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PHYSICS

Standard 11

(Semester I)

PLEDGE

India is my country.
All Indians are my brothers and sisters.
I love my country and I am proud of its rich and varied heritage.
I shall always strive to be worthy of it.
I shall respect my parents, teachers and all my elders and treat everyone with courtesy.
I pledge my devotion to my country and its people.
My happiness lies in their well-being and prosperity.

રાજ્ય સરકારની વિનામૂલ્યે યોજના હેઠળનું પુસ્તક

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PREFACE

The Gujarat State Secondary and Higher Secondary Education Board has prepared new syllabi in accordance with the new national syllabi prepared by N.C.E.R.T. based on N.C.F. 2005 and core-curriculam. These syllabi are sanctioned by the Government of Gujarat.

It is a pleasure for the Gujarat State Board of School Textbooks, to place before the students this textbook of **Physics, Standard 11, (Semester I)** prepared according to the new syllabus.

Before publishing the textbook, its manuscript has been fully reviewed by experts and teachers teaching at this level. Following suggestions given by teachers and experts, we have made necessary changes in the manuscript before publishing the textbook.

The Board has taken special care to ensure that this textbook is interesting, useful and free from errors. However, we welcome any suggestions, from people interested in education, to improve the quality of the textbook.

Dr. Bharat Pandit	**Dr. Nitin Pethani**
Director	Executive President
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FUNDAMENTAL DUTIES

It shall be the duty of every citizen of India :*

(a) To abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) To cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) To uphold and protect the sovereignty, unity and integrity of India;

(d) To defend the country and render national service when called upon to do so;

(e) To promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) To value and preserve the rich heritage or our composite culture;

(g) To protect and improve the natural environment including forests, lakes, rivers and wild life, and to have compassion for living creatures;

(h) To develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) To safeguard public property and to abjure violence;

(j) To strive towards excellence in all spheres of individual and collective activity so that the national constantly rises to higher levels of endeavour and achievement;

(k) To provide opportunities education by the parent or the guardian, To his child or a ward between the age of 6-14 years as the case may be.

* Constitution of India : Section 51-A

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CHAPTER 1

PHYSICAL WORLD

1.1 Introduction

Dear students, welcome to you all to this first lecture on Physics. Till Std. X, you have gained priliminary ideas about optics, electricity, magnetism, motion, force, gravitation, heat, energy, wave, sound, universe, etc. under the subject science. These topics are related to physics.

Now, you may raise a question : **What is Physics ?** Dear students, physics is one of the basic disciplines in the category of natural sciences. Physics is a science to understand nature. The word 'Bhautik Vigyan' (used in Hindi / Gujarati) related to the science of physical world is derived from Sanskrit word 'Bhautiki'. Science of studying the basic laws of nature and their manifestation in different natural phenomena is Physics. Man has always been curious about the world around him. Because of this curiosity, man has observed the physical environment carefully, searched meaningful patterns and relations in natural phenomena; and still continues to do so. Conclusions, facts, theories etc. derived from all these attempts means physics. The english word **'physics'** comes from a Greek word meaning **'nature'**.

Use of mathematics is very wide spread in physics. Mathematical theories, formulae or mathematical models are inseparable from physics. As a famous quotation states **"physics is the king of science while mathematics is the queen."** The Mathematical expressions obtained for some physical event, not only provide logical explanation of that event but also make predictions about many other connected events. Our physical facilities, ever since existence of humans to till date, are also due to physics.

1.2 Physics - Scope and Excitement

Dear students, a teacher speaks in the classroom and you hear it. Have you analyse this event, any time ?

How is sound produced, when a teacher speaks ?

How this sound propagates in the classroom ?

How do your ears receive this propagated sound ?

Similarly, have you thought about the annual cycle of seasons, the eclipse, the tides, the regular repetitions of day and night, the bright celestial objects in the night sky, etc. ?

In physics (i) we not only observe such physical events, taking place in our day to day life, but find out definite mechanisms from the series of systematic observations.

(ii) The quantities involved in such events are to be defined unambiguously and meanigfully,

(iii) To derive laws or principles from such studies, and

(iv) Such derived laws or principles are to be tested in wide perspectives.

Physics involves study of two fundamental constituents of the universe : **matter and radiation,** to find out origin of fundamental particles of matter and radiation, the interaction between them, the laws of nature related to them, etc.

If we think further about matter, then comes the nucleus of the atom. Here interaction among neutrons, protons, mesons etc.; the energies of the nucleus with radius of the order 10^{-14}m, formed due to such interactions; the radiation emitted due to intra nuclear transitions, etc. are the subject matter of nuclear physics.

Electrons revolving around the nucleus in specific orbits of orbit radius 10^{-10}m with specific numbers and electronic configurations, their transitions, their interactions, special properties of atom because of electrons, etc. are all included in physics.

Due to interaction among atoms, molecules are formed. These atoms do not remain stationary inside a molecule, but perform rotational and vibrational motions. By studying the radiations emitted from atoms and molecules, we can get insights into their structure. From such physics, various aspects of chemistry are also understood.

When many molecules and atoms combine together, we get different phases of matter namely **gas, liquid** and **solid;** depending on physical conditions. Under certain temperature and other conditions, we also get the fourth state of matter - **plasma.** The plasma state of matter obtained at extreme high temperature opened up a possiblity as a future source of tremendous energy for use of mankind.

The mechanical, thermal, electrical, magnetic and optical properties of matter are studied in physics. Dear students, now you will realies that to know and understand all such properties of different subdisciplines of physics like mechanics, thermodynamics, electromagnetics, optics, electrodynamics have been developed.

Mechanics is based on Newton's laws of motion and law of gravitation. It is concerned with the motion of particles, rigid and deformable bodies, force, work, etc. Electrodynamics deals with electric and magnetic phenomena associated with charged and magnetic bodies. Here, how we forget the contribution of physicists like Coulomb, Oersted, Ampere, Faraday, Maxwell !!!

The working of optical fibre, telescopes and microscopes; colours exhibited by thin films, rainbow, mirage; images formed by the mirrors and lenses, etc. are explained in optics.

Thermodynamics includes changes in internal energy, temperature, entropy, etc., of the system through external work and transfer of heat. The efficiency of heat engines and refrigerators, the direction of physical and chemical process, etc., are also studied in thermodynamics.

In physics we discuss a space having infinite dimensions. The operators in quantum mechanics in such a space and their operations on vectors are also very exciting.

But this is not the end of physics at all. Studies related to sun and its planetary system is also carried out in physics. Galaxies, and their structure, the large distance of the order of billion light years amongst them, distribution of billions of kilograms of their matter, the intergalactic space; birth, evolution and death of various stars etc. are also studied in astrophysics, a branch of physics.

Dear students, thus we have seen that the span of physics is from almost 'vacuum' to 'infinity'. Even, vacuum is also considered as some definite state in physics.

Thus the scope of physics on the length scale is from very small length of 10^{-14}m (radius of nucleus) to 10^{26}m (length of galaxies). Thus the factor of length scales is of the order of 10^{40}.

The range of the time scale, about 10^{-22} s to 10^{18}s, can be obtained by dividing the length scales by the speed of light.

The range of mass varies from 10^{-30} kg (mass of an electron) to 10^{55} kg (mass of known observable universe).

Table 1.1 to Table 1.3 show the range of these fundamental physical quantities of length, time and mass.

Table 1.1 : Order of the length scale for different objects (only for information)

Size or Distance of object	Order of length scale (m)
Radius of a proton	10^{-15}
Radius of atomic nucleus	10^{-14}
Size of the hydrogen atom	10^{-10}
Thickness of a paper	10^{-4}
Height of a human	10^{0}
Height of the Mount Everest above sea level	10^{4}
Radius of the Earth	10^{7}
Distance of the Sun from the Earth	10^{11}
Size of our galaxy	10^{21}
Distance to the boundary of observable Universe	10^{24}

Table 1.2 : Order of the time interval for various events (only for information)

Event	Order of Time interval (s)
Life span of most unstable particle	10^{-24}
Time required for light to cross a nuclear distance	10^{-22}
Period of atomic vibrations	10^{-15}
Period of a sound wave	10^{-3}
Wink of eye	10^{-1}
Time between successive human heart beats	10^{0}
Rotation period of the Earth	10^{5}
Revolution period of the Earth	10^{7}
Average human life span	10^{9}
Age of the Universe	10^{17}

Table 1.3 : Order of the mass-scale for different objects (only for information)

Object	Order of Mass-scale (kg)
Electron	10^{-30}
Proton	10^{-27}
Dust particle	10^{-9}
Mosquito	10^{-5}
Grape	10^{-3}
Human	10^{2}
Earth	10^{25}
Sun	10^{30}
Milkyway galaxy	10^{41}
Observable Universe	10^{55}

The scope of physics extends over two basic and interesting domains : macroscopic to microscopic. In addition, it covers static and dynamic systems. Thus physics is associated with time, matter and energy.

Dear students, normal events like an object falling on the earth under freefall and flight of a balloon filled with light gas from earth; sinking of a pin or needle in water while floating of a ship on water; or the most advanced projects like Large Hadron Collider (LHC), International Thermonuclear Experimental Reactor (ITER) and moon mission attract us toward the study of physics.

You may be familiar with the names of internationally famous Indian physicists : C. V. Raman, J. C. Bose, M. N. Saha, Homi Bhabha, S. N. Bose, Vikram Sarabhai, S. Chandrashekhar, etc. List of some Indian institutes doing research in the field of physics is given in Table 1.4 for your information, only.

Table 1.4 : List of some of the Indian Institutes doing Research in the Field of Physics (only for information)

Name of the Institutes	Place
Bhabha Atomic Research Centre (BARC)	Mumbai
Physical Research Laboratory (PRL)	Ahmedabad
Institute for Plasma Research (IPR)	Gandhinagar
Institute of Physics (IOP)	Bhuwaneshwar
National Physical Laboratory (NPL)	Delhi
Inter University Consortium for Astronomy and Astrophysics (IUCAA)	Pune
Indian Institute of Science (IISc)	Bangalore
Raman Research Institute (RRI)	Bangalore
Tata Institute of Fundamental Research (TIFR)	Mumbai
Centre for Advance Technology (CAT)	Indore
Nuclear Science Centre (NSC)	Delhi
Indira Gandhi Centre for Atomic Research (IGCAR)	Kalpakamm
Saha Institute of Nuclear Physics (SINP)	Kolkatta
Regional Research Laboratory (RRL)	Bhopal
Inter University Acelerator Centre (IUAC)	Delhi
Veriable Energy Cyclotron Centre (VECC)	Kolkatta
Vikram Sharabhai Space Centre (VSSC)	Banglore
Indian Institute of Astrophysics (IIA)	Banglore
Indian Institute of Geomagnetism (IIG)	Mumbai
Indian Space Research Organization (ISRO)	Various places in India
Space Applications Centre (SAC)	Various places in India
Indian Institute of Technology (IIT)	Various places in India
National Institute of Technology (NIT)	Various places in India
Various Universities	Various places in India

1.3 Physics, Technology and Society

Today, we can reach any corner of the world within few hours, we can talk to a person sitting at any corner of the world within few seconds. While sitting at the home, we can see live telecast of an event or a game taking place at other places in the world. We can take photographs of our solar system and galaxies. All these become possible because of physics.

The progress made in transportation vehicles : bullock cart, bicycle, motor cycle, car, ship, aeroplane; telegram, telephone, mobile, satellite phone used in communication; radio, tape recorder, television used in entertainment and heaters keroseene stove, gas stove, microwave oven etc. used in kitchen, have become possible because of the proper use of laws or theories of physics in technology.

It is physics which introduced us to radiations and taught us to accelerate charge particles, physics gave instruments based on ultrasonic and optical fibres. Very useful medical technology like X-rays, sonography, Electro Cardio Graf (ECG), Electron Spin Resonance (ESR), Nuclear Magnetic Resonance (NMR), Endoscopy are also due to physics.

Instruments like microscope, Electron Microscope (EM), Atomic Force Microscope (AFM) have played a vital role in the development of material technology, nenotechnology and biotechnology.

You would possibly not be unaware of space technology. Rocket, missile, space shuttle, man-made satellite, remote sensing, etc. also, are words in common use. You may have heard about lasers, radar and microwaves.

Physics has taught us the technique of achieving very low temperatures which results in the development of cryogenics. Physics is the mother of subjects like electronics and communication, computer technology, and information technology.

Even after this tremendous development we are still not in a position to say that we have fully understood nature. Many problems are still unresolved for physicists. For example does universe consist of a single element ? Are matter and energy two different aspects of the same thing ? Is unification of various forces in nature possible ? What is the future of universe ?

Forced with severals questions like these, Physicist are attempting to solve two principal thrusts in physics : unification and reduction.

1.4 Fundamental Forces in Nature

Dear students, if we want to roll a ball on the ground (surface) then we have to apply force to give motion to the ball. The ball will stop after travelling a certain distance since frictional force is acting on the surface of the ball. We have to use a force to lift a ball from the surface. We also feel force when someone pulls or pushes us. Thus, we experience forces in day-to-day life in various ways. Starting from such rudimentary concept of force we will learn the scientific concept of force.

Isaac Newton was the first physicist who developed clear concept of force in his famous laws of motion. In addition, he also discovered the universal law of gravitation.

In the macroscopic domain, besides gravitational force, we come across many different types of forces such as frictional force between two surfaces, the restoring force arising in a compressed spring, tension produced in a stretched string the force of surface tension prevailing in the free surface of a liquid, the viscous force in a fluid medium, etc.

In addition to these, force is also generated because of electrically charged and magnetic objects. Electric and magnetic forces, nuclear forces, interatomic and intermolecular forces, etc. are the examples of the forces prevailing in the microscopic domain.

At present, it is understood that there exist only four fundamental forces in nature. Let us understand them qualitatively.

1.4.1 Gravitational Force

The gravitational force is a universal force and every object experiences this force due to every other object in the universe. According to Newton's law of gravitation, **the mutual attractive force is directly proportional to the product of their masses and inversely proportional to the square of the distance**

between them. The gravitational force is the force of mutual attraction between any two objects by virtue of their masses. It is a long range force and it does not require any intervening medium. Compared to other fundamental forces, gravitational force is the weakest force of nature. Yet it is important for a large portion of physical phenomena in the universe. Because of the gravitational force we are able to stand on the earth, a ball thrown up in the air comes down. Even, tides in the sea are belived due to the gravitational force between earth and moon. The effect of the gravitational force cannot be neglected in the phenomena like motion of the satellites around the earth, motion of the planets around the sun, formation of universe, formation and evolution of stars and galaxies, etc.

1.4.2 Electromagnetic Force

The force acting between charged particles is known as the electromagnetic force. In the simple case, when charged particles are at rest, the force between them is known as static electric force. The magnitude of the force obeys Coulomb's inverse-square law. Thus, **the force between two electric charges is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them.** For like charges the force is repulsive while for unlike charges the force is attractive.

When charges are in motion, they produce magnetic effects. This magnetic field gives rise to a force on the moving charge. The intensity of the magnetic field get changed because of the motion of the charged particles. The combined effects of the electric and magnetic fields is, in general, inseparable. Hence the combined effect of the force is known as electromagnetic force. The electromagnetic force between two objects, also depends on the medium prevaling between them. Like the gravitational force, electromagnetic force is also a long range force and does not require any medium. It is very strong compared to gravitiatonal force. The electric force between two stationary protons, for example, is 10^{36} times the gravitational force between them, for any given distance.

The effect of the electromagnetic force is seen in the lightning in the sky, electric bell, etc.

1.4.3 Strong Nuclear Force

We know that the nucleus is made up of protons and neutrons. Proton is a positively charged particle while neutron is a chargeless particle. If we think as per Coulomb's law then repulsive force would act between like charged particles proton-proton and then the nucleus would become unstable. This indicates that within the nucleus there must exist a strong attractive force to bind the proton and the neutron together. The charge independent force acting between proton-proton, proton-neutron and neutron-neutron within the nucleus is known as strong nuclear force. This force is 100 times stronger than the electomagnetic force. As this force exists within the nucleus, it is a short range (10^{-15} m) force. The strong nuclear force is the strongest of all fundamental forces.

Neutrons and protons are thought of as being made of a fundamental particle 'quark'. Hence, according to recent research, this force is believed to be a quark-quark force.

Dear students, please note here that since the electrons are outside the nucleus, the strong nuclear force does not act on them.

1.4.4. Weak Nuclear Force

The weak nuclear force appears only in certain nuclear processes such as the β-decay of a radioactive nucleus. In β-decay, the nucleus emits an electron and an chargeless particle called neutrino. The weak nuclear force arises due to the interaction of neutrino with other particles. This force is stronger than the gravitational force, but much weaker than the strong nuclear force and the electromagnetic force. The range of weak nuclear force is also of the order of 10^{-15} m.

1.4.5 Towards Unification of Forces

In table 1.5, the fundamental forces of nature, their range, and their relative strength, are shown. Since so many years, physicists are thinking on a question that - Do all these fundamental forces arises from a single force ?

Can all the fundamental forces be explained from the hypothesis of a single force ?

Attempts to explore these ideas have opened the door towards unification of forces.

Table 1.5 : Fundamental Forces in Nature

Name	Relative Strength	Range	Operating Among
Gravitational Force	10^{-38}	Infinite	All objects in the Universe
Weak Nuclear Force	10^{-13}	Very short (within the nucleus 10^{-15} m)	Elementary Particles (neutrino)
Electromagnetic Force	10^{-2}	Infinite	Charged Particles
Strong Nuclear Force	1	Very short (within nucleus 10^{-15} m)	Nucleons (Neutron and Proton)

Newton has unified terrestrial and celestial domains under a common law of gravitation.

Oersted and Faraday showed that electric and magnetic phenomena are, in general, inseparable.

Maxwell unified electromagnetism and optics with the discovery that light is an electromagnetic wave.

Einstein attempted to unify gravitational force and electromagnetic force but could not succeed in this.

Glashow, Salam and Weinberg showed that the weak nuclear force and the electromagnetic force can be viewed as a different aspects of a single electro-weak force.

Efforts towards the unification of fundamental forces are still going on. Table 1.6 highlights the attempts towards unification of fundamental forces in nature.

Table 1.6 : Progress in Unification of Different Forces (only for information)

Physicist	Year	Achivement in Unification
Isaac Newton	1687	Unified celestial and terrestrial mechanics. Showed that the same laws of motion and the law of gravitation apply to both the domains.
Hans Christian Oersted Michael Faraday	1820 1830	Electric and magnetic phenomena are inseparable aspects of a Unified domain - electromagnetism.
James Clerk Maxwell	1873	Unified electricity, magnetism and optics by showing light is an electromagnetic wave.
Sheldon Glashow Abdus Salam Steven Weinberg	1979	Showed that the weak nuclear force and the electron agnetic force could be viewed as different aspects of a single electro-weak force.
Carlo Rubia Simon Vander Meer	1984	Experimental verification of the theory of electro-weak force.

1.5 Nature of Physical Laws

Dear students, in any physical phenomenon governed by different forces, several physical quantities may change with time while some special physical quantities remain constant with time. The physical quantities that remain unchanged with time are called conserved quantities. In other words conservation of some physical quantity means that the quantity does not change with time.

Laws of conservation of energy, charge, linear momentum and angular momentum are considered as fundamental laws of physics. The laws of conservation play an important and basic role in physics. These laws are as under :

Law of Conservation of Energy :

The amount of total energy in the universe remains constant. Energy can neither be created nor be destroyed; it can just be converted from one form to the other.

Law of Conservation of Charge :

During any process taking place in an electrically isolated system, the algebraic sum of the charges always remains constant.

Law of Conservation of Linear Momentum :

If the resultant external force on a system is zero, the total linear momentum of the system remains constant.

Law of Conservation of Angular Momentum :

If the resultant external torque acting on a system is zero, the total angular momentum of the system remains constant.

In future you will study these laws in detail. Student friends, in addition to these four laws we have laws of conservation for spin, baryon number, strangeness, hyper charge, etc., in nuclear and particle physics. We will not study these over here.

Now the obvious question is which tacit form of nature is responsible for the existance of such laws of conservation ?

In Physics, studies is carried out of space and time. In classical mechanics space and time are considered independent of each other while according to the theory of relativity given by Einstein, space and time are interrelated. Space is homogeneous and isotropic, as a result of this, we have the law of conservation of linear momentum and the law of conservation of angular momentum. Likewise, time is also homogeneous and isotropic. Because of homogeneity of time we have the law of conservation of energy. But till today physicists are unable to know, what will be the possible result due to isotropicity of time. The great theoretical physicist of 20^{th} century, Dirac was of the opinion that, the law of conservation of charge may be due to isotropic nature of time.

In other words, we know the basic reasons behind the existence of laws of conservation of linear momentum, angular momentum and energy. But the physicists are still putting their efforts to reveal the underlying mystery of nature in the laws of conservation of charge.

Dear students, the doors are still open for you to resolve many such interesting unsolved problems in physics !

SUMMARY

1. The word 'Bhautik Vigyan' (used in Hindi / Gujarati) related to the science of physical world is derived from Sanskrit word 'Bhautiki'.
2. English word 'physics' comes from a Greek word meaning 'nature'.
3. Physics deals with the study of basic laws of nature related to matter, energy and their manifestation in different phenomena.
4. The scope of the physics is extended on two basic domains : macroscopic to microscopic. It also deals with static and dynamic systems.
5. The basic laws of physics are universal and apply in widely different contexts and conditions.
6. The gravitational force, electromagnetic force, strong nuclear force and weak nuclear force are the four fundamental forces in nature. The attempts towards the unification of forces are going on.
7. The physical quantity that remains unchanged with time is called conserved quantity.
8. The laws of conservation of energy, charge, linear momentum and angular momentum are considered as the fundamental laws of physics.

EXERCISES

Choose the correct option from the given options :

1. .... are two fundamental constituents of universe.
(A) Matter and radiation (B) Heat and light
(C) Molecule and atom (D) Electron and proton.

2. .... is the fourth state of matter.
(A) Solid (B) Liquid
(C) Gas (D) Plasma

3. Nucleus of molecule is made up of which fundamental constituents ?
(A) Electron and proton (B) Electron and neutron
(C) Proton and neutron (D) only electron

4. What is a acronym of ECG ?
(A) Electron cardiogram (B) Electron colour graph
(C) Electro cardiograph (D) Electric cordiogram

5. What is full form of NMR ?
(A) Neutron Magnetic Resonance (B) Nuclear Magnetic Resonance
(C) Neutrino Magnetic Resonance (D) Nuclear Motion Resonance

6. What is full form of ESR ?
(A) Electric Spin Resonance (B) Electron Spin Resonance
(C) Electron Spin Radar (D) Electric Space Radar

7. The force exerting between neutron and proton within the nucleus is the
(A) Gravitational force (B) Electromagnetic force
(C) Strong nuclear force (D) Weak nuclear force

8. Which particles are emitted during the β-decay from the nucleus ?
(A) Neutron and proton (B) Electron and proton
(C) Electron and neutron (D) Electron and neutrino

9. Space is isotropic. Which law of conservation is the result of this ?
(A) Law of conservation of energy
(B) Law of conservation of charge
(C) Law of conservation of linear momentum
(D) Law of conservation of angular momentum

10. Space is homogenous. Which law of conservation is the result of this ?
(A) Law of conservation of energy.
(B) Law of conservation of charge
(C) Law of conservation of linear momentum.
(D) Law of conservation of angular momentum

11. Time is homogeneous. Which law of conservation is the result of this ?
(A) Law of conservation of energy
(B) Law of conservation of charge
(C) Law of conservation of linear momentum
(D) Law of conservation of angular momentum

12. The basic reason behind existance of which conservation of law is still not known ?
(A) Law of conservation of energy.
(B) Law of conservation of charge.
(C) Law of conservation of linear momentum.
(D) Law of conservation of angular momentum.

ANSWERS

1. (A)	2. (D)	3. (C)	4. (C)	5. (B)	6. (B)
7. (C)	8. (D)	9. (D)	10. (C)	11. (A)	12. (B)

Answer the following questions :

1. Which forces are considered as the fundamental forces in nature ?
2. What is meant by unification of forces ?
3. Write the law of conservation of energy.
4. Write the law of conservation of charge.
5. Write the law of conservation of linear momentum.
6. Write the law of conservation of angular momentum.
7. Which are the fundamental laws in physics ?
8. Which two fundamental forces are different aspects of electroweak interaction ?
9. What is plasma ?
10. What is cryogenics ?
11. Explain gravitational force ?
12. What is electromagnetic force ?
13. Explain strong nuclear force ?
14. What is weak nuclear force ?

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CHAPTER 2

MEASUREMENT AND SYSTEM OF UNITS

2.1 Introduction

We observe many phenomena occurring in our surrounding; some are natural and some are man-made. To describe any phenomenon, measurement of different physical quantities associated with it are essential. Let us consider a fruit falling from a tree. To understand this natural phenomenon, we should know, from which height does the fruit fall ? How much time does it take to reach the ground ? What is the speed of fall of the fruit ? To answer all these questions, we need to measure physical quantities like distance, time, mass etc. accurately. For measurement of any physical quantity, we require to decide their appropriate units. In this chapter, we shall study how the physical quantities are measured and how different units are defined. We shall also learn the different types of errors associated in the measurement of physical quantities.

2.2 What should be the Unit of a Physical Quantity ?

The standard measure of any quantity is called a unit of that physical quantity.

(1) The measure of a unit should be definite and unambiguous.

(2) The unit should be such that its measure should not change and if a unit is defined with the help of some phenomenon, that phenomenon must be permanent.

(3) The prototype (replica) of a unit should be easily reproducible and easily available.

2.3 Units of Physical Quantities and Systems of Units

Although the number of physical quantities is very large, we need **only a minimum limited** number of physical quantities for which units should be the units of all other quantities can be expressed defined and with their help of them. These minimum physical quantities are known as **fundamental quantities** and their units are called **fundamental or base units.** The other physical quantities can be expressed as a combination of fundamental quantities. Such physical quantities are called **derived physical quantities** and their units are called **derived units.** Different systems of units have come into existence at different places and times. These systems are as under :

(1) (FPS) system (British System) (foot, pound, second system)

(2) CGS system (centimetre, gram, second system)

(3) MKS system (metre, kilogram, second system)

(4) MKSA system (metre, kilogram, second, ampere system)

(5) SI system. (seven base units)

2.4 SI System of Units (Systeme Internationale)

The International System of units was accepted in 1971 by the 14^{th} General conference on Weights and Measures under the leadership of International Bureau of Weights and Measures, located at Paris in France. Seven quantities are accepted as fundamental quantities. These fundamental quantities, their units, symbols and definition are shown in Table 2.1.

Table 2.1 : SI Units

Base quantity	Name	Symbol	Definition
Length	metre	m	The metre is the length of the path travelled by light in vacuum during a time interval of 1/299,792,458 of a second. (1983)
Mass	kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at international Bureau of Weights and Measures, at Sevres, near Paris, France. (1889)
Time	second	s	The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom. (1967)
Electric current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length (1948)
Thermo dynamic Temperature	kelvin	K	The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)
Amount of substance	mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon - 12. (1971)
Luminous intensity	candela	cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian. (1979)

Note : (1) The definitions given in above table are only for information.

(2) When mole is used, the elementary entities must be specified. For example, mol of atoms, mol of molecules, mol of ions or mol of electrons.

2.4.1 Derived Units

All the other units of different physical quantities can be expressed as a combination of these seven base (fundamental) units of SI system. Such units are called derived units.

For example, SI unit of acceleration,

$$= \frac{\text{Unit of displacement}}{(\text{Unit of time})^2} = \frac{m}{s^2} = m\ s^{-2}$$

unit of work = (unit of force) × (unit of displacement)

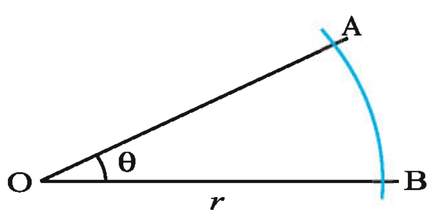
$$= \frac{kg\ m}{s^2} \times (m) = kg\ m^2\ s^{-2}$$

2.4.2 Supplementary Units

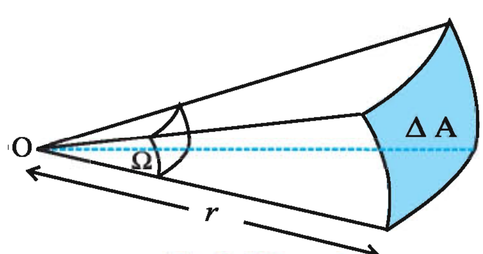
Supplementary physical quantities, their units and symbols in the SI system are shown in Table 2.2.

Table 2.2 : Supplementary Units

No.	Supplementary Physical quantity	SI unit	Symbol	Explanation
1.	Plane angle (θ)	radian	rad	The ratio of length of arc to the radius r of a circle is called plane angle (θ). $\theta = \frac{\text{arc}}{\text{radius}} = \frac{AB}{r}$ (see figure 2.1) one radian is defined as the angle subtended by an arc whose length is equal to the radius. ($1^\circ = \pi/180$ rad)
2.	Solid angle (Ω)	Steradian	Sr	The ratio of the intercepted area of the spherical surface described about the apex O as the centre to the square of radius r is called solid angle. (see fig. 2.1 (b)). $\Omega = \frac{\text{Area } (\Delta A)}{(\text{radius})^2} = \frac{\Delta A}{r^2}$ when $\Delta A = 1 m^2$, $r = 1 m$ then $\Omega = 1$ steradian



(a) Plane angle



(b) Solid angle

Figure 2.1

2.4.3 Practical norms for the use of SI system

(1) Unit of every physical quantity should be represented according to its symbol.

(2) No full stop should be used within or at the end of the symbol of a unit. For example, for kilogram, kg should be written instead of kg. or k.g.

(3) Symbols for units do not take plural form. For example m is used to denote many meters also.

(4) The units of physical quantities in numerator and denominator should be written as one ratio only. For example the SI unit of acceleration should be written either as m/s^2 or $m\ s^{-2}$; but not as m/s/s.

(5) Full name of a unit, when it is named after a scientist, is not written with a capital letter; but the symbol for that unit has a capital letter. For example, the unit of force should be written as newton but in symbol it is written as N. The unit of pressure is pascal in symbol it is written as Pa.

2.5 Measurement of Length

You are already familiar with some direct methods for the measurement of length. For example, a metre scale is used for measuring lengths from 10^{-3} m to 10^{2} m. A vernier callipers is used for measuring lengths to an accuracy of 10^{-4} m. A screw gauge and a spherometer can be used to measure lengths down to 10^{-5} m.

To measure the lengths beyond these ranges, and for astronomical distances, we make use of some indirect methods. Now, we shall study a few such methods.

2.5.1 Measurement of large distances : Parallax Method

Large distances such as the distance of a planet or a star from the Earth can be measured with the help of parallax method.

To measure the distance D of a far away planet, we observe it from two different positions (observatories) A and B on the Earth and fix the directions of observations with respect to distant stars.

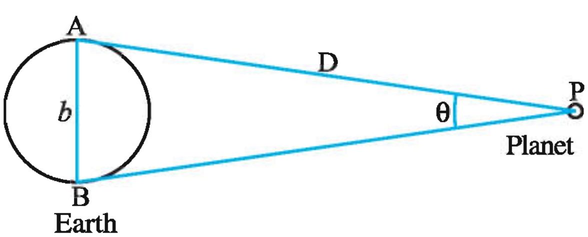


Figure 2.2

For example, observing a planet P simultaneously from two points A and B, situated diametrically opposite on Earth, we get two directions of observations AP and BP.

Since the distance of a planet from the Earth is very large compared to the diameter of the Earth angle θ will be very small (angle θ is called parallax angle). According to the definition of angle in radian,

$$\theta = \frac{\text{arc}}{\text{radius}} = \frac{AB}{AP}$$

$$= \frac{\text{distance between two places of observations, b}}{\text{distance of the planet from the Earth, D}}$$

$$\therefore D = \frac{b}{\theta} \qquad (2.5.1)$$

Illustration 1 : The Moon is observed from two diametrically opposite points A and B on the Earth. The angle subtended at the Moon by the two directions of observations is $1°54'$. Given the diameter of Earth to be about 1.276×10^7m, compute the distance of the Moon from the Earth.

Solution : $D = \frac{b}{\theta}$

$$\theta = 1°54' = 60' + 54' = 114'$$

$$= \frac{114'}{60} \text{ degree}$$

$$= \frac{114}{60} \times \frac{\pi}{180} \text{ rad}$$

$$\therefore \theta = 3.32 \times 10^{-2} \text{ rad}$$

$$b = 1.276 \times 10^7 \text{ m}$$

$$\therefore D = \frac{1.276 \times 10^7}{3.32 \times 10^{-2}}$$

$$= 3.84 \times 10^8 \text{m}$$

2.5.2 Measurement of the size of a planet or a star

If d is the diameter of a planet, the angle subtended by the diameter of the planet at any point on Earth is called the angular diameter of the planet. The angle α can be determined from any given location on Earth by viewing the diametrically opposite points of the planet through a telescope.

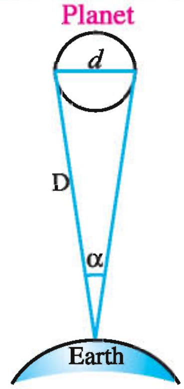


Figure 2.3

$$\alpha = \frac{d}{D} \text{ (in rad)} \qquad (2.5.2)$$

* **Footnote :** 1° (degree) = 60′ (minute) = 3600″ (second)

If the distance D of that planet from the Earth is known, the diameter d of the planet can be determined using equation. (2.5.2)

In practice the angle α is very small.

Illustration 2 : The Sun's angular diameter is measured to be 1920″ as the distance D of the Sun from the Earth is 1.496 $\times 10^{11}$ m. What is the diameter of the Sun ?

($1'' = 4.85 \times 10^{-6}$ rad)

Solution : $\alpha = 1920''$, $D = 1.496 \times 10^{11}$ m

Using the formula $\alpha = \frac{d}{D}$,

$$d = \alpha D$$
$$= (1920)(4.85)(10^{-6})(1.496 \times 10^{11})$$
$$= 1.393 \times 10^{9} \text{ m}$$

2.5.3 Measurement of very small distances, size of molecule

To measure very small distances like the size of a molecule (10^{-8} m to 10^{-10} m), we cannot use a vernier callipers or micrometre screw gauge or similar instruments. We have to adopt special methods. An optical microscope uses visible light. For visible light the range of wavelength is from 4000 Å to 7000 Å (1 Å = 10^{-10} m). Hence an optical microscope cannot resolve particles with sizes smaller than 4000 Å. The electron microscope, uses electron beam instead of visible light. An electron microscope has resolution of 0.6 Å. It can almost resolve atoms and molecules in a material. (Here, you will be surprised to know that, in electron microscope, electron behaves like a wave, instead of a particle.) In recent times, tunneling microscope has been developed during study of nanotechnology. This has very high resolution needed to estimate the sizes of molecules.

One of the methods of finding the size of a molecule is the method of monomolecular layer. In this method, the thickness of a molecular layer is measured to determine the size of a molecule. For example, the thickness of a layer of steric acid cannot be less than a certain definite value. If we assume that the film has one molecular thickness, this becomes the size or diameter of a molecule.

In physics we deal with very small distances as well as with very large distances. e. g. the size of nucleus of the order of 10^{-14}m and size of galaxy is of the order of 10^{21}m. Therefore, we defined special units of length for short and large distances. These are,

1 fermi = 1 fm = 10^{-15} m

1 angstrom = 1 Å = 10^{-10} m

1 astronomical unit = 1 AU = 1.496×10^{11} m

(Average distance of the Sun from the Earth is called 1 AU.)

1 light year = 1 ly = 9.46×10^{15} m

1 parsec = 3.08×10^{16}

A parsec (pc) is the distance at which 1 AU would subtend on angle of exactly 1 second of arc.

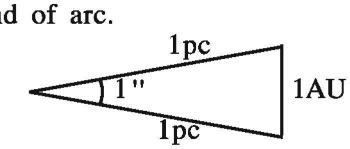


Figure 2.4

$$r = \frac{l}{\theta} = \frac{1 \text{ AU}}{1''}$$
$$= \frac{1.496 \times 10^{11}}{60 \times 60 \times \frac{\pi}{180}} = 3.08 \times 10^{16}\text{m}$$

$\therefore$ 1 pc $= 3.08 \times 10^{16}$m

2.6 Measurement of Mass

The amount of matter in the substance is called mass. Since mass is a internal property of matter, it does not depend on external circumstances like temprature and pressure.

The measurement of mass can be done with the help of a simple balance. In this method, the gravitational force on a given object is compared with the gravitational force on some standard object. Remember that the mass playing role in gravitational force (mg) is called gravitational mass. Hence, the mass determined with the help of a simple balance is gravitational mass. The gravitational mass of an object is the same at all places on Earth.

The gravitational force (mg) on an object, of mass m, is called the weight of the object. Hence we can say that the weight of a body at any place depends on the value of gravitational acceleration of that place. For example, the weight of a body on Moon would be different than that on the Earth.

While dealing with atoms and molecules, the kilogram is an inconvenient unit. Therefore, their mass is measured in 'atomic mass unit' called amu. The $\frac{1}{12}^{th}$ mass of an unexcited atom of C^{12} is called 1 amu. 1 amu = 1.66×10^{-27} kg. It can also be represented as 1u. In physics we come across very wide range of masses from 10^{-30} kg to 10^{55} kg.

Large masses like planets, stars etc. can be determined from Newton's law of gravitation. For measurement of small masses (like atomic particles/atom) we make use of mass spectrograph. (In this method, radius of the trajectory of a charge particle in electric field or magnetic field is proportional to the mass of the charge particle.)

2.7 Measurement of Time

In the early days, time was measured from the length of shadows of objects cast by sunlight. After the invention of the pendulum there is much developement has taken place in the measurement of time. To measure any time interval we need a clock. In order to meet the need for a better standard for time, atomic clocks have been developed. For the measurement of small time intervals, camera, multiflash photography etc. are being used.

2.8 Accuracy and Precision in Measurement

First of all, we shall distinguish between two terms : **accuracy and precision. The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us to what resolution or limit the quantity is measured.**

For example, your digital watch that shows the time as 10 : 11 : 12 AM is very precise because it has least count of 1 second. On other hand, a grand father clock has no second hand and it gives the time as 10 : 13 AM. The least count of watch is 1 minute which means that it is less precise. If the digital watch runs several minutes slow, then time measured with this watch has no accuracy but it is precise. If the grand father watch shows the correct time then time measured with this watch has high accuracy but less precision. Physical quantity should be measured with high accuracy and high precision. Precision depend upon the least count of instrument. Radius of a sphere measured with micrometer screw gauge will be more precise than measured with verniar callipers.

In the measurement of a physical quantity with accuracy.

(1) Skill of the person doing the experiment

(2) Quality of the instrument used

(3) The Method used for measurement

(4) External and internal factors affecting the result of the experiment.

2.9 Errors in Measurement

When different physical quantities are measured in a laboratory with the help of different apparatus, there would be some inaccuracies in the measurement which must be mentioned along with the result. **The inaccuracy in measurement is called error.**

In physics, the errors in measurement can be broadly classified as :

(1) Systematic error

(2) Random error.

(1) Systematic Error : Systematic errors are those errors that tend to be in one direction, either positive or negative. Such errors cannot be both, positive and negative simultaneously. Some of the sources of systematic errors are as follow :

(a) Instrumental Error : This type of errors arise due to imperfect design or improper calibration of the measuring instrument. For example, when no object is suspended from a spring balance, its pointer shows 0.1g instead of zero. Then all the measurements of more than true wieght will systematically contain this error during experiment.

(b) Error due to Imperfection in Experimental Technique or Procedure : For example, while measuring the temperature of a human body, any improper contact of thermometer with the body would produce an error in the measurement. External factors like temperature, pressure humidity can produce systematic error in measurement.

(c) Personal Error : Such an error arises

arises due to an individual's bias, carelessness in taking observations or improper setting of the apparatus of the experiment.

Systematic errors can be minimised by improving experimental techniques, selecting high quality instruments and removing personal bias.

(2) Random Errors : The random errors are those errors, which arise due to irregular and unpredictable fluctuations in the factors affecting the measurement during experiment.

These types of errors can be both positive and negative. Such errors can be estimated by taking many observations and then taking their mean (average).

2.9.1 Estimation of Errors :

(1) Absolute Error and Average Absolute Error : The magnitude of the difference between an individual measurement and the true value of the quantity is called the **absolute error** of the measurement.

If we do not know the true value, then the average value of measurement is considered as true value.

Suppose the values obtained in several measurement of physical quantity a are a_1, a_2, a_3,...., a_n. If their arithmetic mean is $\overline{a}$, then

$$\overline{a} = \frac{a_1 + a_2 + a_3 + ... + a_n}{n}$$

$$\overline{a} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

The absolute error in individual measurement will be,

$$\Delta a_1 = \overline{a} - a_1$$

$$\Delta a_2 = \overline{a} - a_2$$

...

...

$$\Delta a_n = \overline{a} - a_n$$

Here, Δa_1, Δa_2.... Δa_n are called absolute errors in individual measurement which can be either positive or negative. The arithmetics mean average of absolute error is called **average absolute error.**

$$\Delta\overline{a} = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n} \quad \text{or}$$

$$\Delta\overline{a} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$

Thus, the measure of any physical quantity can represented as :

$$a = \overline{a} \pm \Delta\overline{a}$$

This implies that any measurement of physical quantity a is likely to lie between,

$(\overline{a} + \Delta\overline{a})$ and $(\overline{a} - \Delta\overline{a})$

(2) Relative or Fractional Error : The relative error $(\delta\overline{a})$ is the ratio of the mean absolute error $\Delta\overline{a}$ to the mean value $\overline{a}$ of the quantity measured.

$$\therefore \delta a = \frac{\Delta\overline{a}}{\overline{a}}$$

(3) Percentage Error : When the relative error is expressed as a percent age it is called percentage error.

$$\text{Percentage error} = \delta a \times 100\ \%$$

$$= \frac{\Delta\overline{a}}{\overline{a}} \times 100\ \%$$

Illustration 3 : In an experiment, refractive index of glass was observed to be 1·54, 1·53, 1.44, 1.54, 1.56 and 1.45. Calculate (1) Average absolute error (2) relative error and (3) percentage error. Express the result in terms of absolute error and percentage error.

Solution :

(1) Mean refractive index,

$$\overline{n} = \frac{1{\cdot}54 + 1{\cdot}53 + 1{\cdot}44 + 1{\cdot}54 + 1{\cdot}56 + 1{\cdot}45}{6}$$

$$= 1{\cdot}51$$

Here, an accuracy of two digit after decimal point has been considered. Now the absolute error for each observation will be as follow.

$\Delta n_1 = 1{\cdot}51 - 1{\cdot}54 = -0{\cdot}03$	$\Delta n_4 = 1{\cdot}51 - 1{\cdot}54 = -0{\cdot}03$
$\Delta n_2 = 1{\cdot}51 - 1{\cdot}53 = -0{\cdot}02$	$\Delta n_5 = 1{\cdot}51 - 1{\cdot}56 = -0{\cdot}05$
$\Delta n_3 = 1{\cdot}51 - 1{\cdot}44 = +0{\cdot}07$	$\Delta n_6 = 1{\cdot}51 - 1{\cdot}45 = +0{\cdot}06$

To calculate mean absolute error we take only magnitudes.

$$\Delta\overline{n} = \frac{|\Delta n_1| + |\Delta n_2| + ... + |\Delta n_6|}{6}$$

$$= \frac{|-0{\cdot}03| + |-0{\cdot}02| + |+0{\cdot}07| + |-0{\cdot}03| + |-0{\cdot}05| + |+0{\cdot}06|}{6}$$

$$\Delta\overline{n} = \frac{0{\cdot}26}{6} = 0{\cdot}043 \approx 0{\cdot}04$$

Refractive index of glass with absolute error, $n = 1{\cdot}51 \pm 0{\cdot}04$ i.e. the value of refractive index is between 1.51 and 1.47

(2) Relative error $= \dfrac{\Delta \bar{n}}{\bar{n}} = \dfrac{0{\cdot}04}{1{\cdot}51}$

$= 0{\cdot}02649 = 0{\cdot}03$

(3) Percentage error $= 0{\cdot}03 \times 100 = 3\%$

Refractive index of glass with percentage error n = $1{\cdot}55 \pm 3\%$

2.9.2 Combination of Errors

When we do an experiment involving several measurements, we must know how the errors in all the measurements combine. For example, in an experiment to determine the density of any substance, we measure the mass and volume of the substance and there would be errors in each of these two measurements. Then we must know what the error will be in the density of the substance.

(1) Errors in Sum and in Difference : Suppose two physical quantities A and B have measured values. $A \pm \Delta A$ and $B \pm \Delta B$ respectively, where ΔA and ΔB are their absolute errors. We wish to find the absolute error ΔZ in the sum

We have by addition, $Z = A + B$

$\therefore\ Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$

$= (A + B) \pm (\Delta A + \Delta B)$

$\therefore$ The maximum possible absolute error in Z, $\Delta Z = \Delta A + \Delta B$

For the difference,

$Z = A - B$, we have

$\therefore\ Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$

$= (A - B) \pm \Delta A \mp \Delta B$

$\therefore\quad \pm\Delta Z = \pm \Delta A \mp \Delta B$

Here, there are four possible values $(+ \Delta A - \Delta B)$, $(+ \Delta A + \Delta B)$, $(- \Delta A - \Delta B)$, $(- \Delta A + \Delta B)$ in which $(+ \Delta A + \Delta B)$ is the maximum value. thus, the maximum value of absolute error in Z is again $\Delta A + \Delta B$.

Hence the rule : When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

Illustration 4 : Two resistors of $R_1 = 100 \pm 3\Omega$ and $R_2 = 200 \pm 4\Omega$ are connected in series. Find the maximum absolute error in the equivalent resistance of the combination. Express equivalent resistance with percentage error.

Solution :

$R \pm \Delta R = R_1 + R_2$

$= (100 \pm 3) + (200 \pm 4)$

$= 300 \pm 7\Omega$

$\therefore$ Maximum absolute error $= 7\Omega$

Now, percentage error $= \dfrac{\Delta R}{R} \times 100$

$= \dfrac{7}{300} \times 100$

$= 2.3\ \%$

$\therefore$ Equivalent resistance with percentage error $R = 300 \pm 2.3\ \%$

(2) Errors in product and in division :

Suppose $Z = AB$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. Then

$Z \pm \Delta Z = (A \pm \Delta A)\ (B \pm \Delta B)$

$= AB \pm A\Delta B \pm B\Delta A \pm \Delta A\Delta B$

Dividing LHS by Z and RHS by AB we have,

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \cdot \frac{\Delta B}{B}$$

Since $\dfrac{\Delta A}{A}$ and $\dfrac{\Delta B}{B}$ are very small, we shall ignore their product. Hence, the maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

You can easily verify that this is true for division also.

Hence the rule : When two quantities are multiplied or divided, the maximum relative or fractional error in the result is the sum of the fractional errors in each quantity.

Illustration 5 : In an experiment to determine density of an object mass and volume are recorded as, m = (3 ± 0·12) kg and V = (10 ± 1) m^3 respectively. Calculate fractional error and percentage error in mesurement of density. ($\rho = m/V$)

Solution : $\rho = \frac{m}{V}$

Fractional error in density $\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta V}{V}$

$$= \frac{0.12}{3} + \frac{1}{10}$$

$$= 0.14$$

Percentage error = 0.14 × 100 = 14 %

(3) Error due to the power (index) of a measure quantity :

Suppose $Z = A^2 = A \cdot A$

Then, $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta A}{A}$

$$= 2\frac{\Delta A}{A}$$

Hence, the fractional error in $Z = A^2$, is two times the fractional error in A.

Same way, if $Z = A^n$ than

$$\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$$

In general, if $Z = \frac{A^p B^q}{C^r}$ than

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

Note : The quantity in the formula which has large power is responsible for maximum error. Therefore it should be measured with greater accuracy.

Illustration 6 : In an experiment to determine the density of a sphere, the percentage error in measurement of mass is 0.26 % and percentage error in measurement of radius is 0.38 %. What will be percentage error in the determination of its density ?

Solution : $\frac{\Delta m}{m} \times 100 = 0.26$ %;

$\frac{\Delta r}{r} \times 100 = 0.38$ %

Density of sphere $\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi r^3}$

$\therefore$ Error in density $\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 3\frac{\Delta r}{r}$

Percentage error in density = 0.26 % + 3 (0.38 %)

= 1.40 %

Illustration 7 : If the formula for a physical quantity is $W = \frac{a^4 b^3}{c^{\frac{1}{3}}\sqrt{d}}$ and if percentage errors in the measurement of *a*, *b*, *c* and *d* are 1%, 3%, 3% and 4% respectively. Calculate percentage error in W.

Solution : $W = \frac{a^4 b^3}{c^{\frac{1}{3}}\sqrt{d}}$

Percentage error in W,

$$\frac{\Delta W}{W} = 4\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + \frac{1}{3}\frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta d}{d}$$

$$= 4\ (1\ \%) + 3(3\ \%)$$

$$+ \frac{1}{3}(3\ \%) + \frac{1}{2}(4\ \%)$$

$$= 16\ \%$$

Illustration 8 : The period of oscillation of a simple pendulum is given by,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The length *l* of the pendulum is about 10cm and is known to 1mm accuracy. The period of oscillation is about 0.5s. The time of 100 oscillations is measured with a watch of 1s resolution. Calculate percentage error in measurement of *g*.

Solution : $T = 2\pi\sqrt{\frac{l}{g}}$ $\therefore T^2 = \frac{4\pi^2 l}{g}$

or, $g = \frac{4\pi^2 l}{T^2}$

$$\therefore \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

Now, Δl = 1mm = 0.1cm, l = 10cm

Total time $t = n$T = 0.5 × 100 = 50 s and Δt = 1 s

Now, $T = \frac{t}{n}$ and $\Delta T = \frac{\Delta t}{n}$ hence,

$$\frac{\Delta T}{T} = \frac{\Delta t}{t}$$

$$\therefore \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta t}{t}$$

$$\therefore \frac{\Delta g}{g} = \frac{0.1}{10} + 2 \times \frac{1}{50} = 0.05$$

∴ Percentage error in measurement of g = 0.05 × 100 = 5 %.

2.10 Significant Figures

The accuracy of every measurement has some limitation depending on the least count of the instrument used. For example, time measured with the help of a watch having a second hand can measure the time up to accuracy of 1 second.

Suppose you are measuring the length of a pencil. Keep one end of the pencil on the zero scale of metre and suppose its other end lies between 12.3cm and 12.4cm. The least count of metre scale is 0.1cm. Therefore it does not have any marking between 12.3cm and 12.4cm. Hence we estimate the length as 12.38cm. Here we are centain about digit 1, 2 and 3 but are uncertain for the last digit 8.

The number of digit in a measurement about which we are certain plus one additional digit which is uncertain are known as **significant figures.**

In the above example, 12.38cm has four significant digits 1, 2, 3 and 8.

The larger the number of significant figures obtained in a measurement, the greater is the accuracy of measurement. The number of significant digits depends upon the least count of the instrument being used for a measurement. For exmaple, the radius of the rod measured with vernier callipers is r = 0.25cm. For the same rod the radius measured with micrometre screw gauge should be 0.254cm. In the first case, there are two significant digit (2 and 5). While in second case the number of significant digits are three (2, 5 and 4), which shows that the second measurement is more precise.

In mathematics, all the numbers are definite figures. The question of significance arises only when number represents the measured value of a physical quantity.

2.10.1 Rules for Determining Number of Significant Figures :

(1) All the non–zero digits are significant. For example, in measurement of mass 125.63g, there are five significant digits which are 1, 2, 5, 6 and 3.

(2) All the zeros between two non–zero digits are significant, no matter where the decimal point is, if at all.

e.g. in 125.004cm there are six significant figures.

(3) If the number is less than 1, the zero (s) on the right of decimal point but to the left of the first non–zero digit are not significant. e.g., In <u>0.0</u> <u>0</u>1507, the underline zeros are not significant. There are four significant figures in this number.

(4) In a number without decimal point, the zeros on the right side of the last non–zero digit are not significant.

e.g. 132m = 13200cm = 132000mm has three (1, 3 and 2) significant digits. Here there is no change in the number of significant digits because the zeros after decimal point indicate the position value only. Thus by changing the units even if number of zero increases but the number of significant digits does not change.

(5) The trailing zero (s) in a number with a decimal point are significant.

e.g. <u>7.900</u> and 0.0 <u>7</u> <u>9</u> <u>0</u> <u>0</u> have four significant figures each.

Illustration 9 : Write down the number of significant figures/digits in the following :

(1) 0.003 m^2

(2) 0.1570 g cm^{-2}

(3) 2.64 × 10^{24} kg

(4) 7.590 J

(5) 6.032 N m^{-2}

(6) 3.012 × 10^{-4} m^2

Solution :

(1) 0.003 m^2 has only one significant figure (3).

(2) 0.1570 g cm^{-2} has four significant figures. (1, 5 ,7 and 0)

(3) 2.64 × 10^{24} kg has three significant figures. (2, 6 and 4)

(4) 7.590 J has four significant figures. (7, 5, 9 and 0)

(5) 6.032 N m^{-2} has four significant figures. (6, 0, 3 and 2)

(6) 3.012 × 10^{-4} m^2 has four significant figures. (3, 0, 1 and 2)

2.10.2 Significant Figures in Addition, Subtraction, Multiplication and Division

Any physics experiment involves a series of measurement and each of these measurement is made upto a certain degree of accuracy. The number of the significant figures depend upon the least count of the measuring instrument. Suppose the resistance of different resistors are measured using meter (ohm meter) of different least count are :

$R_1 = 5.67\,\Omega$, $R_2 = 12.345\,\Omega$ and $R_3 = 0.7\,\Omega$

Here, total resistance would be,

$$R = 5.67\,\Omega + 12.345\,\Omega + 0.7\,\Omega = 18.715\,\Omega$$

Now, the questions is, are we justified in carrying out sum like this ? In the measurement of R_1 (=5.67 Ω) we do not have the information about the third digits after decimal point. In the measurement of R_3 we do not have information about second and third digits after decimal point, it has only one digit after the decimal point. This shows that precision in the measurement of R_3 less than other two resistors. Therefore, in the sum (i.e. 18.715 Ω) second and third digits become insignificant and answer should be expressed to one decimal place, as 18.7 Ω only.

Thus, any result with more than one insignificant or uncertain digit should be rounded off up to the correct number of significant digits. For this purpose the following rules should be observed :

(1) If the insignificant digit to be dropped is less than 5, then the preceding digit is left unchanged. For example, l = 10.7<u>43</u> cm is rounded off up to three significant digit as 10.7 cm.

(2) If the digit to be dropped is more than 5, then the preceding digit is increased by 1.

For example, l = 10.6<u>8</u> cm = 10.7cm (Rounded off up to three significant digit)

(3) If the digit to be dropped is 5 then the preceding digit is increased by one if it is odd and is left unchanged if it is even.

e.g. l = 10.4<u>5</u> cm = 10.4 cm and

l = 10.5<u>5</u> cm = 10.6 cm

Addition and Subtraction : The following points should be observed while carrying out the addition / subtraction of two significant numbers :

(1) If all the numbers are integers, the summation or subtraction should be carried out in a normal way.

(2) In addition or subtraction, the final result should retain as many decimal places as are there in number with the least decimal places. In above example, the resistance $R_3 = 0.7\,\Omega$ has only one significant digit after decimal place. The total resistance R = 18.715 Ω should therefore be rounded off to R = 18.7 Ω

Multiplication and Divisions : In any measurement the last digit is uncertain. Multiplication of any number with least significant digit is also insignificant. But the final result should contain only one insignificant digit. Therefore, the following points should be observed while carrying out the multiplication / division.

(1) In multiplication or in division, the final result should retain as many significant as are there in the original number with the least significant.

For example : (i) Suppose the length and breadth of a rectangular plate is 2.613 cm and 1.2 cm respectively.

Hence, area of the plate = 2.613 cm × 1.2 cm = 3.1356 cm^2

But 1.2 cm is the least significant number which has only two significant figures. Therefore the area (= 3.1356 cm^2) should be represented by a number with two significant digit.

Hence, 2.613 cm × 1.2 cm = 3.1 cm^2

(ii) Suppose, the mass of an object is m = 3.523 g and volume is V = 1.47 cm^3.

The density of an object $\rho = \frac{3.523g}{1.47cm^3}$

= 2.4296552 = 2.43 g cm^{-3}

Here, the density is respresented with three significant figures because the measurement of volume has three significant figures.

(2) When two numbers are to be multiplied / divided and out of them which is not a measurement is a definite number, the integers and fraction that occurs in general in physics equations are definite number.

e.g. In equation $v^2 - v_0^2 = 2ad$ the coefficient 2 is exactly 2 and it has infinite number of significant figures (2.000........). In such cases we should not consider the significant figures of definite numbers.

Illustration 10 : The diameter of a sphere is 4.24 cm. Calculate the surface area of the sphere to the correct number of significant figures.

Solution : Diameter D = 4.24 cm

Surface area of sphere

$$= 4\pi R^2 = 4\pi\left(\frac{D}{2}\right)^2$$

$$= 4 \times 3.14 \times \left(\frac{4.24}{2}\right)^2$$

$$= 56.478 \text{ cm}^2$$

$$= 56.5 \text{ cm}^2$$

(In the above equation 4 and 2 are definite number. We do not consider them and measure of D has three significant figures. So the answer is rounded to three significant figures.)

2.11 Dimensions and Dimensional Formulae

Any physical quantity (derived quantity) can be expressed in terms of some combination of seven fundamental or base quantities. For convenience the base quantities are represented by a letter symbol. Generally, mass is denoted by 'M', length by 'L', time by 'T' and electric current by 'A'. The thermodynamic temperature the amount of substance and the luminous intensity are denoted by the symbols 'K', 'mol' and 'cd' respectively. When a physical quantity is expressed with appropriate powers (or exponents) of M, L, T, K, A ... then such an expression for physical quantity is called the **dimensional formula.** The power or exponents of M, L, T, ... are called **dimensions** of that quantity. The dimensional formula of physical quantity is expressed by square brackets [ ] along with the symbol of a physical quantity.

For example, (*i*) the dimensional formula of velocity can be obtained as follows :

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\therefore [v] = \frac{\text{Dimension of length}}{\text{Dimension of Time}}$$

$$= \frac{L^1}{T^1}$$

$$= L^1 T^{-1}$$

$$= M^0 L^1 T^{-1}$$

Here, $M^0 L^1 T^{-1}$ is dimensional formula of velocity. The dimensions of velocity are 0 in mass, 1 in length and −1 in time.

(ii) Dimensions of kinetic energy can be obtained as follows :

$$K = \frac{1}{2}mv^2$$

$$[K] = [m] [v]^2$$

(Here, $\frac{1}{2}$ is a number and it is dimensionless)

$$= (M^1) (M^0 L^1 T^{-1})^2$$

$$[K] = M^1 L^2 T^{-2}$$

Dimensional formule of some physical quantities are given in Table 2.3.

2.11.1 Dimensional Analysis

The method of obtaining the solutions to some several problems in physics by using the formula is called **dimensional analysis.**

Uses of Dimensional Analysis :

(a) To obtain the relation between the units of some physical quantity in two different systems of units.

(b) To check the dimensional consistency of an equation connecting different physical quantities.

(c) To derive an equation for a physical quantity in terms of other (related) physical quantities.

(a) To obtain the relation between the units of a physical quantity in two different systems of units :

The unit of work in MKS system is joule (J) and that in CGS system is erg. The relation between joule and erg can be obtained as follows :

Dimensional formula for work :

$[W] = M^1 L^2 T^{-2}$

In MKS system		In CGS system
M(kg)	=	10^3 M (g)
L (m)	=	10^2 L (cm)
T (s)	=	10^0 T (s)

$$M^1L^2T^{-2} = (10^3M)^1 \ (10^2L)^2 \ (10^0T)^{-2}$$
$$= 10^3(M^1) \ 10^4 \ (L^2) \ (T^{-2})$$
$$= 10^7 \ M^1 \ L^2 \ T^{-2}$$

So, MKS Unit of work = 10^7 × CGS unit of work.

$\therefore$ 1 joule = 10^7 erg

(b) To verify the dimensional consistency of an equation connecting different physical quantities :

In any equation relating different physical quantities, if the dimensions of terms on both sides are same then that equation is said to be consistent dimensionally.

For exmaple, the centripetal force acting on an object in uniform circular motion is given by,

$F = \frac{mv^2}{r}$ where, m = mass of the object,

v = velocity of the object and

r = radius of the circular path.

Now, we will check the dimensional consistency of this equation,

For left side of the equation

$[F] = M^1 \ L^1 \ T^{-2}$

For the term on right side of equation,

$$\left[\frac{mv^2}{r}\right] = \frac{[m][v]^2}{r}$$
$$= \frac{(M^1)(L^1T^{-1})^2}{(L^1)}$$
$$= \frac{(M^1)(L^2T^{-2})}{(L^1)}$$
$$= M^1L^1T^{-2}$$

Thus, $[F] = \left[\frac{mv^2}{r}\right]$

Since the dimensions of LHS and RHS are same, the given equation is dimensionally correct.

Note : If an equation has constants which are dimensionless, it can not be verified with dimension analysis.

(c) To obtain the equation for a physical quantity in terms of an other physical quantities :

Suppose we want to obtain the expression for the periodic time of a simple pendulum. The periodic time (T) of simple pendulum depends on the length (l) of the pendulum, the mass (m) of the bob and gravitational acceleration (g).

Suppose, periodic time $T \propto m^a$

$\propto l^b$

$\propto g^c$

$T \propto m^a \ l^b \ g^c$

$$\therefore \ T = km^a \ l^b \ g^c \qquad (2.11.1)$$

where, k is constant of proportionality and it is dimensionless, $a, b, c \in R$

Writing the dimensional formula for both the sides of equation (2.11.1)

$$(M^0L^0T^1) = (M^1)^a \ (L^1)^b \ (M^0L^1T^{-2})^c$$
$$= (M^a) \ (L^b) \ (M^0L^cT^{-2c})$$
$$M^0L^0T^1 = M^a \ L^{b+c} \ T^{-2c}$$

Comparing the dimensions of corresponding quantities on both the sides, we get

$a = 0$ | $\therefore c = -\frac{1}{2}$

$b + c = 0$ | $\therefore b = \frac{1}{2}$

$-2c = 1$

Substituting these values of a, b and c in equation (2.11.1)

$$T = km^0 l^{\frac{1}{2}} g^{-\frac{1}{2}}$$

or $T = k\sqrt{\frac{l}{g}}$

The value of k is obtained experimentally and it is 2π. Hence,

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (2.11.2)$$

which is the formula for periodic time of a simple pendulum.

2.11.2 Limitations of dimensional analysis

(1) In any dimensional formula containing M, L and T, we get at the most three equations by equating power of M, L and T. Hence dimensional analysis cannot be used to derive the exact form of a physical relation if a physical quantity depends upon more than three physical quantities.

(2) Information about dimensionless constant cannot be obtained. e.g. In $T = k\sqrt{\frac{l}{g}}$, the value of $k = 2\pi$ can be obtained with the help of experiment.

(3) Dimensional analysis can not be used to derive relations involving trigonometrical, exponential and logarithmic functions. Such functions are dimensionless. For example, in $\sin\omega t$ and e^{-kx} functions ωt and kx are dimensionless respectively.

(4) This method is not useful if a constant of proportionality is not dimensionless. For example, in the equation $F = G\frac{m_1 m_2}{r^2}$, G has a unit of N m^2 kg^{-2}. Such equations can not be obtained by dimensional analysis.

Illustration 11 : If the velocity of light is taken as the unit of velocity and year as the unit of time, then find the unit of distance. (velocity of light = 3×10^8 m s^{-1})

Solution :

Distance = velocity × time

Unit of distance = unit of velocity × unit of time

$= (3 \times 10^8 \text{ m s}^{-1}) \times (1 \text{ year})$

$= (3 \times 10^8 \text{ m s}^{-1}) \times (365.25 \times 24 \times 3600\text{s})$

$= 9.468 \times 10^{15}$ m

This unit of distance is called **light year.**

Illustration 12 : In a new system, the unit of length, mass and time are chosen to be 10cm, 10g and 0.1s respectively. What will be the new unit of force in newton in this system ?

Solution :

Dimensional formula of force [F]

$= M^1L^1T^{-2}$

Unit of force in new system

$= [(10g)^1 \ (10cm)^1 \ (0.1s)^{-2}]$

$= (10^{-2}kg)^1 \ (10^{-1}m)^1 \ (10^2s^{-2})$

$= 10^{-1}$kg m s^{-2}

= 0.1 newton

Illustration 13 : When a metallic rod through which heat is being conductivity conducting heat through it is in thermal steady state, the amount of heat passing through it in time t is given by $Q = \frac{kA(T_1 - T_2)t}{L}$ where k = thermal conductivity of the material of the rod, A = cross sectional area of the rod, T_1 and T_2 are the temperatures of hot and cold ends, respectively, of the rod, t = time and L = lengh of the rod. Obtain the dimensional formula for k.

Solution :

$$Q = \frac{kA(T_1 - T_2)t}{L}$$

$$\therefore k = \frac{QL}{A(T_1 - T_2)t} \qquad (1)$$

where, heat energy, $[Q] = M^1L^2T^{-2}$

length, $[L] = L^1$

area, $[A] = L^2$

difference of temperature,

$(T_1 - T_2) = [\Delta T] = K^1$

time, $[t] = T^1$

Note that here we have included K (for temperature) along with M, L and T. Substituting these dimensional formula in equation (1), we get,

$$[k] = \frac{M^1L^2T^{-2}L^1}{L^2K^1T^1} = M^1L^1T^{-3}K^{-1}$$

Note : In some books θ is used in place of K.

Illustration 14 : Obtain the dimensional formula of the following physical quantities :

(i) electric charge (Q), (ii) potential difference (V), (iii) capacitance (C) , (iv) resistance (R).

The formulas connecting these physical quantities are as follows :

$Q = It$, $W = VIt$, $Q = CV$, $V = IR$, where I = electric current, t = time, W = energy.

Solution :

(i) $Q = It$

$$\therefore [Q] = M^0L^0A^1T^1$$

Where A is the symbol for ampere.

(ii) $W = VIt$

$$\therefore [V] = \frac{M^1L^2T^{-2}}{AT^1} = M^1L^2T^{-3}A^{-1}$$

(iii) $Q = CV$

$$C = \frac{Q}{V} = \frac{It}{W/It}$$

$$\therefore C = \frac{I^2t^2}{W} \Rightarrow [C] = \frac{A^2T^2}{M^1L^2T^{-2}}$$

$$\therefore [C] = M^{-1}L^{-2}T^4A^2$$

(iv) $V = IR$

$$\therefore R = \frac{V}{I} = \frac{W/It}{I} = \frac{W}{I^2t}$$

$$\Rightarrow [R] = \frac{M^1L^2T^{-2}}{A^2T^1}$$

$$\therefore [R] = M^1L^2T^{-3}A^{-2}$$

Illustration 15 : By taking velocity, time and force as base quantities, obtain the dimensional formula of mass.

Solution :

Use the symbols F for force, T for time and v for velocity

Force = mass × acceleration

$$= \text{mass} \times \frac{\text{velocity}}{\text{time}}$$

$$\therefore \text{mass} = \frac{\text{force} \times \text{time}}{\text{velocity}}$$

$$\therefore [m] = \frac{F^1T^1}{v^1}$$

$$\therefore [m] = F^1T^1v^{-1}$$

Illustration 16 : Heat produced in a current carrying conducting wire depends on current I, resistance R of the wire and time t for which current is passed. Using these facts, obtain the formula for heat energy.

Solution :

Suppose heat energy $H \propto I^a R^b t^c$

$$\therefore H = k\, I^a R^b t^c \qquad (1)$$

(where $a, b, c \in R$ and k is a dimensionless constant)

Writing the dimensional formula for all the physical quantities in eqn. (1),

$$M^1L^2T^{-2} = (A)^a (M^1L^2T^{-3}A^{-2})^b (T)^c$$

$$= A^{a-2b} M^bL^{2b}T^{c-3b}$$

Equating the indices on both the sides, we get,

$a - 2b = 0$, $b = 1$, $-3b + c = -2$

Thus, $a = 2$ and $c = 1$

Substituting these values of a, b and c in eqn. (1),

$$\therefore H = kI^2Rt$$

Experimentally $k = 1$ is obtained

$$\therefore H = I^2Rt$$

Table 2.3 : Some physical quantities, their SI units and dimensional formula :

Sr. No.	Physical Quantity	Relation with other Physical Quantities	Dimensional Formula	SI Unit
1.	Distance (*d*)	–	$M^0L^1T^0$	m
2.	Mass (*m*)	–	$M^1L^0T^0$	kg
3.	Time (T)	–	$M^0L^0T^1$	s
4.	Plane angle (θ)	arc / radius	$M^0L^0T^0$	rad
5.	Solid angle (Ω)	Area / (radius)^2	$M^0L^0T^0$	sr
6.	Area (A)	length × breadth	$M^0L^2T^0$	m^2
7.	Volume (V)	length × breadth × height	$M^0L^3T^0$	m^3
8.	Density (ρ)	mass / volume	$M^1L^{-3}T^0$	kg m^{-3}
9.	Speed/Velocity(*v*)	Distance / time	$M^0L^1T^{-1}$	m s^{-1}
10.	Acceleration (*a*)	Change in velocity / time	$M^0L^1T^{-2}$	m s^{-2}
11.	Force (F)	mass × acceleration	$M^1L^1T^{-2}$	kg m s^{-2} (newton)
12.	Work (W)	Force × distance	$M^1L^2T^{-2}$	joule (J)
13.	Power (P)	work / time	$M^1L^2T^{-3}$	J/s, watt
14.	Energy, (Kinetic energy, Potential energy, Heat energy etc)	work	$M^1L^2T^{-2}$	joule (J)
15.	Momentum (p)	mass × velocity	$M^1L^1T^{-1}$	kg m s^{-1}
16.	Pressure (P)	Force / Area	$M^1L^{-1}T^{-2}$	N m^{-2}, Pa
17.	Periodic time (T)	time	$M^0L^0T^1$	s
18.	Frequency (*f*)	1 / periodic time	$M^0L^0T^{-1}$	s^{-1}, Hz
19.	Angular displacement (θ)	arc / radius	$M^0L^0T^0$	rad
20.	Angular velocity(ω)	angular displacement / time	$M^0L^0T^{-1}$	rad s^{-1}
21.	Angular acceleration (α)	angular velocity / time	$M^0L^0T^{-2}$	rad s^{-2}
22.	Moment of Inertia (I)	mass × (distance)^2	$M^1L^2T^0$	kg m^2
23.	Torque (τ)	Force × Perpendicular distance	$M^1L^2T^{-2}$	N m
24.	Impulse of force	Force × time	$M^1L^1T^{-1}$	N s^{-1}
25.	Surface tension (T)	Force / distance	$M^1L^0T^{-2}$	N m^{-1}

26.	Specific Heat (C)	$\frac{\text{Heat energy}}{\text{mass} \times \text{temperature}}$	$M^0L^2T^{-2}K^{-1}$	$J\ kg^{-1}\ K^{-1}$
27.	Thermal Conductivity	$\frac{\text{Heat energy} \times \text{thickness}}{\text{Area} \times \text{temperature} \times \text{time}}$	$M^1L^1T^{-3}K^{-1}$	$J\ m^{-1}\ s^{-1}\ K^{-1}$
28.	Electric Current (I)	–	$M^0L^0T^0A^1$	A
29.	Electric Charge (Q)	Electric Current × time	$M^0L^0T^1A^1$	C (Coloumb)
30.	Potential Differnce (V)	work / charge	$M^1L^2T^{-3}A^{-1}$	V (Volt)
31.	Resistance (R)	$\frac{\text{Potential difference}}{\text{current}}$	$M^1L^2T^{-3}A^{-2}$	Ω (ohm)
32.	Capacitance (C)	Charge / Potential Difference	$M^{-1}L^{-2}T^4A^2$	F (faraday)

Table 2.4 : Multiples and Submultiples of SI units

Multiples

Value	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
10	deca	da

Submiltiples

Value	Prefix	Symbol
10^{-1}	deci	*d*
10^{-2}	centi	*c*
10^{-3}	mili	*m*
10^{-6}	micro	μ
10^{-9}	nano	*n*
10^{-12}	pico	*p*
10^{-15}	femto	*f*
10^{-18}	atto	*a*

SUMMARY

1. The standard measure of any quantity is called the unit of that physical quantity.
2. The number of physical quantities is large. We select a limited number of physical quantities to express other quantities. These physical quantities are called fundamental or base quantities. The other physical quantities are called derived physical quantities.
3. There are seven fundamental quantities in SI system of units. They are length, mass, time, electric current, thermodynamic, temperature, luminious intensity and amount of substance.
4. There are two supplementary quantities in SI system. They are plane angle (θ) and solid angle (Ω) and their units are radian (rad) and steradian (sr) respectively.

5. Small distances can be measured using either metrescale, vernier callipers or micrometer screw guage. A screw gauge can be used for measuring length in order of 10^{-5}m for the measurement of large distances or astronomical distances indirect methods are used. e.g. Parallax method.

6. Mass and weight : The amount of matter in any substance is called mass (m). It is an internal property of substance. The gravitational force acting on the substance is called weight (W).

7. The accuracy of measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us about the resolution or the limit to which the quantity is measured.

8. Error : Inaccuracy in measurement of physical quantity is called error. There are two types of error. (i) Systematic error (ii) Random error.

9. The magnitude of the difference between the individual measured value of any physical quantity and its mean value is called absolute error.

10. The ratio of the mean absolute error to the mean value of quantity measured is called relative error or fractional error. When the relative error is expressed in percentage, it is called the percentage error.

11. Combination of errors : When more than one physical quantities are measured, then the maximum error that occurs in the final result will be as follows :

Sr. No.	Mathematical operation	Error
1.	Addition : $Z = A + B$	$\Delta Z = \Delta A + \Delta B$
2.	Subtraction : $Z = A - B$	$\Delta Z = \Delta A + \Delta B$
3.	Division : $Z = \frac{A}{B}$	$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$
4.	Multiplication : $Z = A \cdot B$	$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$
5.	Exponential power : $Z = A^n$	$\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$

12. The number of digits in a measurement about which we are certain plus one additional digit which is uncertain are known as significant digits. The greater the numbers of significant digits obtained in a measurement, the more accurate is the measurement.

13. When any physical quantity is expressed with appropriate powers of M, L, T, ... then such an expression for physical quantity is called its dimensional formula.

14. With the help of dimensional analysis, we can obtain relation between the units of different system of units, can verify dimensional consisteney of physical equation as well as relation between different physical quantities can be obtained.

EXERCISES

Choose the correct option from the given options :

1. Which of the following physical quantities is a derived one ?
(A) mass (B) force (C) plane angle (D) time

2. Which of the following physical quantities is not a fundamental physical quantity in the SI system ?
(A) Luminous intensity (B) Electric current
(C) Solid angle (D) Quantity of matter

3. $\frac{1 \mu m}{1 fm}$ = ...
(A) 10^9 (B) 10^{-9} (C) 10^{15} (D) 10^6

4. Unit of plane angle in SI system is ....
(A) degree (B) radian (C) steradian (D) candela

5. The percentage error in the distance 125.0 $\pm$ 0.5 cm is ....
(A) 4 % (B) 0.04 % (C) 0.4 % (D) 40 %

6. In an experiment to determine the density of a cube, the percentage error in the measurement of mass is 0.26 % and percentage error in the measurement of length is 0.38%. What will be percentage error in the determination of its density ?
(A) 14 % (B) 1.40 % (C) 1.04 % (D) 1.44 %

7. If Z = A^3, then relative error in Z is ....
(A) $(\Delta A)^3$ (B) $\frac{(\Delta A)^3}{A}$ (C) $3\frac{\Delta A}{A}$ (D) $\frac{\Delta A}{A}$

8. If $x = ab^{-1}$ and Δa and Δb are the errors in the measurement of a and b respectively, then the maximum percentage error in the value of x will be ....
(A) $\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) \times 100$ (B) $\left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right) \times 100$
(C) $\left(\frac{\Delta a}{a-b} + \frac{\Delta b}{a-b}\right) \times 100$ (D) $\left(\frac{\Delta a}{a-b} - \frac{\Delta b}{a-b}\right) \times 100$

9. The dimensional formula of physical quantity Z is $M^a L^b T^{-c}$. The percentage error in measurement of mass, length and time are α %, β % and γ % respectively. The percentage error in Z would be ....
(A) $(\alpha + \beta + \gamma)$ % (B) $(\alpha + \beta - \gamma)$ %
(C) $(a\alpha + b\beta + c\gamma)$ % (D) $(a\alpha + b\beta - c\gamma)$ %

10. A student performs an experiment for determination of $g\left(= \frac{4\pi^2 l}{T^2}\right)$. The error in length l is Δl and in time T is ΔT and n is the number of times readings were taken. The measurement of g is most accurate for ....

