of a lens is same, and object is at infinite (i.e., $u = \infty$) then $v = f$. From equation (6.9.4)

$$\frac{1}{f} - \frac{1}{\infty} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (6.9.4)$$

Equation (6.9.4) is known as **lens-maker's** formula. It is named so because it enables one to calculate focal length and radii of curvatures of the lens.

When equations (6.9.3) and (6.9.4) are compared, we have, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ (6.9.5)

This equation is known as **Gauss'** formula for a lens.

From equation (6.9.4), if the lens turns around, i.e. R_1 and R_2 get interchanged, then also for proper change in sign, f will be found to be same. Therefore, for a thin lens the focal length is independent of the order of the surfaces. If medium-1 is air (i.e., $n_1 = 1$) and let refractive index of medium-2 be $n_2 = n$, equation (6.9.4) becomes

$$\frac{1}{f} = (n - 1)\cdot\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (6.9.6)$$

For information only : Most general form of lens–maker's formula is,

$$\frac{1}{f} = (n - 1)\cdot\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \left(\frac{n-1}{n}\right)\cdot\frac{t}{R_1 \cdot R_2};$$

where 't' is the thickness of the lens. For thin lens t is negligible and equation (6.9.6) can be recovered. Above equation also suggests that for thick lens, i.e., t is large, and R_1 and R_2 are small, second term contributes significantly. Thus, for thick lens f is small, i.e., **thick lens converges or diverges strongly.**

6.9.3 Newton's Formula : As we have observed that lens-maker's formula relates radii of curvatures and refractive index of the lens to its focal length. We can also derive an expression relating focal length to image and object distances, which we call **lens user's** formula or **Newton's** formula.

On the left of the lens, ΔABF_1 and $\Delta CF_1P'$ are similar triangles. Therefore,

$$\frac{h_1}{x_1} = \frac{h_2}{f_1} \text{ (writing only magnitude)} \quad (6.9.7)$$

Similarly, for right of the lens,

$$\frac{h_1}{f_2} = \frac{h_2}{x_2} \quad (6.9.8)$$

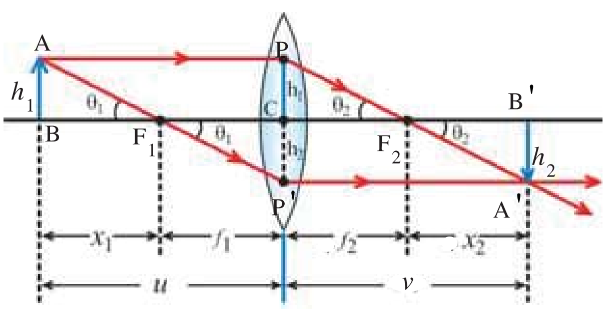


Figure 6.18 Extra Focal Distances of a Convex Lens

Writing combinedly for the ratio $\frac{h_1}{h_2}$,

$$\frac{h_1}{h_2} = \frac{x_1}{f_1} = \frac{f_2}{x_2} \quad (6.9.9)$$

$$\therefore x_1 \cdot x_2 = f_1 \cdot f_2 \quad (6.9.10)$$

Equation (6.9.10) is known as the Newton's lens formula. **Here, x_1 and x_2 are known as extra focal object distance and extra focal image distance.** Since these distances are measured from focii rather than from the lens, Newton's formula can be used equally for thin and thick lenses.

When $f_1 = f_2 = f$ (say), equation (6.9.10) becomes

$$x_1 \cdot x_2 = f^2 \quad (6.9.11)$$

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6.9.4 Conjugate Points and Conjugate Distances

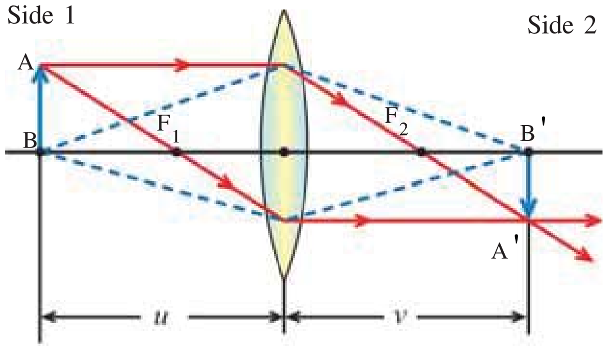


Figure 6.19 Conjugate Points and Distances

As shown in the figure 6.19, all the rays from point A and B are brought to focus at poins A' and B', respectively. Thus, A'B' is the image of an object AB. The principle of reversibility for light rays permits interchange in positions of image and an object. That is, if A'B' is an object, AB becomes the image. Thus, object and image are **conjugate.** Points A and A', and B and B' are called **conjugate points.**

Now, by keeping image distance as the object distance image will form at the object distance. That is, image and object distances are **conjugate distances.**

6.10 Magnification

Convex lenses are used properly to form a magnified image.

$$\text{Magnification, } m = \frac{\text{size of the image}}{\text{size of the object}} \tag{6.10.1}$$

Since for three-dimensional object the image will also three dimensional, correspondingly we have three types of magnifications. Lateral magnification, longitudinal magnification and angular magnification. We discuss only the lateral magnification below.

Lateral Magnification

Lateral magnification is also called as transverse magnification. It is defined as the ratio of height of an image (h_2) to that of the object (h_1) from the figure 6.18,

$$|m| = \frac{h_2}{h_1} \tag{6.10.2}$$

According to Cartesian sign convention, height measured above the principal axis is taken positive and below the principal axis it is negative. Hence, the lateral magnification is positive for erect image and negative for a inverted image. Also, from the figure 6.18,

$$\frac{h_1}{u} = \frac{h_2}{v} \text{ (only magnitude)}$$

$$\therefore\ m = \frac{h_2}{h_1} = \frac{v}{u} \tag{6.10.3}$$

From equation (6.9.10),

$$m = \frac{h_2}{h_1} = \frac{f_1}{x_1} = \frac{x_2}{f_2} \tag{6.10.4}$$

6.11 Power of a Lens

It is defined as the converging or diverging capacity of a lens. General form of lens-maker's formula suggests that the thicker the lens, smaller is the focal length and higher is the convergence or divergence. Thus, converging or diverging ability of a lens is inversely proportional to its focal length.

$$\therefore\ \text{Power of a lens, P} = \frac{1}{f} \tag{6.11.1}$$

For convex lens power is positive, while for the concave lens it is negative.

Its SI unit is m^{-1} or diopter (D).

i.e., 1D = 1 m^{-1}

When an optician prescribes lens of + 2.0 D, it means a convex lens of focal length = $\frac{1}{2}$ = 0.5 m.

6.12 Combination of Thin Lenses in Contact

Consider a simple optical system that consists of two thin lenses L_1 and L_2 in contact and placed on a common axis. Their focal lengths are f_1 and f_2 respectively. For such an optical system, we assume that the image formed by the first lens becomes the object for a second lens, and we get final image due to the system. We now derive formula for focal length of this equivalent lens as follows.

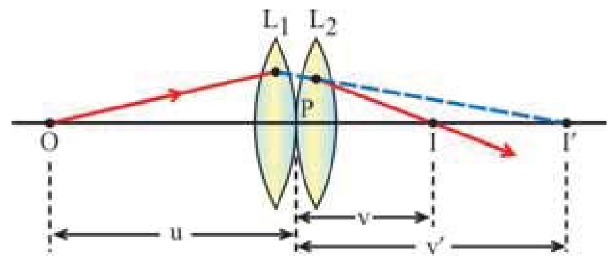


Figure 6.20 **Combination of Thin Lenses**

From the figure 6.20, consider a case of point like object (O) whose final image (I) is formed due to two thin lenses in contact.

Using Gauss' formula for lens L_1, $-\frac{1}{u} + \frac{1}{v'} = \frac{1}{f_1}$ (6.12.1)

For lens L_2, $-\frac{1}{v'} + \frac{1}{v} = \frac{1}{f_2}$ (6.12.2)

Adding these equations, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f_1} + \frac{1}{f_2}$ (6.12.3)

If we assume that the final image is formed by a single equivalent lens of the focal length f, then

$$\frac{1}{f} = \frac{-1}{u} + \frac{1}{v} \quad (6.12.4)$$

$$\therefore \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} \quad (6.12.5)$$

$$\text{or } f = \frac{f_1 \cdot f_2}{(f_1 + f_2)} \quad (6.12.6)$$

Equation (10.12.5) or (10.12.6) is the algebraic relation between f_1, f_2 and f. While using them to find equivalent focal length for different combinations of lenses, proper sign convention should be adopted.

If there are n number of thin lenses in contact, equivalent focal length of them is given by,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + ... + \frac{1}{f_n} \quad (6.12.7)$$

Lenses with Separation : If two thin lenses are not in contact, but having some separation d, then equivalent focal length can be written as,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 \cdot f_2} \quad (6.12.8)$$

Also, $d - (f_1 + f_2)$ is known as the **optical interval** between the two lenses.

Power :

But $\frac{1}{f_1} = P_1$ = power of lens L_1

$\frac{1}{f_2} = P_2$ = power of lens L_2

Therefore, from equation (6.12.8), equivalent power of the combitation is

$P = P_1 + P_2 + ... + P_n$ (6.12.9)

Lateral Magnification :

For two-lens system lateral magnification due to lens L_1 is $m_1 = \frac{v'}{u}$.

That due to lens L_2 is $m_2 = \frac{v}{v'}$.

If resultant magnification is m, then

$$m = \frac{v}{u} = \frac{v'}{u} \times \frac{v}{v'}$$

$$m = m_1 \times m_2$$

For n number of lenses, $m = m_1 \times m_2 \times .... \times m_n$ (6.12.10)

Equation (6.12.10) suggestes that in order to improve magnification one may use combination of lenses (e.g., compound microscope).

6.13 Combination of Lens and Mirror

The combination of lenses are important for achieving proper magnification, focussing of image at a desired point, etc. Similarly combinations of lenses and mirrors are also useful. We consider one such combination of convex mirror and convex lens.

O = Object

O' = Image

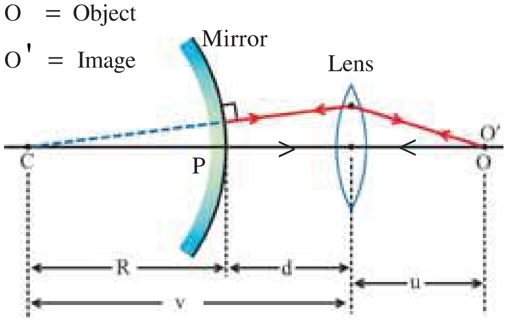


Figure 6.21 Focal Length of a Convex Mirror Using a Convex Lens

As shown in the figure 6.21, image (O') is formed on the same side of the object. For a given object distance (u), we adjust the mirror distance (d) from the lens in such a way that the image is formed at the object position itself (i.e., parallax between an object and image is removed). In this case, rays incident on to the mirror will be normal to the mirror. In absense of the mirror the image would have been formed at C. Its distance from the lens is v. Since rays falling on the mirror are normal, point C is the centre of curvature for the mirror.

Thus, by measuring v and d, we can find focal length of the mirror as,

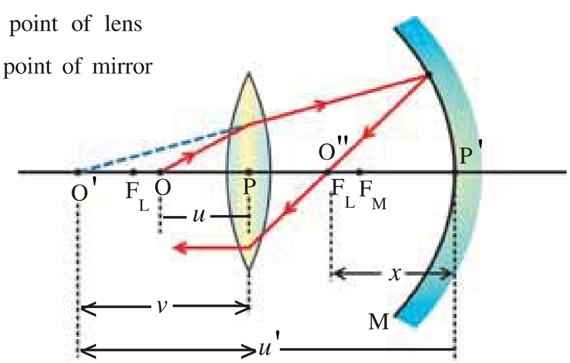
$$f = \frac{R}{2} = \frac{1}{2}(v - d).$$

Illustration 9 : A converging lens of focal length 15 cm and a converging mirror of focal length 20 cm are placed with their principal axes coinciding. Point object is placed at a distance 12 cm from the lens. Refracted ray from the lens gets reflected from the mirror, and again refracted by the lens. It is found that the final ray coming out of the lens is parallel to the principal axis. Find the distance between the mirror and the lens.

Solution :

F_L = Focal point of lens

F_N = Focal point of mirror



Applying Gauss' formula to lens,

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\therefore \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\therefore v = \frac{u \cdot f}{u + f} = \frac{(-12)\times(15)}{-12+15}$$

(Using cartesian sign convention)

$= -60$ cm.

Negative sign indicates that image (O$'$) is virtual. This image works as an object for the mirror. For mirror, object distance,

$u' = O'O'' + O''P' = (PO' + PO'') + O''P'$

$= (60 + 15) + x = (75 + x)$ cm (for mirror, PO$'$ = v is taken positive)

Since image due to mirror is obtained at O$''$, its distance from the mirror is x.

Applying Gauss' formula to the mirror,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\therefore \frac{1}{-(75+x)} + \frac{1}{-x} = \frac{1}{-f}$$

Simplifying , $\frac{(75+2x)}{(75+x)\cdot x} = \frac{1}{20}$

$\therefore x^2 + 35{\cdot}x - 1500 = 0$

$\therefore x = 25$ cm or $x = -60$ cm.

Thus, physically acceptable solution is 25 cm. Therefore, distance of the mirror from the lens is = 25 + 15 = 40 cm.

Illustration 10 : Distance between an object and a screen is d. Prove that for a thin convex lens, there are two positions for the object at which image can be obtained on the screen, and under certain condition only. Derive the condition for the same. When will the image not be formed ?

Solution : Suppose the object distance is u,

$$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

For a convex lens u is negative. So,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

But, $u + v = d$ (given)

$\therefore v = d - u$

$$\therefore \frac{1}{d-u} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore \frac{u+d-u}{u(d-u)} = \frac{1}{f}$$

$\therefore u^2 - ud + fd = 0$

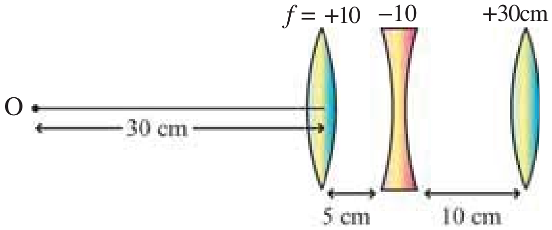
This is the quadratic equation for variable u. It's roots are as given below :

$$u = \frac{d \pm \sqrt{d^2 - 4fd}}{2}$$

Thus, if $d > 4f$, two values of u are possible and if $d < 4f$, u will be a complex number and hence the image will not be formed.

Illustration 11 : Decide the position of the image formed by the given combination of lenses.

Solution : For the image formed by first lens.



$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f}$$

$$\therefore \frac{1}{v_1} - \frac{1}{-30} = \frac{1}{10}$$

$$\therefore v_1 = 15 \text{ cm}$$

Thus image formed by the first lens is formed at 15 cm distance on the right-hand side. This image is on the right-hand side of the second lens at 15 − 5 = 10 cm distance and so it acts as a virtual object for the second lens.

Now for the second lens,

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\therefore \frac{1}{v_2} - \frac{1}{10} = -\frac{1}{10}$$

$$\therefore v_2 = \infty$$

This distance v_2 (= ∞) is the object distance for the third lens. So, the third image formed due to it should be on the principal focus of the third lens. Thus, as the focal length of the third lens is 30 cm, the final image is formed at 30 cm distance on the right side of the third lens.

Illustration 12 : For a thin lens prove that when the heights of the object and the image are equal, object distance and image distance are equal to $2f$.

Solution : Here, $| h | = | h' |$

$\therefore | v | = | u |$

Using the equation for lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-u} = \frac{1}{f}$$

$$\therefore \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\therefore \frac{2}{v} = \frac{1}{f}$$

$\therefore v = 2f$

$\therefore u = v = 2f$

Here, the points at $2f$ distance on both the sides of the lens are called **conjugate focl.**

Illustration 13 : Two converging lenses of powers 5D and 4D are placed 5 cm apart. Find the focal length and power of this combination.

Solution : Focal length of first lens, $f_2 = \frac{1}{5} = 0.2$ m = 20 cm

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Focal length of second lens, $f_2 = \frac{1}{4} = 0.25$ m = 25 cm

Distance between two lenses, d = 5 cm

Now, equivalent focal length of this combination is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$f = \frac{f_1 \cdot f_2}{(f_1 + f_2) - d} = \frac{20 \times 25}{(20+25)-5} = 12.5 \text{ cm}$$

And equivalent power is given by,

$P = (P_1 + P_2) - d.P_1P_2$

$= (5 + 4) - (0.05) \times (5)(4)$ (d is written in meter)

$\therefore$ P = 8D or $P = \frac{1}{f} = \frac{1}{0.125} = 8D$

6.14 Refraction and Dispersion of Light due to a Prism

As shown in the figure 6.22, the cross-section perpendicular to the rectangular surface of a prism is shown. A ray PQ of monochromatic light is incident at point Q on the surface AB. According to Snell's law, it is refracted and travels along the path QR. Thus, it deviates from the incident direction by an amount δ_1. This ray QR is incident on the surface AC at point R, and emerging out as a ray RS. It suffers a deviation δ_2. By extending the incident ray PQ to PQE, total deviation between the incident and the emergent ray is found. When the emergent ray RS is extended backword it meets PE at D. Angle between the incident ray and the emergent ray is called the angle of deviation, δ.

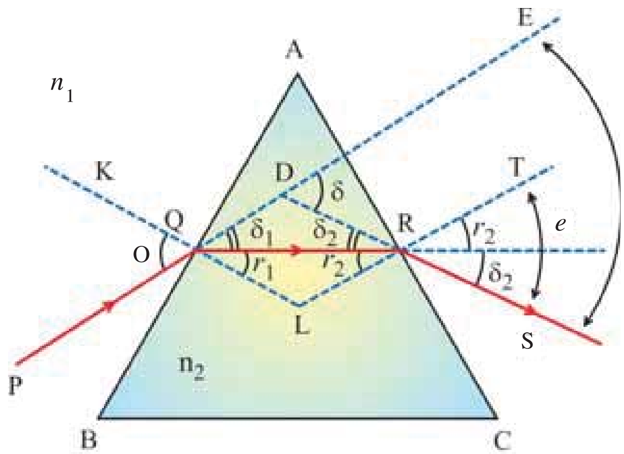

Figure 6.22 Refraction Due to Prism

From figure 6.22, in □AQLR, ∠AQL and ∠ARL are right angles.

$\therefore m\angle A + m\angle QLR = 180°$ (6.14.1)

and for ΔQLR, $r_1 + r_2 + m\angle QLR = 180°$ (6.14.2)

Comparing above equations,

$r_1 + r_2 + m\angle QLR = m\angle A + m\angle QLR$

$\therefore r_1 + r_2 = A$ (6.14.3)

For ΔDQR, ∠EDR ≡ ∠EDS = δ is the exterior angle. Therefore,

$\delta = \angle DQE + \angle DRQ$

$\therefore \delta = \delta_1 + \delta_2$ (6.14.4)

But $\delta_1 + r_1 = i$ ($\because$ vertically opposite angles)

$\therefore \delta_1 = i - r_1$ (6.14.5)

Similarly, $\delta_2 = e - r_2$ (6.14.6)

$\therefore \delta = (i - r_1) + (e - r_2) = (i + e) - (r_1 + r_2)$

Using equation (6.14.3)

$\delta = i + e - A$ or $i + e = A + \delta$ (6.14.7)

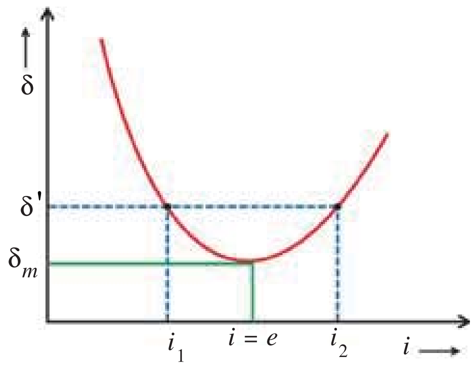

Figure 6.23 Variation of Deviation with Angle of Incidence

Equation (6.14.7) gives the relation between angle of deviation, angle of incidence and angle of emergence and the prism angle. It is known as an **equation for prism.**

It is clear from the above equation that the angle of deviation depends on the angle of incidence. For the sake of understanding, the graph of the measured values of angle of deviation against corresponding angle of incidence for an equilateral prism is shown in the figure 6.23.

We can see from the graph that for two values of angle of incidence (i_1 and i_2) angle of deviation δ is same. This can be understood from the reversibility of the rays.

If the incident ray is SR instead of PQ, then the refracted ray will follow exactly the reverse path, i.e., SRQP, and the emergent ray becomes PQ. In this case also, however, the angle of deviation remains the same. But for a particular value of angle of deviation there exists only one value of angle of incidence. And experimentally, it is found that this angle of deviation is minimum (δ_m). In the condition of minimum deviation of the incident ray the angle of deviation is called the angle of minimum deviation (δ_m). In this situation it is found that $i = e$.

From equation (6.14.7),

$\delta_m = i + i - A = 2i - A$

$$i = \frac{A+\delta_m}{2} \qquad (6.14.8)$$

Applying Snell's law at pont Q,

$$n_1 \sin i = n_2 \sin r_1 \qquad (6.14.9)$$

At point R, considering SR as the incident ray,

$n_1 \sin e = n_2 \sin r_2$

As $i = e$

$$\therefore\ n_1 \sin i = n_2 \sin r_2 \qquad (6.14.10)$$

From equations (6.14.9) and (6.14.10)

$$\therefore\ r_1 = r_2 \qquad (6.14.11)$$

From equation (6.14.3), and let $r_1 = r_2 = r$,

$r + r = A$

$$\therefore\ r = \frac{A}{2} \qquad (6.14.12)$$

Substituting the values of (6.14.8) and (6.14.12) in either in (6.14.9) or (6.14.10),
This gives,

$$\therefore\ n_1 \sin\left(\frac{A+\delta_m}{2}\right) = n_2 \sin\left(\frac{A}{2}\right)$$

$$\text{or}\quad \frac{n_2}{n_1} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \qquad (6.14.14)$$

If the prism is kept in air, i.e. $n_1 = 1$ and $n_2 = n$,

$$\therefore\ n = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \qquad (6.14.15)$$

Equation (6.14.15) shows that value of δ_m depends on the angle of prism. The refractive index of the material of the prism and the medium in which prism is kept.

For equilateral prism, when δ is minimum, refracted ray (QR) through the prism is **parallel to the base BC** of the prism. Equation (6.14.15) is of practical importance to measure refractive index of the material of the prism.

Case : The prisms with small angle of prism are called thin prisms. For such prisms, angle of deviation is also small. In this case equation (6.14.15) gives

$$\delta_m = A(n - 1) \quad (6.14.16)$$

Dispersion :

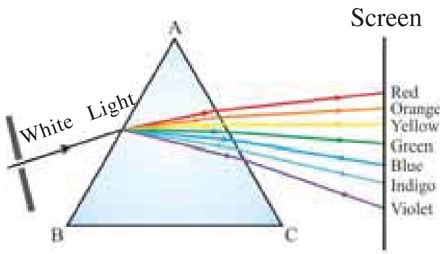

Figure 6.24 Dispersion of White Light

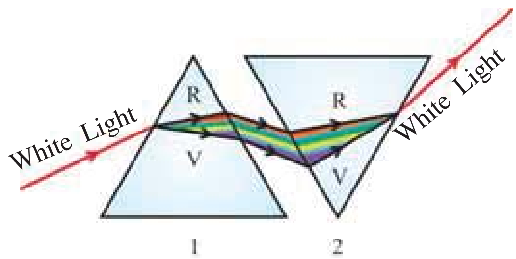

Figure 6.25 Dispersion and Recombination of White Light

As shown in figure 6.24, when a beam of white light or sun light passes through a prism the emergent light is made up of different colours. To understand this phenomenon, Newton has arranged two identical prisms as shown in figure 6.25. A ray of white light is incident on the prism-1, and emergent ray from the prism-2 is observed. It is found that this emergent ray is also white. This experiment explains that the first prism disperses the colours of white light, while the second prism brings them together.

The phenomenon in which light gets divided into its constituent colours is known as **dispersion of light.**

It is found that for the visible part of the electromagnetic spectrum violet colour has the maximum refractive index and red colour has the lowest. From equation (6.14.16), corresponding minimum angle of deviation through the same prism is

$$\delta_v = A(n_v - 1)$$

and $\delta_r = A(n_r - 1)$

It is now clear that as $n_v > n_r$, $\delta_v > \delta_r$.

Thus, deviation of violet colour is more compared to the deviation of red colour.

The total angle through which the spectrum is spread is called as the angular dispersion. It is defined as,

$$\theta = \delta_v - \delta_r = (n_v - n_r) \cdot A \quad (6.14.17)$$

For example, the spectrum obtained by a prism made up of flint glass is wider, more dispersed and more detailed as compared to the one obtained by common crown glass.

Illustration 14 : For a prism, angle of prism is 60° and it's refractive index is 1.5, find (1) angle of incidence corresponding to the angle of minimum deviation and (2) angle of emergence for angle of maximum deviation.

