MATHEMATICS

Class 7

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FOREWARD

Mathematics does not limit itself to the subject alone, but it also plays an important role in understanding other subjects. The main objective of studying mathematics in class VI is to assimilate its universalisation in terms of various characteristics of geometrical figures, to understand the concept of negative numbers (integers) and to utilise the already attained understanding of Mathematics in various fields of life.

Mathematics is not a subject to be spoken of or understand. It is a subject where things have to conceptualised in our minds and when one himself solves several problems related to an area, these concepts are strengthened.

Attempts have been made in this book to allow the student to form concepts, establish them and use these in related environments in different fields of life. To obtain this objective the student has to read the book attentively, follow the ativities described and do them so as to draw conclusions from the experiences. One is also suggested to keep a written record of the experiments and observations.

A book is never complete in itself. Hence, if readers have suggestions regarding difficulties in the book and suggestions for improvement are brought forth, they would be very well taken on in favour for the students of this state in future.

We express our heartiest thankfulness and gratitude towards the teachers of several government and private schools, DIETs, professors from colleges & NGOs as well as senior citizens of the state who have steered and guided through the making of the book.

We again make an appeal to the educated citizens of the state to let the council know about possible improvements and suggestions for the betterment of the textbook before it is finally launched into the mainstream.

The National Council of Educational Research and Training (NCERT) sets some clear and measurable goals for class 1 to 8th. They are known as 'Learning outcomes'.

We have made some necessary changes in this textbook in reference with 'Learning out comes'. Some new contents have been added and some chapters have been transferred from one class to another. Do not let the teachers and the students get confused.

Director
S. C. E. R. T. Chhattisgarh, Raipur

Content

Chapter -1

NUMBERS : REVISION

In previous classes you have studied about Natural, whole, rational and irrational numbers. So, their revision is necessary to make our further studies more easier.

Natural Number :

The number used for counting are called as natural numbers. The group of natural numbers is represented by 'N'

N = 1,2,3,4,5....... etc.

When 1 is added to a natural number then we get its successor and when we subtract 1 from any natural number then we will get its predecessor.

successor of 5 = 5+1
= 6

predecessor of 5 = 5-1
= 4

Every natural number has its successor, and except 1 every number has its predecessor.

- '1' is the first and the smallest natural number.
- None of the number is last or largest natural number.

Properties of Natural number

1. When we add or multiply any two natural numbers then we will get a natural number as result.
2. It is not necessary to get a natural number on subtraction or division of two natural numbers.
3. we can add or multiply any two natural numbers in any sequence. Commutative property is only applied on addition and multiplication of natural numbers but it is not applied on subtraction or division.
4. Associative property is also applied on addition and multiplication of natural number only but not on subtraction and division of natural numbers.
5. Distributive property is applicable for multiplication over addition and subtraction of Natural number.
6. If we multiply or divide a natural number by 1. Then the value of that number will not change.

   If a, b, c are three natural numbers, then

1. (i) (a + b) is a natural number.

   (ii) (a x b) is a natural number.

2. (i) It is not necessary (a – b) will be a natural number.

   (ii) It is not necessary (a ÷ b) will be a natural number.

3- (i) a+b = b+a

   (ii) $a \times b = b \times a$

   (iii) (a¹b)

   (iv) a¸ b ¹ b¸ a (a¹b)

4 (i) a+(b+c) = (a+b)+c

   (ii) $a \times (b \times c) = (a \times b) \times c$

   (iii) a–(b-c) ¹ (a–b)–c

   (iv) a ¸ (b ¸ c) ¹ (a ¸ b) ¸ c (a ¹ b ¹ c ¹ 1)

5 (i)

   (ii) [b>c]

6 (i)

   (ii) a ¸ 1 = a

Whole Number

If we include 0 in the group of natural numbers we get group of whole numbers.

We represent group of whole number by W.

W = 0, 1, 2, 3, 4, 5, 6 ……… etc.

Every whole number has its successor, except 0 every whole number has its predecessor. “0” is the first and smallest whole number. No number is the largest or last whole number. Every natural number is a whole number. But every whole number is not a natural number.

Properties of whole number

1. Properties of whole numbers are as same as natural numbers.
2. On addition or subtraction of zero to any whole number, the actual value of that number does not change. For addition zero is known as additive identity.
3. On multiplying 1 to any whole number the value of that number does not change. For multiplication, 1 is known as multiplicative identitive element.
4. On dividing 0 by any whole number the quotient will be zero itself. But division of any whole number by zero is not defined.

Integer Number

The collection of positive numbers, negative number and zero is called as group of integers. Integers are represented by I or Z.

I = ... -5, -4, -3. -2, -1, 0, 1, 2, 3, 4, 5 ..... etc.

Properties of Integers

1. The properties of Integers are as same as whole numbers.
2. Closure property is applied on addition, subtraction and multiplication of Integers. The resultent of addition, subtraction and multiplication of any two integers is a integer always.
3. Closure property is not always applied on division of integers. The resultant of division of any two integers is not a integer always.
4. The resultant of addition of any two positive integers is always a positive integer and the resultant of addition of any two negative integers is always a negative integer.
5. The resultant of addition of a positive integer and a negative integer is based on higher numeral value of integer. If numeral value of positive integer is higher, resultant will be positive and if numeral value of negative integer is higher than resultant will be negative.
6. The additive inverse of a positive integer is a negative integer and of a negative integer is a postive integer.
7. On multiplication of positive integer and negative integer we get negative integer as result.
8. On multiplication of any two positive integer and two negative integer we get positive integer as result.
9. On dividing any number by number itself we get 1, except zero.
10. On dividing any integer by its additive inverse we get-1 as result, except zero.
11. There is no existence of multiplicative inverse of zero.

Properties of Natural numbers, Whole numbers and Integers -

Properties / Number	Additive Operation			Differencive Operation			Multiplicative Operation			Division operatvie		
	Closure	Commut -ative	Associa -ative	Closure -ative	Commut -ative	Associa -ative	Closure -ative	Commut -ative	Associa -ative	Closure -ative	Commut -ative	Associa -ative
Natural	√	√	√	x	x	x	√	√	√	x	x	x
Whole	√	√	√	x	x	x	√	√	√	x	x	x
Integer	√	√	√	√	x	x	√	√	√	x	x	x

Activity 1

In the given table addition and subtraction of integers are done. do the remaining as per guidense –

S.No.	First Integer	Second Integer	First+Second Integer	Result is a Integer or not	First– Second Integer	Difference is a Integer or not
1	5			Yes		Yes
2	-7			Yes		Yes
3	-4			Yes		Yes
4	13					
5	-9					
6	102					

Activity 2

Complete the following table.

$(-4) + (-4) = -8$

+	– 4	–3	–2	–1	0	1	2	3	4
–4	–8	–7	–6	–5	–4	–3	–2	–1	0
–3	–7	–6	–5						
–2	–6								
–1	–5								
0	–4								
1	–3								
2	–2								
3	–1								
4	0								

Whether the following statement is True of False

(–4) + (–3) = (–3) + (–4) ------------------------

3 + (–2) = (–2) + 3 ------------------------

Activity 3

Complete the following table by difference of integer (A-B)

(– 4) – (– 3) = – 4 + 3 = –1

(– 4) – (– 2) = – 4 + 2 = –2

B→

A↓

–	– 3	– 2	– 1	0	1	2	3	4
–4	– 1	– 2	– 3	– 4	– 5	– 6	– 7	– 8
–3	0	– 1						
–2	1							
–1	2							
0	3							
1	4							
2	5							
3	6							
4	7							

Whether the following statement is True or False.

$(-3) - (-2) = (-2) - (-3)$ ------------------------

$3 - 2 = 2 - 3$ ------------------------

Activity 4

The multiplications of integers are done in the table given below complete the following table as per example.

S.No.	First number	Second Number	First Number x Second Number	Product	Result
01	4	3	4 x 3	12	The product of two positive integer is a positive integer
02	-7	-2	(-70 x (-2)	14	The product of two negative integer is a positive integer
03	-6	3	(-6) x 3	-18	The product of a positive integer and a negative integer is a negative integer
04	5	-4	....	....	....
05	-8	-3	....	....	....
06	-13	6	....	....	....
07	16	-20	....	....	....

Activity 5

In the given table multiplication of integers are given. Now complete the following table.

x	–4	–3	–2	–1	0	1	2	3	4
4	–16	–12	–8	–4	0	4	8	12	16
3	–12	–9	–6	–3	0				
2									
1									
0									
–1									
–2									
–2									
–3									
–4									

Activity 6

In the following table multiplication factor and its respective two division factor are given. Now complete the following table -

S.No.	First number Multiplication factor	Second number Division factor
1.	3 × 5 = 15	15 ÷ 3 = 5 15 ÷ 5 = 3
2.	–8 × 6 = –48	(–48) ÷ 6 = –8 (–48) ÷ (–8) = 6
3.	–5 × –6 = 30	30 ÷ –5 = –6 --------------------
4.	----------------	(–54) ÷ 6 = ? (–54) ÷ (–9) = ?
5.	7×–3= –21	----------------, (–21) ÷ (–3) = 7

Activity 7

Complete the following sentence

1. The division of any positive integer by a positive integer gives a .... integer quotient.
2. On dividing a negative integer by a negative integer we get a .... integer quotient.
3. The division of any negative integer by a positive integer gives a .... integer quotient.
4. On dividing a positive integer by a negative integer we get a .... integer quotient.

Fraction

1. The number in the from of p/q, where p and q both are positive integer, called as fraction number.
2. A fraction is in simplest from if it's numerator and denominator do not have common factor rather than 1.
3. The fractions having denominator greater than numerator are proper friction.
4. The fractions having denominator smaller than numerator are improper friction.
5. A mixed fraction is a whole number and a fraction combined in to one "mixed" number.
6. The fractions which represent equal quantity called equal fraction.
7. By multiplication or dividing the numerator and denominator by the same number except zero. A fraction can be changed into equivalent fraction.
8. For addition of fractions having equal denominator we can directly add the numerator and write the denominator as it is.
9. For addition of fractions having unequal denominator firstly we have to change fractions in to equal fraction of denominator. For this we take the L cm of detonators then we will precede it for addition.
10. Addition of mixed fraction-

First Method

1. Change mixed fraction onto add fraction.
2. By taking L cm we change it into equivalent fraction.

3. Now add the fractions.

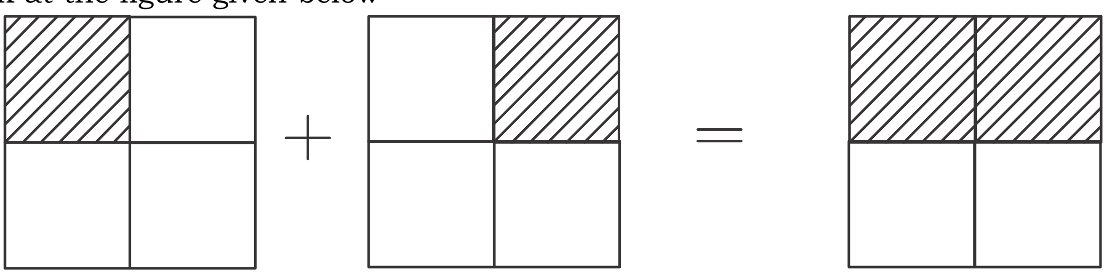
Second Method

1. Add the whole number of mixed fractions.
2. Add the fractional part of mixed fractions.
3. Now add the integer part and fractional part.

11. The process of subtraction of fraction is as same as addition of fractions. The only difference is operation of subtraction between fractions rather than addition.
12. While multiplication of fractions, numerator is directly multiplied to numerator and denominator is directly multiplied to denominator.
13. When we divide a fraction by another fraction, we can directly multiply the reciprocal of divisor to dividend.
14. We get inverse of any fraction by interchanging its numerator and denominator.

Addition of fractions : Pictorial representation

Look at the figure given below-

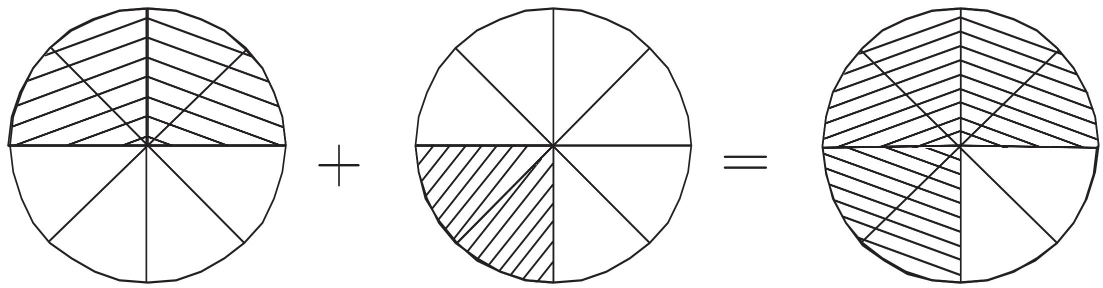


$\frac{1}{4}$ Fig1.1 (a) $\frac{1}{4}$ Fig1.1 (b) $\frac{2}{4}$ Fig1.1 (c)

We can write like this

$$\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$$

Look at the figures



$\frac{2}{4} = \frac{4}{8}$ $\qquad$ $\frac{2}{8}$ $\qquad$ $\frac{6}{8}$

Fig1.2 (a) $\qquad$ Fig1.2 (b) $\qquad$ Fig1.2 (c)

now : $\frac{2}{4}+\frac{2}{8}$

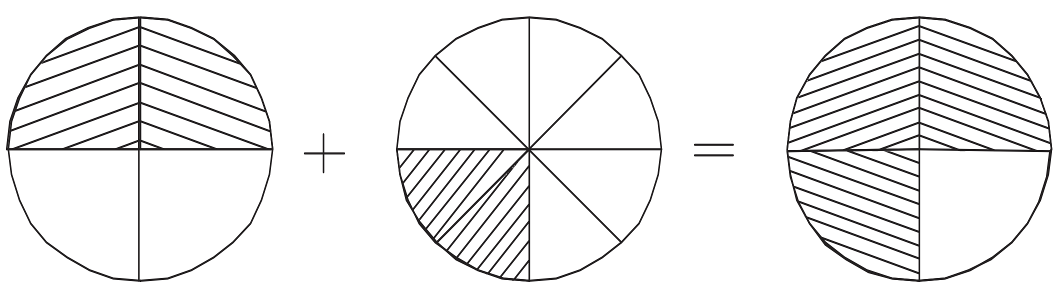
$=\frac{2\times 2}{4\times 2}+\frac{2}{8}$

$=\frac{4}{8}+\frac{2}{8}$

$=\frac{4+2}{8}$

$=\frac{6}{8}$

So, Now tell



$\frac{2}{4}$ $\qquad$ $\frac{2}{8}$ $\qquad$ ---------------

Fig1.3 (a) $\qquad$ Fig1.3 (a) $\qquad$ Fig1.3 (a)

Activity 8

In the following table, addition and subtraction of tractions are given. Now complete the table –

S.no.	Question	LCM of denominator	Changing the fractions into fractions of equal denominator by LCM	Addition of fractions of equal denominator	Solution	Simplest fraction
1	$\frac{2}{3}+\frac{4}{5}$	15	$\frac{10}{15}+\frac{12}{15}$	$10+12=22$		$\frac{22}{15}$
2-	$\frac{3}{4}+\frac{1}{2}+\frac{2}{5}$	20	$\frac{15}{20}+\frac{10}{20}+\frac{8}{20}$	$15+10+8=33$		$\frac{33}{20}$
3-	$\frac{4}{7}-\frac{2}{5}$	35	$\frac{20}{35}-\frac{14}{35}$	$20-14=6$		$\frac{6}{35}$
4-	$\frac{7}{10}-\frac{3}{15}+\frac{1}{2}$	----	------------------	-----------------	--------	--------
5-	$\frac{1}{3}+\frac{3}{5}-\frac{8}{12}$	----	------------------	-----------------	--------	--------

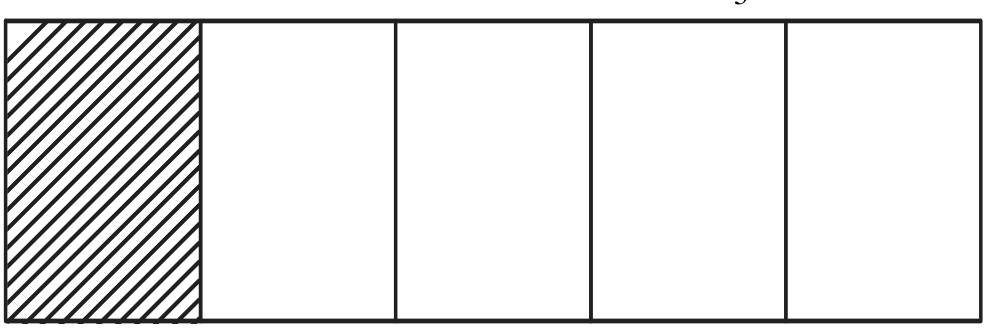
Multiplication of fraction : Pictorial representation

Lets us discuss on $\frac{1}{5}\times\frac{1}{3}$

$\frac{1}{5}\times\frac{1}{3}$ We can tell it as $\frac{1}{5}$ of $\frac{1}{3}$

Representation -

Divide the complete unit into 5 equal parts. Each part represents $\frac{1}{5}$ of this unit. Now shade its one part.



$\frac{1}{5}$

Fig.1.4

Now we have to find its $\frac{1}{3}$, Now divide the outline figure into 3 equal parts. each part represents $\frac{1}{5}$ of $\frac{1}{3}$.

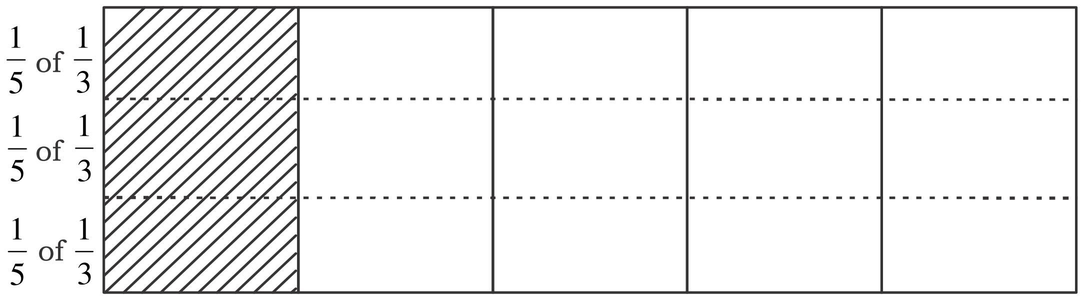
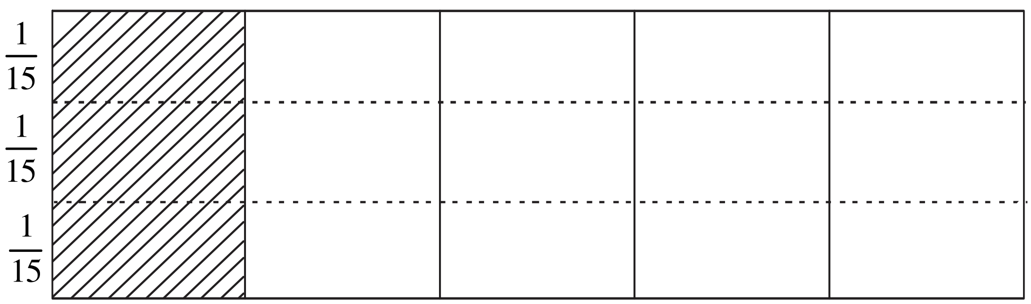


Fig 1.5

Each outline part represent $\frac{1}{3}$ of $\frac{1}{5}$, which is $\frac{1}{15}th$ part 8 complete unit.



$\frac{1}{5}$

Fig 1.6

Now it is clear that $\frac{1}{5}$ x $\frac{1}{3}$ of any unit is $\frac{1}{15}$ part of that complete unit.

Now we can see that.

$$= \frac{1 \times 1}{5 \times 3}$$

$$= \frac{1}{15}$$

Now we find that when we multiply two fractions then its number is directly multiplied to number and denominator is directly multiplied to denominator.

$$\frac{3}{7} \times \frac{2}{5} \quad = \frac{3 \times 2}{7 \times 5} = \frac{6}{35}$$

$$\frac{2}{3} \times \frac{7}{8} \quad = \frac{2 \times 7}{3 \times 8} = \frac{14}{24} = \frac{7}{12}$$

Divisor of fractions : Pictorial representation

The meaning of 6 ÷ 2 is how many groups of 2 are there in 6. (or how many times 2 is included in 6)

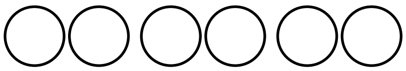


Fig. 1.7 (a)

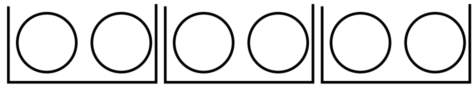


Fig. 1.7 (b)

See

There are three groups of 2 in 6

6 ÷ 2 = 3

Now find out $3 \div \frac{1}{2} = ?$

Meaning of $3 \div \frac{1}{2}$ is how many times $\frac{1}{2}$ is included in 3 or how many parts of $\frac{1}{2}$ is there in 3 ?

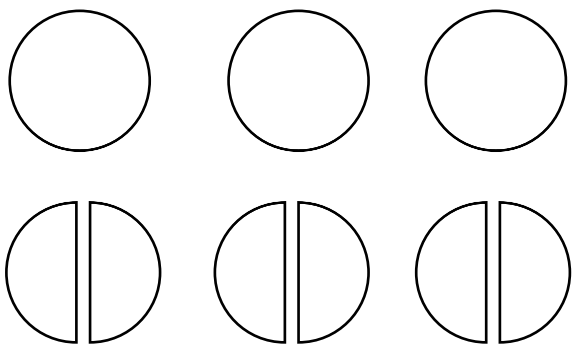


Fig 1.8

It is clear that, there are 6 parts of $\frac{1}{2}$ in 3. Each parts is of $\frac{1}{2}$

$$3 \div \frac{1}{2} = 6$$

Like this, what is the meaning of $\frac{1}{2} \div \frac{1}{4}$

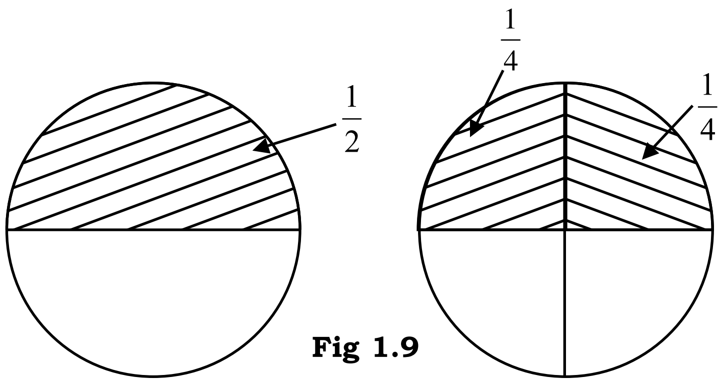


Fig 1.9

We find that

In $\frac{1}{2}$, $\frac{1}{4}$ is included 2 times.

$$\frac{1}{2} \div \frac{1}{4} = 2$$

Another way for division of fractions

$$6 \div 2 = \frac{6}{1} \div \frac{2}{1} = \frac{6}{1} \times \frac{1}{2} = \frac{6}{2}$$

$$3 \div \frac{1}{2} = \frac{3}{1} \div \frac{1}{2} = \frac{3}{1} \times \frac{2}{1} = \frac{6}{1} = 6$$

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$$

In this manner when a fraction is divided by another fraction then the divisor inter change numerator and denominator means reciprocate itself and sign of division is change in to sign of multiplication.