	Δl	ΔT	n
(A)	5mm	0.2s	10
(B)	5mm	0.2s	20
(C)	5mm	0.1s	10
(D)	1mm	0.1s	50

11. When a current of (2.5 ± 0.5) A flows through a wire, it develops a potential difference of (20 ± 1)V. The resistance of the wire is ....

(A) (8 ± 2)Ω (B) (8 ± 1.5)Ω
(C) (8 ± 0.5)Ω (D) (8 ± 3)Ω

12. Numbers of significant figures in 5.055 and 0.005055 are .... respectively.

(A) 4 and 3 (B) 3 and 3 (C) 4 and 4 (D) 4 and 6.

13. The number of significant figures in 0.0060 is ....

(A) 4 (B) 3 (C) 2 (D) 1.

14. The gravitational force F between two masses m_1 and m_2 separated by a distance r is given by $F = G\frac{m_1 m_2}{r^2}$, where G is the universal gravitational constant. What are the dimensions of G ?

(A) $M^{-1}L^3T^{-2}$ (B) $M^1L^3T^{-2}$ (C) $M^1L^3T^{-3}$ (D) $M^{-1}L^2T^{-3}$

15. According to quantum theory, the energy E of a photon of frequency f is given by $E = hf$, where h is planck's constant. What is dimensional formula for h ?

(A) $M^1L^2T^{-2}$ (B) $M^1L^2T^{-1}$ (C) $M^1L^2T^1$ (D) $M^1L^2T^2$

16. The dimensional formula of 'light year' is...

(A) L^{-1} (B) T^{-1} (C) L^1 (D) T^1

17. What is the dimensional formula of a solid angle ?

(A) $M^1L^1T^1$ (B) $M^0L^0T^1$ (C) $M^1L^0T^{-2}$ (D) $M^0L^0T^0$

18. The time dependance of a physical quantity P is given by $P = P_0 \exp(-\alpha t^2)$. where α is a constant and t is the time. P is the pressure. The dimensional formula of α is...

(A) $M^0L^0T^{-2}$ (B) $M^0L^0T^2$
(C) $M^0L^0T^0$ (D) $M^1L^{-1}T^{-2}$

19. If energy (E), momentum (p) and force (F) were to be chosen as fundamental units, what would be the dimensions of mass in the new system ?

(A) $E^{-1}P^2F^0$ (B) $E^1P^{-2}F^0$
(C) $E^{-1}P^2F^{-2}$ (D) $E^{-2}P^1F^2$

20. The number of particles crossing a unit area perpendicular to the X–axis in unit time is given by,

$n = -D\left(\frac{n_2 - n_1}{x_2 - x_1}\right)$ Where n_1 and n_2 are the number of particles per unit volume at $x = x_1$ and $x = x_2$ respectively and D is diffusion constant. The dimensions of D are...

(A) $M^0L^1T^{-2}$ (B) $M^0L^2T^{-4}$ (C) $M^0L^1T^{-3}$ (D) $M^0L^2T^{-1}$

21. The speed of gravity waves in water is proportional to $\lambda^{\alpha}\ \rho^{\beta}\ g^{\gamma}$ where λ is the wavelength, ρ is the density of water and g is acceleration due to gravity. Which of the following relations is correct ?
(A) $\alpha = \beta = \gamma$ (B) $\alpha \neq \beta \neq \gamma$
(C) $\alpha \neq \gamma = \beta$ (D) $\alpha = \gamma \neq \beta$

22. If the distance between two charges is $2a$, then the dipole moment of this system is given by $p = (2a)q$, where q is electric charge. The dimensional formula of p is ....
(A) $M^0L^{-1}T^1A^1$ (B) $M^0L^1T^{-1}A^{-1}$ (C) $M^0L^1T^{-1}A^1$ (D) $M^0L^1T^1A^1$

23. If 1 $gcms^{-1} = x$ N s then x = ...
(A) 1×10^{-1} (B) 3.6×10^{-3} (C) 1×10^{-5} (D) 6×10^{-4}

24. The equation of stationary wave is $y = 2A\sin kx\cos wt$ (in metre). Where A and x are in metre. w is angular frequency. Dimensions of A/k are ....
(A) $M^0L^0T^0$ (B) $M^0L^{-2}T^0$ (C) $M^0L^{-1}T^1$ (D) $M^0L^2T^0$

25. In $\left(P + \frac{a}{V^2}\right)(V - b) = RT$ equation, the dimensional formula of $\frac{a}{b}$ will be .... where, P = Pressure, V = Volume and T is temperature.
(A) $M^1L^2T^{-2}$ (B) $M^1L^2T^{-2}K^1$ (C) $M^1L^{-2}T^2$ (D) $M^1L^2T^{-2}K^{-1}$

ANSWERS

1.	(B)	2.	(C)	3.	(A)	4.	(B)	5.	(C)	6.	(B)		
7.	(C)	8.	(A)	9.	(C)	10.	(D)	11.	(A)	12.	(C)		
13.	(C)	14.	(A)	15.	(B)	16.	(C)	17.	(D)	18.	(A)		
19.	(A)	20.	(D)	21.	(D)	22.	(D)	23.	(C)	24.	(D)	25.	(A)

Answer the following in short :

1. What is a unit ? What are derived units ?
2. Which are the supplementary units of SI system ?
3. What is dimensional formula ?
4. Which physical quantity has the unit amu ?
5. $1g/cm^3$ = ... kg/m^3
6. In any experiment, physical quantity in the formula which has the maximum power should be measured more accurately. Justify the statement ?
7. The mass of an object is $225 \pm 0.05g$. Calculate percentage error in measurement.
8. Write the dimensional formula of capacitance.
9. Give the difference between accuracy and precision.
10. If $\theta_1 = 25.5 \pm 0.1$ °C and $\theta_2 = 35.3 \pm 0.1$ °C calculate $\theta_1 - \theta_2$
11. Subtract with due regard to significant figures : $3.9 \times 10^5 - 2.5 \times 10^4$

Answer the following questions :

1. Which are the fundamental and supplementary units of the SI system ? Give their units with symbols.
2. Explain the parallax method to determine the distance between the earth and a planet.
3. Explain the different types of errors that occur during measurement of a physical quantity.
4. Explain absolute error, average absolute error, relative error and percentage error.
5. Explain the dimensional consistency of an equation can be checked using dimensional analysis ?
6. Give the limitations of dimensional analysis.

Solve the following Problems :

1. In Ohm's experiment, the values of an unknown resistance were found to be 4.12Ω, 4.08Ω, 4.22Ω and 4.14Ω. Calculate absolute error, relative error and percentage error in these measurement.

   [**Ans. :** 0.04, 0.0096, 0.96 %]

2. If the length of a cylinder is $l = (4.00 \pm 0.01) cm$, radius $r = (0.250 \pm 0.001)\ cm$ and mass $m = 6.25 \pm 0.01 g$. Calculate the percentage error in determination of density.

   [**Ans. :** 1.21 %]

3. The acceleration due to gravity (g) is determined by using simple pendulum of length $l = (100 \pm 0.1)$cm. If the time period is $T = (2 \pm 0.01)$s, find the maximum percentage error in the measurement of g.

   [**Ans. :** 1.1 %]

4. The length, breadth and thickness of a metal sheet are 4.234m, 1.005m and 2.01cm respectively. Calculate the total area and volume of the sheet to the correct number of significant figures.

   [**Ans. :** 8.72 m^2, 0.086 m^3]

5. The electric force between two electric charges is given by

   $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$, Where r is the distance between q_1 and q_2. Give the unit and dimensional formula of ε_0.

   [**Ans. :** $N^{-1}C^2\ m^{-2}$; $M^{-1}L^{-3}T^4A^2$]

6. Check the dimensional validity of the following equations :

   (i) Pressure $P = \rho gh$

   ρ = density of matter, g = acceleration due to gravity, h = height

   (ii) $F.s = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

   Where F = force, s = displacement, m = mass, v = final velocity and v_0 = initial velocity

(iii) $s = v_0 t + \frac{1}{2}(at)^2$

s = displacement, v_0= initial velocity, a = acceleration and t = time.

(iv) $F = \frac{m \times a \times s}{t}$

Where m = mass, a = acceleration, s = distance and t = time.

[**Ans. :** (i) and (ii) is dimensionally valid and (iii) and (iv) are not valid.]

7. If the velocity of light, acceleration due to gravity and normal pressure are chosen as the fundamental units, find the unit of mass, length and time. Given that velocity of light, $c = 3 \times 10^8 \text{m s}^{-1}$, $g = 10\text{ms}^{-1}$ and normal atmospheric pressure, $P = 10^5 \text{N/m}^2$

[**Ans. :** unit of mass = 8.1×10^{35}kg, unit of length = L = 9×10^{15}m and unit of time T = 3×10^7s]

8. $v = at + \frac{b}{t + c} + v_0$ is a dimensionally valid equation. Obtain the dimensional formula for a, b and c. where, v is velocity, t is time and v_0 is initial velocity.

[**Ans. :** $[a] = M^0L^1T^{-2}$, $[b] = M^0L^1T^0$, $[c] = M^0L^0T^1$]

9. An object is falling freely under the gravitational force. Its velocity after traversing a distance h is v. If v depends upon gravitational acceleration g and distance has prove with the help of dimensional analysis that $v = k\sqrt{gh}$, where k is a constant.

10. A gas bubble from an explosion under water oscillates with a period T proportional to $P^a \rho^b E^c$ where P is the static pressure, ρ is the density of water and E is the total energy of the explosion. Find the values of a, b and c.

[**Ans. :** $a = -\frac{5}{6}$, $b = \frac{1}{2}$, $c = \frac{1}{3}$]

CHAPTER 3

MOTION IN A STRAIGHT LINE

3.1 Introduction

Motion is common to everything in the universe. Walking, running, cycling, blood circulation in our body, flowing of water, flying birds etc are daily life examples of motion. Some motions are invisible, for example, random motion of gas molecules, flow of electrons in conducting wire. Stationary looking objects such as roads, trees, buildings etc. are in motion with the earth's rotation, also the earth revolves around the Sun and the sun itself is in motion in the milky way. Milky way is also in motion with respect to other galaxies. Thus, we can say that everything in this universe is in motion.

When any object changes its position with respect to another object, we say that the object is in motion. Motion can be of several types, e.g. linear motion, rotational motion, vibrational (oscillatory) motion etc. In this chapter, we shall confine overselves to study the motion of objects along a straight line, also known as rectilinear motion. For this, we shall understand the concept of physical quantities like displacement, velocity and acceleration.

3.2 Kinematics and Dynamics

A branch of physics dealing with motion without considering its causes is known as **kinematics.**

A branch of physics describing motion along with its causes and properties of a moving body is called **dynamics.**

Kinematics and dynamics are collectively known as **mechanics.** In this chapter we shall discuss only kinematics.

3.3 Concept of a Particle

In our dicussion we shall treat the object in motion as a particle. **A particle is a point like object endowed with mass.** In practice it is difficult to get such an object because an object would have some dimensions. But in certain circumstances an object can be treated as a particle. Such circumstances are as follows :

(i) All the particles of a solid body performing linear motion cover the same distance in the same time. Hence, the motion of such a body can be described in terms of the motion of any of its constituent particles.

(ii) If the distance between two objects is very large as compared to their dimensions, these objects can be treated as particles. For exmaple, while calculating the gravitational force between the Sun and the Earth, both of them can be considered as particles.

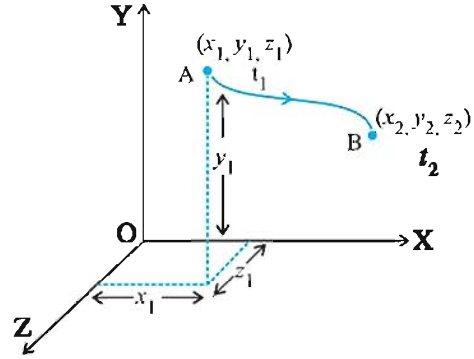
3.4 Frame of Reference

How can we know that the object is at rest or in motion ? In order to answer this question, we shall consider the following illustration. Suppose a bag is lying in a train moving with a constant speed. A passenger in the train would find the bag stationary but a person standing by the roadside would find the bag in motion. Thus, the bag is at rest if it is observed from the train, it is in motion if it is observed from the roadside. If we observe the trees, building etc. from the ground, they are stationary but from the train they will be found to be in motion. Thus, motion is a combined property of the object under study and the observer. In other words, motion is a relative concept.

A place and a situation from where an observer takes his observation is called frame of reference. In the above illustration of the bag, the frame of reference for the passenger is the train moving with constant speed, while for the observer standing by the roadside the frame of reference is the stationary (!) Earth. The frames of reference is of two types : inertial reference frame and non-inertial reference frame. We shall discuss this in detail in Chapter 5.

3.5 Position, Pathlength and Displacement

In order to describe the motion of a particle, its position at every instant of time must be known. To locate the position of a particle we need a frame of reference. There is no rule or restriction on the choice of a frame of reference. We can select a reference frame according to our convenience. We can choose three mutually perpendicular axes and name them (in counter clockwise direction) X, Y and Z axes. The point of intersection of these three axes is called origin (O) and serves as **reference point**. The co-ordinates (x, y, z) of a particle describe the position of the object with respect to this frame of reference or co-ordinate system. To measure the time we put a clock in this system.



Position of a particle in frame of reference

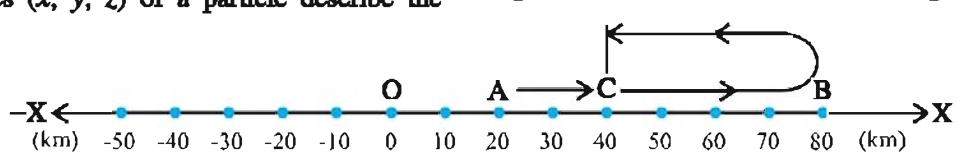
Figure 3.1

As shown in Fig. 3.1, the position co-ordinate of a particle in motion at time t_1 is (x_1, y_1, z_1) and at time t_2 is (x_2, y_2, z_2). This shows the position of particle with respect to the given frame of reference.

If all the three co-ordinates of a particle remain unchanged with time, the particle is considered at rest with respect to time in this frame. If one or more co-ordinates of a particle changes with time, we say that particle is in motion w.r.t. this frame. If any one of the co-ordinates changes with time, the motion of a particle is called one dimensional or linear motion. For example, an object falling freely from the top of the tower, motion of a car along a straight road.

When two or three co-ordinates change with time, the motion of a particle is called two dimensional and three dimensional respectively. For exmple the earth revolving around the sun, a striker moving over the carrom board are examples of two dimensional motion. A butter-fly flying in the garden is an example of three dimensional motion.

To describe motion along a straight line, we can choose an axis, say X–axis, so that it coincide with the path of a particle. We can decide the position of a particle with reference to a conveniently chosen origin, say O, as shown in Fig 3.2. Position, to the right of O are taken as positive and to the left of O, as negative.



Motion of a car in straight line

Figure 3.2

Path length : The total distance travelled by a particle in some time interval is called the path length (or total distance.)

Displacement : The change in position of a particle in some time interval is called displacement.

If x_1 and x_2 be the initial and the final positions of a particle at time t_1 and time t_2 respectively then displacement in time interval $\Delta t = t_2 - t_1$, is given by

Δx = final position – initial position

$= x_2 - x_1$

The SI unit of path length and displacement is metre (m). Now, we shall understand the difference between path length and displacement.

Consider the motion of a car along X–axis. As shown in Fig 3.2 a car is at point A at time t_1, then it goes to B and then comes to point C at time t_2.

In time interval $\Delta t = t_2 - t_1$,

Path length = AB + BC = (80 – 20) + (80 – 40) = +100 km.

Displacement = Final position – Initial position
(Point C) (Point A)
= 40 – 20 = +20 km.

In this case path length and displacement both are positive. Displacement is directed in the positive X axis direction. If the car starts from B and goes to point C then, path length = 80 – 40 = 40 km and displacement = 40 – 80 = –40 km. Thus, displacement can be negative also. Here, displacement of the car is in the X–axis direction. In above illustration if the car starts from A, goes to B and then returns to A, the path length is + 120 km but displacement is zero. From this illustration it is clear that path length is always positive while displacement can be positive, negative or zero.

We do not get correct information about the path of motion of the car from displacement. Displacement gives only the resultant effect of motion of a car.

Displacement has both magnitude and direction. Such physical quantities are represented by vectors. You will learn about vectors in Chapter 4. In one-dimensional motion, there are only two directions (backward and forward or upward and downward) in which a particle can move, and these two directions can easily be specified by + and – signs.

Ilustration 1 : A particle moves along a circle of radius r. It starts from A and moves in anticlockwise direction as shown in Fig 3.3 Calculate the distance travelled by the particle and magnitude of displacement for each of the following cases.

(i) from A to B (ii) from A to C (iii) from A to D (iv) one complete revolution of the particle.

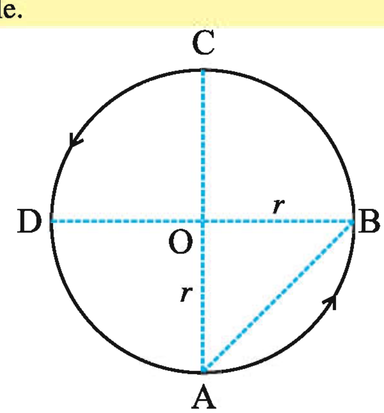


Figure 3.3

Solution : (i) Distance travelled by particle from A to B,

$$\text{Path length} = \frac{2\pi r}{4} = \frac{\pi r}{2}$$

$$\text{Displacement } |AB| = \sqrt{OA^2 + OB^2}$$

$$= \sqrt{r^2 + r^2} = \sqrt{2}\, r.$$

(ii) Distance travelled by the particle from A to C,

$$\text{Path length} = \frac{2\pi r}{2} = \pi r$$

$$\text{Displacement} = |AC| = r + r = 2r$$

(iii) For the motion from A to D,

$$\text{Path length} = 2\pi r \times \frac{3}{4} = \frac{3}{2}\pi r$$

$$\text{Displacement } |AD| = \sqrt{r^2 + r^2}$$

$$= \sqrt{2}\, r$$

(iv) For one complete revolution of the particle, total distance travelled will be equal to circumference of the circle, Path length = $2\pi r$

and displacement = 0, since the particle comes back to the original position.

3.6 Average Speed and Average Velocity

When an object is in motion, its position changes with time. In order to study motion, it is necessary to know how fast the position of the object is changing with time. To describe this, we define the quantity average speed.

The ratio of pathlength (i. e. total distance travelled) to the time interval during which the motion has taken place, is known as **average speed.** It is denoted by symbol $< v >$ or $\overline{v}$. Thus, for a given time interval,

$$\text{Average Speed} = \frac{\text{Path length}}{\text{time interval}}$$

If an object travels a distance Δx in time interval Δt, the average speed in this time interval is,

$$< v > = \frac{x}{t} \qquad (3.6.1)$$

Now, the question arises as to how fast the position is changing with time and in what direction ? To understand this, we define the quantity average velocity.

Average velocity is the ratio of displacement to the time interval, in which the displacement occurs.

$$\text{Average Velocity} = \frac{\text{displacement}}{\text{time interval}} = \frac{x}{t} \qquad (3.6.2)$$

Average speed and average velocity have the same SI unit m s^{-1}, although kmh^{-1} is used in many day to day applications.

The direction is important in average velocity while it is not so in average speed. As explained earlier, for motion in a straight line, the direction of motion can be taken care of by + and – signs. We do not use the vector notation for velocity in this chapter. To understand this, consider the motion of the car as shown in Fig. 3.2.

At t = 0, the car is at point A, it goes to point B and then returns to C in two hours.

Average speed of the car in this time interval,

$$\overline{v} = \frac{\text{Path length}}{\text{time}} = \frac{100}{2} = +50 \text{ kmh}^{-1}$$

$$\text{Average Velocity} = \frac{\text{displacement}}{\text{time}}$$

$$= \frac{40 - 20}{2} = +10 \text{ kmh}^{-1}$$

'+' sign indicates that average velocity of the car is in positive X direction.

Now, if car moves on a path A–B–C–O and comes to O in 3 hours, then

$$\text{Average speed} = \frac{140}{3} = +46.6 \text{ kmh}^{-1}$$

$$\text{Average Velocity} = \frac{0 - 20}{3} = -6.66 \text{ kmh}^{-1}$$

Here, the average velocity of the car is in negative X direction.

Thus, average velocity can be positive, negative or zero. It depends on the sign of displacement. In general, the average speed of an object can be equal to or greater than the magnitude of the average velocity.

We shall understand the reason of using the word 'average' with the help of an illustration. Suppose a car starts from Ahmedabad at morning 10 o'clock and reaches Vadodara at 12 noon. The distance between Ahmedabad and Vadodara is, say 100km, so the average speed of the car will be 50kmh^{-1}. This does mean that, the speed of the car throughout its journey from Ahmedabad to Vadodara remained exactly 50kmh^{-1}. Where the traffic was less the speed of car may have been 80kmh^{-1} and if a railway crossing was closed it may be zero. Now, it should be clear that 50kmh^{-1} is the average speed of the car.

Illustration 2 : A vehicle travels different distances with different speeds in the same direction. Find the expression for the average speed of the vehicle.

Solution : Suppose the vehicle travels distances d_1, d_2, d_3 .... with speeds v_1, v_2, v_3 .... respectively in the same direction.

Total distance travelled,

$D = d_1 + d_2 + d_3 + ...$

Total time taken, $t = t_1 + t_2 + t_3 + ...$

$$= \frac{d_1}{v_1} + \frac{d_2}{v_2} + \frac{d_3}{v_3} + ...$$

$$\text{Average speed} = \frac{D}{t}$$

$$= \frac{d_1 + d_2 + d_3 + \ldots\ldots\ldots}{\frac{d_1}{v_1} + \frac{d_2}{v_2} + \frac{d_3}{v_3} + \ldots\ldots\ldots}$$

Special case : If the vehicle travels two equal distances ($d_1 = d_2 = d$) with different speeds v_1 and v_2, then

$$\text{Average speed} = \frac{d + d}{\frac{d}{v_1} + \frac{d}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}$$

Illustration 3 : A person walking in a straight line, covers half of the distance to be travelled with a speed of v_0. For half of the time, required to complete the remaining distance, he walks with a speed of v_1 and for the remaining half time his speed is v_2. What is the person's average speed during this complete walk ?

Solution : Suppose the total walking distance is d, the time taken to travel half of the distance $\left(\frac{d}{2}\right)$ is t_1 and the time taken to travel the remaining half distance is $2t$.

Using the formula,

distance travelled = average speed × time

$$\frac{d}{2} = v_0t_1 \text{ and hence } t_1 = \left(\frac{d}{2v_0}\right)$$

$$\text{and } \frac{d}{2} = v_1t + v_2t = (v_1 + v_2)t$$

$$\therefore\ 2t = \left(\frac{d}{v_1 + v_2}\right)$$

Thus, the total time taken to walk the total distance d is $t_1 + 2t$

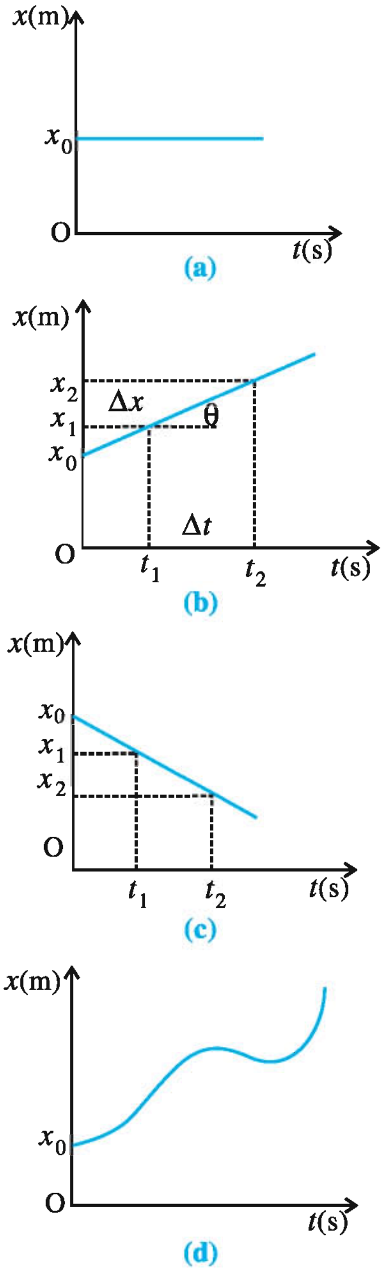
∴ The person's average speed

$$< v > = \frac{d}{t_1 + 2t} = \frac{d}{\left(\frac{d}{2v_0}\right) + \left(\frac{d}{v_1 + v_2}\right)}$$

$$= \frac{2v_0(v_1 + v_2)}{v_1 + v_2 + 2v_0}$$

3.7 Position–Time ($x - t$) Graphs for Motion

Motion of an object can be represented by a position–time graph as you have already learnt it. Such graph is a powerful tool to represent and to analyse different aspects of motion of an object. For motion along a straight line, say X–axis the x co-ordinate varies with time and we have $x - t$ graph. **The slope = $\frac{\Delta x}{\Delta t}$ = $\tan\theta$ of the graph gives the average velocity of an object for a given time interval.** Where, θ is the angle between time–axis and the line. $x - t$ graphs for different types of motion are shown in Fig 3.4.



$x - t$ Graphs for different types of motion

Figure 3.4

(i) If $x - t$ graph is parallel to time-axis then the object is at rest. As shown in Figure 3.4(a), the slope of the graph is zero hence the object is at rest (i.e. its velocity is zero)

(ii) If an object moving along a straight line covers equal distance in equal intervals of time, it is said to be in **uniform motion** (constant velocity) along a straight line. Fig. 3.4 (b) shows position-time graph of such a motion which is a straight line. The slope $= \left(\frac{x_2 - x_1}{t_2 - t_1}\right)$ of this line is positive. It shows that the average velocity of that object is positive and it is moving in positive X direction. If the slope is negative, the object has negative average value and it is moving in negative X direction (see Fig. 3.4 (c)).

(iii) If the $x - t$ graph is a curve instead of straight line, it is said to be non–uniform motion of an object. (See Fig. 3.4 (d))

Illustration 4 : A motorcylist travels in a straight path to a petrol pump in time 60 s, which is at a distance of 120m in the East direction from his house. He stays there for filling up petrol for 120 s and comes back home by the same path in 90 s. After 90 s he gets ready for the office. His office is on a straight way at a distance of 300m in the West direction from his house. He takes 120 s to reach his office. Draw the graph of position–time for the motorcycle and find average velocity for different time intervals.

Solution :

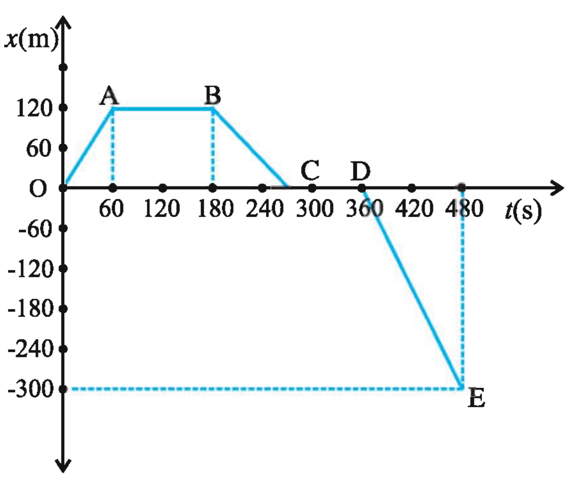


Figure 3.5

Take the house on the origin (O) and consider the displacement in East direction positive (+Y axis) and distance in west direction negative (−Y axis). The position−time graph of a motorcycle will be as shown in Fig 3.5.

Average velocity of motorcycle in OA region

$$= \frac{\Delta x}{\Delta t} = \frac{120 - 0}{60 - 0} = +2\text{m s}^{-1}$$

In AB region, Motorcycle is stationary at petrol pump. ∴ Average velocity = 0

In BC region, average velocity

$$= \frac{\Delta x}{\Delta t} = \frac{0 - 120}{270 - 180} = -1.33\text{m s}^{-1}$$

In CD region, the motorcycle is at home, hence, its velocity is zero.

In DE region, Average velocity

$$= \frac{\Delta x}{\Delta t} = \frac{-300 - 0}{480 - 360} = -2.5\text{m s}^{-1}$$

Here, '+' and '–' sign indicates that the average velocity of the motorcycle is in East and West direction respectively.

3.8 Instantaneous Velocity and Instantaneous Speed

In discussion of average velocity and average speed we saw that the average velocity in any time interval gives information about how fast an object has moved over a given time interval, but does not tell us how fast it moves at different instants of time during that time interval.

Suppose a particle moving in one dimension is at position x and $x + \Delta x$ at time t and $t + \Delta t$ respectively. We can get the average veolocity of this particle in time interval Δt from equation (3.6.2). We do not have the information about its velocity at different instants (theoretically infinite) of time during this interval. During this time interval the velocity of the particle might have increased or decreased. But it is clear that by giving smaller and smaller interval of time to a particle to change its velocity, we get more and more accurate information about its velocity. To understand this clearly let us consider the following illustration.

Suppose, a car starts from rest (at $t = 0$) and moves on a straight line in +X direction. The velocity of the car increases with time. A person sitting in the car takes readings of the speedometer at every second and notes them along with the distance travelled. Speedometer indicates the speed of car at that instant. But our car moves in only one direction, hence the speed and magnitude of velocity will be same. Generally, the reading of a speedometer are in it is km h^{-1}, but we assume that in our car it is in m s^{-1}. Such observations are tabulated in Table 3.1

Table 3.1

Time $t(s)$	Displacement from origin x(m)	Reading of Speedometer v (ms^{-1})
0	0	0
1	1.5	3
2	6	6
3	13.5	9
4	24	12
5	37.5	15
6	54	18

Suppose we want to know the velocity of the car at $t = 3$ second. To calculate the velocity, we must know the displacement of the car and for this displacement car requires some time. Suppose we take time interval Δt = 3 s from t = 3 s to $t + \Delta t$ = 6 s. During this time interval,

Value of average velocity

$$= \frac{(\text{ displacement in 6 s}) - (\text{ displacement in 3 s})}{(6 - 3)\text{ s}}$$

$$= \frac{54 - 13.5}{3} = +13.5\text{m s}^{-1}$$

But from the table we can see that at t = 3s the reading in the speedometer was 9 m s^{-1}. Thus, in this calculation, for a time interval Δt = 3 s, the value of average velocity differs a lot from the true value.

Now, we reduce the time interval Δt = 1 s. In this time interval,

Value of average velocity

$$= \frac{24 - 13.5}{4 - 3} = +10.5\text{m s}^{-1}$$

Here, the car is moving according to position equation, $x = 1.5t^2$. From this we will get x = 13.5 m and x = 14.415 m for t = 3 s and t = 3.1 s respectively. Now, we will calculate the value of average velocity for the time interval Δt = 0.1 s,

The value of average velocity

$$= \frac{14.415 - 13.5}{3.1 - 3} = 9.15\text{m s}^{-1}$$

On decreasing the time interval still further and taking it to be Δt = 0.05 s, the value of average velocity comes out to be 9.07m s^{-1}.

So, as we go on decreasing the time interval, the difference between the calculated value and the true value goes on decreasing. Now, the question arises. How much small time interval is sufficienty small ? you will learn that the symbol for infinitedsimally small (near to zero but not zero) time interval is $\lim_{\Delta t \to 0}$. In such circumstances two instants of time t and $t + \Delta t$ can be considered as almost the same instant and the average velocity obtained is called the value of instantaneous velocity at time t. This can be shown mathematically as follows :

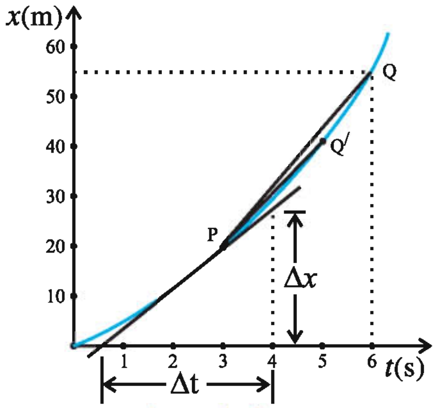
Instantaneous velocity at time t,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (3.7.1)$$

Here, $\frac{dx}{dt}$ is called derivative of x with respect to t. It gives the rate of change of position with time. $\frac{dx}{dt}$ is also denoted as $\dot{x}$. (Explanation of differentiation is given at the end of the chapter.)

Instantaneous velocity from $x - t$ graph :

Now, we shall see how to get the value of instantaneous velocity from the $x - t$ graph. Let us consider the above illustration and plot the position–time graph, as shown in Fig. 3.6.



x − t graph for a car

Figure 3.6

Suppose we want the instantaneous velocity at time $t = 3$ s. For this we select two points P and Q corresponding to $t = 3$ s and $t + \Delta t = 6$ s and join them by a straight line PQ. The slope of the line PQ gives the average velocity of the car in time interval $\Delta t = 6 - 3 = 3$ s.

Similarly the average velocity in time interval from $t = 3$ s to $t + \Delta t = 5$ s, i.e. $\Delta t = 2$ s can be obtained from the slope of PQ'. Thus by reducing the time interval (Δt), the line passing through P goes on leaning towards the tangent to the curve drawn at P and when we take $\lim_{\Delta t \to 0}$, that line would coincide with the tangent.

We know that there can be many lines passing through point P but the line which passes through P and is tangent to the curve at P is unique. Here, the slope of the tangent at P gives the value of instantaneous velocity of a car at time $t = 3$ s. **Note that for uniform motion, $x - t$ graph is a straight line so that value of instantaneous velocity is the same as the value of average velocity.**

Instantaneous speed or simply **speed** is the magnitude of velocity. For example a velocity of +5.0m s^{-1} and velocity of −5.0m s^{-1} both have a speed of 5.0m s^{-1}. The speedometer of car shows the instantneous speed at that particular instant. It should be noted that though an average speed over a finite interval of time is greater or equal to the magnitude of the average velocity, **instantaneous speed at any instant is equal to the magnitude of the instantaneous velocity at that instant.**

Illustration 5 : The position of an object, moving in one dimension, is given by the formula $x(t) = (4.2t^2 + 2.6)$ m. Calculate its (i) average velocity in the time interval from $t = 0$ to $t = 3$ s and (ii) Instantaneous velocity at $t = 3$ s. $\left[\frac{d(x^n)}{dt} = nx^{n-1}\right]$

Solution : (i) $x(t) = 4.2t^2 + 2.6$

For $t = 0$, The position of the object

$x(0) = 4.2(0)^2 + 2.6$

$= 2.6$ m (initial position)

For $t = 3$ s, position of the object,

$x(3) = 4.2(3)^2 + 2.6$

$= 40.4$ m (final position)

Average velocity =

$$\frac{\text{final position} - \text{initial position}}{\text{time interval}}$$

$$= \frac{x(3) - x(0)}{3 - 0} = \frac{40.4 - 2.6}{3}$$

$= 12.6$ m s^{-1}

(ii) To find the instantaneous velocity, differentiate the position equation with respect to 't'.

Instantaneous velocity

$$v = \frac{dx}{dt} = \frac{d}{dt}(4.2t^2 + 2.6)$$

$$= 4.2\frac{d}{dt}(t^2) + \frac{d}{dt}(2.6)$$

$= 4.2\ (2t) + 0$

$= 8.4t$ m s^{-1}

Putting $t = 3$ s, $v = 8.4(3) = 25.2$m s^{-1}

3.9 Acceleration

If the velocity of a particle is the same for different time intervals, then it means that the particle moves with constant velocity. But if the velocity changes, then the particle has accelarated motion. **The time rate of change of velocity is called acceleration.**

Let a particle be moving in a straight line, and at time t_1 and t_2 its velocities are v_1 and v_2 respectively. Thus, the change in velocity of the particle in time interval $\Delta t = t_2 - t_1$ is $v_2 - v_1$.

According to definition of average acceleration,

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$< a > = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \quad (3.9.1)$$

Average acceleration is a vector quantity and its direction is in the direction of change in velocity (Δv). The SI unit of acceleration is m s^{-2}. From average acceleration we cannot know how the velocity of particle changes with time. Taking $\lim_{\Delta t \to 0}$ in equation (3.9.1) we get instantaneous acceleration a at time t. Instantaneous acceleration is also called acceleration in practice,

$\therefore$ Instantaneous acceleration at time t is,

$$a = \lim_{t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \quad (3.9.2)$$

Now, $v = \frac{dx}{dt}$

$$\therefore a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$\therefore a = \frac{d^2x}{dt^2} = \ddot{x} \quad (3.9.3)$$

In other words, second derivative of position with respect to time is acceleration of a particle.

If $\frac{dv}{dt}$ is positive, acceleration is along the positive X–axis and if $\frac{dv}{dt}$ is negative, the acceleration is along the negative X–axis. If the velocity and acceleration are both positive or both negative, the speed increases. In this case a particle has accelerated motion. If they have opposite sign, the speed of particle decreases. When the speed decreases, we say that the particle is decelerating. The direction of deceleration or retardation is opposite to the direction of velocity. Acceleration of a particle +2.5m s^{-2} towards East is same as a deceleration of 2.5m s^{-2} towards West.

Illustration 6 : The position of a particle moving along a straight line is given by $x = 2 - 5t + t^3$. Find the acceleration of the particle at $t = 2$ s. (x is in metre).

Solution :

Position equation : $x = 2 - 5t + t^3$. To get the acceleration, differentiate x two times with respect to t.

$$\therefore \frac{dx}{dt} = \frac{d}{dt}(2 - 5t + t^3) = -5 + 3t^2$$

$\therefore$ Acceleration of particle,

$$a = \frac{d^2x}{dt^2} = \frac{d}{dt}(-5 + 3t^2) = 6t$$

Putting $t = 2$s, $a = 6(2) = 12$m s^{-2}

Illustration 7 : For a moving particle, the relation between time and position is given by $t = Ax^2 + Bx$. Where A and B are constants. Find the acceleration of the particle as a function of velocity.

Solution : $t = Ax^2 + Bx$

$$\therefore \frac{dt}{dx} = 2Ax + B$$

$$\therefore v = \frac{dx}{dt} = (2Ax + B)^{-1} \quad (1)$$

Now, Acceleration, $a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt}$

$$= \left[\frac{d}{dx}(2Ax + B)^{-1}\right](v) \quad (\text{from eq}^n\ (1))$$

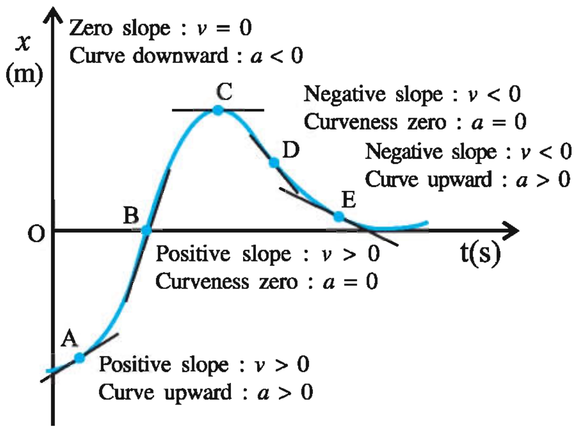
$$\therefore a = (-1)(2A)(2Ax + B)^{-2} \cdot v$$

$$= -2Av^3 \quad (\text{from eq}^n\ (1))$$

3.10 $x - t$ and $v - t$ Graphs for Accelerated Motion

The second derivative of position x with respect to t is acceleration. The second derivative of any function is directly related to the curvature of the graph of that function. In $x - t$ graph, the point at which graph is more curved, the value of acceleration $\frac{d^2x}{dt^2} = a$ is more and point at which graph is less curved, $\frac{d^2x}{dt^2} = a$ is small.

At a point where $x - t$ graph is concave up (curved up ward), the acceleration is positive and velocity of the particle is increasing. At a point where $x - t$ graph is concave down (curved downward) the acceleration is negative and velocity is decreasing. At a point where the $x - t$ graph has no curvature (or is a straight line), the acceleration is zero and the velocity is constant. Fig 3.7 shows all three of these possibilities.



$x - t$ graph for a moving particle

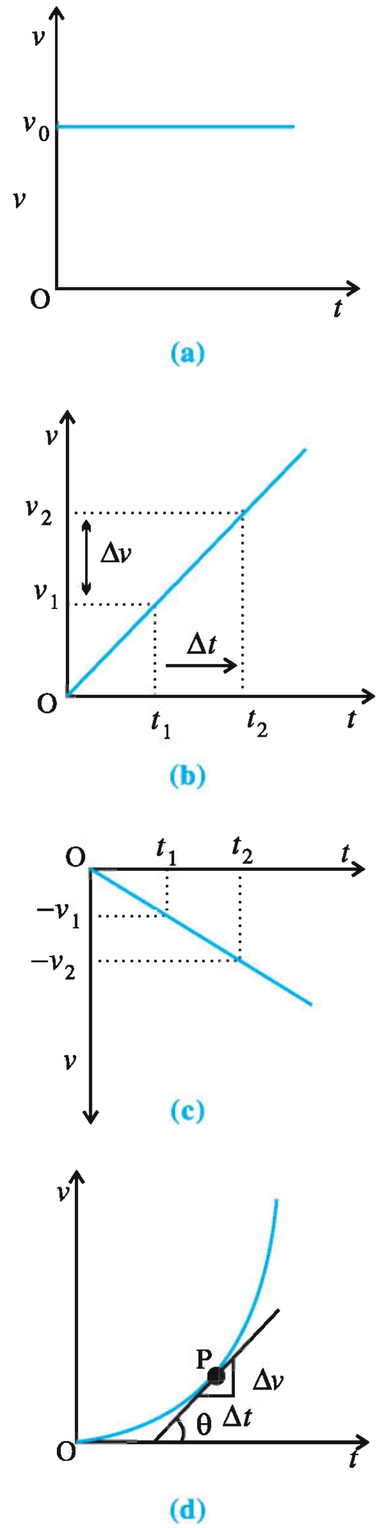
Figure 3.7

Thus, an $x - t$ graph is an easy way to decide what the sign of acceleration is. But we cannot easily obtain the value of acceleration. For this purpose, a $v - t$ graph of the moving particle is useful.

From the $v - t$ graph, we can get average acceleration and instantaneous acceleration. The slope $\left(= \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \right)$ gives the average acceleration for a given time interval Δt. The instantaneous acceleration at any time is equal to the slope of the tangent to the $v - t$ graph at that time.

(i) For a moving particle, if $v - t$ graph is paralled to time–axis, then its slope will be zero. Therefore, the particle is moving with zero acceleration. It means that velocity of the particle is constant. Such a motion is called **uniform motion** (See Fig. 3.8 (a))

(ii) If $v - t$ graph is as shown in Fig. 3.8 (b), (c) and (d) or any other types instead of parallel line to time axis, then motion of the particle is called **non–uniform motion**. As shown in Fig. 3.8 (b), the slope of graph is positive. Hence, the acceleration of the particle will be also positive.



$v - t$ graphs for a moving partcile

Figure 3.8

Here, the graph is a straight line, therefore the slope of the line will be same for any time interval. It is called constant accelerated motion or **uniform accelerated motion**. For such a motion, average acceleration and instantaneous acceleration are same for a given time interval.

(iii) For the motion, as shown in Fig. 3.8 (c) the slope of the graph is negative, we say that particle is decelerating. This is also constant accelerated motion.

(iv) If the velocity of a particle changes continuously with time as shown in Fig 3.8 (d), then the slope of the graph will be different for different time intervals. Hence, the average acceleration will be also be different in different time intervals. This is called **non–uniform accelerated motion**. The acceleration at any time is equal to the slope of the tangent drawn to the curve at that time.

In this chapter, we will confine ourselves to study only the uniform accelerated motion.

The velocity–time graph of a moving particle can be used to find the displacement and distance travelled by the particle during a given time interval. **The area under the curve of $v - t$ graph represents the displacement of a particle over given time interval.** This statement is true for any type of motion. The area of the $v - t$ graph above X–axis is positive and that below the X–axis is negative. The net displacement will be the algebraic sum of the areas under the $v - t$ graph. If we want to find distance travelled by the particle, then negative area of graph should considered positive for algebraic sum. To understand more clearly, consider the following example.

Illustration 8 : The $v - t$ graph of a particle moving in a straight line is shown in Fig. 3.9 Find (a) the distance travelled during the first two seconds. (b) the displacement and total distance during the time 0 to 4 s. (c) acceleration at t = 0.5 s and t = 2 s.

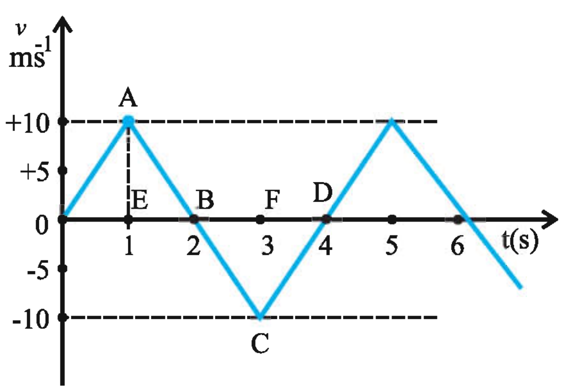


Figure 3.9

Solution : (a) Distance travelled during first two seconds,

x_1 = Area of ΔOAB

$= \frac{1}{2}$(AE) (OB)

$= \frac{1}{2}(+10)(2) = 10$ m

(b) Displacement during 0 to 4 s,

Δx = area of ΔOAB + area of ΔBCD

$= \frac{1}{2}$(AE)(OB) + $\frac{1}{2}$(CF)(BD)

$= \frac{1}{2}(+10)(2) + \frac{1}{2}(-10)(2)$

$= 10 - 10 = 0$

Distance travelled in 0 to 4 s,

= Area of ΔOAB + Area of ΔBCD

= 10 + 10

(Consider area of ΔBCD to be positive)

= 20 m

(c) At t = 0.5 s, acceleration,

a_1 = slope of line OA

$= \frac{10 - 0}{1 - 0} = 10\text{m s}^{-2}$

At t = 2s, acceleration,

a_2 = slope of line AC

$= \frac{(-10) - (+10)}{3-1}$

$= -10\text{m s}^{-2}$

3.11 Kinematic Equations for Uniformly Accelerated Motion

Suppose a particle is moving with a uniform acceleration 'a' along a straight line in X–direction. Let at t = 0, the velocity of the particle be v_0 and at t = t, the velocity of the particle be v. The velocity – time ($v - t$) graph for this motion is shown in Fig. 3.10.

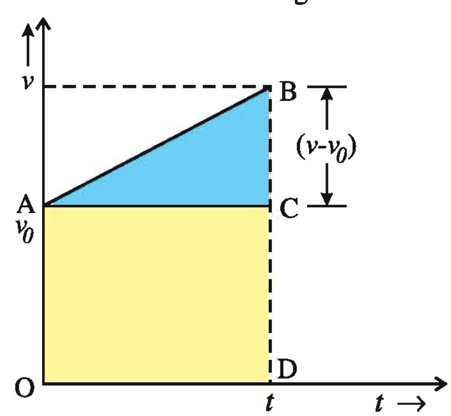


Figure 3.10

Derivation of Kinematic Equation for uniformly accelerated motion from the graph

Here, the acceleration is constant, therefore the average acceleration and instanteneus acceleration will be same over any time interval.

$\therefore$ Acceleration, a = slope of line AB

$$= \frac{v - v_0}{t - 0} = \frac{v - v_0}{t}$$

$$\therefore at = v - v_0 \qquad (3.11.1)$$

$$\text{or} \quad \boxed{v = v_0 + at} \qquad (3.11.2)$$

Now, displacement of a particle during time t is equal to the area of region OABCD in $v - t$ graph.

$\therefore$ x = Area of rectangle OACD + Area of ΔACB

$$= v_0 t + \frac{1}{2}(v - v_0)t \qquad (3.11.3)$$

$$\boxed{x = v_0 t + \frac{1}{2}at^2} \qquad (3.11.4)$$

(Putting, $v - v_0 = a$ from equation 3.11.1)

From equation (3.11.3)

$$x = v_0 t + \frac{1}{2}vt - \frac{1}{2}v_0 t$$

$$\therefore x = \left(\frac{v + v_0}{2}\right)t = \bar{v}\, t \qquad (3.11.5)$$

Where, Average velocity = $\bar{v} = \frac{v + v_0}{2}$

(only for constant acceleration)

From equation (3.11.1), $t = \frac{v - v_0}{a}$,

substituting in equation (3.11.5), we get

$$x = \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = \frac{v^2 - v_0^2}{2a}$$

$$\therefore \boxed{2ax = v^2 - v_0^2} \qquad (3.11.6)$$

Equation (3.11.2), (3.11.4) and (3.11.6) are equations for uniformly accelerated motion.

The set of above equations was obtained by assuming that at $t = 0$, position of the particle, is $x = 0$. We can obtain more general equation if particle at $t = 0$ is at $x = x_0$. In their general form above equation can be written as follows : (replacing x by $x - x_0$) :

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$2a(x - x_0) = v^2 - v_0^2$$

In the above equations the sign of v_0, v and a should be considered according to which the motion of particle is either along positive or along negative direction.

Illustration 9 : A particle with initial velocity v_0 moves with a constant acceleration in a straight line. Show that the distance travelled in n^{th} second is $v_0 + \frac{a}{2}(2n - 1)$,

Solution : Distance travelled in n^{th} second,

$$d = \begin{pmatrix}\text{distance travelled}\\ \text{in } n \text{ second}\end{pmatrix} - \begin{pmatrix}\text{distance travelled}\\ \text{in } (n-1) \text{ second}\end{pmatrix}$$

$$= (v_0 n + \frac{1}{2}an^2) - (v_0(n - 1) + \frac{1}{2}a\,(n - 1)^2)$$

$$= (v_0 n + \frac{1}{2}an^2) - (v_0 n - v_0 + \frac{a}{2}(n^2 - 2n + 1))$$

$$= (v_0 n + \frac{1}{2}an^2 - v_0 n + v_0 - \frac{1}{2}an^2 + an - \frac{a}{2})$$

$$= v_0 + an - \frac{a}{2}$$

$$= v_0 + \frac{a}{2}(2n - 1)$$

Illustration 10 : A train is moving with constant acceleration. When the ends of the train pass by a signal their speeds are u and v respectively. Calculate the speed of the midpoint of the train while passing the signal.

Solution : Suppose length of the train is l. Now, the speed of the front end of the engine while passing through the signal is u (considering the train as a rigid body, the speed of all the points of the train would be same.) and the speed of back end of the last bogey, while passing the signal is v. This means that the speed changes from u to v in travelling the distance l. Hence, using the equation of motion $v^2 - v_0^2 = 2ax$, we get

$$v^2 - u^2 = 2al \quad (1)$$

Now, suppose the speed of the mid point of the train while passing the signal is v'. This means that speed changes from u to v' in travelling a distance $\frac{l}{2}$.

$$\therefore\ v'^2 - u^2 = 2a\left(\frac{l}{2}\right) = al \quad (2)$$

Taking ratio of (1) & (2)

$$\frac{v^2 - u^2}{v'^2 - u^2} = 2$$

$$\therefore\ v' = \sqrt{\frac{u^2 + v^2}{2}}$$

Illustration 11 : Two particles start their motion from points A and B with velocity 252km h^{-1} and 144km h^{-1} and acceleration -4m s^{-2} and 8m s^{-2} respectively in the direction from A to B. Prove that they will meet each other twice. Find where and when they will meet. Consider AB = 36 m.

Solution : $v_1 = 252\frac{\text{km}}{\text{h}} = 70 \text{ m s}^{-1}$,

$v_2 = 144\frac{\text{km}}{\text{h}} = 40 \text{ m s}^{-1}$, $a_1 = -4 \text{ m s}^{-2}$

and $a_2 = 8 \text{ m s}^{-2}$

Suppose these two particles meet at time t at a distance x from point A. We use the formula,

$$x = v_0 t + \frac{1}{2}at^2$$

For the particle starting from A,

$$x = 70t + \frac{1}{2}(-4)t^2$$

$$\therefore\ x = 70t - 2t^2 \quad (1)$$

For the particle starting from B,

$$x - 36 = 40t + \frac{1}{2}(8)t^2$$

$$\therefore\ x - 36 = 40t + 4t^2 \quad (2)$$

Subtracting (2) from (1)

$$t^2 - 5t + 6 = 0$$

$\therefore$ $t = 2$ or $t = 3$ s. Thus, these particles meet each other at these two times. Substituting these values of t in eq.(1) we get $x = 132$ m and $x = 192$ m

Thus, both the particles will meet each other at a distance of 132 m and again at after at 192 m.

Illustration 12 : When brakes are applied to a moving vehicle, the distance it travels before stopping is called braking distance.

Obtain the formula for braking distance.

Solution : Let the distance travelled by the vehicle before it stops, after the application of brake be d_s. From the equation,

$$v^2 - v_0^2 = 2ax$$

$$0 - v_0^2 = 2(-a)d_s$$

$$\therefore\ d_s = \frac{v_0^2}{2a}$$

Thus, braking distance is directly proportional to square of the velocity of the vehicle and its braking capacity. Doubling the initial velocity increases the braking distance four times, for the same deceleration. Braking distance is an important factor considered in setting speed limits in School zones and Hospital zones.

Kinematic Equations for Freely Falling Body : The acceleration produced in the object due to Earth's gravitational force is known as acceleration due to gravity (g). This acceleration is in downword direction. This is called acceleration due to gravity (g). If air resistance is neglected the object can be said to be in free fall. If the height through which the object falls is small compared to the earth's radius, g can be taken to be constant equal to 9.8m s^{-2}. Free fall is thus a case of motion with unifom acceleration.

Consider the upward direction to the along the positive Y–axis, then the due to gravity will be in negative Y direction. Therefore, by putting $a = -g$ in equations of motion,

$$v = v_0 - gt \quad (3.11.7)$$

$$y = v_0 t - \frac{1}{2}gt^2 \quad (3.11.8)$$

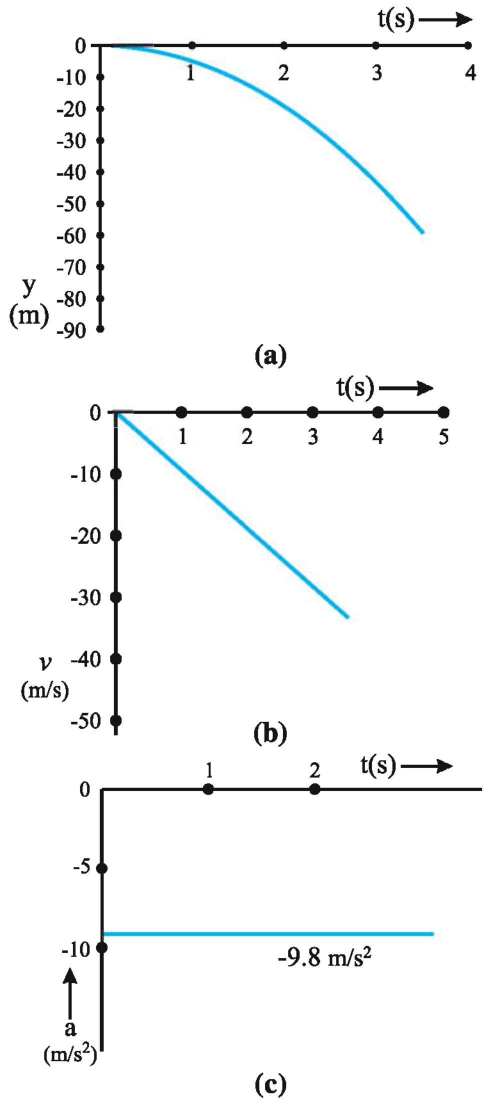
$$v^2 - v_0^2 = -2gy \quad (3.11.9)$$

(Taking $y_0 = 0$)

For a free fall, the initial velocity will be zero i.e. $v_0 = 0$ in above equations.

Sometimes it is convenient to choose vertically downward as the positive Y–axis and upward as negative Y–axis. Then the $a = g$ in equations of motion.

For a free fall, the $y - t$, $v - t$ and $a - t$ graphs are shown in Fig. 3.11.



Graphs of motion for free fall

Figure 3.11

Illustration 13 : An object is thrown in vertically upward direction. Calculate (a) the time to reach maximum height (b) the maximum height.

Solution : (a) Consider the vertically upward direction as positive then v_0 will be positive and the acceleration due to gravity will be negative $(-g)$. At the maximum height $v = 0$.

Therefore, from $v = v_0 + at$ we get,

$$0 = v_0 - gt$$

$$\therefore\ t = \frac{v_0}{g}$$

(b) If maximum height achieved by the object is h, then, put $y = h$, $t = \frac{v_0}{g}$ and $a = -g$ in the equation $y = v_0 t + \frac{1}{2}at^2$

$$\therefore\ h = v_0\frac{v_0}{g} - \frac{1}{2}g\left(\frac{v_0}{g}\right)^2$$

$$= \frac{v_0^2}{g} - \frac{1}{2}\frac{v_0^2}{g}$$

$$h = \frac{v_0^2}{2g}$$

Illustration 14 : A stone after having fallen from rest under the influence of gravity for 6 s crashes through a horizontal glass plate, thereby losing $\frac{2}{3}$ of its velocity. If it then reaches the ground in 2 s, find the height of the plate above the ground.

Solution : The velocity v of the stone just before striking the glass plate is given by.

$$v = v_0 + gt$$

Here, $v_0 = 0$, $g = -9{\cdot}8\text{m s}^{-2}$, and $t = 6$ s.

$$\therefore\ v = 0 + (-9{\cdot}8)(6) = -58{\cdot}8\text{m s}^{-1}$$

After striking, the stone loses $\frac{2}{3}$ of its veloctity, therefore, velocity of the stone after striking,

$$v_0 = \frac{v}{3} = -\frac{58.8}{3} = -19{\cdot}6\text{m s}^{-1}$$

Suppose, the height of the glass plate above the ground is h.

$$\therefore\ h = v_0 t + \frac{1}{2}gt^2$$

Put, $v_0 = -19{\cdot}6\text{m s}^{-1}$, $t = 2\text{s}$, $g = -9{\cdot}8\text{m s}^{-2}$, $h = -h$,

$$-h = (-19{\cdot}6 \times 2) + \frac{1}{2}(-9{\cdot}8)(2)^2$$

$$\therefore\ h = 58{\cdot}8 \text{ m}$$

Illustration 15 : A coin is dropped from a balloon going up with a uniform velocity of 12m s^{-1}. If the balloon was 81m high when the coin was dropped, find its height when the coin hits the ground. Take the g = 10m s^{-2}

Solution : At $t = 0$, the coin was going up with a velocity of 12m s^{-1}. After that it moved as a freely falling body. Take vertically upward direction as positive,

$$y = -81 \text{ m},\ v_0 = 12\text{m s}^{-1},\ a = g = -10\text{m s}^{-2}$$

$$y = v_0 t + \frac{1}{2} g t^2$$

$$-81 = (12)t + \frac{1}{2}(-10)t^2$$

$$\therefore\ 5t^2 - 12t - 81 = 0$$

$$\therefore\ t = \frac{12 \pm \sqrt{(12)^2 - 4(5)(81)}}{2(5)} = \frac{12 \pm 42}{10}$$

$\therefore$ $t = 5{\cdot}4$ s OR $t = -3$ s, which is not possible i.e. the coin reaches the ground after 5.4 s. During this time the balloon has moved uniformly up. The distance covered by it is,

y_1 = (velocity × time) = (12) (5·4) = 64·8m.

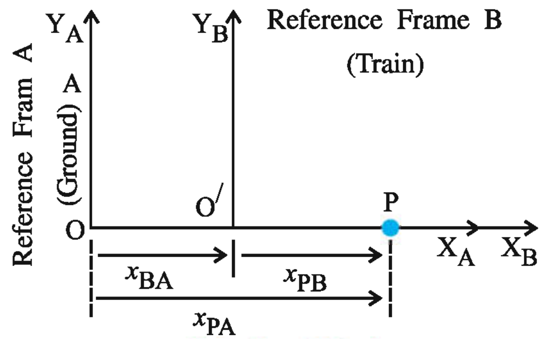
Hence, the height of the balloon when the coin reaches the ground is,

81 m + 64·8 m = 145·8 m.

3.12 Relative Velocity

Motion is a relative concept. We had earlier seen that the velocity of an object is different for different frame of reference. Our common experience is that, for a person sitting in a train, the speed of a fly flying in the train along the direction of moving train appears to be less observed by a person standing on the ground. If the fly is moving in the direction opposite to the motion of the train, the stationary person on the ground will find the speed of the fly to be less than the speed of the train. To understand such observations, we now introduce the concept of a relative velocity.

Let A be the frame of reference associated with person standing on the ground. B is the frame of reference associated with the train moving with uniform velocity. Both these reference frames are inertial frames of reference. Let us denote the fly with P. Then, let x_{PA} be the position of P with respect to origin O of frame of reference A, and Let x_{PB} be the position of P with respect to origin O' of frame of reference B at $t = 0$. (See Fig 3·12)



Relative Velocity

Figure 3.12

From the Fig. 3·12,

$$x_{PA} = x_{PB} + x_{BA}$$

Differentiating with respect to 't',

$$\frac{d(x_{PA})}{dt} = \frac{d(x_{PB})}{dt} + \frac{d(x_{BA})}{dt}$$

$$\therefore\ v_{PA} = v_{PB} + v_{BA} \qquad (3.12.1)$$

$$\text{OR } v_{BA} = v_{PA} - v_{PB} \qquad (3.12.2)$$

Here, v_{PA} = velocity of P (fly) with respect to reference frame A (ground)

v_{PB} = velocity of P with respect to reference frame B (train)

v_{BA} = velocity of reference frame B w.r.t. reference frame A.

In the above illustration, let v_{BA} = + 10m s^{-1} be the velocity of train with respect to the ground, v_{PB} = + 2m s^{-1} be velocity of fly with respect to the train (in the direction of moving train). From equation 3·12·1. The velocity of the fly with respect to the ground will be v_{PA} = 10 + 2 = +12m s^{-1}. This velocity is greater than the velocity of train. If the fly in the train flies in the opposite direction to the motion of the train then v_{PB} = −2m s^{-1} and its velocity with respect to the ground will be v_{PA} = 10 − 2 = 8m s^{-1}

Now, if P is the frame of reference associated with the ground (G) and reference frames A and B are associated with two particles, then from equation 3·12·2, the relative velocity of particle B with respect to particle A will be,

$$v_{BA} = v_{GA} - v_{GB} \quad (3.12.1)$$

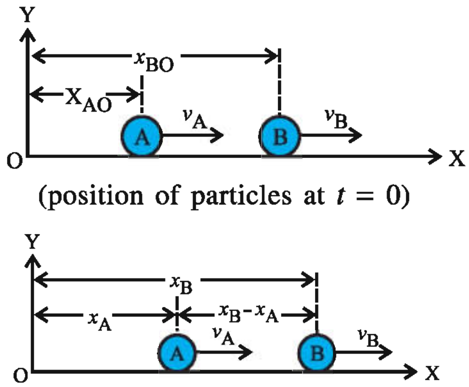
$$v_{BA} = v_{BG} - v_{AG} = v_B - v_A \quad (3.12.3)$$

In the same way, the relative veocity of particle A w.r.t. particle B will be

$$v_{AB} = v_{AG} - v_{BG} = v_A - v_B$$

(v_{AG} and v_{BG} are the relative velocities w.r.t. ground can also be denoted by v_A and v_B respectively)

Relative Displacement : Consider two particles A and B moving along the X–axis with uniform velocities v_A and v_B. Let x_{AO} and x_{BO} be their displacement from the origin O at $t = 0$ as shown in Fig. 3·13.



(position of particles at $t = 0$)

(position of particles at $t = t$)

Figure 3.13

If x_A and x_B are their position co-ordinates then, at $t = t$,

$$x_A = x_{AO} + v_A t$$

$$x_B = x_{BO} + v_B t$$

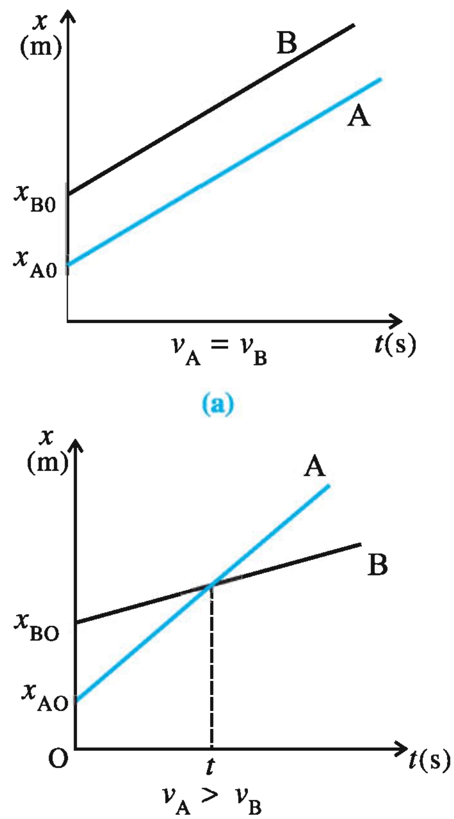
At $t = t$, the displacement of particle B with respect to particle A,

$$x_B - x_A = (x_{BO} - x_{AO}) + (v_B - v_A)t \quad (3.12.4)$$

Where, $x_{BO} - x_{AO}$ is the displacement of particle B with respect to particle A at $t = 0$. $v_B - v_A = v_{BA}$ is the relative velocity of B with respect to particle A.

(i) When $v_A = v_B$, then from equation (3.12.4), $x_B - x_A = x_{BO} - x_{AO}$ This means that the distance between two particles will remain equal to initial distance at any instant of time. (See Fig. 3·14 (a)) Here, relative velocities of both the particles will be $v_{AB} = v_{BA} = 0$.

(ii) If $v_A > v_B$, both the particales will meet at time t. (See Fig. 3·14 (b))



(a)

(b)

Figure 3.14

At this instant, their relative displacement will be $x_B - x_A = 0$. There after particle A overtakes particle B at this time.

(iii) If $v_B > v_A$, then the relative displacement $x_B - x_A$ of both the particles will increase with time. Therefore, they will never meet on their path.

Illustration 16 : When the driver of Shatabdi Express, running with velocity v_1 sights a goods train going ahead of him at a distance x in the same direction on the same track, running with velocity v_2 (where $v_2 < v_1$) he applies brakes. In order to avoid an accident, what should be the magnitude of the deceleration produced by the brakes ?

Solution : It is clear that the velocity of Shatabdi Express relative to goods train is $v_1 - v_2$. If this relative velocity becomes zero in distance x, the accident can be avoided. Let the necessary deceleration for this be a. Using the formula $2ax = v^2 - v_0^2$ we get

$$-2ax = 0 - (v_1 - v_2)^2$$

$$\therefore a = \frac{(v_1 - v_2)^2}{2x}$$

Illustration 17 : Two cars A and B are at positions 100m and 200m from the origin at $t = 0$. They start simultaneously with constant velocities 10m s^{-1} and 5m s^{-1} respectively in the same direction. Calculate the time and position at which they will overtake one another.

Solution :

x_{BO} = 200m, x_{AO} = 100m

v_A = 10m s^{-1}, v_B = 5m s^{-1}

Now, $x_B - x_A = (x_{BO} - x_{AO}) + (v_B - v_A)t$

Suppose, at $t = t$, both the cars overtake one another.

Therefore, $x_B = x_A$ and $x_B - x_A = 0$

$0 = (200 - 100) + (5 - 10)\ t$

$$\therefore\ t = \frac{100}{5} = 20\text{ s}$$

Now, let both the cars overtake one another at distance x from x_{AO}.

$\therefore\ x = x_{AO} + v_A t$

$= 100 + (10)\ (20) = 300\text{m}$

Illustration 18 : The distance between Ahmedabad and Vadodara is 100 km. Two trains set off simultaneously from Ahmedabad and Vadodara towards each other. The speed of these trains are 45 kmh^{-1} and 30 kmh^{-1} respectively. When will they cross each other ?

Solution :

Speed of train set off from Ahmedabad v_A = 45 kmh^{-1}

Speed of train set off from Vadodara v_B = −30 kmh^{-1}

(this train is moving in opposite direction)

$x_{BO} - x_{AO}$ = 100 km

When both the trains meet, the relative displacement will be zero. i.e., $x_B - x_A = 0$.

$\therefore\ x_B - x_A = (x_{BO} - x_{AO}) + (v_B - v_A)t$

$0 = 100 + (-30 - 45)t$

$$\therefore\ t = \frac{100}{75} = \frac{4}{3}\text{ hours.}$$

SUMMARY

1. **Frame of reference :** A place (and a situation) from where an observer takes his observation is called a frame of reference.
2. **Path length :** The distance travelled by a particle in some time interval is called path length. It is always positive.
3. **Displacement :** The change in the position of an object in some time interval is called displacement.

   Displacement = final position − initial position.

   Displacement can be positive, negative or zero. The path length can be equal to or greater than the displacement.
4. **Average Speed and Average Velocity :**

   The ratio of the path length and the time taken for travelling it, is known as average speed.

   The ratio of displacement and the time taken for travelling it is known as average velocity.

   $$\text{Average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{\Delta x}{\Delta t}$$

   Average speed is always positive, the direction is not important. Average velocity is in the direction of displacement. It can be positive or negative. For a given time interval the average speed can be equal or to greater than the average velocity.
5. **Instantaneous velocity :** In a given time interval, average velocity does not give the information about how fast the object moves at different instants. By

taking $\lim_{\Delta t \to 0}$ in the definition of average velocity, it gives the instantanous velocity at time t.

Instantaneous Velocity

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The magnitude of instantaneous velocity is called instantanous speed.

6. When a particle moving on a straight line covers the same distance in the same time intervals, its motion is called uniform motion.

7. In $x - t$ graph, the slope of the line connecting the final position and the initial position gives the magnitude of average velocity for a given time interval. In $x - t$ graph, the slope of the tangent drawn to the curve at a point gives the magnitude of instantaneous velocity at that instant.

8. **Average Acceleration and Instantaneous Acceleration :** If the change in velocity of an object is Δv in time interval Δt, then,

   Average acceleration

$$< a > = \frac{\text{change in velocity}}{\text{time interval}} = \frac{\Delta v}{\Delta t}$$

   Instantaneous acceleration at time t,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \text{ or } a = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

   Acceleration is a vector quantity. It is in the direction of change in velocity The SI unit is m s^{-2}

9. The slope of velocity versus time graph gives the value of average acceleration for a given time interval. The slope of the tangent drawn at a point, gives the value of acceleration at that instant. If velocity and acceleration are in the same direction, the speed of the particle increases. If they are in opposite direction speed decreases. This is known as deceleration of a particle.

10. The area under the $v - t$ graph for a given time interval is equal to the displacement / (total distance) of the particle. For finding the total distance, consider the negative area as positive.

11. **Kinematic Equations for Uniformly Accelerated Motion :**

   $v = v_0 + at$ where, x_0 = initial position of a particle.

   $x = x_0 + v_0 t + \frac{1}{2}at^2$ x = final position of a particle.

   $v^2 - v_0^2 = 2a(x - x_0)$ v_0 = Initial velocity

   v = Final velocity of a particle.

12. Velocity of particle B with respect to particle A is $v_{BA} = v_B - v_A$. Velocity of particle A w. r. t. particle B is $v_{AB} = v_A - v_B$ and $v_{AB} = -v_{BA}$

EXERCISES

Choose the correct option from the given options :

1. A car moves from one end of a semicircular path of radius r, to the other end. For this car, ratio of pathlength to the magnitude of the displacement will be ....

   (A) $\frac{\pi}{2}$ (B) π (C) $\frac{3\pi}{2}$ (D) 2π

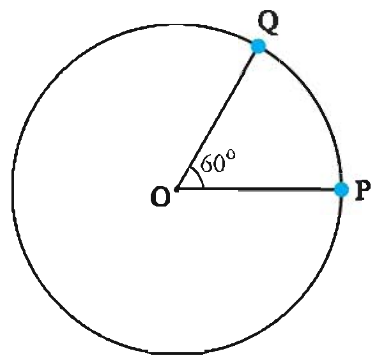
2. A person travelled a distance of 3 km along a straight line in the North direction, then he travelled 2 km in West direction and then 5 km in South direction. The magnitude of the displacement of this person would be ....

   (A) $2\sqrt{2}$ km (B) $3\sqrt{2}$ km (C) $4\sqrt{2}$ km (D) 10 km

3. As shown in Fig. 3.15, an ant moves from point P to Q along the circular track of radius 1m. If the time taken is 1 minute, what is the average velocity of the ant ?

   (A) $\frac{\pi}{40}$ m s^{-1} (B) $\frac{\pi}{60}$ m s^{-1}

   (C) $\frac{3\pi}{160}$ m s^{-1} (D) $\frac{1}{60}$ m s^{-1}



4. An object is thrown in vertically upward direction. Which of the following velocity time graph is appropriate for it ?

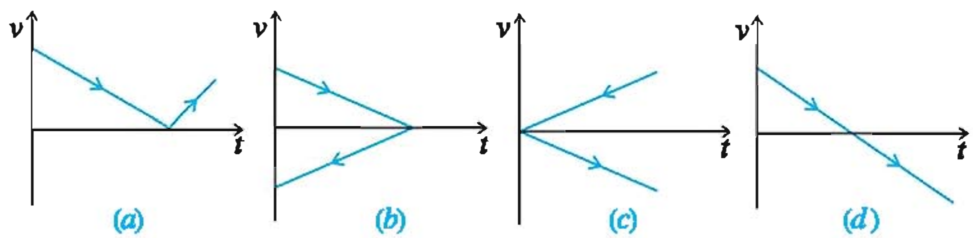


Figure 3.16

5. The change in speed of a car, during a journey of the car for 2 hours, with time is shown in Fig. 3.17. The maximum acceleration of the car is in the region.......... .

   (A) OA

   (B) BC

   (C) CD

   (D) DE

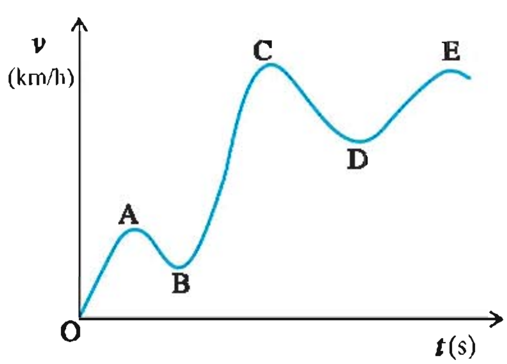


Figure 3.17

6. Fig. 3.18 shows the graph of speed versus time for a moving truck. What will be the distance travelled by truck car in the last two seconds ?