Solution : (1) For minimum deviation,

$r_1 = r_2$ and $A = r_1 + r_2$

$\therefore$ A = $2r_1$

or $r_1 = \frac{A}{2} = \frac{60}{2} = 30°$

Now $n = 1.5$ and

$$n = \frac{\sin i}{\sin r_1}$$

$\therefore n \sin r_1 = \sin i$

$\therefore 1.5 \times \sin 30° = \sin i$

$\therefore 1.5 \times 0.5 = \sin i$

$\therefore i_1 = 48°\ 35'$

(2) For maximum deviation, $i = 90°$

$\therefore 1.5 = \frac{\sin 90°}{\sin r_1} \quad \therefore r_1 = 41°\ 48'$

$\therefore r_2 = A - r_1 = 60 - 41°\ 48' = 18°\ 12'$ ($\because r_1 + r_2 = A$)

$1.5 \sin r_2 = \sin e$ ($\because n \sin r_2 = \sin e$)

$\therefore 1.5 \times \sin 18°\ 12' = \sin e$

$\therefore \sin e = 0.4685$

$\therefore e = 27°\ 56'$

Illustration 15 : An equilateral prism is kept in air and for a particular ray, angle of minimum deviation is 38°. Calculate the angle of minimum deviation if the prism is immersed in water. Refractive index of water is 1.33.

Solution : $\frac{n_g}{n_a} = \frac{\sin\left(\frac{60+38}{2}\right)^{\circ}}{\sin 30^{\circ}}$

Taking $n_a = 1$,

$$n_g = \frac{\sin 49^{\circ}}{\sin 30^{\circ}} = 1.509$$

When prism is immersed in water,

$$\frac{n_g}{n_\omega} = \frac{\sin\left(\frac{60+\delta_m}{2}\right)^{\circ}}{\sin 30^{\circ}}$$

But $n_\omega = 1.33$

$$\therefore \frac{1.509}{1.33} = \frac{\sin\left(\frac{60+\delta_m}{2}\right)^{\circ}}{0.5}$$

$$\therefore \sin\left(\frac{60+\delta_m}{2}\right)^{\circ} = 0.5679$$

$$\therefore \frac{60+\delta_m}{2} = 34°\ 36'$$

$\therefore \delta_m = 9°\ 12'$

6.15 Scattering of Light

Light scattering is one of the two major physical processes that contribute to the visible appearance of most of routine objects, the other being absorption. Broadly, scattering can be classified either as elastic or inelastic. Natural occurrence like, colour of sky during sunrise or sunset and during day time, colour of clouds can be understood by elastic scattering of light due to atmospheric atoms, molecules, water droplets, etc. Light falling on such particles is absorbed by them and immediately radiated in different amount in different directions. As a result, part of the intensity of light ray is diverted to different directions in different proportions.

It is found that the intensity of scattered light depends on the ratio (α) of the size of the particle (i.e. its diameter, for spherical particles) and wavelength of the light.

If $\alpha << 1$: Scattering is known as Rayleigh scattering

$\therefore \alpha \approx 1$: Scattering is known as Mie-scattering.

$\therefore \alpha >> 1$: Geometric scattering.

6.15.1 Rayleigh Scattering : If the size of the particle which scatters the light is smaller than the wavelength of the incident light, the scattering is known as **Rayleigh scattering.**

Lord **Rayleigh** showed theoretically that the intensity of scattering is inversely proportional to the fourth power of the wavelenght of light. Since the wavelength of blue light is 1.7 times smaller than the red light. So, the intensity of scattered blue light is 8 to 9 times more than the intensity of scattered red light. Thus, intense scattered-blue light is responsible for the sky to be bluish.

Another consequence of Rayleigh seattering is the appearance of reddish colour of the sun either at the sunrise or at the sunset

As shown in figure 6.26, at the sunrise or sunset, light from the sun has to travel relatively more distance to reach the observer on the earth as compared to the noon-time. During the passage of light in the atmospheric light of smaller wavelengths scatter more. Hence, only light with high wavelengths (i.e., reddish or yellowish-red) can reach to the observer substantially. Thus, the sun appears reddish. However, if we see vertically upward, sky appears blue. This effect is maximum in the direction perpendicular to the incident light. The same is the reason for reddish full-moon while rising or setting.

It is found that the intensity of the Rayleigh scattered light increases rapidly as the ratio α increases. Further, the intensity of Rayleigh scattered light is identical in the forward and reverse directions.

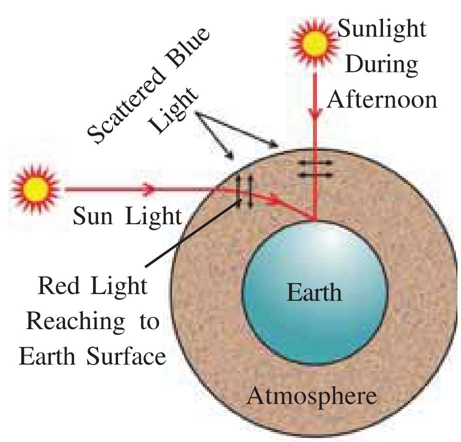

Figure 6.26 Seattering of Sun Light Due to Atmosphere

6.15.2 Mie-Scattering : If the size of scatterer particles are slightly larger than the wave length of the light, scattering is known as Mie-scattering. It was studied by Gustav Mie in 1908. It is found that as the size of the particle increases, the proportion of diffused scattering also increases. Since water droplets in the cloud have size comparable to wavelength of light, scattering of sun light through clouds is diffused scattering. It is independent of incident wavelengths. Hence, all colours scatter equally, and the clouds appear white. Unlike Rayleigh scattering, Mie-scattering is observed in larger amount in the forward direction than in the reverse direction. Also, as the particle size increases, more amount of the light is scattered in the forward direction.

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For information only : The Mie-scattering shows that if the size of the particle lies between two wavelengths of light, then the light having more wavelength is scattered more than the light with smaller wavelength. If dust clouds have such size then the rising sun and moon or setting sun and moon would be seen blue or greeen !

However, such a situation rarely occurs. In the 19th century when the Volcano Krakotoa in Indonesia erupted and in 1950 when there were extensive forest fires in East-Canada and North-East USA, such situation took place.

If earth had no atmosphere, the sky would have been blackish, and stars would have been visible even during day time ! This becomes reality at or above 20 km from the earth surface.

In presense of high pollution in the atmosphere, the sky appears greyish and hazy instead of blue.

6.15.3 Raman-Scattering : The Raman effect was first reported by Indian Nobel laureate C. V. Raman. This inelastic scattering of light was also predicted by Adolf Smekal in 1923. Hence, this effect is also known as **Smekal-Raman effect**.

When a strong beam of visible or ultraviolet light is incident on gas, liquid or transparent solid, a small fraction of light is scattered in all directions. It is found that the scattered light spectrum is made up of lines of incident wavelength (Rayleigh lines) and weak additional lines of changed wave lenghts. These additional lines due to inealstic scattering are called **Raman lines.** Raman lines are found symmetrically on both sides of the central Rayleigh lines. Raman lines with low frequencies (or higher wavelengths) are known as **Stokes lines,** and the one on higher frequency (or low wavelength) sides are known as **Antistokes lines**.

Raman lines are the characteristics of the material.

Raman scattering is the most versatile technique to study characteristics of the material, different excitations in the materials, in optical amplifiers, to study biological organisms and human tissues, etc.

6.16 Optical Instruments : The purpose of most optical instruments is to enable us to see the object better. They are made up of combination of refracting and/or reflecting devices such as lenses, mirrors and prisms. They can be divided into two groups : instruments forming **real images** (e.g., projectors) and instruments forming imaginary images (e.g., microscopes and telescopes).

We first study simple microscope.

6.16.1 Simple Microscope : Suppose we want to see a microscopic object clearly and magnified.

The least distance at which a small object can be seen clearly with comfort is known as **near point** (D) or **distance of most distinct vision**. For normal eye this distance is 25 cm.

Suppose a linear object with height h_0 is kept at near point (i.e., $u \equiv$ D = 25 cm) from eye. Let it subtend an angle θ_0 with the eye (See figure 6.27 (a)).

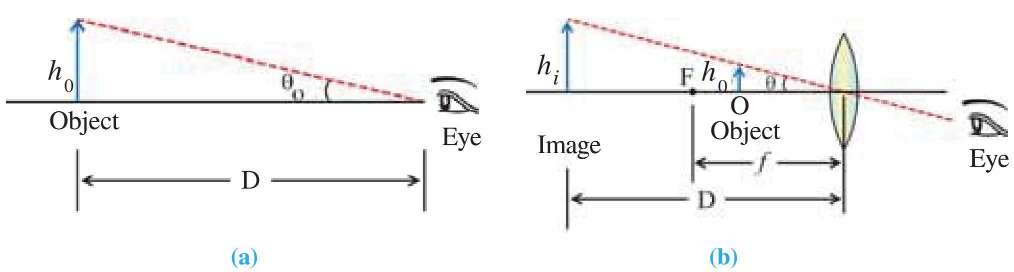

Figure 6.27 Simple Magnifier

Now, if object is kept within the focal length (f) of a convex lens such that its virtual, erect and magnified image is formed at a distance equal to the near point. Since the lens is very close to eye, angle (θ) subtended by the object with lens and eye are almost identical.

The angular magnification is defined as

$$m' = \frac{\tan\theta}{\tan\theta_0} \approx \frac{\theta}{\theta_0} \text{ (for small } \theta \text{ and } \theta_0) \quad (6.16.1)$$

Also, from figures 6.27 (a) and (b),

$$\tan\theta_0 \approx \theta_0 = \frac{h_0}{D}$$

$$\text{and } \tan\theta \approx \theta_0 = \frac{h_i}{D}$$

$$\therefore\ m' = \frac{h_i}{h_0} \quad (6.16.2)$$

But for convex lens, linear magnification,

$$|\ m\ | = \frac{v}{u}$$

$$|\ m\ | = \frac{D}{u} \quad (6.16.3)$$

Using Gauss' formula,

$$-\frac{1}{(-u)} + \frac{1}{(-D)} = \frac{1}{f} \text{ (for this image } v = D \text{ is negative)}$$

$$\therefore\ \frac{1}{u} = \frac{1}{f} + \frac{1}{D} = \frac{D+f}{D\cdot f}$$

$$\therefore\ u = \frac{Df}{D+f} \quad (6.16.4)$$

Using (6.16.3) in equation (6.16.4),

$$|\ m\ | = 1 + \frac{D}{f} \quad (6.16.5)$$

When the image is at a very large distance

$$|\ m\ | \approx \frac{D}{f} \quad (6.16.6)$$

Combinedly equations (6.16.5) and (6.16.6) suggest that the value of m should be between $\frac{D}{f}$ and $\left(1+\frac{D}{f}\right)$

6.16.2 Compound Microscope : We have seen that in a simple microscope magnifying power depends on $\frac{D}{f}$. Thus, we tempted to use a convex lens with small focal length in order to improve magnification. It is found, however, that by reducing the value of focal length, image becomes distorted. Thus, very large and clear image is not possible with a simple microscope. But if magnified image due to one simple microscope is used as an object for another simple microscope, then we get very enlarged image. This is the basic principle of a compound microscope.

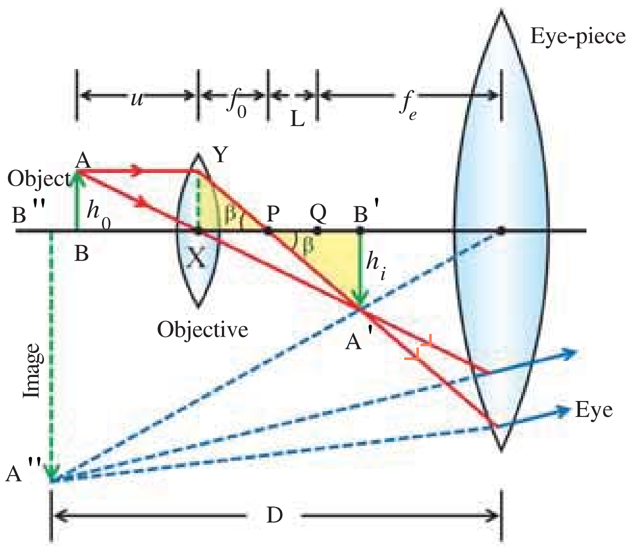

Figure 6.28 Compound Microscope

The lens kept near the object is known as **objective,** while the one nearer to eye is known as **eye-piece.** Distance between the second focal point (P) of the objective and the first focal point (Q) of the eye-piece is known as **tube-length (L)** of the microscope.

It is clear from the figure that the image obtained by the objective is real, inverted and magnified. This image acts as an object for the eye-piece. Eye-piece works as a simple microscope and gives a virtual and highly magnified final image (A''B'').

The image due to objective is observed close to the focal point of an eye-piece. Due to this reason final image is formed at a considerable large distance.

Magnification : Magnification due to the objective,

$$m_0 = \frac{h_i}{h_0} \qquad (6.16.7)$$

From ΔXYP and ΔPA'B', respectively,

$$\tan\beta = \frac{XY}{PX} = \frac{h_0}{f_0} \Rightarrow h_0 = f_0 \cdot \tan\beta$$

and $\tan\beta = \frac{A'B'}{PB'} \approx \frac{h_i}{PQ}$ ($\because$ Q and B' are very close to each other)

$$\therefore\ h_i = PQ \cdot \tan\beta = L \cdot \tan\beta$$

$$\therefore\ m_0 = \frac{L}{f_0} \qquad (6.16.8)$$

Magnification due to eye-piece,

$$m_e = \left(\frac{D}{f_e} + 1\right) \text{ (See Equation (6.16.5)} \qquad (6.16.9)$$

Resultant magnification of a compound microscope is (Equation (6.12.10)),

$$m = m_0 \times m_e$$

$$= \frac{L}{f_0} \times \left(\frac{D}{f_e} + 1\right) \qquad (6.16.10)$$

In practice, eye-piece is so adjusted that image A'B' falls very close to its focus Q. Thus, image obtained by eye-piece will be at very large distance (D). Thus, above equation can be written as,

$$m \approx \frac{L}{f_0} \times \frac{D}{f_e} \qquad (6.16.11)$$

In order to have large magnification, tube length (L) of the microscope should be kept large.

Illustration 16 : An object is 10 mm from the objective of a compound microscope. The lenses are 30 cm apart and the intermediate image is 50 mm from the eyepiece. What overall magnification is produced by the instrument ?

Solution : From the figure 6.28, applying Gauss's formula to the objective,

$$\frac{1}{-u} + \frac{1}{v} = \frac{1}{f_0} \qquad (1)$$

where v = image distance due to objective lens $\approx f_0 + L$ (as Q and B' are very close to each other).

Since image due to objective is formed at 50 mm from the eye-piece, and distance between two lenses is 30 cm = 300 mm (given), image distance from objective

$v = 300 - 50 = 250$ mm

From equation (1),

$$\frac{-1}{-10} + \frac{1}{250} = \frac{1}{f_0} \text{ (using sign convention)}$$

$$\therefore f_0 = \frac{250 \times 10}{(250+10)} = 9.62 \text{ mm} \approx 10 \text{ mm}$$

Since $v \simeq f_0 + L \Rightarrow L = 250 - 10 = 240$ mm

Final image is always close to the object,

D $\approx$ (object distance for objective) + (distance between two lenses)

$= 10 + 300 = 310$ mm

For eye-piece, Gauss' equation,

$$\frac{1}{-u} + \frac{1}{v} = \frac{1}{f_e}$$

$$\therefore \frac{1}{f_e} = \frac{-1}{-50} + \frac{1}{-310} \text{ (For virtual image, } v = -D)$$

$$= \frac{-310+50}{(50 \times 310)}$$

$\therefore |f_e| = 59.6 \approx 60$ mm

thus, resultant magnification is

$$m = \frac{L}{f_0} \times \frac{D}{f_e} = \frac{240}{10} \times \frac{310}{60} = 124$$

Note : Since the final image obtained at a distance 31 cm from the eye-piece is greater than the near-point distance, it can be seen comfortably.

6.16.3 Astronomical Telescope : After observing minute objects using a microscope, now it's time to observe very huge celestial bodies which are crores of kilometers away. Such bodies, in spite of being huge and very far from each other, they are seen to be small and very close to each other by our naked eyes (for example, stars). For observing such objects an Astronomical Telescope is used. It's ray diagram is shown in figure 6.29.

In this telescope two convex lenses are kept in such a way that their principal axis coincide. The lens facing the object is called objective and the lens near the eye is known as eye-piece. Here, the diameter and the focal length of the objective are greater than that of the eye-piece.

The eye-piece can move to and fro in the telescope-tube. When the telescope is focussed on a distant object, parallel rays coming from this object form a real, inverted, and small image

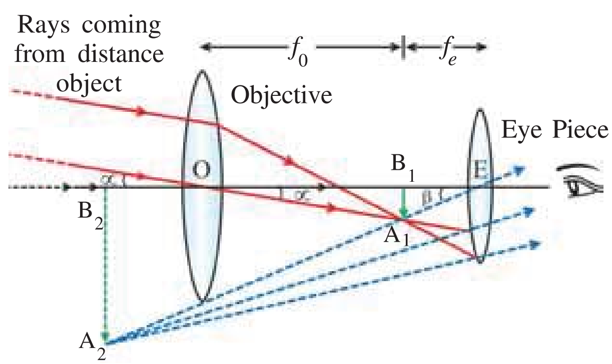

Figure 6.29 Astronomical Telescope

A_1B_1 on the second principal focus of the objective. This image is the object for the eye-piece. Eye-piece is moved to and fro to get the final and magnified inverted image A_2B_2 of the original object at a certain distance.

We obtain the expression for the magnifying power of a telescope, as follows.

Magnification of the telescope,

$$m = \frac{\text{Angle subtended by the final image with eye}}{\text{Angle subtended by the object with the objective or eye}} = \frac{\beta}{\alpha}$$

From figure 6.29

$$\text{Magnification, } m = \frac{\beta}{\alpha}$$

$$= \frac{A_1B_1}{f_e} \times \frac{f_0}{A_1B_1}$$

$$\therefore \; m = \frac{f_0}{f_e}$$

This equation shows that to increase the magnification of the telescope, focal length of the objective should be increased, and focal length of the eye-piece should be reduced. $f_0 + f_e$ is the **optical length** of the telescope. So, length of the tube $L \geq f_0 + f_e$.

If the focal length of the eye-piece is 1 cm and the focal length of the objective is 200 cm, magnification of the telescope would be 200. Using such a telescope, if the stars having angular distance $1'$ are observed, they would be seen at $200 \times 1' = 200' = 3.33°$ angular distance from each other.

For a telescope, light **gathering power** and **resolving power** (power to view two nearby objects distinctly) are very important.

Amount of light entering the objective of the telescope is directly proportional to the square of the diameter of the objective. Also, with increase in the diameter of the objective, resolving power also increases.

Image formed in this type of telescope is inverted. So if we see from the Earth we get an inverted view of the real scene. To get rid of this problem, an extra pair of inverting lenses in the terrestrial telescope are kept, so that the erect image of the distant object is obtained. Such a telescope is called a **terrestrial telescope**. However, Galileo had used a convex lens and a concave lens in such a telescope.

To get rid of the practical problem faced in obtaining high resolution and high magnification in refracting telescopes, mirrors are used in modern telescopes. Such a telescope is known as reflecting telescope. In such a telescope we can get rid of other problems like chromatic aberration and also spherical aberration, if a parabolic mirror is used.

(In chromatic aberration the edge of the image is seen multicoloured due to dispersion of light and in spherical aberration, image of a point like object is seen spread out).

Construction of the telescope made by Cassegrain (reflecting telescope) is shown in figure 6.30.

As shown in the figure, parallel rays coming from a distant object are incident on the reflecting surface of the primary concave mirror. The reflecting surface of the mirror is parabolic. The rays after getting reflected from this surface are focussed on the principal focus (F) of this mirror. (If the eye-piece is kept near F the image can be seen. But as F is inside the tube, it is difficult to place the eye-piece there.) Cassegrain placed a convex mirror. Rays reflected by the secondary mirror are focussed on the eye-piece after passing through the hole kept in the primary mirror. Diameter and focal length of the primary mirror are kept large in such telescope.

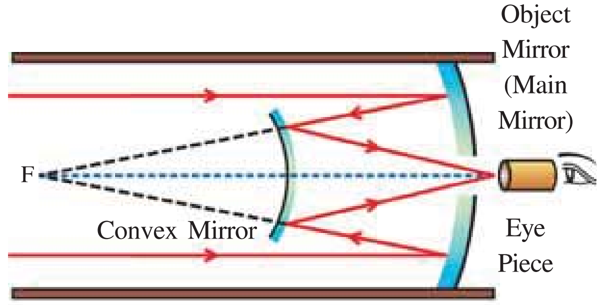

Figure 6.30 Reflecting Telescope

Binoculars used for bird watching or for viewing a cricket match are double telescopes. Here, the final image is erect. In the **binoculars** use of prisms reduces the size of the binoculars. Binoculars are so named because in them viewing is possible by both eyes.

6.16.4 Human Eye : Human eye is the best example of a natural optical appliance. See figure 6.31.

The ray entering the eye is first refracted in the cornea, yet the eye lens is the main factor "culprit" in this case. Due to this lens, inverted and real image is formed on the ratina. This image is processed in the human brain and as a final effect, we feel the image be erect.

Retina has two types of cells :

(1) Rods : These cells give the sensations of less intensity of light.

(2) Cones : These cells give the sensations of colour and high intensity of light.

In case of eye, distance between the retina and the lens is fixed. That is why focal length of the eye lens changes in such a way that the images of the object are always obtained on the retina. (Really, eye lens is smart lens). This becomes possible due to the ciliary muscles attached to the lens. It makes the lens thick or thin as per requirement.

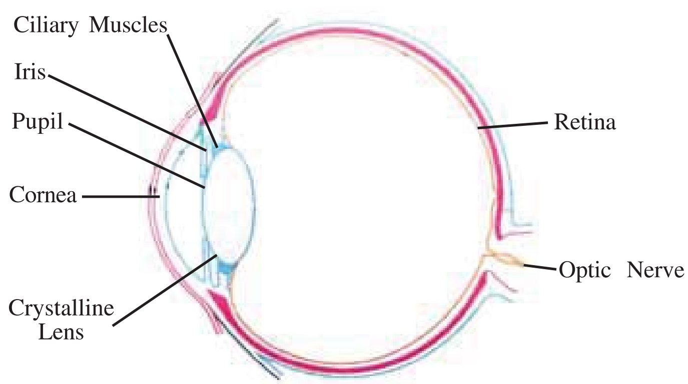

Figure 6.31 Human Eye (For Information Only)

The Iris controls the amount of light entering the eye. It does the work by controlling the size of the pupil. When we see the object kept on the side, lens of the eye rotates and brings the image on the central region of the retina, (fovea).

Defects of Vision : If the lens of eye cannot become thin as per requirement and remains thick only, then rays coming from far objects, which are parallel, undergo extra refraction as shown in figure 6.32, and get focussed in front of the retina. And therefore far off objects cannot be seen clearly. But the image of nearby objects is formed on the retina (figure 6.33). This type of defect is called **Near sightedness (myopia).**

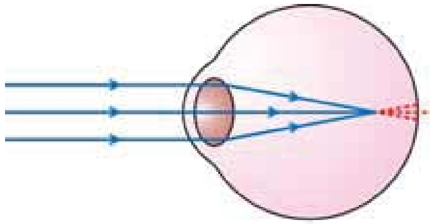

Figure 6.32 Image of Distance Object Falls in front of Retina

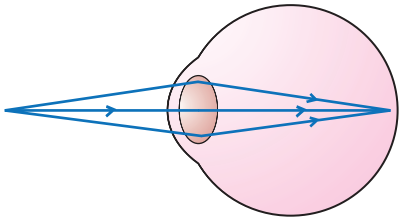

Figure 6.33 Image of Nearby Object Falls on the Retina

This defect can be corrected by using concave lens of proper focal length (figure 6.34).

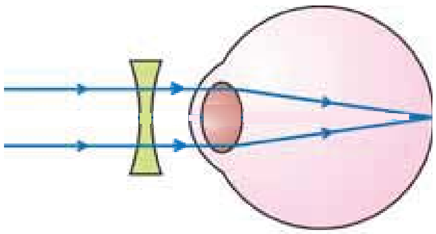

Figure 6.34 To Correct this Defect, Concave Lenses are Used

If the lens remains thin, does not become thick as per requirement, rays coming from a nearby object suffer less refraction and are focussed behind the retina. (figure 6.35). Such an image is not clear. Image of a distant object is formed on the retina only and can be seen clearly, but nearby objects cannot be seen clearly. This defect is called **far sightedness (hypermetropia)**. This type of defect is due to less convergence of rays. To correct this defect a convex lens of proper focal length is used (figure 6.36).

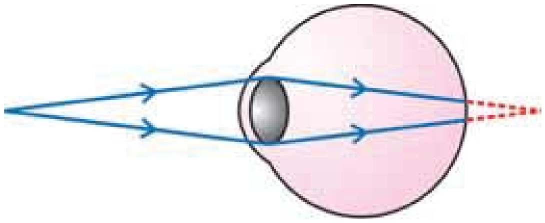

Figure 6.35 Hypermetropia

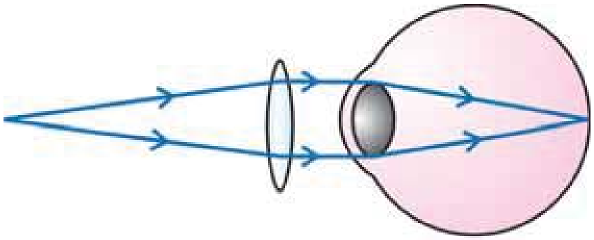

Figure 6.36 Convex Lens Between Object and Eye

Some people, if shown a wire gauge cannot see the vertical and horizontal both wires clearly, but any one is seen clearly. This defect is called **astigmatism**. If the curvature of the lens and the corena are not the same, this defect occurs. E.g., if a person can see horizontal wires but not vertical. Here, horizontal curvatures are same but vertical curvatures are not. So rays are refracted equally in the horizontal plane, but refraction in the vertical plane is not equal. As a result horizontal wires are seen clearly and vertical wires are not seen clearly. To get rid of this defect, cylindrical lens is used. In the above mentioned case a cylindrical lens of proper curvature and horizontal axis can be used to rectify the defect.