Activity 9

1. Change these into pictorial representation.

   (1) $\frac{1}{2} \times \frac{1}{4}$ (2) $2 \times \frac{1}{5}$

   (3) $\frac{2}{3} \times \frac{1}{5}$ (4) $3 \times \frac{1}{2}$

2. Change into pictorial representation.

   1. $\frac{1}{2} \div \frac{1}{8}$

   2. $\frac{3}{4} \div \frac{1}{4}$

Exercise 1

1. Fill up the blank with the help of (>, = or <) suitable signs.

   (i) $(-2) \times 9$ ---------$(-3) \times 9$

   (ii) $3 \times (-5) \times (-2)$--------$(-5) \times 6$

   (iii) 4×9 ---------$(-2) \times 9 \times (-2)$

   (iv) $2 \times (-6) \times 0$ -----------$(-3) \times 4$

   (v) $(-5) \times (-6) \times 2$ ---------$(-2) \times 5 \times (-8)$

2. Find out the Product -

   (i) $(-8) \times 5 \times 4$

   (ii) $(-9) \times 0 \times (-2)$

(iii) $(-42) \times 6 \times 3$

(iv) $5 \times (-75) \times (-7)$

(v) $(-30) \times (-25) \times 8$

(vi) $(-8) \times (-12) \times (-30)$

3. Find out the Quotient

(i) −80 ¸ 16

(ii) $-24 \div (-8)$

(iii) $650 \div (-13)$

(iv) $-170 \div (-17)$

(v) $-256 \div 16$

(vi) $-170 \div (-1)$

(vii) $0 \div (-18)$

(viii) $321 \div (-1)$

(ix) $19 \div (-19)$

(x) $200 \div (-10)$

4. Put the correct sign (< , = or >) to make the statement true.

(i) $(-3)+(-4)$--------------------

(ii) --------------------

(iii) --------------------

(iv) --------------------

(v) --------------

(vi) ---------------------

(vii) --------------------

(viii) ---------------------

(ix) ------------------

(x) ------------------

5. Solve the following and change it into simplest form.

(i) (ii) $\frac{5}{2}\times\frac{3}{10}$ (iii) $\frac{4}{11}\times\frac{22}{8}$ (iv) $\frac{2}{3}\div\frac{8}{5}$

(v) $\frac{3}{7}\div\frac{5}{14}$ (vi) $\frac{3}{4}\div\frac{9}{8}$

6. Radha and Sohan eat $\frac{1}{2}$ and $\frac{1}{4}$ part of the same watermelon respectively. Now tell how much part of watermelon they eat together.

7. Total number of students in Mohan's classroom were 45 in which number of girls is $\frac{1}{5}$ of total students. Find the total number of girls.

8- Prabhat took 500 Rs. to the market. He spends $\frac{1}{4}$ of total money for book and $\frac{1}{5}$ part of total money for sweets. tell the amount he left with him.

9. A businessman is having Rs. 60000 as his total property. He gave $\frac{1}{2}$ part of his property to his wife and $\frac{1}{2}$ part of the remaining to his son and $\frac{1}{2}$ part to his daughter. Now tell the amount everyone get.

Chapter -2

Rational Numbers

Radha asked her friends, "Can you divide the difference of two numbers into three parts?"

Hamid, "Why not? If the numbers are 10 and 9, then 10-9=1 and each of the three equal parts of 1 is $\frac{1}{3}$."

Suresh, "In Chapter on Fractions we have learnt to divide 1 into three equal parts" But 9-10=-1, how can we divide this in three equal parts?

All of them were trying to divide -1 into 3 equal parts.

Radha suggested, "Look at the number line, 1/3 is on the right side of 0 and we get 1/3 upon dividing 1 into 3 equal parts. Similarly, on the left side of O we can get $-\frac{1}{3}$

$-\frac{1}{3}$ upon dividing 1 into 3 equal parts.

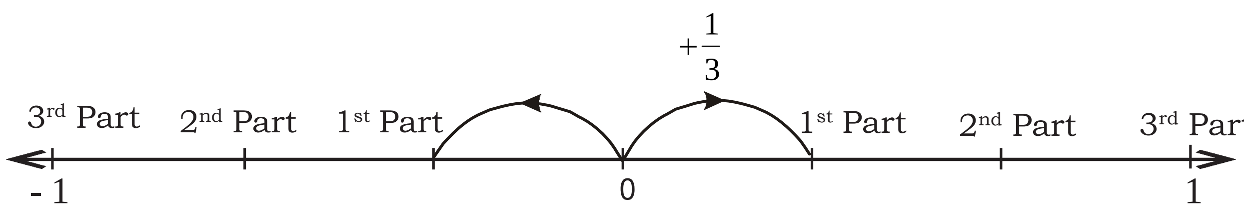


Figure 2.1

Suresh did not understand what Radha had said. He asked, "When we take 2 parts from 3 equal parts, then we get 2/3 but how will we depict $\frac{-2}{3}$?

Hamid, "In the previous class we had read that we can have 5 flowers, 5 goats, 5 leaves and 5 spectacles. There can be any 5 objects. This means that the counting number is not related to any special object. This is an idea which helps us in counting".

Radha said, "Yes! You are right. Positive numbers are used for counting objects but we never used negative numbers for counting. For example, 2,3,5 etc are used for counting but -2, -3, -5 etc are not used for counting".

In the previous class we had also learnt that $\frac{2}{3}, \frac{5}{6}, \frac{7}{9}$...... etc can be shown by taking 2 parts from 3 equal parts or 5 parts from 6 equal parts or 7 parts from 9 equal parts of a rectangle or a circle.

But no negative fraction can be depicted like that.

All the students started thinking of negative fractions along with positive fractions. They did not know how to look at negative fractions in comparison to positive fractions? Are these some different kind of numbers?

They put the problem before their mathematics teacher.

The teacher told them, "We learnt about natural numbers first then we added 'zero' to make whole numbers. Then we thought about fractions and then we learnt negative numbers. All these numbers together with negative fractions are called Rational Numbers. Hence $-\frac{3}{4}, -\frac{1}{2}, 0, \frac{2}{8}, \frac{1}{2}, \frac{15}{7}, \frac{3}{1}$ etc are all rational numbers".

Write 10 examples of rational numbers.

Activity 1

In the following table pairs of integers are given, use one integer as the numerator and the other as the denominator and make rational numbers.

Table 1

S. No.	Integer	Numerat -or	Denomi- nator	Rational Number	Numerat -or	Denomi- nator	Rational Number
1	2 &3	2	3	$\frac{2}{3}$	3	2	$\frac{3}{2}$
2	-5& 7						
3	4& -8						
4	-7&-9						
5	1& 6						

We denote Natural Numbers by N, Whole Number by W, Integers by I. Similarly, Rational Numbers are denoted by Q. A rational number is written in the form of p/q.

What will happen if q = 0 ?

What will be the quotient if the number is divided by 0 ?
Any number divided by 0 gives a infinite quotient.

We cannot calculate the value of $\frac{p}{0}$ as it does not indicate any definite number.

Thus, **rational numbers can be expressed in the form $\frac{p}{q}$ where p, q are integers and q # 0.**

Converting Integers into Rational Numbers

Is every integer a rational number too? Examples of integers are 4, 8, 11, -3, -7 etc. Let us think how we can write 4 as a rational number.

We can write 4 in many forms.

$\frac{4}{1}, \frac{8}{2}, \frac{12}{3}$ .... etc. Do you agree with this? In how many other ways can we write 4?

Similarly – 7 can be written as $\frac{-7}{1}, \frac{-14}{2}, \frac{-21}{3}$ etc. All the numbers are written in fractional form. Since the numerator and denominator are integers therefore these numbers are rational numbers.

Thus, integers can be written as Rational Numbers

Write the following integers as rational numbers.

-3	=	=	=	13	=	=	=
10	=	=	=	100	=	=	=
-11	=	=	=	33	=	=	=
0	=	=	=				

Here, for each integer we have provided 3 blank spaces. In how many more ways can you write each integer?

Here, we can write 0 as ...... etc. Can you think of an integer which can not be written as a rational number?

Think and do – What did you do to write an integer as many different rational numbers? Take any 5 integers and convert them into rational numbers.

Equivalent Rational Numbers

We can write each integer in the form of many rational numbers. The value of all these rational numbers is equal to that integer. We had learnt that we can also write a fraction in more than one form with the same value. Does this happen with rational numbers as well? Let us see.

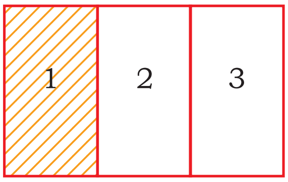


Figure 2.2

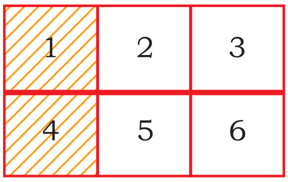


Figure 2.3

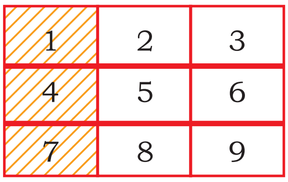


Figure 2.4

In fig 2.2 the coloured part shows

If we divide fig. 2.2 into 2 parts then we have 2 equal coloured parts out of a total 6 equal parts. Fig. 2.2 this means that the coloured part is $\frac{2}{6}$, which is $\frac{1}{3}$ of the total part. Now if we divide fig. 2.2 into 3 equal parts. Then its 3 equal parts are coloured out of 9 equal parts or $\frac{3}{9}$ part is coloured which is $\frac{1}{3}$ of the total. (fig. 2.4)

In fact $\frac{2}{6}$ and $\frac{3}{9}$ are equal to $\frac{1}{3}$.

We have read about equivalent fractions in the 6th standard. If the fractional number is a negative number, we can change the form ofnumber by multiplying the numerator and denominator with the same number and its value remains the same e.g.

$-\frac{1}{2}$ Multiply numerator & denominator with 2, $-\frac{1\times 2}{2\times 2}=-\frac{2}{4}$

$-\frac{1}{2}$ Multiply numerator & denominator with 3, $-\frac{1\times 3}{2\times 3}=-\frac{3}{6}$

$-\frac{1}{2}$ Multiply numerator & denominator with 4, $-\frac{1\times 4}{2\times 4}=-\frac{4}{8}$

Here the form of the rational number is changed but the value $-\frac{1}{2}$ is the same.

Thus, $\frac{-2}{4}, \frac{-3}{6}$ and $\frac{-4}{8}$ are equivalent rational numbers.

Can you choose the groups of equivalent rational numbers from the following numbers?

$\frac{6}{18}, \frac{1}{3}, \frac{2}{5}, \frac{2}{6}, \frac{4}{10}, \frac{12}{30}, \frac{6}{15}$ and $\frac{4}{12}$

:

Choose and fill up the blanks

$\frac{2}{5}$ = __________ = __________= __________=

$\frac{1}{3}$ = __________ = __________= __________=

Write Equivalent Rational Number in the table as directed.

Table 2

S. No.	Rational No.	Equivalent Rational Number
1.	$\frac{2}{5}$	$\frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \frac{10}{25}$ -------
2.	$\frac{-3}{7}$	
3.	$\frac{8}{-11}$	
4.	$\frac{-7}{-9}$	
5.	$-\frac{6}{15}$	

Rational numbers are converted into equivalent rational number by multiplying or dividing the numerator and denominator with the same number.

Simplest or lowest form of Rational Number

While filling above table Anu said that $-\frac{6}{15}$ is equivalent to the rational numbers $-\frac{2}{5}$ and $-\frac{4}{10}$. It means equivalent rational number can be obtained by dividing the numerator and the denominator by the same number.

Seema said that in $\frac{2}{5}$, we do not have a common factor. So we could not divide further. Therefore this is the simplest form. Thus the lowest form of the rational number is obtained by dividing numerator and denominator by the highest common factor. H.C.F. By the same method we can get the simplest from of a rational number.

Ramesh said that in $\frac{28}{35}$, Factors of 28 are 2,4 and 7 and those of 35 are 5 and 7. The highest common Factor is 7.

$\therefore$We can divide the numerator and the denominator by 7 to get the simplest form of $\frac{28}{35}$. $\frac{28}{35} = \frac{28 \div 7}{35 \div 7} = \frac{4}{5}$

This is the simplest form of rational number. All the children said," This is correct. We can not simplify it further".

Anu and Seema's team suggested the method of simplest form. Do you have any other method for this? Using that method find the simplest form of $\frac{24}{36}$ and $\frac{98}{112}$.

Activity 2

Table 3

S. No.	Rational No.	Fac tors of numerators	Factors of denominator	Highest Common factor	$\frac{\text{Numerator} \div \text{H.C.F.}}{\text{Denominator} \div \text{H.C.F.}}$	Simplest form
1.	$\frac{45}{54}$	1,3,5,9,15, 45	1,2,3,6,9, 29,54	9	$\frac{45 \div 9}{54 \div 9}$	$\frac{5}{6}$
2.	$\frac{57}{76}$					
3.	$\frac{18}{36}$					
4.	$\frac{27}{81}$					
5.	$\frac{-63}{85}$					

Exercise 2.1

1. Find rational numbers from the following numbers

   $\frac{4}{1}, \frac{-3}{7}, -27, \frac{24}{0}, \frac{-3}{-5}$

2. Convert the following numbers into rational numbers.

   -38, 17, 0, -100, 79

3. Write 3 equivalent rational numbers for each of the following rational numbers.

   $\frac{1}{5}, \frac{-3}{4}, \frac{-5}{8}, \frac{6}{11}$ and $\frac{4}{3}$

4. Write the lowest form of the following rational numbers..

5. Choose the equivalent rational numbers from the following rational numers.

   (i) $\frac{4}{12}, \frac{8}{24}, \frac{1}{3}, \frac{16}{36}$ and $\frac{25}{75}$

   (ii) $\frac{-3}{5}, \frac{-6}{10}, \frac{-15}{25}, \frac{-27}{45}$ and $\frac{-15}{20}$

6. Are the following pair of rational numbers equivalent? Give reasons.

   29 −91 63 a −15 −48 5 570 2 / 11 10 136 −33 45 96 7 7 100 2, and, (ii) and, (i)

7. Express $\frac{-3}{8}$ into equivalent rational numbers, where

   (i) numerator is – 6

   (ii) numerator is 12

   (iii) denominator is -24

   (iv) denominators is -32

8. Find the value of 'a' if the following pairs are equivalent rational numbers

   (i)

   (ii) $\frac{2}{3}$ and $\frac{8}{a}$

   (iii) $\frac{3}{7}$ and $\frac{a}{35}$

   (iv) $\frac{a}{5}$ and $\frac{18}{30}$

   (v) $\frac{-a}{13}$ and $\frac{-24}{39}$

Rational numbers on the number line

You know how to represent integers on the number line. You also know that positive integers are shown on the right side of zero and negative integers on the left side of zero. +2 and -5 as shown in the following figure.

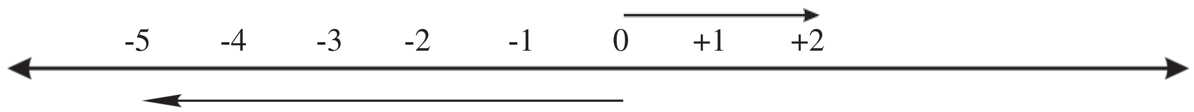


Figure 2.5

Rational number is at the place of +2. All the rational numbers, who have denominator as one, are located at the place of the integers.

Now you can show $\frac{-3}{1}$ and $\frac{+6}{1}$ on the number line. If the denominator is other than 1, the number is shown either on the right side of 0 or on the left side of 0. We divide the number line into as many parts as the denominator e.g. is shown in figure 2.6.

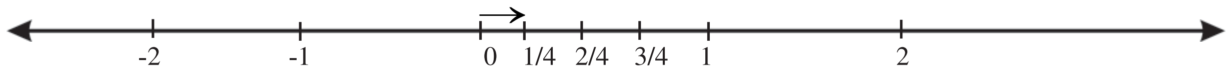


Figure 2.6

Similarly $\frac{-5}{7}$ will be shown on number line like this.

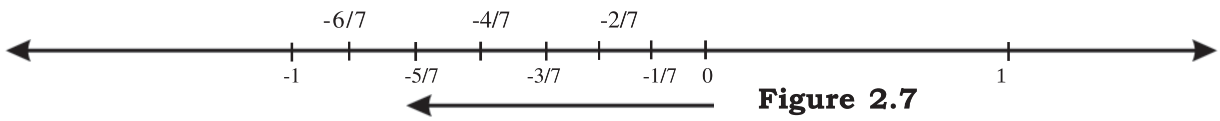


Figure 2.7

is a mixed fraction. We can write this in the forms like

it will be shown on number line as:-

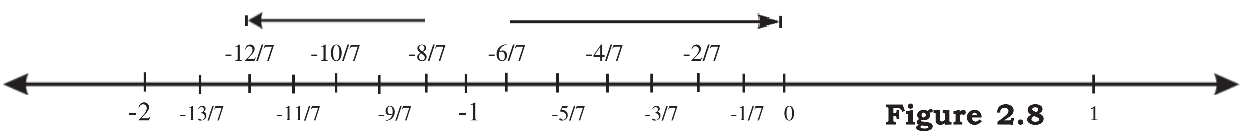


Figure 2.8

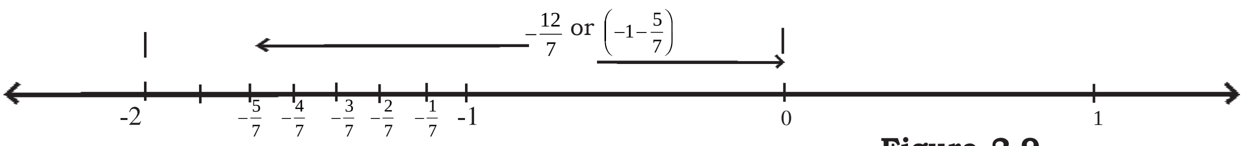


Figure 2.9

Mark the following numbers on number line.

(i) $\frac{3}{5}$ (ii) $\frac{-5}{8}$ (iii) (iv) $\frac{-15}{11}$

Write any three rational numbers and show them on number line.

> **Note:** If the denominator of any rational number is negative, for example $\frac{p}{-q}$ then that number is shown on the number line in the same ways $\frac{-p}{q}$.

You have marked on number line. Now mark on number line.

Activity 3

(i) Mark 4 points between -2 and -3 on number line.

(ii) Find 6 negative numbers between -5 and 3.

$-\frac{175}{816}$ and $\frac{103}{84}$, $-1\frac{5}{9}$, $1\frac{}{8}$

Which number is bigger?

We have already compared Natural numbers, integers and fractional numbers.

For comparing fractions, we have to convert them in such a manner that their values are unchanged and the denominators of all fractions are equal. After this we compare their numerators and then find out the bigger and smaller fraction.

Similarly for comparing rational numbers we convert them in such a manner that their values are unchanged but all of them have the same denominator. Then we compare their numerators and determine the bigger or smaller rational number.

Example 1: Compare $\frac{-5}{8}$ and $\frac{-3}{4}$

Solution: L.C.M. (8, 4) = 8

For converting into same denominator, we multiply the numerator and

denominator of with 1 and multiply the numerator and denominator with 2

$\Rightarrow \dfrac{-5}{8}=\dfrac{-5\times1}{8\times1}=\dfrac{-5}{8}$

$\Rightarrow \dfrac{-3}{4}=\dfrac{-3\times2}{4\times2}=\dfrac{-6}{8}$

Since - 5 > -6

$\therefore \dfrac{-5}{8}>\dfrac{-6}{8}$

or $\dfrac{-5}{8}>\dfrac{-3}{4}$

Example2: Which rational number is smaller, $\dfrac{-4}{7}$ or $\dfrac{-5}{-3}$.

Solution: Firstly, we change the negative fraction into positive.

$\dfrac{-5}{-3}=\dfrac{-5\times(-1)}{-3\times(-1)}=\dfrac{5}{3}$

LCM (7,3) = 21

$\therefore \dfrac{5}{3}=\dfrac{5\times7}{3\times7}=\dfrac{35}{21}$ and $\dfrac{-4}{7}=\dfrac{-4\times3}{7\times3}=\dfrac{-12}{21}$

(The reason for this can be that $\dfrac{-4}{7}$ is a negative number but = $\dfrac{5}{3}$, a positive number. A negative number would always be smaller than a positive number.

Since -12 < 35

$\therefore \dfrac{-12}{21}<\dfrac{35}{21}$

$\Rightarrow \dfrac{-4}{7}<\dfrac{5}{3}$

$\Rightarrow \dfrac{-4}{7}<\dfrac{-5}{-3}$

(The reason for this can be that $\dfrac{-4}{7}$ is a negative number but = $\dfrac{5}{3}$, a positive number. A negative number would always be smaller than a positive number.

Thus $\dfrac{-4}{7}$ is smaller than.

Example 3. Write the following rational number is descending order.

Solution: Firstly, we convert the negative denominators into positive.

$\Rightarrow \dfrac{13}{-24}=\dfrac{13\times(-1)}{-24\times(-1)}=\dfrac{-13}{24}$

and $\frac{-5}{-12}=\frac{-5\times(-1)}{-12\times(-1)}=\frac{5}{12}$

Thus, the given rational numbers are

$\frac{3}{4},\frac{-7}{8},\frac{-13}{24},\frac{5}{12}$

The denominators of the given numbers are 4, 8, 24, 12 and there LCM is = 24.

If we make all the denominators equal then

$\frac{-7}{8}=\frac{-7\times3}{8\times3}=\frac{-21}{24}$

$\frac{-13}{24}=\frac{-13\times1}{24\times1}=\frac{-13}{24}$

$\frac{5}{12}=\frac{5\times2}{12\times2}=\frac{10}{24}$

Since 18 > 10 > -13 > -21

$\frac{3}{4}=\frac{3\times6}{4\times6}=\frac{18}{24}$, $\frac{-1.6}{-5}$

$\frac{3}{4}>\frac{-5}{-12}>\frac{13}{-24}>\frac{-7}{8}$

Activity 4

1. Write any 5 rational numbers and arrange them in order.
2. Write the smallest and the largest rational number without finding out the LCM.

   $\frac{-1}{2},\frac{-5}{1},\frac{3}{2},\frac{2}{7},\frac{17}{12},\frac{2}{1},\frac{-2}{9},\frac{-12}{6}$ Give reasons for your answer.
3. Make 5 such exercises and ask your friends to solve them.

EXERCISE 2.2

1. Mark the following rational numbers on the number line.

2. Which rational number is smaller? Explain by showing on number line.

   (i) $\frac{3}{5},\frac{-7}{8}$ (ii)

3. Fill up the box with appropriate symbols (<, =, >)

(i) $\frac{3}{2}$ ☐ $\frac{5}{4}$ (ii) $\frac{-6}{8}$ ☐ $\frac{-2}{5}$ (iii) $\frac{1}{-2}$ ☐ $\frac{-9}{18}$

(iv) $\frac{-15}{-7}$ ☐ $\frac{3}{7}$ (v) $\frac{-10}{3}$ ☐ -9

4. Between the two rational numbers, which is bigger?

(i) (ii)

(iii) $\frac{-21}{20}, -6$ (iv) $\frac{7}{9}, \frac{3}{7}$ (v) $0, -\frac{1}{2}$

5. Between the two rational numbers, which is smaller?

(i) $5, \frac{13}{3}$, (ii) $\frac{4}{-6}, \frac{-7}{3}$, (iii) $\frac{-17}{11}, \frac{9}{7}$, (iv) $\frac{17}{19}, \frac{-3}{19}$

6. Write the given rational numbers in ascending order ?

$\frac{2}{6}, \frac{-4}{12}, \frac{-9}{-27}, \frac{-5}{18}$

7. Write the given rational numbers in descending order ?

$\frac{-8}{7}, \frac{2}{21}, \frac{-5}{14}, \frac{1}{28}$

8. Julie wrote some of the following statements and asked her friends to find out whether these are true or false statements ?

(i) The rational number is placed at the left side of 0 on number line.

(ii) $\frac{-8}{-3}$ is placed at the right side of 0 on number line.

(iii) $\frac{19}{-5}$ is placed at right side of 0 on number line.

(iv) $\frac{3}{4}$ and $\frac{-2}{7}$ are placed at right and left side of 0 respectively.

Write 4 such statements and ask your friends to check whether they are true.

Exercise 2.3

1. Satish takes houre to reach his school from home, and his sister takes 90 minutes to reach school from home. Now tell who took more time to reach school.

2. Radhika eats $2\frac{1}{2}$ Chapatis in dinner and his sister Gitika eats $\frac{10}{4}$ Chapatis. Tell us whether they both eat equal Chapatis?

3. Ritesh go to the market by walk. After going $\frac{9}{2}$ kilometer in east direction he reminds crossed his destination then he turns back and walks $\frac{1}{2}$ kilometer in west direction by representing on number line tell the distance between his current position and home.