(A) 60 m

(B) 90 m

(C) 20 m

(D) 40 m

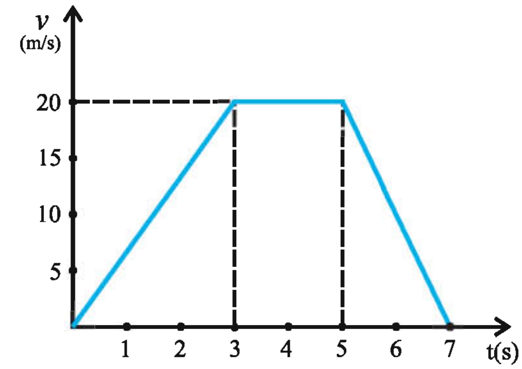


Figure 3.18

7. Fig. 3.19 shows the graph of velocity versus time for a moving particle. The displacement of a particle in time interval from 0 to 20s is ....

(A) 0

(B) 60 m

(C) 120 m

(D) −120 m

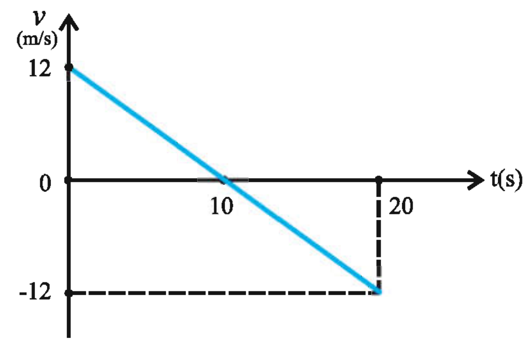


Figure 3.19

8. Fig. 3.20 shows the graph of velocity (v) of a moving object with position (x). Which of the graphs shown in Fig. 3.20 (a) correctly represents the variation of acceleration (a) with position (x) ?

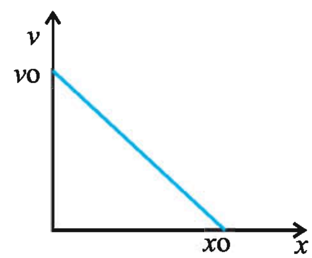
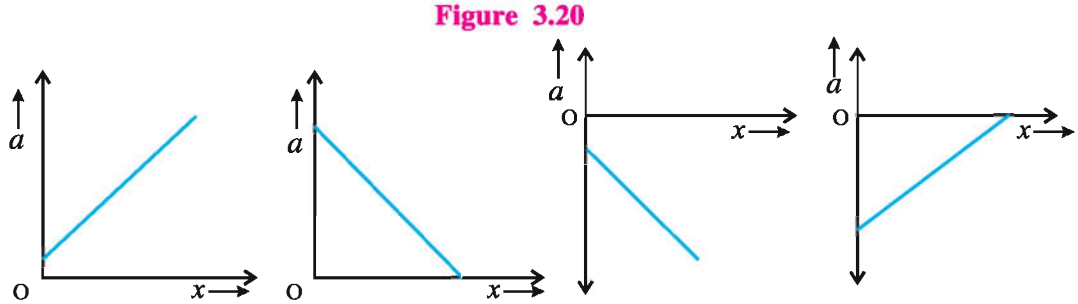


Figure 3.20



(a) (b) (c) (d)

Figure 3.20

Note : From the graph, slope of line will be $m = -{}^{v_0}/_{x_0}$ and intercept will be $C = v_0$. Therefore equation of line will be $v = (-{}^{v_0}/_{x_0})\ x = v_0$ now obtain equation of acceleration.

9. The velocity time graph of a body is shown in Fig. 3.21. What will be the ratio of the average accelerations during the intervals OA and AB ?

(A) 1 (B) $\frac{1}{2}$

(C) $\frac{1}{3}$ (D) 3.

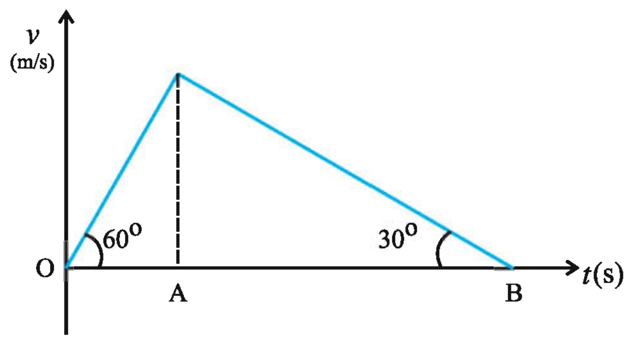


Figure 3.21

10. The displacement of a particle is given by : $y(t) = a + bt + ct^2 - dt^4$. Hence, the initial velocity and initial acceleration of the particle are .... and .... respectively. (a, b, c and d are constants)

(A) $b, -4d$ (B) $-b, 2c$ (C) $b, 2c$ (D) $2c, -4d$.

11. The graph of displacement versus time is shown in Figure 3.22 for a particle performing motion on X–axis. From this graph we can say that...

(A) the particle is moving continuously in X–direction.

(B) the velocity of the particle increases up to time t_0 and then become constant.

(C) the particle is stationary.

(D) the particle travels with constant velocity up to time t_0 and then its velocity becomes zero.

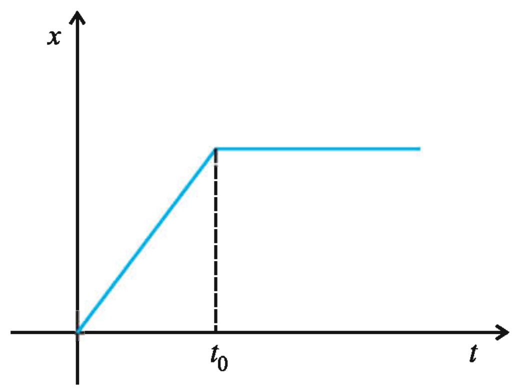


Figure 3.22

12. The displacement (in metre) of a particle varies with time (in second) according to the equation : $y = -\frac{2}{3}t^2 + 16t + 2$. How long does the particle take to come to rest ?

(A) 12 s (B) 8 s (C) 16 s (D) 10 s

13. The position of an object varies with time t as $x = at^2 - bt^3$. At what time will the acceleration of the object will become zero ? (Where a and b are constant)

(A) $\frac{2a}{3b}$ (B) $\frac{a}{b}$ (C) $\frac{a}{3b}$ (D) zero

14. Displacement of an object is related to time as $x = at + bt^2 - ct^3$ where, a, b and c are constants of motion. The velocity of the object when its acceleration is zero is given by ....

(A) $a + \frac{b^2}{c}$ (B) $a + \frac{b^2}{2c}$ (C) $a + \frac{b^2}{3c}$ (D) $a + \frac{b^2}{4c}$

15. An object is allowed to fall freely from a cliff. When it travels a distance 'h', its velocity is v. Hence, in travelling further distance of .... its velocity will become $2v$.

(A) $4h$ (B) $3h$ (C) $2h$ (D) h.

16. When a ball A is thrown in vertically upward direction with velocity v_0, exactly at the same time another ball B is allowed to fall freely, from height h. The velocity of A with respect to B at time t is ....

(A) v_0 (B) $v_0 - 2gt$

(C) $v_0 - gt$ (D) $\sqrt{v_0^2 - 2gh}$

17. The ratio of the distances travelled in the fourth and the third second by a particle, moving over a straight path with constant acceleration, is ....

(A) $\frac{7}{5}$ (B) $\frac{5}{7}$ (C) $\frac{7}{3}$ (D) $\frac{3}{7}$

18. A car, starting from position of rest, moves with constant acceleration x. Then it moves with constant deceleration y and becomes stationary. If the total time elapsed during this is t, what was be the maximum velocity of the car ?

(A) $\frac{xy}{x-y}t$ (B) $\frac{xy}{x+y}t$

(C) $\frac{x^2y^2}{x^2+y^2}t$ (D) $\frac{x^2y^2}{x^2-y^2}t$

19. A car moving over a straight path, covers a distance x with constant speed of v_1 and then the same distance with constant speed of v_2. Average speed of a car would be obtain by .... formula.

(A) $\bar{v} = \frac{v_1 + v_2}{2}$ (B) $\bar{v} = \sqrt{v_1 v_2}$

(C) $\frac{2}{\bar{v}} = \frac{1}{v_1} + \frac{1}{v_2}$ (D) $\frac{1}{\bar{v}} = \frac{1}{v_1} + \frac{1}{v_2}$

20. The $x - t$ graph of a body moving in a straight line is shown in Fig. 3.23. Which one of the graphs shown in Fig. 3.24 represent the $v - t$ graph of the motion of the body ?

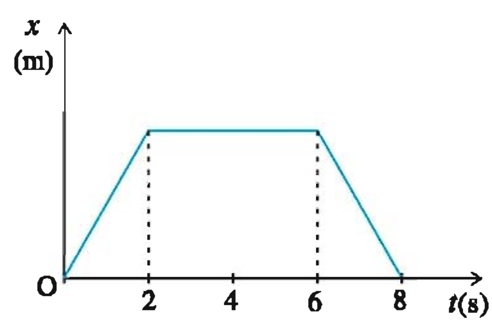


Figure 3.23

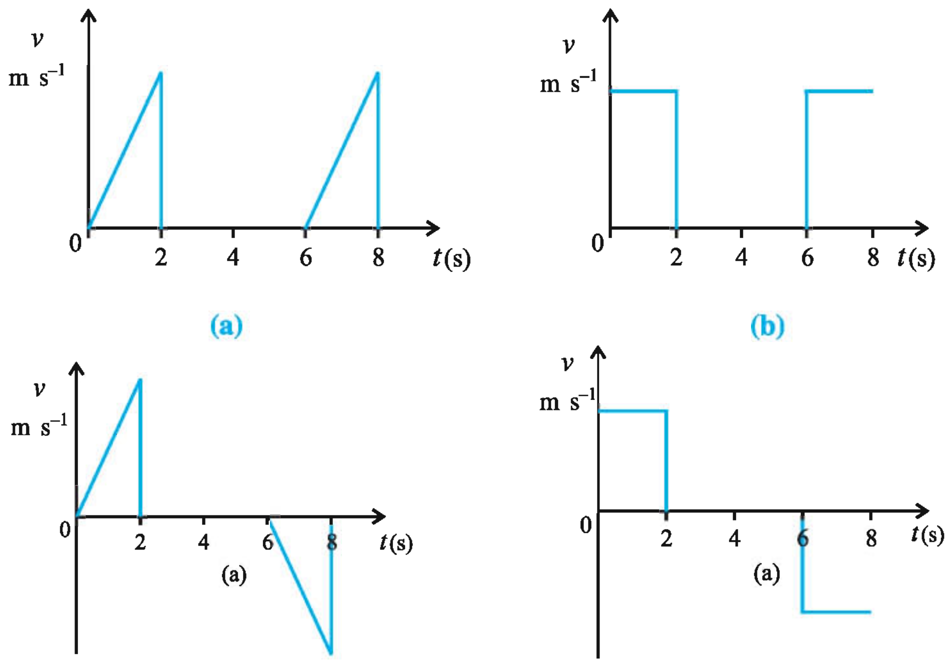


Figure 3.24

21. A ball is thrown in vertically upward direction. Neglecting the air resistance, acceleration of a ball in air will be...

(A) zero

(B) continuously increasing

(C) remain constant

(D) increase when ball is going up and will decrease when it is coming down.

22. A balloon starts rising from the ground with acceleration of 1.25m s^{-2} After 8 s a stone is released from the balloon. How much time will it take to reach the ground ?

(A) 2 s (B) 4 s (C) 6 s (D) 10 s

23. Fig. 3.25 shows the $x - t$ graphs of car A and B. The velocity of car A with respect to car B will be...

(A) +5m s^{-1}

(B) −2.5m s^{-1}

(C) −5m s^{-1}

(D) +2.5m s^{-1}

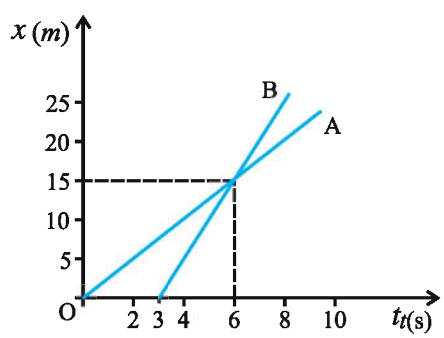


Figure 3.25

24. In above question (No. 23), what would be the position of the car B with respect to the car A at $t = 0$? (Assume that car B stants uniform motion from $t = 0$).

(A) +15 m (B) −15 m (C) −10 m (D) −25 m

ANSWERS

1. (A)	2. (A)	3. (D)	4. (D)	5. (B)	6. (C)
7. (A)	8. (D)	9. (D)	10. (C)	11. (D)	12. (A)
13. (C)	14. (C)	15. (B)	16. (A)	17. (A)	18. (B)
17. (C)	20. (D)	21. (C)	22. (B)	23. (B)	24. (B)

Answer the following questions in short :

1. What is the difference between average speed and average velocity.
2. What is acceleration ? What is its direction ?
3. What is braking distance ?
4. Can the $x - t$ graph of a moving object be paralled to the position axis ?
5. When will the relative velocity of two moving cars be zero ?
6. An object is falling freely under the effect of gravitational force. What will be the distance traveled by the object in 1 s ?
7. What do the slope and the area under a $v - t$ graph represent ?
8. A ball is thrown up in air. What is the acceleration and velocity at the instant it reaches its maximum height ?
9. In one dimensional motion, can an object has zero velocity and non–zero acceleration at any instant ? Give one example.
10. Draw a nature of a graph of a velocity versus time and acceleration versus time a for freely falling body.
11. What does the area under acceleration versus time graph for any time interval represent ?

Answer the following questions :

1. Explain the difference between pathlength and displacement with the help of an example.
2. Explain instantaneous velocity.
3. Explain $x - t$ and $v - t$ graphs for uniform motion.
4. Using the graphical method, obtain the kinematic equations for uniformly accelerated motion.
5. Explain relative velocity.

Solve the following problems :

1. A motorcyclist covers $\frac{1}{3}$ of a given distance with a speed of 10 kmh^{-1}, the next $\frac{1}{3}$ at 20 kmh^{-1} and the last $\frac{1}{3}$ at of 30 kmh^{-1}. What is the average speed of the motorcycle for the entire journey ?

   [**Ans. :** 16.36 kmh^{-1}]

2. The distance between two stations is 40 km. A train takes 1 hour to travel this distance. The train, after starting from the first station, moves with constant acceleration for 5 km; then it moves with constant velocity for 20 km and finally its velocity keeps on decreasing continuously for 15 km and it stops at the other station. Find the maximum velocity of the train.

   [**Ans. :** 60 kmh^{-1}]

3. A monkey standing on the ground wants to climb to the top of a vertical pole 13m tall. He climbs 5m in 1*s* and then slips downwards 3m in the next second. He again climbs 5m in 1s and slips by 3m in the next second and so on. Draw the $x - t$ graph of the motion and hence find the time he will take to reach top of the pole.

[**Ans. :** 9 s]

4. A motorcycle, starting from rest, moves with constant acceleration of +2.6m s^{-2}. After travelling a distance of 120m, it accelerates with –1.5m s^{-2} till its velocity becomes +12m s^{-1} Calculate the total distance travelled by the motorcycle during this journey.

[**Ans. :** 280 m]

5. A ball thrown in vertically upward direction attains maximum height of 16m. At what height would its velocity be half of its initial velocity ?

[**Ans. :** 12 m]

6. An object is allowed to fall freely from a tower of height 39.2m; exactly at the same time another stone is thrown from the bottom of the tower in vertically upward direction with a velocity of 19.6m s^{-1} Calculate when and where these two stones would meet ?

[**Ans. :** 2*s*, 19.6 m]

7. The motion of a body along a straight line is described by the equation :

$x = t^3 + 4t^2 - 2t + 5$

Where x is in metre and t is in second.

(a) Find the velocity and acceleration of the body at t = 4 s.

(b) Find the average velocity and average acceleration during the time interval from t = 0 to t = 4s.

[**Ans. :** (a) v = 78m s^{-1}; a = 32m s^{-2} (b) $\langle v \rangle$ = 30m s^{-1}, $\langle a \rangle$ = 20m s^{-2}]

8. A driver of train A, moving at a speed 30m s^{-1} sights another train B going on the same track and in the same direction with speed 10m s^{-1}. He immediately applies brakes that gives his train a constant retardation of 2m s^{-2}. What must be the minimum distance between the trains in order to avert a collision ?

[**Ans. :** 100 m]

9. An object is moving with constant acceleration. Its velocity is 48m s^{-1} at the end of 10 second and becomes 68m s^{-1} at the end of 15 second. What would be the distance travelled by the object in 15 second ?

[**Ans. :** 570m]

10. The $v - t$ graph of a particle moving in a straight line is shown in Fig. 3.26. Obtain the distance travelled by the particle from (a) t = 0 to t = 10 s, and from (b) t = 2 s to 6 s.

[**Ans. :** 60m , 36m]

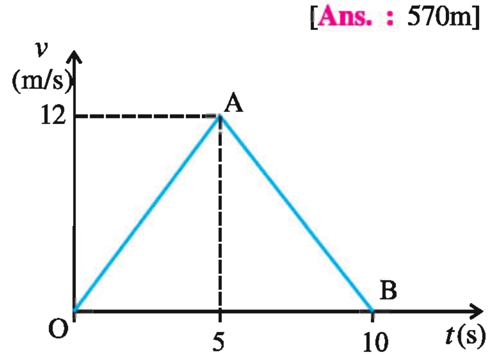


Figure 3.26

11. A 120m long train is going from East to West with velocity 10m s^{-1} A bird, flying due East with velocity 5m s^{-1}, crosses the train. Calculate the time taken by the bird to cross the train.

[**Ans. :** 8 s]

12. If velocity (in m s^{-1}) varies with time as $v = 4t$, find the distance travelled by the particle in time interval of $t = 2$ s to $t = 4$ s.

[**Ans. :** 24m]

APPENDIX 3.1
DIFFERENTIATION

When the value of a quantity changes, it takes some time. For example, when a bowl of water is placed on a hot plate, and suppose the temperature (T) of water incereases from 30°C to 75°C in 5 minute. So, the average rate of increase in its temperature, in this time interval is 9°C/min.

Average rate of change in temperature $= \frac{\Delta T}{\Delta t} = \frac{75^{\circ}C - 30^{\circ}C}{5 \text{ min}} = 9^{\circ}C/min$

9°C/min is the time rate of change in temperature. But in order to find the change in temperature at a particular instant, we have to use the concept of limit.

Suppose, at time t the temperature is T and at time $t + \Delta t$ it is $T + \Delta T$. Thus, the change in temperature is ΔT in the time interval Δt. Now taking the time interval Δt smaller and smaller in the ratio $\frac{\Delta T}{\Delta t}$, we get this rate of change nearer to the instant t, and by taking limit $\Delta t \to 0$, we get the rate of change of temperature at time t. Symbolically this is written as $\frac{dT}{dt}$.

$$\therefore \lim_{\Delta t \to 0} \frac{\Delta T}{\Delta t} = \frac{dT}{dt}$$

Here, $\frac{dT}{dt}$ is called the derivative of temperature (T) with respect to time (t). The operation of obtaining the derivative is called differentiation.

Suppose, a quantity y is a function of another quantity x. i.e. $y = f(x)$. When there is a change in x, y also changes according to the function $f(x)$. In order to find the rate of this change (with respect to x) at any value of x, we must find $\frac{dy}{dx}$ at that particular 'x'. In y versus x graph, $\frac{dy}{dx}$ gives the value of slope of the tangent drawn at point $x = x$.

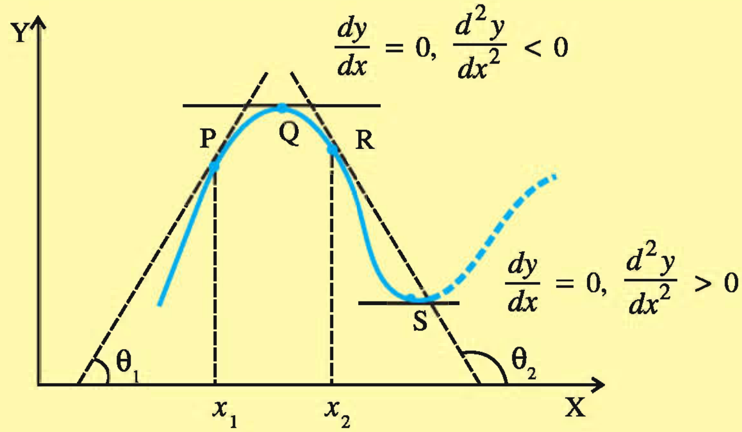


Figure A

For the curve shown in Fig.(A),

the slope of the tangent drawn at a point P = $tan\theta_1 = \left.\frac{dy}{dx}\right|_{x=x_1}$

the slope of the tangent drawn at a point R = $tan\theta_2 = \left.\frac{dy}{dx}\right|_{x=x_3}$

the slope of the tangent drawn at a point Q and S = $tan 180^o = \frac{dy}{dx} = 0.$

From the Figures it is clear that the point at which $\frac{dy}{dx} = 0$, y is maximum (point Q) or minimum (Point S). If $\frac{d^2y}{dx^2} < 0$ then y is maximum and if $\frac{d^2y}{dx^2} > 0$, then y is minimum. Here, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ is called second derivative of y with respect to x.

If we are given the graph of y versus x, we can find $\frac{dy}{dx}$ at any point of curve by drawing tangent at that point and determine its slope. If no graph is drawn but an algebraic relation between y and x is given in the form of an equation, we can find $\frac{dy}{dx}$ algebrically. Let us take an example.

Suppose, A is the area of a square with length L.

Therefore, $A = L^2$.

If we increase the length by ΔL, the area will also increase by ΔA. The new length of the square will be L + ΔL and new area will be A + ΔA.

$A + \Delta A = (L + \Delta L)^2 = L^2 + 2L\,\Delta L + (\Delta L)^2$

$\therefore\ \Delta A = 2\,L\Delta L + (\Delta L)^2\ (\because A = L^2)$

$\therefore\ \frac{\Delta A}{\Delta L} = 2L + \Delta L$

Now, if ΔL is made smaller and smaller, 2L + ΔL will approach to 2L.

Thus, $\frac{dA}{dL} = \lim_{\Delta L \to 0} \frac{\Delta A}{\Delta L} = 2L$

Following table gives the $\frac{dy}{dx}$ for some important functions.

y	$\frac{dy}{dx}$	y	$\frac{dy}{dx}$
x^n	nx^{n-1}	$\sec x$	$\sec x \tan x$
$\sin x$	$\cos x$	$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cos x$	$-\sin^{-2} x$	$\ln x$	$\frac{1}{x}$
$\tan x$	$\sec^2 x$		
$\cot x$	$-\operatorname{cosec}^2 x$	e^x	e^x
$\sin kx$	$k \cos x$	a^x	$a^x \ln a$
$\cos kx$	$-k \sin x$		

Working rules of derivatives :

1. $\frac{d}{dx}(k) = 0$ (where, k is a constant)

2. $\frac{d}{dx}(x) = 1$

3. $\frac{d}{dx}(ky) = k\frac{dy}{dx}$ where k is a constant.

4. If $y = u \pm v$, then $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

5. If $y = uv$, then $\frac{dy}{dx} = u\frac{dv}{dx} \pm v\frac{du}{dx}$

6. If $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

7. $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Illustration : For $y = x^3 + \frac{4}{\sqrt{x}} - \frac{3}{x^2}$, find $\frac{dy}{dx}$

$$y = x^3 + \frac{4}{\sqrt{x}} - \frac{3}{x^2}$$

$$= x^3 + 4x^{\frac{-1}{2}} - 3x^{-2}$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^3) + \frac{d}{dx}(4x^{\frac{-1}{2}}) + \frac{d}{dx}(-3x^{-2})$$

$$= 3x^{3-1} + 4\left(\frac{-1}{2}\right)x^{\frac{-1}{2}-1} + (-3)(-2)x^{-2-1}$$

$$= 3x^2 - 2x^{\frac{-3}{2}} + 6x^{-3}$$

APPENDIX 3.2
INTEGRATION

Standard formulae are available to find out the area of regular shapes like triangle, square, rectangle, circle, etc. Here, we shall study a method to find out the area of an irregular shape and during this study we shall get some idea regarding integration.

Suppose, some quantity y is a function of a variable quantity x, i.e. $y = f(x)$. Suppose the graph of $y \rightarrow x$ is as shown in Fig. (B).

Here, we want to find the area (PQRS) under the curve enclosed between $x = x_0$ and $x = x_N$

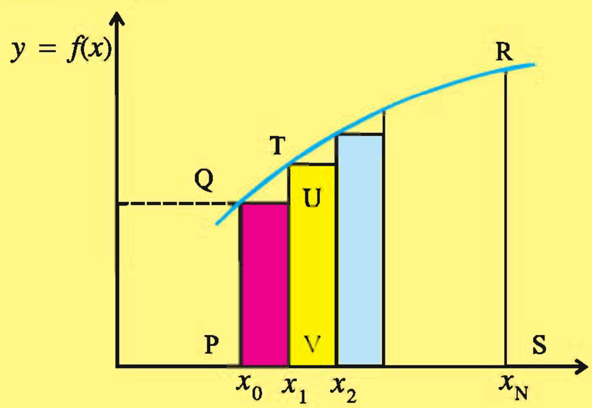


Figure B

For this purpose we divide the area between x_0 and x_N in N strips, each of infinitesimal width Δx.

From the Fig. (B) it is clear that the sum of areas of all such strips is the required area.

For the first strip (x_0 to $x_1 = x_2 + \Delta x$) taking $f(x) = f(x_1)$

area $\Delta A_1 = f(x_1)\Delta x$.

For the second strip (x_1 to $x_2 = x_1 + \Delta x$) taking $f(x) = f(x_2)$,

area $\Delta A_2 = f(x_2)\Delta x$.

Thus, total area obtained by taking sum of the area each strip is,

$A' = f(x_1)\Delta x + f(x_2)\Delta x + ... + f(x_N)\Delta x$

$$= \sum_{i=1}^{N} f(x_i)\Delta x \qquad (1)$$

But this area (A') will be slightly different from the actual area (A) which we want to find. The reason is as follows :

$\Delta A_1 = f(x_1)\Delta x$ is the area of the rectangle PQUV, but the required area is of the strip PQTV. Thus, in the first step the calculated area is somewhat less than the true area of the strip. Same will happen for the other strips also.

It is obvious that as we keep on increasing the number (N) of such strips (i.e. decreasing the width Δx), the difference between the calculated area and the actual value will keep on decreasing. And by taking the width of strip $\Delta x \to 0$, we can get the exact value. Symbolically,

$$A = \lim_{x \to 0} \sum_{i=1}^{N} f(x_i)\ x$$

$$= \int_{x_0}^{x_N} f(x)dx$$

Thus, it can be said that the **limit of summation is integration.**

$\int_{x_0}^{x_N} f(x)dx$ is the continuous sum at $f(x)$ over x, from $x = x_0$ to $x = x_N$.

It is called the (definite) integration of $f(x)$. Over x from x_0 to x_N. Integration is a reverse process of differentiation.

Integrals of Some Standard Functions :

$f(x)$	$F(x) = \int f(x)dx$	$f(x)$	$F(x) = \int f(x)dx$
x^n $(n \neq + 1)$	$\frac{x^{n+1}}{n+1} + c$	$(ax + b)^n$	$\frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + c$
$\frac{1}{x}$	$\ln x + c$	$\sin x$	$-\cos x + c$
e^x	$e^x + c$	$\cos x$	$\sin x + c$
e^{kx}	$\frac{1}{k}e^{kx} + c$	$\sin kx$	$-\frac{1}{k}\cos x + c$
a^x	$\frac{a^x}{\ln a} + c$	$\cos kx$	$\frac{1}{k}\sin kx + c$

In this table c is an integration constant. (definite) integration done between definite limits has got a definite value e.g.,

$$\int_1^4 x^3 dx = \left[\frac{x^4}{4}\right]_1^4 = \frac{1}{4}[(4)^4 - (1)^4]$$

$$= \frac{1}{4}(256 - 1)$$

$$= 63.75$$

Illustration : Evaluate $\int_0^t A\sin\omega t \; dt$

Solution : $\int_0^t A\sin\omega t \; dt = A\left[\frac{-\cos\omega t}{\omega}\right]_0^t = \frac{A}{\omega}(1 - cos\omega t)$

Illustration : Evaluate $\int_R^\infty \frac{GMm}{x^2}dx$. Where G is a constant.

Solution : $\int_R^\infty \frac{GMm}{x^2}dx \quad = GMm\int_R^\infty \frac{1}{x^2}dx$

$$= GMm\left[-\frac{1}{x}\right]_R^\infty$$

$$= GMm\left[-\frac{1}{\infty}-\left(-\frac{1}{R}\right)\right]$$

$$= \frac{GMm}{R}$$

After getting this primary information about differentiation and integration. Now we will now obtain the kinematic equations for uniformly accelerated motion.

Illustration : Obtain the equations of motion for uniformly accelerated motion along a straight line, using method of calculus.

Solution :

(1) Relation Between Velocity and Time :

According to the definition of acceleration,

$$a = \frac{dv}{dt}$$

$$dv = adt$$

Now, at $t = 0$, velocity is $v = v_0$ and at $t = t$, the velocity of an object is $v = v$. Taking integration on both the sides of the equation,

$$\int_{v_0}^{v} dv = \int_0^t adt$$

$$[\; v \;]_{v_0}^{v} = a[\; t \;]_0^t \qquad (a \text{ is constant})$$

$$\therefore \; v - v_0 = at$$

$$\text{or } v = v_0 + at \qquad (1)$$

(2) Relation Between Position and Time :

According to the definition of velocity,

$$v = \frac{dx}{dt}$$

$$\therefore \; dx = vdt$$

At $t = 0$, the position of an object is x_0 and at $t = t$ object is at x. Taking integration on both the sides of the equation,

$$\int_{x_0}^{x} dx = \int_0^t v dt$$

$$= \int_0^t (v_0 + at) dt \qquad \text{(from eq.. (1))}$$

$$= \int_0^t v_0 dt + \int_0^t at \, dt$$

$$\therefore [x]_{x_0}^{x} = v_0 [t]_0^t + a \left[\frac{t^2}{2} \right]_0^t$$

$$\therefore x - x_0 = v_0 t + \frac{1}{2} at^2$$

$$\text{or } x = x_0 + v_0 t + \frac{1}{2} at^2 \qquad (2)$$

(3) Relation Between Velocity and Position :

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = \frac{dv}{dx} \times v \; (\because \frac{dx}{dt} = v)$$

$$\therefore adx = vdv$$

when $x = x_0$ the velocity is $v = v_0$ and when $x = x$, the velocity is $v = v$

$$\therefore \int_{x_0}^{x} adx = \int_{v_0}^{v} vdv$$

$$\therefore a \int_{x_0}^{x} dx = \int_{v_0}^{v} vdv$$

$$a[x]_{x_0}^{x} = \left[\frac{v^2}{2} \right]_{v_0}^{v}$$

$$a(x - x_0) = \left(\frac{v^2}{2} - \frac{v_0^2}{2} \right)$$

$$\therefore 2a(x - x_0) = v^2 - v_0^2 \qquad (3)$$

Equations (1), (2) and (3) are the equations of motion for constant acceleration. The advantage of this method of calculus is that it can be used for motion with non-uniform acceleration also.

CHAPTER 4

MOTION IN A PLANE

4.1 Introduction

Dear students, we have learnt about the concepts of displacement, velocity and acceleration which are necessary to describe the motion of a body on a straight line path (one dimension). We have seen that in one dimension there are only two possible directions and hence the directions are automatically taken care of by using Ace positive (+) and negative (–) signs, But to describe the motion of the body in two dimensions (in a plane) or in three dimensions (in space), a vector is needed. For this, what a vector is how should addition, subtraction and multiplication of vectors be carried out, what is the result on multiplying a vector with a real number, need to understand etc. We will use vector to define velocity and acceleration in a plane. Then we shall discuss the motion of the body in a plane. We will discuss the motion with constant acceleration as a simple case of motion in a plane and the projectile motion in detail. The circular motion being very important in our day to day life, we will study uniform circular motion also in detail.

The equations obtained for the motion in a plane can be easily transformed into those of motion in three dimensions.

4.2 Scalar and Vector Quantities

In physics, quantities are classified as (1) Scalar quantities and (2) Vector quantities. The basic difference between the scalar and vector quantities is that with scalar quantities, direction is not involved while the direction is involved with vector quantities.

The quantities for which, complete information is obtained by knowing their values only are called scalars. e.g. temperature, time, mass, density, volume, work etc. A scalar is represented by a number showing its magnitude in a proper unit. The combination or associations ofr scalar quantities follow the laws of ordinary algebra. Addition, subtraction, multiplication and division can be done like those of usual numbers.

The quantities, which need the direction as well as their values (magnitudes), to be completely known, are called vectors. e.g. velocity, acceleration, force, torque, area, displacement etc.

A vector quantity is represented by putting an arrow on the symbol of that quantity or as a bold letter. For example, the force vector is shown as $\vec{F}$ or **F** the velocity vector is represented as $\vec{v}$ or $\boldsymbol{v}$. The value of the vector quantity is shown by putting the symbol of that quantity in modulas (i.e. between two vertical bars) or by writing that symbol without the arrow. e.g., The value of $\vec{A}$ is shown by $|\vec{A}|$ or A. The vector quantities obey specific laws of combination.

4.3 Presentation of vector by graphical or geometrical method

To represent a vector quantity geometrically an arrow is drawn such that the length of this arrow is equal to the value of this vector quantity on a proper scale. The head of the arrow is put in the direction in which the effect of this vector quantity is prevailing. This arrow can be drawn from any point. Such vectors are called free vectors. e.g. A train moves with a velocity of 40 km/h South to North direction. To represent this velocity vector, as shown in Figure 4.0, draw an arrow from South to North. Keep the length of the arrow proportional to the value of velocity 4.0cm (taking the scale of 10km/h = 1cm). Since the motion is in North direction, put the head of the arrow in North direction. Point O is called the tail of the arrow. Thus this velocity vector is represented as $\vec{v} = \vec{OP}$.

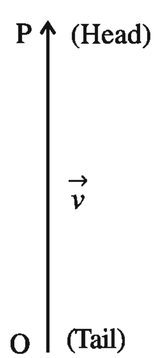


Figure 4.0

4.4 Position and Displacement Vectors

To represent the position of a body we have to mention the reference point which is usually taken as the origin of coordinate axes. Suppose a body moves along the path PQRS as shown in the Figure (4.1). At time t_1 it is at point Q. The vector $\vec{OQ} = \vec{r}_1$ formed by joining the origin O with the point Q is called the position vector of the body at time t_1. Suppose at time t_2 the body reaches the point R. Then, the vector $\vec{OR} = \vec{r}_2$ formed by joining the origin O with the point R is called the position vector of the body at time t_2. During time $t_2 - t_1$ it reaches from Q to R. Hence its displacement vector is shown by $\vec{QR}$.

Here a noteworthy point is that the value of the displacement vector is the minimum distance between the initial position and the final position.

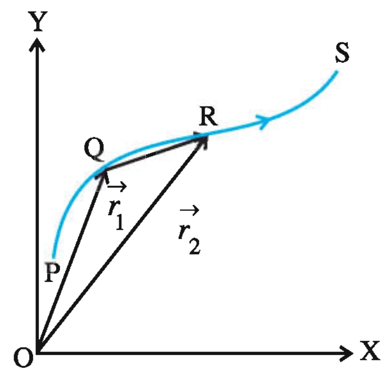


Figure 4.1

4.5 Equality of Vectors

Equal Vectors : If the values and the directions of two vectors are equal, then they are called equal vectors. (Fig. 4.2 *a*)

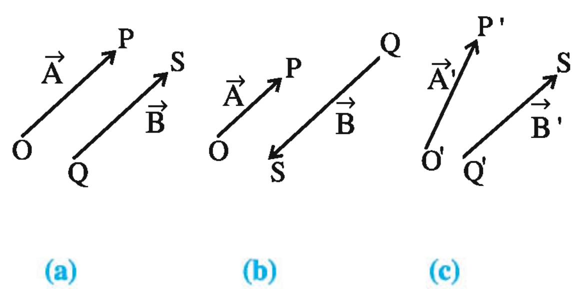


Figure 4.2

Parallel Vectors : The vectors with the same direction are called parallel vectors. (The magnitudes of such vectors can be equal or different). See Fig. 4.2 (a).

Antiparallel Vectors : The vectors having mutually opposite directions are called antiparallel vectors. Fig. 4.2 (b)

Aparallel Vectors : The vectors which are not parallel or antiparallel to each other are called aparallel vectors. Fig. 4.2 (c)

4.6 Vector Algebra

4.6.1 Multiplication of Vectors by real numbers

Multiplying a vector quantity by a real number, the results also in a vector only. Multiplying a vector $\vec{A}$ with a positive number k, the result is vector $k\vec{A}$ whose value is k times that of vector $\vec{A}$.

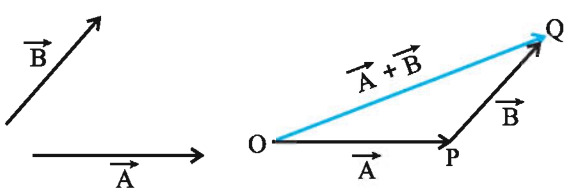
$$|k\vec{A}| = k|\vec{A}| \qquad \text{if } k > 0$$

When we multiply a vector $\vec{A}$ with a negative number $-k$, the result is $-k\vec{A}$; the direction of which is opposite to that of vector $\vec{A}$ and its magnitude is $|k\vec{A}|$.

The coefficient k which is multiplied with vector $\vec{A}$ can also be a scalar with its physical dimensions. Hence the dimensions of the resultant vector $k\vec{A}$, are the product of dimensions of k and the dimensions of $\vec{A}$. e.g. The product of a constant velocity with a time interval gives displacement vector.

4.6.2 Addition and Subtraction of Vectors Graphical or Geometrical Method

Two vectors are geometrically added as shown below



(a) (b)

Figure 4.3

Suppose vector $\vec{A}$ and $\vec{B}$ shown in (Fig. 4.3 *a*) are to be added.

For this, as shown in Figure (4.3 *b*) starting from any point O, draw a vector $\vec{OP}$ which has the same magnitude as that of $\vec{A}$ and is in the direction of $\vec{A}$. Thus $\vec{OP} = \vec{A}$. Now draw $\vec{PQ} = \vec{B}$ by putting the tail of vector $\vec{B}$ on the head P of vector $\vec{OP}$. Then, the vector formed by joining the tail O of vector $\vec{A}$ to the head Q of vector $\vec{B}$ is the resultant vector $\vec{R}$ showing the addition of $\vec{A}$ and $\vec{B}$. i.e. $\vec{A} + \vec{B} = \vec{OQ} = \vec{R}$.

In this method of addition of vectors, the two given vectors and their resultant form the three sides of a triangle, hence it is also called the method of triangle for addition of vectors.

From some point O, draw two vectors $\vec{OP}$ and $\vec{OR}$ representing vectors $\vec{A}$ and $\vec{B}$ respectively. Now considering OP and OR as adjacent sides of a parallelogram, complete the parallelogram OPQR, as shown in the Fig. 4.4.

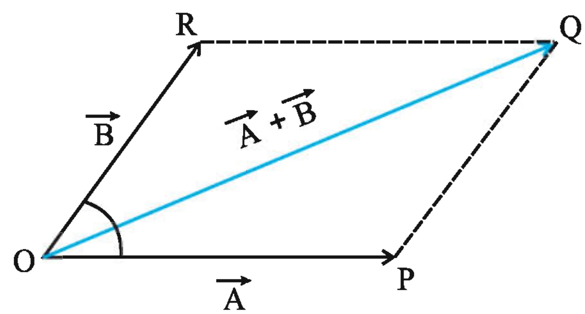


Figure 4.4

Here it is clear that $\vec{OR} = \vec{PQ} = \vec{B}$. The diagonal OQ of this parallelogram drawn from the point O becomes the resultant vector $\vec{R}$ of the addition of vectors $\vec{A}$ and $\vec{B}$ i.e. $\vec{OQ} = \vec{A} + \vec{B}$.

This method is also called the method of parallelogram for addition of vectors. (We will discuss this method in detail in article 4.9.4)

4.6.3 Subtraction of Vectors

Suppose we want to get subtraction of $\vec{A}$ and $\vec{B}$, which are shown in Figure 4.3(a).

Since $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$, the meaning of subtracting vector $\vec{B}$ from vector $\vec{A}$, is to add $-\vec{B}$ (a vector with the same magnitude as that of $\vec{B}$ but in opposite direction) into $\vec{A}$. See Fig. 4.5.

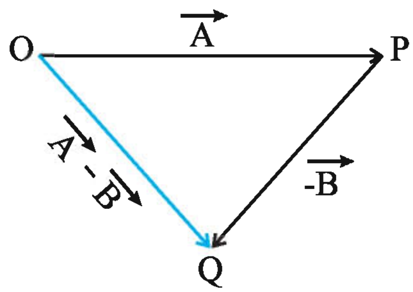
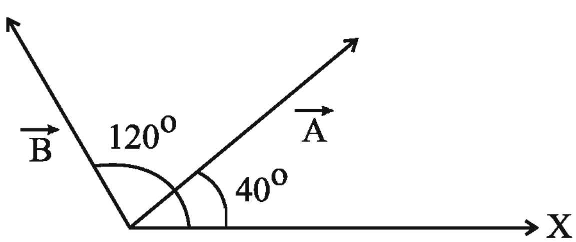


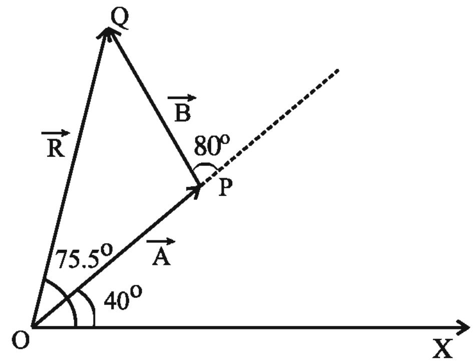
Figure 4.5

In the method of parallelogram, the diagonal formed by joining the points P and Q in Fig. 4.5 shows $\vec{PQ} = \vec{A} - \vec{B}$ (verify this by yourself). Also verify that $\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$.

Illustration 1 : Two vectors $\vec{A}$ and $\vec{B}$, make angles of 40° and 120° respectively with the X–axis. If $|\vec{A}| = 6$ and $|\vec{B}| = 5$ unit, find the resultant vector of these two vectors.



(a)



(b)

Figure 4.6

Solution : The two given vectors $\vec{A}$ and $\vec{B}$ are shown in the Figure 4.6(a). To obtain their addition, draw X–axis from some point O on a graph paper. (See Fig. 4.6 (b)) Taking a proper scale, draw vector $\vec{OP}$ representing $\vec{A}$, and on the point P of this vector put the tail of vector $\vec{B}$ and draw $\vec{PQ} = \vec{B}$.

Joining O and Q, we get $\vec{OQ}$, which in the resultant of $\vec{A}$ and $\vec{B}$, i.e. $\vec{OQ} = \vec{R}$. Measuring the value of the resultant vector $\vec{OQ}$, with a scale, it is found as 8.4 unit. This resultant vector makes an angle of 75.5° with X–axis.

Illustration 2 : River water flows at 40 km/h. In this river a fisherman tries to drive a motorboat at 30 km/h perpendicular to the bank of the river. Find the resultant velocity of the motorboat and its direction with respect to the bank.

Solution : In Figure 4.7, the velocity of flow of water is shown as $\vec{v_r}$ and the velocity of motorboat as $\vec{v_b}$.

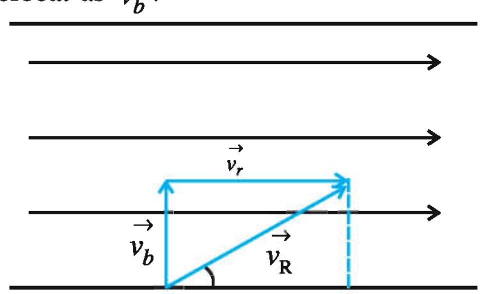


Figure 4.7

Using the method of triangle for the addition of vectors; the resultant velocity of these two velocities is shown as $\vec{v}_R$.

It is clear from the figure that

$$\left|\vec{v}_R\right| = \sqrt{\left|\vec{v}_r\right|^2 + \left|\vec{v}_b\right|^2}$$

$$= \sqrt{(40)^2 + (30)^2}$$

$$= 50 \text{ km/h}$$

If $\vec{v}_R$ makes an angle θ with the bank, it is clear from the figure that,

$$\tan\theta = \frac{v_b}{v_r} = \frac{30}{40} = 0.75$$

$$\therefore \theta = \tan^{-1} 0.75 \approx 37^\circ$$

Thus the resultant velocity of boat is 50km/hr in the direction making an angle of 37° with the river flow.

4.6.4 Properties of vector addition

(1) Addition of vectors is commutative but Subtraction of vectors is not.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}.$$

$$\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$$

(2) Addition of vectors follows associative law.

i.e., $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

4.7 Null or Zero Vectors

The vector obtained by adding two vectors of equal magnitude (value) and but of opposite directions is called null or a zero vector and it is shown as $\vec{0}$. Thus $\vec{A} - \vec{A} = \vec{0}$. As the value of zero vector is zero its direction cannot be shown. e.g. the acceleration of a train moving with constant valocity is zero vector.

4.8 Unit Vector

A vector of unit magnitude is called a unit vector. A unit vector is symbolically expressed as $\hat{n}$ (Read : *n* hat or *n* carat). By dividing any vector by its value, we get a unit vector in the direction of that vector. e.g. in the Fig. 4.8, vector $\vec{A}$ is shown and suppose $|\vec{A}| = 6$.

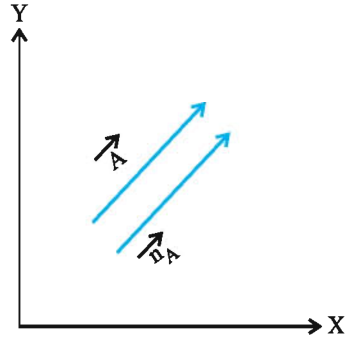


Figure 4.8

If we show the unit vector in the direction of this vector as $\hat{n}_A$, then

$$\hat{n}_A = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{6} \qquad (4.8.1)$$

Thus any vector can be expressed as a product of its value (magnitude) and the unit vector in the direction of that vector.

$$\vec{A} = |\vec{A}|\, \hat{n}_A = A\hat{n}_A \qquad (4.8.2)$$

In the Cartesian co–ordinate system, the unit vectors in the directions of X, Y and Z-axes are expressed respectively as $\hat{i}$, $\hat{j}$ and $\hat{k}$. The vectors shown in the Figure 4.9 can be written as :

$$\vec{B} = 4\hat{i},\ \vec{C} = 2\hat{j}$$

$$\vec{A} = 4\hat{i} + 2\hat{j} \qquad (4.8.3)$$

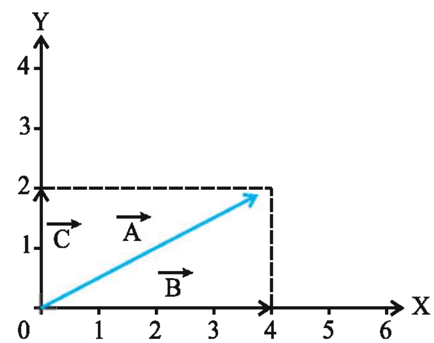


Figure 4.9

4.9 Resolution of a Vector in a Plane

As shown in the Figure 4.10 *a*, consider two non–zero vectors $\vec{a}$ and $\vec{b}$; besides another vector $\vec{A}$ in the same plane. The vector $\vec{A}$ can be represented as combination (addition) of two vectors, one obtained by multiplying the vector $\vec{a}$ with a real number λ and the other by multiplying vector $\vec{b}$ with a real number μ. To verify the above statement, draw a line parallel to $\vec{a}$ and passing through the tail O of vector $\vec{A}$. Similarly draw another line parallel to b and passing through the head P of $\vec{A}$. If these two lines interect in Q, then from Fig. 4.10 *b*.

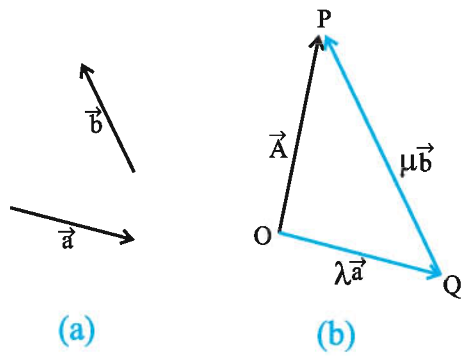


Figure 4.10

$$\vec{A} = \vec{OP} = \vec{OQ} + \vec{QP} \qquad (4.9.1)$$

But $\vec{OQ}$ is parallel to $\vec{a}$ and $\vec{QP}$ is parallel to $\vec{b}$, hence we can write,

$$\vec{OQ} = \lambda\vec{a} \text{ and } \vec{QP} = \mu\vec{b} \qquad (4.9.2)$$

This is called the resolution of vector $\vec{A}$ in the directions of $\vec{a}$ and $\vec{b}$, in the form of vector components $\lambda\vec{a}$ and $\mu\vec{b}$.

Where λ and μ are real numbers.

$$\therefore \vec{A} = \lambda\vec{a} + \mu\vec{b} \qquad (4.9.3)$$

Thus a given vector can be resolved in such a way that its two vector components remain in the direction of two given vectors and all these three vectors remain in a plane.

In the orthogonal co-ordinate system, a given vector can be easily resolved in the directions of the axes using unit vectors.

4.9.1 Perpendicular components of a vector

In Fig. 4.11, a vector $\vec{A}$ is shown in two dimensions. From the head and the tail of this vector perpendiculars are drawn on X– and Y– axes. By doing so, we get PQ = projection of $\vec{A}$ on X–axis or the scalar component (A_x) of vector $\vec{A}$ in the direction of X–axis, MN is the projection of $\vec{A}$ on Y–axis or the scalar component (A_y) of $\vec{A}$ in the direction of Y–axis.

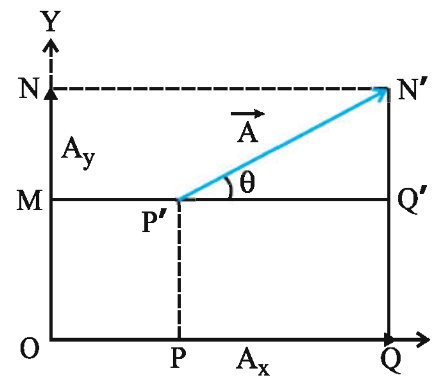


Figure 4.11

Now, from the law of addition of vectors,

$$\vec{A} = \vec{P'Q'} + \vec{Q'N'} = \vec{PQ} + \vec{MN} \qquad (4.9.4)$$

$$\therefore \vec{A} = A_x\hat{i} + A_y\hat{j} \qquad (4.9.5)$$

Here, $A_x\hat{i} = \vec{A_x}$ = vector component of vector $\vec{A}$ in X–direction. $A_y\hat{j} = \vec{A_y}$ = vector component of vector $\vec{A}$ in Y–direction.

From Figure 4.11 in ΔP'Q'N'

$$\cos\theta = \frac{P'Q'}{P'N'} = \frac{A_x}{A} \qquad (4.9.6)$$

$\therefore A_x = A\cos\theta$

Similarly $A_y = A\cos(90^\circ - \theta)$

$$\therefore A_y = A\sin\theta \qquad (4.9.7)$$

From this, we can say that the component of a vector in any given direction is equal to the product of the value of that vector and the cosine of the angle made by that vector with that given direction.

Thus any vector can be resolved in two mutually perpendicular components.

A vector can be described in two ways :

(1) by the magnitude (value) of that vector and the angle made by it with a definite direction or

(2) by the components of that vector.

In the $\Delta P'Q'N'$, in Fig. 4.11,

$$|\vec{A}| = P'N' = \sqrt{(P'Q')^2 + (Q'N')^2}$$

$$= \sqrt{A_x^2 + A_y^2} \quad (4.9.8)$$

Thus the magnitude (value) of any given vector is equal to the square root of the addition of the square of its matually perpendicular components. For the direction,

$$\tan\theta = \frac{N'Q'}{P'Q'} = \frac{A_y}{A_x} \quad (4.9.9)$$

$$\therefore \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right), \quad (4.9.10)$$

where θ = angle between the vector $\vec{A}$ and the x–axis.

In the discussion so far, we have only considered the vector lying in the XY plane. In a similar way a vector in three dimensions can be resolved in three mutually perpendicular components (in directions of X, Y, Z–axes)

The component of a vector representing a physical quantity, in any direction shows the effectiveness of that physical quantity in that direction. e.g. If as shown in Fig 4.12 *a* body makes a displacement of 5 m from A to B. Then it is clear that the distance travelled by it in horizontal direction is (A to C) 4m and the distance travelled by it in the verticale direction (C to B) 3m. From Fig 4.11, in $\Delta P'Q'N'$,

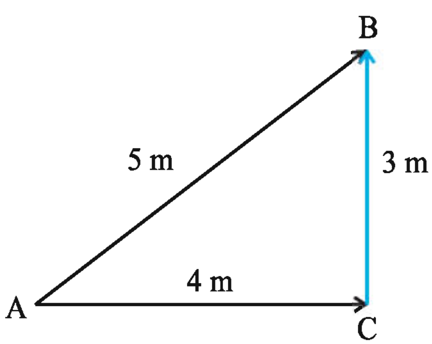


Figure 4.12

In the Fig. 4.13 a vector $\vec{A}$ in three dimensions is shown. The projection of this vector on the XY plane is OQ. By drawing the perpendiculars from the point Q on the X and Y–axes, we get the *x* and *y* components of vector $\vec{A}$ on those axes, as ON = A_x and OM = A_y respectively. Looking three dimensionally we find that PQ = RO = A_z.

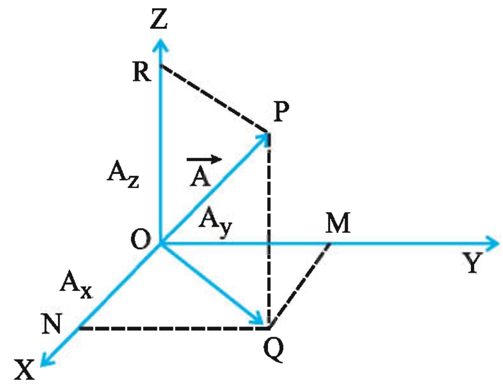


Figure 4.13

Now according to Pythagoras theorem,

$$OQ^2 = MQ^2 + OM^2 = A_x^2 + A_y^2 \quad (4.9.11)$$

and $OP^2 = OQ^2 + PQ^2$

$$\therefore OP^2 = A_x^2 + A_y^2 + A_z^2 \quad (4.9.12)$$

$$|\vec{A}|^2 = A_x^2 + A_y^2 + A_z^2$$

$$\therefore |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} \quad (4.9.13)$$

In three dimensions vector $\vec{A}$ can be written as $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

Another way of writing the very same vector $\vec{A}$, is

$$\vec{A} = (A_x, A_y, A_z)$$

If the co-ordinates of a point are (*x*, *y*, *z*) then, its position vector can be written as

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = (x, y, z) \quad (4.9.14)$$

Here *x*, *y* and *z* are the components of $\vec{r}$ in the directions of X, Y, Z axes respectively. The value of this position vector is

$$\left|\vec{r}\right| = \sqrt{x^2 + y^2 + z^2} \quad (4.9.15)$$

4.9.2 Addition and subtraction of vectors in Algebraic or analytical method :

We have learned the geometrical method for addition of vectors. This method is convenient for addition of two or three vectors, but when large number of vectors are to be added this method is tedious and has limited accuracy. In such circumstances the algebraic method of vector addition is more convenient.

We have already seen that a given vector can be resolved in mutually perpendicular components. Vectors can be easily added by combinity the components of vectors. Suppose $\vec{A}$ and $\vec{B}$ are in the XY–plane, and their components are A_x, A_y and B_x, B_y respectively.

$$\therefore \vec{A} = A_x\hat{i} + A_y\hat{j} \quad (4.9.16)$$

$$\text{and } \vec{B} = B_x\hat{i} + B_y\hat{j} \quad (4.9.17)$$

If the resultant vector of the addition of these two vectors is represented as $\vec{R}$ then,

$$\vec{R} = \vec{A} + \vec{B}$$

$$= (A_x\hat{i} + A_y\hat{j}) + (B_x\hat{i} + B_y\hat{j}) \quad (4.9.18)$$

Addition of vectors is commutative and it also follows the associative law, Therefore,

$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} \quad (4.9.19)$$

Moreover, $\vec{R} = R_x\hat{i} + R_y\hat{j}$

hence $R_x = A_x + B_x$ and $R_y = A_y + B_y$

Thus every component of the resultant vector is equal to the sum of the corresponding components of $\vec{A}$ and $\vec{B}$.

In three dimensions,

$$R = \vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k} \quad (4.9.20)$$

We will now illustrate, how the algebraic method for addition of vectors is easier, then the geomatric mathod.

Ilustration 3 : If $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{B} = 4\hat{i} + 5\hat{j} + 3\hat{k}$; find the magnitudes of $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$

Solution : $\vec{A} + \vec{B} = 6\hat{i} + 8\hat{j} + 7\hat{k}$

$$\therefore \left|\vec{A} + \vec{B}\right| = \sqrt{(6)^2 + (8)^2 + (7)^2}$$

$$= 12.2 \text{ unit}$$

$$\vec{A} - \vec{B} = -2\hat{i} - 2\hat{j} + \hat{k}$$

$$\therefore \left|\vec{A} - \vec{B}\right|$$

$$= \sqrt{(-2)^2 + (-2)^2 + (1)^2}$$

$$= 3 \text{ unit}$$

Illustration 4 : Add vectors $\vec{A}$ and $\vec{B}$ shown in the Fig. 4.14 by the algebraic method. $|\vec{A}| = 10$ unit; $|\vec{B}| = 8$ unit.

Solution : For this we will determine the X and Y components of both the vectors.

$A_x = A \cos 30^\circ = 10 \cos 30^\circ$

$= 10 \times 0.8660 = 8.66$ unit

$B_x = B \cos 60^\circ = 8\cos60^\circ = 8 \times 0.5$

$= 4.0$ unit

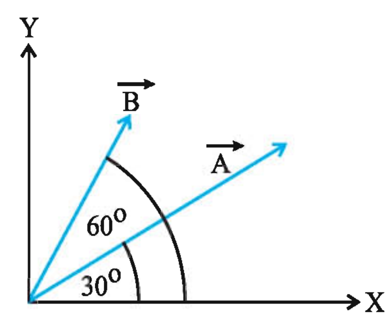


Figure 4.14

$A_y = A \sin 30^\circ = 10 \sin 30^\circ$

$= (10)(0.5) = 5.0$

$B_y = B \sin 60^\circ = 8 \sin 60^\circ$

$= (8)(0.8660) = 6.928 \approx 6.93$

If the resultant of these two vectors is $\vec{R}$, then

$R_x = A_x + B_x = 8.66 + 4.0 = 12.66$

$R_y = A_y + B_y = 5 + 6.93 = 11.93$

∴ The magnitude of the resultant vector $\vec{R}$ is

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(12.66)^2 + (11.93)^2}$$

$$= 17.4 \text{ unit}$$

Suppose the resultant vector makes an angle θ with the X–axis, then

$$\tan\theta = \frac{R_y}{R_x} = \frac{11.93}{12.66} = 0.9423$$

$$\therefore \theta = \tan^{-1} 0.9423 \approx 43^\circ 8'$$

This process can be made easier in the following way. As we can take the X and Y-axes in any way convenient to us,.

Let us take X–axis in the direction of $\vec{A}$. Hence the angle between $\vec{A}$ and X–axis becomes zero and the angle between $\vec{B}$ and X–axis becomes 30° Fig. 4.15.

In this condition,

$$A_x = A\cos 0^\circ = 10\cos 0^\circ = 10$$

$$B_x = B\cos 30^\circ = 8(0.8660) = 6.93$$

$$A_y = A\sin 0^\circ = 10\sin 0^\circ = 0$$

$$B_y = B\sin 30^\circ = (8)(0.5) = 4.0$$

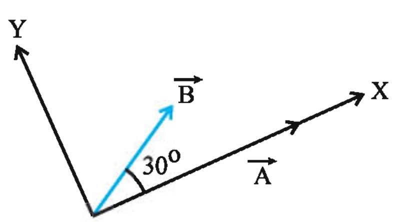


Figure 4.15

$$\therefore R_x = A_x + B_x = 10 + 6.93 = 16.93$$

$$R_y = A_y + B_y = 0 + 4 = 4$$

$$\therefore |\vec{R}| = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(16.93)^2 + (4)^2}$$

$$\approx 17.4 \text{ unit}$$

Varify the direction of the resultant vector yourself.

Illustration 5 : Find the resultant vector of the three vectors shown in the Figure (4.16).

Solution : We will first find the x and y components of all these three vectors and then obtain the resultant vector by addition of the corresponding components.

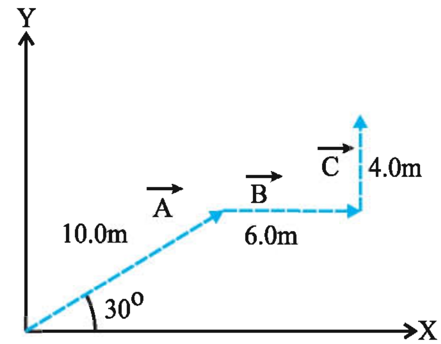


Figure 4.16

Taking x–components of $\vec{A}$, $\vec{B}$ and $\vec{C}$ we get,

$$A_x = A\cos 30^\circ = 10\cos 30^\circ$$

$$= (10)(0.8660) = 8.66$$

$$B_x = B\cos 0^\circ = 6\cos 0^\circ = 6$$

$$C_x = C\cos 90^\circ = 4\cos 90^\circ = 0$$

Taking y components of $\vec{A}$, $\vec{B}$ and $\vec{C}$, we get,

$$A_y = A\cos 60^\circ = (10)(0.5) = 5$$

$$B_y = B\cos 90^\circ = (6)(0) = 0$$

$$C_y = C\cos 0^\circ = (4)(1) = 4$$

If we write the resultant vector as $\vec{R}$, then,

$$R_x = A_x + B_x + C_x$$

$$= 8.66 + 6 + 0 = 14.66$$

$$R_y = A_y + B_y + C_y$$

$$= 5 + 0 + 4 = 9$$

The value of $\vec{R}$ is,

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(14.66)^2 + (9)^2} = 17.2\text{m}$$

If $\vec{R}$ makes an angle θ with the X–axis then,

$$\tan\theta = \frac{Ry}{Rx} = \frac{9}{14.66} = 0.6139$$

$$\therefore \theta = \tan^{-1}(0.6139)$$
$$= 31^{o}\,27'$$

Illustration 6 : If the summation of the three vectors shown in Fig. 4.17 is zero, find magnitudes of the vectors $\vec{B}$ and $\vec{C}$.

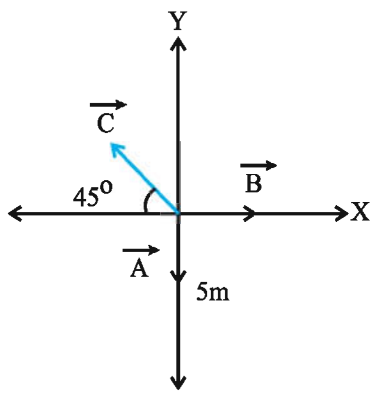


Figure 4.17

Solution : Taking x components of these three vectors

$$A_x = A\cos 270^o = 0$$

$$B_x = B\cos 0^o = B$$

$$C_x = C\cos 135^o = -\frac{1}{\sqrt{2}}C$$

Now, taking the y components of these vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$

$$A_y = A\cos 180^o = -A$$

$$B_y = B\cos 90^o = 0$$

$$C_y = C\cos 45^o = \frac{1}{\sqrt{2}}C$$

If the resultant vector is denoted by $\vec{R}$

and since $\vec{R} = \vec{A} + \vec{B} + \vec{C}$,

$$R_x = A_x + B_x + C_x = 0 + B - \frac{1}{\sqrt{2}}\ C$$

$$R_y = A_y + B_y + C_y = -A + 0 + \frac{1}{\sqrt{2}}\ C$$

It is given that the magnitude of the resultant vector $\vec{R}$ is zero. Hence the magnitudes of its components should also be zero. So,

$$R_x = 0 + B - \frac{1}{\sqrt{2}}C = 0 \Rightarrow B = \frac{1}{\sqrt{2}}C$$

$$R_y = -A + 0 + \frac{1}{\sqrt{2}}C = 0 \Rightarrow A = \frac{1}{\sqrt{2}}C$$

$$\therefore A = B$$

As shown in the figure $|\vec{A}| = A = 5m$

$$\therefore C = A\sqrt{2} = 5\sqrt{2}\ m$$

and B = A = 5m

Illustration 7 : As shown in Fig. 4.18 six vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{D}$, $\vec{E}$, and $\vec{F}$ form a regular haxagon. Using the algebraic method of addition of vectors shows that their resultant is zero.

Solution : Since these vectors form a regular hexagon, their lengths are the same. Suppose, this length is P. Hence, A = B = C = D = E = F = P.

Taking X and Y axes as shown in the figure and taking x and y component of these vectors,

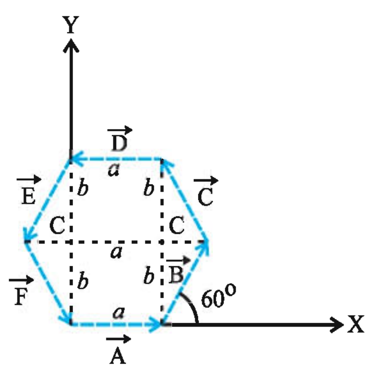


Figure 4.18

From the Fig. 4.18,

$$\vec{A} = a\hat{i}$$

$$\vec{B} = c\hat{i} + b\hat{j}$$

$$\vec{C} = -c\hat{i} + b\hat{j}$$

$$\vec{D} = -a\hat{i}$$

$$\vec{E} = -c\hat{i} - b\hat{j}$$

$\vec{F} = c\hat{i} - b\hat{j}$

$\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} + \vec{F}$

$= (a\hat{i}) + (c\hat{i} + b\hat{j}) + (-c\hat{i} + b\hat{j}) +$

$(-a\hat{i}) + (-c\hat{i} + b\hat{j}) + (c\hat{i} - b\hat{j}) = 0$

When the vectors form a close loop then their vector addition is zero.

4.9.3 Law of parallelogram for addition of two vectors

"If a parallelogram is completed by taking the two given vectors as adjacent sides, then the diagonal of the parallelogram drawn from the common point, gives the addition of the two given vectors." And the other diagonal shows subtraction of the two vectors. As shown in the Fig. 4.19 the given two vectors $\vec{A}$ and $\vec{B}$ are taken as adjacent sides of the parallalogram OP and OQ respectively and parallalogram OQRP is completed. Here θ is the angle between vectors $\vec{A}$ and $\vec{B}$. Diagonal OR is a resultant vector $\vec{R} = \vec{A} + \vec{B} = \vec{OR}$. This can be seen in the following manner.

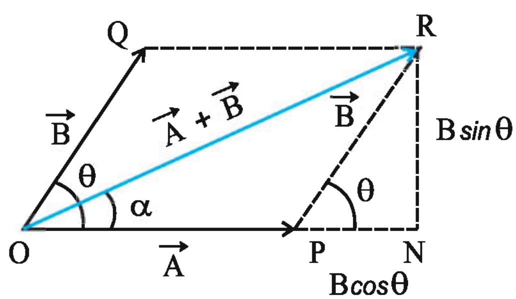


Figure 4.19

Suppose $\vec{A}$ is in the X–direction

$\therefore \vec{A} = A_x\hat{i}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$

∴ Algebraic method gives

$\vec{R} = A_x\hat{i} + B_x\hat{i} + B_y\hat{j}$

$= (A_x + B_x)\hat{i} + B_y\hat{j}$ (4.9.20)

$\therefore \left|\vec{R}\right| = \left[(A_x + B_x)^2 + B_y^2\right]^{\frac{1}{2}}.$

$= \left[A_x^2 + 2A_xB_x + B_x^2 + B_y^2\right]^{\frac{1}{2}}$

If resultant $\vec{R}$ is subtending angle α with vector $\vec{A}$ then $\tan\alpha = \dfrac{B_y}{A_x + B_x}$

$$\therefore \alpha = \tan^{-1}\frac{B_y}{A_x + B_x} \quad (4.9.21)$$

From the geometry of the Figure.

$B_x = PN = B\cos\theta$ and $B_y = NR = B\sin\theta$ (4.9.22)

But $A_x = A$ and $B_x^2 + B_y^2 = B^2$

$$\therefore \left|\vec{R}\right| = \left[A^2 + B^2 + 2AB_x\right]^{\frac{1}{2}}$$

Now $B_x = B\cos\theta$

$$\therefore \left|\vec{R}\right| = \left[A^2 + B^2 + 2AB\cos\theta\right]^{\frac{1}{2}} \quad (4.9.23)$$

If resultant $\vec{R}$ is makes angle α with $\vec{A}$ i.e. with X–axis then from Fig. 4.19.

$$\tan\alpha = \frac{RN}{OP + PN} = \frac{B_y}{A_x + B_x}$$

$$= \frac{B\sin\theta}{A + B\cos\theta}$$

$$\therefore \alpha = \tan^{-1}\frac{B\sin\theta}{A + B\cos\theta} \quad (4.9.24)$$

Thus from equations (4.9.23) and (4.9.24) (using Law of Parallelogram) magnitude and direction of $\vec{A} + \vec{B}$ can be obtained respectively.

Try yourself for $\vec{A} - \vec{B}$ and obtain

$$\left|\vec{R}\right| = \left[A^2 + B^2 - 2AB\cos\theta\right]^{\frac{1}{2}}$$

and $\alpha = \tan^{-1}\dfrac{B\sin\theta}{A - B\cos\theta}$

Where θ is the angle between $\vec{A}$ and $\vec{B}$.

4.10 Multiplication of Two Vectors

Vector quantities have both magnitude and direction. Hence their products do not obey ordinary laws of algebra. By taking product of two vector quantities in a specific way a new physical quantity can be derived. The quantity derived this way may be a vector quantity or a scalar quantity. If the product of two vector quantities results into a vector then the product is called a vector product and if it results in to a scalar then the product called a scalar product. Here one may understand in general that of product of two vectors means specific type of combination of two vectors which looks like a product. Thus vector product can be carried out in two ways (1) scalar product (2) Vector product.

4.10.1 Scalar products of two vectors

The scalar products of the two vectors $\vec{A}$ and $\vec{B}$ is defined as follows :

$$\vec{A} \cdot \vec{B} = |\vec{A}|\ |\vec{B}| \cos\theta$$

$$\therefore\ \vec{A} \cdot \vec{B} = AB \cos\theta \qquad (4.10.1)$$

Where θ is the angle between $\vec{A}$ and $\vec{B}$.

Such product is represented by keeping a dot ($\cdot$) between two vectors, it is also called dot product.

To obtain scalar products of two vectors shown in Fig 4.20.(a), draw these vectors from the common point O as shown in Fig 4.20(b). Now draw perpendicular from the head of the $\vec{A}$ on $\vec{B}$. Hence OM is called projection of $\vec{A}$ on $\vec{B}$.

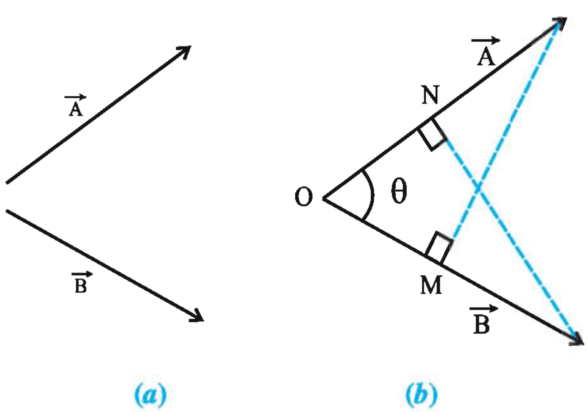


Figure 4.20

From equation (4.10.1)

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

$$\therefore\ \vec{A} \cdot \vec{B} = B(A \cos\theta) \qquad (4.10.2)$$

From Fig. 4.20.(b)

$$\cos\theta = \frac{OM}{A}$$

$$\therefore\ OM = (A)(\cos\theta) \qquad (4.10.3)$$

$$\therefore\ \vec{A} \cdot \vec{B} = B(OM)$$

$$= (\text{magnitude of } \vec{B})(\text{Projection of } \vec{A} \text{ on } \vec{B}) \qquad (4.10.4)$$

$$\text{or } \vec{A} \cdot \vec{B} = A(B \cos\theta) = A(ON)$$

$$= (\text{magnitude of } \vec{A}) \times (\text{projection of } \vec{B} \text{ on } \vec{A}) \qquad (4.10.5)$$

Thus, the scalar product of two vectors is equal to the product of the magnitude of first vector with the projection of second vector on the first vector.

4.10.2 Properties of scalar product

(1) Commutative Law :

$$\vec{A} \cdot \vec{B} = AB \cos\theta = BA \cos\theta = \vec{B} \cdot \vec{A} \qquad (4.10.6)$$

Thus scalar product of two vectors is commutative

(2) Distributive Law :

As shown in Fig. 4.21

$\vec{OP} = \vec{A}$, $\vec{OQ} = \vec{B}$ and $\vec{QR} = \vec{C}$ Now,

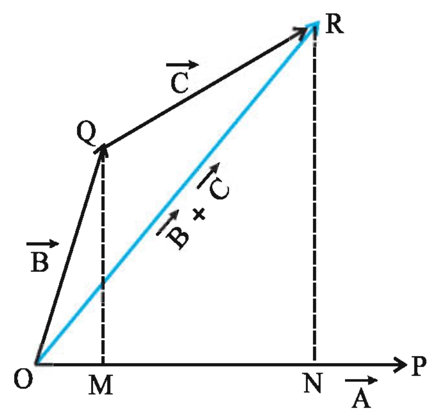


Figure 4.21

$$\vec{A} \cdot (\vec{B} + \vec{C}) = (\text{magnitude of } \vec{A})\ [\text{projection of } (\vec{B} + \vec{C}) \text{ on } \vec{A}]$$

$$= |\vec{A}|(ON)$$

$$= |\vec{A}|(OM + MN)$$

$$= |\vec{A}|(OM) + |\vec{A}|(MN) \qquad (4.10.7)$$

$\therefore\ \vec{A} \cdot (\vec{B} + \vec{C}) = |\vec{A}|$ (projection of $\vec{B}$ on $\vec{A}$) + $|\vec{A}|$ (projection of $\vec{C}$ on $\vec{A}$)

$$\therefore\ \vec{A}\cdot(\vec{B}+\vec{C})=\vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C} \quad (4.10.8)$$

Thus scalar product of vectors is distributive with respect to summation.

(3) If $\vec{A} \parallel \vec{B}$, $\theta = 0°$

$$\therefore\ \vec{A}\cdot\vec{B} = AB\ cos\ 0° = AB \quad (4.10.9)$$

Also $\vec{A}\cdot\vec{A} = |\vec{A}|\ |\vec{A}| = A^2$

$$\therefore\ |\vec{A}| = \sqrt{\vec{A}\cdot\vec{A}} \quad (4.10.10)$$

Thus the magnitude a vector is equal to the square root of the scalar product of the vector with itself.

(4) If $\vec{A} \perp \vec{B}$ $\theta = 90°$:

$$\therefore\ \vec{A}\cdot\vec{B} = AB\ cos\ 90° = 0$$

Thus the scalar product of two mutually perpendicular vectors is zero.

(5) Scalar products of unit vectors in Cartesian co–ordinate system :

$\hat{i}\cdot\hat{i} = \hat{j}.\hat{j} = \hat{k}\cdot\hat{k} = 1$ and

$$\hat{i}\cdot\hat{j} = \hat{j}.\hat{k} = \hat{k}\cdot\hat{i} = 0 \quad (4.10.11)$$

(6) Scalar product in terms of Cartesian Component of vectors :

If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A}\ .\ \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\ .\ (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\therefore\ \vec{A}\ .\ \vec{B} = A_xB_x + A_yB_y + A_zB_z \quad (4.10.12)$$

(7) Angle between two vectors

$$\vec{A}\ .\ \vec{B} = |\vec{A}|\ |\vec{B}|\ cos\theta$$

$$\therefore\ cos\ \theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\ |\vec{B}|}$$

$$= \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\ \sqrt{B_x^2 + B_y^2 + B_z^2}} \quad (4.10.13)$$

The angle between two vectors can be found out using this formula.