SUMMARY

1. For mirrors Gauss' equation is $\frac{1}{u} + \frac{1}{v} = \frac{2}{R} = \frac{1}{f}$, where u = object distance, v = image distance, R = radius of curvature and f = focal length.
2. Lateral magnification for mirrors is given by $m = \frac{h'}{h} = -\frac{v}{u}$
3. For a compound slab of different transparent media, general form of Snell's law is written as, $n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \cdot \sin\theta_3 = ...$
4. Total internal reflection is used as reflectors, e.g. flint glass-prism may be used as high quality reflector. For glass–air interface, critical angle (C) is given by,

   $C = \sin^{-1}\left(\frac{1}{n}\right)$, where n = refractive index of glass.
5. Total internal reflection phenomenon is also used in optical fibres.
6. For thin lens : $\frac{-1}{u} + \frac{1}{v} = \left(\frac{n_2 - n_1}{n_1}\right) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ and $\frac{1}{f} = \frac{-1}{u} + \frac{1}{v}$
7. Since the principle of reversibility suggests that the object and image are conjugate to each other, interchanging the positions of an object, image distance can be determined.
8. Power of lenses in contact is given by
   $P = P_1 + P_2 + ...$
9. Magnification of lenses in contact is given by
   $m = m_1 \times m_2 \; .. \; ....$
10. Focal length of lenses in contact is given by

    $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + ...$
11. Prism equation is given by $\delta = i + e - A$. At minimum angle of deviation, $\delta_m = 2i - A$. For thin prisms, (A < <), $\delta_m = A(n - 1)$, where n = refractive index of the material of prism.
12. Scattering can be classified into two : elastic scattering (Rayleigh and Mie-Scattering) and inealstic scattering (e.g., Raman Scattering). If the size of the particle scattering light is smaller than the wavelength of the incident light, it is known as the Rayleigh scattering, if otherwise, it is known as the Mie-scattering.
13. Compound microscope can be thought of as made up of two cascaded simple microscopes, in which magnified image due to first simple microscope works as an object for the second.
14. For high resolution and magnification, curved mirrors are used in modern telescopes.
15. Retina has two types of cells : rods give the sensations of less intense light and cones give sensations of colour and high intense light
16. Defects of vision can be overcome by proper lenses.

EXERCISES

For the following statements choose the correct option from the given options :

1. An object is placed at a distance of 25 cm on the axis of a concave mirror, having focal length 20 cm. Find the lateral magnification of an image.

   (A) 2 (B) 4 (C) −4 (D) −2
2. A fish in a lake is at a 6.3 m horizontal distance from the edge of the lake. If it is just able to see a tree on the edge of the lake, its depth in the lake is .... m. Refractive index of the water is 1.33.

   (A) 6.30 (B) 5.52 (C) 7.5 (D) 1.55

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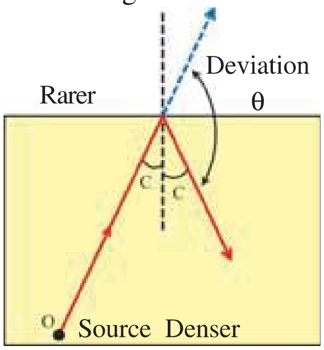
3. For a thin convex lens when the heights of the object is double than its image, its object distance is equal to .... focal length of a lens is f.
(A) f (B) $2f$ (C) $3f$ (D) $4f$

4. A liquid of refractive index n is filled in a tank. A plane mirror is kept at the bottom of the tank. A point like object (P) is kept at a height h from the mirror on the liquid surface. An observer observes the object and its image in the vertically downward direction from top. How much distance will observer note between P and its image ?
(A) $2n{\cdot}h$ (B) $\frac{2h}{n}$ (C) $\frac{2h}{(n-1)}$ (D) $h\left(1+\frac{1}{n}\right)$

5. Depth of a well is 5.5 m and refractive index of water is 1.33. If viewed from the bottom, by how much height would the bottom of the well appear to be shifted up ?
(A) 5.5 m (B) 2.75 m (C) 4.13 m (D) 1.37 m

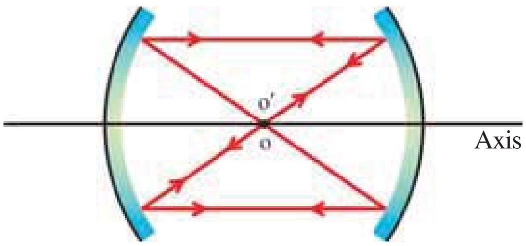
6. A ray of light is travelling from a denser medium to rarer medium. For these media, the critical angle is C. The maximum possible deviation of the ray is ....



(A) $\pi - 2$ (B) $\pi - 2c$
(C) 2C (D) $\frac{\pi}{2} + C$

[**Hint :** The situation at total reflection is shown in the figure.]

7. A point object O is placed midway between on the common axis of two concave mirrors of equal focal length. If the final image is formed at the position of the object, the separation between two mirrors is .... Focal length of mirrors is f.



(A) f (B) $2f$
(C) $\frac{3}{2}f$ (D) $\frac{1}{2}f$.

[**Hint :** A situation is depicted in the figure.]
[**Note :** Another possible situation for which object and its image coincide is when distance between two mirrors is $4f$.]

8. The focal length of a thin lens made from the material of refractive index 1.5 is 20 cm. When it is placed in a liquid of refractive index 1.33, its focal length will be .... cm.
(A) 80.81 (B) 45.48 (C) 60.25 (D) 78.23

9. A tank contains water upto a height of 30 cm and above it an oil up to another 30 cm height. .... cm shifts in the position of bottom of the tank is observed when viewed from the above. Refractive indices of water and oil are 1.33 and 1.28, respectively.
(A) 7.44 (B) 6.46 (C) 14.02 (D) 6.95

[**Hint :** From $\frac{h'}{h} = \frac{n_1}{n_2}$

$$\frac{h'-h}{h} = \frac{-\Delta h}{h} = \frac{n_1 - n_2}{n_2}$$

$$\therefore -\frac{\Delta h}{h} = \left(\frac{n_1}{n_2} - 1\right) \quad \text{(Shift, } \Delta h \text{ is negative because shift, } \Delta h = h - h'\text{)}$$

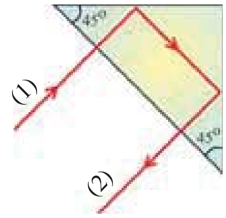
$$\therefore \text{ shift, } \Delta h = h \times \left(1 - \frac{n_1}{n_2}\right) = h \times \left(1 - \frac{1}{n_{21}}\right)$$]

10. For a thin plano convex glass lens with radius of curvature 20 cm, focal length is .... cm. Refractive index (n) of the material of the lens is 1.5 and it is kept in air

(A) 20 (B) 40 (C) 60 (D) 80

[**Hint** : For air – glass lens, $\frac{-1}{u} + \frac{n}{v} = \frac{1}{f} = \frac{(n-1)}{R}$]

11. For right-angled prism, ray-1 is the incident ray and ray-2 is the emergent ray, as shown in the figure. Refractive index of the prism is ....



(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$

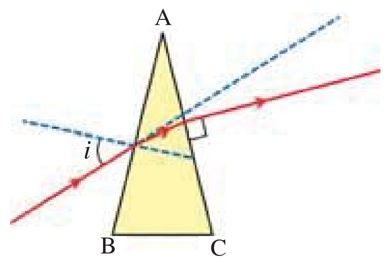
(C) $\frac{2}{\sqrt{3}}$ (D) $\sqrt{2}$

12. A ray of light is incident normally on the surface of an equileteral prism made up of material with refractive index 1.5. The angle of deviation is ....

(A) 30° (B) 45° (C) 60° (D) 75°

[**Hint** : For the present case use the formula $\sin C = \frac{1}{n}$ to understand the phenomenon.]

13. A ray is incident at an angle i on the surface of a prism with very small prism angle A, and emerges normally from the opposite surface. If the refractive index of the prism is μ the angle of incidence i is nearly equal to ....



(A) $\frac{A}{\mu}$ (B) $\frac{\mu A}{2}$

(C) $\frac{A}{2\mu}$ (D) μA

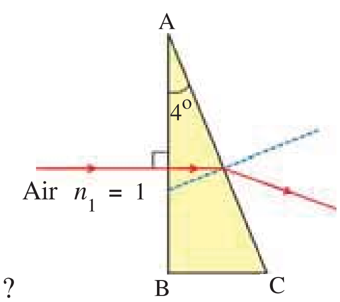
[**Hint** : Use the given figure.]

14. A small linear object of length b is placed on the axis of a concave mirror. The end of the object facing the mirror is at a distance u from the mirror. If f is the focal length of a mirror, the length of the object will be .... approximately.

(A) $b\left(\frac{u-f}{f}\right)^2$ (B) $b\left(\frac{f}{u-f}\right)$ (C) $\left(\frac{u-f}{f}\right)$ (D) $b\left(\frac{f}{u-f}\right)^2$

[**Hint** : Neglect b whenever necessary.]

15. A horizontal ray is incident on a right-angled prism with prism angle of 4°. If the refractive index of material of the prism is 1.5, angle of emergence is .... Use the given figure.



(A) 4° (B) 6°

(D) 10° (D) 0°

16. Which of the following is responsible for glittering of a diamond ?

(A) Interference (B) Diffraction (C) Total internal reflection (D) Refraction

17. The radii of curvature of both the sides of a convex lens are 15 cm and if the refractive index of the material of the lens is 1.5, then focal length of lens in air is .... cm

(A) 10 (B) 15 (C) 20 (D) 30

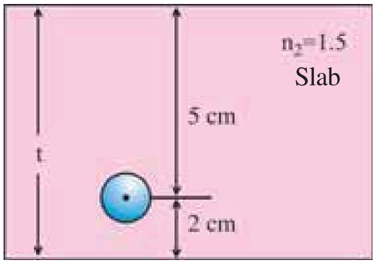
18. An image of an object obtained by a convex mirror is n times smaller than the object. If the focal length of lens is f, the object distance would be ....

(A) $\frac{f}{n}$ (B) $\frac{f}{(n-1)}$ (C) $(n - 1)f$ (D) nf

19. Time taken by the sunlight to pass through a slab of thickness 4 mm and refractive index 1.5 is .... sec.

(A) 2×10^{-8} (B) 2×10^{8} (C) 2×10^{-11} (D) 2×10^{11}

20. An air bubble in a glass slab with refractive index 1.5 is 5 cm deep when viewed from one face and 2 cm deep when viewed from the opposite face. The thickness of the slab is .... cm.



(A) 10.5 (B) 7

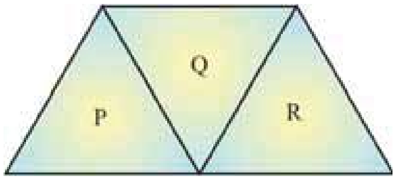
(C) 105 (D) 70

[**Hint :** Use $\frac{h'}{h} = \frac{n_2}{n_1}$]

21. The focal length of an equiconvex lens in air is equal to either of its radii of curvature. The refractive index of the material of the lens is ....

(A) $\frac{4}{3}$ (B) 1.5 (C) 2.5 (D) 0.8

22. A ray of light experiences minimum deviation by an equilateral prism P. Now two prisms Q and R made of the same material as that of P are arranged as shown in the figure. The ray of light will now experience, (The dimensions of P, Q and R are same.) ....



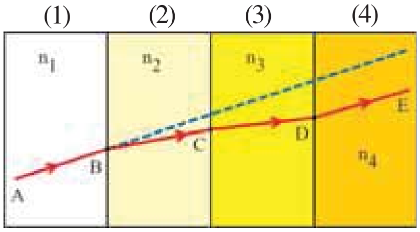
(A) larger deviation

(B) no deviation

(C) same deviation as that due to P

(D) total internal reflection

23. The refractive indices of four media, as shown in the figure, are n_1, n_2, n_3 and n_4. AB is an incident ray. DE, the emergent ray, is parallel to the incident ray AB, then ....



(A) $n_1 = n_2$ (B) $n_2 = n_3$

(C) $n_3 = n_4$ (D) $n_4 = n_1$

24. If the tube length of astronomical telescope is 105 cm and magnifying power is 20 for normal setting, then the focal length of the objective is .... cm.

(A) 10 (B) 20 (C) 25 (D) 100

[**Hint :** Optical length of astronomical telescope is given by L $\geq f_0 + f_e$]

25. The top sky looks blue in morning hours because, ...

(A) red light is absorbed (B) blue light is scattered the most

(C) sun radiates blue light only in the morning.

(D) blue light is absorbed by the sky

26. A defect of vision in which lines in one plane of an object appear in focus while those in another plane are out of focus is called ....

(A) astigmatism (B) distortion (C) myopia (D) hypermetropia

27. Stokes and antistokes lines observed in Raman scattering is due to .... of light.

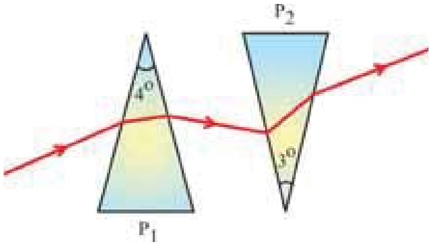
(A) reflection (B) elastic scattering

(C) inelastic scattering (D) dispersion

28. A convex lens of focal length 10 cm is used as a simple microscope. When image of an object is obtained at infinite, magnification is .... Near point for normal vision is 25 cm.

(A) 1.0 (B) 2.5 (C) 0.4 (D) 25

29. As shown in the figure, thin prisms P_1 and P_2 are combined to produce dispersion without deviation. For prism P_1, angle of prism is 4^o and refractive index is 1.54. For prism P_2 angle of prism is 3^o. Refractive index of material of P_2 is ....



(A) 1.72 (B) 1.5

(C) 2.4 (D) 0.58

[**Hint :** For thin prism, $\delta = A(n - 1)$]

30. A spherical convex surface separates an object and image spaces of refractive index 1.0 and 1.5 respectively. If radius of curvature of the surface is 25 cm, its power is .... D.

(A) 13 (B) 33 (C) 3.3 (D) 1.3

[**Hint :** $\frac{-n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$ and $P = \frac{1}{f}$]

31. A light ray is incident at an angle 30^o with normal on a 3 cm thick plane slab of refractive index $n = 2.0$. The lateral shift of the incident ray is .... cm.

(A) 0.835 (B) 8.35 (C) 1.5 (D) 1.197

[**Hint :** Since incident angle θ_1 is not small, lateral shift, $x = \frac{t\sin(\theta_1 - \theta_2)}{\cos\theta_2}$]

ANSWERS

1. (C)	**2.** (B)	**3.** (C)	**4.** (B)	**5.** (D)	**6.** (B)
7. (B)	**8.** (D)	**9.** (C)	**10.** (B)	**11.** (D)	**12.** (C)
13. (D)	**14.** (D)	**15.** (B)	**16.** (C)	**17.** (B)	**18.** (C)
19. (C)	**20.** (A)	**21.** (B)	**22.** (C)	**23.** (D)	**24.** (D)
25. (B)	**26.** (A)	**27.** (C)	**28.** (B)	**29.** (A)	**30.** (D)
31. (A)					

Answer the following questions in brief :

1. What are paraxial rays ?

2. State Snell's Law.

3. What is total internal reflection ?

4. Light is incident normally on a glass slab with refractive index of 1.67. Find percentage reflected intenstity (I_r) compared to the incident intensity.

5. What is the use of cladding in the case of optical fibers ?

6. Define optical centre of a lens.
7. Write one advantage of using Newton's formula over lens-maker's formula.
8. Initially, two thin lenses were kept in contact. Now, if they are separated by d distance, what happens to the focal length of a combination ?
9. What are conjugate foci ?
10. Define near point or distance of most distinct vision.
11. What is the function of rods in retina ?

Answer the following questions :

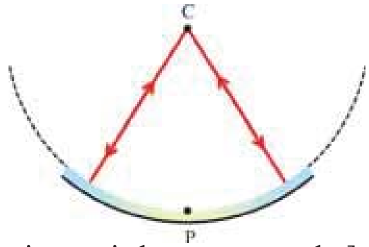
1. Obtain relation between focal length and raidus of curvature for convex mirror.
2. For concave mirror, derive the mirror formula.
3. Define lateral magnification for mirrors. Using cartesian sign convention, derive its relation with image distance and object distance.
4. Obtain an expression for lateral shift due to rectangular slab.
5. Explain the relation between real depth and the virtual depth.
6. Explain total internal reflection.
7. How right-angled prisms are useful as perfact reflecting surface ?
8. Explain how total internal reflection is useful in optical fibre.
9. For a spherically curved surface, derive the relation, $\frac{-n_1}{u} + \frac{n_2}{V} = \frac{(n_2 - n_1)}{R}$
10. Explain the image formation due to thin lens and derive $\frac{-1}{u} + \frac{1}{v} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ relation.
11. Derive lens-maker's formula for thin lens.
12. Derive Newton's formula for thin lens.
13. Explain conjugate points and conjugate distances.
14. Define lateral magnification for lenses. Obtain its relation to extra focal distances.
15. Derive the relation for effective focal length of an optical system made up of two thin lenses in contact.
16. Obtain the relation $f = \frac{1}{2}(v - d)$ for a convex mirror using a combination of convex mirror and convex lens.
17. Derive an equation $\delta = i + e - A$ for equilateral prism.
18. Using $\delta = i + e - A$ for equilateral prism obtain an equation for refractive index (n) of material of the prism.
19. Write note on Rayleigh scattering.
20. What is scattering ? Explian Raman Scattering.
21. Obtain an expression for magnification for simple microscope.
22. With diagram, derive an expression for magnification for compound microscope.
23. Write note on refracting telescopes.
24. What are reflecting telescopes ? What are the advantages of them over refracting telescopes ?
25. Discuss astigmatism defect of human eye.

Solve the following examples :

1. An object moves with uniform velocity (v_0) on the axis of a concave mirror. If it moves towards the mirror, show that when it is at a distance u from the mirror. The velocity of its image is given by $v_i = \left(\frac{R}{2u-R}\right)^2 v_0$, where R is radius of curvature of the mirror.

2. An image of a linear object due to a convex mirror is $\frac{1}{4}^{th}$ of the length of the object. If focal length of the mirror is 10 cm, find the distance between the object and the image. The linear object is kept perpendicular to the axis of the mirror. [**Ans :** 37.5 cm]

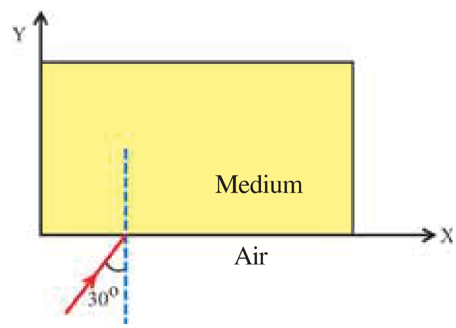
3. A concave mirror has been so placed on a table that its axis remains vertical. P and C are pole and centre of curvature respectively. When a point like object is placed at C, its real image is formed at C. If now, water is filled in mirror. What can be said about the position of the image?



[**Ans :** image is between c and p]

4. The diameter of the sun subtends an angle of 0.5° at the pole of the concave mirror. The radius of curvature of the mirror is 1.5 m. Find the diameter of the image of the sun. Consider the distance of sun from the mirror infinite. [**Ans :** 0.654 cm]

5. A ray, as shown in the figure, is incident at the angle of incidence 30° on the surface and travels in the medium. If the refractive index of the medium is given by $n(y) = 1.5 - ky$. Here, k is constant and it is equal to $0.25 m^{-1}$. At which value of y, will the ray becomes horizontal in the medium ? Here, y is in meter. [**Ans :** $y = 3$ m]

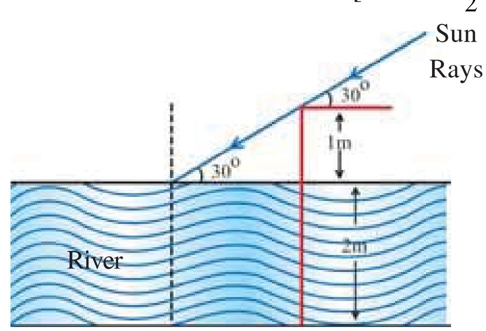


6. A narrow beam of light is incident on a glass plate of refractive index 1.6. It makes an angle 53° with normal to the interface. Find the lateral shift of the beam at the point of emergence, if thickness of the plate is 20 mm. Take sin53° = 0.8. [**Ans :** 9 mm]

7. A real image obtained by a concave mirror is 4 times begger than the object. If the object is displaced by 3 cm away from the mirror, the image size becomes 3 times the object size. Find the focal length of the mirror. [**Ans :** 36 cm]

8. The refractive index of material of a particular optical fibre is 1.75. At what maximum angle a ray can be made incident on it, so that it is totally internally reflected ? Consider air as an external medium with refractive index as 1.0. [**Ans :** $\frac{\pi}{2}$]

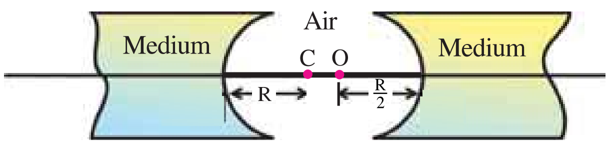
9. A level measuring post (a rod) has been kept in a river of 2 m depth vertically such that its 1 m portion remains outside the river. At this instant, the sun makes an angle of 30° with the horizontal. Find the length of the shadow of the level measuring post on the bottom of the river (see figure). The refravtive index of water is $\frac{4}{3}$.



[**Ans :** 3.44 m]

10. A vessel is fully filled with liquid having refractive index $\frac{5}{3}$. At the bottom of the vessel a point-like source of light is kept. An observer looks at the source of light from the top. Now, an opaque circular disc is kept on the surface of the water in such a way that its centre just rests above the light source. Now liquid is taken out from the bottom gradually. Calculate the maximum height of the liquid to be kept so that light source cannot be seen from outside. Radius of the disc is 1 cm. [**Ans :** 1.33 cm]

11. As shown in the figure, two concave refracting surfaces of equal radii of curvature (R) and refractive indices (n = 1.5) face each other in air (n = 1.0).



A point object (0) is placed midway in between the centre and one of the vertices of the refracting surfaces. Find the distance between image O' formed by one surface and image O'' formed by the other surface in terms of R.

[**Hint :** Use $\frac{-n_1}{u} + \frac{n_2}{v} = (n_2 - n_1)\frac{1}{R}$ for both the refracting surfaces.] [**Ans :** 0.114 R]

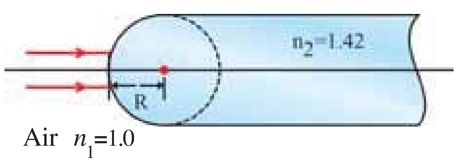
12. (1) If $f = +\ 0.5m$ calculate power of a lens.

(2) The radii of curvature of a convex lens are 10 cm and 15 cm. If its focal length is 12 cm, find the refractive index of the material of the lens.

(3) The focal length of a convex lens in air is 20 cm. What will be its focal length in water. The refractive index of water is 1.33 and that of glass is 1.5.

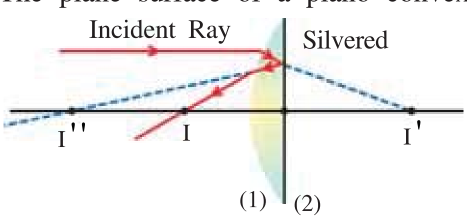
[**Ans :** (1) + 2 D (2) 1.5 (3) 78.2 cm]

13. One end of a cylindrical rod made from the material of refractive index 1.42 is hemispherical.



A narrow beam of parallel rays is incident as shown in the figure. At how much distance will this beam of ray be focussed from the hemispherical surface ? [**Ans :** 3.38 R]

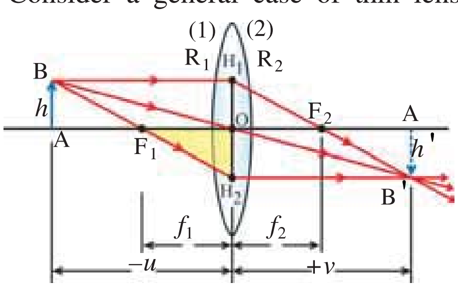
14. The plane surface of a plano convex lens of focal length 20 cm is silvered and made



reflecting, as shown in the figure. Find new focal length of the system.

[**Ans :** 10 cm]

15. Consider a general case of thin lens with first principal focal length (f_1) and second principal focal length (f_2). Obtain the expression for magnification in terms of f_1 and f_2 as $\left(\frac{v - f_2}{f_1}\right)$. Also, for a special case of $f_2 = f_1 = f$, deduce Gauss' equation from the expression for the magnification. Use cartesian sign convention.