4. Saurbha travel $\frac{9}{3}$ kilometer by bus. After that $\frac{2}{3}$ kilometer by walk to reach his home from school. What is the total distance of school and home. Show it on number line.

We have learnt

$1\frac{1}{2}$

1. The numbers which are expressed or can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q ≠ o, are called rational numbers.

2. A rational number p/q is in the simplest form if there is no common factor between p and q except 1.

3. On comparing two or more than two rational numbers, we convert the given rational numbers into another rational number so that their values are unchanged but their denominators are equal then we compare their numerators.

Chapter -3

PROPERTIES OF A TRIANGLE

You know that a figure enclosed by three lines, having three sides and three angles is known as a triangle. The sum of three angles of a triangle is always 180°. We have learnt to classify triangles on the basis of their angles and their sides. Let us first revise those properties of a triangle that we know:-

Opposite sides and opposite angles

See the triangle given below:-

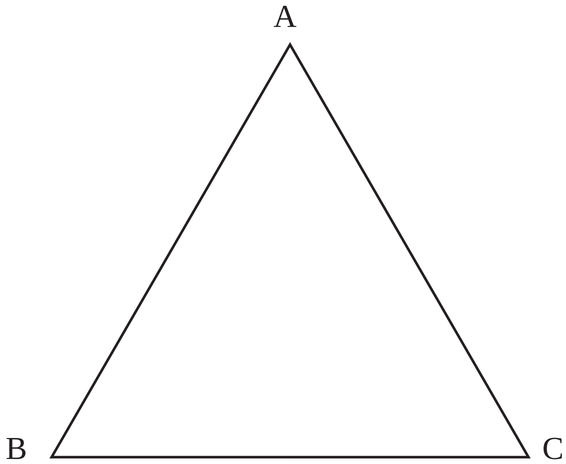


Figure 3.1

Here the opposite angle of sides AB is ∠C.

This angle is not located at the end points of AB (i.e. A and B). Just as the opposite angle of side AB is ∠C, similarly, the opposite side of ∠C is AB.

In the same way, write two other pairs of opposite sides and opposite angles.

Have you ever thought of a relation between angles and their opposite sides and vice-versa? Let us find a relation between them with the help of an activity.

Activity 1

Given are some triangles with different measurements are given. Measure the lengths of the sides and angles opposite to them in the triangles. Fill them in the given table as shown.

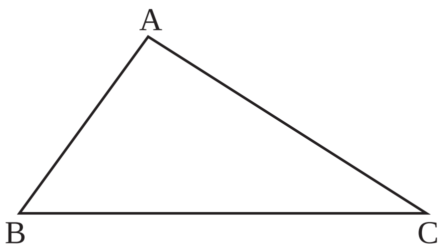


Figure 3.2

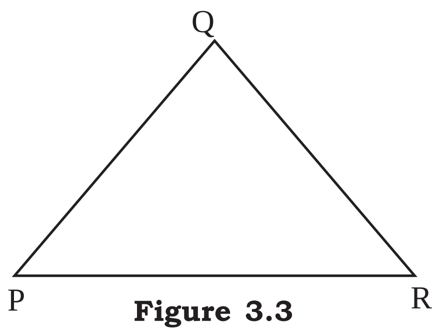


Figure 3.3

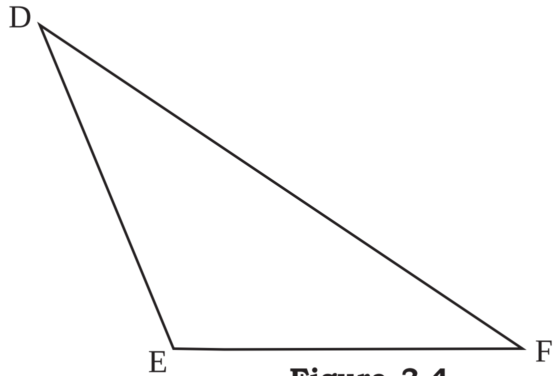


Figure 3.4

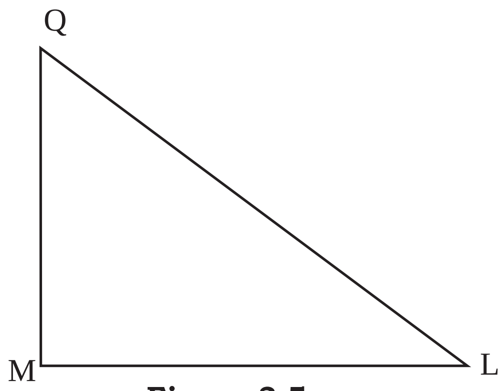


Figure 3.5

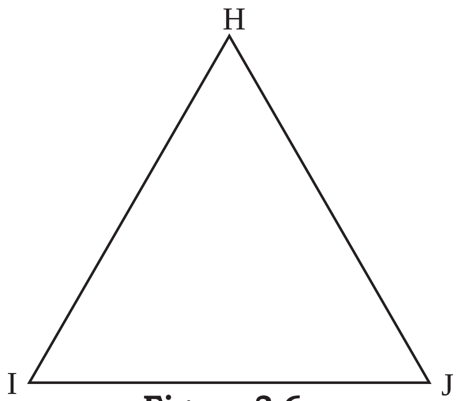


Figure 3.6

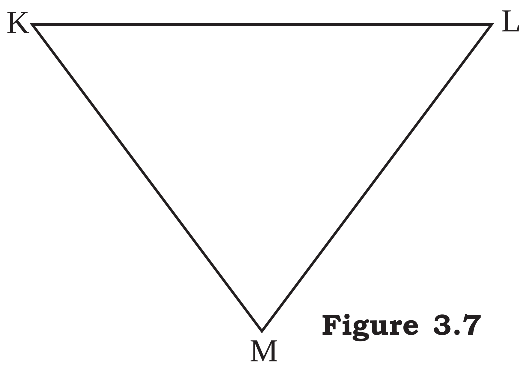


Figure 3.7

Table 1

Fig. No.	Name of Triangle	Measure of sides (Write the length of the sides of fig. 3.2)	Measure of the opposite angle	Sides in descending order	Angles in descending order
3.2	Δ ABC	AB = 2.9 CM BC = 5.4 CM CA = 4.4 CM	$\angle C = 30^0$ $\angle A = 95^0$ $\angle B = 55^0$	BC, CA, AB	$\angle A$, $\angle B$, $\angle C$
3.3	Δ PQR	--------------	--------------	-----------------------	----------------------------
3.4	Δ DEF	--------------	--------------	-----------------------	----------------------------
3.5	Δ QLM	--------------	--------------	-----------------------	----------------------------
3.6	Δ HIJ	--------------	--------------	-----------------------	----------------------------
3.7	Δ KLM	--------------	--------------	-----------------------	----------------------------

Observe the above table, and answer the following questions:-

(i) Is the angle opposite to the longest side, the largest?

(ii) Is the angle opposite to the smallest side, the smallest?

(iii) Is the side opposite to the largest angle, the largest?

(iv) Is the side opposite to the smallest angle, the smallest?

(v) In figure 3.6, are the angles opposite to the two equal sides, equal?

(vi) In figure 3.7, what is the relation between the sides and their respective opposite angles?

You find that the largest side has the largest angle opposite to it and vice versa. Similarly, the smallest angle has the smallest side opposite to it and vice versa.

In ΔHIJ (figure 3.6) equal sides have equal angles opposite to them. In the same manner sides opposite to equal angles are also equal. In figure 3.7, all three sides are of equal lengths and have equal opposite angles. Does this mean opposite sides to the equal opposite angles are always equal in a triangle?

Draw two equilateral and two isosceles triangles of different measures and check the relation and between their sides and their angles.

Examples 1: An isosceles triangle has one angle of 80°. Determine the other two equal angles?

Solution: Isosceles triangle is a triangle in which two sides are of equal length this means, these equal sides has equal angles opposite to them. Let each angle be x.

We known, that sum of three angles of a triangle is 180°.

$\Rightarrow \quad x + x + 80° = 180°$

$\Rightarrow \quad 2x + 80° = 180°$

$\Rightarrow \quad 2x = 180° - 80°$

$\Rightarrow \quad 2x = 100°$

$\Rightarrow \quad x = \dfrac{100°}{2}$

$\Rightarrow \quad x = 50°$

⇒ The two angles are of 50° each.

Examples 2: Determine the angles of an equilateral triangle.

Solution: We know that in an equilateral triangle, all three angles are equal. Let each angle be x°. The sum of the three angles of the triangle is 180°.

$\Rightarrow \quad x° + x° + x° = 180°$

$\Rightarrow \quad x° =$

$\Rightarrow \quad x = 60°$

⇒ In an equilateral triangle, each angle is of 60°.

Example 3: Determine all angles of the triangle given below.

Solution: We know that the sum of three angles of triangle is 180°.

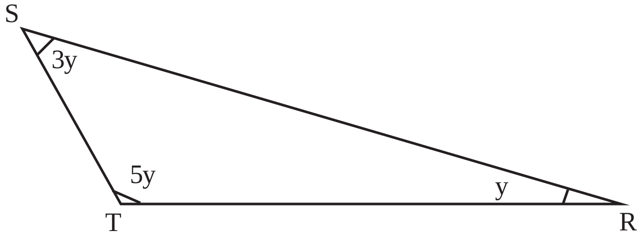


Figure 3.8

∴ In Δ RST, ∠R+ ∠S+ ∠T=180°

⇒ y + 3y + 5y = 180°

⇒ 9y = 180°

⇒ y = $\frac{180°}{9}$

⇒ y = 20°

⇒ ∠R = 20°, ∠S = 3×20°= 60°, ∠T = 5 × 20= 100°.

⇒ The three angle of the given triangle are 20°, 60° and 100°.

EXERCISE 3.1

Q1. Fill in the blanks:

(i) In a triangle, equal sides have .... opposite angles.

(ii) If a triangle has two equal angles, then it is called.........

(iii) Equilateral triangle has three ....angles.

(iv) In an isosceles triangle, if one angle is 100°, then the other two equal angles are ......

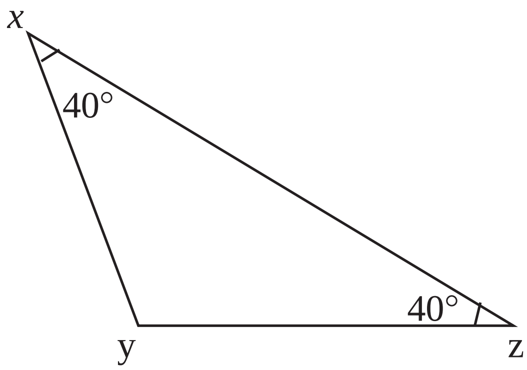
(v) The largest angle in a triangle has the .... opposite side.

(vi) The smallest angle in a triangle has the .... side opposite of it.

Q2. Complete the table given below:-

S. No.	Name of Δ	Length of sides	Angles	Remaining angles
1.	Δ ABC	AB=AC= 4 cm, BC = 5 cm	∠B = 50°	∠C = ----------, ∠A = ------
2.	Δ PQR	PQ=PR= 5 cm, QR = 7 cm	∠R = --------	∠P = -----------, ∠Q = 45°
3.	Δ DEF	DE=DF= 6 cm, FE = 8 cm	∠E = ----------	∠D = 84°, ∠F = ---------
4.	Δ LMN	LM = MN = NL = 5 cm	∠L = ----------	∠M = -------, ∠N = --------

Q3. In the ΔXYZ, given below, write the name of equal sides and determine the third angle ?

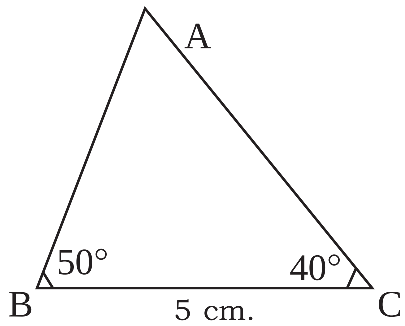


Q4. In the ΔABC given below BC=5cm, ∠C = 40° and ∠B = 50°, then answer the following questions

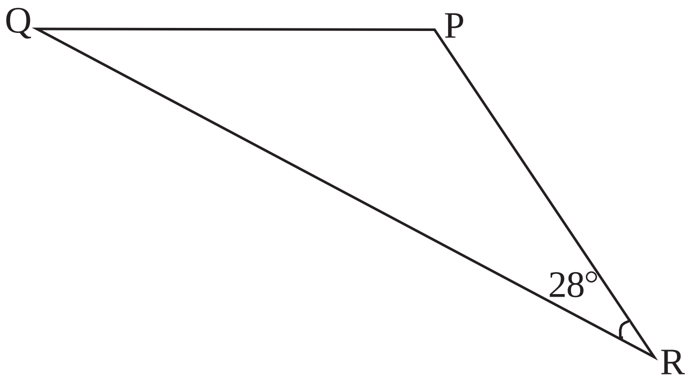
(i) Is AB=AC ? If not, why ?

(ii) Which one is longer side, AB or AC ?

(iii) The largest side is opposite to the smallest angle or to the largest angle?



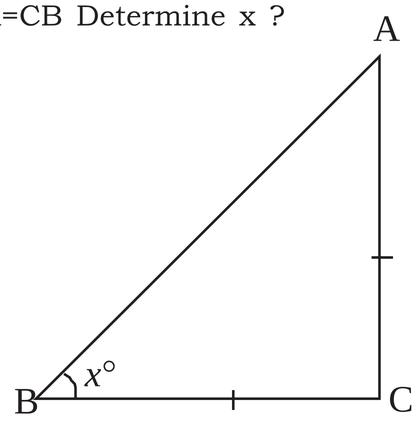
Q5. If ΔPQR has PQ=PR and ∠R=28°. Then calculate the remaining angles.



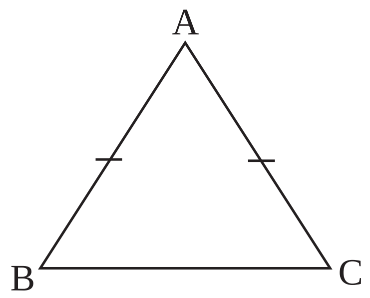
Q6. A triangle has two sides of equal length. If the angles opposite of them are 30°, then determine the third angle ?

Q7. In an isosceles triangle, if the angle at the vertex is 70°, then determine the other two angles opposite to the equal sides ?

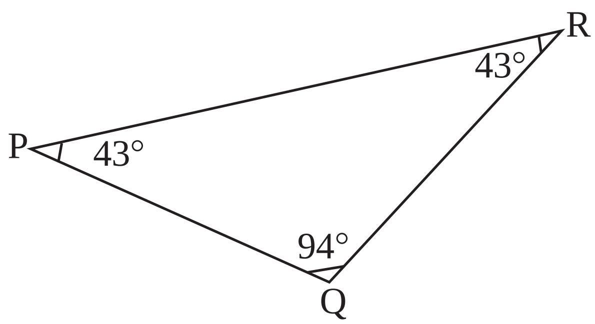
Q8. ΔABC is a right angled triangle with C=90° and CA=CB Determine x ?



Q9. In the figure given below, AB=AC and if ∠B is twice of ∠A, then determine all three angles of the triangle?



Q10. In figure, ΔPQR is given with its three angles. Which are the two equal sides of the triangle? Name the largest side ?



Q11. If three angles of a triangle are in the ratio 2:3:4, find all its three angles ?

MEDIANS OF A TRIANGLE

Cut a triangle from a piece of paper, fold it to keep any two corners of it one over the other. In this way, you have divided one side of the triangle into two equal parts. Label the point from where the side has been folded.

Mark the mid points of all the three sides. Now join all the mid point to the vertices and get the intersection points (fig. 3.9(iii)). Check whether all the points of intersection coincide at the same point? In a triangle, the line segments joining the vertices to the mid point of the opposite sides, are called medians.

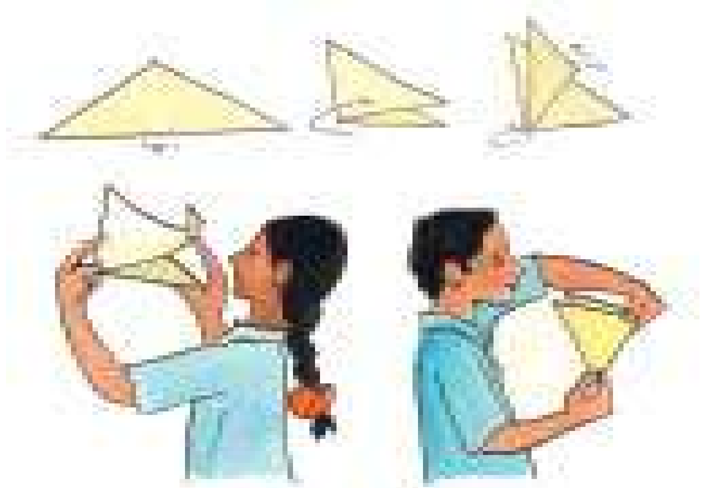


Figure 3.9

You got the mid points of the sides of the triangle made of paper but how will you find out the mid points of a triangle drawn on paper. In class VI, you have learnt to draw perpendicular bisector of a line. Can you find out the mid point of AB? Let us see.

Firstly, open the rounder to more than half of the given line segment AB, place the rounder needle on A and put an arc above and below AB, as shown in figure 3.10 by M and L. Then put the rounder needle on B, and with the same radius mark arcs P and Q above and below AB intersecting the previous arcs.

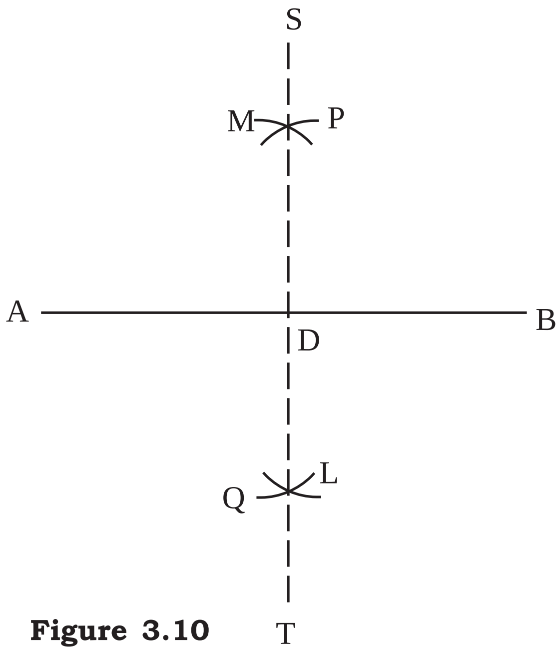


Figure 3.10

Now draw a line joining the points of intersection. This is the perpendicular bisector ST and cuts AB at D. D is the mid point of AB. Similarly, we can get the mid points of the remaining sides of the triangle.

In figure 3.11, ΔABC is given and D is the mid point of BC. A is joined to the mid point D. AD is a median of ΔABC.

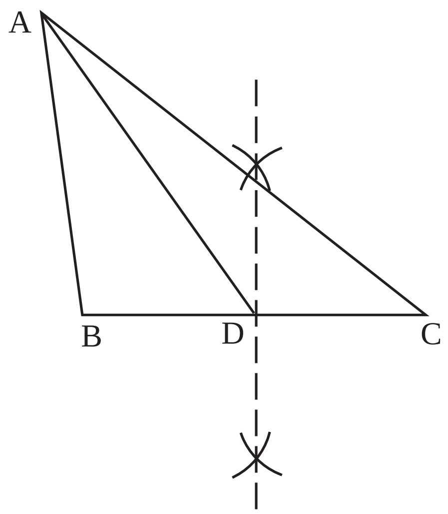


Figure 3.11

In this way draw the three medians of the triangle.

Let us see how we can do this. E and D are mid points of AC and BC respectively. Join these mid points E and D to the vertices B and A respectively.

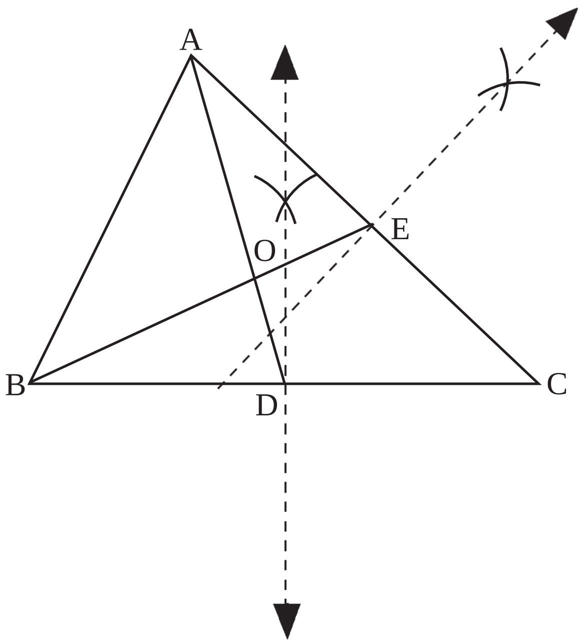


Figure 3.12

The medians AD and BE intersect at the point O. Now you draw a line from C through O and extend it to the point on the side AB. Is that the mid point of AB or not? Does any other median of a triangle exist? You will see that all the three medians of triangle pass through same point or we can say, "The medians of a triangle coincide at a point". The intersecting point of medians is known as **centroid**. O is the centroid of ΔABC.

Now, draw a triangle in your copy and draw its medians to get the centroid. We have seen that if we draw any two medians of a triangle, then the third median also passes through the point where they intersect. So we can say that to find the centroid, only two medians of triangle are required.

Let us learn some more about medians of a triangle.

Activity 2

Some triangles are given below and their medians are drawn. Fill up the blanks in the table accordingly.

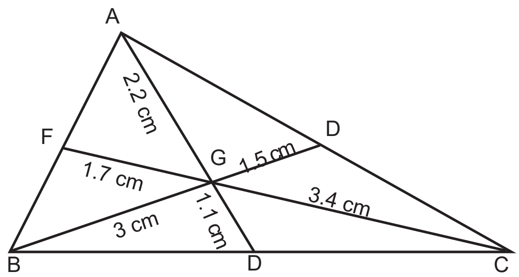


Figure 3.13

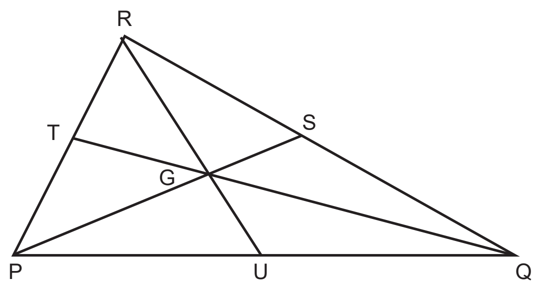


Figure 3.14

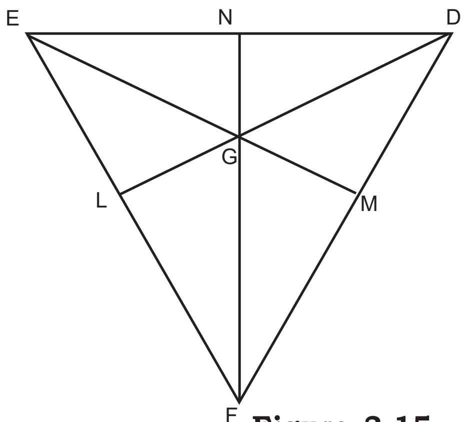


Figure 3.15

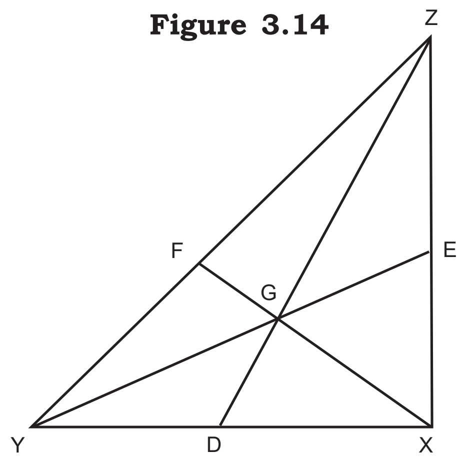


Figure 3.16

Fig.	Name	Distance of centroid G from the vertex	Distance of mid point of opposite side	Ratio
3.13	Δ ABC	AG = 2.2 cm	GD = 1.1 cm	AG = GD 2 : 1
		BG = 3 cm	GE = 1.5 cm	BG = GE 2 : 1
		CG = 3.4 cm	GF = 1.7 cm	CG = GF 2 : 1
3.14	Δ PQR	PG = ---------------	GS = ---------------	PG :GS = --------------------
		QG = ---------------	GT = ---------------	QG :GT = --------------------
		RG = ---------------	GU = ---------------	RG :GU = --------------------
3.15	Δ DEF	DG = ---------------	GL = ---------------	DG :GL = --------------------
		EG = ---------------	GM = ---------------	EG :GM = --------------------
		FG = ---------------	GN = ---------------	FG :GN = --------------------
3.16	Δ XYZ	XG = ---------------	GF = ---------------	XG :GF = --------------------
		YG = ---------------	GE = ---------------	YG :GE = --------------------
		ZG = ---------------	GD = ---------------	ZG :GD = ----------------

From the above table, find out the ratio of the line segments from the vertices to the centroid and from the centroid to the respective mid points? Is the ratio same for all the ratios in a triangle? Is it the same for all triangles? You will find out that the ratio remains 2:1 for all triangles and for all ratios in a triangle. Draw more triangles of different measurements in your copy. Construct their medians and find out whether the centroid divides all the medians in the ratio 2:1.