Illustration 8 : Find the scalar products of two vectors $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{B} = \hat{i} + \hat{j} - 3\hat{k}$

Solution : $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

$= 2 + 3 + 12$

$= 17$ units

Illustration 9 : Find the angle between two vectors $\vec{A} = -2\hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 2\hat{k}$

Solution :

$$cos\ \theta = \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\ \sqrt{B_x^2 + B_y^2 + B_z^2}}$$

$$= \frac{-4 + 8 + 8}{\sqrt{24}\ \sqrt{24}} = \frac{12}{24} = \frac{1}{2}$$

$\therefore \theta = 60°$

Illustration 10 : If vector $\vec{A} = 4\hat{i} - 6\hat{j} + 2\hat{k}$ and $\vec{B} = 6\hat{i} + 8\hat{j} + m\hat{k}$ are mutually perpendicular, find the value of *m*

Solution : As $\vec{A}$ and $\vec{B}$ are perpendicular to each other $\vec{A}\ .\ \vec{B} = 0$

$\vec{A}\ .\ \vec{B} = A_xB_x + A_yB_y + A_zB_z = 0$

$= 24 - 48 + 2m = 0$

$\therefore 2m = 24$

$m = 12$

Illustration 11 : The co-ordinates of a point P in (*x*, *y*) plane are *x* and *y*. The position vector $\vec{r}$, of this point, makes an angle θ with the X–axis. Find the unit vectors $\hat{n}_r$ and $\hat{n}_\theta$ (in XY plane) which are parallel and perpendicular to $\vec{r}$ respectively.

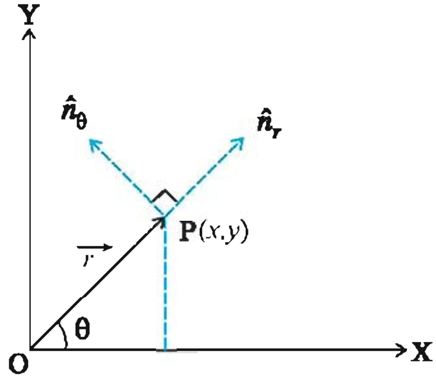


Figure 4.22

Solution : By definition,

$$\hat{n}_r = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{i} + y\hat{j}}{r}$$

$$\therefore \hat{n}_r = \frac{x}{r}\hat{i} + \frac{y}{r}\hat{j}$$

or $\hat{n}_r = \cos\theta\,\hat{i} + \sin\theta,\ \hat{j}$ (From Fig. 4.22)

The vector, obtained by rotating $\hat{n}_r$ by $\frac{\pi}{2}$, would be perpendicular to $\hat{n}_r$. We denote this new vector by $\hat{n}_\theta$.

$$\hat{n}_\theta = \cos\left(\theta + \frac{\pi}{2}\right)\hat{i} + \sin\left(\theta + \frac{\pi}{2}\right)\hat{j}$$

$$\therefore \hat{n}_\theta = -\sin\theta\ \hat{i} + \cos\theta\ \hat{j}$$

Note : Here, θ is increased by $\frac{\pi}{2}$ in anti-clockwise sense.

4.10.3 Vector product of two vectors

The vector product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A} \times \vec{B} = |\vec{A}|\ |\vec{B}|\ \sin\theta\ \hat{n}$$

Where θ is the angle between $\vec{A}$ and $\vec{B}$. $\hat{n}$ is the unit vector in the direction perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$. The direction of $\hat{n}$ can be given by right handed screw rule.

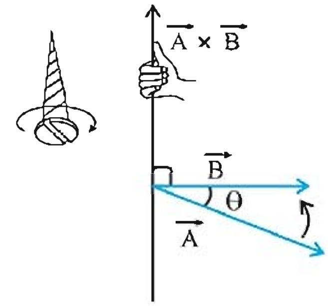


Figure 4.23

As shown in the Fig. 4.23 keep the right handed screw perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$ and rotate it from $\vec{A}$ towards $\vec{B}$. The direction of advancement of the screw is taken as the direction of $\hat{n}$. The direction of $\vec{A} \times \vec{B}$ can be determined using Right hand rule : **Open up your right hand palm and wrap the fingers sweeping from $\vec{A}$ to $\vec{B}$ Your streached thumb points in the direction of $\vec{A} \times \vec{B}$**

A vector product is represented by keeping cross sign (×) between two vectors hence it is also called cross product of vectors.

4.10.4 Properties of vector product of two vectors

(1) $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$, The vector product of two vectors is not commutative

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \qquad (4.10.14)$$

This can be understood from the right handed screw rule.

(2) Distributive law

$$\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C}) \qquad (4.10.15)$$

holds for vector product too.

(3) If two vectors are parallel (θ = 0°) or antiparallel (θ = 180°), their vector product is zero because $\sin(0^\circ) = \sin(180^\circ) = 0$.

(4) If $\vec{A} \perp \vec{B}$, θ = 90°

$$\therefore \sin\theta = \sin 90^\circ = 1$$

$$\therefore \vec{A} \times \vec{B} = AB\sin 90^\circ = AB\,\hat{n} \qquad (4.10.16)$$

(5) Vector products of unit vectors of Cartesian co-ordinate system :

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0 \quad (4.10.17)$$

and $\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{k} = \hat{i},\ \hat{k} \times \hat{i} = \hat{j}$

$$\hat{j} \times \hat{i} = -\hat{k},\ \hat{k} \times \hat{j} = -\hat{i},\ \hat{i} \times \hat{k} = -\hat{j} \quad (4.10.18)$$

(6) Vector products of two vectors,

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{A} \times \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$ is

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k} \quad (4.10.19)$$

$$\therefore \text{Now, } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k} \quad (4.10.20)$$

From equations (4.10.19) and (4.10.20)

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \quad (4.10.21)$$

Illustration 12 : Find the vector product of vectors $\vec{A} = 4\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = \hat{i} + 3\hat{j} + 4\hat{k}$.

Solution : $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 2 & -1 \\ 1 & 3 & 4 \end{vmatrix}$

$= (8 + 3)\hat{i} + (-1 - 16)\hat{j} + (12 - 2)\hat{k}$

$= 11\hat{i} - 17\hat{j} + 10\hat{k}$

Illustration 13 : If vector $\vec{A} = 2\hat{i} - 10\hat{j}$ and vector $\vec{B} = 4\hat{i} - 20\hat{j}$ then show that they are parallel to each other.

Solution : If the two vectors are parallel to each other then their vector product is zero

$$\therefore \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -10 & 0 \\ 4 & -20 & 0 \end{vmatrix} = 8\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -5 & 0 \\ 1 & -5 & 0 \end{vmatrix} = 0$$

Hence $\vec{A}$ and $\vec{B}$ are parallel to each other.

Illustration 14 : Show that the magnitude of cross product of $\vec{A}$ and $\vec{B}$ is equal to twice the area of the triangle of which $\vec{A}$ and $\vec{B}$ are the adjacent sides.

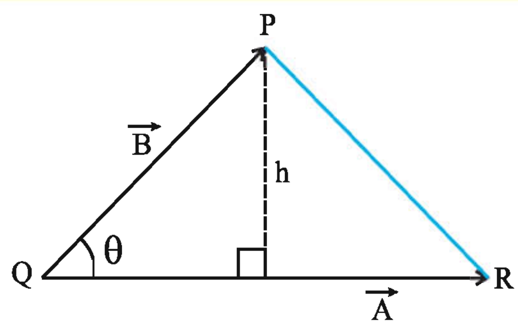


Figure 4.24

Solution : In Fig 4.24, the area of ΔPQR

$= \frac{1}{2}|\vec{A}|h$

$= \frac{1}{2}|\vec{A}||\vec{B}|\sin\theta$

$= \frac{1}{2}\left|\vec{A} \times \vec{B}\right|$

$\therefore \left|\vec{A} \times \vec{B}\right| = 2$ (Area of ΔPQR)

Illustration 15 : Using vector products show that for a plane triangle.

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

Where α, β, and γ are the angles and A, B and C are the lengths of the sides opposite to α, β, and γ respectively.

Solution : We know that the magnitude of a cross product of two vectors is twice the area of the triangle of which the vectors form two adjacent sides. In view of this, we have from Fig. 4.25.

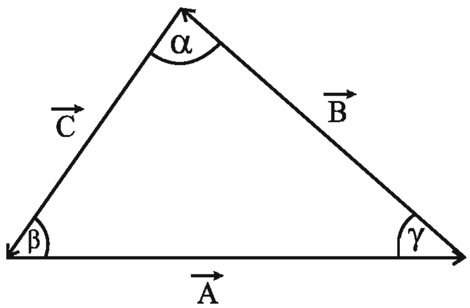


Figure 4.25

$$\left|\vec{A}\times\vec{B}\right| = \left|\vec{B}\times\vec{C}\right| = \left|\vec{C}\times\vec{A}\right|$$

$\therefore$ AB sin $(\pi - \gamma)$ = BC sin $(\pi - \alpha)$ = CA sin $(\pi - \beta)$

$\therefore$ AB $sin\ \gamma$ = BC $sin\ \alpha$ = CA $sin\ \beta$

Dividing each term by ABC, we have,

$$\therefore \frac{sin\ \gamma}{C} = \frac{sin\ \alpha}{A} = \frac{sin\ \beta}{B}$$

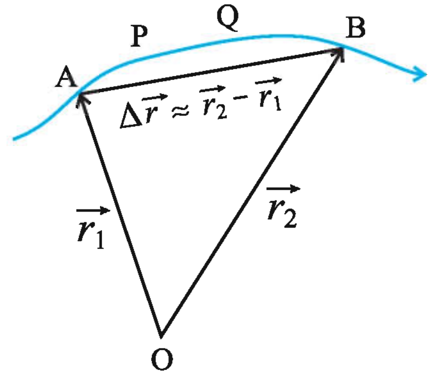
4.11 Instantaneous Velocity

Fig. 4.26(a) shows curved path APQB of a particle moving in the XY plane. Suppose the particle is at point A at time t and reaches point B at time $t + \Delta t$. With respect to certain reference points, the position vectors of these two points are $\vec{r_1} = \vec{OA}$ and $\vec{r_2} = \vec{OB}$ respectively.

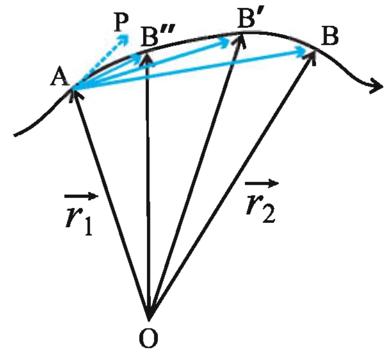
During the motion of the particle from point A to point B, the change in its position is represented by displacement vector $\vec{\Delta r} = \vec{r_2} - \vec{r_1}$. Δt is the time taken for this displacement. By definition the average velocity of the particle in the given time interval Δt is,

$$\text{Average velocity} = \frac{\text{displacement(vector)}}{\text{time(scalar)}}$$

$$\therefore < \vec{v} > = \frac{\vec{\Delta r}}{\Delta t} \qquad (4.11.1)$$



(a)



(b)

Figure 4.26

The average velocity is a vector quantity and its direction is in the direction of $\vec{\Delta r} = \vec{AB}$. If the average velocity of the particle is same during different time intervals then the motion of the particle is said to be a motion with uniform velocity. The displacement vector $\vec{\Delta r}$ in time interval Δt is the vector joining the initial and final positions of the particle in the time interval Δt. Hence it may not show the actual distance covered by the particle. In reality the particle has moved along the path APQB and reached from A to B. Moreover during time interval Δt changes in the velocity might have taken place. **Hence from average velocity of the particle, we do not get the actual path of its motion and information of velocity at various points on the path of motion.**

As shown in the Fig. 4.26(b) if we keep on decreasing the time interval Δt then the particle which is at point A at time t, after time interval

Δt will be at B' instead of B, will be at B'' instead of B' .... and so on. In this manner if we continue to make Δt smaller and smaller, that is we give lesser and lesser time ($\Delta t \to 0$) for change in velocity. Now taking $\lim_{\Delta t \to 0}$ it can be seen from the Fig. 4.26(b) that the displacement vector becomes tangent at point A in the direction AP on the path of the motion of the particle. In these circumstances velocity of the particle has a definite value and direction. This velocity of the particle is called instantaneous velocity ($\vec{v}$) at time t at point A. Symbolically it is respresented as follows.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \qquad (4.11.2)$$

Here $\frac{d\vec{r}}{dt}$ is called derivative of $\vec{r}$ with respect to time t and $\frac{d\vec{r}}{dt}$ is represented symbolically as $\dot{\vec{r}}$. In general instantaneous velocity is termes as velocity. SI unit of velocity is m s^{-1}.

Velocity of a particle at any point on the path of its motion is along the tangent drawn at that point.

To represent the velocity in its components suppose the co-ordinates of points A and B in the Fig. 4.26(a) are (x_1, y_1) and (x_2, y_2) respectively.

$$\therefore \quad \vec{r_1} = x_1\hat{i} + y_1\hat{j} \text{ and } \vec{r_2} = x_2\hat{i} + y_2\hat{j}$$

$$\therefore \Delta\vec{r} = \vec{r_2} - \vec{r_1}$$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

$$= \Delta x\hat{i} + \Delta y\hat{j} \qquad (4.11.3)$$

Where $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$ using equation (4.11.3) in equation (4.11.2)

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t}$$

$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} \qquad (4.11.4)$$

Where $v_x = \frac{dx}{dt} = \dot{x}$ is the X component of velocity $\vec{v}$. (4.11.5)

and $v_y = \frac{dy}{dt} = \dot{y}$ is the Y component of velocity $\vec{v}$. (4.11.6)

If x and y co-ordinates of the particle in motion are functions of time. x and y components (v_x and v_y) of the velocity $\vec{v}$ of the particle can be obtained, by using above formulae and they can be used to obtain the magnitude and direction of the velocity $\vec{v}$ from equations

$$v = \sqrt{v_x^2 + v_y^2} \text{ and } \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

respectively. Here θ is the angle between the X–axis and direction of velocity.

Illustration 16 : Position vector of a particle is given by the formula $\vec{r}(t) = t^2\hat{i} + 3t\hat{j} + 24\hat{k}$.

(i) Obtain formula for the velocity of the particle.

(ii) Find magnitude and direction of its velocity at t = 2 s.

Remember,

$$\left[\frac{d(x^n)}{dx} = nx^{n-1}\right]$$

Solution :

(i) Velocity at any instant of time

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}(t^2\hat{i} + 3t\hat{j} + 24\hat{k})$$

$$\therefore \vec{v}(t) = 2t\hat{i} + 3\hat{j}$$

(ii) To obtain velocity at t = 2 s substitute t = 2 in the above expression

$$\vec{v}_2 = 2(2)\hat{i} + 3\hat{j}$$

$$= 4\hat{i} + 3\hat{j}$$

$\therefore$ v_x = 4m s^{-1} and v_y = 3m s^{-1}

$\therefore$ Magnitude of velocity

$$\left|\vec{v}_2\right| = \sqrt{(4)^2 + (3)^2} = 5\text{m s}^{-1}$$

If direction of velocity is in the direction making angle θ with the X–axis then,

$$\theta = tan^{-1}\left(\frac{v_y}{v_x}\right) = tan^{-1}\left(\frac{3}{4}\right) = tan^{-1} 0.75 \approx 37^o$$

4.12 Acceleration

Time rate of change of velocity is called acceleration.

Suppose a particle is at point P on its path of motion (as shown in Fig. 4.27) at time t and its velocity is v at this point. Now it reaches at point P_1 at time $t + \Delta t$ and its velocity is v' at P_1. Thus the change in velocity of the particle in time

Δt, $\Delta\vec{v} = \vec{v}' - \vec{v}$

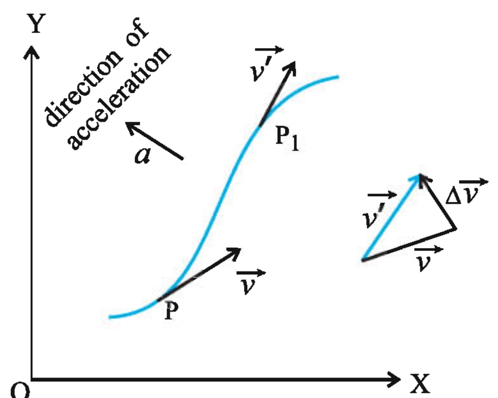


Figure 4.27

As per definition average acceleration

$$= \frac{\text{Change in velocity}}{\text{time}}$$

$$\therefore < a > = \frac{\Delta\vec{v}}{\Delta t} \qquad (4.12.1)$$

Average acceleration $<\vec{a}>$ is vector quantity and its direction is in the direction of the vector representing the change in velocity $\Delta\vec{v}$.

The information regarding how the velocity of the particle change at each moment on its actual path between points P and P_1 cannot be obtained from average acceleration. Taking $\Delta t \to 0$ in equation (4.12.1), instantaneous acceleration ($\vec{a}$) is obtained. Generally instantaneous acceleration is called acceleration. SI unit of acceleration is m s^{-2}.

Instantaneous acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \qquad (4.12.2)$$

Now, $\vec{v} = \frac{d\vec{r}}{dt}$

$$\therefore \vec{a} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}} \qquad (4.12.3)$$

Substituting $\vec{v} = v_x\hat{i} + v_y\hat{j}$ in equation. (4.12.2)

$$\vec{a} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j}) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$\therefore \vec{a} = a_x\hat{i} + a_y\hat{j}$$

Where, $a_x = \frac{dv_x}{dt} = \dot{v}_x$ = X component of the acceraletion $\vec{a}$ of the particle (4.12.4)

$a_y = \frac{dv_y}{dt} = \dot{v}_y$ = Y component of the accelaration $\vec{a}$ of the particle. (4.12.5)

If the co-ordinates x and y of the particle in motion are functions of time, then using equations (4.11.5) and (4.11.6) the X and Y components of velocity of the particle (v_x and v_y) can be obtained. Substituting them in equations (4.12.4) and (4.12.5), X and Y components of the acceleration (a_x and a_y) of the particle can be obtained.

Velocity is a vector quantity, hence, it can be changed in three ways :

(i) by changing its magnitude only (ii) by changing its direction only (iii) by changing both its magnitude and direction.

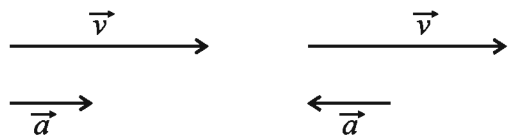


Figure 4.27 (a)

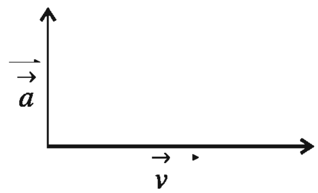


Figure 4.27 (b)

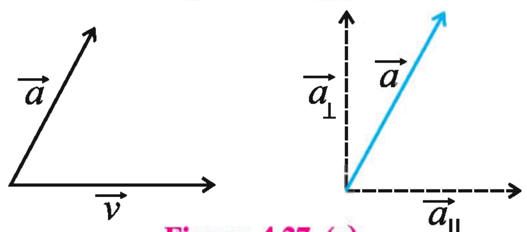


Figure 4.27 (c)

As shown in the Fig. 4.27(*a*) if acceleration $\vec{a}$ is in the direction of velocity ($\vec{v}$) the magnitude of the velocity increases or when $\vec{a}$ is in the direction opposite to velocity ($\vec{v}$) then the magnitude of the velocity decreases respectively.

As shown in Fig 4.27(*b*) if acceleration $\vec{a}$ is in direction perpendicular to the direction of the velocity ($\vec{v}$) then only the direction of the velocity changes.

As shown in the Fig. 4.27(*c*) for some angle between the directions of acceleration ($\vec{a}$) and velocity ($\vec{v}$) of other than 0º, 90º or 180º.Consider the two components of acceleration (i) parallel to the velocity (a_{11}) and (ii) perpendicular to the velocity ($a_{\perp}$). It can be seen that due to component a_{11} the magnitude of the velocity changes and due to component $a_{\perp}$ the direction of the velocity changes.

Illustration 17 : Velocity of a particle at time *t* is $\vec{v}\,(t) = 7t\hat{i} + 16\hat{k}$. Find acceleration of the particle.

Solution : Acceleration $\vec{a} = \dfrac{d\vec{v}}{dt}$

$$\therefore \ \vec{a} = \frac{d}{dt}(7t\hat{i} + 16\hat{k})$$

$$= 7\hat{i} \text{ m s}^{-2}$$

Illustration 18 : The position vector of a moving particle changes with time according to the formula $\vec{r} = \alpha t\hat{i} - \beta t^2\hat{j}$, where α and β are positive constants. Then (*a*) determine the path of motion of the particle, (*b*) obtain the formula for velocity and acceleration as functions of time and also obtain their magnitudes.

Solution :

(a) $\vec{r} = \alpha t\hat{i} - \beta t^2\hat{j}$ is given and

$$\vec{r} = x\hat{i} - y\hat{j}$$

$\therefore$ $x = \alpha t$ and $y = -\beta t^2$. Eliminating *t* from these equations we get,

$y = \dfrac{\beta x^2}{\alpha^2}$ which is similar to the equation of parabola viz, $y = ax - bx^2$ (where $a = 0$ and $b = \dfrac{\beta}{\alpha^2}$). Hence, the path of the said particle is a parabola.

(b) The velocity of the particle $\vec{v} = \dfrac{d\vec{r}}{dt}$

$$\therefore \ \vec{v} = \frac{d}{dt}(\alpha t\hat{i} - \beta t^2\hat{j}) = \alpha\hat{i} - 2\beta t\hat{j}$$

This equation gives the velocity of the particle as a function of time.

The magnitude of velocity

$$\left|\vec{v}\right| = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{\alpha^2 + (-2\beta t)^2}$$

$$= \sqrt{\alpha^2 + 4\beta^2 t^2}$$

Now, the acceleration of the particle $\vec{a} = \dfrac{d\vec{v}}{dt}$

$$\therefore \ \vec{a} = \frac{d}{dt}(\alpha\hat{i} - 2\beta t\hat{j}) = -2\beta\hat{j}$$

Since the expression for acceleration does not contain t we can say that the acceleration of the particle is constant; and it is in the direction of negative Y axis.

The magnitude of acceleration

$$\left|\vec{a}\right| = \sqrt{a_x^2 + a_y^2} = \sqrt{(0)^2 + (-2\beta)^2}$$

$$= 2\beta$$

Illustration 19 : The position vector of on particle, as a function of time, is given by :

$\vec{r} = \vec{b}\,t\,(1 - \alpha t)$; where $\vec{b}$ is a constant vector and α is some positive constant, (i) Obtain the velocity and acceleration of the particle as function of time and (ii) find the time taken by the particle to come back to the same point from where it had started.

Solution : (i) $\vec{r} = \vec{b}\,t\,(1 - \alpha t)$ (1)

$$\therefore \vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}\{\vec{b}\,t\,(1 - \alpha t)\}$$

$$= \frac{d}{dt}\{(\vec{b}\,t - \vec{b}\,\alpha t^2)\}$$

$$\therefore \vec{v}(t) = \vec{b} - 2\vec{b}\,\alpha\,t$$

$$= \vec{b}\,(1 - 2\alpha t) \quad (2)$$

Similarly, acceleration $\vec{a}(t) = \frac{d\vec{v}}{dt} =$

$$\frac{d}{dt}\{\vec{b} - 2\vec{b}\,\alpha t\} = 0 - 2\vec{b}\,\alpha$$

$$\therefore \vec{a}(t) = -2\vec{b}\,\alpha$$

(ii) This particle starts (i.e. at time $t = 0$) its motion from $\vec{r} = \vec{0}$. It can be seen from eqn. (1) that again at time $t = \frac{1}{\alpha}$ it will return back to $\vec{r} = \vec{0}$. Thus, in time interval $\Delta t = \frac{1}{\alpha}$, the particle will return back to the point from where it started its motion.

4.13 Relative Velocity

Uptill now we have discussed the motion of a particle with respect to some given frame of reference. We also noticed that the choice of frame of reference is quite arbitrary. The position vector $\vec{r}$, velocity $\vec{v}$ and acceleration $\vec{a}$ depend upon the frame of reference chosen. Now we shall obtain the relations between such quantities in different frames of reference.

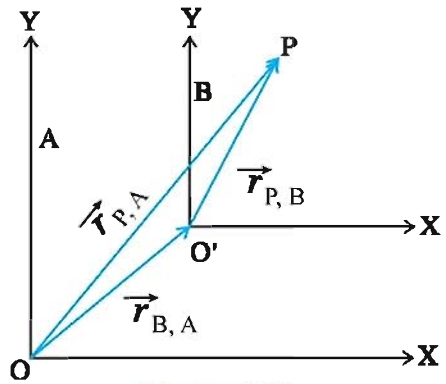


Figure 4.28

In Fig. 4.28 two frames of reference A and B, moving with uniform velocity with respect to (w.r.t.) each other are shown. Such frames of references are called inertial frames of reference and they are discussed in detail in article 5.11. Suppose two observers, one from A and the other form B study the motion of a particle P.

Let the position vectors of particle P at some instant of time with respect to the origin O of frame A be $\vec{r}_{P,A} = \vec{OP}$ and that with respect to the origin O' of frame B be $\vec{r}_{P,B} = \vec{O'P}$.

The position vector of O' w.r.t. O is $\vec{r}_{B,A} = \vec{OO'}$. From Figure 4.28 it is clear that

$$\vec{OP} = \vec{OO'} + \vec{O'P} = \vec{O'P} + \vec{OO'}$$

$$\therefore \vec{r}_{P,A} = \vec{r}_{P,B} + \vec{r}_{B,A} \quad (4.13.1)$$

Differentiating this equation with respect to time we get

$$\frac{d}{dt}(\vec{r}_{P,A}) = \frac{d}{dt}(\vec{r}_{P,B}) + \frac{d}{dt}(\vec{r}_{B,A})$$

$$\therefore \vec{v}_{P,A} = \vec{v}_{P,B} + \vec{v}_{B,A} \quad (4.13.2)$$

Here $\vec{v}_{P,A}$ is the velocity of the particle w.r.t. frame of reference A, $\vec{v}_{P,B}$ is the velocity of the particle w.r.t reference frame B and

$\vec{v}_{B, A}$ is the velocity of frame of reference B with respect to frame A.

Suppose velocities of two particles A and B are respectively $\vec{v}_A$ and $\vec{v}_B$ relative to a frame of reference (suppose earth) then velocity ($\vec{v}_{AB}$) of A relative to B is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B \quad (4.13.3)$$

and velocity $\vec{v}_{BA}$ of B relative to A is

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A \quad (4.13.4)$$

Thus $\vec{v}_{AB} = -\vec{v}_{BA}$

and $|\vec{v}_{AB}| = |\vec{v}_{BA}|$

For example a car is moving with velocity 80 km/h on a highway towards East. A truck is also moving toward East with velocity 60 km/h and a motorbike is moving towards West with velocity 40 km/h . All the velocities are relative to the earth and written as follows.

$\vec{v}_{CG} = 80\hat{i}$ km/h, $\vec{v}_{TG} = 60\hat{i}$ km/h and $\vec{v}_{BG} = -40\hat{i}$ km/h

Now velocity of car relative to motorbike

$\vec{v}_{CB} = \vec{v}_{CG} - \vec{v}_{BG} = 80\hat{i} - (-40\hat{i}) = 120\hat{i}$,

velocity of car relative to truck $\vec{v}_{CT} = \vec{v}_{CG} - \vec{v}_{TG} = 80\hat{i} - 60\hat{i} = 20\hat{i}$ and velocity of motorbike relative to truck $\vec{v}_{BT} = \vec{v}_{BG} - \vec{v}_{TG} = -40\hat{i} - 60\hat{i}$

$= -100\hat{i}$

Generally if we know the velocities of two objects P and Q w.r.t. third X then

$$\vec{v}_{PQ} = \vec{v}_{PX} + \vec{v}_{XQ} = \vec{v}_{PX} - \vec{v}_{QX} \quad (4.13.5)$$

This formula holds true for (a) when the velocities are not very large, (b) if the object is not performing rotational motion and (c) the time interval are the same for all the frames of referance.

Illustration 20 : A boat can move in river water with speed of 8km/h. This boat has to reach to a place from one bank of the river to a place which is in perpendicular direction onthe other bank of the river. Then (i) in which direction should the boat has to be moved ? (ii) If the width of the river is 600 m; then what will be the time taken by the boat to cross the river ? The river is flows with velocity 4km/h.

Solution : Suppose the river is flowing in positive X direction as shown in Fig. 4.29. To reach to a place in the perpendicular direction on the other bank, the boat has to move in the direction making angle θ with Y direction as shown in the Fig. 4.29. This angle should be such that the velocity of the boat relative to the opposite bank is in the direction perpendicular to the bank.

Note : When we say boat can move in water velocity 8 km/h it means that the velocity of boat is 8 km/h relative to water. When air hostess anounces in aeroplance, that the velocity of the plane is 700 km/h then it means that it is relative to atmosphere.

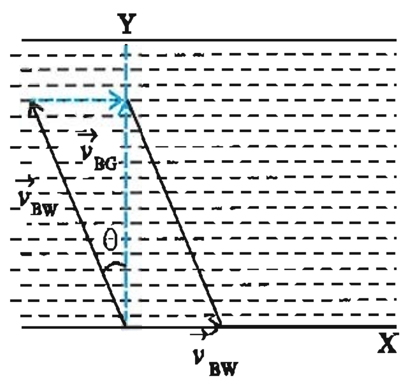


Figure 4.29

Suppose $\vec{v}_{BW}$ = velocity of boat relative to water is 8 km/h in the direction making angle θ with Y–axis.

$\vec{v}_{WG}$ = velocity of water–relative to bank which is 4 km/h in the positive X direction and

$\vec{v}_{BG}$ = velocity of the boat relative to bank which is to be found.

It is clear from the Fig. 4.29

$$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG} \quad (a)$$

Taking x components in this equation

$$0 = 8\ cos\ (90 + \theta) + 4\ cos\ 0 = -8\ sin\ \theta + 4$$

$$\therefore\ sin\ \theta = \frac{4}{8} = \frac{1}{2}\ \therefore\ \theta = 30^o$$

(ii) Taking Y–components in the equation (a)

$v_{BG} = 8\ cos\ 30^o + 0 = 8 \times 0.866 = 6.928$ $\approx$ 6.93 km/h

Thus the velocity of boat relative to bank v_{BG} = 6.93 km/h The time taken with this velocity to cover distance of 600 m.

$$= \frac{\text{displacement in Y-direction}}{\text{velocity Y-direction}}$$

$$= \frac{600\text{km}}{6.93\text{km/h}}$$

= 0.8658 hr $\approx$ 5.2 minute

4.14 Equations of motion in a plane (two dimensions) with uniform acceleration :

Suppose a particle moves in the XY plane with uniform acceleration $\vec{a}$. Its velocities at time $t = 0$ and $t = t$ are v_0 and v respectively. As it is moving with uniform acceleration in any time interval its average acceleration and instantaneous acceleration will be the same.

Now change in velocity in time interval

$\Delta t = t - 0$ is $\Delta \vec{v} = \vec{v} - \vec{v_0}$

Using $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t}$

$$\vec{a} = \frac{\vec{v} - \vec{v_0}}{t - 0} = \frac{\vec{v} - \vec{v_0}}{t} \quad (4.14.1)$$

$$\therefore\ \vec{v} = \vec{v_0} + \vec{a}\,t \quad (4.14.1a)$$

Writing this equation in terms of components (x and y components)

$$v_x = v_{0x} + a_x t \quad (4.14.2)$$

$$v_y = v_{0y} + a_y t \quad (4.14.3)$$

Suppose the positions of the object at time t = 0 and $t = t$ are represented by position vectors $\vec{r_0}$ and $\vec{r}$ respectively. During this time interval $(t - 0)$

$$\text{Average Velocity} = \frac{\vec{v_0} + \vec{v}}{2}$$

$\therefore$ Displacement taking place in time t = average velocity X time

$$\therefore\ \vec{r} - \vec{r_0} = \left(\frac{\vec{v_0} + \vec{v}}{2}\right)t \quad (4.14.4)$$

Substituting the value of $\vec{v}$ from equation (4.14.1)

$$\vec{r} - \vec{r_0} = \left(\frac{\vec{v_0} + \vec{v_0} + \vec{a}\,t}{2}\right)t$$

$$= \vec{v_0}\,t + \frac{1}{2}\vec{a}\,t^2$$

$$\therefore\ r = \vec{r_0} + \vec{v_0}\,t + \frac{1}{2}\vec{a}\,t^2 \quad (14.14.5)$$

Presenting this equation in the form of components (x and y components)

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \quad (14.14.6)$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \quad (14.14.7)$$

It is clear from the equations (4.14.6) and (4.14.7) that motions in X and Y direction can be described independently.

Thus the motion in a plane (two dimension) with uniform acceleration can be considered as a combination of two simultaneous one dimentional motions in mutually perpendicular directions, with different uniform acceleration. This is an important result. (This type of equations can also be used for motion in three dimensions). Selection of two perpendicular directions is arbitrary.

Thus the equations of motion in plane (two dimensions) with uniform acceleration $\vec{a}$ can be written as follows.

$$\vec{v} = \vec{v}_0 + \vec{a}\,t$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}\,t^2$$

Taking dot product of the equations (4.14.1) and (4.14.4)

$$(\vec{a})\cdot(\vec{r} - \vec{r}_0) = (\vec{v} - \vec{v}_0)\cdot\left(\frac{\vec{v}_0 + \vec{v}}{2}\right)$$

$$v^2 - v_0^2 = 2\vec{a}\cdot(\vec{r} - \vec{r}_0)$$

From these equations, the equations for the motion in one dimensions with uniform acceleration a can be written as

$$v = v_0 + at$$

$$d = v_0 t + \frac{1}{2}at^2 \quad \text{Here } d = r - r_0$$

$$v^2 - v_0^2 = 2ad$$

Here, d is the displacement in time t.

Illustration 21 : A particle starts its motion from the origin with velocity $2\hat{i}$ m s^{-1} and moves in the XY plane with uniform acceleration $\hat{i} + 3\hat{j}$, (i) what will be the value of its y co-ordinate when the value of its x co–ordinate is 30 m (ii) at this time what will be its speed ?

Solution : (i) Formula for the displacement of a particle in two dimension is

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}\,t^2$$

Here, $\vec{r}_0 = 0$

$$\therefore \vec{r}(t) = \vec{v}_0 t + \frac{1}{2}\vec{a}\,t^2$$

$\vec{v}_0 = 2\hat{i}$ m s^{-1} and $\vec{a} = \hat{i} + 3\hat{j}$ m s^{-1}

$$\therefore \vec{r}(t) = (2\hat{i})t + \frac{1}{2}(\hat{i} + 3\hat{j})t^2$$

$$= (2t + \frac{1}{2}t^2)\hat{i} + \frac{3}{2}\hat{j}t^2$$

$$\therefore x(t) = 2t + \frac{1}{2}t^2 \text{ and } y(t) = \frac{3}{2}t^2$$

At the instant of time t, $x(t) = 30$m is given

$$\therefore 30 = 2t + \frac{1}{2}t^2$$

$$\therefore t^2 + 4t - 60 = 0$$

$$\therefore (t + 10)(t - 6) = 0$$

$\therefore t = -10$s or $t = 6$s but $t = -10$s is not possible.

$\therefore t = 6$ sec. Substituting $t = 6$ in equation.

$$y(t) = \frac{3}{2}t^2 \Rightarrow y(6) = \frac{3}{2}(6)^2 = 54\text{m}$$

Hence the y co-ordinate is 54 m, when the x co-ordinate is 30 m.

(ii) Velocity at any instant of time

$$\therefore \vec{v}(t) = \frac{d}{dt}(x\hat{i} + y\hat{j})$$

$$\therefore \vec{v}(t) = \frac{d}{dt}[(2t + \frac{1}{2}t^2)\hat{i} + \frac{3}{2}t^2\hat{j}]$$

$$\therefore \vec{v}(t) = (2 + t)\hat{i} + 3t\hat{j}$$

$$\therefore \vec{v}(6) = 8\hat{i} + 18\hat{j}$$

$v_x = 8$m s^{-1} and $v_y = 18$m s^{-1}

$$\therefore v = \sqrt{(8)^2 + (18)^2}$$

$$= \sqrt{64 + 324}$$

$$= 19.698\text{m s}^{-1}$$

4.15 Uniform Circular Motion

The motion of a particle moving on a circular path with constant speed is known as uniform circular motion. As shown in Fig. 4.30 a particle is moving on a circular path with radius r and its speed v is constant.

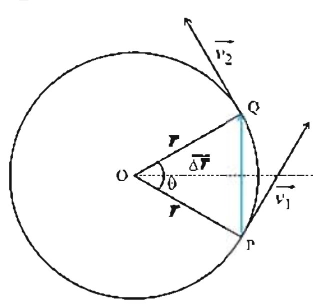


Figure 4.30

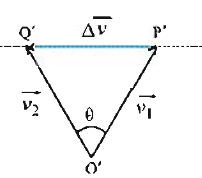


Figure 4.31

Velocity of the particle at a point moving on curved path is in the direction of the tangent drawn to the curved path at that point. Hence it is clear that for the particle moving on the circular path, with constant speed, the direction of velocity changes continuously but its magnitude remains constant.

Since the direction of velocity changes the motion of the particle is an accelerated motion. Thus the uniform circular motion of a particle is an illustration of accelerated motion. (Here the direction of the acceleration vector changes, hence this is also an illustration of the motion of a particle with variable acceleration of constant magnitude.) We have seen in the article 4.12 that in the case in where only the direction of velocity changes, the direction of acceleration is perpendicular to the direction of velocity. Now the velocity is in the direction of the tangent and the direction perpendicular to the tangent is the direction of radius (towards the centre). This acceleration is in the direction along radius towards the centre. This type of acceleration is called **radial acceleration a_r** or **centripetal acceleration a_c**.

To derive formula for radial acceleration, let the velocities of a particle performing uniform circular motion, be $\vec{v}_1$ and $\vec{v}_2$ at points P and Q respectively as shown in Fig 4.30 and the time it takes to go from P to Q be Δt. Thus the change in velocity in time interval Δt is $\overrightarrow{\Delta v} = \vec{v}_2 - \vec{v}_1$ which is as shown in Fig. 4.31.

From the geometry of the figure it is clear that ΔOPQ and ΔO'P'Q' are similar triangles. Hence

$$\frac{P'Q'}{O'P'} = \frac{PQ}{OP} \quad \therefore \quad \frac{\Delta v}{v_1} = \frac{\Delta r}{r}$$

$$\text{but } \left|\vec{v}_1\right| = \left|\vec{v}_2\right| = v$$

$$\therefore \ \Delta v = \frac{v}{r} \cdot \Delta r$$

The magnitude of average acceleration during time interval Δt is

$$< a > = \frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta r}{\Delta t}$$

Taking $\Delta t \to 0$ in this ratio we get the magnitude of instantaneous acceleration at time t.

$$\text{Acceleration } a_c = \lim_{\Delta t \to 0} \frac{v}{r}\frac{\Delta r}{\Delta t}$$

$$= \frac{v}{r}\left(\lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}\right)$$

$$= \frac{v}{r}\frac{dr}{dt}$$

but $\frac{dr}{dt} = v$ = instantaneous speed at time t

$$\text{Acceleration } a_c = \frac{v^2}{r} \qquad (4.15.1)$$

From Fig. 4.31 it is clear that direction of $\overrightarrow{\Delta v}$ is towards the centre. Hence the direction of acceleration (a_c) is towards the centre. Due to this fact this acceleration is called radial or centripetal acceleration. The force corresponding to this acceleration is obviously called centripetal force.

From the above discussion it is clear that in order to make a particle to move on a curved path it should be supplied necessary centripetal force.

The magnitude of centripetal acceleration is constant but its direction keeps on changing continuously so the vector representing the centripetal acceleration is not constant.

Illustration 22 : Nirav ties a small stone at the end of 1 meter long thread. He rotates the stone in the horizontal plane (Here neglect gravitational force). If the stone completes 100 rotations in 314 seconds then (i) what will be its linear speed ? (ii) What will be the magnitude of its centripetal acceleration ? Can we consider the vector representing its acceleration as a constant vector ?

Solution : Here the radius r of the circular path of stone is 1 meter.

(i) Stone completes 100 rotations in 314 seconds.

$\therefore$ time taken to complete one rotation i.e.

periodic time T = $\frac{314}{100}$ = 3.14

Linear speed of the stone $v = \dfrac{\text{distance}}{\text{time}}$

$$= \frac{2\pi r}{T} = \frac{2 \times 3.14 \times 1}{3.14} = 2\text{m s}^{-1}$$

$\therefore\ v = 2\text{m s}^{-1}$

(ii) Magnitude of centripetal acceleration

$a_c = \dfrac{v^2}{r} = \dfrac{(2)^2}{1} = 4\text{m s}^{-2}$. Because the direction of this acceleration changes continuously, the vector representing acceleration can not be considered constant.

4.16 Projectile Motion

When an object is thrown in gravitational field of earth it moves with constant horizontal velocity and constant vertical acceleration. Such two dimentional motion is called a projectile motion and the object is called a projectile. If resistance offered by air is neglected, motion of a football kicked by a player and motion of a cricket ball thrown, in air, by a cricketer can be considered to be the projectile motion and the ball is called a projectile.

The projectile motion can be treated as the resultant motion of two independent component motions taking place simultaneously in mutually perpendicular directions. One component is along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to gravitational force. Galileo was the first to state this independency of the horizontal and vertical components of projectile motion.

In our discussion we will neglect air resistance.

Suppose the projectile is projected with velocity $\vec{v}_0$ and that makes an angle θ_0 with the X–axis (horizontal direction) as shown in Fig. 4.32.

The acceleration acting on the projectile is due to gravity which is directed vertically downward.

$\therefore\ \vec{a} = -g\,\hat{j}$

or $a_x = 0,\ a_y = -g$ (4.16.1)

The components of initial velocity $\vec{v}_0$ are

$v_{0x} = v_0 \cos\theta_0$ (4.16.2*a*)

$v_{0y} = v_0 \sin\theta_0$ (4.16.2*b*)

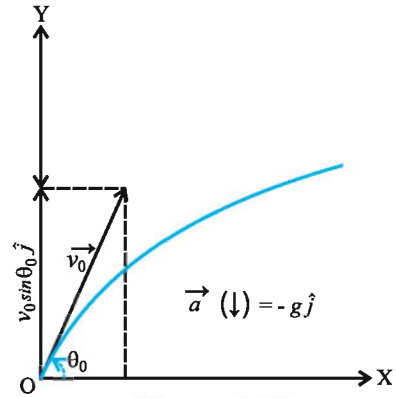


Figure 4.32

If we take the initial position to be the origin of the co–ordinate system, the co–ordinates of the point of projection would be $x_0 = 0$, $y_0 = 0$

Now using equations (4.14.6) and (4.14.7)

$x = v_{0x} t = (v_0 \cos\theta_0)t$ (4.16.3)

and $y = (v_0 \sin\theta_0)t - \dfrac{1}{2}gt^2$ (4.16.4)

The component of velocity at any time t can be obtained from equation (4.14.2) and (4.14.3) as

$v_x = v_{0x} = v_0 \cos\theta_0$ (4.16.5)

$v_y = v_0 \sin\theta_0 - gt$ (4.16.6)

Using equation (4.16.3) and (4.16.4) co-ordinates of the position of the projectile at any time in temrs of two parameters v_0 and θ_0 can be obtained. During the entire motion of the projectile, the x component of its velocity remains constant, while y component of velocity changes, like an object in free fall in the vertical direction.

Equation of trajectory of a projectile :

The equation giving relation between x and y co-ordinates of a projectile is known as equation of trajectory of a projectile.

To obtain the equation of trajectory of a projectile inserting value of t from equations (4.16.3) in equation (4.16.4). we get,

$$y = v_0 \sin\theta_0 \left(\frac{x}{(v_0 \cos\theta_0)}\right) - \frac{1}{2}g\left(\frac{x^2}{(v_0^2 \cos^2\theta_0)}\right)$$

$$y = (\tan\theta_0)\,x - \frac{g}{2(v_0 \cos\theta_0)^2} \cdot x^2 \quad (4.16.7)$$

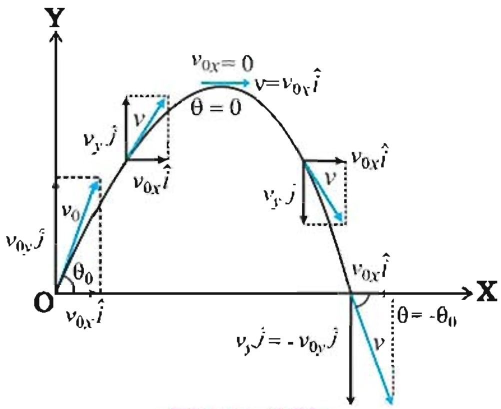


Figure 4.33

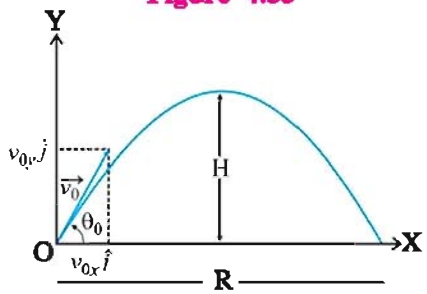


Figure 4.34

In this equation v_0, θ_0 and g are constant it is of the form $y = ax - bx^2$ in which a and b are constants. This is the equation of parabola. Hence we can say that the path of a projectile is a parabola. (See Fig. 4.33 and 4.34)

Time taken to achieve maximum height :

Suppose, the time taken by the projectile to reach maximum height H is t_m. (see Fig. 4.34) When projectile attains the maximum height, the y component of its velocity (v_y) becomes zero (See Fig. 4.33). Hence from equations (4.16.6)

$$v_y = v_0 \sin\theta_0 - gt_m = 0$$

$$\therefore\ t_m = \frac{v_0 \sin\theta_0}{g} \qquad (4.16.8)$$

Maximum height (H) :

The maximum height (H) reached by the projectile can be calculated by substituting

$t = t_m = \frac{v_0 \sin\theta_0}{g}$ in equation (4.16.4).

Thus we get,

$$y = H = (v_0 \sin\theta_0)\left(\frac{v_0 \sin\theta_0}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\theta_0}{g}\right)^2$$

$$\therefore\ H = \frac{v_0^2 \sin^2\theta_0}{2g} \qquad (4.16.9)$$

Time of flight (t_F) :

On substituting $y = 0$ and $t = t_F$ in equation (4.16.4).

$$0 = (v_0 \sin\theta_0)t_F - \frac{1}{2}gt_F^2$$

$$t_F = \frac{2v_0 \sin\theta_0}{g} = 2t_m \qquad (4.16.10)$$

Range of a projectile (R) :

The horizontal distance covered by a projectile from its initial position ($x = y = 0$) to the final position (where it passes $y = 0$ during its fall) is called the range (R) of the projectile.

If is easy to under state that the range in the distance travelled by the projectic during its time of flight

To find the range (R) substitute x = R and $t = t_F$ in equation (4.16.3)

$$R = (v_0 \cos\theta_0)(t_F)$$

$$= (v_0 \cos\theta_0)\left(\frac{2v_0 \sin\theta_0}{g}\right)$$

$$\therefore\ R = \frac{v_0^2 \sin 2\theta}{g}$$

$$\therefore\ R_{max} = \frac{v_0^2}{g} \qquad (4.16.11)$$

It is clear from the above equations, $R = R_{max}$ is the maximum range for $\theta_0 = 45°$ for given v_0.

It is important to note that the magnitude of the range depends upon the projection velocity (v_0) and the projection angle (θ_0) while the maximum range R_{max} depend only on the projection velocity (v_0).

Find t_m and t_F for an object thrown in the vertical direction $\theta_0 = \frac{\pi}{2}$.

Illustration 23 : A football lying on the ground is kicked with velocity 28m s^{-1} in the direction making 30° with horizontal direction. Find (i) maximum height attained (ii) the time to return on the ground and (iii) the distance at which (from initial position) it will return on the earth. (take acceleration due to gravity $g = 9.8$m s^{-2})

Solution : (i) Maximum height (H) attained by the football

Here $v_0 = 28$m s^{-1}, $\theta_0 = 30°$ and $g = 9.8$m s^{-2}

$$\therefore H = \frac{v_0^2 \ sin^2 \ \theta_0}{2g} = \frac{(\ 28\ )^2 \ (sin30)^2}{2 \times 9.8}$$

$$= \frac{(\ 28\ )^2 \ (0.5)^2}{2 \times 9.8} = 10.0\text{m}$$

(ii) Time taken to return on the ground is the time of flight t_F

$$\therefore t_F = \frac{2v_0 \ sin \ \theta_0}{2g} = \frac{2 \times 28 \times sin30}{9.8}$$

$$= \frac{28}{9.8} = 2.9 \text{ s}$$

(iii) The distance at which the football returns on the ground from the place at which it was kicked is the range R.

$$\therefore R = \frac{v^2_0 \ sin \ 2\theta_0}{g}$$

$$= \frac{28 \times 28 \times sin60^o}{g}$$

$= 69$ m

Illustration 24 : Galileo in his book "Dialogues on the Two new sciences", stated that "for elevations which exceed or fall short of 45° by equal amounts, the ranges are equal". Prove this statement.

Solution : Suppose the ranges of two projectiles projected at angles 45° – θ and 45° + θ (having the same difference with 45°) are R_1 and R_2. Now onward θ is in degrees.

Using the formula : $R = \frac{v^2_0 \, sin\, 2\theta^\circ}{g}$, we get

$$R_1 = \frac{v_0^2 \ sin\ 2(45-\theta^\circ)}{g}$$

$$= \frac{v_0^2 \ sin\ (90-2\theta^\circ)}{g}$$

$$= \frac{v_0^2 \ cos\ 2\theta^\circ}{g} \text{ and}$$

$$R_2 = \frac{v_0^2 \ sin\ 2(45+\theta^\circ)}{g}$$

$$= \frac{v_0^2 \ sin\ (90+2\theta^\circ)}{g}$$

$$= \frac{v^2_0 \cos 2\theta^\circ}{g}$$

Thus we can see that $R_1 = R_2$

If two projectiles are throun with same speed with complementary angles of projection ($\theta_1 = \theta_2 = 90^o$) their ranges will be equal.

Illustration 25 : A water pipe lying on the ground has a hole in it. From this hole water stream shoots upwards at an angle 45° to the horizontal. The speed of water stream is 10m s^{-1}. At what height once this stream hit the wall which is 5m away from the hole ?

Solution : $\theta_0 = 45^o$, $v_0 = 10\text{m s}^{-1}$, $x = 5$ m

Using the formula,

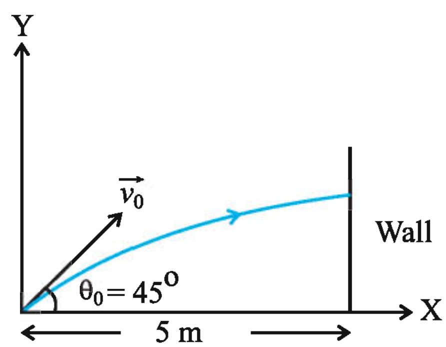


Figure 4.35

$$y = x(tan\ \theta_0) - \frac{g}{2(\ v_0\ cos\ \theta_0\ )^2} \cdot x^2$$

$$y = 5(tan\ 45^o) - \frac{9.8 \times 25}{2 \times (10 \times cos\ 45^o)^2}$$

$$= 5 - \frac{9.8 \times 25}{2 \times 100 \times \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= 5 - \frac{9.8}{4} = 5 - 2.45$$

$= 2.55$ m

Thus water stream will hit the wall at the height of 2.55 m on the wall.

SUMMARY

1. In this chapter we have obtained information regarding vector and scalar quantities in detail. We have learned to represent vectors graphically. We have distinquished between position vector and displacement vector and seen how the displacement vector can be obtained.

2. As vectors do not obey the ordinary laws of algebra, we learnet vector algebra. Zero vector and unit vectors were defined and it was shown that how a vector can be represented using unit vector. How vectors can be resolved in a plain was explained. In the case of product of the vectors, scalar and vector products we are defined. We understood the meaning of instantaneous velocity and derived formula for acceleration. After understanding relative motion we have obtained expression for relative velocity.

3. Equations for motion in a plane were derived.

4. We discussed uniform circular motion in detail and derived expression for centripetal acceleration and shown that its direction is towards the centre along the radius.

5. We also learnt projectile motion and derived the equation for its trajectory is expressions for time required to achieve maximum height, maximum range and time of flight were derived. We have also shown that for any given velocity to obtain the maximum range a projectice should be projected an angle of 45°.

Multiplication of Vector

↓

with scalar

↓

- with number (simple multiplication)

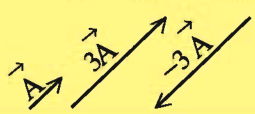
  vector $\vec{A}$ is multiplied by number m, new vector $m\vec{A}$ is obtained.

  • If m is positive, then new vector is in the direction of $\vec{A}$

  • If m is negative, then new vector is in the opposite direction of $\vec{A}$.

  Ex. : $3 \times \vec{A} = 3\vec{A}$

  $-3 \times \vec{A} = -3\vec{A}$



- with physical quantity (simple multiplication)

  (1) mass × velocity = momentum

  (scalar) × (vector) = (vector)

  (2) mass × acceleration = force Thus, when a vector quantity is multiplied by a scalar quantity, new vector quantity is obtained. Which is in the direction of original vector quantity.

with vector

↓

- result is scalar

  (vector)·(vector) = (scalar)

  scalar product or (dot product)

- result is vector

  (vector) × (vector) = (vector)

  vector product or (cross product)

EXERCISES

Choose the correct option from the given options :

1. Which quantity is a scalar from the following physical quantities.
   (A) Acceleration (B) velocity
   (C) linear momentum (D) Temperature

2. If $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = 4\hat{i} + 6\hat{j} - 2\hat{k}$, what will be the angle between $\vec{A}$ and $\vec{B}$.
   (A) π (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) 0^o

3. An object is moving on a circular path with velocity $\vec{v}$, at a given instant. When it completes half rotation, what will be the change in its velocity ?
   (A) $\vec{v}$ (B) $-2\vec{v}$ (C) zero (D) $\sqrt{2}\,\vec{v}$

4. A vector representing a physical quantity is $\vec{C} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, the angle between X–axis and $\vec{C}$ is
   (A) $cos^{-1}\frac{3}{\sqrt{29}}$ (B) $cos^{-1}\frac{4}{\sqrt{29}}$
   (C) $cos^{-1}\frac{5}{\sqrt{29}}$ (D) $cos^{-1}\frac{2}{\sqrt{29}}$

5. Co-ordinates of *a* particle moving in *a* plane at any time *t* are given by equations $x = \alpha t^2$ and $y = \beta t^2$. Magnitude of the velocity of this particle is
   (A) $2t\sqrt{\alpha^2 - \beta^2}$ (B) $2t\sqrt{\alpha^2 + \beta^2}$
   (C) $2t(\alpha + \beta)$ (D) $\sqrt{\alpha^2 + \beta^2}$

6. In a projectile motion if the maximum height H is half the range (R) ($H = \frac{1}{2}R$) then angle of projections θ_0 is
   (A) $tan^{-1}(1)$ (B) $tan^{-1}(2)$
   (C) $tan^{-1}(3)$ (D) $tan^{-1}(4)$

7. An object is projected with velocity $\vec{v}$. If the range (R) of this object is double the maximum height H, then its range is
   (A) $\frac{v^2}{g}$ (B) $\frac{3}{5}\frac{v^2}{g}$ (C) $\frac{4}{5}\frac{v^2}{g}$ (D) $\frac{1}{2}\frac{v^2}{g}$
   [**Note :** Use result of above objective (6)]

8. $(\vec{A} \cdot \vec{B})^2 + |\vec{A} \times \vec{B}|^2 =$ ....
   (A) AB (B) A^2B^2 (C) $\sqrt{AB}$ (C) zero

9. Rain falls in the downward direction with velocity 4 km/h. A man is walking on a straight road with velocity 3 km/h. The apprent velocity of rain relative to this man is
   (A) 3 km h^{-1} (B) 4 km h^{-1} (C) 5 km h^{-1} (D) 7 km h^{-1}

10. For which angle of projection the range and its maximum height will be equal ?

(A) $\theta_0 = 45^\circ$ (B) $\theta_0 = tan^{-1}(4)$

(C) $\theta_0 = tan^{-1}\left(\frac{1}{4}\right)$ (D) $\theta_0 = 30^\circ$

11. A motorcar is moving northwards with velocity 30m s^{-1}. If it turns towards West with the same speed, then change in its velocity is

(A) 60m s^{-1} North–West (B) $30\sqrt{2}$ m s^{-1} North–West

(C) $30\sqrt{2}$ m s^{-1} South–West (D) 60m s^{-1} South–West

12. If the resultant vector of $\vec{A}$ and $\vec{B}$ makes an angle α with $\vec{A}$ and β with B. Then

(A) $\alpha < \beta$ always (B) If A $<$ B, $\alpha < \beta$

(C) If A $>$ B, $\alpha < \beta$ (D) If A = B, $\alpha < \beta$

13. The linear speed of the tip of second arm of a clock is v. The magnitude of change in its velocity in 15 second is

(A) zero (B) $\frac{v}{\sqrt{2}}$ (C) $\sqrt{2}\,v$ (D) $2v$

14. The velocity of a boat with respected to ground is $3\hat{i} + 4\hat{j}$ and the velocity of water with respected to ground is $-3\hat{i} - 4\hat{j}$. Hence the velocity of boat w.r.t. water is .... The quantities are in SI.

(A) $8\hat{i}$ (B) $-6\hat{i} - 8\hat{j}$ (C) $6\hat{i} + 8\hat{j}$ (D) $6\hat{i}$

15. When the angle of projection is 25°, the range of the projectile is R. Now if the angle of projection is .... its range will remain same. (i.e. R)

(A) 40° (B) 45° (C) 65° (D) 60°

16. If the magnitude of the vector products of two vectors $|\vec{A} \times \vec{B}|$ is $\sqrt{3}$ times the magnitude of their scalar product $\vec{A} \cdot \vec{B}$ then the angle between them is ....

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

17. The acceleration of a projectile, at its maximum height is

(A) zero (B) g (C) maximum (D) minimum

18. An object is projected at angle of 45°, with the horizontal, with kinetic energy K. Its kinetic energy at maximum height is .... [$K = \frac{1}{2}mv^2$]

(A) 0 (B) $\frac{K}{2}$ (C) $\frac{K}{\sqrt{2}}$ (D) K

19. The velocity of a boat in a river of width 1.0 km, is 5 km h^{-1}. The boat crosses the river in 15 minutes, moving over the shortest path. Hence, the velocity of the flow of river is .... km h^{-1}.

(A) 1 (B) 3 (C) 4 (D) 5

20. Bullets are fired with the same initial velocity v in different directions on a plane surface. These bullets would fall on the maximum area of .... on this surface.

(A) $\frac{\pi v^2}{g}$ (B) $\frac{\pi v^2}{g^2}$ (C) $\frac{\pi^2 v^2}{g^2}$ (D) $\frac{\pi v^4}{g^2}$

21. For a projectile motion $y(t) = 8t - 5t^2$ and $x(t) = 6t$, where x and y are in metre and t is in second. The initial velocity of this projectile is ....

(A) 6 m/s (B) 8 m/s (C) 10 m/s (D) 14 m/s

22. If $|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is ....

(A) 90° (B) 120° (C) 0° (D) 60°

23. If $\vec{A} + \vec{B} = \vec{C}$ and $A = \sqrt{3}$, $B = \sqrt{3}$ and $C = 3$, then angle between $\vec{A}$ and $\vec{B}$ is ....

(A) 0° (B) 30° (C) 60° (D) 90°

24. The unit vector is perpendicular to the two vectors $3\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} - 2\hat{j} + 4\hat{k}$ is ....

(A) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$ (B) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

(C) $(\hat{i} - \hat{j} - \hat{k})$ (D) $\sqrt{3}(\hat{i} - \hat{j} - \hat{k})$

25. If the vectors $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\hat{j} - \hat{k}$ are mutually perpendicular, then the positive value of 'a' is ....

(A) 3 (B) 4 (C) 9 (D) 13

ANSWERS

1. (D)	2. (D)	3. (B)	4. (D)	5. (B)	6. (B)
7. (C)	8. (B)	9. (C)	10. (B)	11. (C)	12. (C)
13. (C)	14. (C)	15. (C)	16. (C)	17. (B)	18. (B)
19. (B)	20. (D)	21. (C)	22. (B)	23. (A)	24. (A) 25. (A)

Very Short Questions

1. What is the basic difference between vector and scalar ?
2. State names of two vectors quantities and two scalar quantities ?
3. What is needed to be stated to state when the position of an object is to be mentioned ?
4. Which vectors are called equal vectors ?
5. Define parallel vectors.
6. Define antiparallel vectors.
7. Which vectors are called non-parallel vectors.
8. State the two ways in which a vector is described.
9. How is the scalar product of two vectors defined ?
10. How is the vector product of two vectors defined ?
11. If the angle between two vectors is zero, the magnitude of their vector product is ....

12. If the angle between two vectors is 90°, their scalar product will be ....
13. If the angle between two vectors is zero, their scalar product will be ....
14. The direction of velocity of an object at any point on the path of its motion will along the ....
15. Velocity is a vector quantity. In how many ways can this vector be changed ?
16. Component of acceleration parallel to velocity (a_{11}) changes the .... velocity and perpendicular component ($a_{\perp}$) changes the .... velocity.
17. Acceleration in case of uniform circular motion along the tangent to the circular path is ....
18. What is called projectile motion ?
19. At the maximum height of the trajectory of a projectile,its velocity is ....
20. At the maximum height of the trajectory of a projectile,its acceleration is ....
21. To obtain the maximum range the object should be projected at an angle of .... with the horizontal.

Short Questions :

1. Describe the geometrical (graphical) method to represent vector quantities.
2. Distinguish between position vectors and displacement vectors.
3. Explain the multiplication of a vector by a real number.
4. Explain the subtraction of two vectors.
5. State the properties of addition of vectors.
6. Define unit vector and explain it in detail.
7. How are the two perpendicular components of a vector are obtained ?
8. State the properties of the scalar product of vectors.
9. In the case of the vector product of two vectors, explain the right handed screw rule for the direction of the resultant vector.
10. State the properties of vector products.
11. Obtain the formula for time t_m taken to achieve the maximum height of a projectile.
12. Derive formula for the maximum height H achieved by a projectile.
13. Obtain formula for the range of a projectile and using it obtain the formula for the maximum range.
14. Obtain the formula for total time of flight t_F of a projectile.

Answer the following questions in detail :

1. Drawing the necessary figure, explain the method of triangle for the addition of two vectors.
2. Drawing the necessary figure explain addition of two vectors using law of parallelogram. Obtain the magnitude and direction of the resultant vector using the components of the vectors.
3. Explain the resolution of vectors in a plane.
4. Describe the algebraic method for addition and subtraction of vectors.
5. Drawing the necessary figure to explain instantaneous velocity and derive the formula $\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}}$
6. Drawing the necessary figure explain acceleration and derive formula

   $$\vec{a} = \frac{d^2r}{dt^2} = \ddot{\vec{r}}$$
7. Using Derive an appropriate diagram explain relative velocity.
8. Obtain the equations for the motion in a plane.

   $$\vec{v} = \vec{v}_0 + \vec{a}\,t$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}\,t$$

$$v^2 - v_0^2 = 2\vec{a}\cdot(\vec{r} - \vec{r}_0)$$

9. Using an appropriate diagram derive formula for acceleration $a_c = \frac{v^2}{r}$ in the case of uniform circular motion and show that its direction is towards the centre along the radius.

Solve the following problems :

1. Two forces of equal magnitude act on a particle. If the angle between them is θ, find the magnitude of the resultant force. [**Ans. :** $2F\,cos\left(\frac{\theta}{2}\right)$]

2. Find the unit vector of vector $\vec{A} - \vec{B}$. Where vector $\vec{A} = 2\hat{i} - \hat{j} + 2\hat{k}$ unit and $\vec{B} = -\hat{i} - 2\hat{j} + 2\hat{k}$ unit [**Ans. :** $\frac{3\hat{i} + \hat{j}}{\sqrt{10}}$ unit]

3. If vectors $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = 4\hat{i} + 6\hat{j} - 2\hat{k}$, then show that they are parallel vectors.

4. A passenger arriving in a new town has to go to the hotel located 10 km away on straight road from station. Due divergence taxi driver takes him along a circuitous path 23 km long and reaches the hotel in 28 minutes.
(a) What is the average speed of the taxi ?
(b) Find the magnitude of the average velocity. Are these two equal ?
[**Ans. :** (a) 49.3km h^{-1} (b) 21.26km h^{-1} These two are not equal]

5. A particle starts its motion at time $t = 0$ from the origin with velocity $10\hat{j}$ m s^{-1} and moves in the X–Y plane with constant acceleration $8\hat{i} + 2\hat{j}$.
(a) At what time is its x co-ordinate become 16*m* ? And at this time what will be its y co-ordinate ?
(b) What will be the speed of this particle at this time ?
[**Ans. :** (a) at 2s, y co-ordinate 24 m (b) Speed 21.26m s^{-1}]

6. An aricraft is flying at the height of 3600 m from the ground. If the angle subtended at the ground observation point by aircraft positions 10 seconds apart is 30°, what is the speed of the aircraft ? [**Ans. :** 60π m s^{-1}]

7. A bullet fired from a gun at an angle 30° with horizontal direction hits the ground 3 km away on the ground. By adjusting only the angle of projection is it possible to hit the target 5 km away ? Show with calculation. (neglect air–resistance)

8. Prove that the angle of projection $\theta_0 = tan^{-1}\left(\frac{4H}{R}\right)$ for a projectile, projected from the origin, where H = maximum height and R = range of projectile.

9. Three non–zero vector $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the vector equation $\vec{A} + \vec{B} = \vec{C}$ and their magnitudes are related by the scalar equation A + B = C. How would $\vec{A}$ be oriented w.r.t. $\vec{B}$? Account for your answer.

10. If the direction of vector $\vec{A}$ is reversed, find $\Delta\vec{A}$, $|\Delta\vec{A}|$ and $\Delta|\vec{A}|$.
11. By keeping the direction of a vector the same if its magnitude is doubled, would the magnitude of its every component be doubled ?
12. The tail of a vector is on the origin of X Y co-ordinate axes. The vector is in + X direction. If the vector rotates anti-clockwise, find its X and Y components for the rotation of (i) 90° (ii) 180° (iii) 270° and (iv) 360°.
13. Can the magnitude of the relative velocity of either of two objects be more than the magnitude of the velocities of these objects ? Give one example.
14. Determine the magnitude and direction of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$.

    [**Ans. :** magnitude of both = $\sqrt{2}$; 45° and 315° with X–axis]
15. $\vec{A}$ is in positive Y direction and its magnitude is 100 unit. $\vec{B}$ is in the direction making an angle of 60° (in upward direction) with positive X–axis and its magnitude is 200 unit. $\vec{C}$ is in positive X direction and its magnitude is 150 unit. Out of these vectors which one has the maximum value for its (i) x component (ii) y component ? [**Ans. :** (i) $\vec{C}$ (ii) $\vec{B}$]
16. The magnitude of x component of the position vector of a particle is 3 m and it is in negative X direction. The magnitude of y component of this vector is 4 m and it is in negative Y direction. Find the megnitude of this vector and its direction with respect to negative X–axis. [**Ans. :** 5 m, $tan^{-1}\frac{4}{3}$]
17. Which out of the following quantities are independent of the selection of axes ?

    (A) $\vec{A} + \vec{B}$ (B) $\vec{A} - \vec{B}$ (C) $A_x + B_y$

    [**Ans. :** (A) and (B)]
18. Find the angle between $\vec{A}$ and $\vec{B}$ if $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

    [**Ans. :** 90°]
19. Calculate the displacement of a particle, with position vector $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$ metre in 10 s. [**Ans. :** $300\hat{i} + 400\hat{j}$ (m)]
20. Obtain the component of vector $\vec{A} = 2\hat{i} + 3\hat{j}$ in the direction of vector $\hat{i} + \hat{j}$

    [**Ans. :** $\frac{5}{\sqrt{2}}$]
21. Can the magnitude of a component of a vector be zero if the magnitude of the vector is not zero ? Can the magnitude of a vector be zero if one of its components is non–zero ?
22. Three non-zero vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the equation $\vec{A} + \vec{B} = \vec{C}$ and their magnitude satisfy the equation $A^2 + B^2 = C^2$. How would $\vec{A}$ be oriented with respect to $\vec{B}$? Account for your answer.
23. Two obejcts are projected with the same velocity at different angles with the horizontal and if the range is same for both of them prove that $t_1 t_2 = \frac{2R}{g}$ where t_1 and t_2 are their time of flights.

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CHAPTER 5

LAWS OF MOTION

5.1 Introduction

In the last chapter we had discussed displacement, velocity and acceleration of a body during its motion. But we had not considered the causes that produce the motion and the changes in motion. In the present chapter we will consider these aspects. You already know that this branch of Physics in which the motion of a body is discussed along with the causes of motion and the properties of the moving body is called dynamics.

5.2 Force and Inertia

Let us consider our few observations about the motion of different bodies found in our day-to-day life. (1) A book lying on the table remains steady as it is, if we do not apply any force on it from outside. (2) In order to throw a ball upward; we have to exert a push in the upward direction. (3) To bring a lorry into motion from rest, a person has to push it. (4) To stop the ball rolling down the inclined plane we apply force in the direction opposite to the direction of its motion, using our hand. Considering these observations we realise that to bring the body into motion from its rest position and also to slow down or stop its motion; some **external agency** providing some force is required. In all the above cases the external force is in contact with the body. Such a force which is acting (or is applied) by remaining in contact with the body is called a **contact force.** Some illustrations of motion are also found in which the external agency is not in contact with the body and yet it exerts force on the body. e.g. A body released from the top of a building performs an accelerated motion towards earth. Here the Earth is not in contact with that body but the force on the body due to gravitational field of Earth is responsible for its accelerated motion. When a nail of iron is placed a little away from a magnet it moves towards the magnet due to attraction. Here the force on the nail due to the magnetic field of the magnet is responsible for its motion. Such forces (due to fields like gravitational field, electric field, magnetic field.) are called **field forces.** Thus external agencies can exert force on the body even from away, without coming in contact.

From this, we understand that the external agency producing a force that affects the motion of a body may be or may not be in contact with the body. In the cases discussed above, the body either comes in motion from rest position or its velocity changes during the motion. But a question may arise as to-"Does a body in uniform motion (that is, a motion in the same straight line with constant speed) need an external force to continue its uniform motion ?"

Aristotle (384 B. C. to 322 B. C.), a Greek philosopher, was of the opinion that if the body is in motion - uniform or non-uniform- then it needs 'something' external - that is an external force - to maintain its motion. Before we know the truthfulness of this opinion, let us consider an illustration : A bicycle, moving, in the same straight line with uniform speed on a horizontal road that appears smooth will stop after sometime if pedalling is stopped. If we want to keep the bicycle in uniform motion we have to apply external force by pedalling it. This observation may appear to support Aristotle's concept; but actually it does not.

In fact, we all know that the external force of friction due to the road opposes the motion of the bicycle. Hence it comes to a stop. Pedalling is required if we want to continue its motion. Then, what was Aristotle's fallacy ? He had considered his practical experience as a basic law of nature and there he happened to be wrong. To understand the law of nature about forces and motion, we have to imagine a situation in which the frictional force opposing the motion is not present. Galileo did this and acquired a deeper understanding about motion. Most of the Aristotle's concepts of motion are found to be wrong today.

You have studied about Galileo's experiments in standard-9. Galileo observd that

(i) objects moving downward along an inclined plane are accelerated - that is their velocity increases.

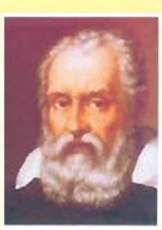
(ii) objects moving upward along an inclined plane are retarded - that is their velocity decreases.

But the motion on the horizontal plane is the intermediate condition of the above two cases. Hence Galileo suggested that a body moving in a straight line on a frictionless horizontal plane should not have either acceleration or retardation. Thus it should move with a constant velocity and it does not need any external force for this.

Friction is an external force on a body, which opposes the motion of the body. If we apply another sufficient external force opposing friction then the net force on the body becomes zero and then its velocity will be maintained constant. Moreover, in the steady state of the body when it remains as it is, also no net external force acts on it. Thus the steady state and the state of uniform motion (motion with constant velocity), are equivalent, as far as force is concerned. This is because in both these states the net external force on the body is zero.

If an external force acts on a body, the state of the body will change. A body does not (and cannot) change of its own, its steady state or the state of uniform motion. This property of the body not to change its state, by itself is called "inertia" of the body. Inertia means to oppose the change. The mass of a body is a measure of its inertia. Out of the two given objects the one with a greater mass is said to have greater inertia also.

Galileo Galilei (1564 - 1642)



Galileo Galilei, born in Pisa of Italy in 1564, was the main figure in the scientific revolution in Europe. He proposed the concept of acceleration. From his studies on the motion of bodies on inclined plane and freely falling bodies, he contradicted the prevailing Aristotelian concepts that to continue a motion some force is needed and heavy bodies fall quickly as compared to lighter bodies under gravity. His idea of inertia later became the starting point of Newton's work.

With the help of a telescope disigned by himself, he made astronomical observations of dark spots on the sun, mountains and depressions on the surface of moon, the moons of Jupiter and phases of Venus. He also proposed that the luminosity of the milky way is due to the light coming from a large number of stars which are not visible to the naked eye.

In his excellent book 'Dialogue on the two Chief World Systems' he advocated the heliocentric theory proposed by Copernicus for the solar system. To day also it is universally accepted.

Due to his great discoveries he is respected and honoured in the world of science.

Isaac Newton

Newton was born in Woolsthorpe, England in 1642, the year Galileo died. During his study at Cambridge University a plague epidemic forced him to return to his mother's farm. Here he had ample scope for thinking deeply, and made many discoveries in mathematics and physics. Calculus, binomial theorem for negative and fractional exponents, inverse square law of gravitation, spectrum of white light...etc. are discoveries due to Newton. On returning to Cambridge he devised a reflecting telescope.

His own book 'The Principia Mathematica' is considered as one of the greatest scientific work. It included his three laws of motion, universal law of gravitation, basic principles of fluid mechanics, mathematics of wave motion, calculation of masses of Sun, Earth and other planets, theory of tides and many other important topics.

His discoveries about light and colours are summarized in his another book 'Opticks'. The scientific revolution initiated by Copernicus, Kepler & Galileo was carried forward and brought to completion by Newton. He proposed that the same laws govern the terrestrial phenomena and the celestial phenomena. e.g. In the fall of an apple on Earth and in the revolution of the moon around Earth the same type of mathematical equations are found applicable. Newton died in 1727.

5.3 Newton's First Law of Motion

Starting with Galileo's logical thoughts, the task of development of mechanics was accomplished almost single-handedly by Isaac Newton, one of the greatest scientists of all times. The three laws of motion developed by Newton form the base of mechanics.

Newton's first law of motion is written as :

"As long as no net external force is acts on the body, a stationary body remains stationary and a body in motion continues to move with constant velocity."

In fact, this is Galileo's law of inertia only. In some cases, we know that the net (resultant) external force on a body is zero and hence we conclude that the acceleration of the body is zero and its velocity is constant. Conversely, sometimes we see a body without acceleration (either in steady state or in motion with constant velocity) and hence we infer that the net external force on the body must be zero.