[**Hint :** From figure ΔBH_1H_2 and ΔF_1OH_2 are similar, and $\Delta B'H_2H_2$ and ΔF_2OH_1 are similar.]

7

Dual Nature of Radiation and Matter

7.1 Introduction

At the end of the nineteenth century, most physicists thought that the Newtonian laws governing the motion of material particles, thermodynamics and Maxwell's theory for electromagnetic waves are complete and fundamental laws of physics. They all together constitute "Classical Mechanics". Classical physics deals primarily with macroscopic phenomena. Most of the effects with which classical theory is concerned are either directly observable or can be made observable with relatively simple instruments. Thus, there is a close link between the world of classical physics and our sense of perception. Almost all known **macroscopic** problems were satisfactorily solved applying the laws of classical mechanics, and therefore scientists have turned their concentration to the study of atomic and subatomic (i.e. microscopic and submicroscopic) systems. Unlike macroscopic system, since these systems are inaccessible to direct observations, the experiments which have generated interest and curiosity studying some microscopic problems are worth mentioning here.

Study of the influence of an electric field to cathode rays by Jean Perin (1895), and experimental demonstration of negatively charged particles have discovered an electron. Just later, J. J. Thomson found the ratio of charge to mass ($\frac{e}{m} = 1.756 \times 10^{11}$ C/kg) for an electron, while Milikan (1909) had estimated the charge of an electron ($e = 1.602 \times 10^{-19}$ C). It was also established that the smallest basic unit of matter is an atom, and it is electrically neutral. Wilhem Rontgen (1885) accidentally discovered X-rays and just few years later, Henry Bacquerel (1896) and Madam Curie (1898) with different compounds have discovered radio activity.

These were the few experiments which provided a foundation to perform series of different experiments yielding results which could not be explained by the laws of classical mechanics. The specific heats of solids and diatomic gases at very low temperatures, large electrical conductivities of metallic solids, structure of an atom and the characteristic wave lengths emitted or absorbed by different elements, the photoelectric effect, the study of black-body radiation were the notable problems which could not be understood in terms of classical mechanics.

For the resolution of the apparent paradoxes posed by these observations and certain other experimental facts, it became necessary to introduce new ideas quite foreign to commonsense concepts regarding the nature of matter and radiation.

Historically to understand how entirely new concepts were emerged, we study the difficulties in explaining the black-body radiation.

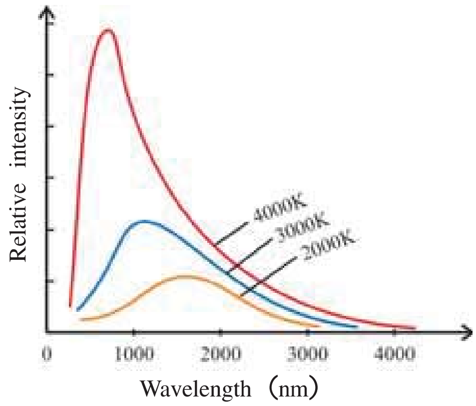


Figure 7.1 Relative Intensity as a Function of Wavelength

Black-body Radiation : In 1897, Lummer and Pringsheim measured the intensities of different wavelengths (i.e., intensity distribution) of black-body or cavity radiations, which is plotted in the figure 7.1.

Scientists were trying to explain these graphs using the laws of electromagnetic theory and thermodynamics.

On the thermodynamic grounds and by using ideas of electromagnetism, Wien gave an expression for energy density as, $u_\lambda = \frac{1}{\lambda^5} \cdot exp\left(-\frac{b}{\lambda \cdot T}\right)$; where b is cosnstant and T is absolute temperature. Such an equation can explain the experimental reuluts only for small wave lengths, but fails to explain the higher wavelengths intensity distribution.

Rayleigh and Jeans determined the number of normal modes of vibration for small intervals of wavelengths, considering the radiations as electromagnetic waves. Each normal mode corresponds to one harmonic oscillator. As the degrees of freedom for harmonic oscillator is two, according to equipartition law for energy, its kinetic energy is $k_B T$. Here, k_B is the Boltzmann constant. Based on this argument, they derived an equation for energy density as,

$$u_\lambda = \frac{8\pi k_B T}{\lambda^4} \qquad (7.1.1)$$

This equation can explain the energy distribution for large wavelengths only. Further, the total energy density (u_{tot}) covering all possible wavelengths must follow the Stefan-Boltzmann's law ($u_{tot} = \sigma \cdot T^4$; where σ = Stefan-Boltzmann's constant). But using equation (7.1.1), if we calculate the total energy density, i.e., $u_{tot} = \int_0^\infty \frac{8\pi k_B T}{\lambda^4} d\lambda$, we get infinite ($\infty$) answer ! This is called ultraviolet catastrophe. On the other hand, Wien's law requires (λ_{max}). T = constant, (b)is called wien's constant. (7.1.2.)

Here, λ_{max} is the wavelength corresponding to the peak value in the intensity distribution graph at that temperature.

Thus, all the attempts based on thermodynamics and electromagnetic theories failed to explain the entire energy distribution curves of black-body radiation.

7.2 Planck's Hypothesis for Radiation

The explanation of energy distribution curves of black-body or cavity radiation was given by Max Planck (1900) at the Academy of Science in Berlin.

He suggested – **"The walls of cavity emitting radiations are made of electric dipoles. According to their temperature, different dipoles oscillate with different frequencies and emit radiations of frequencies equal to frequencies of their oscillations."**

Now, according to the classical physics an oscillator may possess any amount of energy. That is, an oscillator may acquire continuously varying (from zero to maximum available) energy.

Planck presented a revolutionary idea that **"these microscopic oscillators may not possess any arbitrary energy as allowed by the laws of classical mechanics. If the vibrational frequency of such a microscopic oscillator is f, then it may possess energy given by,**

$E_n = nhf,$ (7.2.1)

where n = 1, 2, 3 ..... Here h is known as Planck's universal constant. Thus, according to Planck, energy of such microscopic oscillator depends on its vibrational frequency. This is in contrast to classical oscillator, whose energy depends on its amplitude of oscillation, as per the well known equation $\frac{1}{2}kA^2$. Here, k is the force constant and A is amplitude.

Equation (7.2.1) also suggests that the energy of an oscillator of frequency f is hf, $2hf$, $3hf$, ..., etc. It cannot possess the fractional energy like $0.1hf$, $\frac{1}{2}hf$, $0.06hf$. Thus, energy of microscopic oscillator is an integral multiple of hf. In other words, the smallest quantum of energy of an oscillator of frequency f is 'hf'.

This smallest bundle or packet or quantum of energy is known as **photon.** When an oscillator emits radiation of frequency f, its energy decreases in integral multiple of hf. And quanta of energy hf are emitted. That is, energy is not emitted continuously but in the form of quanta. This phenomenon is known as the **quantization** of energy. (You have also studied the quantization of electric charge.) If an oscillator possesses energy $5hf$, meaning 5 quanta each with energy hf.

Based on his hypothesis Planck could successfully derive the equation of spectral emissive power for a perfect black-body radiation, which is given by

$W_f = \frac{2\pi f^2}{c^2} \times \frac{hf}{\left[e^{\left(\frac{hf}{k_B T}\right)} - 1\right]}$. Here, c = speed of light in vacuum, T = absolute temperature of a perfect balck body, k_B = Boltzmann's constant. (This equation is only for information.)

Above equation gives maximum energy density at the wavelength (λ_{max}) corresponding to Wien's law. Using the experimental values of Stefan-Boltzmann constant σ, Wien's constant b (see equation (7.1.2.)) and Boltzmann's constant k_B, value of Planck's constant (h) can be determined as

$h = 6.625 \times 10^{-34}$ J.s

It can be proved that in the limit $hf \rightarrow 0$, above equation correctly reproduces the classical value $k_B T$, predicted by the law of equipartition of energy. It appears, therefore, that the very small but **non-zero** value of constant 'h' is a measure of the failure of classical mechanics.

Only For Information : If quantum effects are to be observed, the frequency should be high enough so that $\frac{hf}{k_B T}$ becomes comparable to unity. For example, at room temperature (T ≈ 300 K), $\frac{hf}{k_B T} \approx \frac{1}{6}$ for $f = 10^{12}$ Hz. This shows that only when oscillator of at least this frequency or higher, quantum statistical effects become noticeable at room temperature.

7.3 Photoelectric Effect

7.3.1 Emission of Electrons : We know that metals have **free electrons.** However, these free electrons normally cannot come out of the metal surface. The reason is that electrons at the surface experience strong attractive inward force due to positive metallic ions; while virtually no attractive force from the outside. In other words, very close to the surface, potential energy of electrons increase with distance as compared to inside electrons. That is, a potential-barrier exists at the surface. Thus, to bring an electron out, some minimum amount of energy must be supplied to it. This minimum energy required to get emission of an electron is known as **work function** (ϕ_0) of the metal.

The work function of a metal depends on type of the metal, nature of its surface and its temperature.

To bring an electron out of the metal, required energy may be supplied by any of the following ways.

Thermionic Emission : In this method, current is passed through a filament so that it gets heated sufficiently (normally 2500–3000 K). Hence, free electrons in it gain enough energy and get emitted from the metal. Such kind of electron emission is observed in devices like diode, triode, T.V. tube (cathode ray tube), etc.

Field Emission or Cold Emission : When a metal is subjected to strong electric field of the order of $10^8 \frac{V}{m}$, electrons are pulled out of the metal surface.

Photo Electric Emission : When an electromagnetic radiation of enough high frequency is incident on a cleaned metallic surface, electrons can be liberated from the metal surface. This phenomenon is known as the **photoelectric effect** and the electrons so emitted are known as **photo electrons.** To have photo emission, the frequency of incident light should be more than some minimum frequency. This minimum frequency is called the **threshold frequency** (f_0). It depends on the type of the metal. For most of the metals (e.g. Zn, Cd, Mg) threshold frequency lies in the ultraviolet region of electromagnetic spectrum. But for alkali metal (Li, Na, K, Rb) it lies in the visible region.

7.3.2 Hertz's Experiment : The photoelectric effect was discovered accidentally in 1887 by H. Hertz, during his study on the phenomenon of emission of electromagnetic waves by means of spark discharge. In his experiment electromagnetic waves from the transmitter (antenna) induced a potential difference across the spark-gap, as evidence from the jumping spark across it. Hertz noticed that the sparks jumped more easily when the cathode was illuminated by ultraviolet light. This observation suggested that light facilitated the escape of charges from the metallic cathode across the spark-gap. Further, Hallwachs extended this experiment for zinc plate. He connected the negatively charged zinc plate with an electroscope. When this plate was irradiated with ultraviolet light, it was observed that negative charge on the plate decreased. Not only this, even when a neutral plate is irradiated with ultraviolet light it becomes positively charged, while postively charged plates became more positively charged. Hallwachs concluded that under the effect of ultraviolet light, negatively charged electrons are emitted from the zinc plate. These electrons are known as photoelectrons.

7.3.3 Lenard's experiment : The details of the photoelectric phenomenon were studied by P. Lenard, one of Hertz's students. The experimental arrangement to study the photoelectric effect is shown in the figure 7.2.

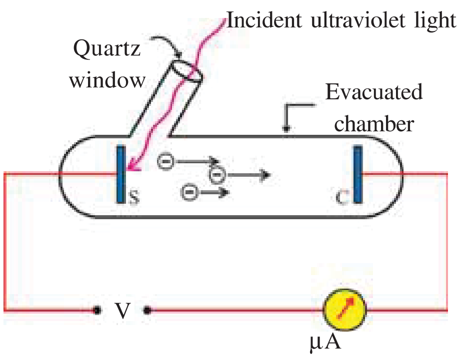


Figure 7.2 Experimental Arrangement to Study Photoelectric Effect

The ultraviolet light entering from quartz window is incident on the cleaned photosensitive surface S. C is the collector, while S is the cathode. C can be kept at different positive or negative voltages with respect to S.

The characteristies of photoelectric effect can be studied in reference to the frequency and the intensity of incident light, and also in terms of number of photoelectrons emitted and their maximum kinetic energy.

When the collector is positive with respect to S, the photo electrons are attracted to it and micro-ammeter registers a current. The amount of current passing through the ammeter gives an idea of the number of photoelectrons. At some value of positive potential difference, when all the emitted electrons are collected, increasing the potential difference further has no effect on the current.

When the collector is made negative with respect to S, the emitted electrons are repelled and only those electrons which have sufficient kinetic energy to overcome the repulsion may reach to the collector, and constitute current. So the current in ammeter falls. On making collector more negative, number of photoelectrons reaching the collector further decreases. For some specific negative potential of the collector, even the most energetic electrons are unable to reach collector, and photoelectric current becomes zero. It remains zero even if the potential is made further negative than the specific value of negative potential. This minimum specific negative potential of the collector with respect to the emitter (photo sensitive surface) at which photoelectric current becomes zero is known as the **stopping potential** (V_0) for the given surface. It is thus the measure of maximum kinetic energy ($\frac{1}{2}mv^2_{max}$) of the emitted photoelectrons. If charge and mass of an electron are e and m respectively,

$$\frac{1}{2}mv^2_{max} = eV_0 \tag{7.3.1}$$

Lenard performed further experiments by varying the intensity (brightness) of the incident light, and measured maximum K.E. and number of photoelectrons via the photoelectric current. He found that by increasing the intensity of the incident light, photoelectric current (i.e. the number of photoelectrons) increases but do not affect the K.E. of the emitted electrons. In the contrast, when he performed the experiment with different frequencies, higher than the threshold frequency of the incident light, changes the stopping potential (V_0) and thereby the K.E. of the emitted electrons, leaving photoelectric current unaltered. It was found that by increasing frequency V_0 and therefore maximum K.E. of the photoelectrons increase, and vice versa. It was also observed that the photo-electrons are emitted within 10^{-9} s after the light is incident.

In summary,

(1) The maximum K.E. of photoelectrons depend on the frequency of incident light, and does not depend on the intensity.

(2) The number of photoelectrons depend directly on the intensity of incident light.

(3) Photoelectric effect is always observed whenever incident light has frequency either equal to or greater than the thereshold frequency for the given surface irrespetive of the intensity.

(4) The phenomenon of photoelectric effect is spontaneous (takes about 10^{-9} sec.).

Above inferences can be depicted in the graphs below : (Figure 7.3 and 7.4)

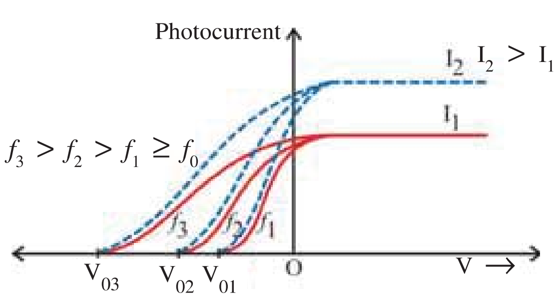


Figure 7.3 Variation of Photoelectirc Current

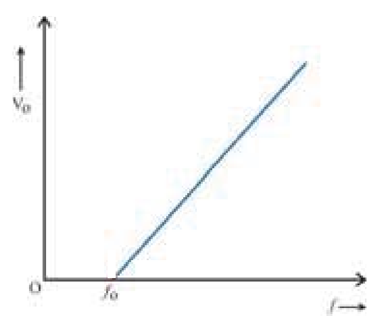


Figure 7.4 Variation of Stopping Potential with Frequency of Incident Light

7.3.4 Explanation from the wave theory of light : Above experimental results cannot be understood with the wave theory of light.

(1) According to the wave theory of light, energy and intensity of wave depend on its amplitude. Hence intense radiation has high energy and on increasing intensity, energy of photoelectrons should also increase. In contradiction to it, experimental results show that the energy of photoelectrons does not depend on the intensity of incident light.

According to the wave theory, energy of light has no relation to its frequency. Hence change in energy of photoelectrons with the change in frequency cannot be explained.

(2) Photoelectrons are emitted immediately (within the 10^{-9} s) on making light incident on the metal surface. Since the free electrons within the metal are withheld under the effect of certain forces, and to bring them out, energy must be supplied.

Now, if the incident energy is showing a wave nature, free electrons in metal get energy gradually and when accumulates energy at least equal to the work function then after they escape from the metal. Thus, electrons get emitted only sometime after the light is incident.

(3) According to the wave theory, less intense light is 'weak' in terms of energy. To liberate photoelectron with such light one has to wait long till electron gathers sufficient energy. Against that experiment shows immediate emission of electron even with diminutive intensity but of course, with sufficiently high frequency.

Thus, wave theory fails to explain the photoelectric effect.

7.3.5 Einstein's Explanation : Einstein gave a successful explanation of the photoelectric effect in 1905 for which he received the Nobel Prize in 1921.

Planck had assumed that emission of radiant energy takes place in the quantized form, the photon, but once emitted it propagates in the form of wave. Einstein further assumed that not only the emission, even the absorption of light takes place in the form of photons.

For Information Only : In the wave nature, the energy is supposed to be spread uniformly across the wave fronts, Einstein proposed that the light energy is not spread over wavefronts but is concentrated in small packets, the photons. He wrote : "According to the assumption considered here, when a light ray starting from a point is propagated, the energy is not continuously distributed over an ever increasing volume, but it consists of a finite number of energy quanta, localized in space, which move without being divided and which can be absorbed or emitted only as a whole."

Suppose frequency of incident light is f, hence energy of its photon is hf. When this photon is incident on the metal, during the interaction with an electron, it is totally absorbed if its frequency (and therefore energy) is greater than threshold frequency or otherwise does not lose energy at all.

As per the laws of classical mechanics (Newtonian mechanics and Maxwell's theory for electromagnetic waves) there is no reason to expect any sensitive frequency dependence of photon-electron interaction. (You will learn its detailed answer in advance course in physics, if you choose physics to shape your career.)

Now if f_0 is the threshold frequency the appropriate photon energy hf_0 will be equal to work function ϕ_0, and at that frequency the photoelectrons are emitted with the minimum (zero) kinetic energy. For frequency $f > f_0$, the maximum kinetic energy of emitted photoelectrons will be,

$$\frac{1}{2}mv^2_{max} = hf - \phi_0$$

From equations (7.3.1), $eV_0 = hf - hf_0$

$$\therefore\ V_0 = \frac{h}{e} \cdot f - \left(\frac{hf_0}{e}\right) \qquad (7.3.2)$$

According to this equation the graph of V_0 versus f is a straight line with a slope $\frac{h}{e}$ and intercept on the X-axis at f_0. This is in excellent agreement with the experimental results shown in the Figure 7.4.

The intensity of light incident on surface is the light energy incident per unit surface area in unit time normal to the surface. According to photon theory (particle nature) of light, if n photons are incident per unit surface area in unit time, intensity of light is $I = nhf$, where hf is the energy of the photon of frequency f. Thus, according to photon theory, more the intensity of light more is the number of photons incident per second and hence more is the photoelectric current. Again showing an experimental trend.

Also, since the interaction between photon and electron takes place as the absorption as a whole or not at all, emission of photoelectron will be instant. Unlike wave nature, where electron has to wait till it gathers enough energy for escape.

Thus, experimental observations for photoelectric effect are reproduced by considering a particle (quantized) nature (photon) of light.

Following table shows work functions and corresponding threshold freqeuncy for some metals.

Table 7.1

Workfunctions and Threshold Frequencies (For information only)

Metal	ϕ_0 (in eV)	f_0 ($\times$ 10^{14} Hz)	Metal	ϕ_0 (in eV)	f_0 ($\times$ 10^{14} Hz)
Cs	1.9	4.60	Fe	4.5	10.89
K	2.2	5.32	Ag	4.7	11.37
Ca	3.2	7.74	Au	4.9	11.86
Cd	4.1	9.92	Ni	5.0	12.10
Al	4.2	10.16	Pt	6.4	15.49

Illustration 1 : Let an electron requires 5×10^{-19} joule energy to just escape from the irradiated metal. If photoelectron is emitted after 10^{-9} s of the incident light, calculate the rate of absorption of energy. If this process is considered classically, the light energy is assumed to be continuously distributed over the wave front. Now, the electron can only absorb the light incident within a small area, say 10^{-19} m^2. Find the intensity of illumination in order to see the photoelectric effect.

Solution : The rate of absorption of energy (power) is

$$P = \frac{E}{t} = \frac{5\times10^{-19}}{10^{-9}} = 5 \times 10^{-10} \frac{J}{s}$$

From the definition of the intensity of light,

$$I = \frac{\text{Energy}}{\text{time}\times\text{area}} = \frac{5\times10^{-10}}{10^{-19}} = 5 \times 10^{9} \frac{J}{s.m^2} \text{ (i.e., 500 billion } \frac{\text{Watt}}{m^2}\text{)}$$

Since, practically it is impossibly high energy, which suggests that explanation of the photoelectric effect in classical term is not possible.

Illustration 2 : Work function of metal is 2 eV. Light of intensity 10^{-5} W m^{-2} is incident on 2 cm^2 area of it. If 10^{17} electrons of these metals absorb the light, in how much time does the photo electric effect start ? Consider the waveform of incident light.

Solution : Intensity of incident light is 10^{-5} W m^{-2}.

$\therefore$ Energy incident on 1 m^2 area in 1 s is 10^{-5} J.

$\therefore$ Energy incident on area of 2 cm^2 = 2×10^{-4} m^2

$$= 2 \times 10^{-4} \times 10^{-5} = 2 \times 10^{-9} \text{ J}$$

This energy is absorbed by 10^{17} electrons.

$\therefore$ Average energy absorbed by each electron $= \frac{2 \times 10^{-9} \text{J}}{10^{17}} = 2 \times 10^{-26}$ J

Now, electron may get emitted when it absorbs energy equal to the work function of its metal. In the given problem work function is 2 eV = $2 \times 1.6 \times 10^{-19}$ J. Thus, electron requires

$(2 \times 1.6) \times 10^{-19}$ J of energy to get emitted.

To absorb 2×10^{-26} J of energy, time required is 1 s, therefore to absorb energy $2 \times 1.6 \times 10^{-19}$ J, time required is,

$$t_e = \frac{2 \times 1.6 \times 10^{-19}}{2 \times 10^{-26}} = 1.6 \times 10^7 \text{ s}$$

Note : If light is considered as wave, photo electron would not be emitted instantaneously as generally seen in the experiments.

7.4 Particle Nature of Light

The photons are considered as discrete amounts of energy (packets) with smallest being the hf. Thus, by nature itself the concept of photon involves the essence of radiation. So, can we consider photon as a real particle ? The compton effect, in which X-rays are scattered by the free electrons, gives the answer. To explain compton effect, photon was considered as a real particle just like a material particle. The way electron collides with any other matter particles, electron may also undergo same type of collision with photon. Also, this collision was considered to follow the laws of conservation of momentum and energy. Thus, as a result of the study of photoelectric effect and compton effect, following properties were attributed to a photon.

(1) Like a material particle, photon is also a real particle.

(2) Energy of a photon of frequency f is hf.

(3) Momentum of photon of frequency f is $\frac{hf}{c}$.

According to Einstein's special theory of relativity the relation between energy (E) and momentum (p) of a particle is given by,

$E = \sqrt{p^2c^2 + m_0^2 \cdot c^4}$, where c = speed of light in vacuum and (7.4.1)

m_0 = rest mass.

Mass of a particle moving with speed v as obtained from equation (7.4.1) is given by,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{7.4.2}$$

Since, in vacuum, photon moves with speed equals to speed of light, its rest mass

$$m_0 = m \times \sqrt{1-\frac{c^2}{c^2}} = 0$$

From equation (7.4.1),

$$E = p.c. \qquad (7.4.3)$$

$$\text{or } p = \frac{E}{c} = \frac{hf}{c} \qquad (7.4.4)$$

(4) Mass of a photon, $m = \frac{E}{c^2}$ ($\because$ $E = mc^2$); where m is given by (7.4.2).

(5) Like a real particle, photon interacts with other particles obeying the laws of conservation of energy and momentum.

For Information Only : To say that electromagnetic radiations propagate as "waves" on one side, at the same time say that in their interaction with matter they exchange energy and momentum as discrete particles (photons) appears contradictory. Let us understand the situation in more details.

Because these cannot be understood in terms of our classical ideas regarding "waves" and "particles". These can be understood only if we accept that :

(1) Light is emitted from a source as described as photons.

(2) Detector records light as discrete photon.

(3) Propagation of light from the source to the detector can be described in terms of "probability waves".

(4) When a "photon" detector is placed in the radiation field of electromagnetic waves, the number of photons detected over the area of the detector is proportional to the square of the amplitude of electromagnetic waves, but the detector interacts with the field as discrete photons.

Illustration 3 : If the efficiency of an electric bulb of 1 watt is 10%, what is the number of photons emitted by it in one second ? The wavelength of light emitted by it is 500 nm. $h = 6.625 \times 10^{-34}$ J s

Solution : As the bulb is of 1 W, if its efficiency is 100 %, it may emit 1 J radiant energy in 1 s. But here the efficiency is 10%, hence it emits $\frac{1}{10}$ J = 10^{-1} J radiant energy in 1 s.

Note : The efficiency of bulb is 10 %. It means it emits 10% of energy consumed in form of light and remaining 90 % is wasted in form of heat energy (due to the resistance of filament.)