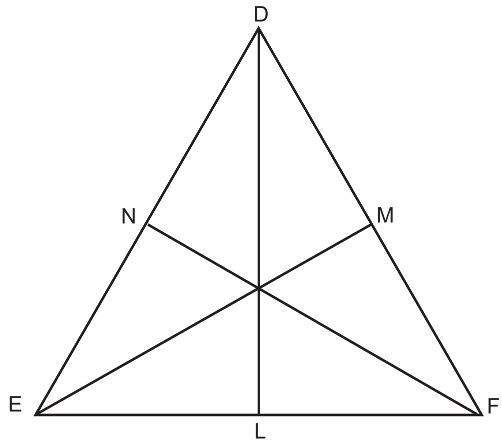


Figure 3.17

Now, consider an equilateral triangle DEF whose medians are DL, EM and FN respectively. Measure the medians and consider the relation between them. Also find out the relation between the angles made with the respective sides by the medians of an equilateral triangle.

You will find out that medians of an equilateral triangle are equal and each median is perpendicular to the respective side of triangle.

Now draw an isosceles triangle and construct medians of its equal sides. Can you see a relation between the two medians? If yes! then write that relation.

Perpendicular from a point on a given line segment

We can draw a perpendicular on a line segment from a point under given two conditions.

1 When point is on the line segment, and

2 When point is outside the line segment

Condition1:- When point is on the line segment.

Steps of Construction

(i) First of all draw a line segment AB, having a point P on it.

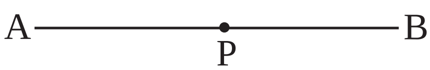


Figure 3.18

(ii) Keep the needle of compass on P and draw a semicircle which cuts AB at Q and R as shown in figure 3.19.

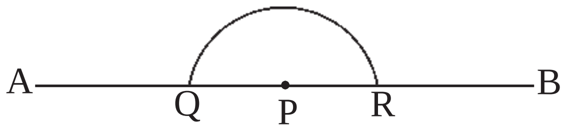


Figure 3.19

(iii) Now keep the needle of compass opened with the same radius first at R. Draw an arc cutting the semicircle QR at T as in figure 3.20. Keeping the needle at T with the same radius draw an arc marking 'S' as shown in figure 3.20.

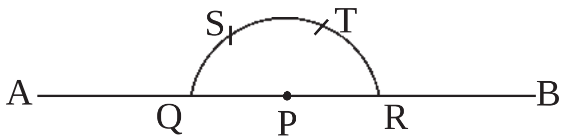


Figure3.20

(iv) Again from points S and T mark arcs of equal radius cutting each other at point M as shown in figure 3.21

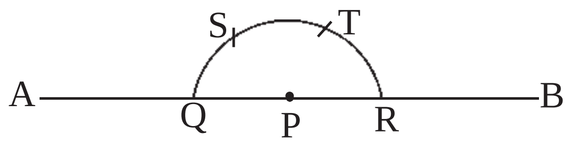


Figure3.21

(v) Join points M and P

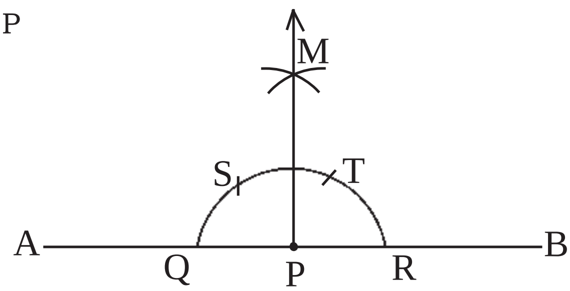


Figure3.22

PM is the required perpendicular on AB from P.

Condition 2: When the point is outside the line segment.

Steps of construction:-

(i) First of all, draw a line AB, take a point P outside AB as shown in figure.

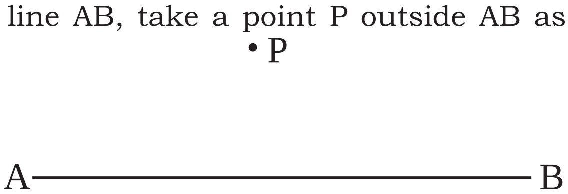


Figure3.23

(ii) Now, from the point P, make an arc QR on AB as shown in figure 3.24

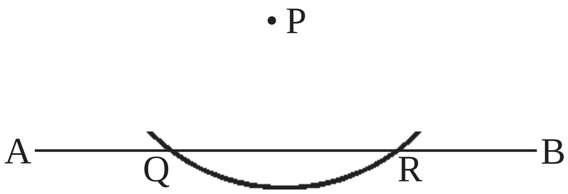


Figure3.24

(iii) Draw two arcs from the point Q and R with equal radius. These arcs intersect at a point S.

(iv) Join S and P and extend. The segments PS and AB meet at M.

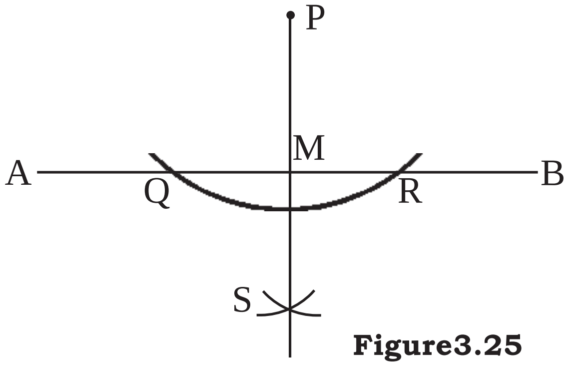


Figure3.25

Thus segment PM is a perpendicular on AB and PM⊥AB.

Altitude of a Triangle

We have learnt to draw a perpendicular to a segment, when the point is on the segment and when the point is out side the segment. Thus we can now easily draw altitudes from the vertices to their opposite side. Altitudes are the perpendiculars drawn from the vertices.

Steps of construction of Altitudes of Triangle

(i) Draw triangle ABC

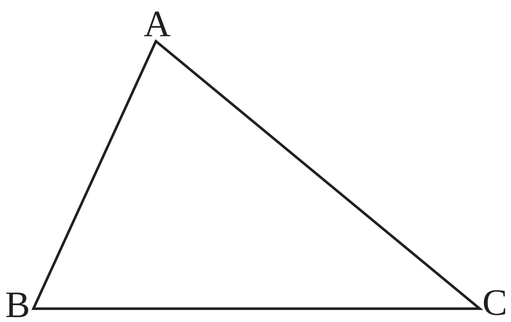


Figure3.26

(ii) We draw perpendiculars from each vertex to their opposite sides. As shown above draw from the point A, a perpendicular on the opposite side. BC meeting it at M and from the point B, a perpendicular on the opposite side AC meeting it at N.

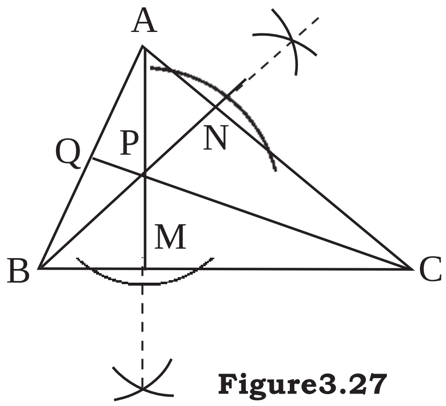


Figure3.27

(iii) Segments AM and BN meet at P

Join P and C. Increase CP to Q at AB. Measure ∠AQC. You will get ∠AQC=90° or CQ ⊥ AB. Thus we get the third altitude as CQ. All three altitudes meet at the point P.

Construct some more triangles. Do the altitudes of the triangle meet at one point?

Yes! **The altitudes of all the triangle meet at one point. The point is called the orthocenter.**

We have seen that the third altitude also passes through the point where any two altitudes meet. Can we conclude that for the orthocenter of the triangle, we need only two altitudes?

Activity 3

Construct an obtuse angle triangle and a scalene triangle in your note book. Draw altitudes from each of the vertices to their opposite sides in both triangles. Similarly draw the altitudes of the right angle triangle. What do you conclude? Write in your notebook.

Exercise 3.2

1. Fill the following blanks:

   (i) Median of a triangle is the segment that joins the vertex to the _________ of the opposite side.

(ii) Altitude of a triangle is the segment that is a ______ from a vertex on the opposite side.

(iii) The medians of a triangle are________________

(iv) Medians of a triangle meet at a point. The point is called ________

(v) Altitudes of a triangle meet at a point. The point is called _________.

(vi) Centroid of the triangles divides the median in the ratio ______________

2. Construct two triangles in your notebook and find their centroid?

3. Construct a right angle triangle and find its orthocenter ?

4. Construct a triangle draw its three medians. Do the medians intersect at one point?

We have learnt

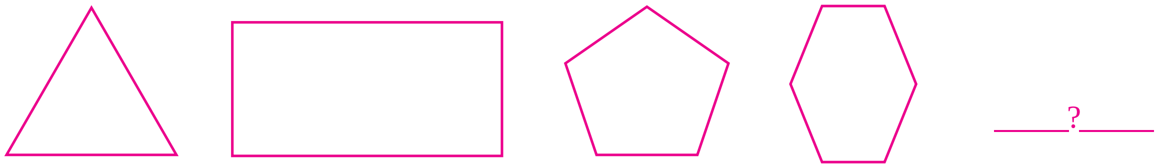
1. Angle opposite to the largest side of the triangle is the largest and the angle opposite to the smallest side of the triangle is the smallest.
2. We get medians by joining the mid points of the sides with their opposite vertices. All the medians intersect at one point and divide each other in the ratio 2:1.
3. The segment drawn perpendicular to a side from the opposite vertex is called the altitude. Altitudes of triangle are concurrent.
4. The point where all the altitudes of a triangle meet is called the orthocenter.

Chapter -4

EQUATIONS

Mental Exercise

Look at the following shapes serially and write answers to the following questions in your notebook:-



(i) (ii) (iii) (iv) (v)

Figure 4.1

a) What can be the figure in the 5th place? Draw it.

b) What did you keep in mind while drawing the 5th shape?

c) Is there a relation between the serial number of the shape and the number of sides of it?

d) Can you tell the number of sides in the 10th shape?

e) How did you answer this?

f) Is it possible to tell the number of sides in a shape if we know its serial number?

g) What relation can you form between the sides and the serial number?

Let us look at the numbers written in the rectangular boxes.

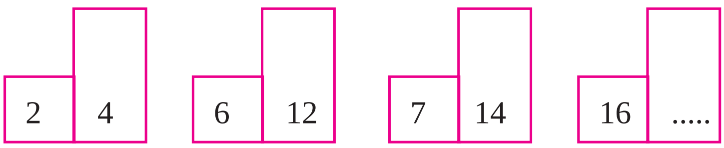


Figure 4.2

In the figure, there are four pairs of boxes. Is there any relation between the numbers in each pair of boxes?

What will be the number in the second box of the fourth pair (Right hand side)? How did you find the solution of this problem?

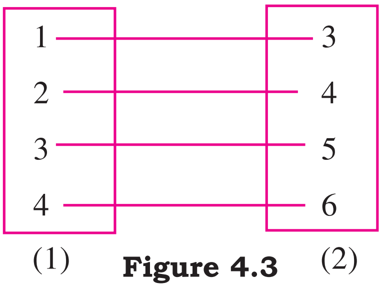


Figure 4.3

Is there any relation between the numbers in the box (1) on the L.H.S. (Left Hand Side) and the numbers on the box (2) on the R.H.S. (Right Hand Side)?

The number in the R.H.S. box is formed by adding 2 to the corresponding number in L.H.S. box i.e. 1+2=3, 2+2=4, 3+2=5, 4+2=6. If 5 is present on L.H.S. then what will be the corresponding number in R.H.S. box? If Y is present in the L.H.S. then what will be needed in R.H.S. box?

Also think about the following stars:-

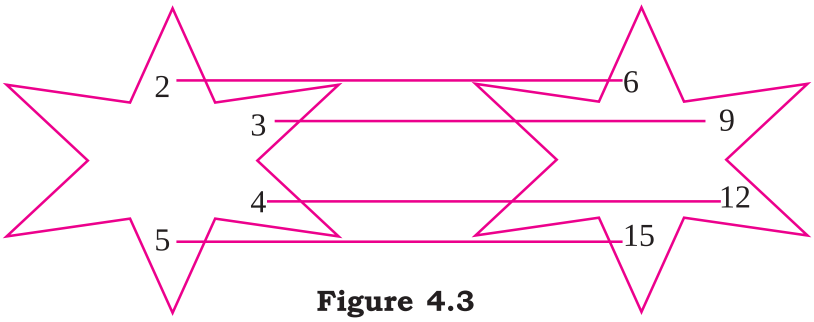


Figure 4.3

Each number of the star is related to a number in the other star. These pairs are shown by lines.

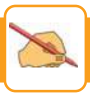
What kind of relation exists between the numbers in the two stars?

If 7 is present in the L.H.S. star then what would be the corresponding number in R.H.S. star?

If x is present in the L.H.S. star, then what would be the corresponding number in R.H.S. star?

On placing 2 in L.H.S. star, we get 6 in R.H.S. star. Similarly, on placing 5 in L.H.S. we get 15 in R.H.S. star.

Now, you can say corresponding to each value of L.H.S. there exist a number 3 times that number in R.H.S.



Activity 1

Take some other sets of numbers and try to establish relation between those numbers.

For example : 1,3,5,7,...... & 2,4,6,8....

Think of some relations and make some groups. Give the stars to your friends and ask them to find the relation between them?

In class-6, we have learnt about both variables and equations. Let us revise it.

We have also seen the following types of questions:-

a) What should be subtracted from 100 such that the remainder is 75?

b) What added to 32 to get sum of 50?

c) What added to half of 12 to get 10?

d) What multiplied by 5 to get 40?

We can find the solution by taking a number to be the unknown variable in each question.

Consider question (a)- On subtracting x from 100 we get 75

i.e. $100-x = 75$

Can we also consider the above question in the following way?

What needs to be added to 75 to get 100?

Let x be added to 75 to get 100.

i.e. $75 + x = 100$

In both cases the value of the unknown variable is the same i.e. on subtracting 25 from 100 we get 75 and on adding 25 to 75 we get 100 i.e. we can analyse the statement in two different ways. We can read of the remaining questions in two ways as well.

Here x is used for the unknown number.

If we use y, z instead of x as the unknown, then would the value remain the same?

$100-x=75$

$100-y=75$

$100-z=75$

Can we write the remaining questions in the same form? Try to do it. How will we find the relations?

Expression and Equation

There are two expressions in each of the above question. The two expressions are equated with the help of a statement. **The statement of equality between two expressions is called an Equation.**

For example, in the above example one expression is 100-x and the other is 75. The symbol of equality implies that the value of L.H.S. and R.H.S. will be equal. Such algebraic statements that have the symbol of equality are called equations. The values present on the left hand side are called **L.H.S.** and the values present on the right hand side are called **R.H.S.** of the equation. Consider some equality statements that have algebraic expressions. State whether the following expressions are equations. If the given expression is not an equation according to you, give reason for that.

(i) $3x + 5 = -9$ (ii) $7x+4>10$

(ii) $x-2<-5$ (iv) $x = 0$

(v) $y = 3x$ (vi) $x+y=3$

(vii) $x= 2y+z+2$

In equation (vi) and (vii), how many unknown quantities or variables are used?

The equation in which the number of unknown quantities or variables is one is called **one variable equation**. If the number of unknown quantities (variables) is two or three then the equations are called **two** or **three variables equation** respectively.

Solution of equation of one variable:

Reeta asked Hameeda "Can you think of a number which when multiplied by seven becomes equal to 3 times the sum of the number and four. Can you guess what the number is?

We will first make an equation to find out the desired number and then solve it.

Let the unknown quantity be x

$3(x+4) = 7x.$

Or $3x+12 = 7x$ (Solving bracket)

Or $3x+12-12=7x-12$ (Subtracting 12 on both sides)

Or $3x=7x-12$

Or $3x-7x=7x-12-7x$ (Subtracting 7x on both sides)

Or $-4x = -12$

Or $\frac{-4x}{-4}=\frac{-12}{-4}$ (Dividing by -4 on both sides)

Or $x=3$

In this way we can find out the value of the unknown quantity. Let us check the answer.

(1) Desired number is 3

(2) On adding 3 to 4 we get 3+4=7

(3) On multiplying 7 by 3 we get $3\times7=21$

(4) 21 is also 7 times the desired number 3.

You have learnt some methods of solving the equations in 6^{th} class.

Let us revise these methods again:-

(1) We can add the same number on both sides of the equation.

(2) We can subtract the same number from both sides of the equation.

(3) We can multiply both sides of equation by the same non-zero number.

(4) We can divide both sides of equation by the same non-zero number.

We use above method in such a way so that the unknown quantity remains on one side only.

Example1 : Solve the equation –

$$3x+2=17$$

[Here L.H.S. is 3x+2 where variable is x. We have to find the value of x].

$\Rightarrow$ $3x+2-2=17-2$ (Subtracting 2 on both sides)

$\Rightarrow$ $3x = 15$

$\Rightarrow$ $\frac{3x}{3}=\frac{15}{3}$ (Dividing by 3 on both sides)

$\boxed{x=5}$

Hence, x=5 is the unique solution of the given equation.

Second method: The above solution of the equation can be written in following manner.

$$3x+2=17$$

$\Rightarrow$ $3x=17-2$ (Changing side of +2)

$\Rightarrow$ $3x=15$

$\Rightarrow$ $X=\frac{15}{3}$ (3 is multiplied by x. Here sign changes in multiplication to division on changing the side)

$\Rightarrow$ $\boxed{X=5}$

Hence, x=5 is the unique solution of the given equation.

In this way we can say that by taking the number on L.H.S. to the R.H.S, the sign of variable (number) changes. While in changing the side the sign of multiplication changes into division and division into multiplication like- In 3x=15 by changing the side of 3 we get x = $\frac{15}{3}$.

Check:-

L.H.S. = 3x + 2

= 3(5) + 2 (Substitute x=5)

= 15+2

=17

R.H.S. = 17

∴ L.H.S. = R.H.S.

Hence, our solution x = 5 is correct.

Example 2. Solve 4x+7 = 2x -11

Solution : Given equation is

⇒ 4x+7 =2x -11

⇒ 4x=2x-11-7 (Changing the side of 7)

⇒ 4x=2x-18

⇒ 4x-2x=-18 (Changing the side of 2x)

⇒ 2x = - 18

⇒ x = $\frac{-18}{2}$ (x is multiplied by 2 on the L.H.S. changing its side, it divides the number R.H.S. changes to divide)

⇒ x = -9

Hence, x=-9 is unique solution.

Check:-

L.H.S. = 4x + 7

= 4(-9) +7 (Substituting the value of x = -9)

= -36 + 7=-29

And R.H.S. = 2x -11

= 2(-9) -11 (Substituting the value of x = -9)

= -18-11

= -29

$\therefore$ L.H.S = R.H.S.

Hence our solution x = -9 is correct.

Example 3. Solve the equation

Solution : Given equation is -

$$\frac{x}{10}+12=17$$

$\Rightarrow \quad \frac{x}{10}=17-12$ (Changing the side of 12)

$\Rightarrow \quad \frac{x}{10} = 5$

$\Rightarrow \quad \frac{x\times10}{10}=5\times10$ (both sides are multiplied by 10)

$\frac{x}{10}+12=17$

$\Rightarrow$ x = 50 (in L.H.S. x is divided by 10 which on changing side multiplies the numbers on R.H.S.)

Check the answer by yourself.

Example 4: $\frac{x}{5}+\frac{x}{20}=10$

Solution : $\frac{x}{5}+\frac{x}{20}=10$

$\Rightarrow \quad \frac{4(x)+1(x)}{20}=10$ (Taking L.C.M. of 5 & 20)

$\Rightarrow \quad \frac{4x+x}{20}=10$

$\Rightarrow \quad \frac{5x}{20}=10$ (The division by 20 on the LHS becomes multiplication on changing the side.)

$\Rightarrow$ 5x = 200

$\Rightarrow \quad x=\frac{200}{5}$ (In L.H.S. 5 is multiplied by x and it changed into division on changing the side.)

$\Rightarrow$ x = 40

Check the solution by yourself.

Example 5: Solve the equation

$$\frac{2}{5}(x+10)=2x+3$$

Solution: Given equation is-

$$\frac{2}{5}(x+10)=2x+3$$

$\Rightarrow \quad \frac{2x}{5}+\frac{2}{5}\times 10=2x+3$ (On solving the L.H.S.)

$\Rightarrow \quad \frac{2x}{5}+4=2x+3$

$\Rightarrow \quad \frac{2x}{5}-2x=3-4$ (Changing the sides of 2x and +4)

$\Rightarrow \quad \frac{2x}{5}-2x=-1$

$\Rightarrow \quad \frac{2x-10x}{5}=-1$

$\Rightarrow \quad \frac{-8x}{5}=-1$

$\Rightarrow \quad -8x = -1 \times 5$

$\Rightarrow \quad -8x = -5$

$\Rightarrow \quad x=\frac{-5}{-8}$

$\Rightarrow \quad \boxed{x=\frac{5}{8}}$

Hence, $\boxed{x=\frac{5}{8}}$ is required solution.

In solving equations we first separate the parts with the variable from the parts without them and take them on the opposite sides of the equation. Then to find the value of the unknown quantity, if the unknown variable is multiplied or divided by a number, it gets converted to division or multiplication on changing the side.

Exercise 4.1

Q 1: Fill in the blanks:-

(i) Solution of equation 2x = 4 is, x = ☐

(ii) Solution of equation $\frac{x}{3} = 3$ is, x = ☐

(iii) Solution of equation 3x + 2 = 8 is, x = ☐

(iv) Solution of equation 5 y = 2y +15 is, y = ☐

Q 2. Solve the equations and check the Answer-

(i) 7x + 15 = 3x +31 (ii) 3 (x-3) = 5 (2x-1)

(iii) $\frac{2y+9}{3} = 3y+10$ (iv) 2(x-1) -3(x-2)=4(x-3)+5(x-4)

(v) (vi) $\frac{x+2}{3} + 5 = 17$

(vii) 3y + (viii) $\frac{3m+2}{3} = \frac{17}{3}$

(ix) 2.5 x + 3.5=6

52x 115 = + = 13
83 86 6

Use of equations for solving problems:

Arithmetic methods take long time to solve daily life problems but these questions can be solved conveniently by algebraic methods.

To establish relation between two quantities we change our language to algebraic language. When we do this, the problem becomes easier to understand and solve. Let us look at this through an example:-

“When 5 is added to a natural number, its value becomes 9. Find the number”.

We will solve it by “hit and trial” method. Since the natural number 9 is less than 10.

∴ Let us start natural numbers from 1 like- 1+5=6

2+5=7

3+5=8

4+5=9

Hence, the required number is 4

If a large number is given then this method will take long time. If we solve it using algebraic language then it becomes easier and also saves time.

Let the required number be x.

According to question $x + 5 = 9$

$\Rightarrow \quad x = 9 - 5$

$\Rightarrow \quad x = 4$

By this method we can solve questions of larger numbers conveniently.