On applying a net force on a body, it comes into motion if it is stationary and if it is aleady in motion, its velocity changes. Thus in both the cases, acceleration is produced in it. Therefore, force comes out to be the cause of producing acceleration.

From Newton's first law we can say that force is a physical quantity due to which, a stationary body comes into motion and a moving body changes its velocity. Thus Newton's first law of motion gives us the definition of force – but it does not give information about the value (magnitude) of force.

5.4 Momentum

The product of the mass (m) of a body and its velocity ($\vec{v}$) is called its momentum ($\vec{p}$).

That is, $\vec{p} = m\,\vec{v}$ (5.4.1)

Momentum is a vector quantity and its direction is in the direction of velocity. The SI unit of momentum is kg m s^{-1} or Ns. Its dimensional formula is [$M^1L^1T^{-1}$].

The momentum of a moving body gives some more information than its velocity. We understand this by examples. Which one out of a bicycle and a car moving with the same velocity can do more harm in colliding with us ? Clearly the car – because due to greater mass its momentum is more.

Thus mementum is an important physical quantity.

5.5 Newton's Second Law of Motion

Newton's second law of motion gives the magnitude (value) of the external force acting on the body.

When force is applied on a body, its velocity changes, hence its momentum also changes. When equal force is applied for equal time on two bodies–one heavier and the other lighter–then it is found that the lighter body acquires a larger velocity but both of them acquire equal momentum.

Suppose you are moving on a bicycle with uniform velocity $\vec{v}$. Now, if you are not in a hurry to stop, you will apply the brakes gently (this produces a small force). Hence the bicycle will gradually slow down, and will stop after some time. But if you want to stop the bicycle quickly, then you will apply the brakes very strongly (this produces a greater force) and then only the bicycle can stop quickly. In both these cases the change in momentum is the same (becomes zero from $m\vec{v}$, where m = mass of yourself + bicycle). But in the second case, greater force had to be applied because that change was to be done faster (Remember that when brakes are applied, pedalling is stopped !)

Thus force has relation with the change in momentum and the time taken for that change; and that relation is obtained from Newton's second law of motion, stated as under :

"The time–rate of change of momentum of a body is equal to the resultant external force acting on it, and this change is in the direction of the force."

Hence, if force $\vec{F}$ is applied on a body of mass m and momentum $\vec{p}$ (= $m\vec{v}$), then,

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{5.5.1}$$

$$= \frac{d}{dt}(m\vec{v}) \tag{5.5.2}$$

If the mass of body remains constant, then

$$\vec{F} = m\frac{d\vec{v}}{dt} \tag{5.5.3}$$

$$\therefore \vec{F} = m\vec{a} \qquad \left(\because \frac{d\vec{v}}{dt} = \vec{a}\right)$$

Thus force,

$$\vec{F} = \text{mass m} \times \text{acceleration } \vec{a} \tag{5.5.4}$$

The SI unit of force is newton (= N)

The force which produces an acceleration of 1 m s^{-2} in a body of mass 1 kg is called 1 N force.

[1 N = 1 kg m s^{-2}]

(The unit of force in CGS system is dyne and 1 N = 10^5 dyne).

The dimensional formula of force is [$M^1L^1T^{-2}$].

From equation (5.5.4), the **value** of the resultant force acting on the body is obtained.

On applying a force $\vec{F}$ for time–interval Δt, on a body of mass m and velocity $\vec{v}_1$; if its velocity becomes $\vec{v}_2$, then from the measured values of $\vec{v}_1$, $\vec{v}_2$, and Δt; we can find $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$ and then from m and $\vec{a}$; force $\vec{F}$ can be calculated with the help of equation (5.5.4).

We note a few important points about this law :

(1) If $\vec{F} = 0$ (that is, the resultant external force is zero), then $\vec{a} = 0$, $\therefore \vec{v}$ = constant. This fact is consistent with Newton's first law.

(2) This law is a vector law. Hence for the three components F_x, F_y, F_z of the force $\vec{F}$, three equations are obtained as under :

$$\left.\begin{aligned} F_x &= \frac{d\,p_x}{dt} = m\,a_x \\ F_y &= \frac{d\,p_y}{dt} = m\,a_y \\ F_z &= \frac{d\,p_z}{dt} = m\,a_z \end{aligned}\right\} \tag{5.5.5}$$

(3) The acceleration $\vec{a}$ of the body at a given point at a given instant is determined by the external force $\vec{F}$ acting on it at that point at that instant, and the body has no memory of its acceleration at previous moments. A body released from an accelerating train does not have any acceleration in the horizontal direction at a moment just after the release – (neglecting the resistance of air).

(4) In equation (5.5.1) the value of $\vec{p}$ is not important but the change in $\vec{p}$ is important. If the body is steady in the beginning then initially $\vec{p} = 0$. But if force $\vec{F}$ acts on it, $\vec{p}$ will change and that change is important in this equation.

Illustration 1 : A body of mass 40 kg is moving in a straight line on a smooth horizontal surface. Its velocity decreases from 5.0 m s^{-1} to 2.0 m s^{-1} in 6 seconds. Find the force acting on this body. How much distance would it travel during this time ?

Solution : Taking the direction of motion of the body as X–axis, we can write $F_x = m\, a_x$ for the value of force.

From $v_x = v_{0x} + a_x t$.

$2 = 5 + a_x(6)$

$\therefore\ a_x = -\ 0.5$ m s^{-2}

$\therefore\ F_x = m\, a_x = (40)\ (-\ 0.5) = -20$ N

Thus, this much force is acting opposite to direction of motion (i. e. acting in negative X – direction)

From, $v_x^2 - v_{0x}^2 = 2a_x x$

$4 - 25 = 2(-0.5)x$

$\therefore\ x = 21$ m.

Illustration 2 : When 45 N force is applied on a body of mass m, acceleration of 4.5 m s^{-2} is produced in it. The same force when applied on a body of mass m', acceleration of 9.0 m s^{-2} is produced. Find the acceleration produced by the same force applied on these two bodies tied together.

Solution :

$F = ma \quad \therefore\ 45 = m\ (4.5)\ \therefore\ m = 10$ kg

$F = m'a' \quad \therefore\ 45 = m'\ (9.0)\ \therefore\ m' = 5$ kg

If after tying these two bodies together, the same force produces acceleration a'', then

$F = (m + m')\ a''$

$45 = (10 + 5)\ a'' \quad \therefore\ a'' = 3$ m s^{-2}

Illustration 3 : A car of mass 1000 kg. is moving with a velocity of 30 m s^{-1} on a horizontal straight road. On seeing red light of a traffic signal, the driver applies brakes to produce a constant braking force of 4 kN. (i) Find the deceleration (or retardation) of the car. (ii) What time will the car take to stop ? (iii) How much distance would it travel during this motion ?

Solution : (i) Taking X–axis in the direction of the car; the force applied on it would be in negative X–direction. 4kN = 4 × 10^3 N

From $F_x = m\, a_x$; $-4 \times 10^3 = (1000)a_x$

$\therefore\ a_x = -4$ m s^{-2}

(ii) The velocity of the car becomes zero when it stops.

$\therefore$ From $v_x = v_{0x} + a_x t$

$0 = 30 + (-4)t$

$\therefore$ t = 7.5 s

(iii) From $v_x^2 - v_{0x}^2 = 2\, a_x x$

$0 - 900 = 2\ (-4)x$

$\therefore\ x = 112.5$ m

5.6 Impulse of Force

The product of force $\vec{F}$ acting on a body and the time–interval for which it acts is called impulse of force. From equation (5.5.1) representing Newton's second law, we can write,

impulse of force $\vec{F}\, dt = d\vec{p}$ = change in its momentum (5.6.1)

When a force of large value acts for a very short time, it is difficult to get the values of force and the time interval. But the change in momentum is measurable. e.g. When a ball of mass m moving with velocity $\vec{v}$ hits a wall and rebounds with velocity $\vec{v}\,'$, the force exerted on the ball, by the wall acts for a very small time. By measuring the velocities of the ball $\vec{v}$ and $\vec{v}\,'$ the change in its momentum can be known.

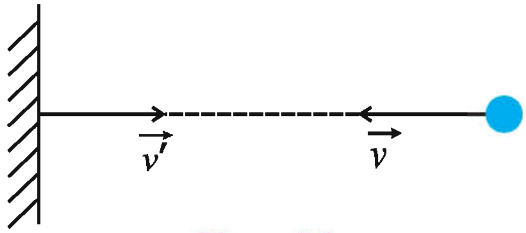


Figure 5.1

Such a force that acts for a very short time is called an impulsive force.

Illustration 4 : A ball of mass 150 g and velocity 12 m s^{-1} coming towards a batsman is hit by him with a force of 480 N in such a way that the ball moves with velocity 20 m s^{-1} in the direction opposite to its original one. Find the time of contact between the ball and the bat.

Solution : If we take the original direction of the ball as negative X–direction, then $\vec{v}_1 = -12\hat{i}$ m s^{-1}, $\vec{v}_2 = 20\hat{i}$ m s^{-1} and $\vec{F} = 480\hat{i}$ N.

Change in momentum of the ball

$$\Delta\vec{p} = \vec{p}_2 - \vec{p}_1$$

$$= m\vec{v}_2 - m\vec{v}_1 = m(\vec{v}_2 - \vec{v}_1)$$

$$= (0.150)\,[\,20\hat{i} - (-\,12\hat{i})\,]$$

$$= 4.8\hat{i} \text{ kg m s}^{-1}$$

From, $|\vec{F}| = \dfrac{|\Delta\vec{p}|}{\Delta t}$, $\quad 480 = \dfrac{4.80}{\Delta t}$

$\therefore\ \Delta t = 0.01$ s.

5.7 Newton's Third Law of Motion

Newton's second law of motion gives the relation between the resultant external force acting on a body and the acceleration of the body, ($\vec{F} = m\vec{a}$). But what is the cause of such a force acting on the body ? Actually the force on the body is exerted due to another body (or bodies). Hence a question may arise – "if the other body applies a force on the given body, does the given body apply force on that another body or not ?" The answer to this question is obtained from Newton's third law of motion, which is stated as under.

"To every action there is always an equal and opposite reaction."

Press a spring with your hand. You will experience that the spring also exerts a force in the opposite direction on the hand. Here the spring and the hand were in contact with each other; so we could feel the force. But when a stone attracted by Earth falls towards it, does the stone also exert a force on the Earth ? And does the Earth also rises up towards the stone ? – Such questions may arise in our mind. The answer to this question, according to Newton is – Yes, the stone also exerts the same force on Earth in opposite direction as Earth has on it. But because of the very large mass of Earth the effect of this force on its motion is extremely small. Therefore we are not able to see or detect or feel it, that is, such an effect is negligible.

Thus from Newton's third law of motion, it comes out that there exists no isolated force in nature. Forces are produced only due to interaction between two bodies. Forces are produced only in pairs, and the forces in a pair are equal in magnitude and opposite in direction. Newton's own wording of this third law of motion as given above, is so crisp and beautiful that it has become a part of day to day conversations. We make a few clarifications about this law to avoid any misunderstanding.

(1) In the interaction between two bodies, any one force is called 'action' and then the other is called 'reaction'.

(2) It is not correct to suppose that first the 'action' takes place and then as a result of it 'reaction' occurs. Such cause-effect relationship is not implied in this third law. The force on A by B and the other on B by A both get simultaneously applied.

(3) Action and reaction act on two different bodies. If we represent the force on A by B as $\vec{F}_{AB}$ and the one on B by A as $\vec{F}_{BA}$, then according to this third law,

$$\begin{bmatrix} \vec{F}_{AB} \\ \text{i.e. Force on A} \\ \text{by B} \end{bmatrix} = -\begin{bmatrix} \vec{F}_{BA} \\ \text{i.e. Force on B} \\ \text{by A} \end{bmatrix}$$

Therefore, if we want to discuss the motion of only one body (e.g. of A) then we have to take the force on that body ($\vec{F}_{AB}$) only, the other force $\vec{F}_{BA}$ is irrelevant and it is not to be taken into account. In discussing the motion of only one body, A or B, if we say that "adding the two forces, the net force obtained is zero", then it is wrong, because these two forces are acting on two different bodies. But if we consider the motion of the system as a whole, consisting of these two bodies, then these two forces ($\vec{F}_{AB}$ and $\vec{F}_{BA}$) will become internal forces and their total force will be zero (How will it become zero will be seen in detail in the chapter on Dynamics of system of particles, in future) Therefore they are not to be considered for the motion of the system as a whole. It is due to this fact that Newton's second law of motion can be applied to the system of particles also.

For the motion of the system as a whole, the force within the system is not responsible. We cannot drive a car by pushing the car by sitting inside the car. For the motion of the car, an external force is required to act on it. Now, you may perhaps feel as to how are we able to drive a car by starting its engine, which is an internal part of the car ! Actually, the external force needed to run the car is provided in the form of friction with the road. This may appear surprising to you, but it is true and you will learn it in future.

5.8 Law of Conservation of Momentum

Newton's second and third laws of motion lead to an important result "the law of conservation of momentum". Let us consider an example of a bullet fired from a rifle. As the bullet fired from the rifle moves forward, the rifle is pushed backward (it is called the recoil of a rifle). If the force on the bullet by the rifle is $\vec{F}$, then the force exerted on the rifle by the bullet is $-\vec{F}$. Both these forces act for the same time interval Δt. Before the bullet is fired, both the bullet and the rifle were steady. Therefore their respective momenta $\vec{p}_b$ and $\vec{p}_r$ both were zero. Hence their total initial momentum,

$$\vec{p}_b + \vec{p}_r = 0 \qquad (5.8.1)$$

Now according to the equation (5.6.1) obtained from Newton's second law of motion,

$$\text{change in momentum of bullet} = \vec{F}\Delta t \qquad (5.8.2)$$

$$\text{change in momentum of rifle} = -\vec{F}\Delta t \qquad (5.8.3)$$

As the initial momentum of each one of them is zero, their respective final momenta ($\vec{p'}_b$ and $\vec{p'}_r$), will be equal to the change in their respective momenta.

$$\text{Thus } \vec{p'}_b = \vec{F}\Delta t \text{ and } \vec{p'}_r = -\vec{F}\Delta t \qquad (5.8.4.)$$

From equations (5.8.4) and (5.8.1), we get

$$\vec{p'}_b + \vec{p'}_r = 0 = \vec{p}_b + \vec{p}_r \qquad (5.8.5)$$

$$\text{i.e.} \begin{bmatrix}\text{Final momentum} \\ \text{of (bullet + rifle)}\end{bmatrix} = \begin{bmatrix}\text{initial momentum} \\ \text{of (bullet + rifle)}\end{bmatrix}$$

Here no external force has acted on the system of (rifle + bullet), hence this system is called an isolated system. The forces which act are only the internal forces; and their resultant is always zero. All these facts are included in the law of conservation of momentum which is stated as under :

"The total momentum of an isolated system remains constant."

This law is fundamental and universal just like the law of conservation of energy and the law of conservation of charge. Moreover, it is equally true for the interactions between large bodies like planets and stars and interactions between micro particles like electrons and protons. No phenomenon or process can occur which violates this law.

Illustration 5 : A soldier fires bullets, each of mass 50 g, from his automatic rifle with a velocity of 1000 m s^{-1}. If he can bear a maximum force of 200 N on his shoulder, find the maximum number of bullets which he can fire in a second.

Solution : Suppose mass of each bullet = m and maximum *n* bullet are fired per second. Before firing the total momentum of bullets and rifle = 0. After firing, the momentum of every bullet, $p = mv$

$\therefore$ The momentum imparted to bullets per second = $(nmv - 0) = nmv$

Since no external force acts during the process of firing, the system consisting of (bullet + rifle) can be considered as an isolated system and so its total momentum should remain constant.

$\therefore$ The momentum received by the rifle, in opposite direction, in 1 second = nmv

Now, since the change in momentum per second is equal to force, we can say that the force on the rifle and hence on the shoulder of the soldier = nmv

$\therefore$ nmv = 200N

$\therefore$ $n\,(50 \times 10^{-3}\text{ kg}) \times 100\text{ m/s} = 200\text{ N}$

$\therefore$ $n = 4\ s^{-1}$

Illustration 6 : A person of mass 60 kg is standing on a raft of mass 40 kg in a lake. The distance of the person from the bank is 30 m. If the person starts running towards the bank with velocity 10 m/s, then what will his distance be from the bank after one second ?

Solution : When the person starts running towards the bank the system (raft + person) moves backwards.

Suppose, mass of person = m_P

mass of raft = m_R

and, velocity of person w.r.t. raft = $\vec{v}_{PR}$

velocity of raft w.r.t. the bank = $\vec{v}_{RB}$

velocity of person w.r.t. the bank = $\vec{v}_{PB}$

Taking the direction of running of person as X–axis,

$\vec{v}_{PR} = 10\hat{i}$ m/s

It is clear that $\vec{v}_{PB} = \vec{v}_{PR} + \vec{v}_{RB}$ (1)

Since no external force is acting on this system of (person + raft) according to the law of conservation of momentum,

$$\begin{bmatrix}\text{initial momentum} \\ \text{of (person + raft)}\end{bmatrix} = \begin{bmatrix}\text{final momentum} \\ \text{of (person + raft)}\end{bmatrix}$$

$$\therefore\ 0 = m_P\vec{v}_{PB} + m_R\vec{v}_{RB}$$

$$= m_P(\vec{v}_{PR} + \vec{v}_{RB}) + m_R\vec{v}_{RB}$$

$$= m_P\vec{v}_{PR} + (m_P + m_R)\vec{v}_{RB}$$

$$\therefore\ 0 = 60\,(10\hat{i}) + (60 + 40)\,\vec{v}_{RB}$$

$$\therefore\ \vec{v}_{RB} = -\,6\hat{i}\ \text{m/s.}$$

$\therefore$ From equation (1)

$$\vec{v}_{PB} = 10\hat{i} - 6\hat{i} = 4\hat{i}\ \text{m/s}$$

Thus the person travels an effective distance of 4 m in one second, towards the bank (because he moves forward by 10 m and the raft moves backward by 6 m)

$\therefore$ After 1 s, his distance from the bank is 30 – 4 = 26 m.

Illustration 7 : A bomb in the steady state explodes into three fragments. Two fragments of equal masses move with velocity 30 m/s in mutually perpendicular directions. The mass of the third fragment is equal to three times the mass of each of these two fragments. Find the magnitude and direction of the velocity of this third fragment.

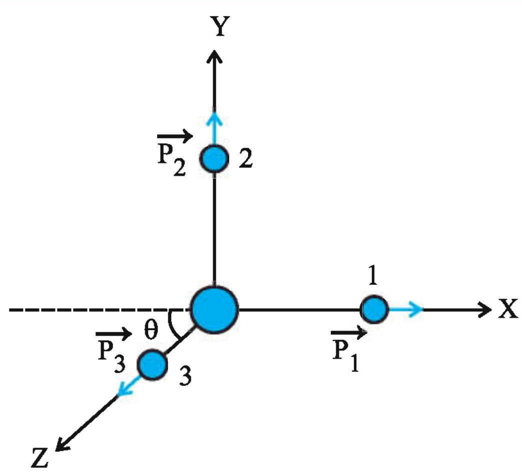


Figure 5.2

Solution : Before explosion, the bomb is steady. $\therefore$ its initial momentum = 0. No external force acts on the bomb during the explosion. Hence, according to the law of conservation of momentum, the vector sum of the momenta of all the fragments after explosion must be zero. Here the mass of each of two fragments is equal to m. ∴ Mass of the third fragment = 3 m. If their respective momenta are $\vec{p}_1$, $\vec{p}_2$ and $\vec{p}_3$.

then, $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$

Taking X– and Y– axes according to Fig. 5.2,

$\vec{p}_1 = m(30)\hat{i}$, $\vec{p}_2 = m(30)\hat{j}$

$\therefore\ m(30)\hat{i} + m(30)\hat{j} + (3m)\vec{v}_3 = 0$

$\therefore\ 3m\,(\vec{v}_3) = -30m\,(\hat{i} + \hat{j})$

$\therefore\ \vec{v}_3 = -10\hat{i} - 10\hat{j}$

$\therefore\ |\vec{v}_3| = \sqrt{(-10)^2 + (-10)^2}$

$= 10\sqrt{2}$ m/s.

And, $\tan\theta = \frac{v_{3y}}{v_{3x}} = \frac{-10}{-10} = 1$

$\therefore\ \theta = 45°$

Thus the third fragment moves at an angle of 45° with negative X axis and nagative Y axis.

5.9. Equilibrium of Concurrent Forces

The forces, of which the lines of action pass through the same point, are called the concurrent forces.

The condition, in which the resultant (net) force of all the external forces acting on a particle, becomes zero, is called equilibrium. Looking from this view point, the steady state and the state of motion with uniform velocity of the body are both equilibrium states.

Thus, for equilibrium $\Sigma\vec{F} = 0$.

If only one external force $\vec{F}$ acts on a particle, then acceleration will be produced in it according to $\vec{F} = m\vec{a}$ and the particle cannot remain in equilibrium. If two external forces $\vec{F}_1$ and $\vec{F}_2$ act on the particle, then for equilibrium (i. e. for $\Sigma\vec{F} = 0$), $\vec{F}_1 = -\vec{F}_2$. Fig. 5.3 (a, b)

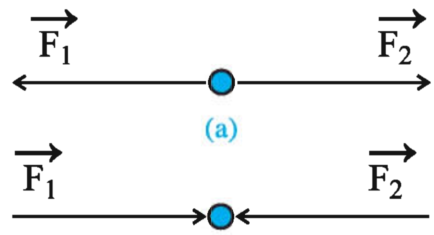


Figure 5.3

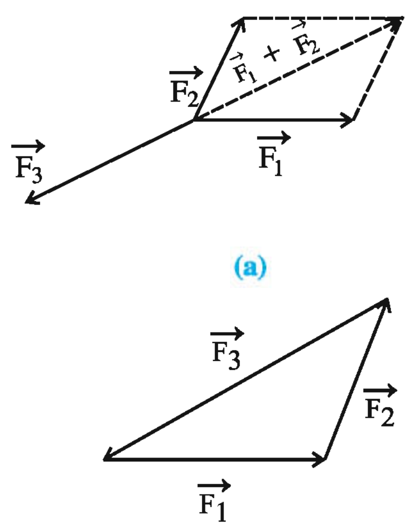
If more than two external forces are acting, then for equilibrium, their vector sum must be zero i.e. $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \dots\dots\dots = 0$.

Moreover, since force is a vector quantity, the sum of the corresponding components of all the forces should also become zero. i.e.

$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0.$

For a particle remaining in equilibrium, under the effect of three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$, the vector sum of the two forces $(\vec{F}_1 + \vec{F}_2)$ has the magnitude equal to that of the third force and direction opposite to it so that $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$. [See Fig. 5.4. (a)]

i.e. $\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$



(b)

Figure 5.4

In other words, if all the three force vectors are arranged tail to head, they should form a closed figure [see Fig. 5.4. (b)] so, the resultant force becomes zero.

Illustration 8 : As shown in Fig. 5.5., two strings AO and BO are tied with a rigid support and a body of 20 kg. mass is suspended with a third string OC. In the equilibrium condition of this entire system, the strings AO and BO make angles of 60° and 30° respectively with the horizontal. Assuming all these strings as massless, find the tensions produced in these strings. [Take $g = 10$ m s^{-2}]

Solution : Here, as the strings are massless, the force applied at one end of the string is communicated undiminished at the other end. (Thus a massless string works like a postman).

In equilibrium condition, the body and the point O are steady.

Suppose the tensions in the strings are T_1, T_2, T_3 as shown in the Fig. 5.5 in equilibrium condition.

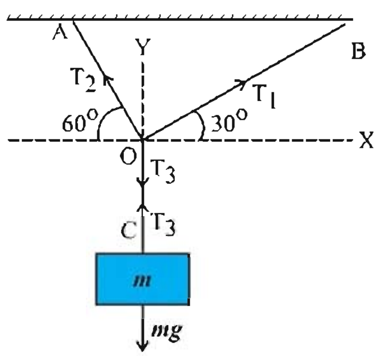


Figure 5.5

For equilibrium of point C, $T_3 - mg = 0$

$\therefore \quad T_3 = mg = (20)(10)$

$= 200$ N (1)

Take X–axis in horizontal direction and Y–axis perpendicular to it.

From Fig., x component of $T_1 = T_1 \cos 30°$

y component of $T_1 = T_1 \sin 30°$

x component of $T_2 = T_2 \cos 60°$

y component of $T_2 = T_2 \sin 60°$

For equilibrium of point 0,

$\Sigma F_x = 0$ gives, $T_1 \cos 30° - T_2 \cos 60° = 0$

$$\therefore \quad \frac{\sqrt{3}}{2} T_1 - \frac{1}{2} T_2 = 0$$

$$\therefore \quad \sqrt{3}\, T_1 - T_2 = 0 \qquad (2)$$

$\Sigma F_y = 0$ gives, $T_1 \sin 30° + T_2 \sin 60° - T_3 = 0$

$$\therefore \quad \frac{1}{2} T_1 + \frac{\sqrt{3}}{2} T_2 - 200 = 0$$

[from eqn. (1)]

$$\therefore \quad T_1 + \sqrt{3}\, T_2 = 400 \qquad (3)$$

Multiplying equation (2) with $\sqrt{3}$ and then adding in equation (3), we get,

$$(3T_1 - \sqrt{3}\, T_2) + T_1 + \sqrt{3}\, T_2 = 400$$

$$\therefore \quad T_1 = 100 \text{ N}$$

Putting this value in equation (3), we get $T_2 = 173$ N.

5.10 Friction

When the bodies are in contact, mutual contact, forces are generated in every pair of particles at their contact surface.This contact forces obey Newton's third law of motion. Consider two components of this contact force – (i) the component perpendicular to the contact surfaces is called **normal reaction** N (sometimes in short, called the normal force) (ii) the component parallel to the contact surfaces is called **frictional force *f***, or in short, the **friction.**

Such contact forces and frictional forces are determined by the roughness of the contact surfaces at the molecular level. Surfaces of objects may appear extremely smooth, but when viewed through a microscope 'ridges' and 'valleys' are found all over the surface. Ridges of one surface and valleys of the other surface get interlocked with each other and a 'cold welding' takes place. Hence when one surface tries to shift on the other, a force opposing it comes into play – which is called the **frictional force *f***.

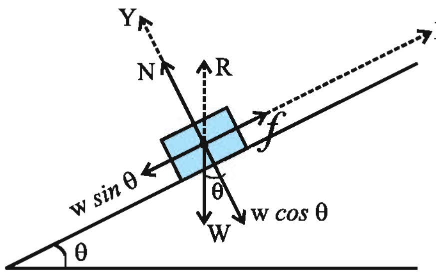
We consider an illustration. Suppose a block is at rest on a slope inclined at an angle θ, to the horizontal. The block exerts force equal to its weight $\vec{W}$ on the surface of slope. This surface exerts an equal and opposite force $\vec{R}$ on the block. $\vec{R}$ is the contact force.

For convinience we choose the X-axis parallal to the inclined surface (see Fig. 5.6(a)) Two mutually perpendicular components of this force $\vec{R}$ are as under :

(1) the component perpendicular to the surface in called the normal force $\vec{N}$

(2) the component parallel to the surface is called the frictional force f. In the absence of such a frictional force f, the block would have slipped down the slope due to the force W sin θ Such a motion (which actually does not take place

due to the presence of frictional force) is called impending motion.



5.6 (a)

Now, $|\vec{R}|^2 = |\vec{N}|^2 + |\vec{f}|^2$ since the block is in equillibrium, $\Sigma F_x = 0$ and $\Sigma F_y = 0$, $\Sigma F_x = 0$ gives $|\vec{f}| - w \sin\theta = 0$ (1)

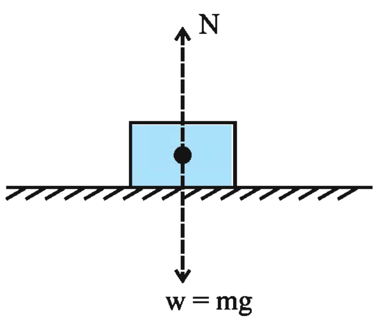
and $\Sigma F_y = 0$ gives $|\vec{N}| - w\cos\theta = 0$ (2)

From eqns. (1) and (2),

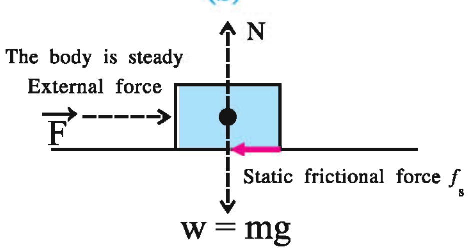
$$\frac{f}{N} = \tan\theta \qquad (3)$$

(a) Static friction : Consider a body of mass m lying steady on the horizontal surface of a table. It exerts a force equal to its weight W (= *mg*) on the surface, and the surface exerts the normal reaction N on the body. From the equillibrium condition of the body, we can say that

[gravitational force on the body in downward direction W (= *mg*)] = [The normal force N by the surface on the body in upward direction] (5.10.1)



(b)



(c)

Figure 5.6

Now suppose a small force $\vec{F}$ is applied on the body in +*x*– direction and yet the body does not come into motion but remains steady. See Fig. 5.6 (b). If this small force would be the only force on the body, then it would come into motion, even with a small acceleration (= F/*m*). But the body is still steady. Hence some other force must be acting on the body in –*x* direction which balances this force $\vec{F}$ and makes total external force zero and thus keeps the body steady.

This other force is the component of contact force parallel to surface and it is called the frictional force f_s. It is also called friction or static friction. This static friction does not exist by itself, but comes into play when the external force acts on the body.

Now on increasing the external force slightly and gradually, the body still does not move. Hence it is clear that the frictional force also must be increasing in the same proportion. Thus this static friction is a self–adjusting force. This happens only upto a certain limit. This static friction opposes the impending motion. The impending motion means, the motion that would have occured (but actually does not occur) under the effect of the external force, if friction were to be absent.

On still increasing the applied external force $\vec{F}$, the value of frictional force increases only up to a certain limit. If the applied external force becomes slightly more than this maximum frictional force, immediately the body starts moving. The frictional force acting on the body when it is on the verge of starting the motion, is called the maximum static friction $f_{s(max)}$ or the limiting frictional force. Experiments show that – (1) maximum static friction does not depend on the contact area of the surfaces. (2) The maximum static friction $f_{s(max)}$ is proportional to the normal reaction N. i.e. $f_{s(max)} \propto N$.

The above facts [stated in (1) and (2)] are called the laws of static friction.

Here it is clear that $f_{s(max)} = \mu_s N$ (5.10.2)

Where μ_s is the constant of proportionality and it is called the **coefficient of static friction.** Its value depends on the nature of surfaces, the materials of surfaces and the temperature. The value of μ_s is smaller for a smooth surface than that for a rough surface. The value of μ_s is in the range between 0.01 to 1.5. As long as the steady body is stationary, we can say that $f_s \le \mu_s N$.

Equation (5.10.2) is the relationship between only the values of $f_{s(max)}$ and N, while their directions are mutually perpendicular.

Illustration 9 : A block of mass 4 kg rests on a horizontal plane. The plane is gradually inclined until at an angle of 15° with the horizontal, the block just begins to slide. What is the co–efficient of static friction between the block and the surface ?

Solution : The forces acting on this block are :

(i) Its weight (gravitational force) acting downwards = mg

(ii) Normal reaction exerted by the surface = N and

(iii) Static frictional force = f_s (parallel to the inclined plane).

see Fig. 5.7

Since the block is in equilibrium, the resultant of all these forces would be zero. Taking two mutually perpendicular components of weight mg, as shown in the figure, we can write

$$mg \sin\theta = f_s \quad \text{and} \qquad ...(1)$$

$$mg \cos\theta = N \qquad ...(2)$$

Taking the ratio of these two equations

$$\tan\theta = \frac{f_s}{N} \qquad ...(3)$$

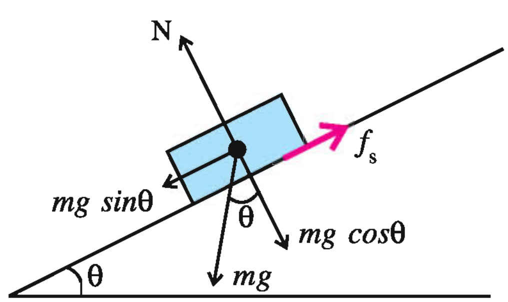


Figure 5.7

As the value of θ increases, the value of $\tan\theta$ also increases and to balance it, the magnitude of frictional force f_s will also increase upto certain value. Suppose for $\theta = \theta_{max}$ the magnitude of static friction becomes maximum, viz. $f_{s\,(max)}$. Hence from eqn. (3).

$$\tan\theta_{max} = \frac{f_{s(max)}}{N} = \mu_s$$

$$\therefore \theta_{max} = \tan^{-1} \mu_s$$

When the value of θ becomes little more than θ_{max}, the block starts sliding down because of (resultant) unbalanced forces acting on it. It is given that $\theta_{max} = 15°$

$$\therefore \mu_s = \tan 15° = 0.27$$

Note that θ_{max} depends only on μ_s, not on the mass of the block.

(b) Kinetic Friction :

In the illustration of body on the table in article (a) discussed above, if the applied external force $\vec{F}$, becomes even slightly more than $f_{s(max)}$, the body immediately comes into motion. Experiments show that as soon as this relative motion starts, the value of frictional force becomes less than that of maximum static friction $f_{s(max)}$. See Fig. 5.8.

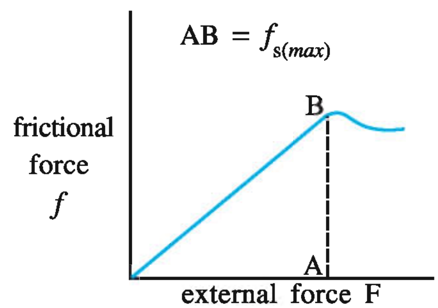


Figure 5.8

The frictional force opposing the relative motion of the contact surfaces is called the kinetic friction. See Fig. 5.9. Like the static frictional force this kinetic friction also does not depend on the contact area and it is proportional to the normal reaction (N). Moreover it is almost independent of the velocity.

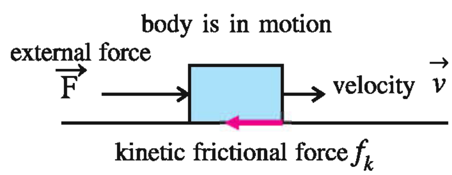


Figure 5.9

Here, it is clear that $f_k = \mu_k$ N (5.10.3)

where μ_k is the constant called coefficient of kinetic friction. Its value depends on the nature of contact surfaces. Experiments show that $\mu_k < \mu_s$.

We have to remember that to bring a body into motion from steady state, the external force has to overcome the maximum static friction but once the body has come into motion, the external force has to face the kinetic friction. The kinetic friction is always less than maximum static friction $f_{s(max)}$.

Illustration 10 : A block of mass 15 kg is lying on an inclined plane of angle 20°. In order to make it move upward along the slope with an acceleration of 25 cm/s^2, a horizontal force of 200 N is required to be applied on it. Calculate (i) frictional force on the block and (ii) co-efficient of kinetic friction.

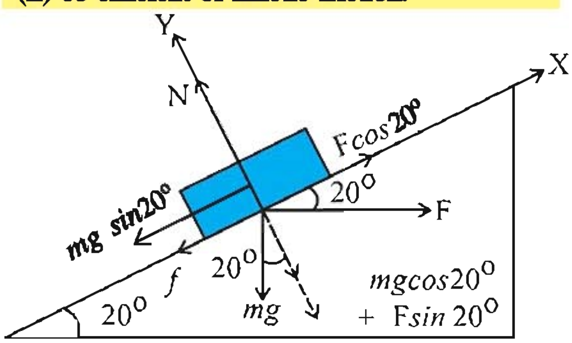


Figure 5.10

Solution : The situation described here is shown in Fig. 5.10. We take X–axis in the direction parallel to the surface of the slope and so Y–axis would be in the direction perpendicular to the slope. Remember that the block performs accelerated motion in X–direction and so the resultant force on it is not zero.

Hence there is no equilibrium in X–direction.

From $\Sigma F_x = ma_x$

$F\cos 20° - f - mg \sin 20° = (15)(0.25)$

$\therefore$ $(200)(0.9397) - f - (15)(9.8)(0.3420) = 3.75$

$\therefore$ $f = 134$ N

Since the block is in equilibrium in the Y–axis

$\Sigma F_y = 0$

$\therefore$ $N - mg \cos 20° - F \sin 20° = 0$

$\therefore$ $N-(15)(9.8)(0.9397) - (200)(0.3420) = 0$

$\therefore$ N = 207 N

Now,

$$\mu_k = \frac{f}{N} = \frac{134}{207} = 0.65$$

Note here that due to the force F the (effective) normal force increases.

(c) Rolling Friction : When a disc, ring or a sphere is rolling without sliding on a surface, the line (or the point) that touches the surface remains momentarily steady. On such a body static or kinetic friction does not act. Then, why does it stop after rolling over a certain distance ? Why do they not continue the motion with constant velocity ? In such motions of the body rolling friction is acting and hence to continue the motion some external force has to be applied. For a body of a given mass and a given surface, the rolling friction is much less (sometimes of thousandth part) than the static and the kinetic friction.

In such rolling phenomena, the contact surfaces are slightly deformed. Hence instead of a line or a point a **small area** comes in contact with the surface. Hence, a component of contact force parallel to the surface (which we call the rolling friction) comes into play, that opposes this relative motion. It depends on the radius, speed and the nature of material of body.

(d) Advantages and Disadvantages of Friction : Under certain conditions, friction is undesirable. Due to the friction opposing the relative motion between different parts in machines, power is dissipated in the form of heat. To decrease the kinetic friction in them, lubricants (e. g. grease, oil, soap, air.... etc.) are used. Another way is to use ball-bearings. Rolling friction will act in them which is very much less than static and kinetic friction. Hence power dissipation is decreased.

In certain conditions friction is necessary also. The kinetic friction dissipates power but it is needed to stop vehicles. It is used in brakes in machines and automobiles (What will happen if we drive a vehicle without brakes ?) It is only due to friction that we are able to walk. It is not possible for a car to move on a very slippery road. Normally the friction between the tyres and road is the necessary external force needed to accelerate the vehicle. Laws of friction are not simple and precise like those of gravitation and electric forces. Laws of friction are empirical and are only apporixamately true.

However, they are useful in solving the problems of mechanics.

The process of friction is very complex at the micro level.

5.11 Dynamics of Uniform Circular Motion

(a) Centripetal force : We have seen in Chapter 4 that the acceleration of a body of mass m, moving with a uniform (constant) speed v on a circular path of radius r, is $\frac{v^2}{r}$ directed towards the centre. It is called centripetal acceleration a_c. Hence according to Newton's second law of motion, a force $F_c = \frac{mv^2}{r}$ (5.11.1)

is required to be acting on it towards the centre for such a motion. This force is called the **centripetal force**. See Fig. 5.11

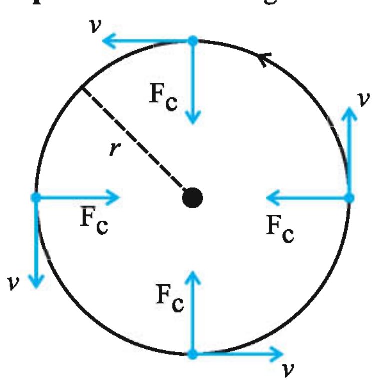


Figure 5.11

The required centripetal force for the circular motion of a planet around the sun is provided by the gravitational force on the planet by the sun.

In the case of an electron moving in a circular path around the nucleus, the required centripetal force is provided by the Coulomb attractive force on the electron by the nucleus.

Uniform circular motion is different from the uniform motion mentioned many times earlier in this chapter. In the uniform motion the velocity-vector ($\vec{v}$) of the body is constant and its acceleration is zero. But in the uniform circular motion the speed of the body on the circular path is constant, its velocity vector ($\vec{v}$) changes and the acceleration is $\frac{v^2}{r}$ towards the centre of the circle.

(b) Motion of a Vehicle on a Level Circular Path : In Fig. 5.12 (a, b) a vehicle is shown moving on a horizontal curved path (such path can be considered as a part of a circle). It can move safely only if sufficient centripetal force is acting on the vehicle on this path, (otherwise it will be thrown towards outside). Here the forces acting on the vehicle are :

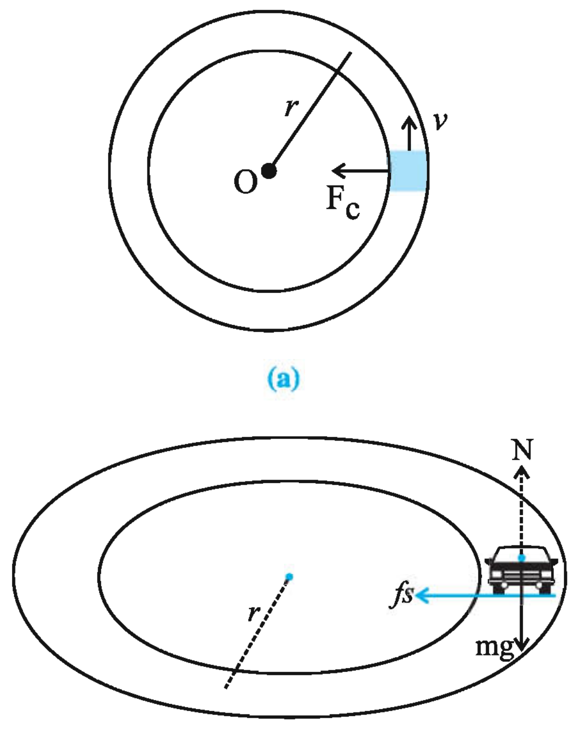


Figure 5.12

(1) Weight of the vehicle (mg) – in downward direction (2) the normal reaction (N) by the road – in upward direction (3) the frictional force (f_s) by the road – parallel to the surface of the road.

Since the vehicle has no acceleration in vertical direction,

$$N - mg = 0$$

$$\therefore N = mg \quad (5.11.1)$$

The required centripetal force F_c for the circular motion of the vehicle on this road must be provided by the frictional force f_s.

$$\therefore F_c = f_s = \frac{mv^2}{r} \quad (5.11.2)$$

This frictional force is the static frictional force, which opposes the impending motion of the vehicle away from the centre of the circle. If the maximum frictional force by the road is $f_{s(max)}$

$f_{s(max)}$ = μ_sN (from equation 5.10.1)

= $\mu_s mg$ (from equation 5.11.1) (5.11.3)

Where μ_s = the coefficient of static friction between the tyres of the vehicle and the road.

From this, we can say that if the speed v of the vehicle is such that

$$\begin{bmatrix}\text{required centripetal} \\ \text{force } \frac{mv^2}{r}\end{bmatrix} \le \begin{bmatrix}\text{maximum frictional} \\ \text{force } \mu_s\ mg\end{bmatrix} \quad (5.11.4)$$

then only the vehicle will move safely on this road.

$$\therefore\ v^2 \le \mu_s\, r\, g \quad (5.11.5)$$

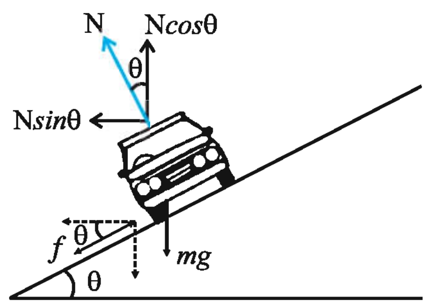
and the maximum speed for the safe motion

$$v_{max} = \sqrt{\mu_s\ r\ g} \quad (5.11.6)$$

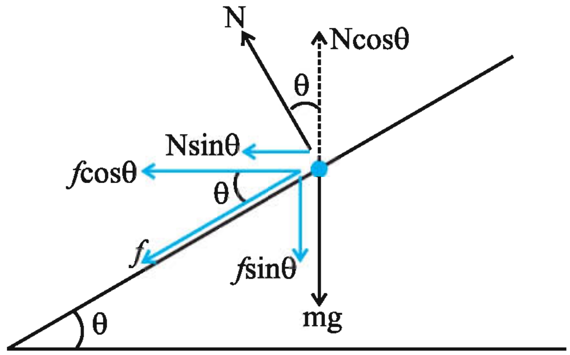
If the speed of the vehicle is more than this v_{max}, it will be thrown away from the road. This v_{max} has the same value for all vehicles – light or heavy. See that mass does not appear in equation (5.11.6)

From this discussion you would be able to understand as to why do we slow down the vehicle while turning on a road.

(c) Motion of a Vehicle on Banked Curved Road : We have seen that the required centripetal force for safe motion of a vehicle on a level circular path is provided by only the friction of the road. But at the curvature of the road (at the turning of the road) if the road is banked (such that the inner edge of the circular road is low and the outer edge is high), then, in the required centripetal force for circular motion; a certain contribution can be obtained from the normal force (N) by the road and the contribution of friction can be decreased, to that extent. In the Fig. (5.13), the section of the road with the plane of paper is shown. This road is inclined with the horizontal at an angle θ. The forces acting on the vehicle are also shown in the Fig. 5.13(b).



(a)



(b)

Figure 5.13

The forces acting on the vehicle are :

(1) weight (mg), in downward direction

(2) normal force (N), perpendicular to road, upward

(3) frictional force (f), parallel to the road.

As the acceleration of the vehicle in the verticle direction is zero,

$$N\cos\theta = mg + f\sin\theta \quad (5.11.7)$$

$$\therefore\ mg = N\cos\theta - f\sin\theta \quad (5.11.8)$$

In the horizontal direction, the vehicle performs a circular motion. Hence it requires centripetal force $\frac{mv^2}{r}$, which is provided by the horizontal components of N and f.

$$\therefore\ \frac{mv^2}{r} = N\sin\theta + f\cos\theta \quad (5.11.9)$$

Dividing equation (5.11.9) by equation (5.11.8), we get,

$$\frac{v^2}{rg} = \frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta} \quad (5.11.10)$$

We put $f = f_{s(max)} = \mu_s$ N in the above equation to get the maximum safe speed v_{max}, on this road.

$$\frac{v^2_{max}}{rg} = \frac{N\,sin\theta + \mu_s\ N\ cos\theta}{N\,cos\theta - \mu_s\ N\ sin\theta} \qquad (5.11.11)$$

$$\therefore\ v^2_{max} = rg\left[\frac{sin\theta + \mu_s\ cos\theta}{cos\theta - \mu_s\ sin\theta}\right] \qquad (5.11.12)$$

Dividing the numerator and the denominator by cosθ; we get

$$v^2_{max} = rg\left[\frac{tan\theta + \mu_s}{1 - \mu_s\ tan\theta}\right] \qquad (5.11.13)$$

$$\therefore v_{max} = \sqrt{rg\left[\frac{\mu_s + tan\theta}{1 - \mu_s\ tan\theta}\right]} \qquad (5.11.14)$$

Comparing equations (5.11.6) and (5.11.14), we find that the maximum safe speed on a banked curved road is **more** than that on horizontal curved road, since *tan*θ is positive here.

By knowing the radius of curvature of the road *r*, by determining the maximum safe speed (e.g. 100 km/h) of the vehicle on this road, and by knowing the coefficient of static friction μ_s between the tyre and the road, the equation (5.11.14) enables us to find the necessary angle of banking θ, of the road and accordingly such roads shoul be constructed. Moreover a board indicating such maximum safe (v_{max}) should be placed at a proper place.

Let us consider following two special cases in this discussion.

(i) In equation (5.11.14), for $\mu_s = 0$ (i. e. friction does not act at all),

$$v_0 = \sqrt{rg\ tan\theta} \qquad (5.11.15)$$

If we drive the vehicle at this speed on the banked curved road the contribution of friction becomes minimum in the required centripetal force, and hence wear and tear of the tyres can be minimised. This speed v_0 is called the optimum speed.

(ii) If $v < v_0$, then the frictional force will act towards the higher (upper) edge of the banked road [See that in the above figure *f* acts towards the lower edge of the road]. The vehicle can be kept stationary i.e. can be parked – on the banked road, only if $tan\theta \le \mu_s$

Illustration 11 : On a smooth, horizontal surface of a table, a body of mass *m* is connected, with a help of a light string passing through the hole on the surface, to a body of mass M suspended at the other end. (See Fig. 5.14)

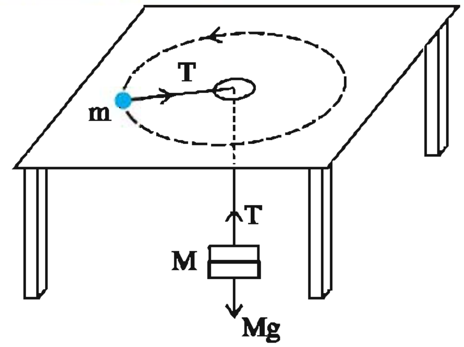


Figure 5.14

(a) In order that the body of mass M remains stationary obtain the condition for the circular motion of the body of mass *m* in terms of *v* and *r*.

(b) In the above case to maintain a uniform circular motion of a body of mass 10 kg, with a speed of 5 m/s, on the path of radius 2m, what should be the mass suspended at the other end ?

(Take $g = 10\ m/s^2$)

Solution :

(a) In this circular motion if the tension in the string is T, then

the required centripetal force $\frac{mv^2}{r} = T$ (1)

Where v = speed, r = radius of path

And for the body of mass M, to remain steady,

$Mg = T$ (2)

$\therefore\ \frac{mv^2}{r} = Mg$ (3)

$\therefore\ \frac{v^2}{r} = \frac{M}{m}g$ is the required condition.

(b) In equation (3) above

$$(10)\frac{(5)^2}{2} = M(10)$$

$\therefore$ M = 12.5 kg

Illustration 12 : A disc is rotating around its centre, in a horizontal plane at the rate of $\frac{100}{3}$ rotations / minute. A coin is placed at a distance of 5 cm and another similar coin at 25 cm from its centre. The coefficient of static friction between the disc and the coins is 0.2. Which coin will be thrown away from the disc ? Which coin will keep rotating with the disc ? (Take $g = 10\ m/s^2$, $\pi^2 = 10$)

Solution :

Suppose the mass of each coin = m.

For uniform circular motion of the coin, if

required centripetal force $\frac{mv^2}{r}$	$\le$	maximum static friction $f_{s(max)}$

then the coin can keep rotating with the disc.

Moreover $f_{s(max)} = \mu_s N = \mu_s mg$

($\because$N = normal force = mg)

Here time for $\frac{100}{3}$ rotations = 60 s

$\therefore$ time for 1 rotation (= T) = ?

$\therefore$ Periodic time $T = \frac{60 \times 3}{100} = 1.8$ s

we know $v = \frac{2\pi r}{T}$

$\therefore$ From above condition

$$\frac{m}{r}\left(\frac{4\pi^2 r^2}{T^2}\right) \le \mu_s mg$$

$$\therefore\ r \le \frac{\mu_s\ gT^2}{4\pi^2}$$

$$\therefore\ r \le \frac{(0.2)(10)(1.8)^2}{(4)(10)}$$

$\therefore\ r \le 0.162$ m

$\therefore\ r \le 16.2$ cm

The coin near the centre will rotate with the disc and the one which is away will be thrown away.

Illustration 13 : A cyclist speeding at 18 km/h on a level road takes a sharp circular turn (without bending) of radius 3 m without reducing the speed. The co–efficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn ?

Solution :

Here $v = \frac{18000}{3600} = 5$ m/s,

$r = 3$ m and $\mu_s = 0.1$

The formula for maximum safe speed, on a horizontal curved road is $v_{max} = \sqrt{\mu_s\ rg}$

$$\therefore\ v_{max} = \sqrt{(0.1)(3)(9.8)}$$

$$= 1.714\ m\ s^{-1}$$

Since the speed of the cyclist (5 m s^{-1}) is more than this value he will slip while negotiating the turn.

Illustration 14 : In Illustration 13 what should the cyclist do in order to prevent slipping while passing over the same road ?

Solution : Here, friction does not provide the necessary centripetal force. If the cycle is inclined at an angle θ with the vertical, the component of contact force towards the centre of the circular path would provide the necessary centripetal force. This is shown in Fig. 5.15.

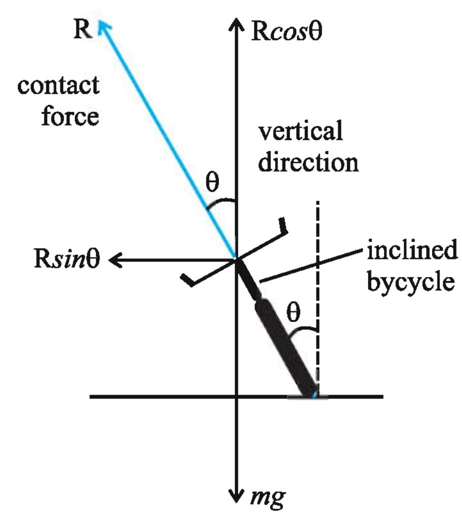


Figure 5.15

Here, R is the contact force between the cycle and the road. Out of two mutually perpendicular components R $cos\theta$ and R $sin\theta$, R $sin\theta$ provides the necessary centripetal force.

$$\therefore\ R sin\theta = \frac{mv^2}{r} \quad (1)$$

From the figure, $R cos\theta = mg$ (2)

Dividing eqn, (1) by eqn. (2),

$$tan\theta = \frac{v^2}{rg} \Rightarrow \theta = tan^{-1}\left(\frac{v^2}{rg}\right) = 40^o23'$$

(What do you think can be the reason for this much inclination ?)

Note : This problem can also be solved by taking moments of the force.

Illustration 15 : A circular racetrack of radius 300 m is banked at an angle of 15^o . If the co–efficient of friction between the wheels of a race–car and the road is 0.2, calculate (i) the optimum speed of the race–car to avoid wear and tear on its tyres, and (ii) the maximum permissible speed to avoid slipping ?

Solution :

(i) Normally, on a banked road the sum of the horizontal components of normal force and frictional force provides the necessary centripetal force to keep a car moving on a circular turn without slipping. At the optimum speed, the horizontal component of normal reaction alone is enough to provide the required centripetal force and the frictional force is not needed.

From the formula $v_0 = \sqrt{rg\ tan\theta}$ for optimum speed,

$$v_0 = \sqrt{(300)(9.8)(tan\,15^o)}$$

= 28.1 m/s.

(ii) From the formula

$$v_{max} = \sqrt{rg\left[\frac{\mu_s + tan\theta}{1 - \mu_s\ tan\theta}\right]}$$

$$v_{max} = \sqrt{(300)(9.8)\left[\frac{0.2 + tan\,15^o}{1 - 0.2 tan\,15^o}\right]}$$

= 38.1 m/s

5.12 Inertial and Non–inertial Frames of Reference

We had seen in Chapter 3 that the place and the situation from where an observer takes his observation is called the frame of reference. Such a frame of reference may be steady, may be moving with a constant velocity or may be moving with an acceleration.

Suppose you are sitting in a bus at rest. When the bus starts with a jerk, you are pushed backward. Now when the same bus is moving with a constant velocity, at that time no such push is experienced by you. When the bus was steady then also you had not experienced the push. If the driver applies brakes suddenly, you are pushed forward. Thus during the accelerated or retarded (decelerated) motion of the bus, you experience such a push, although no apparent force is applied on you. Here, it seems that Newton's first law is not holding good, because according to this law, as long as an external force is not acting on the body, its state of motion should not change.

From this discussion it is clear that Newton's first law of motion holds good in a frame of reference which is either stationary or moving with a constant velocity, but it is not obeyed in an accelerated frame of reference. The frame of reference, in which Newton's first law is obeyed is called the **inertial** frame of reference; and the one in which it is not obeyed is called the **non-inertial** frame of reference. A rotating reference frame is an illustration of non–inertial frame of reference. In the obove illustration, when the bus remains at rest or is moving with a constant velocity, it is an inertial frame of reference; but at the time of accelerated (or retarded) motion, the same bus is a non-inertial reference frame.

In the non-inertial frame of reference, we need to consider one more additional force acting on a body in discussing its motion. This force is called **pseudo or fictitious force F_p**. A force is actually exerted during the interaction between two bodies; but for this pseudo force mentioned here, no other body seems to interact with the given body. This force appears to act only due to the accelerated motion of the frame of reference. Hence it is called fictitious or pseudo force F_p. The direction of this force F_p is opposite to that of acceleration of the reference frame.

A body of mass m, in an accelerated reference frame is given an additional acceleration equal but opposite to the acceleration of the reference frame.

This acceleration is called the pseudo or fictitious acceleration a_p. The motion of body is analyed by giving this force $\vec{F}_p = m\,\vec{a}_p$ in the direction opposite to the acceleration of reference frame, and also considering other forces which are actually acting on the body. We imagine such an additional fictitious force in order to solve such problems in the accelerated reference frame, using Newton's second law of motion. In the inertial frame of reference such a fictitious force F_p is not to be considered.

A merry - go - round is also an accelerated (hence non - inertial) reference frame. For a man sitting in it and rotating with it, a centripetal force is required for circular motion which is provided by the friction between the man and the seat (and / or normal force by the support at the back). This is the actual force.

But the man feels that a force is acting on him towards away from the centre (he feels as if he is being thrown away). This is the fictitious force Fp.

Earth is also a non–inertial frame of reference. But due to its very small acceleration due to its rotational motion whatever error occurs in the measurements is extremely small. Hence, for practical purposes Earth is taken as an inertial reference frame.

The outcome of the discussion above is : If the downward acceleration of a non–inertial (means accelerated) reference frame is ($\vec{a}$), then-for an observer inside it to understand the motion of a body of mass m-all actual forces acting on the body are also to be considered and an additional force $\vec{F}_p = m(\vec{a})$ in the upward direction is also to be considered and then Newton's second law of motion should be used.

Illustration 16 : Ramesh with mass of 60 kg stands on a spring–balance in a lift. (a) If the lift moves with an acceleration of 2 m/s^2 (i) upward and (ii) downward what will be his weight as recorded ? (b) If the cable of the lift breakes what will be his recorded weight ? (g = 10 m/s^2)

Solution : The gravitational force of the Earth on a body is called its weight w and $w = mg$.

The weight recorded by the spring balance is the normal reaction by its surface on the body. When the lift is steady or moving with a constant velocity, it is an inertial frame of reference. At that time, his weight recorded will be $w_1 = mg = (60)\ (10) = 600$ N.

In the accelerated motion of the lift, the man and forces on him are shown below in the Fig. 5.16 (a, b, c).

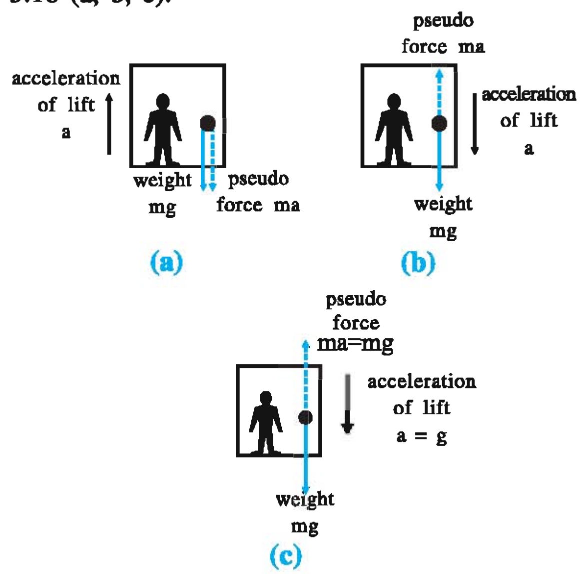


Figure 5.16

(a) (i) Upward acceleration of lift = a

∴ Ramesh inside it is in an accelerated reference frame. Hence, fictitious acceleration ***a*** should be given to Ramesh in the downward direction. Moreover his weight mg also acts in the downward direction. ∴ The resultant force on him in the downward direction is $W_1 = mg + ma = m\ (g + a)$. He will exert this force W_1 on the balance Fig. 5.16 (a) and surface of the balance will apply an equal normal force upwards (normal reaction) on him.

$$\therefore \text{ Weight recorded } = W_1$$
$$= m(g + a)$$
$$= 60(10 + 2) = 720 \text{ N.}$$

(ii) downward acceleration of lift = a.

∴ Ramesh inside it is in an accelerated reference frame. Hence fictitious acceleration **a** should be given to him in the upward direction. Moreover weight mg is acts in the downward direction. Therefore the resultant force on him in downward direction is $W_2 = mg - ma = m\ (g - a)$. He will exert this much force W_2 on the balance Fig. 5.16 (b). The surface of balance will apply an equal normal force upward.

$\therefore$ Weight recorded $= W_2 = m\ (g - a)$

$= 60\ (10 - 2)$

$= 480$ N.

(b) If the cable of the lift breaks, the lift will fall freely with an acceleration, $a = g$.

$\therefore$ recorded weight $W_3 = m\ (g - g) = 0$ N.

This is called the state of weightlessness.

5.13 Guidance for Solving Problems in Dynamics

(Only for information)

(A) In physics many different words like – 'friction', 'normal reaction', 'normal force', 'reaction', 'action', 'air resistance', 'thrust', 'buoyant force', 'pull', 'centripetal force', 'weight', 'tension' etc. – are used in the discussion of many different phenomena and processes. All these terms mean 'force' – in reference to those phenomena and processes.

(B) Tension in a string : When a body (of mass m) is suspended at the end of a string hung from a rigid support, the string becomes tight. In this condition, every region of the string is said to be in tension. The protons – electrons near the lower end of the string exert electromagnetic force on the protons–electrons of the body near the contact point and vise–versa. This force is called the contact force. And due to this contact force; the body does not fall but remains suspended.

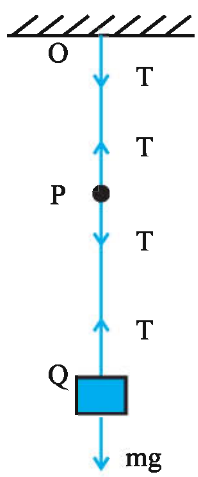


Figure 5.17

Similarly contact forces also act between one region and the other opposite rigion at every point of the string. They are equal and in opposite directions according to Newton's third law of motion. This common magnitude (i.e. equal value) of the contact forces between two regions is called tension produced in the string at each point.

If the string is light (i.e. massless), the tension T is the same at every point in the string, (We shall accept this fact without giving any proof, at present).

In the Figure 5.17 at point P, tension towards PO is T, towards PQ is T. At point Q tension towards QP is T and at O tension towards OP is T.

Moreover, since the body is stationary it is clear that $mg = T$.

(C) Free Body Diagram (FBD) : We can solve different problems of dynamics using Newton's three laws of motion, which we have learned. Sometimes in a given problem more than one body is involved. Such bodies exert force on each other. Moreover every body (object) also experiences gravitational force. In solving such problems, out of the assembly of bodies or systems; we have to take that part as a 'system', of which the motion is to be discussed. And the remaining parts of the assembly and other agencies which affect our system are taken as 'environment'.

To solve the problem, we should proceed acccording to the following steps :

(1) Draw a schematic diagram showing the assembly of different object, other objects connected with them, and those which support these objects.

(2) Select the object (or objects) as 'system', of which we want to discuss the motion.

If the system under consideration is made up of more than one object, then take care that the acceleration vector (magnitude + direction) of all these objects should be the same.

(3) Make a list of the forces acting on the system by the remaining parts of the assembly and by other agencies. In this list, the forces acting inside the system are not to be included.

For example, a coolie standing with a heavy load on his head is shown in Fig. 5.18 (a) (b) (c). The forces acting in this case are as follows : (i) weight of the load mg (acting in downward direction on coolie and on load),

(ii) normal force of reaction (N_1) on the load by the coolie (in the upward direction) (Note clearly that this force is not on the coolie but is on the load) (iii) weight of the coolie, Mg (this force acts in the downward direction on both, the coolie and the ground) (iv) normal force of reaction (N_2) on coolie by the ground (in the upward direction) (v) the force on the ground, (m + M) g. Which of these forces should be considered depends on the choice of the system.

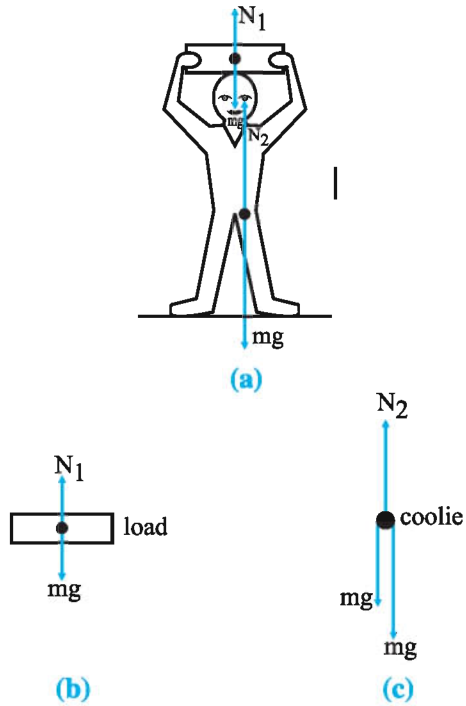


Figure 5.18

If we are interested in the state of motion of the load, we should consider the forces acting on the load only, viz. (i) and (ii).

If we are interested in the state of motion of the coolie only, we should consider coolie as our system and consider the forces acting on the coolie only, viz. (i), (iii) and (iv).

If we are interested in the state of motion of (coolie + load), we should consider the forces acting on both the coolie and the load viz. (a) (m + M) g and (b) N_2. Here, the force by the coolie on the load or the force by the load on the coolie are not considered as they are internal forces between the parts of the system.

(4) Showing a system as a point, all the forces acting on it are depicted as vectors from that point. This figure is called the free body diagram (FBD). This does not mean that the system under consideration is free from forces-actually, only the forces on the system are shown in the figure.

In this figure the forces exerted by the system on the environment are not to be shown.

(5) Now choose X–axis in the direction of actual or likely motion of the system. The direction normal to it becomes the Y–axis.

Now find the resultant of the X–components of the forces on the system. Write an equation showing that this value (resultant) is equal to the product of the mass of the system and its acceleration in X–direction. Similarly the Y–components will give another equation. Such equations are called the equations of motion. By solving such equations we can determine the unknown quantity (or quantities) in them.

(6) If the number of unknown quantities is more than number of equations obtained, then we take another part of assembly as another system, obtain equations from its FBD and hence find the solutions.

Illustration 17 : As shown in Fig. 5.19 two blocks 1 and 2, of the same mass, are in contact with block 3. The co-efficient of friction between the surfaces of 3 and 1 and that between 3 and 2 is μ. The blocks 1 and 2 are tied by a light string and the string is passed over a frictionless pulley. With what minimum acceleration should the block 3 move, in horizontal direction, so that there is no motion of 1 and 2 w.r.t. 3 ?

[This illustration is only for information]

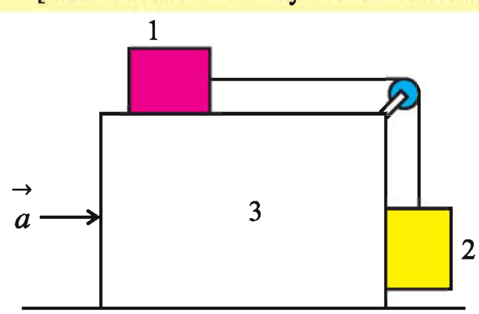


Figure 5.19

Solution : Suppose the minimum acceleration (required) of block 3, in horizontal direction, is a. The forces acting on block 1 are :

(i) The gravitational force of the Earth = mg downward,

(ii) The normal force exerted by the surface of block 3 = N_1 (upward)

(iii) The force of the tension exerted by the string = T (towards right)

(iv) The force of friction = μN_1 (towards left)

(v) Pseudo force = *ma* (towards left)

The FBD for block 1 (considering it as the system) is shown in Fig. 5.20. Since there is no acceleration in the vertical direction $N_1 = mg$ and in the horizontal direction $ma + \mu N_1 = T$

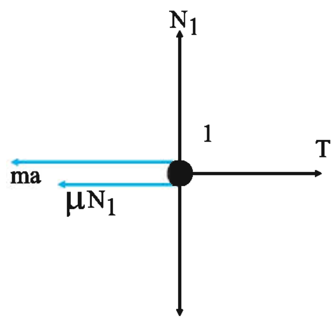


Figure 5.20

$\therefore$ $ma + \mu mg = T$ (1)

The forces on block 2 are :

(i) The gravitational force of the Earth = mg (downward),

(ii) The normal force exerted by the surface of block 3 = N_2 (towards right),

(iii) The force of tension exerted by the string = T (upward),

(iv) The force of friction = μN_2(upward)

(v) Pseudo force = *ma* (towards left)

The FBD for block 2 (considering it as the system) is shown in Fig. 5.21. Since there is no acceleration in the horizontal direction $N_2 = ma$

and in the vertical direction $\mu N_2 + T = mg$

$\therefore$ $\mu ma + T = mg$

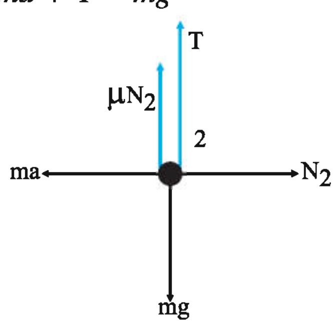


Figure 5.21

Substituting the value of T from eqn. (1),

$\mu ma + ma + \mu mg = mg$

$\therefore$ $a(\mu + 1) = g(1 - \mu)$

$$\therefore a = g\left(\frac{1 - \mu}{1 + \mu}\right)$$

Illustration 18 : In an accelerating goods train, 25 wagons of equal mass are attached to the engine. Show by calculation whether the tension in the coupling between the fourth and the fifth wagon is the same as that between twenty first and twenty second wagon.

Solution : Suppose the pull of the engine on the first wagon = P.

friction on every wagon = *f*

mass of every wagon = *m*

acceleration of entire goods train = *a*.

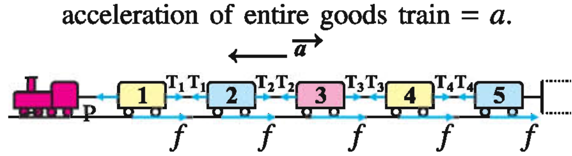


Figure 5.22

Considering the FBD of the first four wagon

$P - 4f - T_4$ = resultant force = $(4m)a$

$\therefore$ $T_4 = P - 4(f + ma)$ (1)

This T_4 is the tension between the fourth and the fifth wagon.

Similarly considering FBD of the first 21 wagons,

$T_{21} = P - 21(f + ma)$ (2)

This T_{21} is the tension between 21st and 22nd wagons.

From equations (1) and (2) it is clear that $T_4 \neq T_{21}$ and $T_4 > T_{21}$.

Illustration 19 : A block (A) of 20 kg is put on a frictionless surface and another object (B) of mass 2 kg is placed over it. The co-effecient of friction between the surfaces of A and B is 0.25. A horizontal force of 2 N is applied on B. Calculate (i) the acceleration of block A and that of object B, (ii) frictional force betwen A and B and (iii) Calculate all these quantities again if the force on B is of 20 N. Take $g = 10$ m s^{-2}.

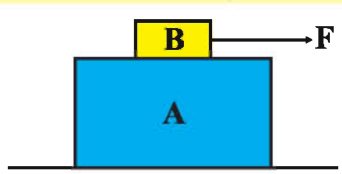


Figure 5.23

Solution : It is clear that as long as the applied force is less than the maximum static force of friction, there would not be any relative motion between A and B this means that both the object would move, under the effect of applied force, as if they are the one object.

In the present case, maximum static frictional force = μmg

$= (0.25)\ (2)\ (10) = 5$ N

(i) When a force of 2 N is applied on B there would not be relative motion between A and B and so the acceleration of both would be the same, say a. mass × acceleration = force

$$\therefore (2 + 20)a = 2 \ \therefore\ a = \frac{1}{11} = 0.09 \text{ m s}^{-2}$$

(ii) The frictional force between A and B $f = F - ma = 2 - (2)\ (0.09) = 1.82$

(iii) When a force of 20 N is applied on B, since it is more than maximum static frictional force (5 N), now there will be relative motion between A and B. Hence the magnitudes of their accelerations would be different. In this position the FBD of both A and B will be as shown in Fig. 5.24.

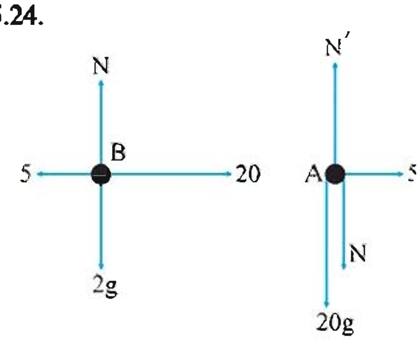


Figure 5.24

From this $20 - 5 = 2\,a_B \ \therefore\ a_B = 7.5$ m s^{-2}

and $5 = 20\,a_A \ \therefore\ a_A = 0.25$ m s^{-2}.

Illustration 20 : Three blocks of masses $m_1 = 1$ kg and $m_2 = 2$ kg and $m_3 = 3$kg are connected by massless strings and placed on a horizontal frictionless surface as shown in Fig. 5.25. A force F = 12 N is applied to mass m_1 as shown. Calculate (i) the acceleration of the system, (ii) tension (T_2) in the string between m_1 and m_2 and (iii) tension (T_3) between m_2 and m_3.

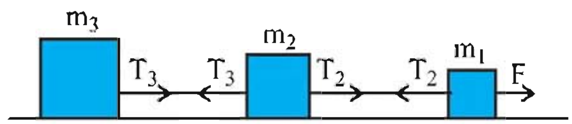


Fig. 5.25

Solution : (i) The acceleration of system

$$a = \frac{\text{Total force}}{\text{Total mass}} = \frac{12}{1+2+3} = 2 \text{ m s}^{-2}$$

(ii) $T_2 = (m_2 + m_3)a = (2 + 3)\ (2)$

$= 10$ N

(iii) $T_3 = m_3 a = 3 \times 2 = 6$ N

Solve this numerically by assuming that the same force (12 N) acts on m_3 towards left and write your conclusion.

SUMMARY

1. We shall consider the causes of motion and changes in motion.
2. Aristotels concept – that force is required to continue the motion of the body is not true. In practice whatever external force that is required to continue the motion with a constant velocity is only to counter the friction (which is also an external force).
3. The law of inertia given by Galileo was represented by Newton as the first law of motion : "If no external force acts on a body, the body at rest remains at rest and a body in motion continues to move with the same velocity." This law gives us the definition of force.
4. The momentum of a body $\vec{p} = m\vec{v}$ is a vector quantity. It gives more information than the velocity.
5. Newton's second law of motion : The time–rate of change in momentum of a body is equal to the resultant external force applied on the body and is in the direction of the external force.

$\vec{F} = d\vec{p} / dt = m\vec{a}$ is the vector relationship.

The SI unit of force is newton (= N). 1 N = 1 kg m s^{-2}. This law gives the value of force. It is consistent with the first law. ($\vec{F} = 0$ indicates that $\vec{a} = 0$) In this equation the acceleration of the body $\vec{a}$ is that which it has when the force is acting on it. (Not of the past !). $\vec{F}$ is only the resultant external force.

6. The impulse of force is the product of force and the time for which it acts.

   When a large force acts for a very small time, it is difficult to measure $\vec{F}$ and Δt but the change in momentum can be measured, which is equal to the impulse of force ($\vec{F}\ \Delta t$)

7. Newton's third law of motion : "To every action there is always an equal and opposite reaction."

   Forces always act in pairs, and $\vec{F}_{AB} = -\vec{F}_{BA}$. The action and the reaction act simultaneously. They act on different bodies, hence they cannot be cancelled by adding. But the resultant of the forces between different parts of the same body becomes zero (You will get the explanation on how this happens, when you study the chapter on "Dynamics of system of particles." in future in the next semester.)

8. The law of conservation of momentum is obtained from Newton's second law and the third law. It is written as – "The total momentum of an isolated system remains constant."

9. The concurrent forces are those forces of which the lines of action pass through the same point. For equilibrium of the body, under the effect of such forces, $\Sigma\vec{F}$ must be = 0. Moreover the sum of the corresponding components also should be zero. ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$)

10. Friction is produced due to the contact force between the surfaces in contact. It opposes the impending or the real relative motion.