$\therefore$ Radiant energy obtained from the bulb in 1 s. = 10^{-1} J

If it consists of n photons,

$$nhf = 10^{-1} \text{ J}$$

$$\therefore n = \frac{10^{-1}}{hf} = \frac{0.1}{6.625 \times 10^{-34} \times \frac{c}{\lambda}} = \frac{\lambda \times 10^{-1}}{6.625 \times 10^{-34} \times 3 \times 10^{8}} \qquad (\because f = \frac{c}{\lambda})$$

$\therefore\ n = \dfrac{0.1 \times 500 \times 10^{-9}}{6.625 \times 10^{-34} \times 3 \times 10^{8}}$ ($\because$ velocity of light, $c = 3 \times 10^8$ m s^{-1})

$\therefore\ n = 2.53 \times 10^{17}$ photons.

Illustration 4 : 11×10^{11} photons are incident on a surface in 10 s. These photons correspond to a wavelength of $10\,\mathring{A}$. If the surface area of the given surface is 0.01 m^2, find the intensity of given radiations. Velocity of light is 3×10^8 m s^{-1}, $h = 6.625 \times 10^{-34}$ J.s.

Solution : Number of photons incident in 10 s = 11×10^{11}

$\therefore$ Number of photons incident in 1 s = 11×10^{10}

Now, these photons being incident on area 0.01 m^2

Number of photons being incident on 1 m^2 in 1 s,

$$n = \frac{11 \times 10^{10}}{0.01} = \frac{11 \times 10^{10}}{10^{-2}} = 11 \times 10^{12}$$

Energy associated with n photons,

$$= nhf = \frac{nhc}{\lambda} = \frac{11 \times 10^{10} \times 6.6 \times 10^{-34} \times 3 \times 10^{8}}{10 \times 10^{-10}} = 2.18 \times 10^{-3}$$

$\therefore$ Intensity of incident radiation = 2.18×10^{-3} W m^{-2}

Illustration 5 : A beam of photons of intensity 2.5 W m^{-2} each of energy 10.6 eV is incident on 1.0×10^{-4} m^2 area of the surface having work function 5.2 eV. If 0.5 % of incident photons emits photo-electrons, find the number of photons emitted in 1 s. Find minimum and maximum energy of these photo electrons.

1 eV = 1.6×10^{-19} J

Solution : Here, intensity of incident radiation is 2.5 W m^{-2}.

$\therefore$ Energy incident per 1 m^2 in 1 s = 2.5 J

$\therefore$ Radiant energy incident on area 1.0×10^{-4} m^2 in 1 s = $2.5 \times 1.0 \times 10^{-4} = 2.5 \times 10^{-4}$ J

Suppose there are n number of photons in this energy.

$\therefore\ nhf = 2.5 \times 10^{-4}$ (1)

but hf = energy of photon = 10.6 eV = $10.6 \times 1.6 \times 10^{-19}$ J

($\because$ 1 eV = 1.6×10^{-19} J)

Replacing it in equation (1) and making n the subject of equation,

$$n = \frac{2.5 \times 10^{-4}}{hf} = \frac{2.5 \times 10^{-4}}{10.6 \times 1.6 \times 10^{-19}}$$

As 0.50 % of these photons emits photo electrons,

$$\begin{bmatrix} 100 : 0.5 \\ n : ? \end{bmatrix}$$

$\therefore$ Number of photo electrons emitted in 1 sec is,

$$N = \frac{0.50 \times n}{100} = \frac{0.5 \times 2.5 \times 10^{-4}}{100 \times 10.6 \times 1.6 \times 10^{-19}}$$

$$= 7.37 \times 10^{11}\ s^{-1}$$

The minimum energy of photo electon is = 0 J. Such photo electrons spend all the energy gained from the photon against the work function.

Maximum energy of photo electorn :

$E = hf - \phi_0 = 10.6\ eV - 5.2\ eV \qquad (\because\ hf = 10.6\ eV \text{ and } \phi_0 = 5.2\ eV)$

$= 5.4\ eV$

Illustration 6 : Radius of a beam of radiation of wavelength 5000 Å is 10^{-3} m. Power of the beam is 10^{-3} W. This beam is normally incident on a metal of work function 1.9 eV. What will be the charge emitted by the metal per unit area in unit time ? Assume that each incident photon emits one electron.

$h = 6.625 \times 10^{-34}$ J s

Solution : Power of the beam of light = 10^{-3} W

$\therefore$ Amount of energy incident in unit time = 10^{-3} J

If the number of photons corresponding to this energy is n,

$$nhf = nh\frac{c}{\lambda} = 10^{-3} \Rightarrow n = \frac{10^{-3}\times\lambda}{hc}$$

$$\therefore\ n = \frac{10^{-3}\times5000\times10^{-10}}{6.625\times10^{-34}\times3\times10^{8}} \qquad (\because\ \lambda = 5000\ \text{Å} = 5000 \times 10^{-10}\ m)$$

These photons are incident on the surface of radius 10^{-3} m in one second.

$\therefore$ Number of photons incident per unit area in one second,

$$n_1 = \frac{10^{-3}\times5000\times10^{-10}}{6.625\times10^{-34}\times3\times10^{8}\times\pi\times(10^{-3})^2}$$

Each photon emits one electron and charge on electron being $e = 1.6 \times 10^{-19}$ C

$\therefore$ amount of charge emitted per unit area in unit time.

$$Q = n_1 e = \frac{10^{-3}\times5000\times10^{-10}\times1.6\times10^{-19}}{6.6\times10^{-34}\times3\times10^{8}\times3.14\times10^{-6}} = 128.6\ C$$

Illustration 7 : Work function of some metals are Na : 1.92 eV, K : 2.2 eV, Cd : 4.1 eV, Ni : 5 eV. A laser beam from He-Cd of wavelength 3300 Å is incident on it. From which of the metals photo electrons will be emitted, if the distance of the source is initially 1 m from the metals. If it is brought to the distance of 10 cm will there be any change in emission ?

$h = 6.625 \times 10^{-34}$ J s. $c = 3 \times 10^{8}$ m s^{-1}, 1 eV = 1.6×10^{-19} J.

Solution : For photo-electric effect to be observed, energy of each photon should be at least equal to or more than work function of the metal.

$$\therefore\ hf = h\frac{c}{\lambda} \geq \text{work-function, } \phi_0$$

$$\text{Energy of Incident radiation} = \frac{6.625\times10^{-34}\times3\times10^{8}}{3300\times10^{-10}}\ J = \frac{6.625\times10^{-34}\times3\times10^{8}}{3300\times10^{-10}\times1.6\times10^{-19}}$$

$(\because\ 1\ eV = 1.6 \times 10^{-19}\ J)$

Energy of Incident radiation = 3.76 eV

This result shows that the metal which has the work function 3.76 eV or less, may produce photoelectric effect. In the given list of metals Na and K may produce photoelectric effect, while in Cd or Ni this effect is not observed.

While the source is brought nearer, from 1 m to 10 cm, the intensity of incident light will of course increase, but its frequency will remain same. Hence, Na and K will emit more number of photo electrons and photo electric cuurent will increase, but still photo electric effect will not be seen in Cd and Ni.

Illustration 8 : U. V. light of wavelength 200 nm is incident on polished surface of Fe. Work function of the surface is 4.5 eV. Find, (1) stopping potential (2) maximum kinetic energy of photo-electrons (3) maximum speed of photo electrons.

$h = 6.625 \times 10^{-34}$ J s, $c = 3.00 \times 10^{8}$ m s^{-1}, 1 $eV = 1.6 \times 10^{-19}$ J.

$m = 9.11 \times 10^{-31}$ kg

Solution : $eV_0 = \frac{1}{2} mv^2_{max} = hf - \phi_0 = \frac{hc}{\lambda} - \phi_0$

First we find $\frac{hc}{\lambda}$, to calculate V_0.

$$\frac{hc}{\lambda} = \frac{6.625\times10^{-34}\times3\times10^{8}}{200\times10^{-9}} = 9.94 \times 10^{-19} \text{ J} = 6.21 \ eV$$

Now, $eV_0 = \frac{hc}{\lambda} - \phi_0 = 6.21 - 4.5 \ (\because \phi = 4.5 \ eV) = 1.71 \ eV$

$\therefore V_0 = 1.71$ V

Now,

$$\therefore \frac{1}{2}mv^2_{max} = eV_0 = 1.71 \ eV = (1.71)\ (1.6 \times 10^{-19}) \text{ J} = 2.74 \times 10^{-19} \text{ J}$$

$$\therefore v^2_{max} = \left(\frac{2.74\times10^{-19}\times2}{9.11\times10^{-31}}\right) = 6.0 \times 10^{11}$$

$$\therefore v_{max} = 7.75 \times 10^{5} \text{ m s}^{-1}$$

Illustration 9 : A crystal of Cu emits $8.3 \times 10^{10} \frac{\text{photo-electrons}}{m^2 \ s}$. Atomic mass of Cu is 64 g mol^{-1} and its density is 8900 kg m^{-3}. Supposing that photo electrons are emitted from first five layers of atoms of Cu, will one electron be emitted per how many (average) atoms ? Consider the crystal to be a simple cubic lattice.

Solution : As the number of photo electrons are given as photo electrons/m^{2}s, consider the cube of crystal of length 1 m. Volume of such a crystal = $1 \times 1 \times 1 = 1$ m^3. Now density is 8900 kg m^{-3}. Hence, the mass of such crystal is 8900 kg. As the atomic mass is 64 g mol^{-1}, number of atoms in 64×10^{-3} kg of Cu will be same as Avogadro number.

64×10^{-3} kg : 6.02×10^{23}

$\therefore$ 8900 kg : number of atoms (?)

$\therefore$ Number of atoms in 8900 kg of Cu, N = $\dfrac{6.02\times10^{23}\times8900}{64\times10^{-3}}$ (1)

These atoms form simple cubic lattice.

If in one row there are n number of atoms, in one layer there may be n^2 number of atoms.

In 5 layers number of atoms = $5n^2$

Note that total number of atoms in the given cube is n^3

$\therefore$ N = n^3

$\therefore$. From equation (1),

$$n^3 = \frac{6.02\times10^{23}\times8900}{64\times10^{-3}}$$

$$\therefore\ n = \left(\frac{6.02\times10^{23}\times8900}{64\times10^{-3}}\right)^{\frac{1}{3}} = 4.37 \times 10^9$$

$\therefore$ $5n^2 = 5 \times (4.37 \times 10^9)^2 = 9.55 \times 10^{19}$

$\therefore$ 8.3×10^{10} photo electrons are emitted from 9.55×10^{19} atoms.

If 8.3×10^{10} photo electrons are emitted from $5n^2$ atoms, from how many atoms one electron is emitted ?

8.3×10^{10} : $5n^2$

1 : ? (number of atoms)

$$\therefore\ \frac{5n^2}{8.3\times10^{10}} = \frac{9.55\times10^{19}}{8.3\times10^{10}}$$

$\therefore$ Number of atoms emitting one photo-electrons = 1.15×10^9

Illustration 10 : Light of 4560 Å of 1 mW is incident on photo-sensitive surface of Cs (Cesium). If the quantum efficiency of the surface is 0.5 %, what is the amount of photo-electric current produced ?

Solution : Meaning of light of 1 mW is that 1 mJ = 10^{-3} J of energy is being incident on the surface in 1 s. This light is being incident in the form of photons of energy hf. If n photons are incident.

$nhf = 10^{-3}$ (1)

Out of n photons only 0.5% photons emit photo-electrons, as the quantum efficiency is 0.5%.

Now, 0.5% of n, is

$$\begin{bmatrix} 100 : 0.5 \\ n\ :\ ? \end{bmatrix}$$

$\therefore$ Number of photo-electrons = $\dfrac{n\times0.5}{100}$

Photo electric current is produced by these electrons is being emitted in 1 s.

Photoelectric current, I = Number of photo–electrons emitted in 1 s × charge of electron

$\therefore$ I = $\frac{n \times 0.5}{100} \times 1.6 \times 10^{-19}$ A (2)

But from equation (1),

$$n = \frac{10^{-3}}{hf} = \frac{10^{-3}}{6.625 \times 10^{-34} \times \frac{c}{\lambda}} \quad (\because f = \frac{c}{\lambda})$$

$$\therefore n = \frac{10^{-3} \times 4560 \times 10^{-10}}{6.625 \times 10^{-34} \times 3 \times 10^{8}} = 2.303 \times 10^{15}$$

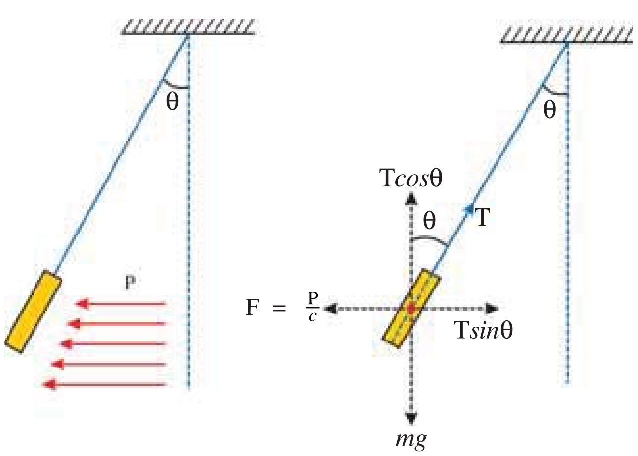
Replacing the value of n in equation (2),

$$I = \frac{2.303 \times 10^{15} \times 0.5 \times 1.6 \times 10^{-19}}{100}$$

$\therefore$ I = 1.84×10^{-6} A = 1.84 μA

Illustration 11(a) : As shwon in the figure, light of energy P(joule) is incident on a small, flat strip of metal of mass m, suspended with the help of weightless string of length l in 1 s. All the energy incident on it is absorbed and the strip remains in equilibrium at an angle θ with respect to vertical. If the light is monochromatic, find angle θ.

Solution : When electromagnetic radiations are incident on a surface, force is produced due to pressure. Here, P joule of energy is incident in 1 s. If this radiation is made of photons and n photons are incident in 1 s,



nhf = P (1)

Now, momentum of each photon, $p = \frac{hf}{c}$ (2)

Replacing the value of hf from (1) in (2),

$$p = \frac{P}{nc}$$

$\therefore$ momentum of n photons = $np = \frac{P}{c}$

The strip gains this much momentum every second.

$\therefore$ Rate of change of momentum = $\frac{P}{c}$ = Force

$\therefore$ F = $\frac{P}{c}$ (3)

This force is shown in the Figure.

As the strip is in equilibrium, equating their vertical and horizontal components,

$$\left.\begin{aligned} T\cos\theta &= mg \\ \text{and } T\sin\theta &= \frac{P}{c} \end{aligned}\right\} \quad \therefore \tan\theta = \frac{P}{cmg} \Rightarrow \theta = \tan^{-1}\left(\frac{P}{cmg}\right)$$

Illustration 11(b) : If the strip is slightly displaced from its state of equilibrium, find the period of it's simple harmonic oscillations.

Solution : Here, effective gravitational acceleration = $\vec{g_e} = \frac{\vec{P}}{mc} + \vec{g}$

$$\therefore |\vec{g_e}| = \sqrt{\left(\frac{P}{mc}\right)^2 + g^2}$$

$$\text{Now, } T = 2\pi\sqrt{\frac{l}{g_e}} = \sqrt{\frac{l}{\sqrt{\left(\frac{P}{mc}\right)^2 + g^2}}}$$

$$\therefore T = 2\pi\left[\frac{l}{\left\{\left(\frac{P}{mc}\right)^2 + g^2\right\}^{\frac{1}{2}}}\right]^{\frac{1}{2}}$$

7.5 Photocell

A Photocell (which is also known as electric eye) is a technological application of the photoelectric effect. In some photocells single layer of photosensitive material is used. A schematic diagram of a typical photocell is shown in the figure 7.5.

The wall of the photocell is made of glass or quartz. When the light (of suitable frequency) is incident on the photosensitive surface, a photocurrent of few micro ampere is normally obtained. When intensity of incident light is changed the photo electric current also changes. Using this property of photocell, control systems are operated and the intensity of light can be measured.

They are used in light meters, photographic camera, electric bell, burglar alarm, fire alarm. In astronomy, they are used to study the spectra of stars and their temperatures.

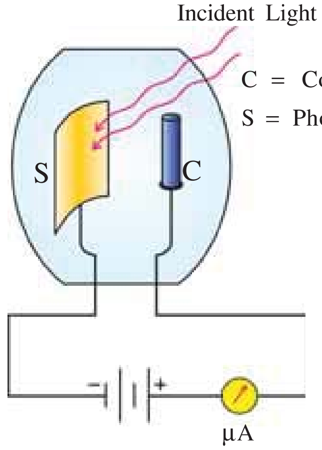


Figure 7.5 A Photocell

7.6 Matter Waves - Wave Nature of Particles

The photoelectric and compton effect have confirmed that light behaves as a collection of particles and not as a wave. At the same time, we also know that the phenomena of diffraction, interference and polarization can be understood only when light behaves as wave. This is a paradox of the existence of two quite different (the wave and the particle) nature of the same physical quantity (light). One possibility is to suppose that light propagates in the form of wave but assumes particle character at the instant of absorption or emission (i.e. during the interaction with matter). This explanation suggests that radiation shows dual nature; (continuous) wave-like extended and (discrete) quantized particle behaviour under the suitable conditions.

According to the theory of relativity, Lorentz transformation for a change of reference frame requires that relation like between E and f must necessarily hold for momentum (p) and wave-vector (k). Since for photon rest mass (m_0) is zero, its momentum is given by (see equation 7.4.4),

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \quad (\because\ c = f\lambda) \tag{7.6.1}$$

$$\text{or } \lambda = \frac{h}{p} \tag{7.6.2}$$

Based on this requirement, in 1924, Louis de Broglie arugued that if light (which consists of waves according to classical mechanics) can sometimes behave like particles. Then it should be possible for matter (which consists of particles according to classical picture) to exhibit wave-like behaviour under favourable circumstances. **"Nature should be symmetric with respect to radiation and particles."** The dual nature of radiation and particle must be a part of some general law of nature. That is, radiation and matter both show dual nature : particle and wave.

Thus, according to de Broglie, equation (7.6.2) is also true for material particles. For a particle with mass m and moving with a speed v (i.e., momentum, $p = mv$), when showing wave nature, corresponding wavelength can be found by using equation (7.6.2), as

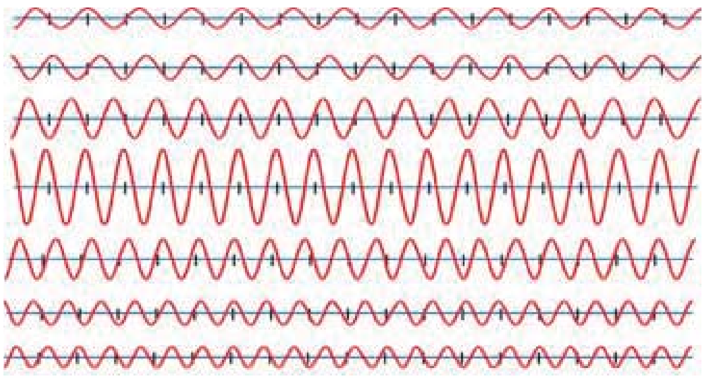
$$\lambda = \frac{h}{mv} \tag{7.6.3}$$

This wavelength is known as de Broglie wavelength of the particle. We must remember that it is not that any kind of wave is attached to the matter particle. Under some circumstances, the behaviour of the particle can be explained by its wave nature.

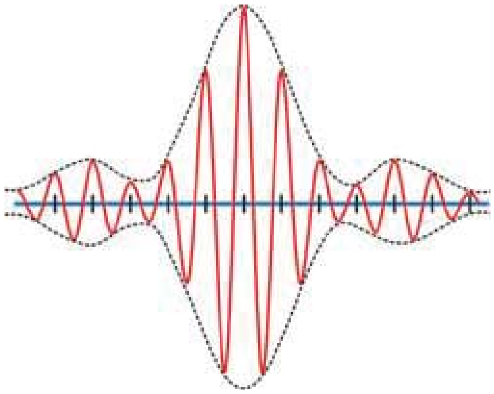
Actually, the concept of matter particle as a wave was well supported by Erwin Schroedinger (1926) through his differential wave equation. He showed that this wave equation (for matter waves associated to particles) together with some physically-required conditions leads to quantized (discrete) nature of various physical quantities which supports the wave nature of particles. While the experimental evidences for matter as a wave were due to Davisson-Germer experiment, (which we will study in the next section), Kikuchi's diffraction experiment and Thomson's experiment showing associated de Broglie waves of electrons.

However, the most serious problem raised by the discovery of the wave nature of matter concerns the very basic definition of a 'particle'. Classically, particle means a point-like object endowed with a precise position and momentum. The de Broglie's hypothesis, which also supports wave-like (i.e. an extended in space) behaviour of matter, questions about how to measure accurately position and momentum of a material particle.

A pure harmonic wave extending in space obviously cannot represent point-like particle. This suggests that the wave activity of a wave representing a particle must be limited to (or nearby to) the space occupied by the particle. For this reason an idea of wave packet (i.e. a wave which is confined to a small region of space) is introduced.



(a) Harmonic Waves with Slightly Differing Wave Lengths.



(b) Amplitude Variation Due to Superposition of Harmonic Waves.

Figure 7.6 Construction of Wave Packat

We know that when many harmonic waves with slightly varying wave lengths are superposed (Hey, don't forget superposition principle), non-zero displacement of resulting wave is limited to small part of the space (See figure 7.6). In this sense, it would seem reasonable to suppose that the particle is within the region of the packet. Further, the probability of finding the particle is more in a region in which the displacement of the resulting wave is greater. If we use a single harmonic wave to represent a particle, the probability of finding a particle anywhere from $-\infty$ to $+\infty$ is equal. (This is because amplitude of a harmonic wave is finite and equal everywhere.) In other words, the position of the particle becomes totally uncertain. But, since the harmonic wave has unique wave length (λ), according to equation (7.6.3), its momentum is unique and certain.

If the concept of wave-packet (a group of superimposing waves of different wave lengths) is used to represent particle, position of the particle is more certain and is proportional to the size of the wave-packet. But as several waves of different wave lengths are used to represent a particle, its momentum is no longer unique and becomes uncertain.

Thus, the fundamental dual nature of radiation and particle introduces uncertainty in the simultaneous measurement of physical quantities.

Heisenberg's Uncertainty Principle : According to Heisenberg's uncertainty principle, if the uncertainty in the x-coordinate of the position of a particle is Δx and uncertainty in the x-component of its momentum is Δp (i.e. in one dimension) then

$$\therefore \ \Delta x \cdot \Delta p \geq \frac{h}{2\pi} \geq \hbar \text{ (Read as } h \text{ cut or } h \text{ cross).} \qquad (7.6.4)$$

Now, if $\Delta x \rightarrow 0$ then $\Delta p \rightarrow \infty$

and $\Delta p \rightarrow 0$ then $\Delta x \rightarrow \infty$

Similarly, one finds uncertainty principle associated in measuring energy of a particle and time as,

$$\therefore \ \Delta E \cdot \Delta t \geq \hbar \qquad (7.6.5)$$

Only for Information : We discussed about the probability of the particle to be at a definite point. In fact the wave functions representing a particle can be mathematically obtained in the form of solutions of typical differential equations (Schroedinger's equation). These wave functions may be real or complex according to the situation. According to Max Born, the probability of finding a particle at any point in the space in one dimension is proportional to the square of the magnitude of such a wave function ($|\psi|^2 = \psi^*\psi$). Hence, we have to deal with such probabilities while discussing about microscopic particles. This branch of physics is called **wave mechanics**.

You might have noted that the approach of physics based on quantum mechanics is not deterministic like classical physics.

So, for a microscopic particle like an electron, it is meaningless to question whether it is a particle or a wave. Actually it is neither a wave nor a particle. It is more fundamental physical reality whose behaviour can be understood with particle mechanics in some situation and with wave mechanics in the other. The mathematical studies developed in reference to the wave and particle nature are merely two disciplines to understand the nature.

Noted writer Margenau compares the question : "wave or particle ?" with the question "what is the colour of an egg of an elephant ?" This question is meaningful only if an egg of an elephant exists !

Illustration 12 : Find the certainty with which one can locate the position of (1) a bullet of mass 25 g and (2) an electron, both moving with a speed 500 m/s, accurate to 0.01 %. Also, draw inferences based on your results. Mass of an electron is 9.1×10^{-31} kg.

Solution : (1) Uncertainty in measurement of momentum of a bullet is 0.01% of its exact value. i.e., $\Delta p = 0.01\%$ of mv.

$$= \left(\frac{0.01}{100}\right) \times (25 \times 10^{-3}) \times (500)$$

$$= 1.25 \times 10^{-3} \text{ kg m s}^{-1}$$

Therefore, corresponding uncertainty in the determination of position is

$$\therefore \ \Delta x = \frac{\hbar}{\Delta p} \text{ (using equation 7.5.4)}$$

$$= \frac{6.625\times10^{-34}}{2\times3.14\times(1.25)\times10^{-3}} \times 10^{-3} \qquad \left(\because \ \hbar = \frac{h}{2\pi}\right)$$

$$= 8.44 \times 10^{-32} \ m.$$

Conclusion : The value of Δx is too small compared to the dimension of the bullet, and can be neglected. That is, position of the bullet is determined accurately.