Example : 6

Nilima goes from one place to another. In the first hour she covers a fixed distance. In the second hour she covers 5 km lesser. In the 3rd hour she covers 8 km less than the distance covered in second hour. If the total distance travelled is 48 km, then find out the distance covered by Nilima in the first hour ?

Solution: Let the distance covered by Nilima in first hour be x km.

Then distance covered by Nilima in the second hour = $x - 5$

and distance covered by Nilima in third hour = $x - 5 - 8 = x - 13$

$\Rightarrow \quad x + x - 5 + x - 13 = 48$

$\Rightarrow \quad x + x + x = 48 + 5 + 13$

$\Rightarrow \quad 3x = 66$

$\Rightarrow \quad x = \frac{66}{3}$

$\Rightarrow \quad x = 22$ km

Thus, the distance covered by Nilima in first hour is 22 km.

Example :7: One number is greater than the second by 5 and 9 times of the second number is equal to 4 times of the first number. Then find the numbers ?

Solution: Let the second number be x.

Then the first number = $x + 5$

And 9 times of the second number = $9x$

4 times of first number = $4(x + 5)$

According to the question

$9x = 4(x + 5)$

$\Rightarrow \quad 9x = 4x + 20$

$\Rightarrow \quad 9x - 4x = 20$

$\Rightarrow \quad 5x = 20$

$\Rightarrow \quad x = \frac{20}{5}$

$\Rightarrow \quad x = 4$

$\therefore$ Second number = 4

First number = x + 5

= 4 +5 = 9

$\therefore$ Required numbers are 4 and 9

Example : 8. The sum of three consecutive natural number is 63. Then find the numbers.

Solution: Let three consecutive natural numbers be x, x+1 and x+2.

(Because there is a difference of 1 in consecutive natural numbers)

According to question

$x + x + 1 + x + 2 = 63$

$\Rightarrow \quad x + x + x + 1 + 2 = 63$

$\Rightarrow \quad 3x + 3 = 63$

$\Rightarrow \quad 3x = 63 - 3$

$\Rightarrow \quad 3x = 60$

$\Rightarrow \quad x = \frac{60}{3} = 20$

x = 20 therefore, x +1 = 21 and x +2 =22

Hence the numbers are 20, 21, and 22 respectively.

Example 9: In a two digit number the number at tenth place is double the number at unit place. If the sum of two numbers is 9 then find the number.

Solution : Let unit place number be x. Then tenth place number is 2 x

According to question,

$x + 2x = 9$

$\Rightarrow \quad 3x = 9$

$\Rightarrow \quad x = \frac{9}{3} = 3$

∴ Unit place number = 3

Tenth place number $= 2 \times x$

$= 2 \times 3$

$= 6$

Tenth	Unit
6	3

Hence, required number is 63.

Example 10: The base of an isosceles triangle is 3 cm less than the measure of each of the equal side. If the perimeter of triangle is 21 cm then find the length of each side ?

Solution: Let the length of each equal side be x cm.

Length of base is x – 3 cm.

Perimeter of triangle = sum of three sides

$\Rightarrow \quad 21 = x + (x-3) + x$

$\Rightarrow \quad 21 = 3x - 3$

$\Rightarrow \quad 21 + 3 = 3x$

$\Rightarrow \quad 24 = 3x$

$\Rightarrow \quad \frac{24}{3} = x$

$\Rightarrow \quad 8 = x$

Therefore, x = 8

Therefore, the sides of triangle are 8, 5, 8 respectively.

Example 11: Sheela is twelve years older than Ranjeet. After 6 years the age of Sheela will be two times the age of Ranjeet. Then find the present ages of Sheela and Ranjeet ?

Solution: Let present age of Ranjeet be x years.

Present age of Sheela is x + 12 years.

Age of Ranjeet after 6 years = x + 6 years and

Age of Sheela after 6 years= x + 12 + 6

= (x + 18) years.

According to question:-

Age of Sheela after 6 years = 2 (Age of Ranjeet after 6 years)

$x + 18 = 2 (x + 6)$

$\Rightarrow \quad x + 18 = 2x + 12$

$\Rightarrow \quad x - 2x = 12 - 18$

$\Rightarrow \quad -x = -6$

$\Rightarrow \quad x = 6$

Therefore, present age of Ranjeet = 6 years

And the present age of Sheela = 6 + 12

= 18 years.

Example 12: The numbers of boys and girls studying in a class are in the ratio 3:5. If the total number of boys and girls are 80 then find the actual number of boys and girls in the class ?

Solution: Let the number of boys and girls be 3 x and 5 x respectively.

(In order to solve ratio question we take variable with ratio)

Therefore, $3x + 5x = 80$

$\Rightarrow \quad 8x = 80$

$\Rightarrow \quad x = \frac{80}{8}$

$\Rightarrow \quad x = 10$

Therefore, number of boys in class = $3x = 3 \times 10 = 30$

Number of girls in a class = $5x = 5 \times 10 = 50$

Therefore, the number of boys and girls are 30 and 50 respectively.

Exercise 4.2

1. Convert the statements into equations.

   (i) $\frac{2}{3}$ of a certain number is 24

   (ii) The age of father is double the age of the son and the sum of their ages is 51.

   (iii) $\frac{1}{10}$ of a certain amount is Rs. 2500

   (iv) The sum of two consecutive numbers is 15.

   (v) The denominator of a rational number is 5 more than the numerator and the rational number is $\frac{19}{24}$?

2. When 3 is added to 7 times of a number, then the sum becomes 31. Find the number ?

3. Divide Rs. 300 between Ram, and Shyam such that Ram gets Rs. 100 less than 3 times the amount Shyam gets ?

4. Find the number which on multiplying by 4, is 42 more than the number ?

5. The length of a rectangle is 3 more than its width. If the perimeter of the rectangle is 30 cm then find the length and width of the rectangle ?

6. The length and breadth of a rectangle are in the ratio 2:3. If the perimeter of the rectangle is 90 cm then find the length and breadth of rectangle ?

7. In a class of 35 students, the number of girls is $\frac{2}{5}$ times the number of boys. Find the number of boys in the class ?

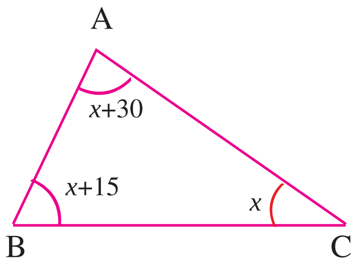
8. The sum of one fourth of a number and 12 is 20. Find the number ?

9. The sum of two consecutive numbers is 35. Find the numbers ?

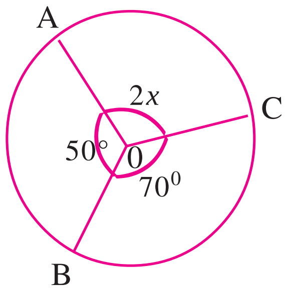
10. The age of Namrata's father is 3 times the age of Namrata. If the sum of their ages is 48 years then find the ages of both ?

11. A plot reserved for playground is in the ratio of length & breath 11:4. Gram Panchayat spends Rs. 45000 for boundary at the rate of 100 per m. Find the dimensions of the plot.

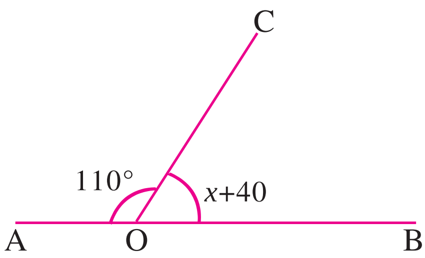
12. Find the value of x in degree in the following figures.



(i)



(ii)



(iii)

We have learnt

1. The quantity whose numerical values are not fixed are called variable quantities (example x = 1,2,.... 3 etc. and y = 1,2,3....) Here x and y are variables.
2. If there is equality between algebraic expressions, then it is called an equation.
3. In any equation, to determine the value of the unknown quantity is to solve the equation.
4. Adding, subtracting, multiplying and dividing the same number on both sides of an equation does not change the equation.
5. The value of the unknown variable that satisfies the equation is called a solution of the equation.
6. In an equation taking any quantity to other side is called changing the side.

Chapter -5

USE OF BRACKETS

Radha had taken Rs 80. She purchased a pen for Rs 15 and a compass box for Rs 23. She spent Rs 15 + 23 = Rs 38. Now Radha has 80-38= Rs 42. If you have to record this, how will you write it? Write in your notebook.

Akanksha had written–

80-15-23=Rs42. Answer is correct.

Julie asked, “You have subtracted Rs 15 and then Rs 23 from Rs 80. Why don’t you add Rs 15 and Rs 23 and then subtract from Rs 80?” But how do we represent this? Do you have an answer to Julie’s question?

Let us discuss this problem-

Let the rectangle be of 5 units length and 3 units width, the area of the rectangle = 5×3=15 square units (Fig. 5.1). If we increase the length by 2 units and do not change the width, what will be the area of the rectangle?

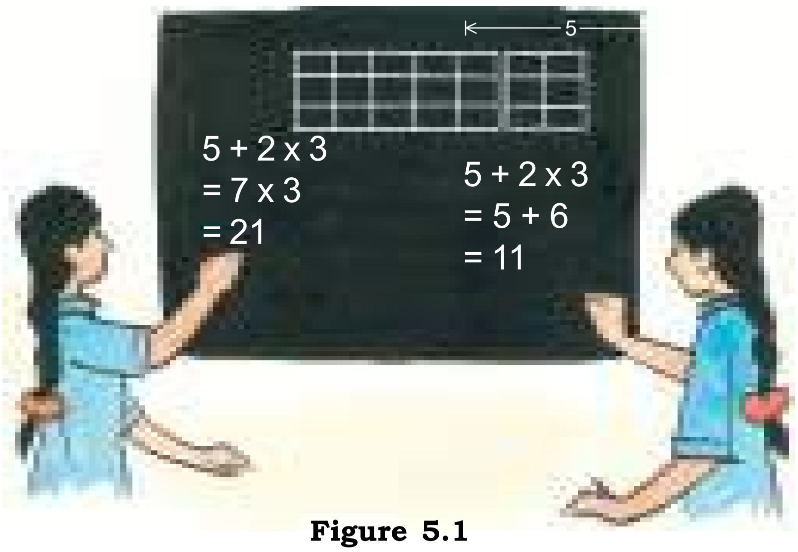


Figure 5.1

The students took the length to be 5+2 and width 3 and solved using the following two ways.

First Method -

5 + 2 × 3 = 7 × 3 = 21 square units.

Second Method -

5 + 2 × 3 = 5 + 6 =11 square units.

Find the reason 'why we get these two answers?' Write the reason in your notebook.

In first method, we added 5 and 2 and then multiplied with 3.

In second method, we multiplied 2 and 3 and then added 5.

Here, the numbers have been grouped differently. The first has put 5 with 2 and the second has put 2 with 3. Both groups have got different answers. Therefore, when a question has more than one operation (addition, subtraction, multiplication, division), we may need brackets.

Since, the length is increased and width is unchanged. The length 5 + 2 needs to be together hence we write (5 + 2) for it.

Therefore, area = (5 + 2) × 3= 7 × 3=21 square units.

Julie has got an answer to the question. Let us see some more examples-

80 - (23 + 15) = 80 – 38 = 42

(80 – 23) + 15 = 57 + 15 = 72

(5 + 2) × 3 = 7 × 3 = 21

5 + (2 × 3) = 5 + 6 = 11

Example 1: Use brackets to write the following sentence in numbers. "Multiply seven with the sum of five and three."

Solution: Add five and three and then multiply with seven.

Therefore, we write (5 + 3) × 7 = 8×7 = 56

Example 2: Divide the difference between 28 and 15 by the sum of 12 and 4.

Solution: Find the difference between 28 and 15.

Add 12 and 4.

Divide the difference ;by the sum.

Therefore (28-15) ÷ (12 + 4)

Example 3: Add $\frac{4}{11}$ to twice the sum of $\frac{7}{9}$ and $\frac{3}{5}$.

Solution: $\left(\frac{7}{9}+\frac{3}{5}\right)\times 2+\frac{4}{11}$ Or $2\times\left(\frac{7}{9}+\frac{3}{5}\right)+\frac{4}{11}$ (Commutative Property)

Exercise 5.1

1. Use brackets in the following sentences.

   (i) Divide the difference of ten and two by thirty.

   (ii) Multiply the difference of twelve and five by twenty seven.

   (iii) Divide the sum of 4.5 and 2.3 by 3.8.

   (iv) Divide $\frac{8}{27}$ by the sum of $\frac{2}{3}$ *and* $\frac{7}{15}$.

2. Write the following sentences using brackets.

   (i) Multiply the difference between 8 and 6 by the sum of 15 and 27.

   (ii) Add the result of the division of 11 by 29 to the product of 37 and 28.

   (iii) Multiply the difference between 8.45 and 6.75 by the sum of 3.2 and 2.4.

   (iv) Subtract the difference between 8 and 3 from twice the sum of 5 and 11.

   (v) Divide the sum of $\frac{4}{27}$ *and* $\frac{5}{9}$ by $\frac{7}{8}$.

   (vi) Add the sum of 5 and 10 , the difference of 7 and3 and the product of 8 and 25.

Let us do some examples-

Example 4: Solve 2(5+3).

Solution 2(5+3) = 2(8)

= 2 × 8

= 16.

We have done this type of questions earlier. Here 2 is multiplied by 8. **If we have no symbol between the number and the bracket, we multiply the numbers of the bracket with the number outside.**

Example 5: Solve a - (b - c).

Solution a - (b - c) = a – b + c.

Example 6: Solve p – (q + r – s).

Solution : p – (q + r – s) = p – q - r + s.

If the negative operation is before the bracket, positive number in the bracket is converted into negative and the negative number is converted into positive.

If before the bracket we have the positive operation, will there be any change? Just think about?

Fill up the blank boxes with the correct symbol-

(i) 13 - (7 - 5) = 13 ☐ 7 ☐ 5

(ii) 8 + (10 - 6) = 8 ☐ 10 ☐ 6

(iii) 20 - (8 - 5 - 1) = 20 ☐ 8 ☐ 5 ☐ 1

(iv) (a*x* - by) - (cz+d) = a*x* ☐ by ☐ cz ☐ d

(v) 0.75 + (0.25 - 0.30 + 0.05) = 0.75 ☐ 0.25 ☐ 0.30 ☐ 0.05

Example 7: Solve

a + 2 (2a – 3b).

Solution a + 2 (2a – 3b)

= a + 4a – 6b (Removing Brackets)

= 5a - 6b.

Example 8: Solve

3x - 4y – (2x -3y).

Solution 3x - 4y – (2x -3y)

= 3x - 4y – 2x +3y

= 3x – 2x - 4y + 3y

= x – y.

Exercise 5.2

Solve the Following:

1. 2a + 4 (a + 5b)
2. (3a – 4b) – 2b
3. (4x+3) – (2x+3)
4. 2(5x+3 – 4x+2)

5. 30 – 15 (4x - 2y)
6. 4.5 + 2.5 (3.5 + 8.5)
7. 12.8 – 3.2 (4 - 2.8)
8. 8a + 3 (5a +6b – 3)
9. $\frac{3}{4}+\frac{11}{19}\left(\frac{6}{11}+\frac{7}{22}\right)\frac{3}{4}+\frac{11}{19}\left(\frac{6}{11}+\frac{7}{22}\right)$

We often perform more than one operations (addition, subtraction, multiplication, division) in one question. Let us solve some examples-

Example 9: Simplify 15-4×3+16÷8.

15-4×3+16÷8=15-4×3+2(division)

=15-12+2(multiplication)

= 3+2 (subtraction)

=5 (addition)

Anu told Suresh, "My age is $\frac{1}{3}$ of my father's age. It means my father's age is 39 years and my age is 13 years. That is we use 'of' for multiplication. Can you give any other example of 'of'?" Suresh said, "Why not? In our class numbers of boys are double that of girls. If girls are 24, boys are 2 ×24=48."

Let us think of another example-

Example 10: Akanksha's book has 120 pages. She daily reads $\frac{1}{5}$ of the total number of pages. How many pages does she read daily?

Solution: Here, we have to find $\frac{1}{5}$ of 120 pages

Therefore, of 120 pages= 120×$\frac{1}{5}$ =24 pages

Hence she daily reads 24 pages.

Thus here also 'of' is used for multiplication (×).

Example 11: Length of a room is 10m. If the width is $\frac{3}{5}$ of the length, find the width.

Solution: Width of the room = $\frac{3}{5}$ of the length of the room.

= $\frac{3}{5}$ of 10

$= \frac{3}{5} \times 10$

= 6m.

Example 12. There are 200 boys and 150 girls in a school. If the number of teachers is $\frac{1}{25}$ of the number of students, how many teachers are there in the school?

Number of teachers = $\frac{1}{25}$ of (number of boys + number of girls).

$= \frac{1}{25}$ of (200+150)

$= 350 \times \frac{1}{25} = 14$

Thus the number of teachers is14.

Example 13: Simplify 5 + 6 of 3 ÷ 9 + 8 – 2 × 3.

Solution: 5 + 6 of 3 ÷ 9 + 8 – 2 × 3 ('of' means '×' therefore, 6 of 3=6×3=18)

= 5 + 18 ÷ 9 + 8 – 2 × 3 (Division Operation)

=5 +2 +8 – 2 × 3 (Multiplication Operation)

= 5 + 2 + 8 - 6 (Addition Operation)

= 15 - 6 (Subtraction Operation)

= 9

Thus, we observed that the following steps are used in solving the above examples-

Step1. Simplify 'of'.

Step2. Do Division.

Step3. Do Multiplication.

Step4. Do Addition.

Step5. In the end, do subtraction.

We use "BODMAS" in simplifying the question. "BODMAS" is a rule which is used when the brackets and the mathematical operations are given simultaneously.

Here, B = Bracket

O = Of

D = Division

M = Multiplication

A = Addition

S = Subtraction.

Types of Brackets

Till now we have learnt use of one bracket ( ). Sometimes we need to use more than one type of bracket. Generally, we use the following kinds of brackets. The different shapes are only to distinguish each expression that has a bracket.

Brackets with their symbol

	Types of Brackets	Symbol
1.	Bar Bracket	" —— "
2.	Parentheses	"( )"
3.	Curly Brackets or Braces	"{ }"
4.	Square Brackets	"[ ]"

We simplify the brackets in the following order.

Step1. Simplify Bar Brackets "——"

Step2. Simplify Parentheses "( )"

Step3. Simplify Curly Brackets or Braces "{ }"

Step4. Simplify Square Brackets "[ ]"

While constructing expression using brackets this order has to be kept in mind.

Example 14: Simplify

$7- \{ 13- 2 (4+ \overline{4-2})\}$

Solution:

$7- \{ 13- 2 (4 + \overline{4-2})\}$

$= 7-\{13-2(4+\overline{4-2})$

$= 7- \{13-2(4+2)\}$ (Removing the bar)

$=7- \{13-2\times6\}$ (Removing the parenthesis)

$=7- \{13-12\}$ (Removing the curly brackets.)

$=7-1=6.$

Example 15: Simplify

5x – [2x – 4 + {7x- 3 (3 + 2x)}]

Solution 5x – [2x – 4 + {7x- 3 (3 + 2x)}]

=5x-[2x-4+ {7x- 9- 6x}] (Removing the parentheses)

= 5x-[2x-4+ {7x-6x-9}]

= 5x- [2x-4+{x-9}]

= 5x – [2x-4+ x-9] (Removing the Curly Brackets)

= 5x- [3x-13]

= 5x – 3x +13 (Removing the Square Brackets)

= 2x + 13

Example 16: $\frac{3}{4}+\left\{\frac{1}{2}-(\frac{2}{3}-\frac{1}{3})\right\}$

Solution: $\frac{3}{4}+\left\{\frac{1}{2}-(\frac{2}{3}-\frac{1}{3})\right\}$

$=\frac{3}{4}+\left\{\frac{1}{2}-(\frac{2-1}{3})\right\}$

$=\frac{3}{4}+\left\{\frac{1}{2}-(\frac{1}{3})\right\}$

$=\frac{3}{4}+\left\{\frac{1}{2}-\frac{1}{3}\right\}$

$=\frac{3}{4}+\left\{\frac{3-2}{6}\right\}$

$=\frac{3}{4}+\frac{1}{6}$ (LCM (4, 6) =12)

$=\frac{9+2}{12}=\frac{11}{12}$

Example 17: Simplify

3.4 + {4.6 – (1.5 +0.6)}

Solution: 3.4 + {4.6 – (1.5 +0.6)}

$= 3.4 + \{4.6 - 2.1\}$

$= 3.4 + 2.5$

$= 5.9$

Example 18: Simplify

$5a + \{3b - (2a - 4b)\}$

Solution $5a + \{3b - (2a - 4b)\}$

$= 5a + \{3b - 2a + 4b\}$

$= 5a + \{3b + 4b - 2a\}$

$= 5a + \{7b - 2a\}$

$= 5a + 7b - 2a$

$= 5a - 2a + 7b$

$= 3a + 7b$

Example 19: Simplify

$5x^2 - \{ 3x + (3x^2 - 2x)\}$

Solution : $5x^2 - \{ 3x + (3x^2 - 2x)\}$

$= 5x^2 - \{ 3x + 3x^2 - 2x \}$

$= 5x^2 - \{ 3x - 2x + 3x^2 \}$

$= 5x^2 - \{x + 3x^2 \}$

$= 5x^2 - x - 3x^2$

$= 5x^2 - 3x^2 - x$

$= 2x^2 - x$

Sometimes brackets make multiplication easier by simplifying the expression.

Example 20: Simplify

88×95

Solution: 88×95

$= 88 \times (100 - 5)$

$= 88 \times 100 - 88 \times 5$ (Using Distributive Property)

$= 8800 - 440$

= 8360

Distributive Property is used to solve the above question.

Example 21: Find the value of 23.5 × 9.9

Solution : 23.5 × 9.9

= 23.5 × (10 – 0.1)

= 23.5 × 10 - 23.5 × 0.1

= 235 – 2.35

= 232.65

Exercise 5.3

1. Find the value of the following expressions-

(i) $\frac{1}{5}$ of (4+6)

(ii) (-13) + 6 ÷ (7 - 4).

(iii) 20 – (5 × 3)

(iv) 15- (2 of 2) +4

(v) 16 ÷ (6 – 5)

(vi) (-20) × (-2) + (-14) ÷ 7.

(vii) 15 + (-3) of (-4)-6

2. Simplify-

(i) 3x- [4x + {x + (5x -3x) }]

(ii) $2-[\ 2-\{\ 2-(2-\overline{2-2})\}\ ]$

(iii) $36 \div (8-\overline{4+2})$

(iv) (19 – 6) [19 + { 15 + 8 – 3}]

(v) $3a^2 + \{\ 5a^2 - (2a + 2a^2\ )\ \}$

(vi) $5\frac{3}{4} \div 4\frac{3}{5} + 2\frac{1}{2}$

(vii) 4a + [2a – {3b + (3a -2b) }]

3. Write true or false.

(i) $18 - 3 \times 5 = 75$

(ii) $5 \times 4 + 2 = 22$

(iii) $4 - 2 - 2 = 0$

(iv) $18 \div \frac{1}{6} \div 3 = 1$

4. Simplify by using "Distributive Property" –

(i) 347×101

(ii) 429×98

(iii) 5.8×1.5

(iv) 48×0.9

5. Ankit has Rs 50. He gave $\frac{7}{10}$th of it to Vinita for her requirements. How much money did he give to Vinita?

6. Mayank with Rs 20 and Pankaj with Rs. 30 visited a fair. Both of them together spent $\frac{2}{5}$th part of their total money. While returning from fair they paid Rs10 more for exhibition. How much amount did they spend in fair?

7. Puja got Rs 60 from her father, Rs 40 from her mother and Rs 20 from her brother. She purchased a pen for Rs 15. She distributed the remaining amount equally among her five friends. How much money did each of her friends get?

We have Learnt

1. We use brackets for representing together two or more than two operations.

2. If 'Of' is given with four operations, we simplify 'Of' first, 'Division' second, 'Multiplication' third, ' Addition' fourth and 'Subtraction' last.

3. Generally, we use four brackets. We simplify expressions in the following order-

"——", "( )", "{ }", "[ ]".

4. If the brackets are given with mathematical operations, we simplifying that in the order of 'BODMAS'.

5. Operation 'Of' means multiplication. We simplify 'Of' before any other mathematical operation.

Chapter -6

EXPONENTS

Introduction

An old proverb says "The news spreads like fire in the forest". Does this happen to news in reality?