    Static frictional force $f_s \le f_{s(max)} = \mu_s$ N and

    the kinetic friction is $f_k = \mu_k$ N

    μ_s = coefficient of static friction

    μ_k = coefficient of kinetic friction and $\mu_k < \mu_s$.

11. On a body performing uniform circular motion a force equal to mv^2 / r acts towards the centre of the circular path. This is called the centripetal force.

    The maximum safe speed on level curved road is $v_{max} = \sqrt{\mu_s\ rg}$

    The maximum safe speed on a banked curved road is

    $$v_{max} = \sqrt{rg\left(\frac{\mu_s + tan\theta}{1 - \mu_s tan\theta}\right)}$$

12. The reference frame, in which Newton's first law of motion is obeyed is called the inertial frame of reference and the one in which it is not obeyed is called non–inertial frame of reference. The frame of reference with constant velocity is an inertial frame of reference and one which has acceleration is non–inertial frame of reference.

An additional (fictitious) acceleration equal but opposite to that of the non–inertial reference frame is considered on the body to solve problems on motion for accelerated frame of reference.

EXERCISES

Chose the correct option from the given options :

1. When a force acts on a body of mass 100 g, the change in its velocity is 20 cm s^{-1} per second. The magnitude of this force is .... N.

(A) 0.2 (B) 0.02 (C) 0.002 (D) 2.0

2. A bullet of mass m, moving horizontally with velocity v hits and gets embedded in a wooden block of mass M resting on a horizontal frictionless surface. What will be the velocity of this composite system ?

(A) $\frac{mv}{M - m}$ (B) $\frac{Mv}{M - m}$

(C) $\frac{Mv}{M + m}$ (D) $\frac{mv}{M + m}$

3. The accelerated motion of the vehicle on a horizontal road is due to what ?
(A) Engine of the vehicle
(B) Driver
(C) Earth's gravitational force
(D) Friction between the road and the vehicle.

4. An object of mass 8 kg is suspended through two light spring balances as shown in the figure Then,

(A) both the balances will read 8 kg.

(B) both the balances will read 4 kg.

(C) The upper balance will read 8 kg and the lower balance will read zero.

(D) The balances will read any value but their sum will be 8 kg.

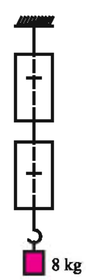


Figure 5.26

5. A block of mass m is placed on a smooth inclined plane of angle θ. The normal force exerted by the surface of the plane would be ....

(A) mg (B) $\frac{mg}{\cos\theta}$ (C) $mg \cos\theta$ (D) $mg \sin\theta$

6. A block of mass m is placed on a smooth slope of angle θ. The whole system (slope + block) is moved horizontally with acceleration a in such a way that the block does not slip on the slope. Hence, a = ...

(A) $g \tan\theta$ (B) $g \sin\theta$ (C) $g \cos\theta$ (D) $g / \sin\theta$

7. A block is placed on the top of a smooth inclined plane of inclination θ, kept on the floor of a lift. When the lift descends with a retardation a, the relative acceleration of the block, parallel to the surface of the slope, is ....

(A) $g \sin\theta$ (B) $a\sin\theta$

(C) $(g - a)\sin\theta$ (D) $(g + a)\sin\theta$

8. Which of the following statement is correct ?

(A) A body has a constant velocity but a varying speed.

(B) A body has a constant speed but a varying value of acceleration.

(C) A body has a constant speed and non-zero constant acceleration.

(D) A body has a constant speed but varying velocity.

9. A reference frame attached with a geostationary satellite moving around the earth can be regarded as ....

(A) non-inertial (B) inertial

(C) any one of them (D) none of above

10. A person standing on the floor of a lift drops a coin. The coin reaches the floor of the lift in time t_1 if the lift is stationary and in time t_2 if it is accelerated in upward direction. Then

(A) $t_1 = t_2$ (B) $t_1 < t_2$

(C) $t_1 > t_2$ (D) cannot say anything.

11. A person standing on the floor of a lift drops a coin. The coin reaches the floor of the lift in time t_1 if the lift is stationary and in time t_2 if it is moving with uniform velocity in upward direction. Then

(A) $t_1 = t_2$ (B) $t_1 < t_2$

(C) $t_1 > t_2$ (D) cannot say anything.

12. N bullets, each of mass m, are fired normally towards a wall at the constant rate of n bullets per second with velocity v. They stop on the wall. Hence, the reaction on bullets by the wall is ....

(A) nmv (B) $\frac{Nmv}{n}$ (C) $\frac{nNm}{v}$ (D) $\frac{nNv}{m}$

13. A force acts on an object of mass 1.5 kg at rest, for 0.5 s. After the force stops acting, the object travels a distance of 5 m in 2 s. Hence, the magnitude of the force will be ....

(A) 5 N (B) 7.5 N (C) 10 N (D) 12.5 N

14. A force of 4 N acts on an object of mass 2 kg in X–direction and another force of 3 N acts on it in Y–direction. Hence, the magnitude of the acceleration of the object will be ....

(A) 1.5 m s^{-2} (B) 2.0 m s^{-2} (C) 2.5 m s^{-2} (D) 3.5 m s^{-2}

15. A rope which can withstand a maximum tension of 400 N hangs from a tree. If a monkey of mass 30 kg climbs on the rope in which of the following cases will the rope break ? (Take g = 10 m s^{-2} and neglect the mass of the rope.)

(A) When the monkey climbs with constant speed of 5 m s^{-1}

(B) When the monkey climbs with constant acceleration of 2 m s^{-2}

(C) When the monkey climbs with constant acceleration of 5 m s^{-2}

(D) When the monkey climbs with constant speed of 12 m s^{-1}

16. A ball with momentum 0.5 *kg* m s^{-1} coming towards a batsman is hit by him such that it goes on the same path in opposite direction with momentum 0.3 *kg* m s^{-1}. If the time of contact of the ball with the bat is 0.02 s, find the force on the ball by the bat.

(A) 10 N (B) 40 N (C) 75 N (D) 30 N

17. A block of mass 1000 *kg* lying steady on the horizontal surface of a table needs 200 N horizontal force to come into motion. What is the coefficient of static friction between the block and the surface of table ?

[take g = 10 m s^{-2}]

(A) 0.2 (B) 0.02 (C) 0.5 (D) 0.05.

18. As shown in the figure, a horizontal force of 70 N is applied on a system of blocks of masses 4 *kg*, 2 *kg* and 1 kg placed on a frictionless horizontal surface. If the tension in one string is T_1 = 60 N, find the tension T_2 in the second string.

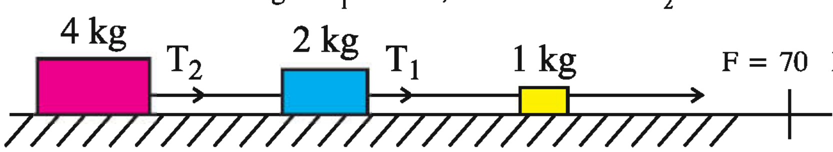


Figure 5.27

(A) 40 N (B) 60 N (C) 20 N (D) 10 N

19. A body of mass 30 *kg* at one end and another of 50 *kg* at the other end of a string passing over a frictionless pully are suspended as shown in the figure. What is the acceleration of this system ?

[take g = 10 m s^{-2}]

(A) 8 m s^{-2} (B) 6 m s^{-2}

(C) 2.5 m s^{-2} (D) 2 m s^{-2}

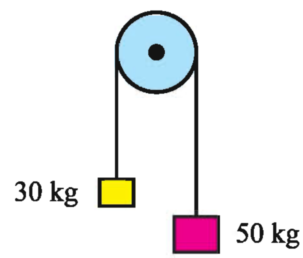


Figure 5.28

20. As shown in the figure blocks of masses 2 kg, 5 kg and 3 kg are arranged with light strings and frictionless pully fitted on frictionless horizontal surface. What is the acceleration of this system ?

[take g = 10 m s^{-2}]

(A) 1 m s^{-2} (B) 2 m s^{-2}

(C) 5 m s^{-2} (D) 8 m s^{-2}

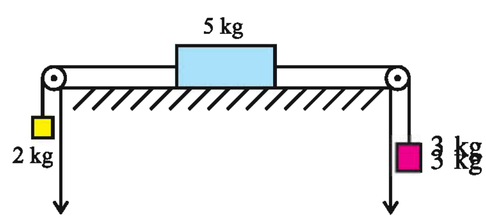


Figure 5.29

21. What is the value of the force $\vec{F}$ to be applied horizontally on a block of mass 5 kg which is in contact with a wall, as shown in the figure. (take g = 10 m s^{-2}) such that it does not fall down. The coefficient of friction between the block and the wall is 0.4.

(A) 200 N (B) 20 N

(C) 12.5 N (D) 125 N

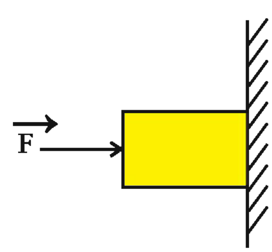


Figure 5.30

22. A stationary bomb explodes into three pieces. If the momenta of two pieces are $2\hat{i}$ unit and $3\hat{j}$ unit respectively, then what is the value of the momentum of the third piece ?

(A) $\sqrt{13}$ unit (B) 5 unit (C) 6 unit (D) 13 unit

23. As shown in the figure, one block of 2.0 kg at one end and the other of 3.0 kg at the other end of a light string are connected. If this system remains stationary, find the magnitude and direction of the frictional force. (take g = 10m s^{-2})

(A) 20 N, downward on slope

(B) 20 N, upward on slope

(C) 10 N, downward on slope

(D) 10 N, upward on slope.

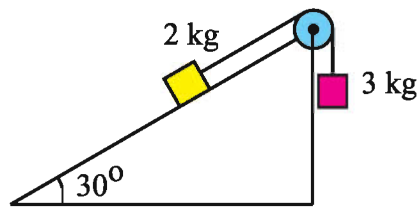


Figure 5.31

24. A man sitting in a train moving with uniform (constant) velocity tosses a coin upward from his hand, which comes back in his hand after sometime. What will be the nature of motion of the coin observed by a steady observer on the ground outside the train ?

(A) parabola (B) horizontal

(C) Straight line upward and then straight line downward. (D) circular

25. Consider a pendulum suspended from the ceiling of a room and oscillating in the vertical plane suppose that the string breaks when (i) the bob is at the end position of its path, (ii) bob is in the mean position of its path. What will be the nature of the path of bob, till it touches the ground.

(A) (i) curved towards downward; (ii) straight line downward

(B) (i) straight line downward; (ii) parabola

(C) (i) straight line upward; (ii) parabola

(D) (i) straight line upward; (ii) curved towards downward

26. Seven blocks each of mass 10 kg are arranged one above the other as shown in Fig. 5.32. What are the values of the contact forces exerted on the third block; by the fourth block and the second block respectively ?

(Take g = 10 m s^{-2})

Figure 5.32

(A) 40 N, 50 N (B) 50 N, 40 N

(C) 40 N, 20 N (D) 50 N, 30 N

ANSWERS

1. (B)	2. (D)	3. (D)	4. (A)	5. (C)	6. (A)
7. (D)	8. (D)	9. (A)	10. (C)	11. (A)	12. (A)
13. (B)	14. (C)	15. (C)	16. (B)	17. (B)	18. (A)
19. (C)	20. (A)	21. (D)	22. (A)	23. (A)	24. (A)
25. (B)	26. (A)				

Answer the following questions in short :

1. A ball of mass 0.2 *kg* is thrown in the vertical direction with a velocity of 2 m s^{-1}. At the top of its path (i) what is the value of its velocity ? (ii) What is the value of its acceleration ? (iii) what is the value of the force acting on it ? [Take g = 10 m/s^2] [**Ans. :** 0, 10 m/s^2, 2N]
2. Define inertia
3. What is meant by non–inertial frame of reference ?
4. What is similar from the dynamics point of view between a book lying stationary on the horizontal table and a raindrop falling downward with constant speed ?
5. F → *t* graph for a body is shown in the figure. What is the change in the value of momentum in the initial time interval of 0.03 s ?

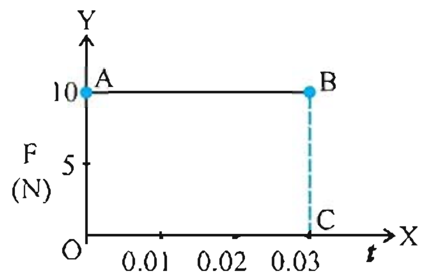


Figure 5.33

[**Ans. :** 0.3 kg m s^{-1}]

6. What is impending motion ?
7. Give the dimensional formula of impulse of force.
8.

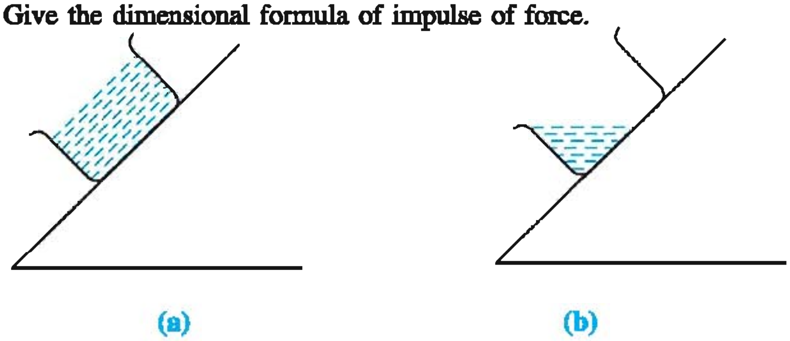


Figure 5.34

Can you tell which beaker is steady and which is coming down with acceleration, in the above figure ? [Hint : The level of a steady liquid remains horizontal. On the liquid in the accelerating beaker fictitious force acts opposite to acceleration of beaker.]

9. In uniform circular motion (i) only the value of velocity is constant (ii) velocity vector is constant (iii) direction of velocity is constant Select the correct. [**Ans. :** (i)]

[Similar questions for acceleration, momentum and force can be formed.]

10. Which out of (i) value of velocity, (ii) value of acceleration, (iii)value of force, (iv) the momentum vector of the body is not constant during uniform circular motion ? [**Ans. :** (iv)]

Answer the following questions :

1. Define momentum. Write Newton's second law of motion and hence derive the equation $\vec{F} = m\vec{a}$
2. Write the law of conservation of momentum and explain with an illustration.
3. Give Newton's first and second laws of motion. State which information they provide about force.
4. Explain about the static friction and give its laws.
5. Obtain the formula for the maximum safe speed (v_{max}) of a vehicle on a level curved road.
6. For a vehicle moving on a banked curved road, using free body diagram (FBD), obtain the formula for the maximum safe speed (v_{max})
7. State advantages and disadvantages of friction.

Solve the following problems :

1. Two balls, each of mass 80 g, moving towards each other with a velocity 5 m s^{-1}, collide and rebound with the same speed. What will be the impulse of force on each ball due to the other ? What is the value of change in momentum of each ball ? [**Ans. :** 0.8 N S, 0.8 kg m s^{-1}]
2. Two blocks of masses 6 kg and 2 kg are placed in contact on a horizontal frictionless surface. If a horizontal force of 2 N is applied to mass 6 kg. to move them together, what will be the acceleration of 2 kg block ? What will be the force on this block ? [**Ans. :** 0.25 m s^{-2}, 0.5 N]
3. Three blocks of masses 1 kg, 2 kg and 3 kg are placed in contact with each other on a horizontal frictionless surface as shown in Fig. 4.28. A force of 12 N is applied as shown in the figure.

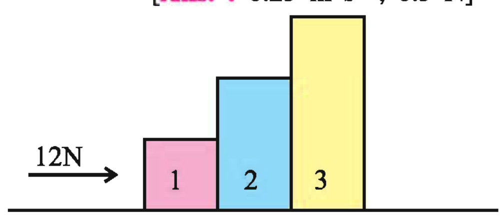


Figure 5.35

Calculate (i) the acceleration of the system of these three blocks, (ii) the contact force acting on 2 kg block by first block of 1 kg and (iii) the contact force on 3 kg block.

[**Ans. :** (i) 2 m s^{-2} (ii) 10 N (iii) 6 N]

4. A block of 50 kg on a smooth plane inclined at 60° and another block of 30 kg on a smooth plane inclined at 30° with horizontal, are cannected by a light string passing over a frictionless pully as shown in the Fig. 5.35.

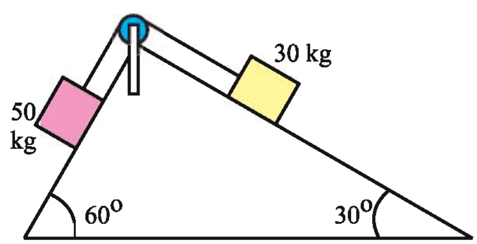


Figure 5.36

Calculate the acceleration of the blocks and tension in the string.

[Take g = 10 m s^{-2}, $\sqrt{3}$ = 1.7] [**Ans. :** 3.437 m s^2 , 253.11 N]

5. Two blocks, each of mass 3 kg, are connected by a light string and are placed on a horizontal surface. If a force of 20 N is applied in the horizontal direction on either of these blocks, the acceleration of each block is 0.5 m s^{-2} Assuming that the frictional forces on the two blocks are equal, calculate the tension produced in the string. [**Ans. :** 10 N]

6. As shown in Fig. 5.37 unequal forces F_1 and F_2 ($F_2 < F_1$) act on a rod of length L. Calculate the tension at a point situated at a distance y from end A.

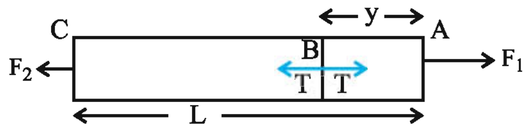


Figure 5.37

[**Ans. :** $T = F_1\left(1 - \frac{y}{L}\right) + F_2\left(\frac{y}{L}\right)$]

7. For a body of mass 2.0 kg moving in a straight line, the graph of its distance x from the starting point → time t is shown in the Fig. 5.38 Find the value of impulse of force at. (i) t = 2 s and (ii) t = 6 s for very small time intervals.

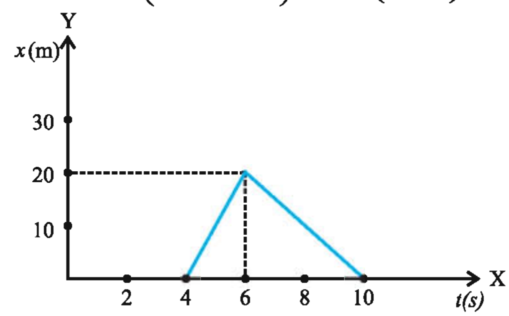


Figure 5.38

[**Ans. :** (i) 0, (ii) 30 N s]

8. Two objects of masses m_1 and m_2; start moving towards each other under the effect of only the gravitational force on each other. If the distances travelled by them are s_1 and s_2 respectively when they meet, find the ratio $\frac{s_1}{s_2}$. [**Ans. :** $\frac{s_1}{s_2} = \frac{m_2}{m_1}$]

9. The upper half of an inclined plane of inclination θ is perfectly smooth while the lower half is rough. A block starting from rest from the top of the plane; if comes back to rest at the bottom, what is the coefficient of friction between the surface of the block and the rough surface of the inclined plane ?

[**Ans. :** $\mu = 2\tan\theta$]

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CHAPTER 6

WORK ENERGY AND POWER

6.1 Introduction

Dear students, we all are familiar with the words work, energy and power. When a teacher is teaching, student is studying or a man is pushing a table, in all these cases they are said to be working. But in physics 'work' has a very precise meaning. The meaning of 'work' in physics is very much different from the picture that arises in our mind when we hear the word 'work'. In daily life, in order to work we spend energy. e.g. moving a table. For this purpose we have to apply force and then 'work' is done. From the physics perspective, for 'work' to be done, displacement in the direction of force is necessary. No work is done, as far as the physics point of view is concerned, if one keeps reading only, sitting only at one place. (Perhaps it can be considered to be mental work). Some times in order to assess the efficiency of persons, we compare the work done by two or more persons in the same period of time. Now let us understand, what exactly is the meaning of 'work' in physics.

6.2 Work and Work done by a Constant Force

As mentioned earlier, from the physics point of view, work is said to be done only if some displacement is in the direction of a force or a component of displacement is in the direction of the force. We can have a rough idea of work done by knowing the change in the magnitude of velocity.

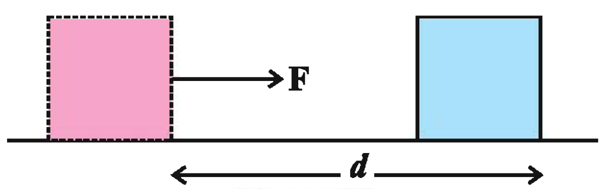

Figure 6.1

As shown in Fig. 6.1 suppose a block lying on a horizontal surface, when acted upon by force $\vec{F}$, undergoes displacement $\vec{d}$ in the direction of the force. It is clear that if the block travels more distance under the influence of this force the change in the magnitude of its velocity will be more ($v^2 - v_0^2 = 2ad$). Also if magnitude of the force is more then change in the magnitude of velocity will be more ($\because$ F = *ma*). Thus change in the magnitude of velocity depends on the magnitude of displacement and the magnitude of force. If the force and displacement are in the same direction 'work' can be defined as follows.

The product of magnitude of force and displacement (in the direction of force) during which the force acts is called work.

Thus, work is given by

W = (F) × (*d*)

Unit of work is N m or joule. Its dimensional formula is $M^1 L^2 T^{-2}$ (When is 1J work said to be done ?)

For information only :

Unit of work, 'Joule' is named in memory of the British Physicist James Prescott Joule. His main contribution is in the field of Heat. He estabished equivalence between heat and work. After a series of experiments he proved that in order to produce 1 calorie heat 4.186 Joule work is required. i.e. 1 calorie = 4.186 Joule. This constant is also known as mechanical equivalent of heat (or Joule's constant). It is noteworthy that heat is measured in terms of calorie also besides Joule. (Amount of heat energy required to be given to 1 g pure water to increase its temperature from 14.5°C to 15.5°C is known as 1 calorie.)

So this is the definition of work, if force and displacement are in the same direction. But force and displacement are not found to be in the same direction always. So the definition of work is generalised as follows.

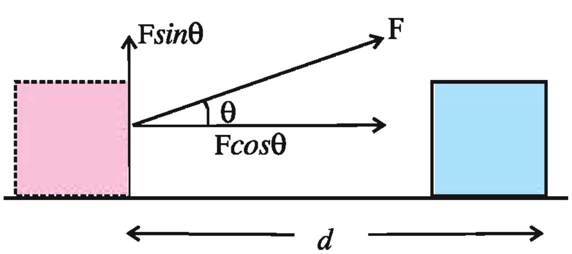

Figure 6.2

Product of magnitude of displacement due to force and magnitude of component of force in the direction of displacement is known as work. As shown in Fig. 6.2 angle between force $\vec{F}$ and displacement $\vec{d}$ of the block is θ. So the magnitude of displacement is d and magnitude of the component of the force F in the direction of displacement is F cos θ. Thus, work,

$$W = F\cos\theta \times d \quad (6.2.1)$$
$$= F\,d\cos\theta$$

Here F and *d* are magnitudes of force $\vec{F}$ and displacement $\vec{d}$. Note that in spite of force $\vec{F}$ and displacement $\vec{d}$ being vectors, work W is a scalar quantity. So, equation 6.2.1 can also be written as,

$$W = \vec{F} \cdot \vec{d} \quad (6.2.2)$$

Now let us see some special cases of work.

(i) If θ = 0, as shown earlier force and the displacement are in the same direction. Thus, work,

W = F*d*

For example the gravitational force acts in the downward direction on a freely falling body, and displacement is also in the same direction. Thus work done is,

$$W = F \times d$$
$$= mgd$$

Where *d* is the magnitude of displacement, *m* is the mass of the object and *g* is gravitational acceleration.

(ii) If θ = π/2, the force is perpendicular to the displacement and so work done is,

$$W = F\cos\pi/2 \times d$$
$$= F(0) \times d = 0$$

Thus, when the force and the displacement are perpendicular to each other, no work is done by the force on the body. The acceleration produced by the force, in this case, is perpendicular to the velocity, and so it is capable of changing only the direction of the velocity. In the case of a uniform circular motion the centripetal force acting on the body is perpendicular to the the instantaneous velocity and hence to the instantaneous displacement also. So no work is done. Geostationary satellites move in a definite orbits, work done by the gravitational force on earth is zero.

(iii) If θ = π, force $\vec{F}$ and displacement $\vec{d}$ are in opposite directions, thus

$$W = F\cos(\pi) \times d$$
$$= F(-1) \times d$$
$$= -Fd$$

This shows that when force and displacement are in opposite directions the work done is 'negative', and work is said to be done by the body against the force. When brakes are applied to a car moving with high speed, the force of friction, produced by the brakes, is in the direction opposite to displacement and so work is said to be done by the car against the frictional force.

If $0 \le \theta < \pi/2$, since value of $\cos\theta$ is positive, work done is considered to be positive. But if $\pi/2 < \theta \le \pi$ as value of $\cos\theta$ is negative, work done is negative.

Here, it is noteworthy that, if the force and displacement are in the same direction, presence of some other force is also indicated. It is also possible that body might be having initial velocity which is not in the direction of the force.

Illusration 1 : When force (3, 2, 1) N acts on a body displacement of the body in the direction of X–axis is $5m$. Calculate work done.

Solution :

Here $d = 5\hat{i}$

$$\therefore \quad W = \vec{F} \cdot \vec{d}$$
$$= (3\hat{i} + 2\hat{j} + \hat{k}) \cdot (5\hat{i})$$
$$= 15J$$

[What would have been the work done if same displacement were in direction of y and z axis. Calculate yourself.]

Illustration 2 : Natvarlal's bicycle gets dragged on a road through a distance of 10 m before it stops. During this, a frictional force of 200 N acts on the bicycle in the direction opposite to its motion, by the road. Find (1) the work done by the frictional force on the bicycle and (2) the work done by the bicycle on the road.

Solution : Here, the force of friction and the displacement of the bicycle are in mutually opposite directions and so $\theta = \pi$. Thus, the work done on the cycle by the frictional force due to the road is $W = Fd\cos\theta = (200)(10)(-1) = -2000$ J.

According to Newton's third Law of Motion the bicycle also exerts an equal and opposite force on the road, but the road does not get displaced due to this force. Hence, the work done by the cycle on the road is zero.

This example shows that if a body A exerts a force on a body B, the body B also exerts an equal and opposite force on A; but the work done by A on B is not necessarily equal to the work done by B on A.

Illustration 3 : A block lying on a rough horizontal surface (as shown in Fig. 6.3) is displaced through a distance d by a force $\vec{F}$ acting at an angle θ with the horizontal. If μ is the coefficient of friction between the block and the surface, find the work done. The mass of the block is M.

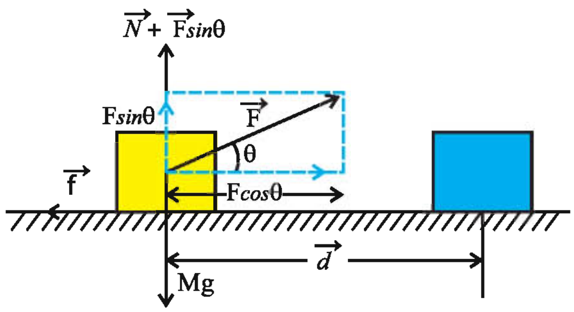

Figure 6.3

Solution : FBD (free body diagrams) of the block is shown in Fig. 6.4.

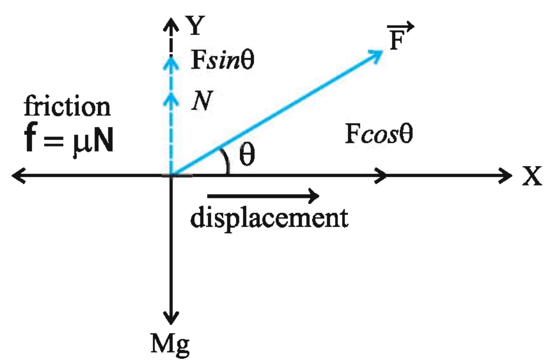

Figure 6.4

As there is no displacement in Y–direction,

$$N + F\sin\theta = Mg$$
$$\therefore \quad N = Mg - F\sin\theta \qquad (1)$$

The force responsible for horizontal displacement is

$$= F\cos\theta - \mu N = F\cos\theta - \mu(Mg - F\sin\theta)$$
(from eqn. (1))

$$\therefore \text{Work} = [F\cos\theta - \mu(Mg - F\sin\theta)]\, d$$
$$= [F(\cos\theta + \mu\sin\theta) - \mu Mg]\, d$$

6.3 Work done by Variable Force

In many situations of the work done, a variable force is responsible. For example a spring is fixed at one end on a horizontal surface. If at the open end it is compressed using a block, work is done under the effect of a variable force. We will consider this situation later in this chapter.

Consider a particle moving along a curved path AB, as shown in Fig. 6.5 under the effect of a variable force.

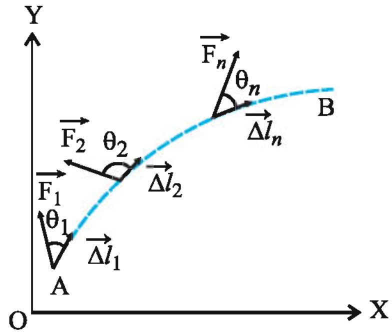

Figure 6.5

The magnitude and direction of force at different points on the curved path are different.

In order to calculate the work done, the whole curved path is assumed to be divided into small segments $\Delta\vec{l}_1$, $\Delta\vec{l}_2$,...., $\Delta\vec{l}_n$. Each segment is so small (infinitesimal) that it can be regarded as a straight line segment and can be considered as a vector.

Let $\vec{F}_1$, $\vec{F}_2$,...., $\vec{F}_n$ be the forces at the respective line segments. The force over each such segment can be considered constant because the segments are very small.

Therefore, for the displacement over each line segment, the work can be calculated by taking scalar product of the local force with the small displacement. The total work, for the motion of the particle from A to B, can be obtained by taking the sum of all such scalar products.

Total work,

$$W = \vec{F}_1 \cdot \Delta\vec{l}_1 + \vec{F}_2 \cdot \Delta\vec{l}_2 + ... + \vec{F}_n \cdot \Delta\vec{l}_n$$

$$\therefore \ W = \sum_{A}^{B} \vec{F}_i \cdot \Delta\vec{l}_i \qquad (6.3.1)$$

If we take $\lim\limits_{|\Delta\vec{l}|\to 0}$, the above summation gets converted into an integral, giving

$$W = \int_{A}^{B} \vec{F} \cdot \vec{dl} = \int_{A}^{B} F cos\theta dl \qquad (6.3.2)$$

Here $\int_{A}^{B}$, is the line integral of the force over the curved path from A to B.

If the motion of the particle is one dimensional and force also acts in the direction of the motion (here along X axis),

$$W = \int_{A}^{B} F dx \ cos 0 = \int_{A}^{B} F dx$$

If the X co–ordinates of A and B are x_1 and x_2,

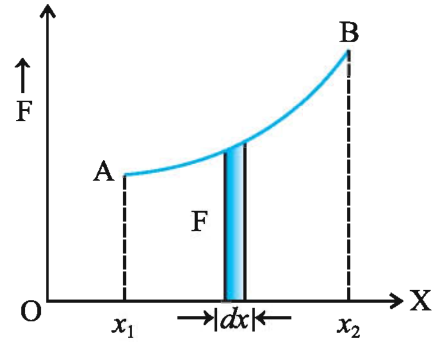

Figure 6.6

$$W = \int_{x_1}^{x_2} F dx \qquad (6.3.3)$$

Fig. 6.6. shows the graph of F versus x in a special case. It can be seen from the figure that the work done during a small displacement dx (that is Fdx) is equal to the area of the strip. This suggests that total work done for motion of the particle from x_1 to x_2, can be obtained by summing the areas of such strips. In other words, the work done during the motion from x_1 to x_2 is the area under the curve of F $\rightarrow$ x between x_1 and x_2.

If force remains constant along a curved path, the calculation of work becomes simple. Suppose a particle moves from $\vec{r}_1$ to $\vec{r}_2$, as shown in Fig. 6.7 under the effect of a constant force $\vec{F}$.

Now, the work done, $W_{12} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot \vec{dr}$

Since, $\vec{F}$ is constant,

$$W_{12} = \vec{F} \cdot \int_{\vec{r}_1}^{\vec{r}_2} \vec{dr} = \vec{F} \cdot (\vec{r}_2 - \vec{r}_1)$$

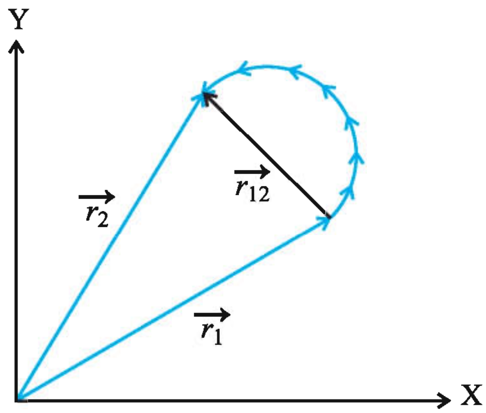

Figure 6.7

Thus, the work done by a constant force along a curved path can be obtained from the scalar product of the force and the displacement vector of the particle.

Illustration 4 : A particle moves from $x = 0$ to $x = 10$ m on X axis under the effect of a force $\vec{F}(x) = (3x^2 - 2x + 7)\hat{i}$ N. Calculate the work done. $\left[\int x^n dx = \frac{x^{n+1}}{n+1}\right]$

Solution :

$$W = \int_0^{10} F dx \qquad \text{(from eqn. (6.3.3))}$$

$$\therefore W = \int_0^{10} (3x^2 - 2x + 7)dx$$

$$W = \left[\frac{3x^3}{3}\right]_0^{10} - \left[\frac{2x^2}{2}\right]_0^{10} + [7x]_0^{10}$$

$W = 1000 - 100 + 70 = 970$ J.

6.4 Kinetic Energy

The capacity of an object to do work, by virtue of its motion, is known as kinetic energy of the object. It can be thought logically that a body moving with more speed would have more kinetic energy compared to the kinetic energy of the same body moving with comparatively lesser speed.

When a body is acted upon by a force, acceleration is produced in it. Thus, velocity of the body changes and hence the kinetic energy of the body also changes. Also, a force acting on a body displaces the body and so work is said to be done on the body by the force. These facts indicate that there should be some relation between the work done on a body and change in its kinetic energy. Let us obtain such a relation.

The work done by the force $\vec{F}$,

$$W = \vec{F} \cdot \vec{d} \qquad \text{(where } \vec{d} = \text{displacement)}$$

But $\vec{F} = m\vec{a}$

$$\therefore W = m\vec{a} \cdot \vec{d} \qquad (6.4.1)$$

Also, $v^2 - v_0^2 = 2\vec{a} \cdot \vec{d}$. Using the result in eqn. (6.4.1), we have

$$W = m\left(\frac{v^2 - v_0^2}{2}\right)$$

$$\therefore W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \qquad (6.4.2)$$

Here, v_0 and v are the speeds before and after the application of force respectively.

The right hand side of eqn. (6.4.2) is the difference of two terms with the dimensions of energy (work). **Half of the product of the mass of a body and the square of its velocity is defined as kinetic energy (K) of the body,**
So, kinetic energy,

$$K = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m} \qquad (6.4.3)$$

where, p is linear momentum of the body.

The right hand side is the change in kinetic energy of the body.

$$W = K - K_0 = \Delta K \qquad (6.4.4)$$

where K_0 and K are respectively the initial and final kinetic energies. Thus, **"the work done by the resultant force on a body, is equal to the change in the kinetic energy of the body."** This statement is known as **work energy theorm.** From equation (6.3.4) it is clear that the unit of kinetic energy is same as the unit of work (joule in SI).

If the speed of a body is constant, its kinetic energy remains constant. Since the speed of a particle performing uniform circular motion is constant, its kinetic energy also remains constant throughout.

Work energy theorm for variable force in one dimension :

Suppose force F(*x*) acts on a body of mass *m*. [F(*x*) shows that F is a function of *x* or its value depends on *x*]

Work done under the influence of this force.

$$W = \int_i^f F(x)\, dx$$

$$= \int_i^f m \frac{dv}{dt} dx$$

$$= \int_i^f m\ dv \frac{dx}{dt} \quad (\because \frac{dx}{dt} = v)$$

$$= m \int_i^f v dv$$

If initial velocity of the body and final velocity of the body are v_1 and v_2

$$\therefore W = m \int_{v_1}^{v_2} v dv$$

$$= m \left[\frac{v^2}{2} \right]_{v_1}^{v_2} = \frac{m}{2} [v_2^2 - v_1^2]$$

$$\therefore W = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \qquad (6.4.5)$$

$$\therefore W = \Delta K$$

Illustration 5 : A proton and an electron are in motion with both having kinetic energy equal to 100 eV. Which of these two particles have more speed ? ($m_e = 9.1 \times 10^{-31}$ *kg*, $m_p = 1.67 \times 10^{-27}$ kg)

[**Note :** Here eV (electron volt) is a unit of energy only. [eV = 1.6×10^{-19}J]

Solution : Kinetic energy of electron

$$= 100\ keV = \frac{1}{2} m_e v_e^2$$

Kinetic energy of proton

$$= 100\ keV = \frac{1}{2} m_p v_p^2$$

$$\therefore m_e v_e^2 = m_p v_p^2$$

$$\therefore \frac{v_e}{v_p} = \sqrt{\frac{m_p}{m_e}}$$

$$= \sqrt{\frac{1.67 \times 10^{-27}}{9.1 \times 10^{-31}}}$$

$$= 42.84$$

Thus, if an electron and a proton have equal kinetic energy, the speed of the electron is 42.84 times the speed of the proton (Why ? Think !)

Illustration 6 : A body of mass 2 kg is at rest on a smooth horizontal surface. When a horizontal force of 0.5 N acts on this body, it is displaced in the direction of the force. Find the work done by the force in 8.0 s. Show that this work is equal to the change in kinetic energy of the body.

Solution : According to Newton's second law of motion, $a = \frac{F}{m}$

$$\therefore a = \frac{0.5}{2} = 0.25 \text{ m/s}^2$$

The velocity of the body after 8 s,

$v = v_0 + at = 0 + 0.25 \times 8.0 = 2$ m/s

The displacement of the body in 8s,

$$d = \frac{1}{2} at^2 = \left(\frac{1}{2} \right)(0.25)(64) = 8.0\text{m}$$

The work done by the force,

$W = 0.5 \times 8.0 = 4$J (1)

Now, Initial kinetic energy = 0

Final kinetic energy

$$= \frac{1}{2} mv^2 = \frac{1}{2} \times 2.0 \times [2.0]^2 = 4 \text{ J}$$

$\therefore$ Change in kinetic energy = ΔK = 4 J (2)

From equations (1) and (2)

$W = \Delta K$

Here, the work done on the body has been converted into kinetic energy of the body.

6.5 Potential Energy

In mechanics there is another important form of energy other than kinetic energy, it is potential energy. **"The capacity to do work or energy possessed by a body due to its position in a force field or due to its configuration is called potential energy of the body/system.** When a force acts on a body there is change in its position or configuration or both. So there can be a change in its potential energy also.

Gravitational Potential Energy : Acceleration produced in a body, due to earth's gravitational force is called gravitational acceleration (g). Value of g can be treated to be constant for a small height compared to the radius of the earth. In this case a force equal to mg acts on the body of mass m towards the centre of the earth.

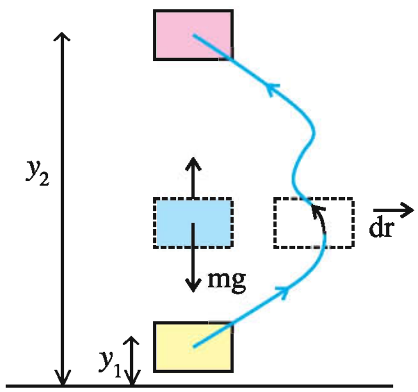

Figure 6.8

As shown in the Fig. 6.8 an object is taken to height y_2 from initial height y_1. For simplicity let our co-ordinate system be such that Y–axis is with vertical direction.

The body can be taken from y_1 height to y_2 height along vertical direction or along any other path. In the figure two such paths are shown. As a general case we will consider motion of the body along the curved path from initial position to final position. For this, a very small displacement $\vec{dr}$ on the path, as shown in the figure, can be thought of;

Work done for this displacement

$$dw = \vec{F} \cdot \vec{dr}$$

Since gravitational force is in downward direction $\vec{F} = -mg\,\hat{j}$

$$\therefore\ dw = -mg\,(\hat{j}) \cdot (dx\hat{i} + dy\,\hat{j} + dz\hat{k})$$

$$= -mg\ dy$$

So the work done during, its motion from the initial position to the final position is

$$W = \int dw$$

$$= -mg \int_{y_1}^{y_2} dy$$

$$= -mg[\ y\ ]_{y_1}^{y_2}$$

$$= -mg(y_2 - y_1)$$

$$= -(mgy_2 - mgy_1) \qquad (6.5.1)$$

The above equation indicates that work done in taking a body from one position to the other position depends only on the initial and the final positions, and not on paths connecting them. The body can travel along any path. The same amount of work shall be done. A force having such property is called conservative force and a force field with such a property is called conservative field.

If v_1 and v_2 are the magnitudes of velocities of a body at heights y_1 and y_2 respectively, the change in its kinetic energy will be $\frac{1}{2}mv_2^{\ 2} - \frac{1}{2}mv_1^{\ 2}$. According to work–energy theorem this must be equal to the work done.

$$\therefore\ W = \frac{1}{2}mv_2^{\ 2} - \frac{1}{2}mv_1^{\ 2} \qquad (6.5.2)$$

Comparing eqns. (6.5.1) and (6.5.2),

$$(\frac{1}{2}mv_2^{\ 2} - \frac{1}{2}mv_1^{\ 2}) = -(mgy_2 - mgy_1) \quad (6.5.3)$$

$$\text{or } (\frac{1}{2}mv_1^{\ 2} - \frac{1}{2}mv_2^{\ 2}) = mg(y_2 - y_1) \quad (6.5.4)$$

The terms on the left hand side of the equation is represent the kinetic energy and hence the terms on the right hand side must also correspond to some energy. In fact these terms represent the potential energies of the body at heights y_1 and y_2 from the surface of the Earth, in the gravitational field of the Earth.

The product of the weight of a body and its height, from some reference level in the gravitational feiled of the Earth, is called the gravitational potential energy U of the body in Earth's gravitational field.

Thus, gravitational protential energy of a body of mass m at height h from the surface of the Earth,

$$U = mgh \qquad (6.5.5)$$

In practice the potential energy at the reference level is taken as zero; because it is the change in potential energy which is important, not its absolute value.

From equation (6.5.4),

$$\frac{1}{2}mv_1^{\,2} + mgy_1 = \frac{1}{2}mv_2^{\,2} + mgy_2 \qquad (6.5.6)$$

Thus, in a conservative field the sum of kinetic energy (K = $\frac{1}{2}mv^2$) and potential energy (U = mgh) remains constant.

The sum of kinetic energy and potential energy is called mechanical energy (E).

$\therefore$ E = K + U

In other words, the mechanical energy of an isolated body is conserved in a conservative field.

In this example the decrease in kinetic energy of the body as it moves up, equals increase in its potential energy. (Here frictional force or air resistance acting on the body is neglected)

It is clear from this discussion that **"under influence of conservative forces the mechanical energy of a mechnically isolated system remain constant."** This statement is known as law of conservation of mechanical energy.

[**Note :** In this discussion we have considered a system comparising of the earth and a body. There is no external force acting on the system. In this sense it can be considered to be a mechanically isolated system. The total energy mentioned above is in fact the energy of the system. But there is no change in the potential energy or the kinetic energy of the earth. So a convention we have discussed potential energy or kinetic energy of the body only.]

Illustration 7 : As shown in the figure a bob of mass m is tied to the end of a light (mass less !) string. At its lowest position it is given velocity v, so that it moves along a circular path. It can just reach the point C, the highest position, as the string becomes tensionless. Prove that $v = \sqrt{5gl}$.

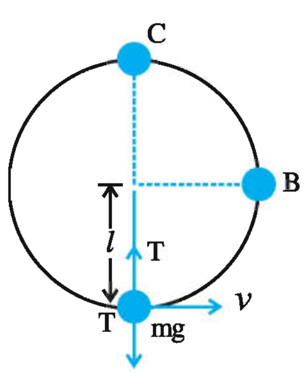

Figure 6.9

Solution : In figure forces acting on the body when it is at its lowest position are shown in the fig. Considering its potential energy zero, conveniently at this point; its mechanical energy is,

$$E = \frac{1}{2}mv^2 + 0$$

$$= \frac{1}{2}mv^2 \qquad (1)$$

By Newton's second law of motion, the centripetal force acting on it is $mv^2/l = T - mg$. At position 'C' as string becomes tensionless, tension becomes zero and its velocity here is v'. Since P. E. is $2mgl$ the mechanical energy

$$E = \frac{1}{2}mv'^2 + 2mgl \qquad (2)$$

$$\text{and } mg = mv'^2 / l \qquad (3)$$

From equation (2) and (3)

$$E = \frac{1}{2}mgl + 2mgl = 5/2\ mgl \qquad (4)$$

As per the law of conservation of mechanical energy and from equation (1) and (4)

$$\frac{1}{2}mv^2 = \frac{5}{2}mgl$$

$$\therefore v = \sqrt{5gl} \qquad (5)$$

[Thinking little more here, if its velocity near B is v''

$$E = \frac{1}{2}mv''^2 + mgl \qquad (6)$$

As per the law of conservation of mechanical energy and from equation (1) and (6)

$$\frac{1}{2}mv^2 = \frac{1}{2}mv''^2 + mgl$$

Inserting value of v from equation (5) we get $\frac{1}{2}m\,(5gl) = \frac{1}{2}mv''^2 + mgl$

$$\therefore v'' = \sqrt{3gl}\ ]$$

6.6 Elastic Potential Energy (Potential energy due to the configuration of a system)

Consider an elastic spring, obeying Hooke's law, with negligible mass whose one end is tied rigidly to a wall, as shown in Fig. 6.10. At the other end of the spring a block of mass M has been tied. We shall, for the sake of simplicity, restrict the motion of the block in the X–direction only. The increase or decrease in the length of the spring will also be only in X–direction.

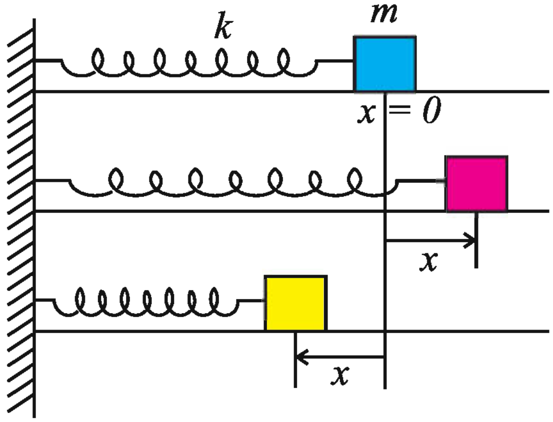

Figure 6.10

In the normal position of the spring (i.e., without extension or compression), the position of the block is taken as $x = 0$.

When the block is pulled and the length of the spring is increased a restoring force is produced in the spring which tries to bring the spring back to its normal position. The restoring force is also produced when the spring is compressed.

In the present case, the restoring force is directly proportional to the change in the length of the spring and is in the direction opposite to the change in the length,

$\therefore F \propto -x$

$$\therefore F = -kx \qquad (6.6.1)$$

The constant of proportionality, k is known as the force constant of the spring.

If x is the increase in the length of the spring (or the displacement of the block) the work done by the applied force is,

$$W = \int_0^x kx dx = k\int_0^x x dx$$

$$= k\left[\frac{x^2}{2}\right]_0^x$$

$$\therefore W = \frac{1}{2}kx^2 \qquad (6.6.2)$$

This work done on the spring is stored in the form of potential energy of the spring. This potential energy is called the elastic potential energy of the spring.

Taking potential energy in the normal position of the spring as zero arbitrarily, the potential energy in the condition when the change in length is equal to x will be,

$$U = \frac{1}{2}kx^2 \qquad (6.6.3)$$

Value of potential energy can also be obtained from $F - x$ graph as shown in figure.

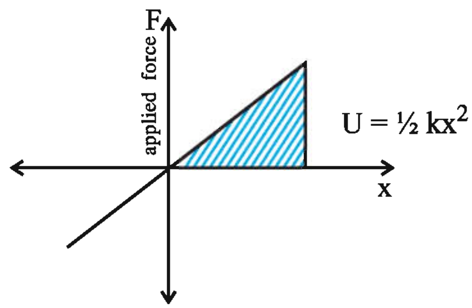

Figure 6.11

Here it is clear that work done on the system by the external force (for compression or extension of spring) is stored in the form of potential energy and kinetic energy.

6.7 Conservative Forces and Relation between force and potential energy :

Suppose a conservative force F acts on a particle and as a result the particle is displaced through a small distance Δx. The work done by the force,

$\Delta W = F\Delta x$

Now, the change in kinetic energy of the particle, according to work–energy theorem, is given by,

$\Delta K = W = F\Delta x$

But, from the law of conservation of mechanical energy, $\Delta K + \Delta U = 0$

Using this fact in the above equation,

$F\Delta x + \Delta U = 0$

$$\therefore F = -\frac{\Delta U}{\Delta x}$$

Taking $\Delta x \to 0$, this relation can be written as :

$$\therefore F = -\frac{dU}{dx} \qquad (6.7.1)$$

Thus, in the case of conservative force, the negative gradient of potential energy gives the force.

Using equation (6.7.1) we can get the restoring force in the case of a spring as under :

Potential energy, $U = \frac{1}{2}kx^2$

$$\therefore -\frac{dU}{dx} = -\frac{1}{2}k(2x) = -kx$$

$\therefore$ F = $-kx$

The above discussion is in context to conservative forces only.

If the forces are non-conservative, the work done is not converted fully into potential energy. In the case of non–conservative forces like friction, the work done is dissipated in the form of heat energy. Also, the law of conservation of mechanical energy does not hold true in the case of non–conservative forces and hence in such cases force can not be obtained by differentiating potential energy.

Illustration 8 : A block of mass 1 kg. falls freely on a spring from a height of 20 cm as shown in the Fig. 5.13. Find the compression in the spring if its force constant is 600 N/m. Take g = 10.0 m/s^2.

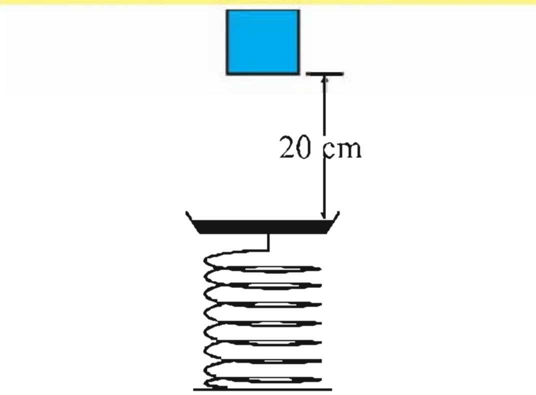

Figure 6.12

Solution : Let x be the compression in the spring. In this condition, block is considered to fall through the height $(x + 0.2)$m. The potential energy of the block at height $(x + 0.2)$m is used up in compressing the spring by x m, and is converted in the configurational potential energy $\frac{1}{2}kx^2$ of the spring.

$$\therefore mg(h + x) = 1 \times 10(0.2 + x)$$

$$= \frac{1}{2}kx^2$$

$$\therefore \frac{1}{2}kx^2 = 1 \times 10\ (0.2 + x)$$

$$300x^2 = 10x + 2.0$$

$$\therefore 300x^2 - 10x - 2.0 = 0$$

$$\therefore 150x^2 - 5x - 1 = 0$$

$$\therefore x = \frac{5 \pm \sqrt{25 - 4(150)(-1)}}{300}$$

$$\therefore x = 0.0167 \pm 0.0833$$

Here, 0.0167 shows the value of the compression at which the block can remain in equilibrium. In practice the block oscillates about this equilibrium position with an amplitude of 0.064 m and maximum displacement would be or 0.1m or 10 cm

6.8 Power

So far we have not considered the time taken in doing work. In raising a body to a definite height from a given position, if the time taken is 1 s, 1 hr or different, the work done in each case is same, but the rate at which the work is done in each case is different. In many cases the rate of doing work is more important than the total work done. Hence, a physical quantity known as power is defined as :

The time rate of doing work is known as power. or Power is defined as work done per unit time. If ΔW is the work done in time interval Δt, the average power in time interval Δt is,

$$\langle P \rangle = \frac{\Delta W}{\Delta t}$$

$\therefore$ Instantaneous power at time t is,

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

$$\therefore P = \frac{dW}{dt} \qquad (6.8.1)$$

If dW is the work done by a force $\vec{F}$ during the displacement $d\vec{r}$,

$$dW = \vec{F} \cdot d\vec{r}$$

In this case, the instantaneous power is given by,

$$P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

$$\therefore P = \vec{F} \cdot \vec{v} \qquad (6.8.2)$$

Power is a scalar quantity like work and energy. Its dimensional formula is $M^1L^2T^{-3}$. Its unit is J s^{-1}. This unit is named as watt in the honour of inventor of steam engine, James Watt. 1W = 1 J s^{-1}. Since watt is a small unit, to measure large power in practice, kilowatt and megawatt are used.

1 kW = 10^3W

1 MW = 10^6W

To measure large powers of vehicles and water pumps, in practice, another big unit known as horse power is used which is originally from the British system of units.

1 horse power [hp] = 746 W

According to equation (6.8.1), work can be expressed as a product of power and time. This fact gives rise to another unit of work, known as kilowatt hour (kWh).

"The work done at the rate of 1 kilowatt in 1 hour is called 1 kilowatt–hour."

The electrical energy consumed in our homes is measured in kWh which is popularly known as " Unit ".

1 Unit = 1 kWh = 3.6 × 10^6 J

Remember that "Unit" or kWh is the unit of energy and not the unit of power. If an electric bulb of 100 W is kept 'ON' for 10 hours, 1 unit of electrical energy is consumed.

Illustration 9 : A particle of mass m moves on a circular path of radius r. Its centripetal acceleration is kt^2, where k is a constant and t is time. Express power as function of t.

Solution : Centripetal acceleration or radial acceleration, $\frac{v^2}{r} = kt^2$

Differentiating with respect to time,

$$2v\frac{dv}{dt} = 2ktr$$

$$mv\frac{dv}{dt} = mktr$$

$\therefore$ Fv = $ktmr$ [$\because$ F = $m\frac{dv}{dt}$, $\frac{dv}{dt}$ is the tangential acceleration.]

$\therefore$ P = $ktmr$

6.9 Elastic and Inelastic Collisions

During collision between two objects, the total energy and the total linear momentum of the colliding bodies are conserved.

If total kinetic energy before and after the collision is the same, the collision is termed as an elastic collision. In other words, in an elastic collision total kinetic energy of the colliding bodies is conserved.

During many collisions the kinetic energy of the colliding bodies is converted fully or partly into internal energy of the bodies. **In such collisions the kinetic energy is not conserved. Collisions of this kind are known as inelastic collisions.** It may be noted that total energy and total linear momentum are, indieed, conserved in both the types of collisions.

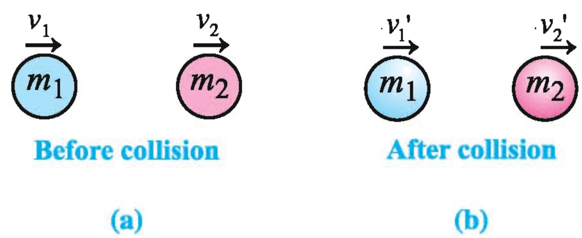

Figure 6.13

Now we will consider elastic collision in one dimension, as shown in Fig. 6.13 (a)

Suppose a body of mass m_1 moving with velocity v_1, along X–axis, undergoes an elastic collision with a body of mass m_2 moving with velocity v_2 along X–axis. Their final velocities are v_1' and v_2'

As per the law of conservation of momentum

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \quad (6.9.1)$$

$$\therefore m_1(v_1 - v_1') = m_2(v_2' - v_2) \quad (6.9.2)$$

Also, as the collision is elastic

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

$$\therefore m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2) \quad (6.9.3)$$

Dividing equations (6.9.3) by (6.9.2) we get

$$v_1 + v_1' = v_2 + v_2' \quad (6.9.4)$$

Multiplying equation (6.9.4) by m_1 and adding it to equation (6.9.1) we get

$$m_1v_1 + m_2v_2 + m_1v_1 + m_1v_1' = m_1v_1' + m_2v_2' + m_1v_2 + m_1v_2'$$

$$\therefore 2m_1v_1 + (m_2 - m_1)v_2 = (m_1 + m_2)v_2'$$

$$\therefore v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 \quad (6.9.5)$$

Substituting value of v_2' in equation (6.9.4)

$$v_1' = \left(\frac{2m_1}{m_1 + m_2} - 1 \right) v_1 + \left(1 + \frac{m_2 - m_1}{m_1 + m_2} \right) v_2$$

$$\therefore v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_1 + \left(\frac{2m_2}{m_1 + m_2} \right) v_2 \qquad (6.9.6)$$

Equations (6.9.5) and (6.9.6) are equations of elastic collision in one dimension.

Special cases : (i) For $m_1 = m_2$

If $m_1 = m_2$, $v_1' = v_2$ and $v_2' = v_1$. This means that at the time of collision both the bodies interchange their velocities.

(ii) For $m_2 >> m_1$. In this case the lighter body collides with the hevier body while moving in the same direction. Now m_1 can be neglected comparison with m_2 in equations (6.9.5) and (6.9.6) and we get,

$$v_1' = -v_1 + 2v_2$$

and $v_2' \approx v_2$

This shows that velocity of heavy body is almost unchanged the velocity of lighter body changes. In other words heavier body does not respond to coillision.

Think about the results obtained in this article 6.9 with $v_2 = 0$ (Really think about it)

What can be said about the magnitudes of the relative velocity of approach and the relative velocity of separation ? (**Hint :** consider equation (6.9.4)]

Now let us consider a very special case of inelastic collision. Suppose a bullet fired from a gun hits a comparitively large wooden block and gets embeded in it, and then both of them move together as a single body. Such a collision is known as a completely elastic in collision.

Suppose a body of mass m_1 moving with velocity v_1 hits another body of mass m_2 moving with velocity v_2 in the same direction of v_1. Since the collision is completely inelastic after the collision the combined body moves with velocity v. According to law of conservation of momentum.

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v$$

$$\therefore v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \qquad (6.9.7)$$

6.10 Elastic Collision in Two Dimensions

As shown in Fig. 6.14, suppose an object of mass m_1 moving in X-direction with velocity $\vec{v}_1$ collides elastically with a stationary $[\vec{v}_2 = 0]$ object of mass m_2. After the collision these objects move in the directions making angles θ_1 and θ_2 with the X axis with velocities $\vec{v}_1'$ and $\vec{v}_2'$ respectively.

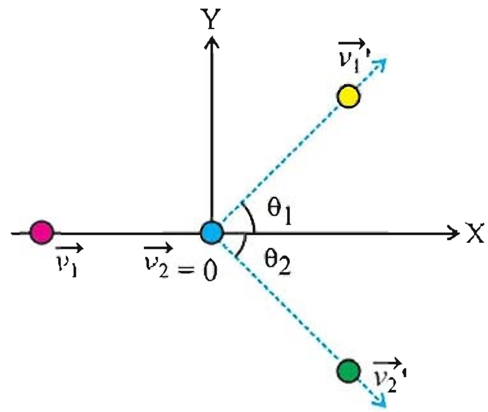

Figure 6.14

According to the law of conservation of momentum,

$$m_1\vec{v}_1 = m_1\vec{v}_1' + m_2\vec{v}_2' \qquad (6.10.1)$$

Equating the X–components of the momenta of these objects,

$$m_1v_1 = m_1v_1' cos\theta_1 + m_2v_2' cos\theta_2 \qquad (6.10.2)$$

Similarly, equating Y–components of the momenta, we get,

$$0 = m_1v_1' sin\theta_1 - m_2v_2' sin\ \theta_2 \qquad (6.10.3)$$

Since the collision is elastic,

$$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 \quad (6.10.4)$$

Usually, m_1, m_2 and v_1 are known and v_1', v_2', θ_1 and θ_2 are to be determined. But we have only three equations (6.9.2, 3, 4) for four unknown quantities. Thus, for the solution of the problem at least one of the four unknown quantities must be known, as three equations can give the values of only three unknown quantities.

Illustration 10 : A ball moving with a velocity of 12 ms^{-1} collides with another identical, stationary ball. After the collision they move in XY plane as shown in Fig. (6.15).Find the speeds of the balls after the collision. Also, decide whether the collision is elastic or not.

Solution :

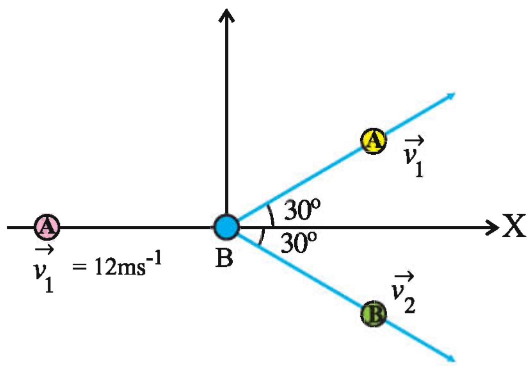

Figure 6.15

Let mass of each ball be m. According to the law of conservation of momentum,

$$mv_1 = mv_1'cos30^o + mv_2'cos30^o \quad (1)$$

and

$$0 = mv_1'sin30^o + mv_2'sin30^o \quad (2)$$

$$\therefore \; v_1' = v_2' \quad (3)$$

From eqns. (1) and (2),

$$12 = 2v_1' \times \frac{\sqrt{3}}{2}$$

$$\therefore \; v_1' = \frac{12}{\sqrt{3}} = 4\sqrt{3} \text{ m/s}$$

Total kinetic energy before collision

$$K_1 = \frac{1}{2}mv_1^2$$

$$K_1 = \frac{1}{2}m(12)^2 = 72 \; m \text{ J} \quad (4)$$

Total kinetic energy after collision

$$K_2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2$$

$$= \frac{1}{2}m\,(48 + 48)$$

$$\therefore \; K_2 = (48\; m)\text{J} \quad (5)$$

It is clear from (4) and (5) that $K_1 > K_2$. So kinetic energy is not conserved and so collision is not an elastic collision.

Let us end this chapter with an example of work done by a non-conservative force.

Illustration 11 : A tube is fixed in a vertical plane as shown in the figure. From point A a sphere of mass 0.314 kg is released. During its motion in the tube it faces a constant resistive force R. At B its velocity becomes zero. Calculate (i) the constant resistive force R and (ii) work done by resistive force (Average radius of semicircular path is 1 m)

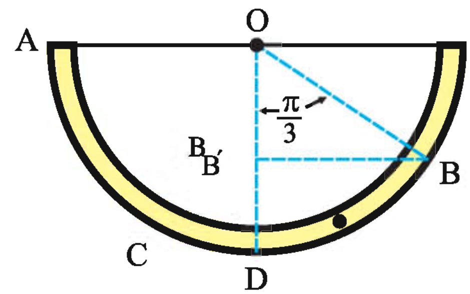

Figure 6.16

Solution : Let D be the lowest point on its path. Let us assume that the potential energy at D to be zero. So, potential energy at A is

$$U_A = mgr \quad (1)$$

Its potential energy at B is

$$\therefore \; U_B = mg\,(B'D)$$

$$\text{Since } OB' = OB\cos\frac{\pi}{3} = \frac{r}{2}$$

$$\therefore \; B'D = OB' = \frac{r}{2}$$

$$\therefore \; U_B = mg\frac{r}{2} \quad (2)$$

As the resistive force is constant, the work done against resistive force W_R is given by

$$W_R = R \times \frac{5\pi}{6} \times r \; (\because \text{arc} = \theta \times r) \quad (3)$$

From (1), (2) and (3)

$$mgr = \frac{mgr}{2} + R\frac{5\pi r}{6}$$

$$\therefore \; \frac{mgr}{2} = \pi r R\left(\frac{5}{6}\right)$$

$$\therefore \; R = \frac{3mg}{5\pi} = \frac{3 \times 0.314 \times 10}{5 \times 3.14} = 0.6\text{N}$$

Work done by the resistive force

$$\text{is } W_R = R \times \frac{5\pi r}{6}$$

$$= 0.6 \times \frac{5 \times 3.14 \times 1}{6} = 1.57 \text{ J}$$

INFORMATION FOR COMPITITIVE EXAMINATIONS ONLY

Newton's law of impact : When the collision is 'head on' or direct i.e. when the relative velocities of two colliding bodies at the instant of the collision, lie along the common normal at the point of impact, the relative velocity after the impact bears a constant ratio to that before impact but in opposite direction.

This constant ratio in known as co-efficient of restitution (e).

From definition, in terms of terminology used here.

$$e = \frac{v_2' - v_1'}{v_1 - v_2}$$, 'e' depends on the types of materials of bodies colliding.

For perfectly elastic collision e = 1 and for perfectly inelastic collision e = 0.

If coefficient of restitution is also considered for general case, equations of velocities after collision of two bodies can be written as :

$$v_1' = \frac{(m_1 - m_2 e)}{m_1 + m_2} v_1 + \frac{(1 + e)m_2}{m_1 + m_2} v_2 \text{ and } v_2' = \frac{(1 + e)m_1}{m_1 + m_2} v_1 - \frac{(m_1 e - m_2)}{m_1 + m_2} v_2$$

SUMMARY

1. The concept of work in physics is different from the normal concept of work.
2. The product of the magnitude of the displacement during the action of a force and the magnitude of the component of the force in the direction of displacement is known as work. Its unit in joule and its dimensional formula is $M^1L^2T^{-2}$.
3. If angle between force and displacement is θ
   (i) for $\theta = 0$ $W = Fd$
   (ii) for $\theta = \pi/2$ $W = 0$
   (iii) for $\theta = \pi$ $W = -Fd$
   If θ is an acute angle work is positive and work is done by the force on the body. If θ is an obtuse angle, work is negative and work is done by the body against the force.
4. The work done by a variable force is given by $W = \int_i^f \vec{F} \cdot \vec{dl}$
5. If a variable force and displacement due to it are in the same direction the area below the $F - x$ graph gives value of work.
6. The ability of a body to do work by virtue of its motion is known as its kinetic energy. If the velocity of a body of mass 'm' is 'v' its kinetic energy is $K = \frac{1}{2} mv^2 = p^2/2m$.
7. **Work Energy Theorem :** Work done by a resultant force on a body is equal to the change in its kinetic energy.
8. When a body has the ability to do work due to its position in a force field or its configuration. It is known as potential energy.
9. Potential energy is a relative physical quantity. It is impossible to get its absolute value. Also only the changes in potential energy are important.

10. If the gravitational potential energy, due to the gravitational field of Earth, is randomly taken to be zero on its surface, the potential energy of a body of mass *m*, at height *h* is *mgh*, where *g* is the gravitational acceleration. The value of '*h*' is negligible compared to the radius of the earth.
11. The sum of the potential energy and the kinetic energy of a substance is called the mechanical energy.
12. Considering potential energy of a spring as zero in its normal state, if its length is changed by *x*, the potential energy of the spring is $U = \frac{1}{2}kx^2$. Here *k* is the spring constant. Unit of *k* is N/m and dimensional formula is $M^1L^0T^{-2}$.
13. **Conservative forces :** The forces for which work done is independent of the path of motion of the body but depends only on initial and final positions, are called conservative force. The force of gravitation or the restoring force developed in a spring due to its compression or extension are conservative forces.
14. The relation between the conservative force and the potential energy is $F = -\frac{dU}{dx}$
15. The time–rate of doing work is called power. Its unit is watt (J/s). Its dimensional formula is $M^1L^2T^{-3}$.

    Thus power $P = W/t$ or $P = \vec{F} \cdot \vec{v}$

    1 horse power $\simeq$ 746 watt.

    Unit of electric energy for domestic use is 1 unit = 1 kWh = 3.6×10^6J
16. If during collision of two bodies, the kinetic energy is conserved the collision said to be elestic.
17. A body of mass *m* moving with velocity v_1 undergoes elastic collision with a body of mass m_2 moving with velocity v_2 in the direction of v_1. If after collision their velocities are v_1' and v_2'.

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2}v_1 + \frac{2m_2}{m_1 + m_2}v_2 \text{ and } v_2' = \frac{2m_1}{m_1 + m_2}v_1 + \frac{m_2 - m_1}{m_1 + m_2}v_2$$

18. In case of complete inelastic collision bodies colliding move together after colision with a common velocity *v*. In this case

$$v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

19. A body of mass m_1 moving with velocity v_1 collides with a stationary body of mass m_2 elastically. They move with velocities v_1' and v_2' making angles θ_1 and θ_2 with direction of v_1 then

$$m_1v_1 = m_1v_1'\cos\theta_1 + m_2v_2'\cos\theta_2$$

$$0 = m_1v_1'\sin\theta_1 - m_2v_1'\sin\theta_2$$

and $m_1v_1^2 = m_1v_1'^2 + m_2v_2'^2$

EXERCISES

Choose the correct option from the given options :

1. A wall, acted upon by a force of 20 N, does not move. The work done in this process is ....
(A) 20 J (B) 0 J
(C) 10 J (D) Nothing can be said

2. If linear momentum of a body is increased by 1 %, its kinetic energy increases by .... %
(A) 10 % (B) 0 % (C) 2 % (D) 100 %

3. With what velocity should a student of mass 60 *kg* run so that his kinetic energy becomes 270 J ?
(A) 10 m/s (B) 3 m/s (C) 20 m/s (D) 2.5 m/s

4. A spring is compressed by 1 cm by a force of 3.92 N. Find the potential energy of the spring when it is compressed by 10 cm.
(A) 1.96 J (B) 2.45 J (C) 19.6 J (D) 196.0 J

5. How much power is required to lift a body of mass 100 *kg* to a height of 60 m in 1 minute ? (g = 9.8 m/s^2)
(A) 100 W (B) 980 W (C) 9.8 W (D) 1980 W

6. A body is displaced by 2 m in Y–direction by a force (–4, 2, 6) N. Find the work done.
(A) 2 J (B) 4 J (C) 1 J (D) 4.5 J

7. The angle between $\vec{F}$ = (1, –3, 1) and $\vec{d}$ = (2, –3, –11) is .... rad.
(A) π (B) 0 (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

8. The mass of a bus is 2000 kg. How much work is required to be done on it to make it move with a speed of 50 km/h ?
(A) 1.6×10^5 J (B) 1.6×10^6 J (C) 1.93×10^6 J (D) 193 J

9. The potential energy of a projectile at its highest point is $\left(\frac{3}{4}\right)^{th}$ the value of its initial kinetic energy. Therefore its angle of projection is ....
(A) 30° (B) 45° (C) 60° (D) 75°

10. A body is moved along a straight line by a machine delivering a constant power. The velocity gained by the body in time t is proportional to ....
(A) $t^{\frac{3}{4}}$ (B) $t^{\frac{3}{2}}$ (C) $t^{\frac{1}{4}}$ (D) $t^{\frac{1}{2}}$

11. A particle moves in a straight line with retardation proportional to displacement. Its loss of kinetic energy for any displacement x proportional to ....
(A) x^2 (B) e^x (C) x (D) $log e^x$

12. A body of mass m is accelerated uniformly from rest to a speed v is time T. The instantaneous power delivered to the body in tems of time is given by ....
(A) $\frac{mv^2}{T^2}t$ (B) $\frac{mv^2}{T^2}t^2$ (C) $\frac{mv^2t}{2T^2}$ (D) $\frac{mv^2t^2}{2T^2}$

13. A spherical ball of man 20 kg. is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to ground then climb up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above ground. The velocity attained by the ball is .... (g = 10 ms^{-2}) Ignore friction.

(A) 40 m/s (B) 20 m/s (C) 10 m/s (D) $10\sqrt{30}$ m/s

14. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with initial vertical direction is ....

(A) $Mg(\sqrt{2} + 1)$ (B) $Mg\sqrt{2}$

(C) $Mg / \sqrt{2}$ (D) $Mg(\sqrt{2} - 1)$

15. A boy holding a gas filled balloon relases it. The balloon starts going up then its potential energy will ....

(A) increase (B) decrease

(C) First increase and then decrease (D) remain constant

16. For a conservative force $\vec{F}$, $\int_{\text{closed path}} \vec{F} \cdot d\vec{l}$ ....

(A) $\neq 0$ (B) < 0 (C) > 0 (D) $= 0$

17. Which one of the following isn't a conservative force ?

(A) Gravitational force (B) Restoring force in spring

(C) Frictional force (D) All

18. The velocity of a body of mass 0.8 kg is $3\hat{i} + 4\hat{j}$ *m/s*. So its kinetic energy is ....

(A) 10 J (B) 40 J (C) 32 J (D) 16 J

19. The potential energy of 1 kg object, free to move along x axises is given by $U(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$J. If its mechanical energy is 2 J its maximum speed is .... m/s.

(A) $\frac{3}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) 2

20. Kinetic energy of a body, moved by a machine, is directly proportional to time t. So, distance travelled by the body is proportional to ....

(A) $t^{\frac{3}{2}}$ (B) $t^{\frac{2}{3}}$ (C) $t^{\frac{1}{4}}$ (D) $t^{\frac{1}{2}}$

ANSWERS

1. (B)	2. (C)	3. (B)	4. (A)	5. (B)	6. (B)
7. (D)	8. (C)	9. (C)	10. (D)	11. (A)	12. (A)
13. (A)	14. (D)	15. (B)	16. (D)	17. (C)	18. (A)
19. (A)	20. (A)				

Answer the following questions as briefly as possible :

1. What is the work done, on a body performing uniform circular motion by the the centripetal force ?
2. What does area under F – x graph give ?
3. How many joule equals 1 eV ?
4. Two bodies with unequal mass, have equal momentum which one has larger value of kinetic energy ?

5. A body is thrown vertically upwards with initial velovity 7 m/s. At how much height will its kinetic energy be half ?
6. What is the sum of K.E. and P.E. ?
7. Give dimesional formula of spring constant.
8. How many horsepower make 1 W ?
9. If momentum of a body is doubled, what will be the increase in percentage of its kinetic energy ?
10. What is meant by a non–conservative force ?
11. Define elastic collision.
12. What is necessary for work to be done, when a force is acting ?
13. Give expression of momentum in terms of mass and kinetic energy.
14. In which circumstances force and displacement aren't in same direction ?
15. State the work energy theorem.

Answer the following question briefly

1. Discuss the factors on which work done, depends, and hence define work.
2. Explain work done by a variable force, on a particle.
3. State and prove the work energy theorem.
4. What is elastic potential energy ? Explain with the example of spring and the required equation.
5. For conservative force prove that $F = -\frac{dU}{dx}$
6. Discuss elastic collision between two bodies moving along x–axis with necessary equations.
7. Discuss elastic collision in two dimensions.