(2) Uncertainty in measurement of momentum of an electron is

$$\therefore \ \Delta p = \left(\frac{0.01}{100}\right) \times (9.1 \times 10^{-31}) \times (500) = 4.55 \times 10^{-32} \text{ kgms}^{-1}$$

Corresponding uncertainty in position is

$$\Delta x = \frac{6.625\times10^{-34}}{2\times3.14\times4.55\times10^{-32}} = 0.23 \times 10^{-2} \text{ m} = 2.3 \text{ mm}$$

Conclusion : Uncertainty in position for an electron (2.3 mm) is too large compared to the dimension of an electron, when it is assumed to be as a particle. Consequently, the concept of an electron as a tiny particle does not hold.

7.7 Davisson-Germer Experiment

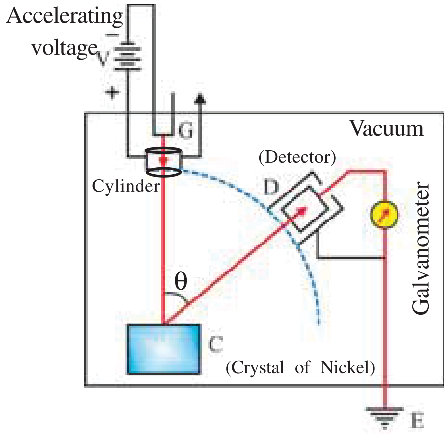


Figure 7.7 Arrangement for Davisson-Germer Experiment

Till 1927, De Broglie's hypothesis did not get any experimental confirmation. In 1927, two scientists named Davisson and Germer performed series of experiments at Bell laboratory to study scattering of electron by a piece of Nickle placed in vacuum.

The device used by them is shown in figure 7.7.

Here, G is the electron gun having tungsten filament coated with barium oxide. Filament is heated with L. T. (Low Tension = low p.d.). Hence, it emits electrons.

Now, these electrons can be accelerated under appropriate electric field produced by H. T. (High Tension). These electrons pass through a cylinder having a small hole and form a thin beam of electrons which is incident on a piece of Nickel and get scattered by it (in fact by its atoms). To detect the electrons scattered in different directions a detector D is arranged which can be moved on a circular scale as shown in figure 7.7. The output current from this detector passes through a galvanometer. The amount of current represents the number of electrons scattered in that direction.

According to classical physics, number of electrons scattered in different directions does not depend much on the angle of scattering. Also, this number hardly depends on the energy of incident electrons. Davisson and Germer tested these predictions of classical physics using the piece of Nickel as the scatterer.

During one of their experiments the bottle filled with liquefied air burst and the surface of the piece of Nickel was damaged. They heated the piece of Nickel to a high temperature and then cooled it to level its surface. Again when the experiment was repeated they found "something unusual". They found that the results of diffraction of electrons by Nickel are similar to the diffraction of X-rays by a crystal. This can happen only if electrons act as waves. This happened because when the piece of Nickel was heated and then cooled it was converted into a single crystal.

In this experiment the intensity of electron beam scattered at different angles of scattering, can be measured for the given accelerating voltage. Angle of scattering (θ) is the angle between the incident beam and scattered beam of electrons. The graphs of intensity $\rightarrow \theta$ for the observations taken by Davisson and Germer between 44 V to 68 V are shown qualitatively in figure 7.8.

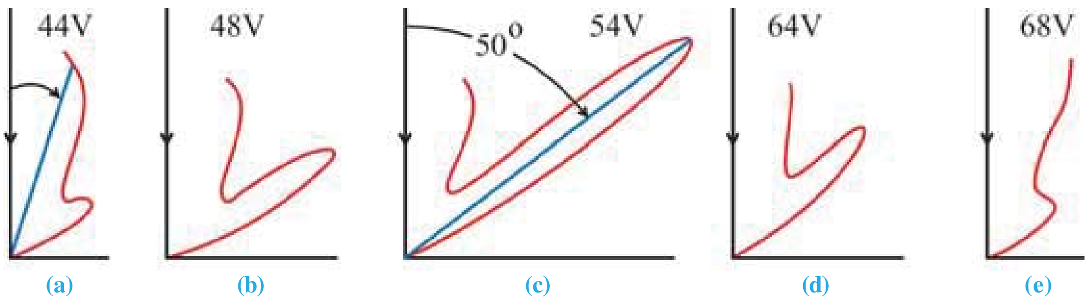


Figure 7.8 **Results for Davisson Germer Experiment**

The graphs indicate that the number of electrons scattered at a specific angle of scattering is maximum for the given accelerating voltage. See the graph of 54 V carefully. Here, the number of electrons scattered at an angle of 50^o is found to be maximum. These experimental results can be understood if the electrons are considered as the waves having de Broglie wavelength and if we accept that electrons are scattered just as X-rays by a crystal. The interatomic distance of Nickel is known. With this information and using the equation of scattering wavelength of electron can be obtained experimentally.

If the accelerating voltage is V and charge of an electron is e, energy of electron is

$$\frac{1}{2}mv^2 = e\text{V}$$

$$\therefore\ m^2v^2 = 2me\text{V}$$

$$\therefore\ mv = \sqrt{2me\text{V}}$$

But wavelength, $\lambda = \frac{h}{mv}$

$$\therefore\ \lambda = \frac{h}{\sqrt{2me\text{V}}} \qquad (7.7.1)$$

In above equation substituting V = 54 V, $h = 6.625 \times 10^{-34}$ Js, $m = 9.1 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ C, we get $\lambda = 1.66 \times 10^{-10}$ m. The value of λ obtained in the experiment was 1.65×10^{-10} m. **Thus, accidentally it was proved that an electron behaves as wave also.**

For Information Only : The development of quantum physics is very interesting. This is the magnificent knowledge of mankind struggling to know the nature. Not only that but it is the confluence of rivers like science, mathematics and philosophy. Diving in it we realize how magnificent is the nature !

Illustration 13 : Suppose you are late in reaching the school, and you are going at the speed of 3.0 m s^{-1}. If your mass is 60 kg. assuming that you are a particle find your de Broglie wavelength $h = 6.625 \times 10^{-34}$ J S.

Solution : $p = mv = 60 \times 3.0 = 1.8 \times 10^2$ kg m s^{-1}

$$\text{Now, } \lambda = \frac{h}{p} = \frac{6.625\times10^{-34}}{1.8\times10^{2}} = 3.68 \times 10^{-36} \text{ m}$$

Note : This wavelength is even smaller than the radius of the nucleus (~ 10^{-15} m) by 10^{-21} times. If you want to make your wave properties "regular", your mass should be reduced to unimaginable level.

Illustration 14 : A proton falls freely under gravity of Earth. What will be its de Broglie wavelength after 10 s of its motion ? Neglect the forces other than gravitational force.

$g = 10$ m s^{-2}, $m_p = 1.67 \times 10^{-27}$ kg, $h = 6.625 \times 10^{-34}$ J s

Solution : From $v = v_0 + gt$,

$v = gt$

$\therefore$ momentum, $p = m_p v = m_p gt$

$$\therefore \lambda = \frac{h}{p} = \frac{h}{m_p gt}$$

$$\therefore \lambda = \frac{6.625\times10^{-34}}{1.67\times10^{-27}\times10\times10}$$

$$\therefore \lambda = 3.96 \times 10^{-9} \text{ m} = 39.6 \text{ Å}$$

Illustration 15 : An electron is at a distance of 10 m from a charge of 10 C. Its total energy is 15.6×10^{-10} J. Find its de Broglie wavelength at this point.

$h = 6.625 \times 10^{-34}$ J s; $m_e = 9.1 \times 10^{-31}$ kg; $k = 9 \times 10^9$ SI,

$e = 1.6 \times 10^{-19}$ C

Solution : Potential energy of an electron, $U = -k\frac{(q)(e)}{r}$

$$\therefore U = -\frac{9\times10^{9}\times10\times1.6\times10^{-19}}{10}$$

$$\therefore U = -14.4 \times 10^{-10} \text{ J} \quad (1)$$

Now total energy E = Kinetic energy K + Potential energy U

$\therefore$ K = E − U

$= 15.6 \times 10^{-10} + 14.4 \times 10^{-10}$

$\therefore$ K = 30 × 10^{-10} J

But, $K = \frac{p^2}{2m_e}$

$\therefore p = \sqrt{2Km_e}$

$$\lambda = \frac{h}{\sqrt{2Km_e}} = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 30 \times 10^{-10} \times 9.1 \times 10^{-31}}}$$

$\therefore \lambda = 8.97 \times 10^{-15}$ m

Illustration 16 : Compare energy of a photon of X-rays having 1 Å, wavelength with the energy of an electron having same de Broglie wavelength. $h = 6.625 \times 10^{-34}$ J s; $c = 3 \times 10^8$ m s^{-1}; $m_e = 9.1 \times 10^{-31}$ kg

Solution : For photon,

Energy, $E_p = hf = \frac{hc}{\lambda}$ $\qquad\qquad \lambda = 1$ Å $= 10^{-10}$ m

$$\therefore E_p = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{10^{-10}} = 19.87 \times 10^{-16} \text{ J}$$

For an electron;

Energy, $E_e = \frac{p^2}{2m}$

According to de Broglie relation, $p = \frac{h}{\lambda}$

$$\therefore E_e = \frac{h^2}{\lambda^2(2m)} = \frac{(6.625 \times 10^{-34})^2}{(10^{-10})^2 \times 2 \times 9.1 \times 10^{-31}} = 2.41 \times 10^{-17} \text{ J}$$

$$\therefore \frac{E_p}{E_e} = \frac{19.87 \times 10^{-16}}{2.41 \times 10^{-17}}$$

$$\therefore \frac{E_p}{E_e} = 82.4$$

Thus, energy of photon is 82.4 times the energy of electron having same wavelength.

Illustration 17 : Wavelength of an electron having energy E is $\lambda_0 = \frac{h}{\sqrt{2mE}}$, where m is the mass of an electron. Find the wavelength of the electron when it enters in X-direction in the region having potential V(x). If we imagine that due to the potential, electron enters from one medium to another, what is the refractive index of the medium ?

Solution : Energy of electron in the region having potential

E = (Kinetic energy)K + (Potential energy)U

$\therefore E = \frac{p^2}{2m} - eV(x)$

$$\therefore\ p = [2m(E + eV(x)]^{\frac{1}{2}}$$

$$\therefore\ \lambda = \frac{h}{p} = \frac{h}{[2m(E + eV(x)]^{\frac{1}{2}}}$$

Now, refractive index $= \dfrac{\lambda_0}{\lambda} = \dfrac{[2m(E + eV(x)]^{\frac{1}{2}}}{(2mE)^{\frac{1}{2}}}$ $\left(\because\ \lambda_0 = \dfrac{h}{\sqrt{2mE}}\right)$

$$\therefore\ \text{Refractive index} = \left[\frac{E + eV(x)}{E}\right]^{\frac{1}{2}}$$

Illustration 18 : Consider the radius of a nucleus to be 10^{-15} m. If an electron is assumed to be in such nucleus, what will be its Energy ?

Mass of electron $= 9.1 \times 10^{-31}$ kg; $h = 6.625 \times 10^{-34}$ J s

Solution : As the electron acts as a wave in this situation, the maximum uncertainty in its position.

$\therefore\ \Delta x = 2r = 2 \times 10^{-15}$ m $\qquad$ r = radius of the nucleus $= 10^{-15}$ m

Now, according to Heisenberg's principle

$$\therefore\ \Delta x \cdot \Delta p \approx \frac{h}{2\pi}$$

$$\therefore\ \Delta p \approx \frac{h}{2\pi\Delta x} = \frac{6.625\times10^{-34}}{2\times3.14\times2\times10^{-15}} = 0.5274 \times 10^{-19}$$

Now, if this uncertainty is (approximately) taken as the momentum ($p \approx \Delta p$), energy of electron

$$E = \frac{p^2}{2m}$$

$$= \frac{(0.5274\times10^{-19})^2}{2\times9.1\times10^{-31}}\ \text{J} = \frac{(0.5274\times10^{-19})^2}{2\times9.1\times10^{-31}\times1.6\times10^{-19}}\ eV = 9.55 \times 10^3\ \text{MeV}$$

Now, the binding energy of a nucleus is several MeV. As compared to it the energy of an electron in the nucleus is very large. Hence, electron can not reside in a nucleus.

Illustration 19 : Find the wave packet formed due to the superposition of two harmonic waves represented by $y_1 = A\ sin(\omega t - kx)$ and $y_2 = A\ sin[(\omega + d\omega)t - (k + dk)x]$

Solution : According to the principle of superposition,

$$y = y_1 + y_2$$

$$= A\ sin(\omega t - kx) + A\ sin[(\omega + d\omega)t - (k + dk)x]$$

Using the relation $sin\ A + sin\ B = 2\ sin\left(\dfrac{A+B}{2}\right) \cdot cos\left(\dfrac{A-B}{2}\right)$

$$y = 2A\ cos\left(\frac{xdk - td\omega}{2}\right) \cdot sin\left[(\omega t - kx) + \left(\frac{td\omega - xdk}{2}\right)\right]$$

As amplitude of the wave packet $= 2A\ cos\left(\dfrac{xdk - td\omega}{2}\right)$, it depends both on the position and time.

SUMMARY

1. Difficulties by the classical theoretical explanation of certain experimental observations like, energy distribution in black-body radiation, stability of an electrically neutral atom and its spectra, specific heats of solids and diatomic molecules at low temperatures, etc., have forced scientists to think totally differently.
2. Planck with his revolutionary idea that energy of microscopic oscillating dipoles is quantized to hf. And total energy is always an integral multiple of the smallest quantum of energy (hf), the photon. Here, h is known as the Planck's constant. The photon possesses all the properties of a material particle.
3. Planck's hypothesis could solve black-body radiation problem successfully.
4. To bring an electron out of the metal, some minimum amount of energy must be supplied to an electron, which is known as work function of the metal. The work function depends on the type of metal, nature of its surface and its temperature.
5. Corresponding to work function minimum frequency required to eject photoelectron is known as the threshold frequency.
6. Dependence of photoelectric current on the intensity of incident light, value of maximum kinetic energy of an emitted photoelectron on frequency of incident light and not on its intensity, instantaneous (within 10^{-9} sec) emission of photoelectrons cannot be explained by the wave nature of light.
7. Assuming light as a particle, Einstein could solve the mystery of the photoelectric effect. His photoelectric equation $\frac{1}{2}mv^2_{max} = eV_0 = hf - \phi_0$ is in accordance with the energy conservation law.
8. Photoelectric effect and Compton effect have confirmed the dual nature of radiation.
9. On the symmetry argument, de Broglie had further proposed dual nature for material particles. Which was supported by experimental observations (e.g., Davisson-Germer experiment) as well as by theoretical calculations (e.g., Schroedinger wave equation).
10. This confirms the dual (particle and wave) nature for both radiation and matter particles.
11. The non-zero value of Planck's constant (h) alongwith Heisenberg's uncertainty principle measures the inadequacy of the classical mechanics.

EXERCISES

For the following statements choose the correct option from the given options :

1. Cathode rays ....
   (A) are the atoms moving towards the cathod.
   (B) are electromagnetic waves.
   (C) are negative ions travelling from cathode to anode.
   (D) are electrons emitted by cathode and travelling towards anode.
2. Which of the following statement is not true for a photon ?
   (A) Photon produces pressure (B) Photon has energy hf.
   (C) Photon has momentum $\frac{hf}{c}$ (D) Rest mass of photon is zero
3. The velocity of photon emitted in photo-electric effect depends on the properties of photosensitive surface and..........
   (A) frequency of incident light (B) state of polarization of incident light
   (C) time for which the light is incident (D) intensity of incident light
4. Photoelectric effect represents that
   (A) electron has a wave nature (B) light has a particle nature
   (C) (1) and (2) both (D) none of the above

5. De Broglie wavelength of a particle moving with velocity 2.25×10^8 m s^{-1} is same as the wavelength of photon. The ratio of kinetic energy of the particle to the energy of photon is..........
Velocity of light = 3×10^8 m s^{-1}

(A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{7}{8}$

6. Energy of photon is $E = hf$ and its momentum is $p = \frac{h}{\lambda}$, where λ is the wavelength of photon. With this assumption speed of light wave is ....

(A) $\frac{p}{E}$ (B) $\frac{E}{p}$ (C) Ep (D) $\left(\frac{E}{p}\right)^2$

7. Wavelength λ_A and λ_B are incident on two identical metal plates and photo electrons are emitted. If $\lambda_A = 2\lambda_B$, the maximum kinetic energy of photo electrons is ....

(A) $2K_A = K_B$ (B) $K_A < \frac{K_B}{2}$ (C) $K_A = 2K_B$ (D) $K_A > \frac{K_B}{2}$

8. Cathode rays travelling in the direction from east to west enter in an electric field directed from north to south. They will deflect in ....
(A) east (B) west (C) south (D) north

9. If photoelectric effect is not seen with the ultraviolet radiations in a given metal, photo electrons may be emitted with the ....
(A) infrared waves (B) radio waves (C) X-rays (D) visible light

10. Photons of energy 1 *e*V and 2.5 *e*V successively illuminate a metal whose work function is 0.5 *e*V, The ratio of maximum speed of emitted electron is ....
(A) 1 : 2 (B) 2 : 1 (C) 3 : 1 (D) 1 : 3

11. When frequencies f_1 and f_2 are incident on two identical photo sensitive surfaces, maximum velocities of photo electrons of mass m are v_1 and v_2, hence ....

(A) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$ (B) $v_1 + v_2 = \left[\frac{2h}{m}(f_1+f_2)\right]^{\frac{1}{2}}$

(C) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2)$ (D) $v_1 - v_2 = \left[\frac{2h}{m}(f_1+f_2)\right]^{\frac{1}{2}}$

12. A proton and an α-particle are passed through same potential difference. If their initial velocity is zero, the ratio of their de Broglie's wavelength after getting accelerated is.\
(A) 1 : 1 (B) 1 : 2 (C) 2 : 1 (D) $2\sqrt{2}$: 1

13. Mass of photon in motion is ....

(A) $\frac{c}{hf}$ (B) $\frac{h}{\lambda}$ (C) hf (D) $\frac{hf}{c^2}$

14. Wavelength of an electron having energy 10 keV is .... Å.
(A) 0.12 (B) 1.2 (C) 12 (D) 120

15. If the momentum of an electron is required to be same as that of wave of 5200 Å wavelength, its velocity should be .... m s^{-1}.
(A) 10^3 (B) 1.2×10^3 (C) 1.4×10^3 (D) 2.8×10^3

16. The uncertainty in position of a particle is same as it's de Broglie wavelength, uncertainty in its momentum is ....

(A) $\frac{\hbar}{\lambda}$ (B) $\frac{2\hbar}{3\lambda}$ (C) $\frac{\lambda}{\hbar}$ (D) $\frac{3\lambda}{2\hbar}$

17. A proton and electron are lying in a box having unpenetrable walls, the ratio of uncertainty in their velocities are .... [m_e = mass of electron and m_p = mass of proton.]

(A) $\frac{m_e}{m_p}$ (B) $m_e \cdot m_p$ (C) $\sqrt{m_e \cdot m_p}$ (D) $\sqrt{\frac{m_e}{m_p}}$

18. When α-particles are accelerated under the p.d. of V volt, their de Broglie's wavelength is .... Å [Mass of α-particle is 6.4×10^{-27} kg and its charge is 3.2×10^{-19} C.]

(A) $\frac{0.287}{\sqrt{V}}$ (B) $\frac{12.27}{\sqrt{V}}$ (C) $\frac{0.103}{\sqrt{V}}$ (D) $\frac{1.22}{\sqrt{V}}$

19. De Broglie wavelength of a proton and α-particle is same, ... physical quantity should be same for both.

(A) velocity (B) energy (C) frequency (D) momentum

20. To reduce de Broglie wavelength of an electron from 10^{-10} m to 0.5×10^{-10} m, its energy should be ....

(A) increased to 4 times (B) doubled

(C) halved (D) decreased to fourth part

21. The de-Broglie wavelength of a proton and α–particle is same. The ratio of their velocities will be ....

[α–particle is the He-nucleus, having two protons and two neutrons. Thus, its mass $m_\alpha \approx 4m_p$; where m_p is the mass of the proton.]

(A) 1 : 4 (B) 1 : 2 (C) 2 : 1 (D) 4 : 1

22. The de-Broglie wavelength associated with a particle with rest mass m_0 and moving with speed of light in vacuum is ....

(A) $\frac{h}{m_0 c}$ (B) 0 (C) ∞ (D) $\frac{m_0 c}{h}$

23. An image of sun is formed by convex lens of focal length 40 cm on the metal surface of a photoelectric cell, and a photoelectric current I is produced. If now another lens with half the focal length but with same diameter is used to focus the sun image, on the photoelectric cell, photoelectric current becomes ....

(A) $\frac{I}{4}$ (B) 2 I (C) I (D) $\frac{I}{2}$

24. In quantum mechanics, a particle ....

(A) can be regarded as a group of harmonic waves.

(B) can be regarded as a single wave of definite wave-length only

(C) can be regarded as only a pair of two harmonic waves

(D) is a point-like object with mass.

25. Which of the following physical quantity has the dimension of planck constant (h) ?

(A) Force (B) Angular momentum

(C) Energy (D) Power

ANSWERS

1. (D)	**2.** (A)	**3.** (A)	**4.** (B)	**5.** (B)	**6.** (B)
7. (B)	**8.** (D)	**9.** (C)	**10.** (A)	**11.** (A)	**12.** (D)
13. (D)	**14.** (A)	**15.** (C)	**16.** (A)	**17.** (A)	**18.** (C)
19. (D)	**20.** (A)	**21.** (D)	**22.** (B)	**23.** (C)	**24.** (A)
25. (B)					

Answer the following questions in brief :

1. What is photon ?
2. What is ultraviolet catastrophe ?
3. Write Planck's hypothesis to explain energy distribution for cavity radiation.
4. Write Planck's revolutionary idea to explain energy distribution for cavity radiation.
5. Define work function of metal.
6. On which factors work function of metal depends ?
7. What is thermionic emission ?
8. Define field emission.
9. Give definition of photoelectric emission.
10. What is threshold frequency ? On which factor does threshold frequency depend ?
11. What is stopping potential ?
12. Which physical quantity can be infered from the knowledge of shopping potential ?
13. On what factor does the stopping potential depend ?
14. Write de Broglie hypothesis.
15. Define wave packet.
16. State Heisenberg's Uncertainty principle.
17. Write the conclusion of Davisson-Germer's experiment.
18. If the threshold wave length of Na element is 6800 $\overset{o}{A}$, find its work function in eV.
19. Calculate the energy of photon in eV for a radiation of wavelength 5000 $\overset{o}{A}$?

Answer the following questions :

1. Write the characteristics of photoelectric emission.
2. How wave theory fails to explain the experimental results of photoelectric effect ?
3. Explain Einstein's explanation for photoelectric effect.
4. Write the properties of a photon.
5. Write a short note on photo cell.
6. Explain the experimental arrangement of Davisson-Germer experiment.
7. Explain the conclusions of Davisson-Germer experiment.
8. Calculate the maximum kinetic energy (eV) of a photo electron for a radiation of wave length 4000 $\overset{o}{A}$ incident on a surface of metal having work function 2 eV ?
9. A light beam of 6000 $\overset{o}{A}$ wavelength and 39.6 w/m^2 intensity is incident on a metal surface. If 1 % photon of the incident photon comitts the photo electron, calculate the number of photo electron emitted per second ?

Solve the following examples :

1. A small piece of Cs (work function = 1.9 eV) is placed 22 cm away from a large metal plate. The surface charge density on the metal plate is 1.21×10^{-9} C m^{-2}. Now, light of 460 nm wavelength is incident on the piece of Cs. Find the maximum and minimum energies of photo electrons on reaching the plate. Assume that no change occurs in electric field produced by the plate due to the piece of Cs.

   [**Ans. :** Minimum energy = 29.83 eV, Miximum energy = 30.63 eV)

2. Threshold wavelength of tungsten is 2.73×10^{-5} cm. Ultraviolet light of wavelength 1.80×10^{-5} cm is incident on it. Find, (1) threshold frequency, (2) work function (3) maximum kinetic energy (in joule and eV) (4) stopping potential and (5) maximum and minimum velocity of an electron.

   [**Ans. :** (1) $f_0 = 1.098 \times 10^{15}$ Hz $\approx 1.1 \times 10^{15}$ Hz, (2) $\phi = 4.54$ eV, (3) $K_{max} = 3.76 \times 10^{-19}$ J = 2.35 eV, (4) $V_0 = 2.35$ V, (5) $v_{max} = 9.09 \times 10^5$ m $s^{-1} \approx 9.1 \times 10^5$ m s^{-1}, $V_{min} = 0$ m s^{-1})

3. Wavelength of light incident on a photo-sensitive surface is reduced from 3500 Å to 290 nm. Find the change in stopping potential $h = 6.625 \times 10^{-34}$ J s. [**Ans. :** 73.42×10^{-2} V]

4. An electric bulb of 100 W converts 3% of electrical energy into light energy. If the wavelength of light emitted is 6625 Å, find the number of photons emitted is 1 s. $h = 6.625 \times 10^{-34}$ J s. [**Ans. :** 10^{19}]

5. When a radiation of wavelength 3000 Å is incident on a metal, stopping potential is found to be 1.85 V and on making radiation of 4000 Å incident on it the stopping potential is found to be 0.82 V. Find (1) Planck's constant (2) Work function of the metal (3) Threshold wavelength of the metal.. [**Ans. :** (1) $h = 6.59 \times 10^{-34}$ J s (2) $\phi_0 = 2.268$ eV (3) $\lambda_0 = 5440$ Å.]