Let us estimate how the news spreads as fast as fire spreads in the forest.

One man reached his home town from the capital with a news. He told the news to 3 persons. He took 5 minutes in narrating the news. Each of the three narrated the news to 3 other persons in the next 5 minutes. This way we get a sequence. In the first 5 minutes, 3 persons knew the news, in the next 5 minutes 3× 3 = 9 more persons also knew the news, in the next 5 minutes the news will be reach to 9 × 3 persons = 27 new persons and in next 5 minutes it will be reached to 27 ×3 =81 new persons. So, how many persons will know the news in 60 minutes?

Let us calculate-

With in 5 minutes, the news reached to:	3	new persons
Between 5 to 10 minutes, the news reached to:	3×3=9	new persons
Between 10 to 15 minutes, the news reached to:	3×3×3=27	new persons
Between 15 to 20 minutes, the news reached to:	3 ×3×3×3=81	new persons
Between 20 to 25 minutes, the news reached to:	3 ×3×3×3×3=243	new persons
Between 25 to 30 minutes, the news reached to:	3×3×3×3×3×3=729	new persons
Between 30 to 35minutes, the news reached to:	3×3×———7times=2187	new persons
Between 35 to 40 minutes, the news reached to:	3×3×———8times=6561	new persons
Between 40 to 45 minutes, the news reached to:	3×3×———9times=19683	new persons
Between 45 to 50 minutes, the news reached to:	3×3×———10times=59049	new persons

Between 50 to 55 minutes, the news reached to: 3×3×———11times=177147 new persons

Between 55 to 60 minutes, the news reached to: 3×3×———12times=531441 new persons

Thus, news reached to –

1+ 3+9+27+81+243+729+2187+6561+19683+59049+177147+531441 =797161 persons within one hour.

Here, we have assumed that everyone is spreading the news and narrating to 3 new persons. You can see the spread would much faster now as 797161 would be spreading the news in the next five minutes.

This way news started from one persons reached to about 8 lakh people in an hour.

"Chess" was invented in India. There is an interesting story related to the game. The story is—when the king wanted to reward the scholar who was the inventor, and he called him and said, "I have enough money. I can fulfill your wish. So, speak, what do you want?"

The scholar said, "Oh King! You are great. Please give me one grain for the first square, two grains for the second square, 4 grains for the third square, 8 grains for the fourth square, 16 grains for the fifth square, 32............."

The king angrily said, "Stop! this. You will get the grains for all 64 squares. Each square will get the double of the previous square. This is what you want. But you have insulted my greatness by asking for such a small gift."

Do you know how many grains will be needed for the sixty-fourth square?

Multiplication will become larger and larger. Each time 2 is multiplied by 2.

Grain for first square	:	1
Grains for second square	:	2
Grains for third square	:	2×2
Grains for fourth square	:	2×2×2
Grains for fifth square	:	2×2× 2×2
Grains for sixth square	:	2×2× .... 5 times
____________________		___________
___________________		___________

Thus, grains for sixty-fourth square : 2×2×.........63 times.

Certainly, this is a very large number. You do not want to know the end of the story. Will the king be able to give the gift?

The king would have to give 18446744073709551615grains to the scholar. If the whole earth was cultivated, still the king would not be able to give that gift. You can see it is a very-very large number.

We get a very large number by simplifying 2×2 ×.......... 63 times. Is, there a short form to write the product when a number is multiplied by itself again and again?

Exponents in Natural Number.

All the students were trying to think where they had used the multiplication of a number by itself repeatedly.

Anu told Ashu, " We write unit cm × cm = cm^2 in calculating area. Similarly, we write cm × cm× cm = cm^3 in finding the volume. So, can't we write 2 × 2 ×2 as =2^3?

Yes! Anu gave the correct short form for writing a number which multiplied by itself again and again.

Can you write 5×5×5×5×5×5 in short?

Thus, 5×5×5×5×5×5=5^6

Similarly, a×a×a×a×a×a =a^6 and x×x×x×x =x^4

Write the following exressions (expanding form) in the short form.

(i) $x×x×x×x×x×x×x×x×x×x$ =...............

(ii) r×r×r×r×r =...............

(iii) 17×17×17×17×17×17×17×17×17 =...............

(iv) 101×101×101×101×101 =...............

We have learnt to write a short form of a number which is multiplied by itself again and again. This short form is called 'Exponent Notation' or 'Power Notation'. Let us learn "To read it".

3×3×3×3 = 3^4. Here, 3 is base and 4 is exponent.

p×p×p×p×p×p = p^6. Here, p is base and 6 is exponent.

r×r×.........17 times= r^{17}. Here, r is base and 17 is exponent.

Activity 1.

Fill up base and exponent in the following number.

3^5 , base = 3 and exponent 5

7^{19}, base =....... and exponent.......

x^a , base =...... and exponent......

p^q , base =...... and exponent......

x^y , base =...... and exponent......

You would have understood that the use of exponent is for writing a very large number in a short form. For example the distance between Sun and Earth is 150000000 km.

Clearly, this is a larger number. This can be written as –

150000000 km = $15\times10\times10\times10\times10\times10\times10\times10 = 15\times10^7$ km .

We have learnt to write the expanded form in short.

Now write the exponent notation in expanded form.

1. $a^5 = a\times a\times a\times a\times a$.
2. 3^6 =...............
3. 5^5=...............
4. r^7=................
5. 2^m=..............

Rahim was confused in writing the expanded form of 2^m. He was not able to write the number as m did not have a fixed value. Do you have answer of Rahim's question?

We have seen earlier that the chess has 64 squares. King had to give $2\times2\times$.......... 63 times or 2^{63} grains.

Similarly, we can write-

$2^m = 2\times2\times$.......... m times.

$x^m = x\times x\times$.......... m times

$y^n = y\times y\times$.......... n times.

Laws of Exponents.

We know that $2^5 = 2\times2\times2\times2\times2$. We can write this exponent notation in different ways by using brackets, like—

$2^5 = 2\times(2\times2\times2\times2) = 2^1\times2^4$

$2^5 = (2\times2)\times(2\times2\times2) = 2^2\times2^3$

$2^5 = (2\times2\times2)\times(2\times2) = 2^3\times2^2$

$2^5 = (2\times2\times2\times2)\times2 = 2^4\times2^1$

Here the exponent notation 2^5 is written in different ways with base 2.

Write the following exponent notation expressions with the same base and add the exponents of both.

Table

S.No.	Exponential expression	Write the expanded form as groups by using brackets (make groups as per choice)	Write the exponent notation for each bracket	Add Exponents
1.	a^7	$a\times a\times a\times a \times a\times a\times a$	$a^4\times a^3$	$4+3=7$
2.	x^5			
3.	y^{10}			
4.	27^7			
5.	7^{12}			

Look at the expanded form of exponent notation in the table above and fill the boxes.

$a^7 = a^5 \times$ [a^2] $x^5 = x^3 \times$ [ ] $y^{10} = y^7 \times$ [ ]

$27^7 = 27^4 \times$ [ ] $7^{12} = 7^8 \times$ [ ]

Is there any relation between the exponents of the two numbers with the same base and the exponent of their product? Write.

Let us discuss how to multiply the exponent expression having the same base number.

$$x^3\times x^4 = x\times x\times x\times x\times x\times x\times x = x^7 = x^{(3+4)}$$

$$x^5\times x^3 = x\times x\times x\times x\times x\times x\times x\times x = x^8 = x^{(5+3)}$$

$y^{19} \times y^{21} = y \times y \times .....19 times \times (y \times y \times .....21 times)$

Can you answer how many times y is multiplied by itself? What will be the exponent if we take base y?

We multiplied y with y (19+21) = 40 times. So the multiple will be y^{40}.

Thus, we can say that **"When we multiply two exponent expressions having the same base, the multiple has the same and the exponents are added."**

Like:-

$$3^{99} \times 3^{13} = 3^{(99+13)} = 3^{112}$$

Can you tell the multiple of $x^m \times x^n$?

$x^m \times x^n = x \times x \times .....$ m times and n times.

Or it is multiplied (m + n) times.

Thus, **Rule1** $x^m \times x^n = x^{m+n}$

You think of another two exponent expressions whose bases are same and multiply them by using rule1.

1. $3^5 \times 3^9 = 3^{14}$ (*Example*)
2. $3^{10} \times 3^4 = 3^{14}$ (*Example*)
3. × =
4. × =
5. × =

You have learnt to write a number multiplied by itself again and again in exponent form and you can easily multiply the two exponent expressions of the same base.

Can you convert 8^2 into an exponent expression having base2?

Raju solved this in the following way:-

$8^2 = 8 \times 8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$

Radha solved this question differently. She wrote $8 = 2^3$ and multiplied 2^3 *with* 2^3 .

$8^2 = 8 \times 8 = 2^3 \times 2^3 = 2^{(3+3)} = 2^6$

After seeing both the solutions Rahim said, "We get the same answer from both methods. But, if we have a bigger exponent expression like-8^{12} then what will we do?"

Radha was trying to solve the question in her own way:-

$$8^{12} = 8 \times 8 \times 8 \times ....12 times$$
$$= 2^3 \times 2^3 \times 2^3 \times ....12 times$$
$$= 2^{(3+3+3+............12 times)}$$
$$= 2^{(3\times12)}$$
$$= 2^{36}$$

Radha solved the question but in Raju's method there were so many 2's that he also gave up his method and solved the question using Radha's method.

Radha gave some other questions to her friends and asked them to write those as the exponent expressions with given base.

1. Write $(27)^6$ as the exponent expression of base3.
2. Write $(25)^5$ as the exponent expression of base5.
3. Write $(64)^6$ as the exponent expression of base4.

Let us discuss the questions given by Radha:-

For writing as the exponent expression of base 4, we have to convert 64 as the exponent expression of base 4. Or

$$(64)^6 = (4 \times 4 \times 4)^6$$
$$= (4^3)^6$$
$$= 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3$$
$$= 4^{(3+3+3+3+3+3)}$$
$$= 4^{18}$$

In the above question, we can write 64 as 4^3 and $(64)^6$ as $(4^3)^6$ or we can say that 4^3 is multiplied by 4^3 6times.Or $4^{(3\times6)} = 4^{18}$.

Clearly while solving 4 raisedto the power 3 and the whole raised to the power 6, i.e. $(4^3)^6$,we multiply the exponents of 4.

Look at another example –

$$(25)^5 = (5^2)^5$$
$$= 5^2 \times 5^2 \times 5^2 \times 5^2 \times 5^2$$
$$= 5^{(2+2+2+2+2)}$$
$$= 5^{10}$$

Activity 2

Simplify the following:-

(i) $(2^3)^5 =$

(ii) $(14^2)^4 =$

(iii) $(10^2)^5 =$

(iv) $(x^2)^3 =$

(v) $(a^7)^9 =$

(vi) $(x^2)^m =$

(vii) $(y^n)^6 =$

(viii) $(x^m)^n =$

How did you solve question (viii)? Describe it in your notebook.

$$(x^m)^n = x^m \times x^m \times \ldots\ldots\ldots\ldots n \text{ times}$$
$$= x^{(m+m+\ldots\ldots\ldots\ldots n \text{ times})}$$

We know that:-

m + m =2×m

m + m + m = 3×m

m +m+........8 times 8×m

m + m +........12 times 12×m

Therefore m + m +.......n times=n × m=m × n.

Thus **Rule2** $(x^m)^n = x^{m\times n}$

Similarly, $(p^q)^r = p^{q\times r} = p^{qr}$

Thus when we have an exponent number raised to a certain exponent, we multiply the exponents or powers.

Activity 3

Solve the following questions with rule2.

(i) $(7^5)^9 = 7^{5\times 9} = 7^{45}$

(ii) $(2^9)^{13} =$

(iii) $(a^b)^c =$

(iv) $(x^2)^3 =$

(v) $(31^{12})^3 =$

Let us discuss the following numbers which can be written as products:-

(i) $6^3 = (2 \times 3)^3$
$= (2 \times 3) \times (2 \times 3) \times (2 \times 3)$
$= (2 \times 2 \times 2) \times (3 \times 3 \times 3)$
$= 2^3 \times 3^3$

(ii) $35^4 = (5 \times 7)^4$
$= (5 \times 7) \times (5 \times 7) \times (5 \times 7) \times (5 \times 7)$
$= (5 \times 5 \times 5 \times 5) \times (7 \times 7 \times 7 \times 7)$
$= 5^4 \times 7^4$

(iii) $77^5 = (7 \times 11)^5$
$= (7 \times 11) \times (7 \times 11) \times (7 \times 11) \times (7 \times 11) \times (7 \times 11)$
$= (7 \times 7 \times 7 \times 7 \times 7) \times (11 \times 11 \times 11 \times 11 \times 11)$
$= 7^5 \times 11^5$

While considering these questions, Rahim thought that if the number is 26^m then how would it be written?

$26^m = (2 \times 13)^m = (2 \times 13) \times (2 \times 13) \times (2 \times 13)$ -------------- m times
$= (2 \times 2 \times 2$ -------- m times$) \times (13 \times 13 \times 13$ -------- m times$)$
$= 2^m \times 13^m$

Rajia asked Rahim, "How will we write $(ab)^m$?"

Rahim answered, "We will simplify it as we have simplified the above questions."

$(ab)^m = (ab) \times (ab) \times (ab) \times$ ----------------m times
$= (a \times a \times a$ ------------- m times$) \times (b \times b \times b$ ------------- m times$)$
$= a^m b^m$

Thus, **Rule3.** $(ab)^m = a^m b^m$

Exercise 6.1

1. Write the following in exponent form-
   (a) $3\times3\times3\times3\times3=$
   (b) $5\times5\times5\times5\times5\times5\times5\times5=$
   (c) $a \times a \times a \times a \times a \times a \times a=$
   (d) $b \times b \times b\times b=$

2. Factorize the following and write in the exponent form:-

   (a) 15^4 (b) 42^5 (c) 51^3 (d) 21^m (e) 65^6

3. Prove the following:-

(a) $(a \times b \times c)^p = a^p \times b^p \times c^p$

(b) $30^5 = 2^5 \times 3^5 \times 5^5$

(c) $616^9 = 7^9 \times 8^9 \times 11^9$

4. Write the following in exponent form by using rule3:-

(a) $6^8 \times 7^8$ (b) $a^3 \times b^3$

(c) $p^9 \times q^9 \times r^9$ (d) $a^n \times b^n \times c^n \times d^n$

5. State true or false:-

(a) $2^3 \times 2^4 = 2^7$ (b) $5^{15} \times 5^5 = 5^{20}$

(c) $2^4 \times 3^2 = 2^6$ (d) $(27)^2 = (3^3)^2$

(e) $(2^3)^4 = (2^4)^3$

Division with exponent in Natural Numbers

Fatima asked Monu, "We have learnt multiplication of numbers expressed in form of exponents having the same base, but how can we divide the quantities having the same base?"

Monu said, "Let us do it."

$$\frac{2^5}{2^3} = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} = 2 \times 2 = 2^2$$

Kamli and Ashu solved such questions also:-

(i) $\frac{5^8}{5^4} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5} = 5 \times 5 \times 5 \times 5 = 5^4$

(ii) $\frac{7^9}{7^6} = \frac{7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7}{7 \times 7 \times 7 \times 7 \times 7 \times 7} = 7 \times 7 \times 7 = 7^3$

Fatima looked at all the solutions and then said to her friends, "As we add exponents (powers) when multiplying two exponent expressions with the same base, in the same manner we subtract exponents (powers) when dividing the two exponent expressions with the same base."

Example:-The exponent (power) of the quotient $2^5 \div 2^3$ is 5 - 3 = 2, the exponent (power) of the quotient $5^8 \div 5^4$ is 8 - 4 = 4 and the exponent (power) of the quotient $7^9 \div 7^6$ is 9 - 6 =3.

Or the exponent (power) of the quotient $a^m \div a^n$ is m-n.

Thus **Rule4** $\frac{a^m}{a^n} = a^{m-n}$

All of a sudden Monu said, "This is ok! But if the numerator and the denominator have the same number as exponent then, what will happen?"

"Let us try to do"

Example:- $\dfrac{7^5}{7^5} = \dfrac{7\times7\times7\times7\times7}{7\times7\times7\times7\times7} = 1$

$\therefore\ 7^0 = 1$

So, if any number has as the exponent the number 0, the value of that number is 1.

Like $\dfrac{p^n}{p^n} = 1$ and by rule 4 $\dfrac{p^n}{p^n} = p^{n-n} = p^0$.

Thus **Rule5** **$p^0 = 1$**

Let us discuss the following numbers:-

$\dfrac{1}{5^2} = \dfrac{5^0}{5^2} = 5^{0-2} = 5^{-2} \quad (\because 5^0 = 1)$

$\dfrac{1}{6^{35}} = \dfrac{6^0}{6^{35}} = 6^{0-35} = 6^{-35} \quad (\because 6^0 = 1)$

$\dfrac{1}{4^{90}} = \dfrac{4^0}{4^{90}} = 4^{0-90} = 4^{-90} \quad (\because 4^0 = 1)$

In thinking about such questions, Fatima concluded that if in the exponent expressions, the denominator becomes the numerator then the positive integer power (exponent) is converted into negative integer and the negative integer is converted into positive integer.

Thus $\dfrac{1}{a^4}$ will be:-

$\dfrac{1}{a^4} = \dfrac{a^0}{a^4} = a^{0-4} = a^{-4}$

If the number is $\dfrac{1}{a^m}$ then $\dfrac{1}{a^m} = \dfrac{a^0}{a^m} = a^{0-m} = a^{-m}$

Thus **Rule6** $\dfrac{1}{a^m} = a^{-m}$ OR $\dfrac{1}{a^{-m}} = a^m$

But, if we take the numerator to the denominator then what will happen?
As we have seen earlier that

$\dfrac{1}{7^{-3}} = 7^3$ or $\dfrac{1}{a^{-4}} = a^4$ or $a^m = \dfrac{1}{a^{-m}}$

Exercise 6.2

(1) Solve the following questions with the help of rule4.

(a) $6^5 \div 6^3$ (b) $27^8 \div 27^2$

(c) $13^m \div 13^n$ (d) $(mn)^7 \div (mn)^2$

(e) $x^{11} \div x^4$

(2) Prove the following:-

$(a)\ \dfrac{6^5}{6^4} = 6 \quad (b)\ \dfrac{p^9}{p^4} = p^5 \quad (c)\ \dfrac{16^{2m}}{16^m} = 16^m$

(3) Write the following numbers as numbers with positive integer power (exponent).

(a) 12^{-3} (b) 19^{-5}

(c) 3^{-4} (d) 5^{-3}

(4) Take the denominator to the numerator.

$(a)\ \dfrac{1}{35^4} \qquad (b)\ \dfrac{1}{x^5}$

(5) Change the numerator into denominator.

$(a)\ \dfrac{3^{-9}}{1} \quad (b)\ \dfrac{a^m}{b^m} \quad (c)\ \dfrac{p^{-8}}{p}$

(6) Simplify -

$(a)\ 273^0 \qquad (b)\left(\dfrac{x^5}{x^2}\right)^0 \qquad (c)\left(\dfrac{27p^{21}}{p^{11}}\right)^0$

(7) Find the value of x

(i) $2^x = 4$ (ii) (iii) (iv) (v) (vi) $3^{2x+4} = 343$

[8] The population of the world is 5×10^9 approx. and its area is 4×10^{11} Sq km. Find number of persons per sq. km.

We have learnt

1. Writing an expression that has a number multiplied by itself again and again in a short form is called **Exponential Expression**.

2. When we multiply two exponent numbers having the same bases, then their product has the same base and their powers (exponents) are added.

$$x^m \times x^n = x^{m+n}$$

3. If the numerator and the denominator have an exponent expression with the same base then on solving the base remains the same and we subtract the power of the denominator from the power of the numerator. $\frac{x^m}{x^n} = x^{m-n}$.
4. If the exponent expression has also an exponent then the powers are multiplied. $(x^m)^n = x^{m \times n}$.
5. If any number has exponent 0, the value of the number is 1. $x^0 = 1$, $5^0 = 1$.
6. If the number has no exponent then the exponent of that number is one. $x = x^1$, $6 = 6^1$.
7. If we carry the exponent expressions from numerator to the denominator or from denominator to the numerator, the positive power integer is converted into a negative integer and the negative integer power is converted into a positive integer. Like: $\frac{1}{x^{-n}} = x^n$ and $x^m = \frac{1}{x^{-m}}$.

Chapter -7

CONSTRUCTION OF PARALLEL LINES & TRIANGLES

In the Previous class you have studied about construction of different geometrical figures with the help of Set Square and compass. In which line bisector, perpendicular, angle of different measurement, angle bisectors etc. are there.

In this chapter you are going to know about how to draw parallel lines and construction of some triangles.

Construction of parallel lines

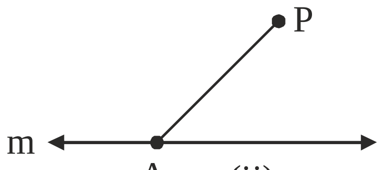
Drawing a line through the point parallel to a given line which is not located on that line.

Construction1

1- Draw a line m and take a point P outside the line.

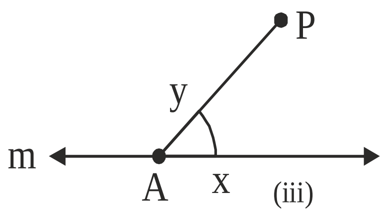
(i)

2. Take a point A on line m and join P and A.



(ii)

3. By taking point A as center and a convinent radius, draw an arc which intersect line m at X and AP at Y.



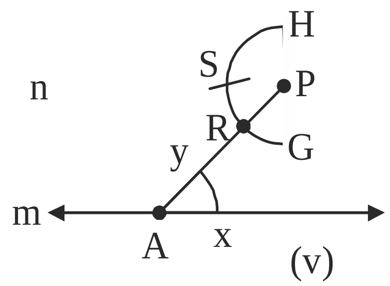
(iii)

4. Now, taking P as center and radius of equal measurement taken in step 3 draw an arc GH which intersect AP at R.

m

5. Place the pointed side of compass on X and expand the fitted pencil arm upto Y.

6. By taking R as center and radius as same of step 5 we draw an arc which cuts the arc GH on S.



7. Now draw a line n through SP, which is desired parallel line.

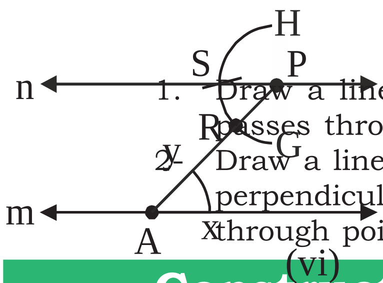


Figure 7.1

EXERCISE 7.1

1. Draw a line l. Take a point A outside of this. Now draw a parallel line which passes through point A and parallel to line l.
2. Draw a line m.Take any point P on it. Draw a perpendicular on point P. On this perpendicular take a point Q at a distance of 3 cm. Draw a parallel line n passes through point Q.

Construction of triangles

You have read about properties of triangles in previous classes. Now we will discuss about the construction of different types of triangles. You know that a closed figure with 3 sides is called a triangle. We also know that these three lines have some length.

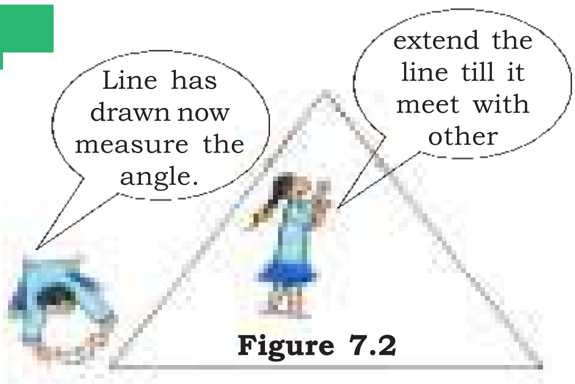


Figure 7.2

Constructing a triangle with three given sides?

If you are asked to draw a triangle whose two sides are of lengths 3cm and 4 cm then what is the difficulty you may face? How many triangles can you form with the given parameters?

Now, if we say that third side is of length 5cm, is it sufficient to draw a unique triangle? Let us see:-

Steps to be followed:- First, draw a rough sketch of the triangle keeping in mind the parameters of triangle (i.e. lengths of sides of the triangle) Write the given length of sides on the rough sketch.