Solve the following problems :

1. A block of mass 2 kg is tied at one end of a light inextensible string. At the other end of the string another block of mass 1 kg is tied. Initially the block of mass 1 kg is on the ground and the system of these blocks is stationary as shown in Fig. 6.17. Now the masses are released. Find the common velocity of the blocks when the mass of 2 kg touches the ground. The pulley is smooth and weightless. The initial height of block of 2 kg from the ground is 3 m. (g = 9.8 m/s^2)
[**Ans. :** 4.43 m/s]

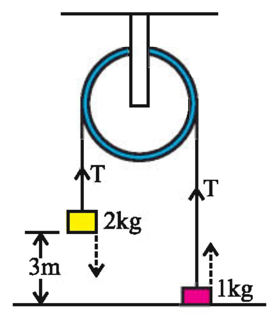

Figure 6.17

2. A particles of mass m, moving with velocity $\vec{v}_1$ collides elastically (in two dimensions) with another particle of mass m at rest. If $\vec{v}_1{}'$ and $\vec{v}_2{}'$ are the velocities of the particles after the collision, show that these velocities are perpendicular to each other.
3. A sphere of steel of mass 15 kg moving with a velocity of 12 m/s, along X–axis collides with a stationary sphere of mass 20 kg. If velocity of the first sphere after the collision is 8 m/s and is moving at an angle of 45° with X–axis, find the magnitude and direction of the second sphere after the collision.
[**Ans. :** 6.37 m s^{-1}, 41°44']
4. The relation between position x and time t for a particle, performing one dimensional motion, is $t = \sqrt{x} + 3$, Here x is in metre and t is in second.
(1) Find the displacement of the particle when its velocity becomes zero.
(2) If a constant force acts on the particle, find the work done in first 6 seconds. [**Ans. :** (1) –9 m, (2) 0 J]

5. As shown in Fig. 6.18, two spheres at rest are released from point A and one of them moves over path AB while the other moves over path AC. Will these spheres reach the ground simultaneously ? Calculate the speed of these spheres at the bottom and the time taken by them to reach the bottom. Take $\theta_1 = 30^o$, $\theta_2 = 60^o$ and $h = 10$ m. $g = 10$ m/s^2.

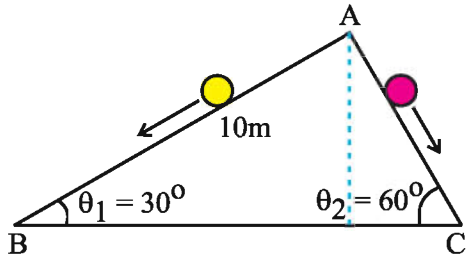

Figure 6.18

[Ans. : No, $10\sqrt{2}$ m/s, $2\sqrt{2}$ s, $\frac{2\sqrt{2}}{\sqrt{3}}$ s]

6. A spring of force constant k is kept in a compressed condition between two blocks of masses m and M on the smooth surface of a table as shown in Fig. 6.19. When the spring is released, both the blocks move in opposite directions. When the spring attains its original (normal) position, both the blocks lose the contacts with spring. If x is the initial compression of the spring find the speeds of block while getting detached from the spring.

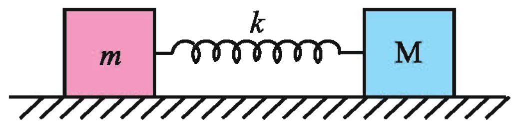

Figure 6.19

[Ans. : Velocity of the block of mass m is $\sqrt{\frac{kM}{m(M+m)}} \cdot x$;

Velocity of the block of mass M is $\sqrt{\frac{km}{M(M+m)}} \cdot x$]

7. Two beads of masses m_1 and m_2 are threaded on a smooth circular loop, of wire, of radius R. Initially both the beads are in positions A and B in a vertical plane. Now, the bead A is pushed slightly so that it slides on the wire and collides with B. If B rises to the height of the centre of the loop (O) on the wire and A becomes stationary after the collision, prove that $m_1 : m_2 = 1 : \sqrt{2}$

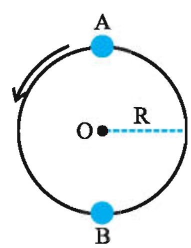

Figure 6.20

8. For the illustration 11 calculate speed of the sphere at the lowest position during its motion from (i) A to B (ii) B to C. Take g = 10 m/s^2.

[Ans. : $\sqrt{14}$ m/s and $\sqrt{6}$ m/s]

9. A bullet is fired in to a huge wooden block. The bullet while moving inside the block loses half the velocity when it travels 6 cm inside the block. How far, then, would it go inside the block ? Resistive force is constant.

[Ans. : 2 cm]

10. Solve sum-9 of exercises of chapter-5 using work-energy theorem.

[Ans. : $2tan\theta$]

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CHAPTER 7

HEAT TRANSFER

7.1 Introduction

Dear students, we have studied the fundamental concepts of heat earlier. When two bodies with different temperature are brought in contact, heat flows from the body with higher temperature to the body at lower temperature. But in the same solid, how does heat flow from the part at higher temperature to the part at lower temperature ? How does a small fraction of the enormous heat energy produced in the sun reach the earth ? Using solar energy we can cook 'dal' and 'rice'in a solar cooker, then why couldn't Birbal cook (intentionally) his 'khichdi'? Why does a hot substance, when kept open, cools down ? Perhaps, you would be able to answer such questions by the time you complete this chapter.

7.2 Thermal Conduction

The flow of energy between the adjacent parts of a body due to temperature difference between them is called **thermal conduction or heat conduction.** The constituent particles (atoms, molecules, or ions) in solids vibrate about their mean positions with an amplitude depending on their temperature. As the temperature rises the amplitude of their vibration increases. Thus when heat is given to a solid it causes an increase in kinetic energy of vibrational motion. Special type of forces (intermolecular forces) acting between the molecules of a solid are responsible for transfering the effect of this increased vibrational kinetic energy to neighbouring particles. So now the amplitude of 'neighbours' also increases. Thus, heat transfer takes place in solids. Heat transfer taking place in this way is known as thermal conduction.

In metals free electrons also play an important role in heat transfer.

Consider a solid–slab of uniform cross–section, as shown in Fig. 7.1. Suppose temperatures of two cross sections ABCD and EFGH, at distance x and $x + \Delta x$, from one end of the solid, are $T + \Delta T$ and T respectively. Thus for the seperation Δx the temperature difference is ΔT.

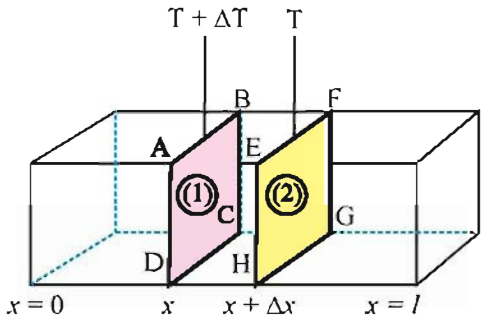


Figure 7.1

$\frac{\Delta T}{\Delta x}$ is known as **temperature gradient.** For small value of Δx and ΔT the heat ΔQ flowing between the two cross–section, perpendicular to the cross–sections in time Δt is directly proportional to time Δt, temperature gradient $\frac{\Delta T}{\Delta x}$ and cross–section area A. Thus,

$$\Delta Q \propto A \frac{\Delta T}{\Delta x} \Delta t$$

$$\therefore \ \Delta Q = -kA \frac{\Delta T}{\Delta x} \Delta t$$

$$\therefore \ \frac{\Delta Q}{\Delta t} = -kA \frac{\Delta T}{\Delta x} \quad (7.2.1)$$

Here k is constant of proportionality. It is called thermal conductivity of a given substance. Its value depends on the type of the material and upto some extent on the temperature. Good thermal conductors have high values of thermal conductivity. Under ordinary circumstances if the temperature difference between various parts of the object is not very high, thermal conductivity can be considered to be constant.

Aren't you perturbed about the negative sign appearing in the equation above ? It may puzzle you but it is necessary because as x increases T decreases and so $\frac{\Delta T}{\Delta x}$ is negative but since $\frac{\Delta Q}{\Delta t}$ is positive, negative sign is kept in the equations above.

If distance between two adjacement layer is way small the value of Δt would also be very small hence. Taking $\Delta x \to 0$ and $\Delta t \to 0$ in equation (7.2.1) it can be written as

$$\frac{dQ}{dt} = -kA \frac{dT}{dx} \quad (7.2.2)$$

$$\therefore \ H = -kA \frac{dT}{dx} \quad (7.2.3)$$

Here $\frac{dQ}{dt}$ denoted by H is known as heat current. Heat current is the heat flowing perpendicularly through to any cross-section in unit time. In equation (7.2.1) if $A = 1m^2$ and $\frac{dT}{dx} = -1 \ Km^{-1}$, $\frac{dQ}{dt} = k$.

Thus, **the amount of heat flowing per unit time perpendicularly between the planes having unit temperature gradient between them per unit area is defined to be thermal conductivity.**

Unit of thermal conductivity is Cal s^{-1} m^{-1} K^{-1} or Wm^{-1} K^{-1}

Thermal Conduction in a Bar :

In Figure 7.2 a bar having thermally insulated sides (this means exchange of heat is possible at ends not on sides) is shown. Length of the bar is L and its cross section is A. Temperatures of red is T_1 and T_2. At $t = 0$ a heat source of temperature T_1 is placed at the end having $x = 0$.

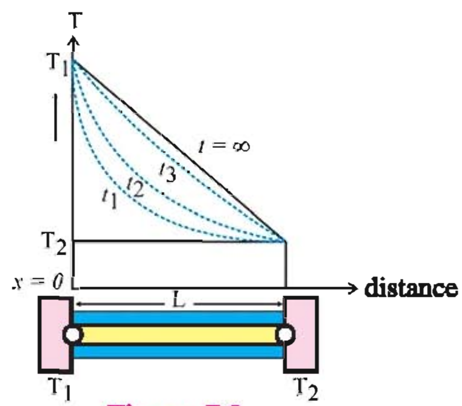


Figure 7.2

Gradually due to thermal conduction the temperature of each portion starts rising. Variation in temperature of each part of the bar with time is shown in the graphs in Fig. 7.2.

After a sufficiently long time (theoretically $t = \infty$) the temperatures of all parts of the rod become steady. These steady temperatures decrease along the length of the rod from its hot end to its cold end. In this situation, the amount of heat energy received by the hot end in some time interval is equal to the amount of heat lost by the cold end in the same time interval. The sides of the rod are thermally insulated, hence the rod does not lose any heat through its sides. So every part of the rod passes the same amount of heat towards the neighbouring colder part, as what it receives from the neighbouring hotter part. Hence, any cross–section of the rod, along its entire length, has the same value of heat current $\frac{dQ}{dt}$. Further, along the entire length of the rod the value of the temperature gradient $\frac{dT}{dx}$ is also the same along the length. Now both $\frac{dQ}{dt}$ and $\frac{dT}{dx}$ remain constant with time. This condition of the rod is called **"thermal steady state"** of the rod.

In the thermal steady state, the temperatures of the two ends of the rod are T_1 and T_2 with $T_1 > T_2$. As $\frac{dT}{dx}$ is same all along the length of the rod,

$$\frac{dT}{dx} = -\left[\frac{T_1 - T_2}{L}\right] \qquad (7.2.4)$$

Eqn. (7.2.1) now takes the form :

$$\frac{dQ}{dt} = kA\left[\frac{T_1 - T_2}{L}\right] \qquad (7.2.5)$$

As $\frac{dQ}{dt}$ is constant, we write $\frac{Q}{t}$ for $\frac{dQ}{dt}$

$$\therefore \frac{Q}{t} = kA\left[\frac{T_1 - T_2}{L}\right]$$

$$\therefore Q = kA\left[\frac{T_1 - T_2}{L}\right]t \qquad (7.2.6)$$

Equation (7.2.6) gives the amount of heat flowing through the rod in a steady thermal state, in time t.

Table : 13.1

Thermal conductivity of some substances (only for informatin)

Substance	Thermal conduction W m^{-1} K^{-1}
Silver	406
Copper	385
Aluminum	205
Brass	109
Iron	50.2
Lead	34.7
Mercury	8.3
Glass	0.8
Water	0.8
Wood	0.12–0.04
Body fat	0.2
Hydrogen gas	0.14
Air	0.024

The information in the table above shows that most of the metals are good conductors of heat. Metals are also good conductors of electricity. Free electrons, present in metals, are responsible for conductivity of both the types in metals.

Thermal Resistance

From equation (7.2.5)

$$H = kA\left[\frac{T_1 - T_2}{L}\right]$$

$$\therefore H = \left[\frac{T_1 - T_2}{L / KA}\right]$$

Comparing this equation with equation $I = \frac{V}{R}$ for electric current we find that H and I are heat current and electric current respectively. V is electric potential differences where as $T_1 - T_2$ is temperature difference. This indicates that L/kA is thermal resistance. Thus, thermal resistance is given by.

$R_H = L / kA$

Unit of thermal resistance (R_H) is kelvin/watt and its dimensional formula is $M^{-1}L^{-2}T^3K$.

Formula for effective thermal resistances when thermal conductors are connected in series or parallel are similar to those for electrical resistances. Thus, for series connection

$(R_H)_s = (R_H)_1 + (R_H)_2$

and for parallel connection

$$\frac{1}{(R_H)_P} = \frac{1}{(R_H)_1} + \frac{1}{(R_H)_2}$$

(verify yourself)

Here $(R_H)_s$ and $(R_H)_p$ are effective thermal resistances for series and parallel connections.

One point may be added to the above discussion that, as electrical potential difference is necessary for electric current, temperature difference is necessary for heat current.

Only for information :

Note : If you refer some other books, thermal resistance is also defined as $R = \frac{l}{k}$ for industrial purpose, which is known as R–value. R–value is used to indicate thermal resistance of building material.

Illustration 1 : A 25 cm long rod of a cross–section of 1.5 cm^2 has one of its end in thermal contact with steam at 100 °C and the other end is in thermal contact with ice at 0 °C in its thermal steady state. Find (1) the temperature gradient along the rod (2) the rate of heat conduction and (3) the temperature at a point 18 cm away from the hot end. (Thermal conductivity of the material of the rod is $k = 0.9$ cal s^{-1} cm^{-1} $°C^{-1}$)

Solution :

$A = 1.5\ cm^2 \quad T_1 = 100\ °C \quad T_2 = 0\ °C$

$L = 25\ cm \quad \frac{dT}{dx} = ? \quad \frac{dQ}{dt} = ?$

(1) Temperature gradient,

$$\frac{dT}{dx} = -\left[\frac{T_1 - T_2}{L}\right]$$

$$= -\left[\frac{100-0}{25}\right]$$

$$= -4\ °C\ cm^{-1}$$

(2) Rate of heat conduction

$$\frac{dQ}{dt} = kA\left[\frac{T_1 - T_2}{L}\right]$$

$$= 0.9 \times 1.5 \times \left[\frac{100-0}{25}\right]$$

$$\therefore \frac{dQ}{dt} = 5.4\ cal\ s^{-1}$$

(3) Suppose T_l is the temperature at a point $l = 18$ cm away from the hot end. In the thermal steady state, as $\frac{dT}{dx}$ remains the same all along the length of the bar, the temperature at a distance l from the end with temperature T_1 is

$$T_l = T_1 + \left(\frac{dT}{dx}\right)l$$

$$= 100 - 4 \times 18 = 28\ °C$$

OR

The decrease in temperature per centimeter distance from the hot end is 4 °C from (I) above. Therefore, the decrease in temperature at a distance of 18 cm is 72 °C.

$\therefore$ Required temperature = 100 − 72 = 28 °C

Illustration 2 : A compound slab consists of two slabs of thickness L_1 and L_2 with thermal conductivities k_1 and k_2 respectively and possess equal area of cross−section ($A_1 = A_2 = A$). If the temperature of the two ends of the compound slab are T_1 and T_2 and temperatures at their contact surface is T_x, show that in the thermal steady state,

$$T_x = \frac{\frac{L_2 T_1}{k_2} + \frac{L_1 T_2}{k_1}}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}$$ and heat current

$$\frac{dQ}{dt} = \frac{A(T_1 - T_2)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}$$

(Neglect the loss of heat)

Solution :

Thermal resistance of block (1)

$$R_1 = \frac{L_1}{k_1 A}$$ and

thermal resistance of block (2)

$$R_2 = \frac{L_2}{k_2 A}$$

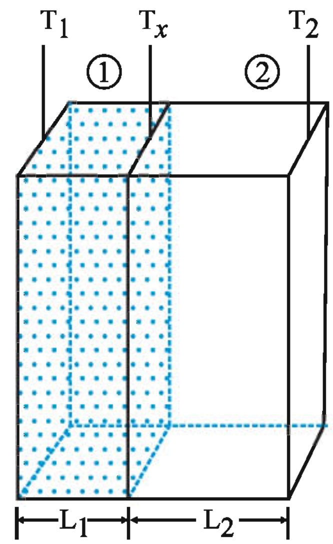


Figure 7.3

The blocks are in series,

therefore the total thermal resistance is

$R = R_1 + R_2$

$$\therefore \frac{dQ}{dt} = \frac{T_1 - T_2}{R}$$

$$= \frac{T_1 - T_2}{R_1 + R_2}$$

$$\therefore \frac{dQ}{dt} = \frac{T_1 - T_2}{\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A}}$$

$$= \frac{A(T_1 - T_2)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}$$

The temperature at contact surface,

$$T_x = T_1 - \frac{dQ}{dt} \times R_1$$

$$= T_1 - \frac{(T_1 - T_2)}{(R_1 + R_2)} R_1$$

$$= \frac{T_1R_2 + T_2R_1}{R_1 + R_2}$$

$$= \frac{\frac{L_2T_1}{k_2A} + \frac{L_1T_2}{k_1A}}{\frac{L_1}{k_1A} + \frac{L_2}{k_2A}}$$

$$\therefore T_x = \frac{\frac{L_2T_1}{k_2} + \frac{L_1T_2}{k_1}}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}$$

Illustration 3 : A spherical thermocol container contains 5 kg ice. Thickness of its wall in 23.14 cm. Inner radius of the box in 20 cm. If, 335 k J heat energy in required to melt 1 kg of ice. How much ice will melt in a day ? Temperature outside the box in 30 °C. Thermal conductivity of thermocol in 0.0275 SI unit. Consider wall of container in thermal steady state.

For a spherical shell, in theermal steady state, heat current is given by :

$$\frac{Q}{t} = \frac{4\pi k r_1 r_2 (T_1 - T_2)}{r_1 - r_2}$$

Here, T_1 and T_2 are temperatures of outter and inner surface repectively. r_1 and r_2 are outter and inner radies respectively.

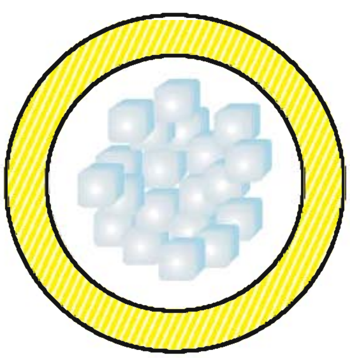


Figure 7.4

Solution :

Suppose m kg ice melts in a day. As for melting 1 kg 335×10^3 J heat energy is required.

Total heat energy required

$Q = m \times 335 \times 10^3$ (1)

Now,

$$\frac{Q}{t} = \frac{4\pi k r_1 r_2 (T_1 - T_2)}{r_1 - r_2}$$

$$\therefore \frac{m \times 335 \times 10^3}{24 \times 3600} =$$

$$\frac{4 \times 3.14 \times 0.0275 \times 20 \times 10^{-2} \times 23.14 \times 10^{-2} \times (30-0)}{3{\cdot}14 \times 10^{-2}}$$

$$\therefore m = \frac{4 \times 0.0275 \times 20 \times 10^{-2} \times 23.14 \times 30 \times 24 \times 3600}{335 \times 10^3}$$

$= 3.939$ kg.

Note : For a cylinderical shell of outter and inner radii r_1 and r_2 respectively and temperatures of outter and inner surfaces T_1 and T_2 respectively, if in a thermal steady state heat current in given by

$$\frac{Q}{t} = \frac{2\pi k L (T_1 - T_2)}{ln\ r_1 - ln\ r_2},$$

L is the length.

Illustration 4 : Two rods, one of iron and another of aluminium, of equal length and equal cross–sections are connected with each other. The free end of the iron rod is kept at 100 °C and the free end of the aluminium rod is kept at 0 °C. If thermal conductivity of aluminium is four times that of iron, find the temperature of their contact surface in the thermal steady state.

Solution : Suppose the length of both the rods is L and area of cross–section is A.

Let thermal conductivity of iron be k.

∴ Thermal conductivity of aluminium will be 4k.

Suppose temperature at the surface of contact is T_x.

In thermal steady state of the compound rod,

$$\left(\frac{dQ}{dt}\right)_{Iron} = \left(\frac{dQ}{dt}\right)_{Aluminium}$$

$$\therefore \frac{kA(100 - T_x)}{L} = \frac{4kA(T_x - 0)}{L}$$

$\therefore 100 - T_x = 4T_x$

$\therefore\ 100 = 5T_x$

$\therefore\ T_x = 20\ °C$

Illustration 5 : The cross-sectional area of a rod is 12.56 cm^2. One end of this rod is placed in a boiler. Thermometers separated by 13 cm on the rod indicate 56 °C and 45 °C temperatures respectively. At the other end of the rod a copper tube is wound around it and temperature difference of water entering the tube and flowing out of the tube is 30 °C. 800 *g* of water flows through the tube in 3 minutes. Find the thermal conductivity of the material of the rod.

(Specific heat of water = 1 cal g^{-1} °C^{-1})

Solution :

A = 12.56 cm^2	m = 800 g
L = 13 cm	$\theta_1 - \theta_2 = 30$ °C
T_1 = 56 °C	t = 3 min = 180 s
T_2 = 45 °C	$\therefore\ T_1 - T_2 = 11$ °C

Since $Q = mc\Delta\theta$ and $Q = \dfrac{kA(T_1 - T_2)t}{L}$

$$k = \frac{mc(\theta_1 - \theta_2)L}{A(T_1 - T_2)t}$$

$$= \frac{800 \times 1 \times 30 \times 13}{12.56 \times 11 \times 180}$$

$$= 12.54 \text{ cal s}^{-1} \text{ cm}^{-1}\ °\text{C}^{-1}$$

Illustration 6 : A metal rod of 1 m length is in a thermal steady state at atmospheric pressure. Its one end is placed in water at 100 °C and the other end is placed in ice at 0 °C. At what distance, from its hot end, a flame of temperature 2000 °C should be placed so that the same amount of water that evaporates at the hot end is the same as the amount of ice that melts at the cold end in unit time ? The latent heat of evaporation of water is 540 cal g^{-1} and the latent heat of melting of ice is 80 cal g^{-1}.

[**Hint :** If temperature charges, heat exchanged in $\Delta Q = mc\ \Delta T$, c is sp. heat. If there is a change in physical state at constant temperature heat exchanged is $\Delta Q = mL$ whtere L is latent hent.]

Solution : Let the flame be placed at x cm from the hot end.

Also assume that in 1 second m gm of water evaporates at the hot end and the same amount of ice melts at the cold end.

$$\therefore\ m(540) = kA\left[\frac{2000 - 100}{x}\right]$$

$$= \frac{1900kA}{x} \qquad (1)$$

$$\text{and } m(80) = kA\left[\frac{2000 - 0}{100 - x}\right]$$

$$= \frac{2000kA}{100 - x} \qquad (2)$$

Dividing (1) by (2), we get,

$$\frac{540}{80} = \frac{1900(100 - x)}{2000(x)}$$

$$\therefore\ \frac{27}{4} = \frac{19}{20}\left(\frac{100 - x}{x}\right)$$

$\therefore\ 540x = 7600 - 76x$

$\therefore\ 616x = 7600$

$\therefore\ x = 12.33$ cm

7.3 Convection

In the phenomenon of thermal conduction the constituent particles of solids vibrate about their mean positions and thermal conduction takes palce with the help of inter-molecular forces. In thermal convection the constituent particles actually move from one place to the other. This fact indicates that convection is observed only in gases and liquids. i.e. in fluids and not in solids. It is also true that in fluids thermal conduction plays a very small part in heat transfer.

In convection, fluid at the bottom becomes hot and so its volume increases, due to which its density decreages. So due to buoyant force this 'lighter' fluid moves upwards and due to gravitational effect the fluid with greater density is transferred from the top to the bottom. Due to such a continuous process the fluid is heated. You can buy potassium permanganate from a chemist and add it to water while heating it in a flask to observe this phenomenon.

Convection can be natural as well as forced. If the motion of the material is only due to the difference in density it is natural convection. To understand this let us examine the phenomenon of 'cool current' near a sea-shore. The soil becomes hot due to sun-rays and so the air in contact with this soil also gets heated and its volume increases and its density decreage. As a result of this, this air moves up. So the air pressure near the soil reduces due to which the cool air from sea-side rushes towards the soil. Thus, cool-currents of air are formed. What happens during night ? (Think yourself)

In forced convection a fluid is forced to move using some appliance like, a pump, or a stirrer. In human body heart (of the size of a fist) acts as a pump to keep blood circulation going on. Due to this blood circulation body temperature is maintained.

The volume of water increases instead of decreasing when the temperature of water is reduced from 4 °C to 0 °C. This is called anomalous expansion of water. The existence of fish and other aquatic animals is possible because of natural convection and anomalous expansion of water. In winter, as temperature of the atmosphere decreases, water at the surface cools down, becomes densel and moves downwards. Less cool water at the bottom moves to the surface and becomes cooler. In this way the temperature of the whole quantity of water decreases to 4 °C through the process of convection. Now as water at 4 °C on the surface cools down further, its volume increases instead of decreasing due to which its density decreases. Hence such water remains on the surface while cooling further and at 0 °C temperature, it transforms into ice. After the formation of ice, the water below the ice loses heat energy through heat conduction. The thermal conductivity of ice is very low and so the process of cooling becomes very slow. The temperature of water at the bottom is therefore maintained at 4 °C for a very long time. In normal conditions, after such a long time interval, the temperature of the atmosphere starts increasing and hence aquatic animals survive.

7.4 Radiation

In thermal conduction and convection particles of the medium play a very active role. Most of the region between the sun and the earth consist of vacuum only. How does the solar energy then reach the earth ? During winter even if we stand away from bonfire, we feel its heat. The third type as well as of heat transfer is responsible for making the solar heat reach the earth the heat of bonfire to us. This type of heat transfer is known as thermal radiation.

Every substance emits electromagnetic radiation of certain frequencies in accordance with its temperature. This radiation is known as thermal radiation. Thermal radiations are electromagnetic radiations only. The energy associated with these electromagnetic radiations is called scientist radiant-energy.

According to **Prevost** each substance continuously emits thermal radiations at any temperature. With increase in temperature the rate of emission of thermal radiation also increases. Also the same substance absorbs other radiations incident on it. If the rate of absorption of thermal radiation is more, than the rate of emission of thermal radiation the temperature of the substance increases and if the rate of absorption of the thermal radiation is less then the rate of emission of thermal radiation, temperature of the substance decreases. When the temperature of a substance is the same as that of the environment in which it is kept, rates of absorption and emission of thermal radiation are equal.

The proportion of various frequencies of electromagnetic radiation in thermal radiation depends on the type of material and the temperature of the material. For example in a candle-flame or flame of a bunsen burner the temperature of the inner part is less so, it is yellow in colour. As we go out temperature in the flame increases so the colour of the outermost part is blue or violet.

7.5 Perfect Black Body and Black Body Radiation

The body which absorbs all the radiant energy incident on it is called a perfect black body. It is impossible to get 100 % perfect black body in nature on the earth. The substance very close to a perfect black body available in daily life is lamp black or soot. It absorbs about 98 % of radiant energy incident on it. In this sense it is 98 % perfect black body and silver is 2 % perfect black body. Remember the black colour has hardly anything to do with a black body.

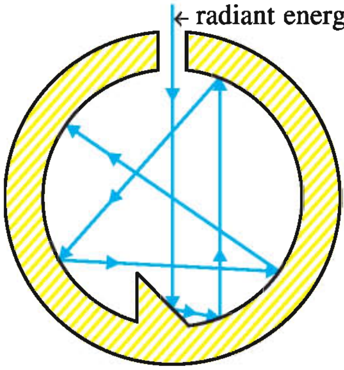


Figure 7.5

To study black body radiations consider a spherical cavity as shown in the diagram. Its inner surface is blackened and rough. It has a small (compared to the dimension of the cavity) hole. A radiation entering this cavity through this entarance hole undergoes many reflections and each time it is partly absorbed and partly reflected. When it reaches the hole again it is almost left with no energy. Also the portion of cavity just opposite to hole is such that radiation can not be reflected back immediately after it enters the cavity in the opposite direction to come out. In this sense this pin hole on the cavity can be considered a perfect black body. If such a cavity is uniformly heated, radiations coming out of it can be considered to be black body radiation. These radiations are also called cavity radiations.

Solar radiations contain radiations of all wave langths and so the Sun can be considered to be a perfect black body. Its temperature is about 5800 K. Cavity radiations from a cavity kept at the same temperature would be similar to solar radiations.

So the Sun behaves like a perfect black body at 5800 K temperature. Now it should be clear that the black colour doesn't have much relation with a black body.

Properties of thermal radiation emitted from any substance depend on the type of the substance and its temperature, whereas black body radiation depends only on the temperature. In this sense the black body radiation possess universal properties. This fact indicates the importance of the study of cavity radiations.

7.6 Kirchoff's Law

Two spheres A and B of the same material and of equal surface area are suspended in a room. Suppose surface of A is polished and that of B is black. Equal amount of radiant energy is incident on them. It is clear that sphere A reflects most of the radiations while B absorbs most of it. But as their temperatures remain same (same as the temperature of the room) the rate of emission of B must be more than rate of emission of A. Thus we can conclude that the surface which is a good absorber must also be a good emitter. This fact is also stated by Kirchoff's law. But let us first clearly define some terms.

Absorptivity : On irradiating a surface, the ratio of the radiant energy absorbed to the amount of radiant energy incident on the surface is called absorptivity (a) of that surface at a given temperature.

$$\therefore\ a = \frac{\text{radiant energy absorbed}}{\text{incident radiant energy}}$$

For a completely black body $a = 1$.

Total emissive power : The amount of radiant energy emitted per unit area per second from a surface at a given temperature for all possible wavelengths is called the total emissive power (W) of serface of that temperature.

Radiations of all frequencies are included in the definition of total emissive power.

Spectral emissive power : In the definition of total emissive power, radiations of all frequencies are considered. We can also define emissive power related to a particular frequency. Emissive power defined in this manner is called the spectral emissive power (W_f) at frequency f.

"At a given temperature, the amount of radiant energy emitted per second per unit surface area, in a unit frequency interval about a given frequency (f) is called spectral emissive power (W_f) of the given surface at the given temperature, corresponding to that frequency."

If we choose to use wavelength λ instead of frequency f, we should write W_λ instead of W_f. Here λ is the wavelength corresponding to frequency f.

Further it is clear that the sum of the spectral emissive powers for all the frequencies gives the total emissive power.

$$\therefore\ W = \sum_f W_f = \sum_\lambda W_\lambda$$

The magnitude of W_f depends on the temperature, material of the surface and frequency *f*.

Emissivity : The ratio of the total emissive power of a surface to the total emissive power of the surface of a perfectly black body kept under the same conditions is called emissivity (*e*) of that surface.

For the surface of a completely black body, $e = 1$.

Kirchhoff's law : "The values of emissivity and absorptivity are equal for any surface."

$\therefore a = e$

Thus, from this law it is clear that, the surface which is a good absorber is also a good emitter and the surface which is a good reflector (i.e. a poor absorber) is also a poor emitter. Now, you can understand why the surface of a glass bottle of a thermo–flask is kept shining (like a mirror).

7.7 Wien's Displacement Law

The thermal radiation emitted by a body consists of electromagnetic waves of different wavelengths (frequencies) and the wavelengths of these waves form a continuous spectrum, but the number of electromagnetic waves of certain definite frequencies is more. For instance, in the radiation emitted by a black body at room temperature (300 K) majority of the radiation consists of electromagnetic waves of wavelength 9,550 Å. (called infrared waves). On increasing its temperature, the number of waves of smaller wavelengths increases. At about 1100 K, as the waves of wavelength corresponding to the red colour are more, the body appears red. See the graphs of spectral emissive power W_λ versus wavelengths at different temperatures to know the relative intensity of different wavelengths in the radiation emitted by a black body. It can be seen from the graphs that with the increase in temperature, the wavelength (λ_m) corresponding to the maximum value of W_λ decreases. A physicist named Wien showed that this wavelength λ_m is inversely proportional to the absolute temperature of the emitting surface.

$$\lambda_m T = \text{constant} \qquad (7.6.1)$$

This equation represents mathematic form of Wien's displacement law.

The constant in the equation is called Wien's constant and its value is 2.9×10^{-3} m–K.

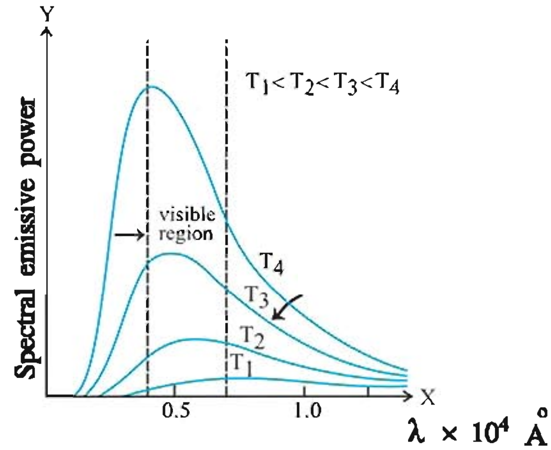


Figure 7.6

7.8 Stefan – Botzmann's Law

In 1879 AD a scientist named Stefan showed experimentally and in 1884 AD Boltzmann proved theoretically that **"the amount of radiant energy emitted by a surface per unit area in unit time (i.e. total emissive power) is directly proportional to the fourth power of its absolute temperature."** This statement is known as **Stefan – Boltzmann law.**

$$\therefore W = \sigma e T^4 \qquad (7.8.1)$$

Here T is the absolute temperature *e* is the emissivity of the surface and σ is the Stefan–Boltzmann constant. Its value is $\sigma = 5.67 \times 10^{-8}$ Wm^{-2} K^{-4}.

If the body of temperature T is kept in an environment with temperature T_s ($T > T_s$), using equation (7.8.1) it can be proved that the net rate at which the body loses heat is given by

$$\frac{dQ}{dt} = e\sigma A\left(T^4 - T_s^4\right) \qquad (7.8.2)$$

where A is area of the surface. (How can you get this result from equation ? (7.8.1) Think.

7.9 Newton's Law of Cooling

If the temperature of a body is T °C and that of its surrounding is T_s °C and $T > T_s$ then it continuously loses heat energy and its temperature goes on decreasing with time. The law presented by Newton for explaining how much the temperature of a body decreases with time through forced convection is known as Newton's law of cooling.

"The rate of loss of heat by a body and hence the rate of decrease of its temperature (i.e. the rate of cooling of a body) is directly proportional to the difference of temperatures of the body and its surrounding."

We know that the amount of heat required to change the temperature of a body of mass m and specific heat c, by ΔT is,

$\Delta Q = mc\Delta T$

Therefore, the rate of loss of heat,

$$\frac{dQ}{dt} = -mc\frac{dT}{dt} \qquad (7.9.1)$$

According to Newton's law, the rate of loss of heat by a body depends on the difference of temperature $(T - T_s)$ between the body and its surrounding.

$$\therefore \frac{dQ}{dt} = -mc\frac{dT}{dt} \propto (T - T_s) \qquad (7.9.2)$$

$$\therefore \frac{dT}{dt} = -k'(T - T_s) \qquad (7.9.3)$$

Here $\frac{dT}{dt}$ is the rate of decrease in the temperature of a body at temperature T. Equation. (7.9.3) represents the Newton's law of cooling. The constant k' depends on the mass and the specific heat of the cooling body. Here, the negative sign indicates that the temperature of the body decreses with time, as it loses heat energy.

Note that Newton's law of cooling is true only for small intervals of difference of temperature between the body and its surrounding. If the amount of heat lost by the body due to radiation is very small, this law holds true for a large interval of temperature also. Moreover, the law can be used only when a body is cooling due to forced convection.

For natural convection, the law of cooling given by Langmuir–Lorentz is as under.

$$-\frac{dT}{dt} \propto (T - T_s)^{\frac{5}{4}} \qquad (7.9.4)$$

Illustration 7 : A body at 80 °C cools down to 64 °C in 5 minutes; and in 10 minutes it cools down to 52 °C. What will be its temperature after 20 minutes ? What is the temperature of the environment ?

Solution : For the first 5 minute

$\Delta T = T_2 - T_1 = 64 - 80 = -16$ and $\Delta t = 5$

$$\therefore \frac{+16}{5} = +k'\left(\frac{80 + 64}{2} - T_s\right) \qquad (1)$$

Here we have taken the average of initial and final temperatures as the temperature of the body.

Similarly for the next 5 minutes,

$\Delta T = 52 - 64 = -12$

$$\therefore \frac{12}{5} = k'\left(\frac{52 + 64}{2} - T_s\right) \qquad (2)$$

Dividing equ. (1) by eqn. (2), we get,

$$\frac{16}{5} \times \frac{5}{12} = \frac{72 - T_s}{58 - T_s}$$

$$\therefore \frac{4}{3} = \frac{72 - T_s}{58 - T_s}$$

$\therefore 232 = 4T_s = 216 - 3T_s$

$\therefore 232 - 216 = T_s$

$\therefore T_s = 16$ °C

Substituting this value of T_s in eqn. (1),

$$\frac{16}{5} = k'(72 - 16)$$

$$= k'(56)$$

$$\therefore k' = \frac{16}{5 \times 56} = \frac{2}{35}$$

Now, for the third stage $\Delta t = 10$ minute

$\Delta T = T - 52$, where T is final temperature.

$$\therefore \frac{52 - T}{10} = \frac{2}{35}\left(\frac{52 + T}{2} - 16\right)$$

$$\therefore 52 - T = \frac{4}{7}\left(\frac{52 + T - 32}{2}\right)$$

$$\therefore 52 - T = \frac{2}{7}(20 + T)$$

$\therefore 364 - 7T = 40 + 2T$

$\therefore 364 - 40 = 9T$

$$\therefore T = \frac{324}{9} = 36 \text{ °C}$$

7.10 Green House Effect

A green house is a structure used for proper and rapid growth of saplings. It consists of transparent walls and roof made from a material like plastic or glass. Solar radiations entering through these walls and roof are absorbed by plants and the soil in the structure. This energy absorbed by plants and soil is re-emitted in form of infrared radiation (wavelength 8000A° to 20,000A°). Walls and roof of a green house partly allow the outward passage of this infrared radiations. So a major part of the infrared radiation is trapped in the air inside the green house. And thus 'warmth' is produced air inside the green house.

From this perspective our earth and the atmosphere around it, behaves like a green house. Solar radiations contain all wave lengths. Our atmosphere allows visible light to pass through and the infrared radiations are absorbed in the atmosphere. During the day the surface of the earth and other objects become hot and later they emit infrared radiations. This infrared radiation cannot penetrate through the atmosphere. Molecules of air, like molecules of CO_2 and H_2O, absorb there radiations and re-emit them. Some part of it comes down on earth's surface and thus, some part of heat energy is trapped in earth's atmosphere and as a result its temperature is maintained. This phenomenon is known as the green house effect. Infrared rays are responsible for the effect of 'warmth' and so, they are called heat-rays. This is the reason behind 'warmth' felt even during night. Some pollutant gases add to the green house effect. If the green house effect would not have been there, average temperature of lower part of earth's atmosphere would have been very low and there would have been very large difference between the temperatures of day and night. (Would life ever have been possible ?) So, like other things, green house effect should also be in proper proportion ! As green house effect is increasing, due to pollutants, glaciers are melting and so there is rise in sea level. This would lead to reduction in the land meant for terrestrial life. So steps should be taken to reduce pollutants.

॥ अति सर्वत्र वर्जयेत ॥

SUMMARY

1. Heat transfer takes place in three ways :
   (1) Thermal Conduction (2) Convection (3) Radiation
2. Thermal conduction is usually seen in solids. Here, heat transfer takes place due to the difference in temperature between two adjacent parts. If temperatures of the portions at $x = 0$ and $x = x + \Delta x$ distance from one end are $T + \Delta T$ and T, then heat current (H) is given by $H = \frac{\Delta Q}{\Delta t} = -kA\frac{\Delta T}{\Delta x}$

   Here A in the area of cross section and k is thermal conductivity.
3. $\frac{\Delta T}{\Delta x}$ is called temperature gradient.
4. Thermal conductivity of a substance depends on the type of substance and upto certain extent on the temperature also. Its unit is $W\ m^{-1}\ k^{-1}$.
5. If in a substance, through which heat flows, the temperatures of each part are constant (not same) the substance is said to be in thermal steady state.

   In a steady thermal state, $H = \frac{Q}{t} = \frac{kA(T_1 - T_2)}{L}$ $(T_1 > T_2)$
6. Good conductors of electricity are also good conductors of heat.
7. **Thermal Resistance (R_H) :** $\frac{L}{kA}$ is called thermal resistance.
8. The laws of thermal resistance for series and parallel connection are same as those for electrical resistance.
9. In thermal conduction the net displacement of constituent particles is zero. Also intermolecular forces play a very important role in thermal conduction.
10. In convection constituent particles of fluid actually move and go to some other position. So convection is possible in fluids only.

11. There are two types of convection, (1) Natural convection and (2) Forced convection.
12. Convection plays an important role in saving the lives of aquatic animals.
13. Medium is not needed for heat transfered by radiation.
14. Each substance emits radiation in accordance with its temperature.
15. At higher temperatures the rate of thermal radiation in higher.
16. A substance which can absorb or emit all types of radiations is called perfectly black body.
17. Perfectly black body does not exist in nature.

    Sun behaves like a perfectly black body with temperature equal to 5800 K.
18. On irradiating a surface, the ratio of the radiant energy absorbed to the amount of radient energy incident is called absorptivity a of the surface.
19. **Total emissive power :** The amount of radiant energy emitted per unit area per unit time from a surface at a given temperature for all possible wave lengths is called total emissive power (W).
20. **Spectral emissive power :** At a given temperature the amount of radient energy emitted per second per unit surface are in a unit frequency interval about a given frequency (f) is called spectral emissive power (W_f).

    If spectral emissive power is W_f total emissive power is

    $$W = \sum_f W_f$$
21. **Emissivity :** The ratio of total emissive power of a surface to the total emissive power of the surface of perfectly black body under sume condition is called emissivity (e) of the surface.
22. **Kirchhoff's law :** The values of emissivity and absorptivity are equal for any surface. i.e. $a = e$ for perfectly black body $a = e = 1$
23. **Wien's displacement law :** In black body radiation product of Wavelength of a radiation having maximum spectral emissive power and absolute temperature is constant.

    $\lambda_m T = \text{constant}$

    The value of this constant is 2.9×10^{-3} m K
24. **Stefan Boltzmann's law :** The amount of radiant energy emitted by a surface per unit area per unit time (i.e. total emissive power of a surface) in directly proportional to fourth power of its absolute temperature.

    $W = \sigma e T^4$

    σ is Stefan–Boltzmann constant. Its value is $5.67 \times 10^{-8} wm^{-2}K^4$.
25. **Newton's law of cooling :** The rate of loss of heat by a body depends on temperature difference $(T - T_s)$ between body and its surrounding.

    $$\frac{dT}{dt} \propto (T - T_s)$$
26. **Langmuir–Lorentz law :** For natural convection the rate of cooling is proportional to $\left(\frac{5}{4}\right)^{th}$ power of difference of temperature between the body and its sorroundings.

EXERCISES

Choose the correct option from the given options :

1. One end of metal rod is kept in boiling water and the other is in contact with ice then ....

(A) all the parts of the rod are in thermal equilibrium with each other.
(B) the rod is said to have one definite temperature.
(C) the rod is said to have one specific temperature when it attains thermal steady state.
(D) the thermal state of the rod does not change after it attaines thermal steady state.

2. A compound slab is made up of two slabs. If thermal conductivity of their material are k_1 and k_2 respectively and their cross–sectional areas are the same, the equivalent thermal conductivity of the slab is .... (Consider series connection)

(A) $k_1 + k_2$ (B) $\frac{k_1 - k_2}{2}$
(C) $\frac{k_1 + k_2}{k_1 k_2}$ (D) $\frac{2k_1 k_2}{k_1 + k_2}$

3. A black body at T K temperature emits E amout of radiant energy from surface of 1 m^2 in 1 s. If its temperature is halved, the amount of energy emitted will be ....

(A) $\frac{E}{16}$ (B) $\frac{E}{4}$ (C) $\frac{E}{2}$ (D) 2E

4. In thermal steady state the temperatures of two ends of a meter scale are 30 °C and 20 °C. Temperature of the part of the scale at 60 cm from the hot end is ....

(A) 25 °C (B) 24 °C (C) 23 °C (D) 22 °C

5. Temperature of a steel block changes from 100 °C to 90 °C in time t_1, from 90 °C to 80 °C in time t_2, and from 80 °C to 70 °C in time t_3 then ....

(A) $t_1 < t_2 < t_3$ (B) $t_1 > t_2 > t_3$
(C) $t_1 = t_2 = t_3$ (D) $t_3 = \frac{t_1 + t_2}{2}$

6. The wavelengths corresponding to maximum intensity (spectral emissive power) emitted from two black bodies A and B are 11×10^{-5} cm and 5.5×10^{-5} cm respectively then $\frac{T_A}{T_B}$ = ...

(A) 2 (B) 4 (C) $\frac{1}{2}$ (D) 1

7. Equivalent thermal resistance of a parallel connection of two rods having thermal resistances R_1 nad R_2, is ....

(A) $\frac{R_1 R_2}{R_1 + R_2}$ (B) $\frac{R_1 + R_2}{R_1 R_2}$
(C) $R_1 + R_2$ (D) none of the above.

8. A big piece of glass is first heated and then is allowed to cool. On cooling down, a crack is developed in it. One of the possible reasons for this is ....

(A) small thermal conductivity (B) large thermal conductivity
(C) large specific heat (D) high melting point

9. A steel ball is brought in contact with an identical ball of wood, then they will be equally hot or cold at ....
(A) 98.4 °C (B) 98.4 K
(C) 98.4 °F (D) room temperature.

10. Which of the following is closest to a black body ?
(A) Black board paint (B) Green leaves
(C) Lamp soot (D) Black hole

11. Two spheres of the same material have radii 1 m and 4 m and temperature 4000 K and 2000 K respectively. Ratio of the energy radiated per second by the first sphere to that by the second is ....
(A) 1 : 1 (B) 16 : 1 (C) 4 : 1 (D) 1 : 9

12. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta T)^n$. Where ΔT is the difference of the temperature of the body and the surroundings and then n = ...
(A) 2 (B) 3 (C) 4 (D) 1

13. If the temperature of the Sun were to increase from T to 2T and its radius from R to 2R. They the ratio of radiant energy recieved on earth to what it was previously will be ....
(A) 4 (B) 16 (C) 32 (D) 64

14. Three rods of equal dimensions are arranged as shown in the figure. Their thermal conductivities are 5k, 3k and 2k. Temperature of junction O is ..... .

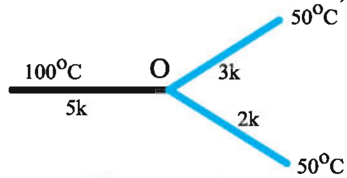


Figure 7.7

(A) 75 °C (B) $\frac{200}{3}$ °C
(C) 40 °C (D) $\frac{100}{3}$ °C

15. A sphere, a cube and a thin circular plate, all of same material and same mass are at the same temperature. Which of the following would cool fastest ?
(A) Circular plate (B) Sphere
(C) Cube (D) All of them

16. A body heated to 1000K, uses surface area 10cm^2. If radiates 340.2 J energy per minute. So emissivity is .... ($\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4})
(A) 0.1 (B) 0.02 (C) 0.01 (D) 0.2

17. The temperature of the two outer surfaces of a composite slab, consisting of two matrials with thermal conductivity k and 2k and thickness x and $4x$ respectively are T_2 and T_1 ($T_2 > T_1$).

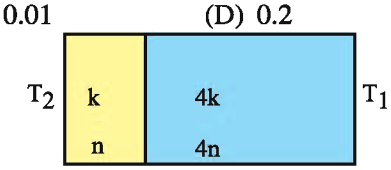


Figure 7.8

The rate of heat transfer through the slab is $\frac{A(T_2 - T_1)k}{x} f$. So f = ...
(A) 1 (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

18. In a system of two concentric spherical shells of radii r_1 and r_2 temperature of the inner sphere is T_1 and that of outer sphere is T_2. The radial rate of flow of heat in a substance between these two concentric spheres is proportional to ..... ($r_1 < r_2$)
(A) $\frac{(r_2 - r_1)}{r_1 r_2}$ (B) $ln\left(\frac{r_2}{r_1}\right)$
(C) $\frac{r_1 r_2}{(r_2 - r_1)}$ (D) $r_1 - r_2$

ANSWERS

1. (D)	2. (D)	3. (A)	4. (B)	5. (A)	6. (C)
7. (A)	8. (A)	9. (C)	10. (C)	11. (A)	12. (D)
13. (D)	14. (A)	15. (A)	16. (A)	17. (D)	18. (C)

Answer as briefly as possible :

1. What is meant by thermal conduction ?
2. Give the dimensional formula for temperature gradient.
3. Give SI unit of heat current.
4. Name the physical quantity which has unit similar to heat current.
5. State the dimensional formula for heat conductivity.
6. What is meant by thermal resistance ?
7. What is forced convection ?
8. On which factors does the frequency of thermal radiation depend ?
9. Temperature of the Sun's surface is 5800 K. What is the wave length which has the maximum spectral emissive power for the Sun ?
10. What is the unit of emissivity ?
11. In the radiations emitted by a black body at 27 °C temperature which radiations have the maximum spectral emissive power ?
12. As per Wien's displacement law $f_m \propto$ ...... f_m is the frequency corresponding to maximum spectral power.
13. The total emissive power at 0 °C is W_1. So total emissive power at 546 °C is ....
14. On which factor does the constant k' in Newton's law of Cooling depend ?

Answer briefly :

1. Discuss the factors on which amount of thermal energy passing perpendicularly between two nearby cross–sections depend. Hence obtain expression for heat current.
2. Explain thermal steady state using proper example.
3. Explain heat convection in fluids.
4. Define absorptivity, emissivity and hence explain Kirchoff's law of radiation.
5. Explain cavity and cavity radiation.
6. Explain total emissive power and spectral emissive power.
7. State and explain Wein's displacement law.
8. State Newton's law of cooling. Also obtain its expression.

Solve the following problems :

1. A and B are two rods of equal lengths and different materials. Temperatures of their two ends are T_1 and T_2 for each rod. Which condition must be satisfied so that the rate of heat flow becomes equal for both ?

   [**Ans. :** $\frac{k_A}{k_B} = \frac{A_B}{A_A}$]

2. The dimensions of the ceiling of a room are 4 m × 4 m × 10 *cm*. Thermal conductivity of the concrete in the ceiling is 1.26 W m^{-1} $°C^{-1}$. At one moment, the temperature outside and inside the room are 46 °C and 32 °C respectively. (i) Find the amount of heat flowing through the ceiling in one second. (ii) A layer of bricks of thickness 7.5 cm and thermal conductivity 0.65 W m^{-1} $°C^{-1}$ is laid on the ceiling. Find the new rate of flow of heat.

   [**Ans. :** (1) 2822 J (2) 1150 W]

3. The thickness of ice layer on the surface of a lake is 5 cm. Temperature of enviroment is –10 °C. Find the time required for the thickness of the ice layer to become double. Thermal conductivity of ice is 0.004 cal/cm s °C, density of ice is 0.92 g/cm^3 and latent heat of fusion is 80 cal/gm. (**Hint :** $\int x\,dx = \frac{x^2}{2}$)

[**Ans. :** 19 hour and 10 min]

4. From 1 m^2 area of surface of the Sun 6.3×10^7 J energy is emitted per second $\sigma = 5.669 \times 10^{-8}$ Wm^{-2} K^{-4}. Find the temperature of the surface of the Sun.

[**Ans. :** 5773K]

5. How many times faster the temperature of a cup of tea will decrease by 1 °C at 373 K, then that at 303 K ? Consider tea as a black body. Take room temperature as 293 K. ($\sigma = 5.7 \times 10^{-8}$ Wm^{-2} K^{-4})

[**Ans. :** 11.3]

6. For the composite slab shown, find effective thermal conductivity.

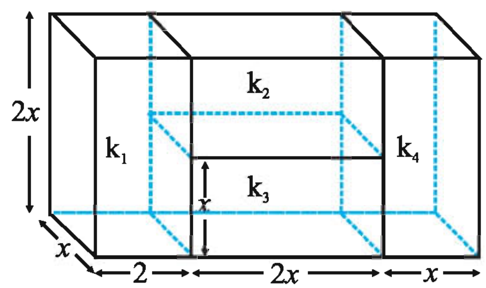


Figure 7.9

[**Ans. :** $\frac{4k_1k_4(k_2 + k_3)}{(k_1 + k_4)(k_2 + k_3) + 4k_1k_2}$]

7. A thermally insulated rod of length L, has conducting ends. In its thermal steady state, its ends are maintained at T_1 and T_2 temperature ($T_1 > T_2$). If the temperature dependence of thermal conductivity of rod in different regions is given by $k = a + bT$,
Obtain the formula for its heat current
Its cross sectional area is A.

[**Ans. :** $\frac{Q}{t} = \frac{A}{L}(T_1 - T_2)\left(a + b\left(\frac{T_1 + T_2}{2}\right)\right)$]

8. Solar constant is the amount of solar energy incident perpendicularly per unit time on unit surface area of the earth at an average distance between the Sun and the earth. Its value in $S_0 = 1340$ w/m^2. Hene calculate temperature of the Sun's surface.
Radius of the Sun $R_s = 7 \times 10^8$m
Average distance between the earth and the Sun
$R_0 = 1.5 \times 10^{11}$m
$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} [**Ans. :** 5739 K]

CHAPTER 8

KINETIC THEORY OF GASES

8.1 Introduction

Dear students, Boyle, Newton and other scientists tried to explain the behaviour of gases by considering that gas is made up of tiny molecules. Any physical form of matter is made up of constituent particles like atoms, molecules or ions which are in continuous motion. In solids, the molecules are closely spaced and oscillate about their mean position. In liquids, the distance betwen constituent particles is more in comparison to that in solids. Because the distance between the constituent particles in gases is more, the interactive forces between them are negligible in gases, in comparison to those in solids and liquids. Hence the constituent particles of gases can move freely in perpetual motion.

Out of these three states of matter, the study of behaviour of gases in terms of its constituent particles is easy, which is studied in kinetic theory of gases. The kinetic theory of gases is based on the gas laws and on Avogadro's hypothesis.

Physical quantities like pressure, temperature, volume, internal energy are associated with a gas. These quantities are obtained as an average combined effect of the processes taking place at the microscopic level in a system known as **macroscopic quantities.** Macroscopic quantities can be measured directly, or can be calculated with the help of other measurable macroscopic quantities. For example, the pressure of a gas can be measured directly, where as its internal energy can be calculated from the macroscopic quantities like its pressure, volume and temperature. The description of a system and events associated with it in context to its macroscopic quantities is known as **macroscopic description.**

The understanding about the macroscopic quantities and their inter relationship can be obtained from the processes occuring between constituent particles of the system at microscopic level. For example, the pressure of a gas can be understood in terms of random motion of its molecules, their collisions with the walls of the container, and the associated changes in their momenta. This way, the physical quatities like speed, momentum, kinetic energy etc. associated with the constituent particles at microscopic level, are known as **microscopic quantities.** When the system and events associated with it are described in context to microscopic quantities, this description is known as **microscopic description.**

In the kinectic theory of gases, by applying the laws of mechanics statistically to the constituent particles of the system, the macroscopic quantities are obtained in terms of microscopic quantities with the help of mathematical scheme.

The kinetic theory of gases gives information about the pressure, temperature and volume of the gases, as well as other parameters like viscosity, conduction, diffusion and specific heat.

8.2 Behaviour of Gases

It has been observed from experiments that, for very low densities, the pressure, volume and temperature of a gas are interrelated by some simple relations.

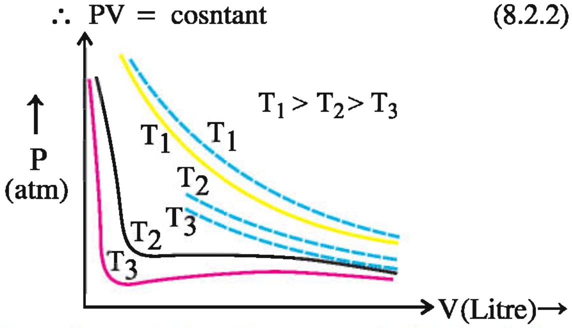
8.2.1 Boyle's law

At constant temperature and low enough density, the pressure of a given quantity (mass) of gas is inversely proportional to its volume, i.e.

$$P \propto \frac{1}{V} \quad (8.2.1)$$

(constant temperature, constant mass)

$$\therefore PV = \text{cosntant} \quad (8.2.2)$$



Experimental P – V curves (solid lines) for steam at three temperature compared with Boyle's law (broken lines)

Figure 8.1

Fig. 8.1 shows the experimentally obtained curves (solid lines) of pressure against volume for steam at three temperatures, and theoretically obtained curves (broken lines) using Boyle's law. Experimental curves are in agreement with the curves obtained using Boyle's law at high temperatures and low pressures.

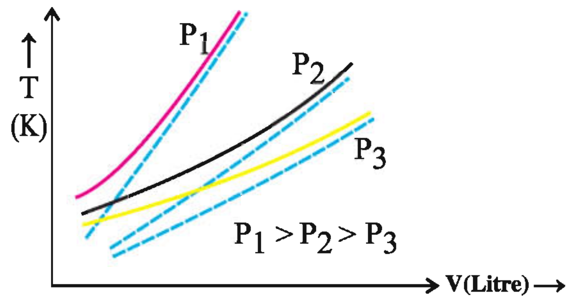
8.2.2 Charle's law

At constant pressure and low enough density, the volume of a given quantity (mass) of a gas is proportional to its absolute temperature, i.e.

$$V \propto T \quad (8.2.3)$$

(constant pressure, constant mass)

$$\therefore \frac{V}{T} = \text{constant} \quad (8.2.4)$$



Experimental T – V curves (solid lines) for CO_2 gas at three pressures compared with Charle's law (broken lines)

Figure 8.2

From Fig. 8.2 it can be observed that, at constant pressure and enough low density, the given quantity (mass) of gas obey Charle's law.

Gay Lussac's Law : For a given volume and low enough density the pressure of a given quantity of gas is proportional to its absolute temperature, i.e.

$P \propto T$ (constant volume, constant quantity)

$$\therefore \frac{P}{T} = \text{constant} \quad (8.2.5)$$

8.3 Avogadro Number

If P, V and T are same, then the number of molecules (N) is also same for all gases, which is Avogadro's hypothesis. Hence,

"For given constant temperature and pressure, the number of molecules per unit volume is the same for all gases."

At standard temperature (273 K) and pressure (1 atm), the mass of 22.4 litres of any gas is equal to its molecular mass (in grams). This quantity of gas is called 1 mole. (Avogedro had guessed the equality of number of gas molecules in equal volumes of gases at a fixed temperature and pressure from chemical reactions. Kinetic theory justifies this hypothesis.)

The number of particles (atoms or molecules) in one mole of a substance (gas) is called Avogadro number, which has a magnitude $N_A = 6.023 \times 10^{23}$ mol^{-1}.

If the number of gas molecules in a container is N, then the number of mole of given gas is

$$\mu = \frac{N}{N_A} \quad (8.3.1)$$

Thinking in other way, if the total mass of gas in a container is M, and the mass of 1 mole of the gas (called atomic mass or molar mass of the gas) is M_0, then the number of moles of the gas is

$$\mu = \frac{M}{M_0} \quad (8.3.2)$$

8.4 Ideal gas-state equation and its different forms

If we combine the Boyle's law and Charle's law, we get

$$\frac{PV}{T} = K \text{ (contant)} \quad (8.4.1)$$

(for given quantity of gas)

Which shows that, by keeping the pressure and temperature of a gas to be constant, if the quantity (mass) of a gas is varied, then the volume of the gas is proportional to the quantity of the gas. Thus the constant term on the right side of equation (8.4.1) depends on the quantity of the gas. If the quantity of gas is represented in mole, then

$$\frac{PV}{T} = \mu R$$

or $\quad PV = \mu RT \quad (8.4.2)$

Where μ = number of mole

R = Universal Gas constant

$= 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$

If a gas completely obeys equation PV = μRT at all values of pressure and temperature, then such a (imaginary) gas is called an ideal gas. In actual practice no gas is an ideal gas in every situations.

As the thermodynamic state of a gas at very low devity can be verified by putting the magnitudes of macroscopic quantities P, V and T in equation (8.4.2), this equation is called **an ideal gas-state equation.**

If we put $\mu = \frac{N}{N_A}$ in equation (8.4.2),

we get,

$$PV = \frac{N}{N_A} RT \quad (8.4.3)$$

Putting $\frac{R}{N_A} = k_B$ in equation (8.4.3)

$$R = N_A k_B \quad (8.4.4)$$

Where k_B = Boltzmann's constant

$= 1.38 \times 10^{-23} \text{ J K}^{-1}$

which is defined for one molecule

$$\therefore PV = k_B NT \quad (8.4.5)$$

$$\text{or} \quad P = k_B \frac{N}{V} T = k_B nT \quad (8.4.6)$$

Where, $n = \frac{N}{V}$ = number density of gas

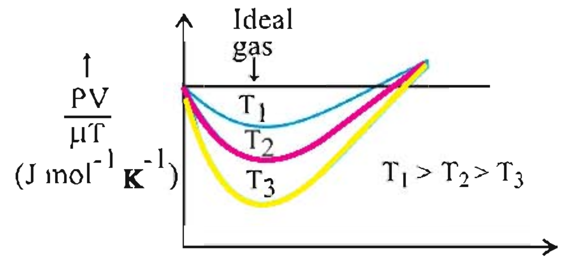
= number of molucules per unit volume of the gas

Further, from equations (8.3.2) and (8.4.2)

$$PV = \frac{M}{M_0} RT$$

$$\therefore P = \frac{M}{V} \frac{RT}{M_0} = \frac{\rho RT}{M_0} \quad (8.4.7)$$

Where $\rho = \frac{M}{V}$ = mass density of the gas



Real gases approach ideal gas behaviour at low pressures and high temperatures

Figure 8.3

In Fig. 8.3 the behaviour of a real gas at three different temperatures is shown, which differs from ideal gas behaviour. It can be seen from the figure that at low pressure and high temperature, the real gas behaves like an ideal gas. Ideal gas is actually a theoretical model of a gas.

The work done during the change in volume of the gas :

Dear friends, in Chapter-6, we obtained the work from the graph of F – *x*. Using this procedure, the work done during change in volume of the gas can be obtained from the graph of P – V. This way we get the relation

$W = \int_{V_i}^{V_f} P\, dV$ for the work. We will discuss about it in detail in future in the chapter of thermodynamics.

Illustration 1 : The surface area of the floor of a room is 20 m^2 and the height of its walls is 3 m. If the temperature in the room is 27° C and pressure is 1 atm, then find the mass of air inside the room. Consider the mass of 1 mole of air to be 29 g. (**Given :** 1 atm = 1.01×10^5 N m^{-2} = 1.01×10^5 *Pa*, R = 8.31 J mol^{-1} K^{-1})

Solution : P = 1 atm = 1.01×10^5 N m^{-2}

V = volume of room = 20 × 3 = 60 m^3

R = 8.31 J mol^{-1} K^{-1}.

T = 27 °C = 273 + 27 = 300 K.

According to ideal gas state equation

$PV = \mu RT$

$$\therefore \mu = \frac{PV}{RT}$$

$$= \frac{1.01 \times 10^5 \times 60}{8.31 \times 300}$$

$\therefore \mu = 2.43 \times 10^3$ mole (1)

Hence the mass of air is

$m = 29\mu = 29 \times 2.43 \times 10^3$

$= 70.5 \times 10^3$ g

$\therefore m = 70.5$ kg (2)

which is the mass of air inside the room.

Illustration 2 : In a vessel of 20 L volume, if oxygen is filled at a pressure of 2.5×10^5 N m^{-2}, find the mass of oxygen in the vessel. The temperature of oxygen is 27 °C, and molar mass of oxygen is 32 g mol^{-1}.

R = 8.31 J mol^{-1} K^{-1}.

Solution : M_{O_2} = molar mass of oxygen

= 32 g mol^{-1}

= 32×10^{-3} kg mol^{-1}

P = 2.5×10^5 N m^{-2}

V = 20 L = 20×10^{-3} m^3

μ = number of moles

R = 8.31 J mol^{-1} K^{-1}

T = 27 + 273 = 300 K.

According to ideal gas state equation

$PV = \mu RT$

$$\therefore \mu = \frac{PV}{RT}$$

$$\therefore \frac{m_{O_2}}{M_{O_2}} = \mu = \frac{PV}{RT}$$

$\therefore m_{O_2}$ = mass of oxygen

$$= M_{O_2} \times \frac{PV}{RT}$$

$$= \frac{32 \times 10^{-3} \times 2.5 \times 10^5 \times 20 \times 10^{-3}}{8.31 \times 300}$$

= 0.064

= 64×10^{-3} kg

$\therefore m_{O_2}$ = 64 g.

Illustration 3 : A gas of 15 kg is filled in a cylinder at a pressure of 10^7 N m^{-2}. The gas is leaking out of the cylinder. At one moment the pressure of the gas is found to be 3×10^6 N m^{-2}. How much gas would have leaked out by then ? Consider the temperature to be constant.

Solution : $P_1 = 10^7$ N m^{-2}

$P_2 = 3 \times 10^6$ N m^{-2}

M_1 = 15 kg

M_2 = ?

According to the ideal gas state equation, at constant volume and temperature

$P_1V = \mu_1RT$

$P_2V = \mu_2RT$

$$\therefore \frac{P_1}{P_2} = \frac{\mu_1}{\mu_2} = \frac{M_1}{M_2}$$

$$\therefore M_2 = \frac{P_2}{P_1} \times M_1 = \frac{3 \times 10^6 \times 15}{10^7}$$

= 4.5 kg

Thus out of 15 kg, 4.5 kg gas is left. Therefore the amount of gas leaked out is

15 – 4.5 = 10.5 kg

Illustration 4 : What will be the relative mass of air in your class room in winter at temperature of 7 °C, as compared to that in summer at temperature of 37 °C ?

(Consider the pressure to be constant)

Solution : T_1 = 7 + 273 = 280 K

T_2 = 37 + 273 = 310 K

According to the ideal gas state equation

$PV = \mu_1RT_1$,

$PV = \mu_2RT_2$

$\therefore \mu_1T_1 = \mu_2T_2$

$$\therefore \frac{\mu_1}{\mu_2} = \frac{T_2}{T_1}$$

$$\therefore \frac{\mu_1}{\mu_2} = \frac{M_1}{M_2} = \frac{T_2}{T_1} = \frac{310}{280} = 1.1$$

Hence, in comparison to summer, the mass of air in winter will be 1.1 times.

Illustration 5 : A bottle is closed by a cork which contains air at temperature of 7 °C and pressure of 1 atm. The cork can withstand a pressure of up to 1.3 atm. Then up to what minimum temperature can we heat up the bottle so that the cork gets ejected ? Neglect the thermal expansion of the bottle.

Solution : $P_1 = 1$ atm

$P_2 = 1.3$ atm

V = constant

$T_1 = 7$ °C $= 280$ K

$T_2 = ?$

According to the ideal gas state equation

$$P_1V = \mu RT_1$$

$$P_2V = \mu RT_2$$

$$\therefore \frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\therefore T_2 = \frac{P_2}{P_1} T_1 = \frac{1.3 \times 280}{1}$$

$$\therefore T_2 = 364 \text{ K}$$

$$\therefore T_2 = 364 - 273 = 91 \text{ °C}$$

Illustration 6 : Calculate the volume of an ideal gas at 0 °C temperature and 1 atm pressure.

Solution : $T = 0$ °C $= 0 + 273 = 273$ K

$\mu = 1$ mole

$P = 1$ atm $= 1.01 \times 10^5$ N m^{-2}

$R = 8.31$ J mol^{-1} K^{-1}

$V = ?$

According to the ideal gas state equation

$$PV = \mu RT$$

$$\therefore V = \frac{\mu RT}{P} = \frac{1 \times 8.31 \times 273}{1.01 \times 10^5}$$

$$= 0.0224$$

$$\therefore V = 22.4 \times 10^{-3} \text{ m}^3 = 22.4 \text{ L.}$$

8.5 Kinetic Theory of Gases

Till now we gave macroscopic definition of an ideal gas in terms of inter relations of its macroscopic quantities like volume, pressure, temperature etc. Actually, according to the kinetic theory, these calculations are given in terms of the microscopic quantities like speed, momentum and kinetic energy of molecule of the constituent particles of the gas. To understand the macroscopic quantities from these calculations, scientists developed a theoretical model for an ideal gas which is based on a hypothesis. The hypothesis made for an ideal gas gives the definition of the ideal gas based on its microscopic description. The theoretical model based on these hypothesis is called molecular model of the ideal gas.

8.5.1 Molecular model of an ideal gas : Postulates :

(1) Gas is made up of tiny particles. These particles are called molecules of the gas.

The molecules of gas may be monoatomic, diatomic or polyatomic. If the gas is made up of a single element or a compound and is chemically stable, then all of its molecules are similar.

(2) The molecules of gas may be regarded as completely rigid spheres without any internal structure.

(3) The molecules of a gas perform incessant random motion.

During such motion molecules collide with each other and the walls of the container.

(4) The molecules of a gas follow Newton's Laws of motion.

During the random motion, between two consecutive collisions, molecules move freely in a straight line with constant speed according to Newton's first law. During collision, the speed and direction of molecules of gas varies in accordance with the Newton's second and third laws.

(5) The number of molecules in a gas is very large.

The reason for stipulating a very large number of molecules is that we can not study the motion of individual molecules. Further, considering a large number of molecules, there will be a large number of collisions among molecules and an incessant random motion will continue.

(6) The volume occupied by a molecule of a gas is negligible in comparison with the volume of the container.

It means that in comparison to the average distance between molecules, the diameter of the molecule is very small, so that the interactions among molecules can be neglected.

(7) Intermolecular forces act only when two molecules come close to each other or collide.

(8) The collisions between the molecules and between a molecule and the wall of container are elastic. The time interval of collision is negligible in comparison with the interval between successive collisions.

Kinetic energy is conserved during elastic collision.

Illustration 7 : If the volume of steam of a given mass of water is 10^3 times the volume of water of the same mass, determine the available space to the water molecules of steam. Take radius of water molecule R = 2Å.

Solution : As the volume of steam of a given mass of water is 10^3 times the volume of water, considering the molecules to be spherical, and the surrounding to be spherical,

$$V = \frac{4}{3}\pi R^3 \Rightarrow R \propto V^{\frac{1}{3}}$$

$$\therefore\ R \propto (10^3)^{\frac{1}{3}} = 10 \text{ times more.}$$

Thus the total available (radius) space to every molecule of steam is

= 10 × radius of water molecule

= 10 × 2 Å

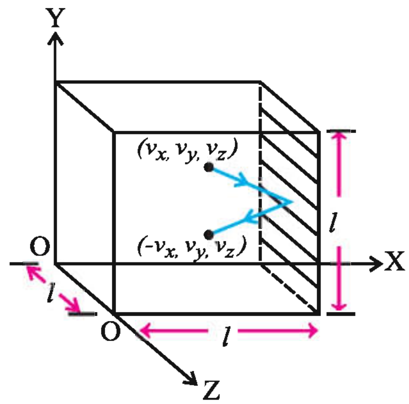
= 20 Å

Hence the average distance between adjacent steam molecules is 2 × 20 = 40 Å

8.6 Pressure of an ideal gas and rms speed of gas molecules

As shown in Fig. 8.4 consider that an ideal gas is filled in a cubical container of sides l, and that the three sides of the container are parallel to x, y, z axes. As shown in the Figure, a molecule of gas with velocity (v_x , v_y , v_z) hits the planer wall of the container parallel to yz plane of area A = l^2. As the collision is elastic, the components of velocity along y and z directions do not change, but only the component along x direction changes its direction. Thus, after collision, the velocity of the gas molecule is ($-v_x$, v_y , v_z).

Therefore the change in the momentum of the molecule = $- mv_x - (mv_x) = -2mv_x$



Elastic collision of a gas molecule with the wall of the container.

Figure 8.4

Hence according to law of conservation of momentum, the momentum transferred to the wall is $+2mv_x$.

To calculate the force (and hence pressure) on the wall, we have to calculate the momentum imparted to the wall in unit time. In time Δt, the molecules with velocity v_x, will hit the wall only if they are within the distance $\Delta d = v_x \cdot \Delta t$ from the wall.

Thus only the molecules within the volume $Av_x\Delta t$ can collide with the wall in time Δt. But, on an average, half of these molecules will be moving towards and other half away from the wall. Thus the number of molecules with velocity (v_x, v_y, v_z) hitting the wall in time Δt is $\frac{1}{2} nAv_x\Delta t$.

where n = number of molecules in unit volume.

The total momentum transferred to the wall in time Δt by these molecules is

$$p_1 = (2mv_x)\left(\frac{1}{2} nAv_x\Delta t\right) \quad (8.6.1)$$

$$\therefore\ p_1 = nmAv_x^2\Delta t \quad (8.6.2)$$

Thus in time Δt the force acting on the wall is $F = \frac{p_1}{\Delta t}$, and hence the pressure per unit area acting on the wall is,

$$P = \frac{F}{A} = \frac{p_1}{\Delta t \cdot A} = nmv_x^2 \quad (8.6.3)$$

Actually, all molecules in the gas do not have same velocity. Hence equation (8.6.3) is applicable for the pressure exerted by n number of molecules per unit volume with velocity v_x.

Taking average value of v_x^2, we get the total pressure

$$P = nm < v_x^2 > \qquad (8.6.4)$$

$$\therefore \ P = \rho < v_x^2 > \qquad (8.6.5)$$

where, $nm = \rho$ = density of the gas and $< v_x^2 >$ = average value of v_x^2.

As the gas is isotropic, the gas molecules will be moving with different velocities in different directions, and hence the average values of squared velocities in every direction will be identical, i.e.

$$< v_x^2 > = < v_y^2 > = < v_z^2 > \qquad (8.6.6)$$

$$\therefore \ < v^2 > = < v_x^2 > + < v_y^2 > + < v_z^2 >$$

$$= 3 < v_x^2 > = 3 < v_y^2 > = 3 < v_z^2 >$$

$$\therefore \ < v_x^2 > = < v_y^2 > = < v_z^2 > = \frac{1}{3} < v^2 > \qquad (8.6.7)$$

Where, $< v^2 >$ = mean of the squared velocity of gas molecules.

Using equation (8.6.7) in (8.6.5)

$$\therefore \ P = \left(\frac{1}{3}\right) \rho < v^2 > \qquad (8.6.8)$$

Equations (8.6.4) and (8.6.8) represent the pressure of an ideal gas.

v_{rms} **: The square root of the mean squared velocity $< v^2 >$ is called v_{rms} (root mean squared speed).** This is a particular type of molecular speed. From equation (8.6.8).

$$v_{rms} = \sqrt{\frac{3P}{\rho}} \qquad (8.6.9)$$

Two remarks :

(1) We considered the container of gas to be cubical, but actually the shape of the vessel is immaterial. For a vessel of arbitrary shape, we may choose a small (infinitesimal) planar area and carry out the above calculations. In equation (8.6.8) both A and Δt do not appear. According to Pascal's law, the pressure is same everywhere for a gas in equilibrium.

(2) In the above calculations we have neglected the inter–atomic collisions. The number of molecules hitting the wall in time Δt is $\frac{1}{2} nAv_x\Delta t$. In steady state also the molecules collide with each other randomly. When a molecule with velocity (v_x, v_y, v_z) collides with another molecule of different velocity, then the other molecule acquires velocity (v_x, v_y, v_z) and the velocity of first molecule changes. Thus, during any collision, there is no change in the value of $< v_x^2 >$ (i.e. $< v^2 >$). Thus, on a whole, there is no effect of intermolecular collisions on the pressure P. (Equation 8.6.8).