6. Work function of Zn is 3.74 eV. If the sphere of Zn is illuminated by the X-rays of wavelength 12 Å, find the maximum potential produced on the sphere.

   $h = 6.25 \times 10^{-34}$ J s. [**Ans. :** 1032.2 V]

7. Find the energy of photon in each of the following :

   (1) Microwaves of wavelength 1.5 cm (2) Red light of wavelength 660 nm

   (3) Radiowaves of frequency 96 MHz (4) X–rays of wavelength 0.17 nm

   [**Ans. :** (1) 8.3×10^{-5} eV (2) 1.9 eV (3) 4×10^{-7} eV (4) 7.3 keV]

8. Human eye can experience minimum 19 photons per second. Light of 560 nm wavelength is required for it. What is the minimum power necessary to excite optic nerves ?

   [**Ans. :** 67.4×10^{-19} W]

9. Power produced by a star is 4×10^{28} W. If the average wavelength of the emitted radiations is considered to be 4500 $\mathring{A}$, find the number of photons emitted in 1 s.

[**Ans. :** 9.054×10^{46} photons/s]

10. What should be the ratio of de Broglie wavelenghts of an atom of nitrogen gas at 300 K and 1000 K. Mass of nitrogen atom is 4.7×10^{-26} kg and it is at 1 atm pressure. Consider it as an ideal gas. [**Ans. :** 1.826]

11. Monochromatic light of wavelength 3000 $\mathring{A}$ is incident normally on a surface of area 4 cm^2. If the intensity of light is 150 $\frac{mW}{m^2}$, find the number of photons being incident on this surface in one second. [**Ans. :** 9.05×10^{13} s^{-1}]

12. A star which can be seen with naked eye from Earth has intensity 1.6×10^{-9} W m^{-2} on Earth. If the corresponding wavelength is 560 nm, and the diameter of the lens of human eye is 2.5×10^{-3} m, find the number of photons entering in our eye in 1 s.

[**Ans. :** 9×10^4 photons/s]

13. Find the velocity at which mass of a proton becomes 1.1 times its rest mass, $m_p = 1.6 \times 10^{-27}$ kg. Also, calculate corresponding temperature. For simplicity, consider a proton as non-interacting ideal-gas particle at 1 atm pressure.

[$c = 3 \times 10^8$ ms^{-1}, $k_B = 1.38 \times 10^{-23}$ SI] [**Ans. :** $v = 0.42$ C, 6.75×10^{11} K]

14. Output power of He-Ne LASER of low energy is 1.00 mW. Wavelength of the light is 632.8 nm. What will be the number of photons emitted per second from this LASER ? $h = 6.25 \times 10^{-34}$ J s. [**Ans. :** 3.18×10^{15} s^{-1}]

• • •

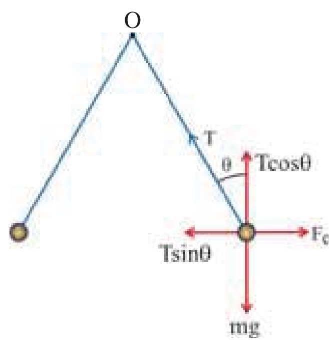
SOLUTIONS

CHAPTER 1

1. Suppose charge on a sphere A and sphere B is q_1 and q_2 respectively. In first case force between two spheres is $F = k\frac{q_1 q_2}{d^2}$. When sphere A is brought in contact with sphere C the charge on sphere A will be, $q_A = \frac{q_1}{2}$ and charge on sphere B will be $q_B = \frac{q_2}{2}$.

Now, force between sphere A and sphere B at distance $\frac{d}{2}$ will be

$$F' = \frac{k(q_1/2)(q_2/2)}{(d/2)^2} = \frac{kq_1 q_2}{d^2} = F$$



2. Suppose the density of sphere = ρ and density of kerosene = ρ'. When two spheres are suspended in air forces acting on it are shown in figure. In equilibrium pasition,

$F_e = T\sin\theta$ and $mg = T\cos\theta$. From these,

$$\tan\theta = \frac{F_e}{mg} \qquad (1)$$

Now, the sphere is immersed in kerosene, due to buoyant force acting on it, its weghit will be $(m - m')g$ instead of mg and electric force. $F_e = \frac{F_e}{2}$.

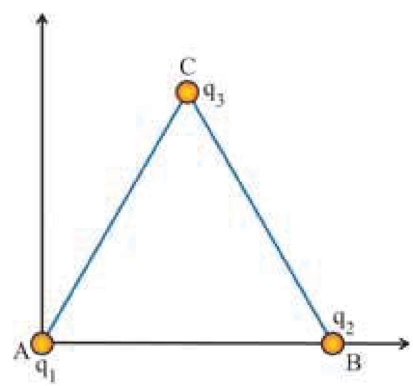
Because dilectric constant of kerosene is, K = 2.

$$\therefore \tan\theta = \frac{F_e/2}{(m-m')g} \qquad (2)$$

Comparing equation (1) and (2), m = 2m'

or $\rho V = 2(\rho' V)$

$\therefore \rho = 2\rho' = 2 \times 800 = 1600$ kg m^{-3}



3. Suppose $q_1 = 0.5 \times 10^{-6}$C

$q_2 = -0.25 \times 10^{-6}$C, $q_3 = 0.1 \times 10^{-6}$C

From the figure,

Position vector of q_1 is $\vec{r_1} = (0, 0)$m

Position vector of q_2 is $\vec{r_2} = (5 \times 10^{-2}, 0)$m

Position vector of q_3 is $\vec{r_3} = (2.5 \times 10^{-2}, 2.5 \times 10^{-2} \times \sqrt{3})$m

Now, force on q_3, $\vec{F_3} = \vec{F_{31}} + \vec{F_{32}}$

$$= kq_3\left[\frac{q_1(\vec{r_3}-\vec{r_1})}{|\vec{r_3}-\vec{r_1}|^3} + \frac{q_2(\vec{r_3}-\vec{r_2})}{|\vec{r_3}-\vec{r_2}|^3}\right]$$

Substituting values in above equation and calculate $\vec{F_3}$.

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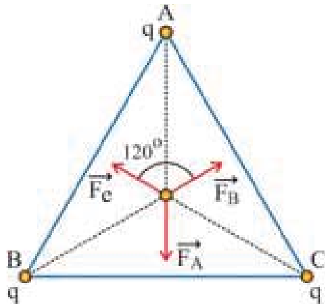
4. 

As shown in figure, the net force on charge $2q$ is $\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C$ and $|\vec{F}_A| = |\vec{F}_B| = |\vec{F}_C|$ because of equilateral triangle. All three forces are at 120° respectively. Vector addition of $\vec{F}_A$, $\vec{F}_B$ and $\vec{F}_C$ using triangular method forms close loop. So, the resultant force is zero.

5. Torque on dipole, $\tau = \text{PE}\sin\theta = \text{PE}\theta$ ($\because$ θ is small)
Torque is in clockwise direction, therefore $\tau = -\text{PE}\theta$
Now, $\tau = \text{I}\alpha$ and $\alpha = -\omega^2\theta$

$$\omega = \sqrt{\frac{\text{PE}}{\text{I}}} \quad \therefore f = \frac{1}{2\pi}\sqrt{\frac{\text{PE}}{\text{I}}}$$

6. Suppose an electron is thrown from the distance r from surface with 150 eV energy. Work done on electron against the force,

$$\text{W} = \vec{F}\cdot\vec{r} = (-e\text{E})(r) = \left(\frac{-e\sigma}{2\varepsilon_0}\right)(r)$$

Now substitue the values in this equation and find the value of r.

7. Suppose charge on two spheres is q, and q_2 respectively.
In the first case According to Coulomb's Law.

$$0.108 = 9 \times 10^9 \frac{q_1 q_2}{(0.5)^2}$$

$$\therefore q_1 q_2 = 3 \times 10^{-6} \qquad (1)$$

Now, when both the spheres are bring in contact, the charge on both spheres will be is $\frac{q_1 - q_2}{2}$. In this case, force between them, $0.036 = \frac{9\times10^9\left(\frac{q_1 - q_2}{2}\right)^2}{(0.5)^2}$

$$\therefore q_1 - q_2 = 2 \times 10^{-6} \qquad (2)$$

Equating equation (1) and (2)
$q_1 = 3 \times 10^{-6}$C and $q_2 = 1 \times 10^{-6}$C

8. Acceleration of $2q$ charge, $a_1 = \frac{\text{F}_1}{m} = \frac{2q\text{E}}{m}$

Velocity of charge after t time, $v_1 = a_1 t = \frac{2q\text{E}}{m} t$

So, Kinetic energy $\text{K}_1 = \frac{1}{2} m v_1^2 = \frac{2q^2\text{E}^2}{m} t^2$ (1)

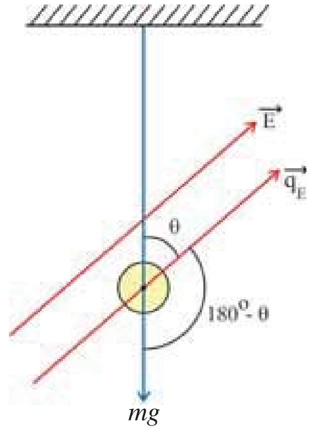
Similarly, calculating kinetic energy of charge q will be

$\text{K}_2 = \frac{q^2\text{E}^2}{4m} t^2$ (2)

From, equation (1) and (2)

$$\frac{\text{K}_1}{\text{K}_2} = \frac{8}{1}$$

9. Two forces are acting on a simple pendulum (1) Electric force $q\vec{E}$ (2) Gravitational force $\text{m}\vec{g}$.
Resultant force, $\vec{F} = \text{m}\vec{g} + q\vec{E}$



$\therefore \; |\vec{F}| = \sqrt{(mg)^2 + (qE)^2 + 2(mg)(qE)cos(180^o - \theta)}$

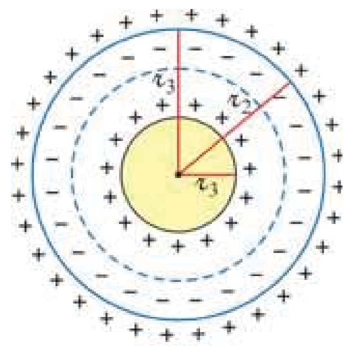
Taking effective acceleration g_e of sphere,

$mg_e = \sqrt{(mg)^2 + (qE)^2 - 2(mg)(qE)cos\theta}$

$$\therefore \; g_e = \left(g^2 + \frac{q^2E^2}{m^2} - \frac{2gqE}{m}cos\theta\right)^{\frac{1}{2}}$$

Periodic time of pendulum $T = 2\pi\sqrt{\frac{l}{g_e}}$. Substitute the value of g_e in it.

10. Due to charge q on a sphere of radius 1 cm, $+q$ charge is induced on outer region of sphere having radius r_3 = 5 cm and $-q$ charge is induced in inner region. Now, draw the spherical Gaussian surface of radius r_2 = 2 cm. Applying Gauss's Law,



$$\int_s \vec{E}\cdot d\vec{a} = \frac{\Sigma q}{\varepsilon_0}$$

$$E(4\pi r_2^{\,2}) = \frac{q}{\varepsilon_0}$$

$$\therefore \; E = \frac{q}{4\pi\varepsilon_0 r_2^{\,2}}$$

Calculate the value of E by substituting the values in above equation.

11. Solved according to illustration 15.

12. When charge q is established on a particle and if electric force qE acting vertically downward on it and gravitational force mg acting on it will be same then particle will be in equalibrium.

$qE = mg$

$$\therefore \; q = \frac{mg}{E} = \frac{mg}{\sigma/\varepsilon_0}$$

Calculate q by substituting the values in above equation.

13. Force between the electron and the proton is,

$$F = k\frac{q_1q_2}{r^2} = \frac{9\times10^9(1.6\times10^{-19})^2}{(0.53\times10^{-10})^2} = 8.2 \times 10^{-8}N$$

The force F on a revolving electron will be equal to centripetal force.

$$\frac{mv^2}{r} = F.$$

Now, putting $v = r\omega$,

$mr\omega^2 = F$

$$\therefore \text{ Radial acceleration } r\omega^2 = \frac{F}{m} = \frac{8.2\times10^{-8}}{9.1\times10^{-31}} = 9 \times 10^{22} \text{ m/s}^2$$

$$\therefore \; \omega = \sqrt{\frac{8.2\times10^{22}}{0.53\times10^{-10}}} = 3.9 \times 10^{16} \text{ rad/s.}$$

CHAPTER 2

1. **Figure**

$q_1 = 2$ C $\qquad q_2 = -3$ C

(i) $V = \frac{Kq_1}{x_1} + \frac{Kq_2}{(100 - x_1)} = 0$ $\qquad$ (ii) $V = \frac{Kq_1}{x_2} + \frac{Kq_2}{(100 + x_2)} = 0$

Hence find x_1. $\qquad$ Hence find x_2.

(Take $q_1 = 2$ C and $q_2 = -3$ C)

2. When both the spheres are joined by a conducting wire; the charges distribute in such a way that their potentials become equal.

$\therefore \frac{KQ_a}{a} = \frac{KQ_b}{a}$. But $Q_b = Q - Q_a$. Hence find Q_a. Similarly find Q_b.

3. $E_x = \frac{-\partial V}{\partial x} = -(4xy - 4z^4)$, $E_y = \frac{-\partial V}{\partial y} = -(2x^2 + 9y^2z)$, $E_z = \frac{-\partial V}{\partial z} = -(3y^3 - 16z^3x)$

For point (1, 1, 1) put $x = 1$, $y = 1$, $z = 1$ to find E_x, E_y, E_z. Hence find $\vec{E}$.

4. Find r from $V = \frac{Kq}{r}$.

If the radius of big drop is r', $\frac{4}{3}\pi r'^3 = (8)(\frac{4}{3}\pi r^3)$ $\therefore r' = 2r$

Now, find $V' = \frac{Kq'}{r'}$ where, $q' = 8q$.

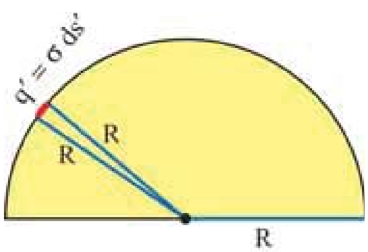
5. When Q = 0, the potential = 0, When Q = Q, the potential $= V = \frac{KQ}{R}$.

$\therefore$ Average potential $= \frac{O+V}{2} = \frac{V}{2}$.

Now, find potential energy = (average potential) (charge)

6. Electric charge in small surface element $= \sigma ds'$. Potential due to it at 0

$dV' = \frac{1}{4\pi\varepsilon_0} \frac{\sigma ds'}{R}$.



$\therefore$ Total potential $V = \int dV' = \frac{1}{4\pi\varepsilon_0} \frac{\sigma}{R} \int ds'$

$\int_{\text{on semi-sphere}} ds' = 2\pi R^2$. Hence find V.

7. Potential on the sphere $V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R}$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\sigma(4\pi R^2)}{R}$$

$$= \frac{\sigma R}{\varepsilon_0}$$

Hence find V_A, V_B, V_C and $V = V_A + V_B + V_C$

8. $\frac{C_1}{C_2} = \frac{C_3}{C_4}$ $\therefore$ Potential difference for $C_5 = 0$

$\therefore$ It is not in action (not effective)

$\therefore \frac{1}{C'} = \frac{1}{1} + \frac{1}{3} = \frac{4}{3}$. find C'.

$\frac{1}{C''} = \frac{1}{2} + \frac{1}{6} = \frac{8}{12}$ find C''.

Find C = C' + C''.

9. Initially the charge on capacitor = Q.

Find initial energy $U = \frac{1}{2}CV^2$. It can also be written as $= \frac{Q^2}{2C}$.

When it is joined with other uncharged capacitor, the charge on each one will be $= Q' = \frac{Q}{2}$.

Now find energy of each one $U' = \frac{Q'}{2C}$

Find total energy = U' + U' = 2U'

10. If capacitance on path MNOP is C',

$\frac{1}{C'} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \therefore C' = \frac{10}{3} pF.$

C' and C_4 are in parallel. ∴ Their equivalent capacitance $C'' = \frac{10}{3} + 10 = \frac{40}{3} pF.$

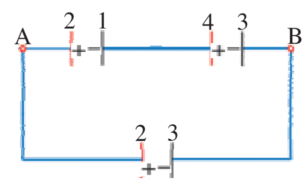
Find charge coming from battery Q'' = C''V

Find $Q_4 = C_4V$. Find equal charge on

C_1, C_2, C_3 as $= Q'' - Q_4$.

11. If capacitance on the path B C_2 C_3 D, is C', $\frac{1}{C'} = \frac{1}{C_2} + \frac{1}{C_3}$. Find C'. Now between A and C, C_1, C' and C_4 are in parallel. ∴ $C'' = C_1 + C' + C_4$.

12. The equivalent connections are as shown here.

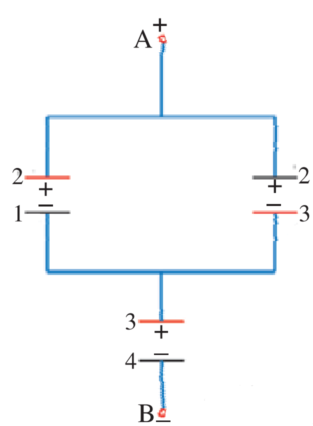


C_{21} and C_{43} are in series. Their equivalent $C' = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)$

With this combination C_{23} is in parallel.

find $C_{AB} = C' + C_{23}$.

13. C_{21} and C_{23} are in parallel



∴ Their equivalent $C' = 2\left(\frac{\varepsilon_0 A}{d}\right)$

C' and C_{34} are in series.

Use $\frac{1}{C_{AB}} = \frac{1}{C'} + \frac{1}{C_{34}}$ to find C_{AB}.

CHAPER 3

1. If the number of electrons striking the screen per second = n

   Using the equation of current, $I = \frac{Q}{t} = \frac{ne}{t}$, calculate n.

   Electric charge striking the screen in t = 1 minute = 60 s can be calculated from $Q = It$.

2. The speed of electron in a circular orbit $v = \frac{2\pi r}{T} = 2\pi rf$

   Substituting given values of v and r, calculate f.
   Now, apply the equation $I = ef$.

3. (i) Calculate the potential difference across the ends of a wire from equation,

   $$V = IR = I\left(\rho\frac{l}{A}\right)$$

   (ii) From equation $I = Av_d ne$, drift velocity $v_d = \frac{I}{Ane}$

   where, number density of electron $n = \frac{dN_A}{M}$

4. Area of cross-section of a semiconductor
   $A = bh$
   $A = (4 \times 10^{-3})(25 \times 10^{-5}) = 10^{-6}m^2$

   Find the current density from $J = \frac{I}{A}$

   Now, using equations $J = ne\ v_d$ and $v_d = \frac{l}{t}$ calculate time t.

5. New length of a wire $l' = l + 10\ \%$ of l

   $$l' = 1 + 0.1\ l = 1.1\ l$$

   $$\frac{l'}{l} = 1.1$$

   Initially, $R = \rho\frac{l}{A}$

   After stretching the wire, $R' = \rho\frac{l'}{A'}$

   volume of the wire is constant.

   $\therefore\ Al = A'l' \Rightarrow \frac{A}{A'} = \frac{l'}{l} = 1.1$

   Now, $\frac{R'}{R} = \frac{l'}{l}\cdot\frac{A}{A'} = \left(\frac{l'}{l}\right)^2 = 1.21$

   Percentage increase in resistance $= \frac{R'-R}{R} \times 100 = (1.21 - 1) \times 100 = 21\%$

6. Let the length of P part of wire = l,
   and Q part of wire = $(1 - l)$
   If the resistances of P, Q and R part of wires be R_P, P_Q and R_R respectively then,

   $R_P = \rho.\frac{l}{A}$, $R_R = \frac{\rho(2l)}{A/2}\ 4\frac{\rho l}{A} = 4R_P$

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and $R_Q = \frac{\rho(1-l)}{A}$

given that, $R_R = R_Q$

$4 \cdot \frac{\rho l}{A} = \frac{\rho(1-l)}{A}$

$\Rightarrow l = \frac{1}{5}$ m

7. Given that, $R_{Al} = R_{CU}$

$\rho_1 \cdot \frac{l_1}{A_1} = \rho_2 \cdot \frac{l_2}{A_2}$

$\Rightarrow \frac{\rho_2}{\rho_1} = \frac{A_2}{A_1}$

Now, mass of Al wire $m_{Al} = A_1 l_1 d_1$ and mass of C_{Cu} wire $m_{Cu} = A_2 l_2 d_2$

Their ratio gives, $m_{Cu} = 2.15\ m_{Al}$

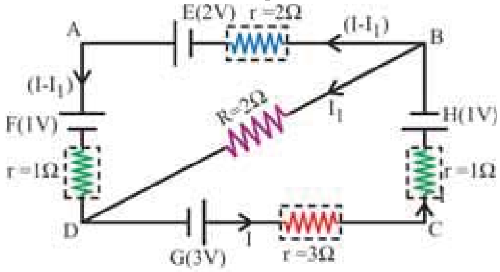
$\therefore m_{Al} < m_{Cu}$

8. (i) Applying Kirchhoff's second rule to the closed loop BADB, $-3I + 5I_1 + 1 = 0$(1)

(ii) For a closed loop DCBD, $-2I - I_1 + 1 = 0$ (2)

Solving equation (1) and (2) we get, $I_1 = \frac{1}{13}$A

and $V_{BD} = I_1 R = \left(\frac{1}{13}\right)(2) = \frac{2}{13}$V



9. (i) Applying Kirchhoff's second rule to the closed loop ACDBMNA, $r(2x + y) = \varepsilon$ (1)

(ii) Similarly for a closed loop ACEFDBMNA, $2r(3x - 2y) = \varepsilon$ (2)

From equation (1) and (2), $y = \frac{4}{5}x$ (3)

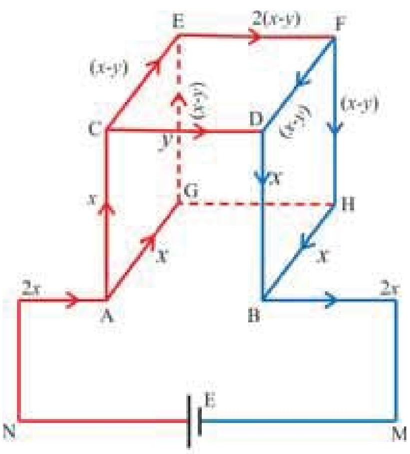
From equation (1) and (3), $\varepsilon = \left(\frac{14}{5}x\right)r$ (4)

If the effective resistance between A and B is r' then,

$\varepsilon = 2xr'$ (5)

comparing equation (4) and (5)

$r' = \frac{7}{5}r$



10. Let the required equivalent resistance be X. The network is infinite. Therefore adding one more stage to the network does not affect the value of X.

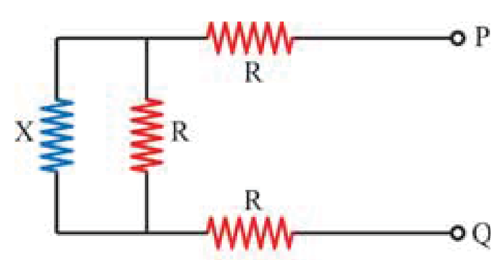
The equivalent resistance of the above circuit should be equal to X.

$\frac{XR}{X+R} + 2R = X$

$\therefore X^2 - 2RX - 2R^2 = 0$

Solving the above equation by the method of quadratic equation, we get,

$X = R(1 + \sqrt{3})$



11. Length of the potentiometer wire $L_1 = 200$ cm

$\Rightarrow$ null point length $l_1 = 80$ cm

$$\therefore\ \varepsilon = \sigma\ l_1 = \left(\frac{IR}{L_1}\right)l_1 \qquad (1)$$

Length of potentiometer wire $L_2 = 300$ cm

$\Rightarrow$ null point length $l_2 = ?$

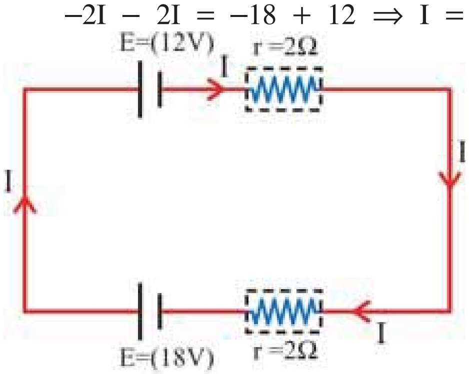
$$\varepsilon = \sigma l_2 = \left(\frac{IR}{L_2}\right)l_2 \qquad (2)$$

Comparing equation (1) and (2),

$l_2 = 120$ cm

12. (1) Applying Kirchhoff's second rule to the closed circuit shown in Figure,

$-2I - 2I = -18 + 12 \Rightarrow I = 1.5A$



(2) Calculate electrical power in Battery using equation $P = \varepsilon I$

(3) Terminal voltage of a 18 V battery

$V = \varepsilon - Ir \Rightarrow V = 15$ V

Terminal voltage of a 12 V battery

$V = \varepsilon + Ir \Rightarrow V = 15$ V ($\because$ The battery of 12 V is being charged.)