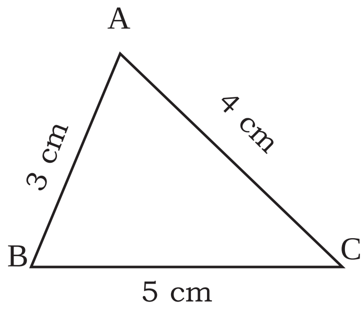


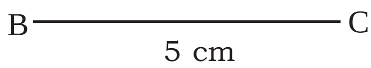
Figure 7.3

Let's now draw a triangle as per the parameters.

Construction1

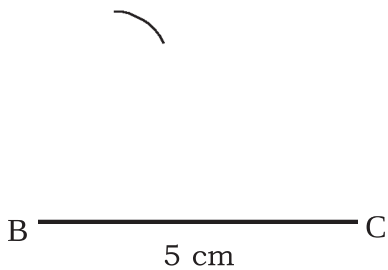
STEP 1

Draw a line segment BC of length 5 cm with the help of scale.



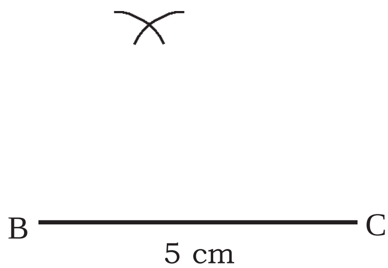
STEP-2

Mark an arc of radius 3 cm from B, with the help of compass as shown in figure.



STEP-3

Now from c mark an arc of radius 4 cm, intersecting the arc drawn in step 2. Name the point of intersection as A Join AB and AC.



Δ ABC is the required triangle as shown in figure 7.4

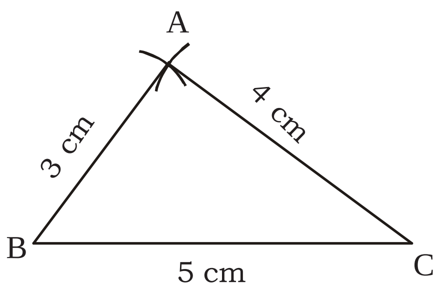


Figure 7.4

EXERCISE 7.2

1. Draw the triangles using the given parameters:-

    (i) AB = 4 cm, BC = 7 cm, AC = 5cm

    (ii) AB = 5 cm, BC = 6 cm, CA = 5 cm

    (iii) AB = 4 cm, BC = 6 cm, CA = 7 cm

Determine angle C in the above three cases? Which triangle has the largest value of angle C?

2. Draw the triangles using the parameters given below:-

    (i) AB = 8 cm, BC = 8 cm, CA = 8 cm

    (ii)AB = 4 cm, BC = 2 cm, CA = 2 cm

    (iii) AB = 8 cm, BC = 3 cm, CA = 4cm.

    (iv) AB = 5 cm. BC = 6 cm.

Do you find any difficulty in drawing the above triangles?

What was the difficulty?

What can you do to overcome these difficulties? Have a look at the measures given in (ii) and (iii) again. Can you draw triangles using above parameters?

We know that in a triangle, sum of any two sides is greater than the third side. In question (ii) sides are of lengths 4 cm, 2 cm and, 2 cm. If we consider sides of lengths 2 cm, 2 cm, their sum is 4 cm (2 cm + 2 cm) which is equal to the length of third side (of length 4 cm) that is why, we are not able to form this triangle.

While, in (iii) what should be the length of CA so that the triangle may be formed?

As AB = 8 cm, BC = 3 cm, then should the value of CA be more than 5cm? Think over it. What is the maximum possible value of CA?

If CA = 11 cm, then is it possible to form the triangle?

If not, why?

This means, that if AB = 8 cm, BC = 3 cm, then to form the triangle the length of CA must be greater than 5 and less than 11 (i.e. CA $>$ 5 and CA $<$ 11).

Give such questions to your friends too? And try some other such question to determine the possible values of the third side.

In (iv), the length of the third side is not given. In this case, as discussed above third side can be of any length greater than 1 and less than 11.

ONE MORE SITUATION

We have seen that it is not possible to form a unique triangle with the lengths of only two sides given. If instead of the length of the third side, we are given the angle between the two lines, is it possible to form a triangle? Let us try to construct it.

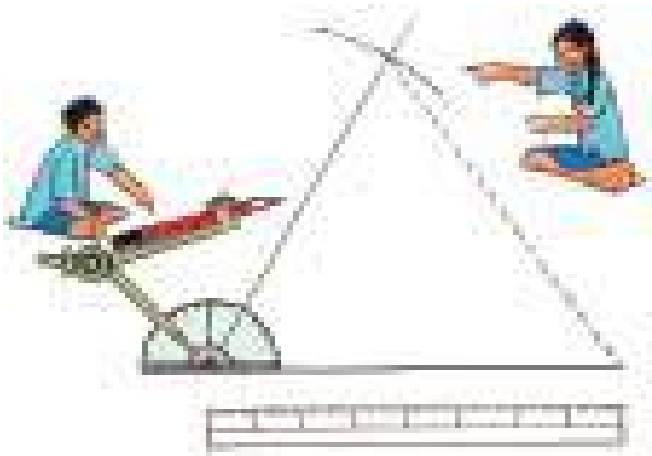


Figure 7.5

Construction of a triangle with two sides and the embedded angle

(A) Let us consider that the two sides are of lengths 5 cm and 6 cm and the angle between the two sides is 60°.

Keeping the question in mind draw a rough sketch of the triangle without any measurement on one side of page. Write the given measurement on the triangle. A sketch not drawn to measurement is called a rough sketch.

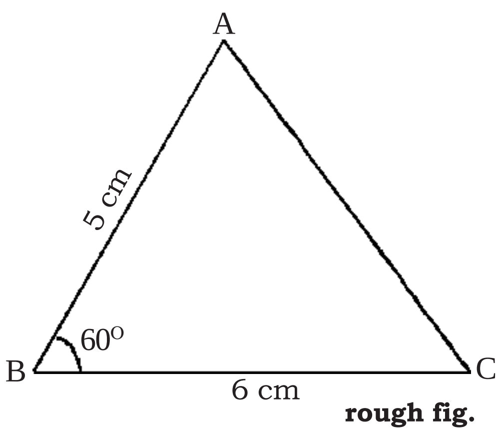


Figure 7.6

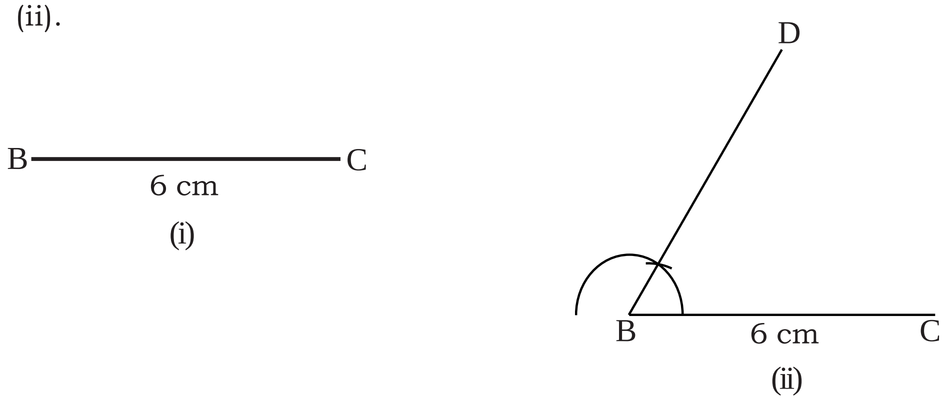
Keeping these steps in mind let us construct the triangle with proper measure.

Construction 2

STEPS

(i) Draw a line segment BC of length 6 cm as shown in figure (i).

(ii) Make an angle of 60° at B i.e. ∠DBC with the help of compass as shown in figure (ii).



(iii) Now with the help of compass, mark a cut of 5 cm on BD from B as shown in figure (iii).

(iv) Name that point A. Join A and C as in figure (iv).

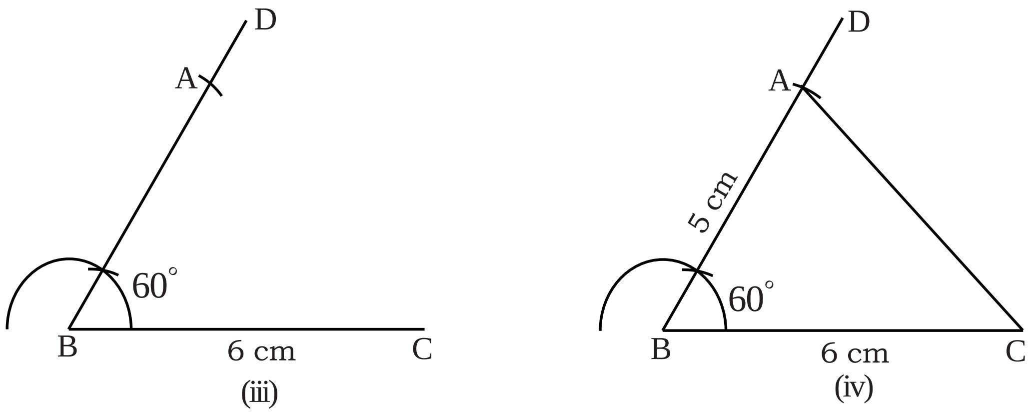


Figure 7.7

The required ΔABC is formed. This has BC = 6 cm, AB = 5 cm and ∠ABC = 60°.

(B) In the above figure when we measure AC, we get AC = 5.5 cm. and ∠ABC = 60°. So can you construct a triangle ABC with AC = 5.5 cm, BC = 6 cm and ∠ B = 60°?

Construction3

Draw a rough sketch of ΔABC. Write the given measurements on this rough sketch as shown in figure 7.8

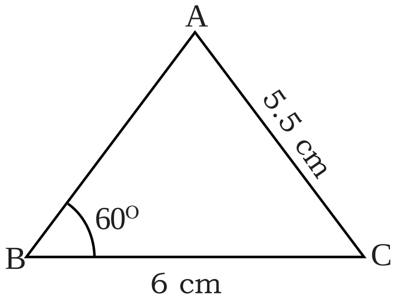


Figure 7.8

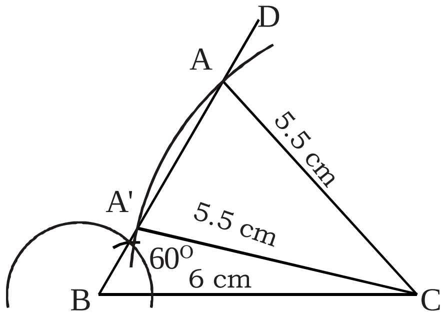


Figure 7.9

STEPS

(i) Draw a line segment BC = 6 cm

(ii) With the help of the compasses construct an angle of 60° on B i.e. ∠DBC = 60°.

(iii) From C, mark a cut of 5.5 cm on DB.

(iv) You would see that, the arc of 5.5cm radius from C cuts DB on two points, A and A' as shown in figure 7.9 Here two triangles ABC and A'BC are formed.

But if the arc AC is greater than 6cm, then would we still get two triangles?.

ACTIVITY 1

In the same figure 7.9 instead of taking length 5.5 cm, take a length of your choice and mark an arc from C on DB. Check whether you get two points of intersection or not? Follow the same activity at point B, taking the angle DBC to be less than 60° and write your conclusion.

(C) Now draw ΔDEF with EF = 4 cm, FD = 5 cm and ∠E = 90°.

Construction 4

Draw a rough sketch of ΔDEF and mark all the given details of required Δ DEF as shown in figure 7.10

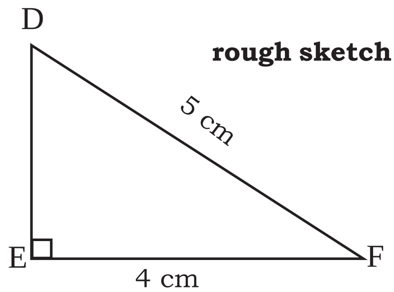


Figure 7.10

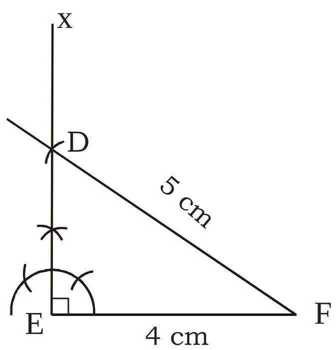


Figure 7.11

STEPS:

(i)Draw a line segment EF= 4 cm

(ii) Draw an angle of 90° on E i.e. ∠XEF= 90.

(iii) With the help of compass, mark a cut of length 5 cm, from F to line EX.

(iv) Name that point D. Join DF. Δ DEF is the required triangle as shown in figure 7.11

ACTIVITY 2

(1) Draw a line segment BC = 6 cm, in your copy.

(2) Draw an angle of 90° on B i.e. ∠CBD = 90°.

(3) With the help of compasses, mark a cut of length greater than 6 cm from C to DB. For different values of length (i.e. AC) check if you get two intersection points on DB?

ACTIVITY 3

Do the same activity for angle ∠B greater than 90° and write the conclusion you get.

From the above activities, we conclude that when instead of the embedded angle some other angle is given, formation of triangle is possible if the length of the second side is greater than the length of the side on which the angle is formed.

EXERCISE 7.3

Draw the triangle with the following given information:-

(i) BC = 5 cm, ∠ B = 60°, AB = 3 cm

(ii) BC= 8 cm, ∠B = 70°, AB = 4 cm

(iii) Choose some more values of your choice and form triangles on that basis.

Figure 7.12

Draw the following:-

Now think, if you are given one side and two angles of triangle, would you be able to construct the triangle? Let us try to draw that.

Draw a triangle with one side and two angles:-

BC= 6.5 cm, $\angle B = 60°$, $\angle C = 45°$.

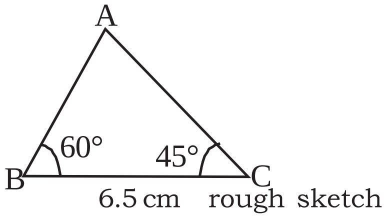


Figure 7.13

Construction 5

STEPS

(1) Draw a line segment BC = 6.5 cm.

(2) Draw a line BD from B such that angle $\angle CBD = 60°$.

(3) Now draw an angle ECB = 45° on C, such that it cut DB at point A as shown in figure 7.14.

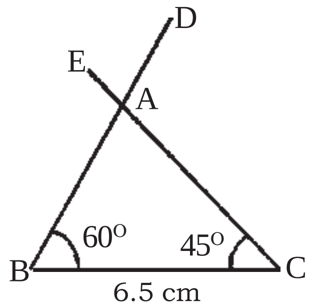


Figure 7.14

(4) ΔABC is the required triangle

What is the measure of $\angle CAB$?

How will you find out $\angle CAB$?

Is it possible to determine the third angle if we know the measure of the other two angles? Think of some more pairs of two angles of triangles and find the third angle.

See one more situation:-

Let BC = 6.5 cm, $\angle C = 60°$, $\angle A = 75°$.

In this example, we know only one angle on BC i.e. $\angle C$. This means we must know $\angle B$ to draw ΔABC.

In your previous class, you have learnt that the sum of the three interior angles of a triangle is 180°

In the above example, the given two angles are 60° and 75°.

$\therefore$ Third angle, $\angle B = 180° - (75° + 60°)$

$= 180° - 135°$

$\Rightarrow \angle B = 45°$

Are you able to form the triangle now? If yes, try to draw it.

EXERCISE 7.4

(i) Draw Δ PQR with PQ = 4 cm, QR=3 cm and RP = 5.5 cm.

(ii) Draw Δ UVW with WU = UV = 5.5 cm, and ∠VUW= 45°.

(iii) Draw Δ ABC with BC = 3.5 cm, ∠B = 30° and ∠ A = 45°.

Also write the steps of construction in each ?

WE HAVE LEARNT

(i) We can form a triangle if we know the lengths of three sides.

(ii) We can form a triangle if we know lengths of two sides and the embedded angle between the two sides.

(iii) We can form a triangle if we know the length of one side and two angles.

(iv) We can form a triangle, only when sum of any two sides is greater than the third side.

Chapter -8

CONGRUENCE

Introduction

Radha was returning home from school with her friends. Due to a storm the previous night, leaves that had fallen from the tree were scattered on the road. Radha picked up a leaf that she liked very much. She tried to find a leaf exactly similar to the one she had picked up. She asked her friends to play a game. In this game, we will collect leaves of same shape and size. The person who finds the maximum number of leaves of the same type till the count of hundred is reached, would be the winner.

Radha started counting and all of them began to collect leaves. Rajesh collected three, Hari –four, Anu- two, and Radha – three. Now the time was to examine the leaves. How do we check if leaves are of exactly the same shape? Can you think of a method to verify if the leaves collected were of the same shape?

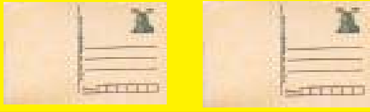
Anu said – "the leaves that I have collected are exactly same. I have checked that the two leaves exactly cover each other. The leaf that is above can cover the leaf lying below and vice-versa." Everyone checked their leaves using this method and found that each had only two leaves that were similar.

Figure8.1

Activity 1

Now you find out from things around you, those that are of the same size and shape. Add those things to the table of such things given below:-

Like-

1.		2.	
3.		4.	
5.		6.	
7.		8.	
9.		10.	

Figures that completely overlap or overlap each other are called congruent figures. This property is called **congruence**. Congruence is shown by '≅' sign.

Can you draw two congruent figures in your notebook?

Radha traced the outline of a leaf twice using a pencil and made two identical figures.

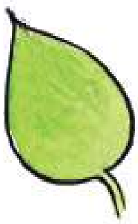


Figure 8.2

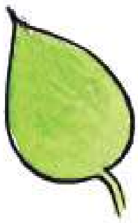


Figure 8.3

Anu traced the outline of a postage stamp with a pencil and made the following figures.

Figure 8.4

Figure 8.5

Rajesh had a carbon paper. He placed the carbon paper under a sheet of the notebook and drew a shape on the sheet on top. He found that the same shape was traced on the page over which the carbon was placed.

Hari took a one rupee coin and drew the outline around the coin with a pencil and made two similar figures.

Everyone made the two figures in their copies but the question was how can the two figures be compared. Two leaves or two notes can be compared by placing them one on the other to see if they are congruent or not. But how can we check if the two figures are congruent?

How can you check whether the figures in Figure 8.4 and figure 8.5 are congruent?

CONGRUENCE IN GEOMETRY

A leaf can be transferred from one place to another without changing its shape. Similarly, in geometry one figure can be transferred from one place to another without changing its shape or size. This axiom is called the axiom of super position.

For example two straight lines are always congruent because when one straight line is placed on the other straight line, they cover each other completely.

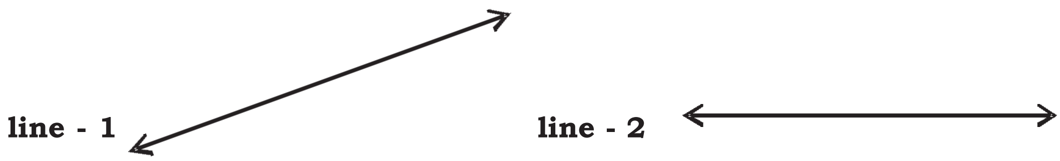


Figure 8.6

Will two line segments be congruent also?

For example:-

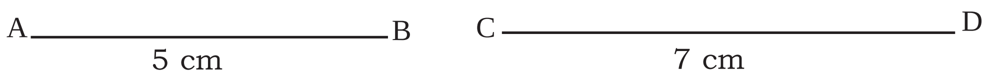


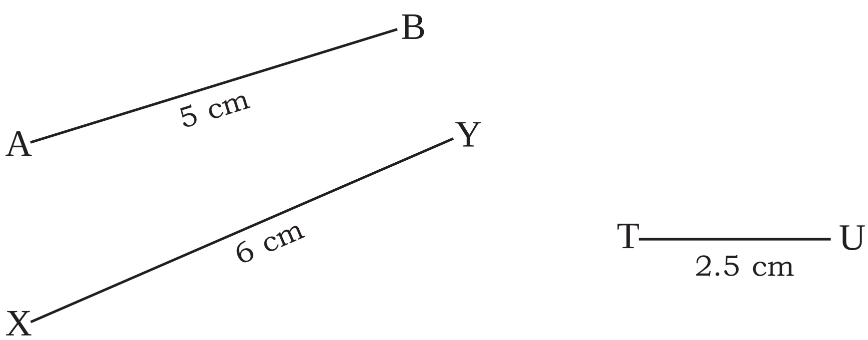
Figure 8.7

The length of a line is infinite, therefore any two straight lines can cover each other. But the length of line segments is finite and a 5cm line segment cannot completely cover a 7 cm line segment?

So, the line segments will only be congruent if their lengths are equal.

Activity 2

Some line segments are given below, find the congruent line segments from among them.



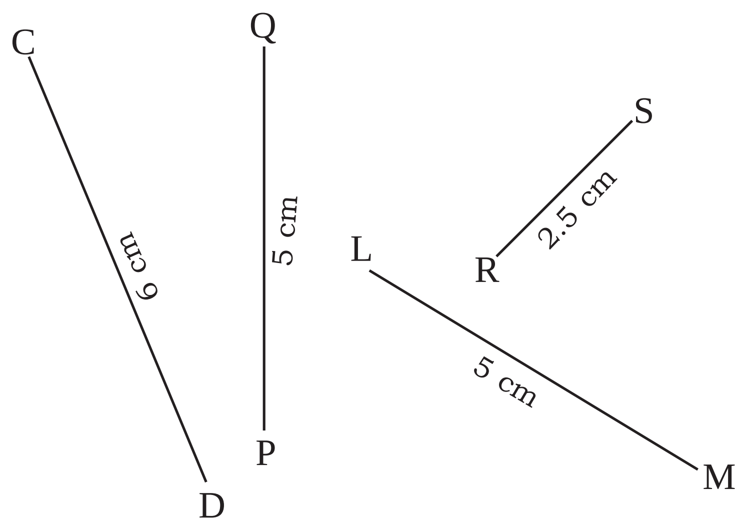


Figure 8.8

Here all line segments with lengths of 5 cm, 2.5 cm and 6 cm are congruent to each other respectively. Thus $\overrightarrow{AB}$ ≅ ≅ , ≅ and ≅ .

CONGRUENCE IN ANGLES

Two angles are given below. Can you say whether they are congruent or not?