8.7 Kinetic Energy and Temperature

Equation (8.6.8) can be written as

$$PV = \frac{1}{3} nVm < v^2 > \qquad (8.7.1)$$

$$\therefore \ PV = \frac{2}{3} N \cdot \frac{1}{2} m < v^2 > \qquad (8.7.2)$$

where $N = nV$ = number of molecules of the gas in volume V, and $\frac{1}{2} m < v^2 >$ = average translational kinetic energy of gas molecules.

But since the internal energy of an ideal gas is purely kinetic, i.e.

$$E = N \cdot \frac{1}{2} m < v^2 > \qquad (8.7.3)$$

From equations (8.7.2) and (8.7.3)

$$PV = \frac{2}{3} E \qquad (8.7.4)$$

Comparing equation (8.7.4) with the ideal gas state equation (8.4.5) (i.e. $PV = k_BNT$),

$$\frac{2}{3} E = k_BNT$$

$$\therefore \ E = \frac{3}{2} k_BNT \qquad (8.7.5)$$

From equations (8.7.3) and (8.7.5),

$$\frac{E}{N} = \frac{1}{2} m < v^2 > = \frac{3}{2} k_BT \qquad (8.7.6)$$

Thus the average kinetic energy of a gas molecule is proportional to the absolute temperature of the gas. Further it is independent of pressure, volume or nature of the gas. This fundamental result relates temperature (which is a macroscopic quantity, and is related with the average combined effect of the processes occuring at microscopic level in the system) with the average kinetic energy of the molecules. Equation (8.7.5) shows that the internal energy of the (ideal) gas depends only on its temperature and not on its pressure or volume. From this interpretation of temperature, the kinetic theory of an ideal gas is seen to be completely consistent with the ideal gas state equation and the various gas laws based on it.

8.8 Law of Equipartition of Energy and Degrees of Freedom

The average kinetic energy per molecule of a gas in a container is

$$< E > = \frac{1}{2} m < v_x^2 > + \frac{1}{2} m < v_y^2 > + \frac{1}{2} m < v_z^2 >$$

$$= \frac{3}{2} k_B T \quad (8.8.1)$$

But as the gas is isotropic

$$\therefore < v_x^2 > = < v_y^2 > = < v_z^2 > = \frac{1}{3} < v^2 >$$

$$\therefore < E > = \frac{3}{2} m < v^2 > = \frac{3}{2} k_B T \quad (8.8.2)$$

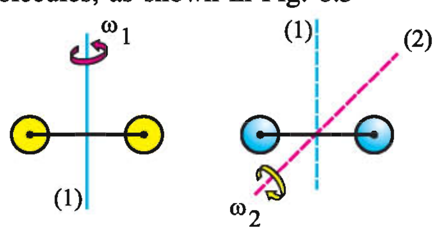
$$\therefore \frac{1}{2} m < v^2 > = \frac{1}{2} k_B T \quad (8.8.3)$$

Thus, the energy associated with each possible independent motion of a molecule in a container is $\frac{1}{2} k_B T$.

As explained in equation (8.8.1), the molecules of gas have independent translational motion in x, y and z directions.If the gas is made up of diatomic molecules, then the molecules have rotational and vibrational motion along with the translational motion. The rotational motion of such molecules is possible in two different directions :

(1) about a perpendicular axis passing through the mid point of a line joining both the molecules, and,

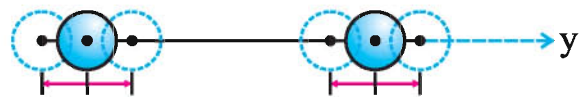
(2) about an axis perpendicular to both, the above mentioned axis and the line joining both the molecules, as shown in Fig. 8.5



Rotational motion of diatomic molecule in two independent directions.

Figure 8.5

Further the atoms of diatomic molecules perform vibration (oscillation) about their mean position along the line joining them. (See Figure 8.6)



The vibration of the atoms of diatomic molecule about their mean position along the line (y) joining them

Figure 8.6

During vibration (oscillation), the atoms posses potential energy and kinetic energy.

The number of different types of independent motion possessed by a molecule is called degrees of freedom of the molecules of the gas system. A monoatomic molecule has 3 degrees of freedom, where as a diatomic molecule (CO) has 7 degrees of freedom. If the diatomic molecule is taken as a rigid rotor, then its degrees of freedom are 5 (e.g. at moderate temperature O_2). According to the law of equipartition of energy, the energy associated with each degree of freedom is $\frac{1}{2} k_B T$ (where k_B = Boltzmann's constant).

Illustration 8 : Find the energy associated with thermal motion of 200 g of oxygen at 27 °C. (Consider the molecule of O_2 as a rigid rotator.)

Solution : Since oxygen is a diatomic molecule (and a rigid rotator), it has 5 degrees of freedom.

The energy associated with each degree of freedom is $\frac{1}{2} k_B T$.

Now, 32 g O_2 contains 6.02×10^{23} molecules.

Hence, the number of molecules in 200 g O_2 is

$$= \frac{6.02 \times 10^{23} \times 200}{32}$$

$$= 3.76 \times 10^{24}$$

$\therefore$ Total thermal energy of 200 g of O_2 is $= 3.76 \times 10^{24} \times \frac{5}{2} k_B T$

$$= 3.76 \times 10^{24} \times \left(\frac{5 \times 1.38 \times 10^{-23} \times 300}{2} \right)$$

$$= 3.8 \times 10^4 \text{ J.}$$

8.8.1 Estimation of the specific heat of a gas from the law of equipartition of energy

Dear students, the number of independent types of motion a gas molecule (an ideal) possess, is called the number of degrees of freedom of the gas molecule. The energy associated with each degree of freedom is $\frac{1}{2} k_B T$. It means that, if the degrees of freedom of a gas molecule is f, then each molecule of the gas can store the energy in f different ways. If the degree of freedom of the gas molecule is f then the average heat energy of each molecule of the gas is

$$E_{avg} = f \times \frac{1}{2} k_B T = \frac{f}{2} k_B T$$

If the number of mole of an ideal gas is μ, then the number of molecules in the gas is μN_A. Therefore the internal energy of μ mole of ideal gas is,

$$E_{int} = \mu N_A E_{avg}$$

$$= \mu N_A \frac{f}{2} k_B T$$

$$= \frac{f}{2} \mu N_A k_B T$$

$$\therefore E_{int} = \frac{f}{2} \mu R T \qquad (8.8.4)$$

Where, $R = N_A k_B$ = universal gas constant.

Equation (8.8.4) shows that the internal energy of given amount of an ideal gas directly propertional to the absolute temperature of the gas.

The amount of heat energy required to change the temperature of a unit mass of substance by unity, is called **specific heat** of the material of the substance.

For the gases, 1 mole quantity is taken as unit mass.

Hence, **"The quantity of heat required to change the temperature of 1 mole of gas by 1 K (or 1 °C) is called molar specific heat of the gas."**

Out of many methods for variation of temperature of the gas, two methods are important.

(i) Specific heat at constant volume (C_V) :

The amount of heat required to change the temperature of 1 mole of gas by 1 K, keeping its volume constant, is called the specific heat of the gas at constant volume.

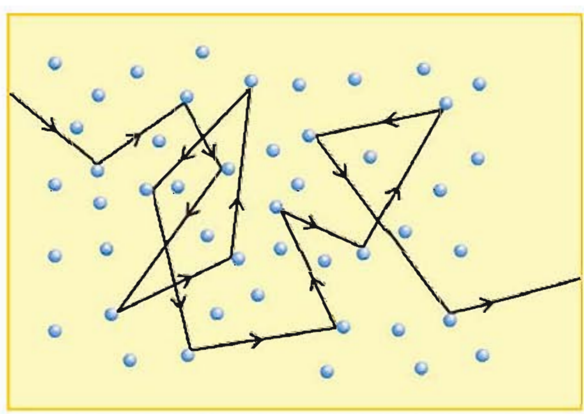
(ii) Specific heat at constant pressure (C_P) :

The amount of heat required to change the temperature of 1 mole of gas by 1 K, keeping its pressure constant, is called the specific heat of the gas at constant pressure.

In future we will discuss the specific heat in detail.

8.9 Mean Free Path

The path of motion of a molecule of a gas is shown in Fig. 8.7. When this molecule collides with another molecule on its path, the direction and magnitude of its velocity changes. After this collison the molecule moves on a straight path until it collides with another molecule. In gases at NTP the mean free path is of the order of 1000 Å.

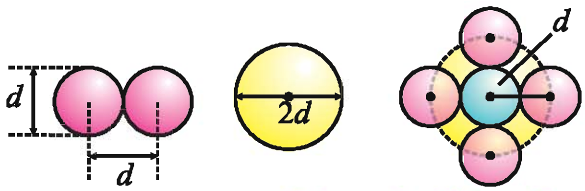


Path of motion of a molecule of a gas

Figure 8.7

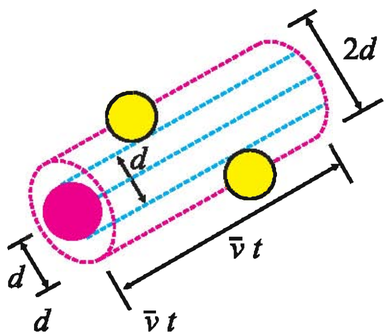
The linear distance travelled by a molecule of gas with constant speed between two consecutive collisions (between molecules) is called free path. The average of such free paths travelled by a molecule is called mean free path.

As shown in Fig. 8.8 (a), suppose that the molecules of a gas are spheres of diameter d. Thus when the centres of two molecules come together at a distance of d, then the molecules collide.



Gas molecules of diameter d
(a)

Sphere of collision of diameter $2d$
(b)



Imaginary cylinder of radius d and length $\bar{v}\,t$
(c)

Figure 8.8

Now consider a molecule of diameter d moving with average speed $\bar{v}$, and the other molecules to be stationary. The molecule under consideration will suffer collision with any molecule that comes within a distance d between their centres. To count the number of such collisions in time t, imagine a **"sphere of collision"** of radius d (or diameter $2d$) around the centre of the given molecule (see Fig. 8.8 (b)).

In time t, the molecule will sweep a cylinder (imaginary) of area of cross–section πd^2 and length $\bar{v}\,t$ (see Fig. 8.8 (c)). Thus the molecule will pass through an imaginary cylinder of volume $\pi d^2 \bar{v} t$ in time t. If the number of molecules per unit volume is n, then the number of molecules in this cylinder of volume $\pi d^2 \bar{v} t$ is $n\pi d^2 \bar{v} t$. Hence, the molecule will undergo $n\pi d^2 \bar{v} t$ collisions in time t.

The mean free path $\bar{l}$ is the average distance between two successive collisions.

$\therefore$ Mean free path =

$$\frac{\left(\text{Distance travelled by a molecule in time t with average speed } \bar{v}\right)}{\text{Total number of collisions in time } t}$$

$$\therefore\ \bar{l} = \frac{\bar{v}t}{n\pi d^2 \bar{v}t} \qquad (8.9.1)$$

$$\therefore\ \bar{l} = \frac{1}{n\pi d^2} \qquad (8.9.2)$$

In this derivation, other molecules are assumed to be stationary. In actual practice, all gas molecules are moving and their collision rate is determined by the average relative velocity $< v_r >$ in equation (8.9.1). Hence

$$[\text{ Mean free path }]\ \bar{l} = \frac{1}{\sqrt{2}n\pi d^2} \qquad (8.9.3)$$

Illustration 9 : At normal temperature and pressure (NTP) the number of molecules per cubic meter of Nitrogen gas is 2.7×10^{25}. Find the mean free path of molecules of Nitrogen. (Diameter of a molecule of Nitrogen is = 3.2×10^{-10} m)

Solution :

$n = 2.7 \times 10^{25}$ molecule m^{-3}

$d = 3.2 \times 10^{-10}$ m.

$$\therefore \text{ Mean free path } \bar{l} = \frac{1}{\sqrt{2}n\pi d^2}$$

$$\therefore\ \bar{l} = \frac{1}{1.41 \times 2.7 \times 10^{25} \times 3.14 \times (\ 3.2 \times 10^{-10}\ )^2}$$

$$\therefore\ \bar{l} = 8.17 \times 10^{-8} \text{ m.}$$

Illustration 10 : The radius of a molecule of Argon gas is 1.78 Å. Find the mean free path of molecules of Argon at 0 °C temperature and 1 atm pressure.

$k_B = 1.38 \times 10^{-23}$ J K^{-1}

Solution :

$r = 1.78\ \text{Å} = 1.78 \times 10^{-10}$ m

$d = 2r = 3.56 \times 10^{-10}$ m

T = 0ºC = 237 K

P = 1 atm = 1.01×10^5 N m^{-2}

From equation (8.4.6)

$P = nk_BT$

$\therefore\ n = \dfrac{P}{k_BT}$

The mean free path of Argon molecule is

$$\therefore\ \bar{l} = \frac{1}{\sqrt{2}n\pi d^2} = \frac{k_BT}{\sqrt{2}\pi P d^2}$$

$$\therefore\ \bar{l} = \frac{1.38 \times 10^{-23} \times 273}{1.414 \times 3.14 \times 1.01 \times 10^5 \times (3.56 \times 10^{-10})^2}$$

$$\therefore\ \bar{l} = 6.65 \times 10^{-8}\ \text{m}$$

SUMMARY

1. The description of a system and events associated with it in context to its macroscopic quantities (like pressure, temperature, volume etc.) is known as macroscopic description.
2. **Gay Lussac's Law :** For given volume, the pressure of a given quantity of gas is proportional to its absolute temperature.
3. **Avogadro's Hypothesis :** For a given constant temperature and pressure, the number of molecules per unit volume is the same for all gases.
4. The mass of 22.4 litres of any gas at standard temperature (273 K) and pressure (1 atm), STP, is equal to its molecular mass in gram. This amount of gas is called **1 mole.**
5. **Ideal Gas :** A gas that satisfies the equation PV = μRT exactly at all temperatures and pressures is called an ideal gas.
6. **Boyle's Law :** At constant temperature and low enough density, the pressure of a given quantity (mass) of gas is inversely proportional to its volume.
7. **Charles' Law :** At constant pressure and low enough density, the volume of a given quantity (mass) of a gas is proportional to its absolute temperature.
8. The average kinetic energy of gas molecules is proportional to the **absolute temperature** of the gas, and it is independent of pressure, volume or nature of the gas.
9. The internal energy of (an ideal) gas depends only on its **temperature,** and not on its pressure or volume.
10. The number of independent and different types of motions that the molecules of gas posses are called **degrees of freedom of the gas system.** Each type of motion has an associated energy of $\frac{1}{2}k_BT$.
11. **Free Path :** The linear distance travelled by a molecule of gas with constant speed between two consecutive collisions (between molecules) is called free path.
12. **Mean free path :** The average of the free paths travelled by a molecule of gas is called mean free path.
13. **Molar specific heat :** The quantity of heat required to change the temperature of 1 mole of gas by 1K (or 1 ºC) is called molar specific heat of the gas.
14. **Specific heat at constant volume (C_V) :** The amount of heat required to change the temperature of 1 mole of gas by 1 K, keeping its volume constant, is called the specific heat of the gas at constant volume.

15. **Specific heat at constant pressure (C_P) :** The amount of heat required to change the temperature of 1 mole of gas by 1 K, keeping its pressure constant, is called the specific heat of the gas at constant pressure.

EXERCISES

Choose the correct option from the given options :

1. To increase the given volume of a gas by four times,
   (A) its temperature should be increased 4 times
   (B) at constant pressure, its temperature should be increased 4 times
   (C) its pressure should be decreased to one fourth
   (D) its pressure should be increased 4 times
2. 2 kg of air is filled in a container. Its pressure is 10^5 Pa. On adding another 2 kg of air in the container at constant temperature, its pressure becomes.....
   (A) 10^5 Pa (B) 0.5×10^5 Pa (C) 2×10^5 Pa (D) 10^7 Pa
3. SI unit of the universal gas constant is ....
   (A) cal mol^{-1} (B) J mol^{-1} (C) J mol^{-1} K^{-1} (D) J mol^{-1} K
4. The volume of an ideal gas is V, pressure is P and temperature is T. The mass of each molecule is m. Hence the density of gas is ....
   (A) mk_BT (B) $\frac{P}{k_BT}$ (C) $\frac{P}{k_BTV}$ (D) $\frac{Pm}{k_BT}$
   (Where, k_B = Boltzmann's constant)
5. When the absolute temperature of a gas is increased 3 times, v_{rms} of its molecules becomes ....
   (A) 3 times (B) 9 times (C) $\frac{1}{3}$ times (D) $\sqrt{3}$ times
6. At some definite temperature, the mean kinetic energy of the molecule of O_2 (molar mass = 32 g) is 0.048 *e*V. The mean kinetic energy of the molecule of N_2 (molar mass = 28 g) at the same temperature is .... *e*V.
   (A) 0.048 (B) 0.042 (C) 0.056 (D) 0.42
7. 1 mole O_2 gas (molar mass = 32 g), filled in a container has temperature T and pressure P. Hence the pressure of 1 mole He (molar mass = 4 g), filled in an identical container, at temperature 2T is ....
   (A) P (B) 2P (C) 4 P (D) $\frac{P}{2}$
8. In a sample of Chlorine gas, the average kinetic energy per molecule at 300 K is 6.21×10^{-21} J and v_{rms} is 325 m s^{-1}. What will be the values of these quantities at 600 K ?
   (A) 12.42×10^{-21} J, 650 m s^{-1}
   (B) 6.21×10^{-21} J, 650 m s^{-1}
   (C) 12.42×10^{-21} J, 325 m s^{-1}
   (D) 12.42×10^{-21} J, 459.6 m s^{-1}
9. Air is filled in an open vessel at 60 °C. On heating this vessel to temperature T, $\frac{1}{4}^{th}$ of air comes out of the container, then T = ...
   (A) 80 °C (B) 444 °C (C) 333 °C (D) 171 °C

10. If a fan is kept switched on in a closed room then the room ....

(A) gets cooled (B) gets warm
(C) remains at the same temperature (D) may get cooled or warmed

11. The interatomic forces between constituent particles of gases are .... in comparision with the interatomic forces in solids and liquids.

(A) more (B) equal
(C) negligible (D) much more

12. The amount of heat energy required to change the temperature of unit mass of substance by unity is called .... of the material of the substance.

(A) specific heat (B) kinetic energy
(C) heat energy (D) internal energy

13. The internal energy of given amount of an ideal gas depends on .... of the gas.

(A) pressure (B) temperature
(C) volume (D) molar mass

14. The mean free path in gases is of the order of ....

(A) 1 Å (B) 10 Å (C) 10^3 Å (D) 10^5 Å

15. The volume occupied by a molecule of a gas is .... in comparison with the volume of the container.

(A) more (B) negligible (C) much more (D) double

16. If the degrees of freedom of a gas molecule is f, then each molecule of the gas can store the energy in .... different ways.

(A) $2f$ (B) f (C) $f / 2$ (D) f^2

17. For given constant temperature and pressure, the number of molecules per unit volume of a gas ....

(A) is different for different gases.
(B) varies with the volume of molecules of the gas.
(C) is proportional to the molecular mass of the gas.
(D) same for all gases.

18. The pressure of a gas in a container .... due to inter–moleculer collisions of the gas.

(A) does not change (B) changes continuously
(C) increases slowly (D) decreases slowly.

19. The degrees of freedom of CO gas are ....

(A) 3 (B) 5 (C) 7 (D) 9

20. The average kinetic energy of gas molecules is ....

(A) proportional to the absolute temperature of the gas.
(B) proportional to the pressure of the gas.
(C) proportional to the volume of the gas.
(D) depends on the nature of the gas.

21. The degrees of freedom of inert gases like Ar, Ne, He are ....

(A) 3 (B) 5 (C) 7 (D) 9

22. At moderate temperature the degrees of freedom of O_2 are ....

(A) 3 (B) 5 (C) 7 (D) 9

23. The length of a straight path between two consecutive collisions of gas molecules is called ....

(A) degrees of freedom (B) free path
(C) jan marg (D) mean free path

ANSWERS

1. (B)	2. (C)	3. (C)	4. (D)	5. (D)	6. (A)
7. (B)	8. (D)	9. (D)	10. (B)	11. (C)	12. (A)
13. (B)	14. (C)	15. (B)	16. (B)	17. (D)	18. (A)
19. (C)	20. (A)	21. (A)	22. (B)	23. (B)	

Answer the following questions in brief :

1. Which macroscopic quantities are explained by kinetic theory of gases ?
2. What is the value of standard temperature and pressure ?
3. At which temperature and pressure does a real gas behave like an ideal gas ?
4. What is the basis for molecular model of an ideal gas ?
5. What should be the shape of a container of ideal gas for calculating its pressure ?
6. Which quantity is conserved during elastic collision of gas molecules ?
7. On what factors does the average kinetic energy of gas molecules not depend ?
8. What are the macroscopic quantities ?
9. What are the microscopic quantities ?
10. Define the degrees of freedom of a molecule of a gas.
11. Define specific heat of gas at constant volume.
12. Define specific heat of gas at constant pressure.

Answer the following questions :

1. Write the ideal gas state equation and explain its different forms.
2. State the Boyle's law and explain it using graphs.
3. Give the kinetic interpretation of temperature of gas.

OR

Explain the interpretation of temperature of gas in terms of kinetic energy of its molecules.

4. Obtain an expression for the momentum of a gas transferred to the walls of a container gas in time Δt.
5. If the momentum transferred by gas molecules to the wall of a container of gas is $p_1 = nmAv_x^2\Delta t$, obtain an expression for the pressure of gas in the container.
6. Explain the average heat energy and internal energy of an ideal gas in terms of its temperature.

Solve following examples :

1. The volume of a definite amount of a gas at 3 *atm* pressure is 12 L. At constant temperature what should be its pressure so that the volume of the gas may reduce to 9 L ? [**Ans. :** 4 atm]
2. Find the number of molecules in a room of volume 27 m^3 at temperature 27 °C and 1 atm pressure. ($k_B = 1.38 \times 10^{-23}$ J K^{-1})

   [**Ans. :** 6.58×10^{26} molecules]
3. Find the mean translational kinetic energy of a molecules of He at 27 °C.

   ($k_B = 1.38 \times 10^{-23}$ J K^{-1}) [**Ans. :** 6.21×10^{-21} J]
4. At what temperature v_{rms} of O_2 is equal to v_{rms} of H_2 gas at 27 °C ?

   ($m_{O_2} = 32$ g mol^{-1}, $m_{H_2} = 2$ g mol^{-1}) [**Ans. :** 4800 K]

5. An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27 °C. After some oxygen is withdrawn (released) from the cylinder, the gauge pressure drops to 11 *atm* and its temperature drops to 17 °C. Estimate the mass of oxygen taken out of the cylinder. (R = 8.31 J mol^{-1} K^{-1}, Molar mass of O_2 = 32 g mol^{-1}) [**Ans. :** 0.141 kg]

6. At what temperature, is v_{rms} of the molecules of a given gas double of the v_{rms} at 16 °C ? [**Ans. :** 1156 K]

7. Compare v_{rms} of the molecules of hydrogen and oxygen at 27 °C temperature and 1 atm pressure. (Molecular mass of hydrogen = 2 g mol^{-1}, Molecular mass of oxygen = 32 g mol^{-1}) [**Ans. :** $(v_{rms})_{H_2} = 4(v_{rms})_{O_2}$]

8. Find v_{rms} of hydrogen at 0 °C temperature and 1 atm pressure. Density of hydrogen gas is 8.9×10^{-2} kg m^{-3}. [**Ans. :** 1845 m s^{-1}]

9. If the molecular radius of hydrogen molecule is 0.5 Å, find the mean free path of hydrogen molecules at 0 °C temperature and 1 atm pressure.

($k_B = 1.38 \times 10^{-23}$ J K^{-1}) [**Ans. :** $\bar{l} = 8.4 \times 10^{-7}$m]

10. If the mean free path of the molecules of a gas at temperature T and pressure P is l, show that the mean free path of the molecules of the same gas at temperature 2T and pressure $\frac{P}{2}$ is 4 l.

11. The atmosphere contains 3×10^{25} molecules of air in one cubic meter and the mean speed of the moluecules is 10^3 m s^{-1}. Find their mean free path and from that calculate the number of collisions of a molecule with other molecules in 1 s. (i. e. collision frequency). (Take the diameter of air molecules to be 2 Å) [**Ans. :** 1.88×10^{-7} m, and 3.77×10^9 collisions per second]

• • •

SOLUTION

CHAPTER 2

1. $\overline{R} = \dfrac{4{\cdot}12 + 4{\cdot}08 + 4{\cdot}22 + 4{\cdot}14}{4} = 4{\cdot}14\ \Omega.$

Now, take $\Delta R_1 = \overline{R} - R_1$, $\Delta R_2 = \overline{R} - R_2$....... and calculate the average absolute error using formula,

$$\Delta\overline{R} = \frac{1}{n}\sum_{i=1}^{n} |\Delta R_n| = 0.04\,\Omega.$$

Relative error $= \dfrac{\Delta\overline{R}}{\overline{R}} = \dfrac{0{\cdot}04}{4{\cdot}14} = 0.0096$

Percentage error = 0.0096 × 100 = 0.96%

2. Density of cylinder $\rho = \dfrac{m}{V} = \dfrac{m}{\pi r^2 l}$

Now, use the formula $\dfrac{\Delta\rho}{\rho} = \dfrac{\Delta m}{m} + 2\dfrac{\Delta r}{r} + \dfrac{\Delta l}{l}$ and hence calculate

percentage error using formula $\dfrac{\Delta\rho}{\rho} \times 100\ \%$

3. $T = 2\pi\sqrt{\dfrac{l}{g}}\ \therefore\ g = 4\pi^2\dfrac{l}{T^2}$

$$\therefore\ \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T} = \frac{0.1}{100} + 2\frac{0.01}{2} = 0.011$$

Percentage error = 0.011 × 100 = 1.1 %

4. Total area of sheet $= 2(\ (l \times b) + (b \times t) + (t \times l)\ )$

put $l = 4.234$ m, $b = 1.005$ m, $t = 2.01 \times 10^{-2}$ m

Area = 2(4.3604739) = 8.7209478 m^2 = 8.72 m^2

Volume of a sheet $= l \times b \times t$ = 0.0855289 = 0.086 m^3

($t = 2.01 \times 10^{-2}$ m has the least number of significant figures (3). Therefore the answer is rounded off up to 3 significant digits.

5. $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1\ q_2}{r^2}\ \therefore\ \varepsilon_0 = \dfrac{1}{4\pi} \times \dfrac{q_1\ q_2}{F\cdot r^2} = \dfrac{C^2}{N\ m^2} = N^{-1}\ C^2\ m^{-2}$

$$[\varepsilon_0] = \frac{[q_1]\ [q_2]}{[F]\ [r]^2} = \frac{(A^1T^1)\ (A^1T^1)}{(M^1\ L^1\ T^{-2})(L^1)^2} = M^{-1}\ L^{-3}\ T^4\ A^2$$

7. $[c] = [L\ T^{-1}] = 3 \times 10^8\ m\ s^{-1}$

$[g] = [L\ T^{-2}] = 10\ m\ s^{-2}$

$[P] = [M^1\ L^{-1}\ T^{-2}] = 10^5\ N\ m^{-2}$

Now, Solve the above equations for M, L and T,

7. $[c] = [t] = M^0 L^0 T^1$

Now, $[at] = [v]$ $\therefore$ $[a] = \left[\frac{v}{t}\right] = M^0 L^1 T^{-2}$

and $\left[\frac{b}{t+c}\right] = [v]$ $\therefore$ $[b] = [v]\ [t + c] = M^0 L^1 T^0$

9. $v \propto kg^a h^b$

$(M^0 L^1 T^{-1}) = (M^0 L^1 T^{-2})^a (M^0 L^1 T^0)^b$

From this, we will get $a = \frac{1}{2}$, $b = \frac{1}{2}$

$\therefore\ v \propto kg^{\frac{1}{2}} h^{\frac{1}{2}}$

10. $T \propto p^a \rho^b E^c$

Write the dimensional formulae of both the sides and compare them. You will get

$a = -\frac{5}{6}$, $b = \frac{1}{2}$, $c = \frac{1}{3}$.

CHAPTER 3

1. $t_1 = \frac{x/3}{10} = \frac{x}{30}$ h, $t_2 = \frac{x/3}{20} = \frac{x}{60}$ h, $t_3 = \frac{x/3}{30} = \frac{x}{90}$ h

Average velocity $= \frac{x}{t_1 + t_1 + t_1} = \frac{x}{\frac{x}{30} + \frac{x}{60} + \frac{x}{90}} = 16.36$ km h^{-1}

2. The time required to cover the first 5 km $t_1 = \frac{10}{v}$ h

The time required to cover the next 20 km, $t_2 = \frac{20}{v}$ h

The time required to cover last 15 km $t_3 = \frac{30}{v}$ h

Total time $= \frac{10}{v} + \frac{20}{v} + \frac{30}{v} = 1$ h

From that $v = 60$ km h^{-1},

3. Consider the upward motion of the monkey as positive and downward motion as negative,

$x = 5\text{m} + (-3\text{ m}) + 5\text{ m} + (-3\text{ m}) + 5\text{ m} + (-3\text{ m}) + 5\text{ m} + (-3\text{ m}) + 5\text{ m}$

$= 13\text{m}$

The required time

$t = (1 + 1) + (1 + 1) + (1 + 1) + (1 + 1) + 1 = 9$ s

4. For, first 120m, $v_0 = 0$, $x = 120$m, $a = 2.6$m s^{-2} from $2ax = v^2 - v_0^2$,

$v = \sqrt{624}$ m s^{-2} for the rest of the distance, $v_0 = \sqrt{624}$ m s^{-1}, $v = 12$m s^{-1},

$a = -1.5$m s^{-2} from $x = \frac{v^2 - v_0^2}{2a}$, $x = 160$ m

Total distance travelled = 120 + 160 = 280 m

5. Take $x = 16$ m, $v = 0$ and $a = -9.8$ m s^{-2} and from $2ax = v^2 - v_0^2$ calculate v_0. Suppose at height h', initial velocity becomes half $\left(\frac{v_0}{2}\right)$ Now take $x = h'$, $v = \frac{v_0}{2}$ and use above euqations to find h'.

6. Suppose both the objects meet at height h (from the bottom of the tower) and at time t. For a freely falling object, $v_0 = 0$, $x = (39.2 - h)$ m and for an object thrown in vertically upward directions.

$v_0 = 19.6$ m s^{-1}, $x = h$ m. Now use equation $x = v_0 t + \frac{1}{2}at^2$ and calculate t and h.

7. $$v = \frac{dx}{dt} = \frac{d}{dt}(t^3 + 4t^2 - 2t + 5) = 3t^2 + 8t - 2$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 8t - 2) = 6t + 8$$

Putting $t = 4$ s in the above equation we shall get
$v = 78$ m s^{-1} and $a = 32$ m s^{-2}
Now put $t = 0$ and $t = 4$ s in $x = t^3 + 4t^2 - 2t + 5$ and find out $v(0)$ and $v(4)$.
Now put $t = 0$ and $t = 4$ s in $v = 3t^2 + 8t - 2$ and find out $v(0)$ and $v(4)$.

Now, $< a > = \frac{v(4) - v(0)}{4 - 0} = 20$ m s^{-2}

8. The relative speed of train A w.r.t. train B is $v_A - v_B = 30 - 10 = 20$ m s^{-1}
Now, put $v = 0$, $a = -2$ m s^{-2}s in
$2ax = v^2 - v_0^2$ and find out x.

9. For both the cases use the equation $v = v_0 + at$. You will get $a = 4$ m s^{-2} and $v_0 = 8$ m s^{-1}. Put these values in $x = v_0 t + \frac{1}{2}at^2$, the displacement will be $x = 570$ m.

10. (a) The area under curve of $v - t$ graph gives the distance travelled by a particle

$\therefore$ distance $= \frac{1}{2}(12)(10) = 60$ m

(b) The slope of the line OA during the time interval 0 – 5 s will give acceleration $a = 2.4$ m s^{-2}. Now put $a = 2.4$ m s^{-2} in $x = v_0 t + \frac{1}{2}at^2$ and calculate the distance travelled in the time interval of 2 s to 5 s. Same way the slope of line AB gives acceleration $a = -2.4$ m s^{-2}. Again, use $x = v_0 t + \frac{1}{2}at^2$ and calculate the distance for time interval of 5 s to 6 s.

11. The relative velocity of the bird w.r.t. train will be $v_b - v_t = 5 - (-10) = 15$ m s^{-1}. Time required to cross the train $= \frac{120 \text{ m}}{15 \text{ m s}^{-1}} = 8$ s.

12. $v = 4\ t$, Therefore, $a = \dfrac{v}{t} = 4 \text{ m s}^{-2}$.

Now, use $x = v_0 t + \dfrac{1}{2} at^2$

$t = 2$ s, distance travelled $x(2) = 0 + \dfrac{1}{2}(2)(2)^2 = 8$ m

t = 4 s, distance travelled $x(4) = 0 + \dfrac{1}{2}(2)(4)^2 = 32$ m

Therefore, distance travelled in $t = 2$ s to $t = 4$ s will be = 32 m – 8 m = 24 m

Second Method :

$v = 4t$

$\therefore \dfrac{dx}{dt} = 4t$

$\therefore dx = 4t\ dt$

$\therefore \int_0^x dx = \int_2^4 4t\ dt$

$x = 4\left[\dfrac{t^2}{2}\right]_2^4 = 2(4^2 - 2^2) = 24$ m

CHAPTER 4

1. For the resultant force, use the formula $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ where A = F, B = F and θ = θ

2. Find $\vec{A} - \vec{B}$. For the unit vector of $\vec{A} - \vec{B}$ use the formula. $\hat{n} = \dfrac{\vec{A} - \vec{B}}{\left|\vec{A} - \vec{B}\right|}$

3. Calculate according to illustration 13.

4. (a) Average speed $= \dfrac{\text{distance}}{\text{time}} = \dfrac{23\text{km}}{\left(\dfrac{28}{60}\right)\text{hr}}$

(b) Magnitude of average velocity $= \dfrac{\text{Magnitude of displacement}}{\text{time}}$

$= \dfrac{10\text{km}}{\left(\dfrac{28}{60}\right)\text{hr}}$

5. (a) at $t = 0$, $\vec{v_0} = 10\hat{j}$,

$\therefore v_{0x} = 0\ ms^{-1}$, $v_{0y} = -10 \text{ m s}^{-1}$

Constant acceleration $\vec{a} = 8\hat{i} + 2\hat{j}$, $\therefore a_x = 8 \text{ m s}^{-2}$, $a_y = 2.0 \text{ m s}^{-2}$

$$x = v_{0x}t + \frac{1}{2}a_x t^2, \therefore\ 16 = 0 + \frac{1}{2}8t^2, \quad \therefore\ t = 2\ sec$$

$$y = v_{0y}t + \frac{1}{2}a_y t^2, \text{ Substitute } t = 2\ sec \Rightarrow y = 24 \text{ m}$$

(b) $\vec{v} = \vec{v}_0 + \vec{a}\ t; \qquad \therefore\ \vec{v}_0 = 10\hat{j},$

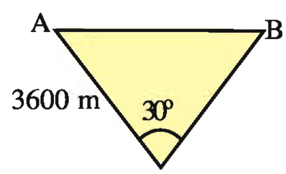
$\vec{a} = 8\hat{i} + 2\hat{j}$

$\therefore\ \vec{v} = 16\hat{i} + 14\hat{j};\ v_x = 16 \text{ m s}^{-1},\ v_y = 14 \text{ m s}^{-1}$

$|\vec{v}| = \sqrt{v_x^2 + v_y^2} \Rightarrow |\vec{v}| = 21.26 \text{ m s}^{-1}$

6. Suppose the distance covered by the plane at the height 3600 m in 10 seconds. Here (as shown in the Figure) the distance AB can be considered as an arc of a circle with radius 3600 m which subtends angle 30° at the centre.

Arc = Radius × Angle (in Radians)



$\therefore$ AB $= 3600 \times \left(\frac{30\pi}{180}\right)$

$v = \frac{\text{distance}}{\text{time}} = \frac{\text{AB}m}{10s} \Rightarrow 60\pi \text{ m s}^{-1}$

7. $\theta_0 = 30°$, R = 3 km, using formula $R = \frac{v_0^2 sin2\theta}{g}$

Find v_0. Using v_0 in formula for the maximum range find out the maximum range

$$R_{max} = \frac{v_0^2}{g}$$

If the target is farther away then the maximum range, the bullet will not hit the target.

8. $\frac{H}{R} = \frac{v_0^2 sin^2\theta_0}{2g} \times \frac{g}{v_0^2 2sin\theta_0\ cos\theta_0} = \frac{sin\theta_0}{4cos\theta_0}$

$\therefore\ tan\ \theta_0 = \frac{4H}{R} \quad \therefore\ \theta_0 = tan^{-1}\frac{4H}{R}$

9. $\vec{C} = \vec{A} + \vec{B}$

$\therefore\ |\vec{C}| = C = |\vec{A} + \vec{B}|$

Substitute R = C in $R = \sqrt{A^2 + B^2 + 2ABcos\theta}$

$\therefore\ C^2 = A^2 + B^2 + 2ABcos\theta$ (1)

But A + B = C is given

$\therefore\ (A + B)^2 = C^2 \Rightarrow A^2 + B^2 + 2AB = C^2$ (2)

Using (1) and (2)

$2AB = 2AB\cos\theta$

$\therefore \ \cos\theta = 1 \Rightarrow \theta = 0^\circ$

Hence $\vec{A}$ and $\vec{B}$ are in the same directin (i.e. $\vec{A}$ and $\vec{B}$ are parallel)
Here A + B = C is given. Hence the three vectors would be in the same direction. Hence $\theta = 0$

10. Changing the direction of $\vec{A}_1 = \vec{A}$; $\vec{A}_2 = -\vec{A}$

$\therefore \ \Delta\vec{A} = \vec{A}_2 - \vec{A}_1 = -\vec{A} - \vec{A} = -2\vec{A}$

$\Delta\vec{A} = |-2\vec{A}| = 2A$

$\Delta\vec{A}$ means change in the magnitude of vector $\vec{A}$. Now changing the direction of the vector, the magnitude of the vector does not change.

$\therefore \ \Delta\vec{A} = |\vec{A}_2| - |\vec{A}_1| = A - A = 0$

11. Suppose $\vec{A}_1 = A$ and $\vec{A}_2 = 2\vec{A}$

X and Y components of $\vec{A}_1$, $A_{1x} = A\cos\theta$ and $A_{1y} = A\sin\theta$

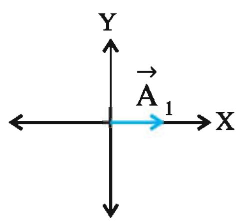
X and Y components of $\vec{A}_1$, $A_{2x} = 2A\cos\theta$ and $A_{2y} = 2A\sin\theta$

$\therefore \ A_{2x} = 2A_{1x}$ and $A_{2y} = 2A_{1y}$

12. Suppose the given vector is $\vec{A}_1$

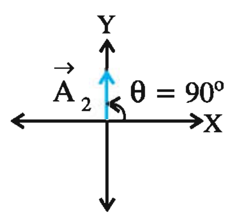
$\therefore \ A_{1x} = \cos 0^0 = A_1$, $A_{1y} = A_1\sin 0^0 = 0$

If only the direction of the vector is changed, the magnitude of the vector do not change.



(i) when $\vec{A}_1$ is rotated through 90° it becomes $\vec{A}_2$

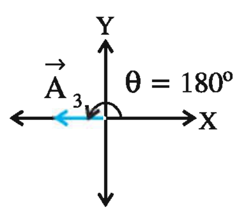
$\therefore \ A_{2x} = A_1\cos 90^\circ = 0$, $A_{2y} = A_1\sin 90^\circ = A_1$



(ii) $\vec{A}_1$ is rotated through 180° becomes $\vec{A}_3$

$A_{3x} = A_1\cos(180^\circ) = -A_1$,

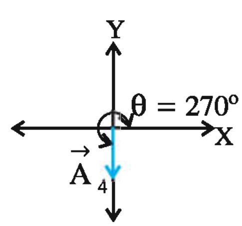
$A_{3y} = A_1\sin(180^\circ) = 0$



(iii) $\vec{A}_1$ is rotated through 270° becomes $\vec{A}_4$

$A_{4x} = A_1\cos(270^\circ) = A_1\cos(180^\circ+90^\circ) = A_1\cos 90^\circ = 0$

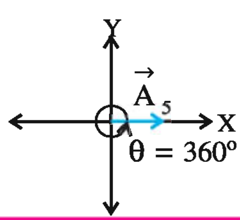
$A_{4y} = A_1\sin(270^\circ) = A_1\sin(180^\circ+90^\circ) = A_1\sin 90^\circ = -A_1$



(iv) $\vec{A}_1$ is rotated through 360 becomes $\vec{A}_5$,

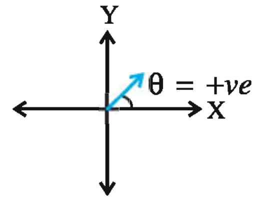
which is nothing but $\vec{A}_1$

$\therefore \ A_{5x} = A_{1x} = A_1$; $A_{5y} = A_{1y} = 0$

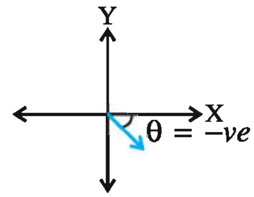


13. Yes, when two objects are moving opposite to each other, their relative velocity will be the summation of the velocities of the two objects, which will be greater than the velocity of each object.

14. $|\hat{i} + \hat{j}| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$



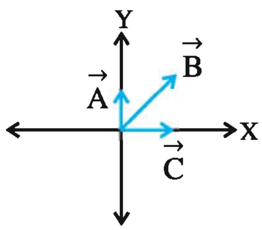
$$\theta = tan^{-1}\left(\frac{y}{x}\right) = tan^{-1}\,\theta = 45°$$



$|\hat{i} + \hat{j}| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$

$$\theta = tan^{-1}\frac{-1}{1} = tan^{-1}\,\theta = -45°$$

15. A = 100 units, B = 200 units, C = 150 units.



X = Components	Y = Components
$A_x = Acos90° = 0$ units	$A_y = Asin90° = 100$ units
$B_x = Bcos60° = 100$ unit	$B_y = Bsin60° = 173$ units
$C_x = Ccos0° = 150$ units	$C_y = Csin0° = 0$ units

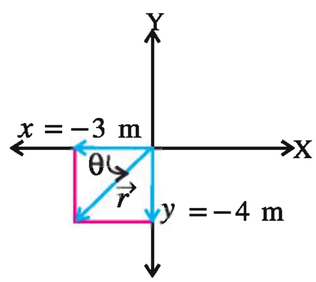
16. Position vector $\vec{r} = x\hat{i} + y\hat{j}$

$x = -3, \quad y = -4$

$\vec{r} = -3\hat{i} - 4\hat{j}$

$\vec{r} = \sqrt{(-3)^2 + (-4)^2} = 5$ m

$tan\,\theta = \left(\frac{-4}{-3}\right) = 1.333$



$\theta = tan^{-1}\,1.333$

17. With the change in the direction of the axis, the magnitude and the direction of a vector do not change. Of course the value of the components along the axis change.

$\therefore\ \vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ are independent of the selection of axis. But the values $A_x + B_y$ will change with the change in the position of the axis. i.e. It will depend on the selection of axis.

18. $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

$\therefore\ A^2 + B^2 + 2ABcos\theta = A^2 + B^2 - 2ABcos\theta$

$\therefore\ 2ABcos\theta = -2ABcos\theta$

$\therefore\ 4ABcos\theta = 0 \Rightarrow\ cos\theta = 0$

$\therefore\ \theta = 90$

19. In the formula for $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$, substituting $t = 0$ gives position vector $\vec{r_0}$ at $t = 0$; and on substituting $t = 10$ s in the same formula gives $\vec{r_{10}}$, the position vector at the end of the 10[th] second.

$\therefore$ displacement during 10 seconds $\Delta\vec{r} = \vec{r_{10}} - \vec{r_0}$

20. The component of the vector $\vec{A}$ in the direction of vector $\hat{i} + \hat{j}$ is given by A $cos\ \theta$ where θ is the angle between $\vec{A}$ and vector $\hat{i} + \hat{j}$ given by

$$A\ cos\theta = \frac{\vec{A} \cdot (\hat{i} + \hat{j})}{|\hat{i} + \hat{j}|}$$

21. The component of a non-zero vector taken in the direction perpendicular to the non-zero vector is zero. Hence the non-zero vector has zero component.
If a given vector has non-zero component, it indicates that the vector has some magnitude (value) because the magnitude of a vector is always greater than the value of its component. Hence, vector with non-zero component cannot be a zero vector.

22. $\vec{A} + \vec{B} = \vec{C}$

$\therefore |\vec{A} + \vec{B}|^2 = |\vec{C}|^2$

$C^2 = A^2 + B^2 + 2ABcos\theta$

Now $A^2 + B^2 = C^2$

$\therefore 2ABcos\theta = 0$

$\therefore cos\theta = 0$

$\therefore \theta = 90^o$

Hence the vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

23. As the range of both the objects is same, we have $\theta_{01} + \theta_{02} = \frac{\pi}{2}$

time of flight $t = \frac{2v_0 sin\theta_0}{g}$

$$t_1 \times t_2 = \frac{2v_{01} sin\theta_{01}}{g} \times \frac{2v_{01} sin\theta_{02}}{g}$$

But $\theta_{02} = \frac{\pi}{2} - \theta_{01}$

$$\therefore\ t_1\ t_2 = \frac{2v_{01}^2}{g^2} 2sin\theta_{01}\ sin(\frac{\pi}{2} - \theta_{01})$$

$$= \frac{2v_{01}^2}{g^2}\ 2sin\theta_{01} cos\theta_{01}$$

$$= \frac{2v_{01}^2}{g^2}\ sin2\theta_{01}$$

$$= \frac{2}{g} R$$

CHAPTER 5

1. Impulse of force $\vec{F}\Delta t = m\Delta\vec{v}$ = change in momentum $\Delta\vec{p}$

$\Delta\vec{p}_1 = m_1 \vec{v}_1{}' - m_1 \vec{v}_1$ $\qquad m_1 \vec{v}_1 = (0.08)(5)\ \hat{i}$ kg m/s

$\Delta\vec{p}_2 = m_2 \vec{v}_2{}' - m_2 \vec{v}_2$ $\qquad m_2 \vec{v}_2 = (0.08)(5)(-\hat{i})$ kg m/s

$m_1 \vec{v}_1{}' = (0.08)(5)(-\hat{i})$ kg m/s

$m_2 \vec{v}_2{}' = (0.08)(5)(-\hat{i})$ kg m/s

Now proceed further.

2. Acceleration of entire system $= a = \dfrac{F}{(m_1 + m_2)}$

$F = 2$ N, $m_1 = 6$ kg, $m_2 = 2$ kg

Force on 2 kg mass $= (m_2)(a)$

3. Acceleration of entire system $= a = \dfrac{F}{(m_1 + m_2 + m_3)}$

$F = 12$ N, $m_1 = 1$ kg, $m_2 = 2$ kg, $m_3 = 3$ kg

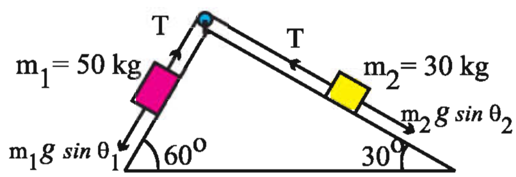
contact force on 2 kg block by the first block.

$F_2 = (m_2 + m_3)a$ **or** $F_2 = F - m_1 a$

Contact force on 3 kg block $F_3 = (m_3)a$ **or** $F_3 = F - (m_1 + m_2)a$

4. Find $m_1 g sin60°$ and $m_2 g sin60°$.

The greater value will decide the direction of motion.



$m_1 g sin\theta_1 - T = m_1 a$ ...(1)

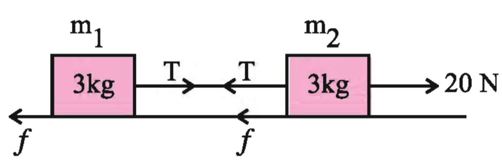
$T - m_2 g sin\theta_2 = m_2 a$ ...(2)

Adding we get,

$m_1 g sin\theta_1 - m_2 g sin\theta_2 = (m_1 + m_2)a$

Find a from this equation. Substitute its value in equation 1 or 2 to find T.

5.



$F = 20$N, $m_1 = m_1 = 3$ kg, $a = 0.5$ m/s^2

Think of FBD,

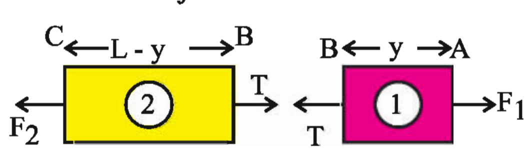
for block with mas m_1; $T - f = m_1 a$ ...(1)

for block with mas m_2; $F - T - f = m_2 a$

Adding we get, $F - 2f = (m_1 + m_2)a$

From this find f and then substitute its value in equation (1) or (2) to find T.

6.



Let the total mass of rod = M

$\therefore$ mass per unit length $\lambda = \dfrac{M}{L}$ $\therefore$ $M = \lambda L$

$\therefore$ mass of part 1 of rod, $m_1 = y\lambda = (y)\dfrac{M}{L}$

and mass of part 2 of rod, $m_2 = (L - y)\lambda = (L - y)\dfrac{M}{L}$

For part 1, FBD gives $F_1 - T = m_1 a = \left(\frac{My}{L}\right) a$

For part 2, FBD gives $T - F_2 = m_2 a = \left[\left(\frac{M}{L}\right)(L - y)\right] a$

Adding we get $F_1 - F_2 = Ma$

$\therefore\ a = \frac{F_1 - F_2}{M}$

$T - F_2 = m_2 a$ gives

$T = F_2 + m_2 a = F_2 + \left(\frac{M}{L}\right)(L - y)\left(\frac{F_1 - F_2}{M}\right)$

Now proceed.

7. (i) For small time interval before 2 s; x is constant. $\therefore\ \vec{v}_1 = 0$ Similarly for small interval after 2 s also, x is constant $\therefore\ \vec{v}_2 = 0$

$\therefore$ Impulse of force $\vec{F}\Delta t = m\Delta\vec{v} = m\,(\vec{v}_2 - \vec{v}_1) = 0$

(ii) Here $\vec{v}_1 = \frac{20\,\hat{i}}{2} = 10\hat{i}$ m/s

$\vec{v}_2 = \frac{-20\,\hat{i}}{4} = -5\hat{i}$ m/s

$\therefore$ Impulse of force $\vec{F}\Delta t = m\Delta\vec{v} = m\,(\vec{v}_2 - \vec{v}_1)$

$= (2)\,(-5\hat{i} - 10\hat{i})$

$= -30\hat{i}$ N s

$\therefore\ |\vec{F}\Delta t| = 30\hat{i}$ N s

$\therefore\ |\vec{F}\Delta t| = 30$ N s

8. Mutual gravitational force have equal magnitude = F

$\therefore\ F = m_1 a_1 = m_2 a_2$ (in magnitude)

Since $v_0 = 0$ for both, $\frac{s_1}{s_2} = \frac{\frac{1}{2}a_1 t^2}{\frac{1}{2}a_2 t^2} = \frac{a_1}{a_2} = \frac{m_2}{m_1}$.

9. (i) For first half length (d), $v_0 = 0$

acceleration $a = g\sin\theta\ \ \therefore\ v^2 - 0 = 2\,(g\sin\theta)d$ (1)

(ii) For second half length (d), the frictional force f is greater than $mg\sin\theta$, Hence, the downword motion along the slope is a retarded motion. This frictional force, $f = \mu N = \mu mg\cos\theta$

∴ The value of deceleration (retardation) is,

$$a' = \frac{f - mg\sin\theta}{m} = \frac{\mu mg\cos\theta - mg\sin\theta}{m} \quad g(\mu\cos\theta - \sin\theta)$$

For this downward motion,

$$0 - v^2 = 2(-a')d \qquad (2)$$

From equations (1) and (2)

$$-2gd\sin\theta = 2[-g(\mu\cos\theta - \sin\theta)d]$$

This gives $\mu = 2\tan\theta$.

CHAPTER 6

1. Find the mechnical energy for the situation shown in the figure. Now draw the figure for the situation in which the block of 2 kg touches the reference surface. For this situation also calculate the mechnical energy. Compare both values of mechanical energy.

2. As per law of conservation of momentum

$$m_1\vec{v_1} = m_1\vec{v_1'} + m_1\vec{v_2'}$$

Now $m_1 = m_2 = m$

$$\therefore \vec{v_1} = \vec{v_1'} + \vec{v_2'}$$

$$\therefore v_1^2 = v_1'^2 + v_2'^2 + 2v_1'\,v_2'\,cos\,\theta$$

3. Use law of conservation of momentum

4. $x = t^2 - 6t + 9$ find $v = \frac{dx}{dt}$

for $v = 0$ find x.

Displacement $= x - x_0$

Find acceleration $a = \frac{dv}{dt}$ which is zero

∴ Force = 0 ∴ work = ?

5. To find v at bottom use the law of conservation of mechenical energy. Acceleration parallel to slope is $a = g\ sin\theta$. Now use the equations of motion.

6. As the system is stationary the initial mometum = θ

$\therefore MV + mv = 0 \therefore |M\ V| = |-m\ v| = P$

As per law of conservation of mechanical energy

$$\frac{1}{2}kx^2 = \frac{1}{2}MV^2 + \frac{1}{2}mv^2$$

$$= \frac{P^2}{2M} + \frac{p^2}{2m}$$

Now try further.

7. For the motion from A to B, use the conversion of PE into KE to determine the velocity of A and B. For collision between A and B use law of conservation of momentum and find the velocity of B. For the final position again think about conversion of KE into PE.

8. PE near A = mgr.

Near D while moving from A to B

$$mgr = \frac{1}{2}mv^2 + \frac{\pi}{4} \times r \times R$$

Similarly think about the motion from B to C.

9. Initial kinetic energy = $\frac{1}{2}mv_0^{\,2}$

KE when velocity redues to half of original value.

$$= \frac{1}{2}\frac{mv_0^{\,2}}{4}$$

$$\therefore \frac{1}{2}\frac{mv_0^{\,2}}{4} - \frac{1}{2}mv_0^{\,2} = F \times 6$$

$$\therefore F = -\frac{3}{4}\left(\frac{1}{2}mv_0^{\,2}\right) \times \frac{1}{6}$$

Now try further.

10. Since initial and final speeds are same $K - K_0 = W = 0$

Also W = work done against frictional force + work done by graviational force. Now try further.

CHAPTER 7

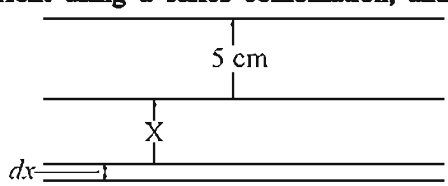
1. Write the equations for the heat current for both and explain.
2. First write each data in SI unit.

(i) $H = kA\,\frac{T_2 - T_1}{L}$

(ii) Layer of bricks and slab from composite slab.

Calculate the thermal resistance for current using a series combination, and then calculate heat current.

3. Consider a layer of thickness dx and surface area A. Heat required to be taken away to form such layer is



$dQ = A\,dx\,\rho L'$ ρ in density and L′ latent heat of melting

If for passage of this much heat through $5 + x$ cm thick slab is dt

$$dQ = kA\,\frac{\Delta T}{5 + x}\,dt$$

Compare the equation, integrate and get the answer.

4. In $H = \frac{dQ}{dt} = \sigma e A T^4$ as $A = 1\ m^2$, $\frac{dQ}{dt} = 6.3 \times 10^7 W$

$e = 1$ and value of σ

5. Use $H = \sigma e A\,(T^4 - T_s^{\,4})$
6. For the composite slab, determine the effective thermal resistance using formulae for series and parallel connections. Compare with $R = L'/A'k$ where $L' = 4x$, $A' = 2x^2$
7. $K = a + bT$

∴ K and T bear a linear relationship.

∴ T can be replaced by the avarage value of the maximum and minimum values of T. i.e. $\left(\dfrac{T_1 + T_2}{2}\right)$

$\therefore K = a + b\left(\dfrac{T_1 + T_2}{2}\right)$

Insert value of K in the equation.

Note : Instead of using the avarage value of T, same result canbe obtained by integration.

8. Consider unit surface area on earth to be a point of sphere of rudius R_0, at average distance between the earth and sun.

Now $S = \dfrac{H}{4\pi R_0^2}$ and $H = \sigma\ 4\pi\ R_s^2\ T^4$.

CHAPTER 8

1. According to the ideal gas state equation

$PV = \mu RT$

Hence at constant temperature

$P_1V_1 = P_2V_2 = \mu RT$

$\therefore\ P_2 = \dfrac{P_1V_1}{V_2}$

2. $PV = k_B NT$

$\therefore\ N = \dfrac{PV}{k_B T}$

3. $< E > = \dfrac{3}{2}\ m < v^2 > = \dfrac{3}{2}\ k_B T$

4. $\dfrac{1}{2} m < v^2 > = \dfrac{3}{2}\ k_B T$

$\therefore\ < v^2 > = \dfrac{3k_B T}{m}$

But $(v_{rms})_{O_2} = (v_{rms})_{H_2}$

$\therefore\ \dfrac{3k_B T_{O_2}}{m_{O_2}} = \dfrac{3k_B T_{H_2}}{m_{H_2}}$

$\therefore\ T_{O_2} = \left(T_{H_2} \times m_{O_2}\right) \Big/ m_{H_2}$

5. $PV = \mu RT = \dfrac{M}{M_0}\ RT$

$\therefore\ M = \dfrac{M_0 PV}{RT}$

$\therefore\ \Delta M = M_1 - M_2 = \dfrac{M_0 VP_1}{RT_1} - \dfrac{M_0 VP_2}{RT_2} = \dfrac{M_0 V}{R}\left[\dfrac{P_1}{T_1} - \dfrac{P_2}{T_2}\right]$

6. $(v_{rms})_2 = 2(v_{rms})_1$

$$\frac{1}{2}\ m < v_{rms}{}^2 >_1 = \frac{3}{2}\ k_B T_1$$

$$\frac{1}{2}\ m < v_{rms}{}^2 >_2 = \frac{3}{2} k_B T_2$$

$$\therefore \frac{< v_{rms}{}^2 >_2}{< v_{rms}{}^2 >_1} = \frac{T_2}{T_1}$$

$$\therefore T_2 = \frac{< v_{rms}{}^2 >_2 \times T_1}{< v_{rms}{}^2 >_1}$$

7. $\frac{1}{2}\ m_{H_2} < v^2 >_{H_2} = \frac{3}{2}\ k_B T$

$$\frac{1}{2}\ m_{O_2} < v^2 >_{O_2} = \frac{3}{2} k_B T$$

$$\therefore \frac{m_{H_2} < v^2 >_{H_2}}{m_{O_2} < v^2 >_{O_2}} = 1$$

$$\therefore < v_{rms}{}^2 >_{H_2} = \frac{m_{O_2} < v^2 >_{O_2}}{m_{H_2}}$$

$$\therefore (v_{rms})_{H_2} = \sqrt{\frac{m_{O_2}}{m_{H_2}}}\ (v_{rms})_{O_2}$$

8. $v_{rms} = \sqrt{\frac{3P}{\rho}}$

9. $\bar{l} = \frac{1}{\sqrt{2}\pi n d^2}$

But $P = n k_B T$

$$\therefore n = \frac{P}{k_B T}$$

$$\therefore \bar{l} = \frac{k_B T}{\sqrt{2} P \pi d^2}$$

10. Use $\bar{l}_1 = \frac{k_B T_1}{\sqrt{2} P_1 \pi d^2}$

$$\bar{l}_2 = \frac{k_B T_2}{\sqrt{2} P_2 \pi d^2}$$

11. $\bar{l} = \frac{1}{\sqrt{2}\pi n d^2}$

Number of collissions per second $= n\pi d^2 \bar{v}\, t$

APPENDICES

APPENDIX 1

THE GREEK ALPHABET

Alpha	Α	α	Iota	Ι	ι	Rho	Ρ	ρ
Beta	Β	β	Kappa	Κ	κ	Sigma	Σ	σ
Gamma	Γ	γ	Lambda	Λ	λ	Tau	Τ	τ
Delta	Δ	δ	Mu	Μ	μ	Upsilon	Υ	υ
Epsilon	Ε	ε	Nu	Ν	ν	Phi	Φ	ϕ
Zeta	Ζ	ς	Xi	Ξ	ξ	Chi	Χ	χ
Eta	Η	η	Omicron	Ο	ο	Psi	Ψ	ψ
Theta	Θ	θ	Pi	Π	π	Omega	Ω	ω

APPENDIX 2

SOME IMPORTANT CONSTANTS

Name	Symbol	Value
Speed of light in vacuum	c	2.9979×10^{8} m s^{-1}
Charge of electron	e	1.602×10^{-19} C
Universal constant of gravitation	G	6.673×10^{-11} N m^{2} kg^{-2}
Planck's constant	h	6.626×10^{-34} J s
Boltzmann constant	k	1.381×10^{-23} J K^{-1}
Avogadro number	N_A	6.022×10^{23} mol^{-1}
Universal gas constant	R	8.314 J mol^{-1} K^{-1}
Mass of electron	m_e	9.110×10^{-31} kg
Mass of neutron	m_n	1.675×10^{-27} kg
Mass of proton	m_p	1.673×10^{-27} kg
Electron charge to mass ratio	e/m_e	1.759×10^{11} C/kg
Faraday constant	F	9.648×10^{14} C/mol
Rydberg constant	R	1.097×10^{7} m^{-1}
Bohr radius	a_0	5.292×10^{-11} m
Stefan-Boltzmann constant	σ	5.670×10^{-8} W m^{-2} K^{-4}
Wien's Constant	b	2.898×10^{-3} m K
Permitivity of free space	ε_0 $1/4\pi\varepsilon_0$	8.854×10^{-12} C^{2} N^{-1} m^{-2} 8.987×10^{9} N m^{2} C^{-2}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T m A^{-1} 1.257×10^{-6} Wb A^{-1} m^{-1}

APPENDIX 3

TRIGONOMETRY

In right angle triangle, if a, b and c are opposite sides of angles A, B and C respectively than trigonometric functions are defined as follows :

(i) $sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{b}{c} = cos\phi$

(ii) $cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{a}{c} = sin\phi$

(iii) $tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}} \cdot = \dfrac{b}{a} = cot\phi$

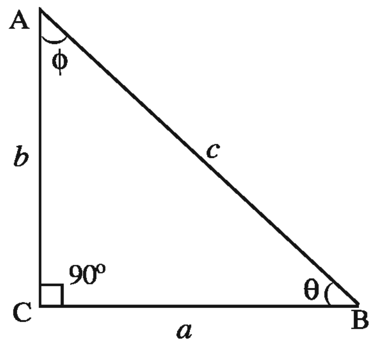
(iv) $cot\theta = \dfrac{\text{adjacent side}}{\text{opposite side}} = \dfrac{a}{b}$

(v) $sec\theta = \dfrac{1}{cos\theta} = \dfrac{c}{a}$

(vi) $sec\theta = \dfrac{1}{sin\theta} = \dfrac{c}{b}$

(vii) $tan\theta = \dfrac{sin\theta}{cos\theta}$

(viii) $cot\theta = \dfrac{cos\theta}{sin\theta}$



$\phi = 90^o - \theta$

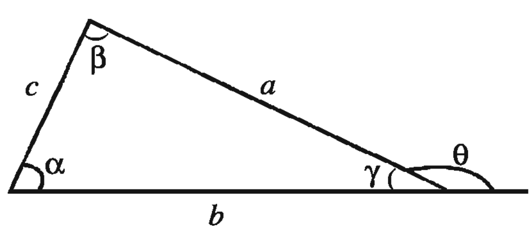
SINE AND COSINE RULES

(i) $\dfrac{sin\ \alpha}{a} = \dfrac{sin\ \beta}{b} = \dfrac{sin\ \gamma}{c}$

(ii) $c^2 = a^2 + b^2 - 2\ abcos\ \gamma$

(iii) Exterior angle, $\theta = \alpha + \beta$

TRIGONOMETRIC IDENTITIES



(i) $sin^2\theta + cos^2\theta = 1$

(ii) $1 + tan^2\theta = sec^2\theta$

(iii) $1 + cot^2\theta = cosec^2\theta$

(iv) $sec^2\theta - tan^2\theta = 1$

(v) $cosec^2\theta - cot^2\theta = 1$

(vi) $sin2\theta = 2sin\theta\ cos\theta$

(vii) $cos2\theta = cos^2\theta - sin^2\theta = 2cos^2\theta - 1 = 1 - 2sin^2\theta$

(viii) $sin(\alpha \pm \beta) = sin\alpha cos\beta \pm cos\alpha sin\beta$

(ix) $cos(\alpha \pm \beta) = cos\alpha cos\beta \mp sin\alpha sin\beta$

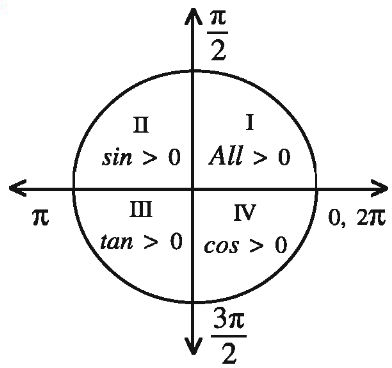
(x) $\sin\alpha \pm \sin\beta = 2\sin\left(\frac{\alpha \pm \beta}{2}\right)\cos\left(\frac{\alpha \mp \beta}{2}\right)$

(xi) $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$

(xii) $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$

Sign of $\sin\theta$, $\cos\theta$ and $\tan\theta$ in different quadrants :

Quadrant	*sin*	*cos*	*tan*
I	+	+	+
II	+	−	−
III	−	−	+
IV	−	+	−



$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\sin(90^\circ - \theta) = \cos\theta$

$\cos(90^\circ - \theta) = \sin\theta$

$\tan(90^\circ - \theta) = \cot\theta$

$\sin(90^\circ + \theta) = \cos\theta$

$\cos(90^\circ + \theta) = -\sin\theta$

$\tan(90^\circ + \theta) = -\cot\theta$

Values of sine and cosine for special angles :

Function	0° 0 rad.	30° $\frac{\pi}{6}$ rad	45° $\frac{\pi}{4}$ rad	60° $\frac{\pi}{3}$ rad	90° $\frac{\pi}{2}$ rad	180° π rad	270° $\frac{3\pi}{2}$ rad	360° 2π rad
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0

Quandratic Formula :

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Formulae of Log :

1. If $log\ a = x$, then $a = 10^x$
2. $log(ab) = log(a) + log(b)$
3. $log\left(\frac{a}{b}\right) = log(a) - log(b)$
4. $log(a^n) = n\ log\ a$
5. $log_a a = 1$
6. $ln\ a = log_e^{\ a} = 2.303\ log_{10} a$

Important Expansions :

1. Bionomial Expansion

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)x^2}{2!} \pm \text{......} (x < 1)$$

$$(1 \pm x)^{-n} = 1 \mp nx + \frac{n(n+1)x^2}{2!} \mp \text{......} (x < 1)$$

2. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^2}{3!} + ...$

   when $x << 1$, then $e^x = 1 + x$

3. $ln(1 + x) = x - \frac{x^2}{2} + \frac{x^2}{3} + ...(|x| < 1)$

   when $x << 1$, then $ln(1 \pm x) = \pm x$.

4. Trigonometric Expansion (θ in radian)

   (i) $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + ...$

   (ii) $\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + ...$

   (iii) $\tan\theta = \theta + \frac{\theta^3}{3} + \frac{\theta^5}{15} + ...$

   If θ is very small, then $sin\theta \approx \theta$; $cos\theta \approx 1$ and $tan\theta \approx \theta$ rad.

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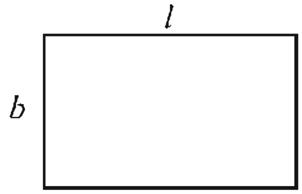
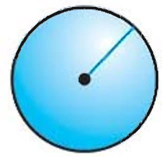
GEOMETRIC FORMULAE



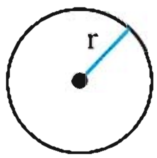
Rectangle

Area = lb


Sphere

Surface area = $4\pi r^2$

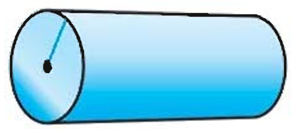
Volume = $\frac{4}{3}\pi r^3$



Circle

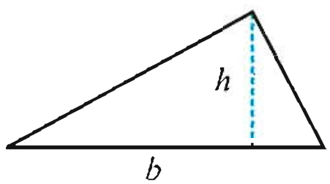
Area = πr^2

Circumference = $2\pi r$


Cylinder

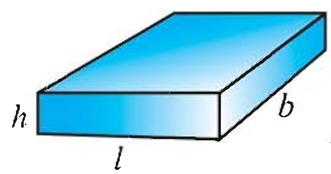
Area of curved surface = $2\pi rl$

Volume = $\pi r^2 l$



Triangle

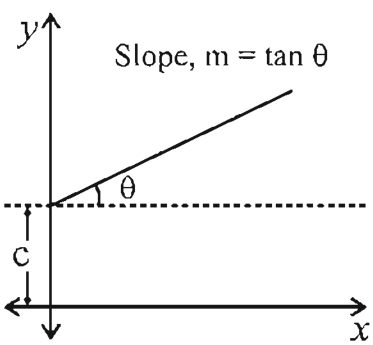
Area = $\frac{1}{2}bh$



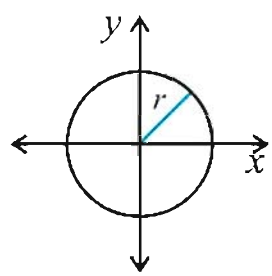
Cuboid

Area = $2(lb + lh + bh)$

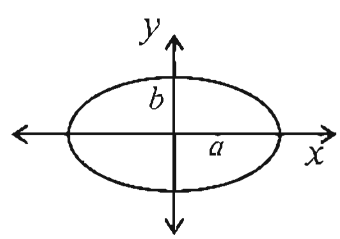
Volume = lbh



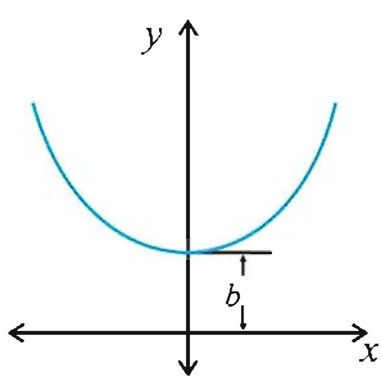
Straight line : $y = mx + c$



Circle : $x^2 + y^2 = r^2$

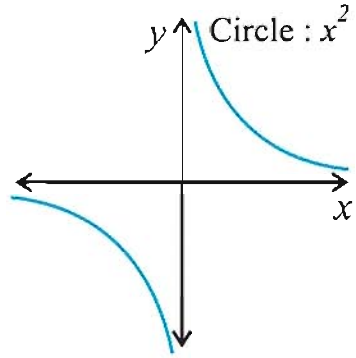


Ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Parabola : $y = ax^2 + bx + c$

Circle : $x^2 + y^2 = r^2$



Hyperbola : xy = constant

REFERENCE BOOKS

1. PHYSICS, Part 1 and 2, Std. XI, GSBST
2. PHYSICS, Part 1 and 2, Std. XI, NCERT
3. Fundamentals of PHYSICS by Halliday, Resnick and Walker
4. Universty Physics by Young, Zemansky and sears
5. CONCEPTS OF PHYSICS by H. C. Verma
6. Advanced PHYSICS by Tom Duncan
7. Advanced LEVEL PHYSICS by Nelkon and Parker
8. FUNDAMENTAL UNIVERSITY PHYSICS by Alonso and Finn
9. COLLEGE PHYSICS by Weber, Manning, White and Weygand
10. PHYSICS FOR SCIENTIST AND ENGINEERS by Fishbane, Gasiorowicz, Thornton
11. PHYSICS by Cutnell and Johnson
12. COLLEGE PHYSICS by Serway and Faughn
13. UNIVERSITY PHYSICS by Ronald Reese
14. CONCEPTUAL PHYSICS by Hewitt
15. PHYSICS FOR SCIENTIST AND ENGINEERS by Giancoli
16. Heat Transfer by Holman

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LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference 1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5566	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6345	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference 1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7769	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9843	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Antilogarithms

	0	1	2	3	4	5	6	7	8	9	Mean Difference 1	2	3	4	5	6	7	8	9
00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1289	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Antilogarithms

	0	1	2	3	4	5	6	7	8	9	Mean Difference 1	2	3	4	5	6	7	8	9
50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
82	6607	6622	6637	6653	6668	6683	6699	6715	6730	6745	2	3	5	6	8	9	11	12	14
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	12	14	16
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

NATURAL SINES

Degree	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	Mean Deifferences 1'	2'	3'	4'	5'
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	.0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	.0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	14
5	.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	.1045	1063	1080	1197	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	.1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	.1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	.1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	11	14
11	.1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	.2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	3	6	9	11	14
13	.2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	.2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	.2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	.2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	.3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	.3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	.3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	.3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	.3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	.4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	.4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	.4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	.4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	.4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	.5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	.5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	.5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	.5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	.5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	9	12
36	.5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	.6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	.6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	.6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	.6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	.6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	.6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	.6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES

Degree	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	Mean Deifferences 1'	2'	3'	4'	5'
45	.7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	.7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	.7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	.7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	.7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	.7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	.7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	.7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	.7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	.8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	.8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	.8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	.8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	.8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	.8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	.8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	.8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	.8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	.8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	.8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	.9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	.9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	.9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	.9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	.9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	.9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	.9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	.9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	.9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	.9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	.9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	.9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	.9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	.9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	.9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	.9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	.9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	.9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	.9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	.9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	.9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	.9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	.9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	.9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0

NATURAL TANGENTS

Degree	0 0.0°	6 0.1°	12 0.2°	18 0.3°	24 0.4°	30 0.5°	36 0.6°	42 0.7°	48 0.8°	54 0.9°	Main Deifferences 1	2	3	4	5
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	.0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	.0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	.1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	.1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	.1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	.1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	.1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	.2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	.2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	.2493	2512	2530	2549	2568	2686	2605	2623	2642	2661	3	6	9	12	16
15	.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	.2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	.3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	.3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	.3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	.3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	.3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	.4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	.4245	4265	8286	4307	4327	4748	4369	4390	4411	4431	3	7	10	14	17
24	.4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	.4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	.4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	.5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	.5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	.5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	.6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	.6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	.6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	.6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	.7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	.7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	.7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	.8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	.8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	.9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	.9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	.9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

Degree	0 0.0°	6 0.1°	12 0.2°	18 0.3°	24 0.4°	30 0.5°	36 0.6°	42 0.7°	48 0.8°	54 0.9°	Main Deifferences 1	2	3	4	5
45	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1.0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1.0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1.1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1.1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1.1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1.2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1.2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1.3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1.3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1.4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1.4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1.5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1.6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1.6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1.7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1.8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1.8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1.9626	9711	9797	9883	9970	2.0057	2.0145	2.0233	2.0323	2.413	15	29	44	58	73
64	2.0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2.1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2.2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2.3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2.4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2.6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2.7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2.9042	9208	9375	9544	9714	9887	3.0061	3.0237	3.0415	3.0595	29	58	87	116	145
72	3.0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3.2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3.4824	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3.7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4.0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4.3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4.7046	7453	7867	8288	8716	9152	9594	5.0045	5.0504	5.0970					
79	5.1446	1929	2422	2924	3465	3955	4486	5026	5578	6140					
80	5.6713	7297	7894	8502	9124	9758	6.0405	6.1066	6.1742	6.2432					
81	6.3138	3859	4596	5350	6122	6912	7720	8548	9395	7.0264					
82	7.1154	2066	3002	3962	4997	5958	6996	8062	9158	8.0285					
83	8.1443	2636	3863	5126	6427	7769	9152	9.0579	9.2052	9.3572					
84	9.514	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20					
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95					
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27					
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0					

Mean Differences no longer sufficiently accurate