(4) Calculate power consumed in battery from equation $I^2 r$.

13. V and H is same for both the coils

$$\text{For coil-1,}\quad H = \frac{I_1^2 R_1 t_1}{J} = \left(\frac{V^2}{R_1^2}\right)\frac{R_1 t_1}{J} = \frac{V^2 t_1}{R_1 J}$$

$$\Rightarrow \frac{1}{R_1} = \frac{JH}{V^2 t_1} \qquad (1)$$

$$\text{For coil-2,}\quad \frac{1}{R_2} = \frac{JH}{V^2 t_2} \qquad (2)$$

when two coils are connected in parallel,

$$\frac{1}{R} = \frac{JH}{V^2 t} \qquad (3)$$

where R = equivalent resistance of a parallel connection

From equations (1), (2) and (3)

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{JH}{V^2 t} = \frac{JH}{V^2 t_1} + \frac{JH}{V^2 t_2}$$

$$\therefore\ \frac{I}{t} = \frac{I}{t_1} + \frac{I}{t_2} \Rightarrow t = 3.43 \text{ minute}$$

14. Heat required to melt the fusewire

$$H = mc\Delta\theta$$

$$= (Ald)\ c\Delta\theta \qquad (1)$$

where, d = density of fusewire
c = specific heat of fusewire
m = Ald = mass of fusewire
$\Delta\theta$ = Increase in temperature required to melt the fusewire

Heat produced in the fusewire by passing current I through it for time t,

$$H = \frac{I^2Rt}{J} \qquad (2)$$

For melting of a fusewire,

$$\frac{I^2Rt}{J} = Aldc\Delta\theta$$

$$I^2\left(\rho\frac{l}{A}\right)\frac{t}{J} = Aldc\Delta\theta$$

$$\therefore\ I^2\rho t = JA^2dc\Delta\theta$$

$$\therefore\ t = \frac{JA^2dc\Delta\theta}{I^2\rho} \qquad (3)$$

From this equation both the fuse wire will melt in the same time for the same value of current flowing through them.
($\because$ J, A, d, C, $\Delta\theta$, ρ are constant for both the fusewires)

15. Using $P = \frac{V^2}{R}$, R_A = 302.5 Ω and R_B = 121 Ω

Similarly from equation P = VI,
I_A = 0.3636 A and I_B = 0.9091 A
Same current will flow through each of the bulb when two bulbs are connected in series with a supply of 220 V.

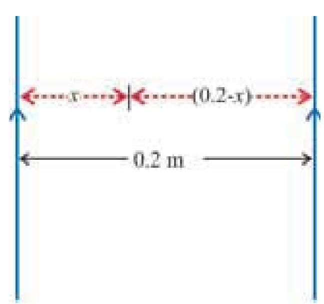
$$\therefore\ I = \frac{V}{R_A + R_B} = \frac{220}{302.5+121} = 0.519 \text{ A}$$

Here I > I_A, therefore A bulb will fuse.

CHAPTER 4

1. $\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi(0.2 - x)}$ (in opposite directions)

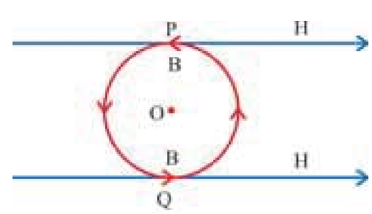
Hence find x.



2. At P, B = H (in opposite directions)

$$\frac{\mu_0 I}{2\pi y} = H$$

Hence find I.
At Q, B = H (in the same direction)
$\therefore$ Total magnetic field = B + H = 2B = 2H.



3. $S = \frac{GI_G}{I - I_G}$ I = 100 unit and I_G = 2 unit, find S.

4. Find velocity v from $qV = \frac{1}{2}Mv^2$ ...... V = voltage. Put it in $\frac{Mv^2}{R} = Bqv$ and then Make R the subject, to get $R = \left(\frac{2MV}{q}\right)^{\frac{1}{2}} \times \frac{1}{B}$. V, q, B are same for both particles.

$\therefore \frac{M_1}{M_2} = \left(\frac{R_1}{R_2}\right)^2$

5. $\vec{\tau} = \vec{\mu} \times \vec{B} = NI\vec{A} \times \vec{B}$ gives $\tau_{max} = NIAB$,

Length of wire $L = N(2\pi R) \Rightarrow R = \frac{L}{2\pi N}$; $A = \pi R^2 = \pi\frac{L^2}{4\pi^2 N^2}$, Put A in τ_{max}.

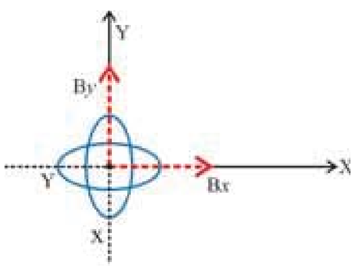
6. $\frac{mv^2}{r} = Bqv$ gives $r = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2mE}}{Bq}$ $\therefore E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ $\therefore p = \sqrt{2mE}$

$\frac{r_d}{r_p} = \frac{p_d}{Bq_d} \times \frac{Bq_p}{p_p}$, ... $(q_p = q_d)$,

$\frac{p_d}{p_p} = \frac{\sqrt{2m_d E}}{\sqrt{2m_p E}}$ (But $m_d = 2m_p$; E is same)

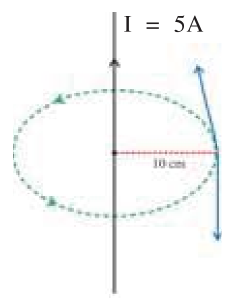
$\therefore \frac{r_d}{r_p} = \sqrt{2}$

7. $k\phi = NABI$; Take $\phi = \left(36 \times \frac{\pi}{180}\right)$rad, $k = \frac{B_I N}{\phi}$. Find k.

8. 

$B_x = \frac{\mu_0 I_x}{2a}$ and $B_y = \frac{\mu_0 I_y}{2a}$.

Resultant magnetic field is $B' = \sqrt{B_x^2 + B_y^2}$.



9. $\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{y}$ Find $\frac{F}{l}$.

10. $B = \frac{\mu_0 I}{2\pi y}$. The velocity of electron is perpendicular to this B.

Use $F = Bqv\sin\theta = Bev$

11. Use formula obtained in illustration-1 $B = \frac{\mu_0 I\theta}{2\pi R}$; θ in radian $= 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$, I = 6A and R = 0.02. Find B.

CHAPTER 5

1. Average radius $r = \frac{r_1 + r_2}{2}$, $n = \frac{N}{2\pi r}$

Now $H = nI_f$, Hence use $\mu_r = \frac{B}{\mu_0 H}$

2. $m_{atom} = 1.5 \times 10^{-23}$ A m^2

$\therefore\ m_{net} = m_{atom} \times$ number of atoms per unit volume

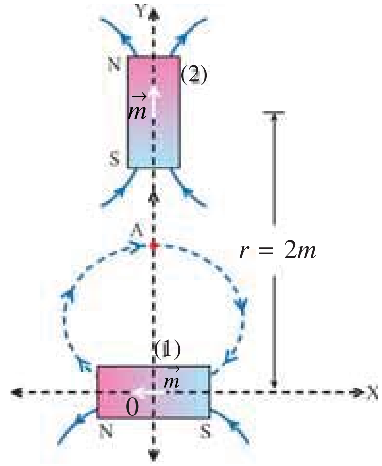
Hence use $M_{max} = \dfrac{m_{net}}{V}$ (1)

Thermal energy of the atom of gas $= \dfrac{3}{2}kT$ (2)

Maximum potential energy of atom $= m_{net}$ B (3)

Find the ratio $\left(\dfrac{\frac{3}{2}kT}{m_{net}B}\right)$ and give answer.

3. For magnet (1) the equatorial magnetic field at A (1 m from its centre) is $B_1 = \dfrac{\mu_0}{4\pi}\dfrac{m}{r^3}\hat{i}$



For magnet (2) the axial magnetic field at A (1 m from its centre) is

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2m}{r^3}\hat{j}$$

Hence resultant magnetic field at A is

$$B = \sqrt{B_1^2 + B_2^2}$$

4. Axial magnetic field of magnet is

$$B(z) = \frac{\mu_0}{4\pi}\frac{2m}{z^3} = \frac{\mu_0}{4\pi}\frac{2(l\,p_b)}{z^3}$$

l = length of magnet, p_b = pole strength

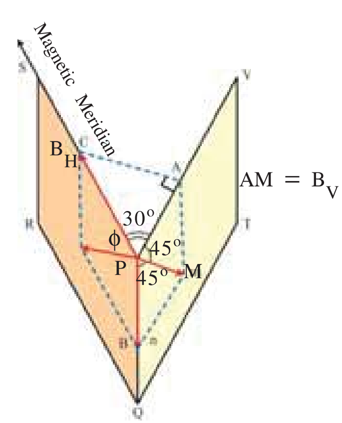
Hence calculate force on magnetic pole as $F = p_b\,B(z)$

5. Work done for rotating magnet of magnetic dipole moment m by angle θ is

$$W(\theta) = \int_0^\theta mB\sin\theta\, d\theta = \left[-mB\cos\theta\right]_0^\theta$$

Now $W(90^o) = nW(60^o) \Rightarrow n = \dfrac{W(90^o)}{W(60^o)}$

6. Magnetic moment of magnet is shown by $\overrightarrow{PM}$ making angle of 45^o in PQTV plane. The plane PQTV makes an angle of 30^o with magnetic meridian plane represented by PQRS.



For rectangular triangle PAC, PA = $B_H\cos 30^o$.

Hence in plane PQTV, for rectangular triangle

$$PAm\ \tan 45^o = \frac{AM}{PA}$$

Now calculate B_v in terms of B_H.

In plane PQRS, $\tan\phi = \dfrac{B_V}{B_H}$

7. Use $m = \frac{1}{2}evr$, and $L = m_e vr$

8. (a) Use $B(x) = \frac{\mu_0}{4\pi}\frac{2m}{x^3}$ to calculate m.

(b) Using the value of m from (a), calculate $B(y) = \frac{\mu_0}{4\pi}\frac{m}{y^3}$

9. Volume of cylindrical rod $V = \pi r^2 l$

Then use, $m_{net} = M \times V$

10. Number of electrons (n_e) = number of ions (n_i)

Average kinetic energy of electron = K_e

Average kinetic energy of ion = K_i

Total kinetic energy of the gas is $K = (n_e \times K_e) + (n_i \times K_i)$

When the gas is completely magnetized, the resultant magnetic moment of gas is equal to its magnetization M or M_{max},

When $U = \vec{M}\cdot\vec{B} = K$, $(\vec{M} = n_e\vec{m} + n_i\vec{m} = 2n_e\vec{m})$

$\therefore$ $K = MB\cos\theta$, Calculate M when $\theta = 0^\circ$

11. Length of solenoid = l, Number of turns per unit length = n

Hence, magnetic moment of solenoid $m = NIA = nlIA$

Also pole strength of solenoid $p_S = \frac{m}{l}$

CHAPTER 6

1. Using Gauss' formula for mirror $\frac{2}{R} = \frac{1}{u} + \frac{1}{v}$, obtain $v = \frac{u \cdot R}{2u - R}$

Differentiate it with respect to time, i.e. obtain $\frac{dv}{dt}$ and simplify.

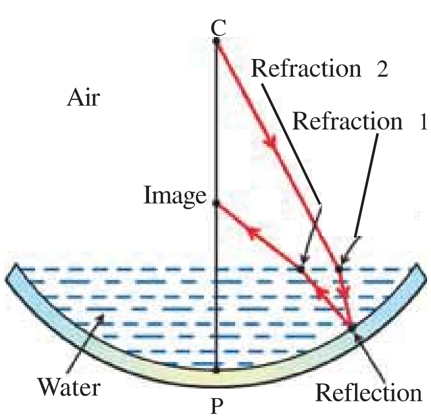
Let $\frac{dv}{dt} = v_i$ = velocity of image

and $\frac{du}{dt} = v_0$ = velocity of object.

2. Use $m = \frac{-v}{u} = \frac{h'}{h}$ and $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ with proper sign convention.

[**Ans.** : 37.5 cm]

3. This is the combination of plano-convex lens formed by water and a concave mirror. Focal length of a lens,



$$\frac{1}{f_1} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For plano-convex lens, $R_1 = \infty$, $R_2 = -R$ (say), n is the refractive index of the material forming lens, here water.

$$\therefore f_1 = \frac{R}{(n-1)} \qquad (1)$$

If focal length of a mirror is f_2 then effective focal length of this combination

$$\frac{1}{f'} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

where f_3 focal length corresponding to emerging ray, which is from denser (water) to rarer (air). f_2 is the focal length of the mirror.

Here, $f_2 \rightarrow -f_2$ and $f_3 \rightarrow f_1$

$$\therefore \frac{1}{f'} = \frac{2}{f_1} - \frac{1}{f_2}$$

$$= \frac{2}{\left(\frac{R}{(n-1)}\right)} - \frac{1}{\left(\frac{2}{R}\right)} \quad \left(\because f = \frac{2}{R} \text{ for mirror and using equation (1)}\right)$$

$$\therefore f' = \frac{R}{2(n-2)}$$

$\therefore$ effective radius of curvature,

$$R' = 2f' = \frac{R}{2(n-2)}$$

Since $n = 1.33 \Rightarrow |R'| > |R|$

i.e. image will form between C' and pole.

5. Since Snell's law is applicable to all points, first apply it to the point of incident.

i.e., $n_1 \cdot sin\theta_1$ = const, A (say)

Let at a distance y in the medium, refracted ray is horizontal (i.e. $\theta_2 = 90°$). Then again using Snell's law at this point,

$n_2 \cdot sin\theta_2 = A$;

Where $n_2 = (1.5 - 0.25y)$. This gives value of y. **[Ans. : $y = 3$m]**

6. For large incidence angle, lateral shift,

$$x = \frac{t \cdot sin(\theta_1 - \theta_2)}{cos\,\theta_2}, \text{ where } \theta_1 = 53°$$

Find θ_2 using Snell's law. t is given. **[Ans. : $x = 9$ mm]**

7. Use $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ to obtain $m = \frac{f}{u-f}$

(1) When $m = 4$, obtain object distances $u_1 = \frac{3}{4}f$.

(2) Now, on displacing object by 3 cm away from the mirror,

$u_2 = u_1 + 3$ (in cm)

Now, calculate f. **[Ans. : $|f| = 36$ cm]**

8. For optical fibre, requirement for total internal reflection is $(90° - \theta_f) > C$

$\therefore sin(90° - \theta_f) > sinC$

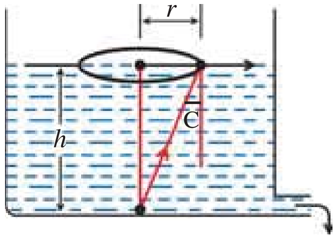
$\therefore cos\theta_f > \frac{1}{n}$ ($\because$ for air-medium interface $sinC = \frac{1}{n}$)

Now apply Snell's law to find maximum incident angle. **[Ans. : $\frac{\pi}{2}$]**

9. Apply Snell's law at the point of incidence to the river. Then use simple trigonometry.

[Ans. : length of a shadow = 3.44 m]

10. $\sin C = \frac{1}{n}$ gives C. From the Figure, $\tan C = \frac{r}{h}$ find h.



[**Ans. :** 1.33 cm]

11. For image due to surface on right,

$$\frac{-n_1}{u} + \frac{n_2}{v} = (n_2 - n_1) \times \frac{1}{R_1} \quad (1)$$

Where $n_1 = 1$, $n_2 = 1.5$, $R_1 = -R$, $u = -\frac{R}{2}$

$$\therefore\ v = \frac{-3R}{5} \quad (2)$$

The image due to this surface is the object for the second. For surface on the left, $n_1 = 1$, $n_2 = |1.5\ R_2 = +R|$.

From equation (2), image of right-surface is at a distance $\left(R - \frac{3}{5}R\right)$ from the centre towards the right-surface. Therefore, u for left-surface is $\frac{3}{2}R$. Using equation (1), image distance due to left-surface from the centre is $\frac{2R}{7}$.

$\therefore$ distance between two images due to both surfaces is $\frac{2}{5}R - \frac{2}{7}R = 0.114R$

14. For plano-convex lens,

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = (n - 1)\cdot\left(\frac{1}{R_1} - \frac{1}{\infty}\right)$$

$\therefore$ $R_1 = 10$ cm, $f = 20$ cm, $n = 1.5$

This lens would have given image at I'. But back plane reflecting surface gives image at I''. This I'' image is the virtual object for the curved surface. Using

$\frac{-n_1}{u} + \frac{n_2}{v} = \left(\frac{n_2 - n_1}{R}\right)$ formula for incident

rays : $u \to -\infty$, $v' = ?$, $n_2 = 1.5$, $n_1 = 1$.

$\therefore$ $v' = 30$ cm.

For emerging ray (second refraction),

$n_1 = 1.5$, $n_2 = 1$, $R = -10$ cm, $u = +30$ cm, $v = ?$

$v = 10$ cm.

Since object was at infinite, final image distance (v) gives the focal length of the system.

15. From the similarity of ΔBH_1H_2 and ΔF_1OH_2,

$$\frac{OF_1}{OA} = \frac{OH_2}{H_1H_2}$$

$$\therefore \frac{-f_1}{-u} = \frac{-h'}{(-h'+h)} \text{ (sign convention)}$$

Similarly, for $\Delta B'H_1H_2$ and ΔF_2OH_1,

$$\frac{f_2}{v} = \frac{h}{(-h'+h)}$$

Adding these equations,

$$\frac{f_1}{u} + \frac{f_2}{v} = \frac{-h'+h}{(-h'+h)} = 1 \quad (1)$$

$$\therefore |m| = \frac{v}{u} = \frac{(v-f_2)}{f_1}$$

From equation (1) for special case, $f_2 = -f_1 = f$

$$\frac{-f}{u} + \frac{f}{v} = 1$$

$\therefore \frac{-1}{u} + \frac{1}{v} = \frac{1}{f}$. This is the Gauss' formula.

CHAPTER 7

1. Minimum Energy, $W = Fd = (eE)(d)$

$$= e\left(\frac{\sigma}{\varepsilon_0}\right)d$$

$$= 29.83 \ eV$$

Minimum Energy $= \frac{1}{2}mV^2 + W$

$$= (hf - \phi_0) + W$$

$$= \frac{hc}{\lambda} - \phi_0 + W$$

$$= 30.63 \ eV$$

2. (1) Threshold Frequency, $f_0 = \frac{c}{\lambda_0} = 1.098 \times 10^{15} Hz \approx 1.1 \times 10^{15}$ Hz

(2) Work Function $\phi_0 = hf_0 = 4.54 \ eV$

(3) *max* K.E., $\frac{1}{2}mV^2_{max} = hf - hf_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$

$$= 2.35 \ eV$$

(4) Stopping Potential $V_0 = (\frac{1}{2}mV^2_{max})$

$= 2.35\ eV$

(5) $\frac{1}{2}mV_{max}{}^2 = eV_0$

$\therefore\ V_{max} = \sqrt{\frac{2eV_0}{m}}$ and $V_{min} = 0$ m/s

3. $eV_0 = \frac{hc}{\lambda} - \phi_0$

$\therefore\ eV_0 = \frac{hc}{\lambda_1} - \phi_0$ (1) and $\frac{hc}{\lambda_2} - \phi_0$ (2)

$\therefore$ Subtract (1) from (2)

Change in stopping potential, $V_{0_2} - V_{0_1} = \frac{hc}{e}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$

4. $p = \frac{3}{100} \times = 100 = 3$ J/s

$p = \frac{E}{t} = \frac{nhf}{t} \Rightarrow n = \frac{p\lambda t}{hc}$

5. $eV_0 = \frac{hc}{\lambda} - \phi_0$

$\therefore\ eV_{0_1} = \frac{hc}{\lambda_1} - \phi_0$ (1) and $eV_{0_2} = \frac{hc}{\lambda_2} - \phi_0$

$\therefore\ e(V_{0_1} - V_{0_2}) = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$

Now, subject the formula for h and calculate if.
From equation (1)

$\phi_0 = \frac{hc}{\lambda_1} - V_{0_1}e = ...$.

$\phi_0 = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = \frac{hc}{\phi_0} = ...$.

6. $Ve = \frac{hc}{\lambda} - \phi_0$

$\therefore\ V = \frac{hc}{\lambda e} - \frac{\phi_0}{e} = ...$.

7. For (1) and (2)

Energy of photon $E = \frac{hc}{\lambda}$

for (3) and (4) $E = hf$

8. $p = \frac{E}{t} = \frac{nhf}{t} = \frac{nhc}{t\lambda} = ...$.

9. $p = \frac{E}{t} = \frac{nhf}{t} = \frac{nhc}{t \cdot \lambda} \Rightarrow$ no. of photon emitted per second is $n = \frac{p\lambda t}{hc} = ...$.

10. $\frac{1}{2}mv^2 = \frac{3}{2}k_B T$

$\therefore \; m^2v^2 = 3k_B Tm$

$\therefore \; p = \sqrt{3k_B Tm}$

$\lambda = \frac{h}{p}$

$\therefore \; \lambda \; \alpha \; \frac{1}{\sqrt{T}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_2}{T_1}} = ...$.

11. $I = \frac{E}{At} = \frac{nhf}{At} = \frac{nhc}{At\lambda}$

$\Rightarrow n = \frac{IAT\lambda}{hc} = ...$.

12. $I \; \frac{E}{At} = \frac{nhf}{At} = \frac{nhc}{At\lambda}$

$\Rightarrow n = \frac{I\lambda At}{hc}$

13. $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ and $m = 1.1 \; m_0$ and calculate V.

$\frac{1}{2}mv^2 = \frac{3}{2}k_B T \Rightarrow T = \frac{mv^2}{3k_B} = ...$.

14. $p = \frac{E}{t} = \frac{nhf}{t} = \frac{nhc}{\lambda t} \Rightarrow$ no. of photons emitted per second is, $n = \frac{p\lambda t}{hc} = ...$.

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16. Heat Transfer by Holman

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LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5566	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6345	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7769	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9843	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Antilogarithms

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											Mean Difference								
00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	2
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1289	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Antilogarithms

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											Mean Difference								
50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
82	6607	6622	6637	6653	6668	6683	6699	6715	6730	6745	2	3	5	6	8	9	11	12	14
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	12	14	16
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

NATURAL SINES

Degree	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	Mean Deifferences 1'	2'	3'	4'	5'
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	.0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	.0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	14
5	.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	.1045	1063	1080	1197	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	.1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	.1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	.1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	11	14
11	.1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	.2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	3	6	9	11	14
13	.2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	.2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	.2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	.2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	.3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	.3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	.3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	.3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	.3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	.4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	.4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	.4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	.4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	.4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	.5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	.5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	.5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	.5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	.5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	9	12
36	.5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	.6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	.6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	.6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	.6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	.6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	.6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	.6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES

Degree	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	Mean Deifferences 1'	2'	3'	4'	5'
45	.7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	.7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	.7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	.7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	.7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	.7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	.7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	.7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	.7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	.8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	.8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	.8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	.8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	.8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	.8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	.8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	.8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	.8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	.8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	.8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	.9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	.9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	.9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	.9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	.9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	.9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	.9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	.9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	.9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	.9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	.9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	.9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	.9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	.9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	.9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	.9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	.9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	.9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	.9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	.9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	.9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	.9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	.9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	.9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0

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NATURAL TANGENTS

Degree	0	6	12	18	24	30	36	42	48	54	Main Deifferences				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	.0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	.0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	.1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	.1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	.1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	.1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	.1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	.2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	.2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	.2493	2512	2530	2549	2568	2686	2605	2623	2642	2661	3	6	9	12	16
15	.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	.2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	.3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	.3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	.3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	.3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	.3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	.4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	.4245	4265	8286	4307	4327	4748	4369	4390	4411	4431	3	7	10	14	17
24	.4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	.4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	.4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	.5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	.5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	.5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	.6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	.6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	.6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	.6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	.7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	.7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	.7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	.8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	.8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	.9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	.9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	.9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

Degree	0	6	12	18	24	30	36	42	48	54	Main Deifferences				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
45	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1.0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1.0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1.1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1.1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1.1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1.2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1.2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1.3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1.3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1.4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1.4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1.5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1.6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1.6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1.7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1.8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1.8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1.9626	9711	9797	9883	9970	2.0057	2.0145	2.0233	2.0323	2.413	15	29	44	58	73
64	2.0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2.1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2.2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2.3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2.4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2.6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2.7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2.9042	9208	9375	9544	9714	9887	3.0061	3.0237	3.0415	3.0595	29	58	87	116	145
72	3.0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3.2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3.4824	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3.7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4.0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4.3315	3662	4015	4374	4737	5107	5483	5864	6252	6646	Mean Differences no longer sufficiently accurate				
78	4.7046	7453	7867	8288	8716	9152	9594	5.0045	5.0504	5.0970					
79	5.1446	1929	2422	2924	3465	3955	4486	5026	5578	6140					
80	5.6713	7297	7894	8502	9124	9758	6.0405	6.1066	6.1742	6.2432					
81	6.3138	3859	4596	5350	6122	6912	7720	8548	9395	7.0264					
82	7.1154	2066	3002	3962	4997	5958	6996	8062	9158	8.0285					
83	8.1443	2636	3863	5126	6427	7769	9152	9.0579	9.2052	9.3572					
84	9.514	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20					
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95					
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27					
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0					