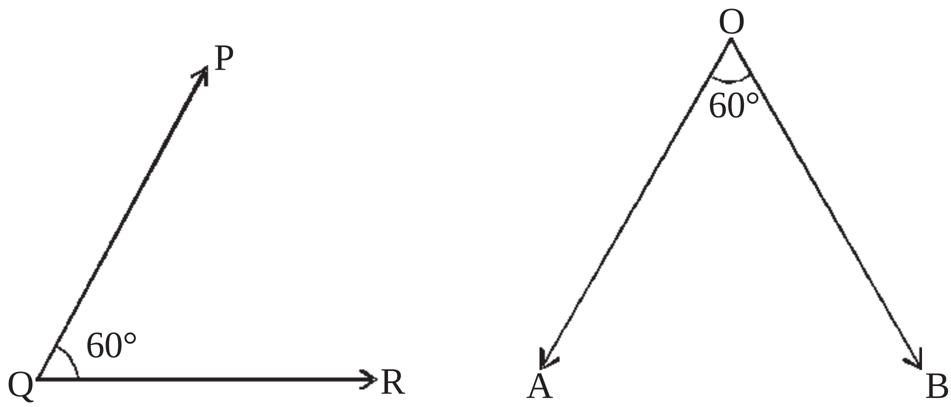
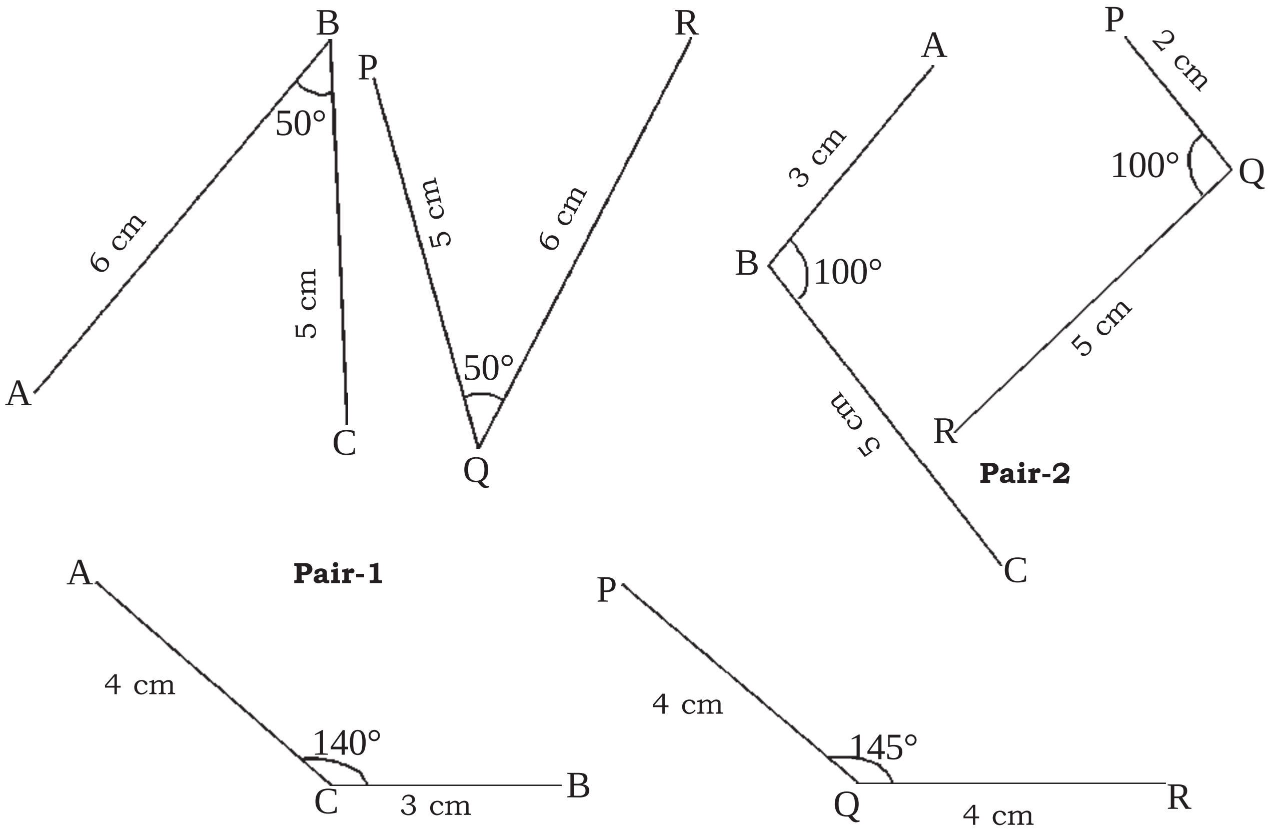


Figure 8.9

According to the axiom of superposition any geometrical object can be transferred from one place to another without changing its shape and size, if ∠PQR is placed on over ∠AOB then they will completely cover each other. The angles between and $\overrightarrow{OB}$ and $\overrightarrow{QP}$ and $\overrightarrow{QR}$ are equal. So say $\overrightarrow{OA}$ would have the same inclination with $\overrightarrow{QR}$ as $\overrightarrow{OB}$ has with the ray $\overrightarrow{QP}$ would fall on and on . Since $\overrightarrow{QR}$, $\overrightarrow{OA}$, , $\overrightarrow{QP}$ are all rays, they have infinite length. Thus rays will cover each other till infinity.

Pair of angles are given below, out of these some are congruent. Find the congruent pairs.



Pair-3

Figure 8.10

Two angles are congruent if and only if they have same measure. The length of the arms making the angles do not make any difference.

Activity 3

In the following pairs choose the congruent pairs and mark (✓) against them.

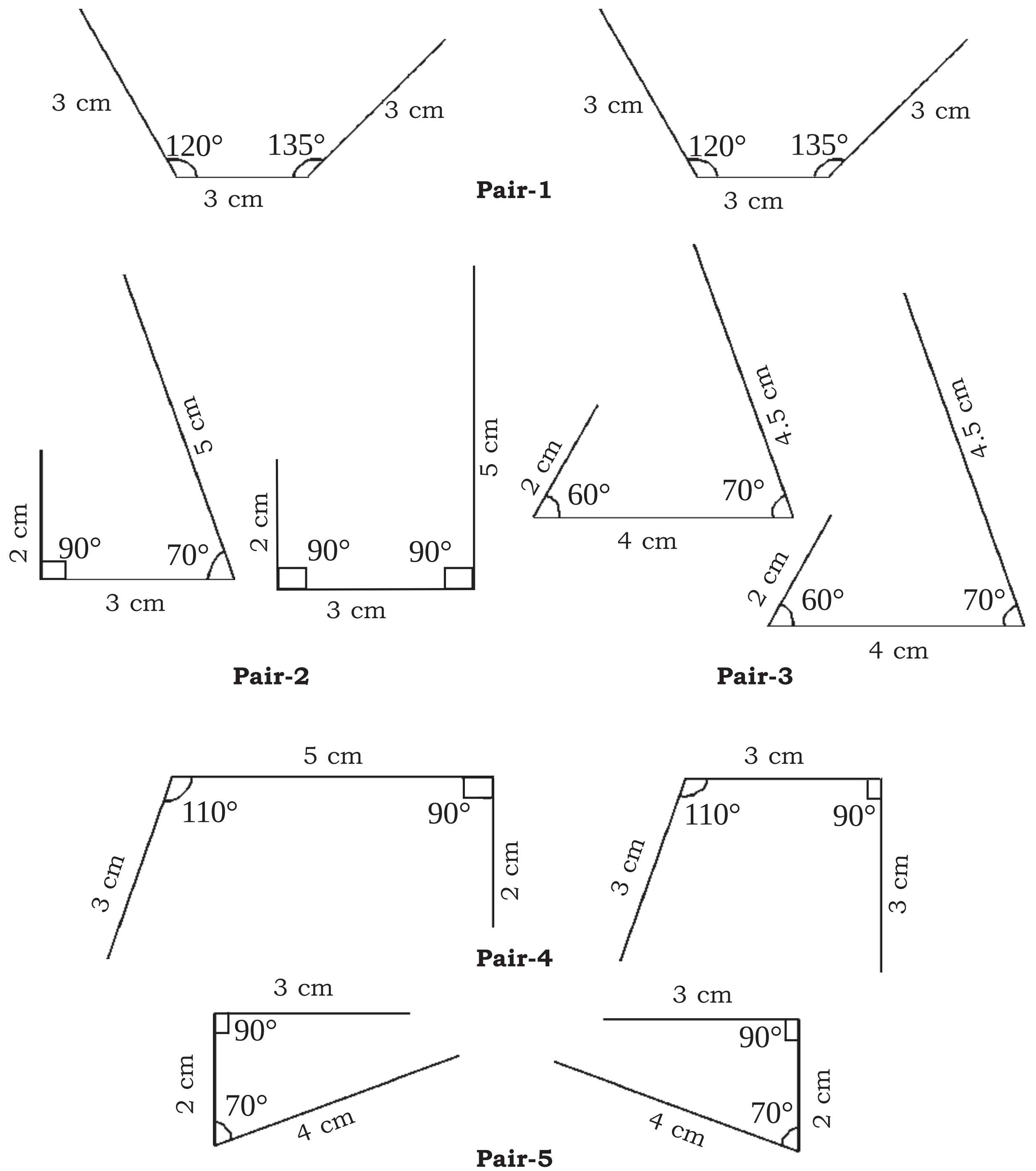


Figure 8.11

In the above figures the shapes in pair 1, pair 3 and pair 5 are congruent but pair 2, 3, 4 are not congruent. Can you now tell when any two shapes would be congruent?

Two figures would be congruent if they are equal in measure and shape, only their orientations are different. It means if we will keep the two shapes one above the other they will completely cover each other. Measure being equal means that the lengths of the arms and the measure of the angle is equal to the corresponding arms and measure of the angle of the second shape. Correspondence is denoted by↔. For example in pair-5 the angles of the first figure are 90°& 70°, the second figure also has the same angles. Both figures have an equal base of 2 cm and the segments making angle of 90° have arms with length 2cm and 3cm. In the same way the arms making the angle of 70° are 2 cm and 4 cm. In pair-5 if one figure is placed over the other, they will cover each other completely. This means the figures are congruent.

Are the following figures congruent? If not then why not?

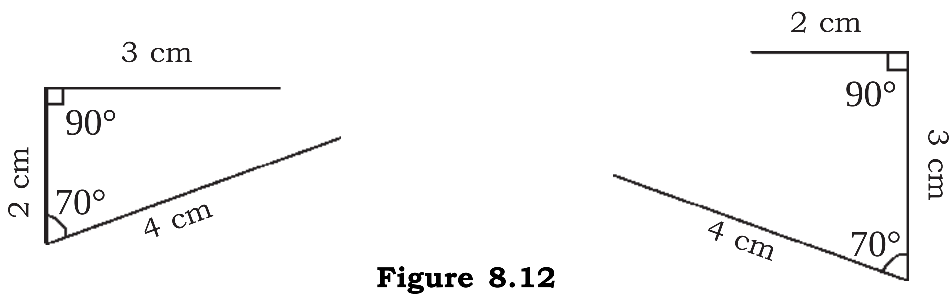


Figure 8.12

Here, both figures have angles of 90° and 70° but the corresponding sides are not equal. Similarly, the length of one line segment is 3 cm in the first figure, this is not equal to the corresponding line segment of the second figure length 2cm. Hence, both figures are not congruent.

Activity 4

Are the following figures congruent? If not, why not not, give reason?

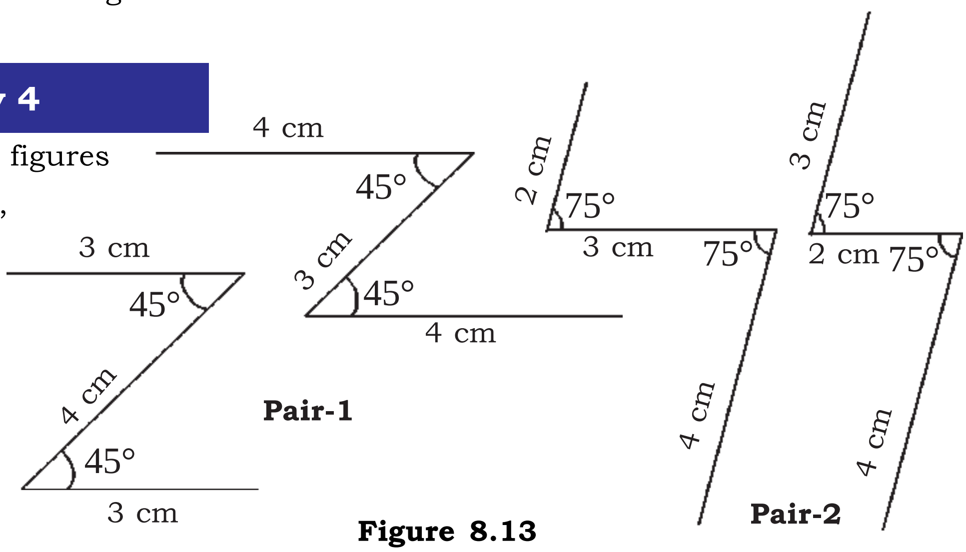


Figure 8.13

Are two squares having equal sides congruent?

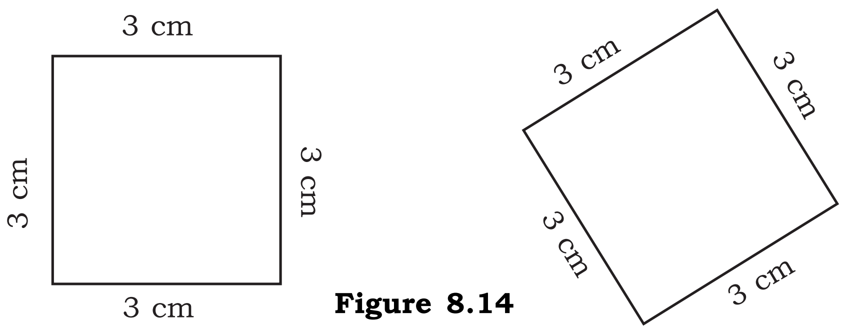


Figure 8.14

In a square all angles are of 90° and all sides are of equal length, hence the two squares are congruent if their sides are equal.

Similarly two circles are congruent if they have equal radius.

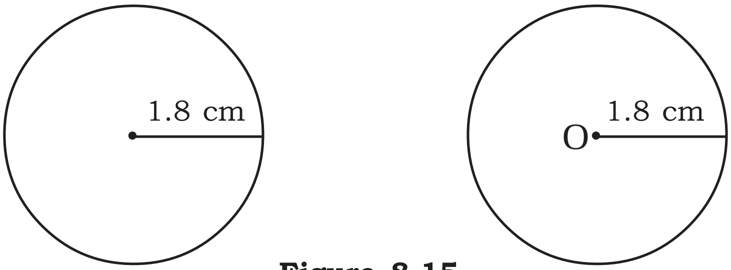


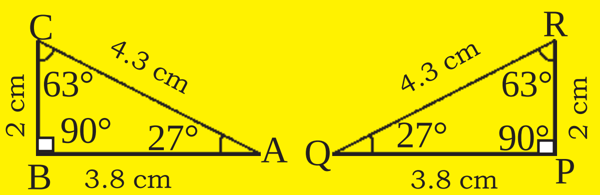
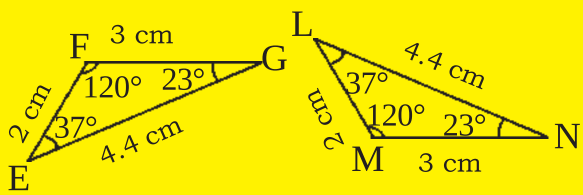
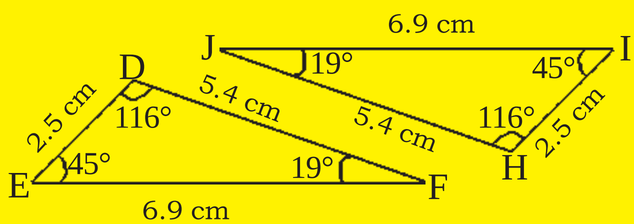
Figure 8.15

Congruence in triangles

Now, you would have understood that two shapes made from two or more line segments are congruent, only if all sides of the first shape are equal to the corresponding sides of the second shape. Similarly all the angles of the first shape have to be equal to all the corresponding angles of the second shape.

Activity 5

Some pairs of congruent triangles are given below.
Which of the sides and angles of the first triangle are equal to the corresponding sides and angles of the second triangle?

Fig.No.	Congruent Triangle	Equal Sides	Equal Angles
8.16		AB = PQ BC = PR CA = RQ	∠CBA = ∠RPQ ∠BCA = ∠PRQ ∠CAB = ∠RQP
8.17			
8.18			

When two congruent triangles are placed over each other then the vertices of the first triangle that cover the vertices of the other are the corresponding vertices. Similarly, the sides of the first triangle that cover the sides of the second triangle are corresponding to them and the angles of first triangle that completely cover the angles of second triangle are their corresponding angles.

For example - Two congruent triangles ABC and DEF are kept over each other and if,

Vertex A falls on vertex D

Vertex B falls on vertex E

Vertex C falls on Vertex F

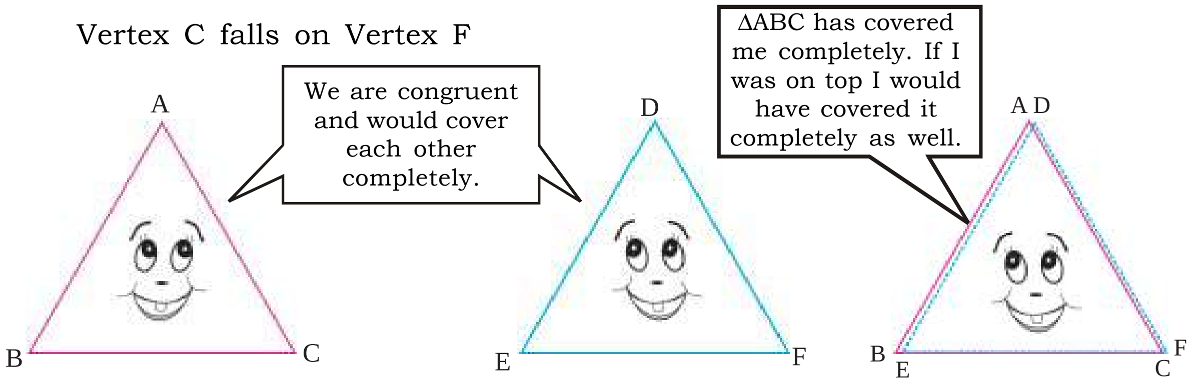


Figure 8.16

then Δ ABC is congruent to Δ DEF. It is not congruent to Δ EDF or Δ FDE or Δ FED. This is because vertex A vertex D, vertex B ↔ vertex E and vertex C ↔ vertex F. Thus we can say that Δ BAC ≅ ΔEDF Now, find out if the following are true:

1. Δ CAB ≅ Δ FDE
2. Δ CBA ≅ Δ FED
3. Δ BCA ≅ Δ FED
4. Δ ACB ≅ Δ DFE

Give reasons in support of your answer.

Example 1: In figure 8.20, Δ ABC ≅ Δ EFG. Find the following measures:

1. Length of EF
2. Length of BC
3. Measure of ∠G
4. Measure of ∠F

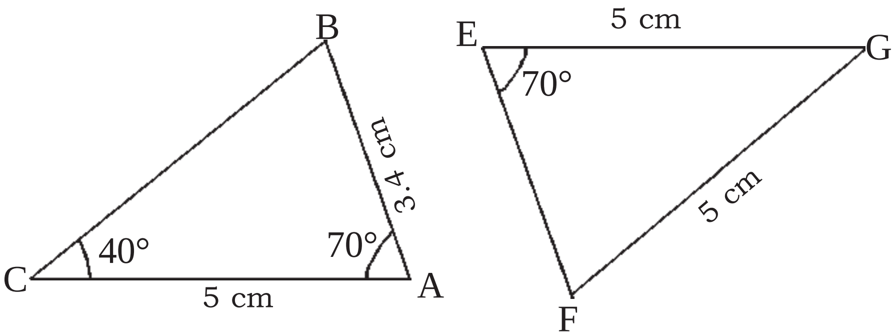


Figure 8.20

Solution

It is given that Δ ABC ≅ Δ EFG i.e. all the elements of Δ ABC are equal to the corresponding elements of Δ EFG.

1. Since EF ↔ AB, ∴EF = 3.4cm
2. Since BC↔ FG, ∴BC = 5cm
3. Since ∠G ↔ ∠C, ∴ ∠G=40°
4. In Δ EFG, ∠E + ∠F + ∠G = 180°

⇒ 70°+ ∠F + 40° = 180° (∠G=40°)

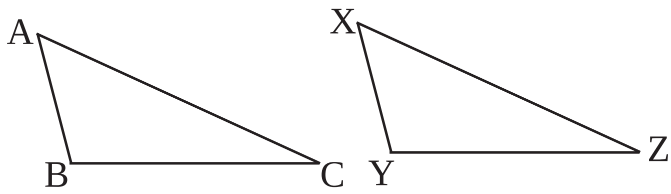
⇒ ∠F + 110° = 180°

⇒ ∠F = 180° – 110°

⇒ ∴ ∠F = 70°

Exercise 8.1

Q1. If Δ ABC ≅ Δ XYZ, then find



i. ∠A = ...

ii. _______________ = ∠Y

iii. ________________= ∠Z

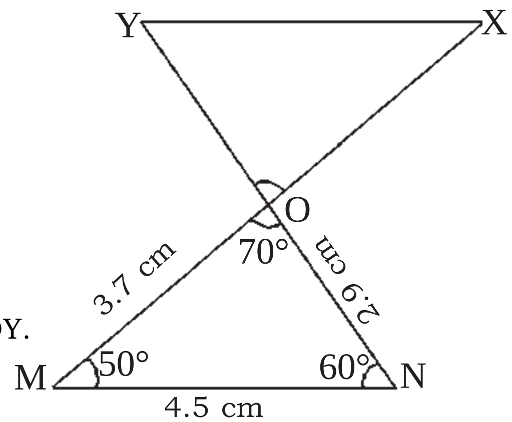
iv. AB = ___________

v. _______________ = YZ

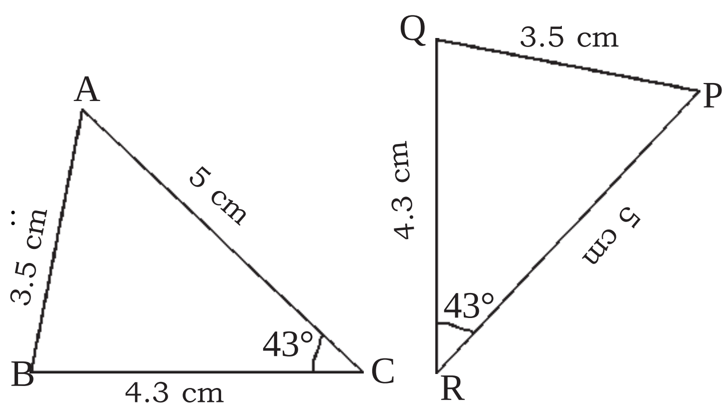
vi. ________________= XZ

Q2. If ΔMON ≅ ΔXOY then find

the length of sides and angles of ΔXOY.



Q3. If ΔABC ≅ΔPQR then put ✓ or ⊗ marks for true and false statements respectively :



i. ΔABC ≅ ΔPQR ☐

ii. ΔBCA ≅ΔRPQ ☐

iii. ΔCAB ≅ΔRPQ ☐

iv. Side AC = Side QR ☐

v. ∠B = ∠Q ☐

vi. ΔPRQ ≅ΔACB ☐

vii. ∠P = C ☐

Rules to check congruence in triangles.

Corresponding angles of two congruent triangles are equal. But if the corresponding angles of two triangles are equal, would that mean that the triangles are congruent?

Here, ∠A = ∠P

∠B = ∠Q

∠C = ∠R

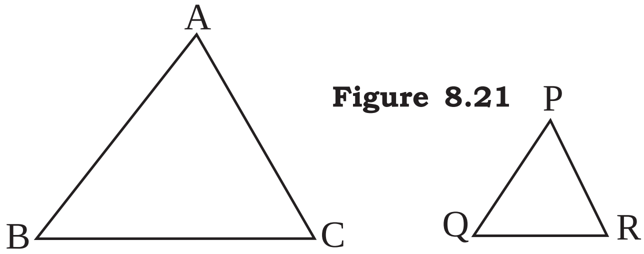


Figure 8.21

In Δ ABC and ΔPQR corresponding angles are equal. But the two triangles are not congruent. Why? **"Measure of corresponding angles being equal is not a sufficient condition for two triangles to be congruent as their corresponding sides must be also equal."** Therefore the three angles and three sides of a triangle should be equal to the corresponding sides and corresponding angles of the second triangle.

You have read about congruence in triangles. You also know how to construct a triangle. Can you draw two congruent triangles?

The students knew how to draw a triangle but not how to draw two congruent triangles? They started thinking. Then Rajesh said, "A triangle is constructed using some measurements."

If we construct triangles with equal measure, then all sides and angles of the triangles will be equal, thus the two triangles constructed would be congruent".

Anu said "We have learnt to draw triangles in three ways. First- when the three sides are given, second when two sides and the angle between them is given and third when one side and two angles are given. In all the three methods we can construct two triangles with equal dimensions, these will be congruent triangle. Let us take equal dimensions and construct two congruent triangles.

Write questions of constructing triangles and ask your friends to make congruent triangles from the given measures.

Activity 6

Radha formed a question – "Construct congruent triangles with sides having the length 5 cm, 6 cm and 7 cm."

Anu made a question – "Construct congruent triangles with sides 4cm. and 7 cm with the angle between them as 60°".

Hari made this question – "Construct congruent triangles in which one of the sides of the triangle is 7 cm and the angles formed on the side are 50° and 70° respectively."

According to the questions set by Radha, Anu and Hari, three pairs of triangles are made below, find all dimensions of the triangles and see whether they are congruent or not?

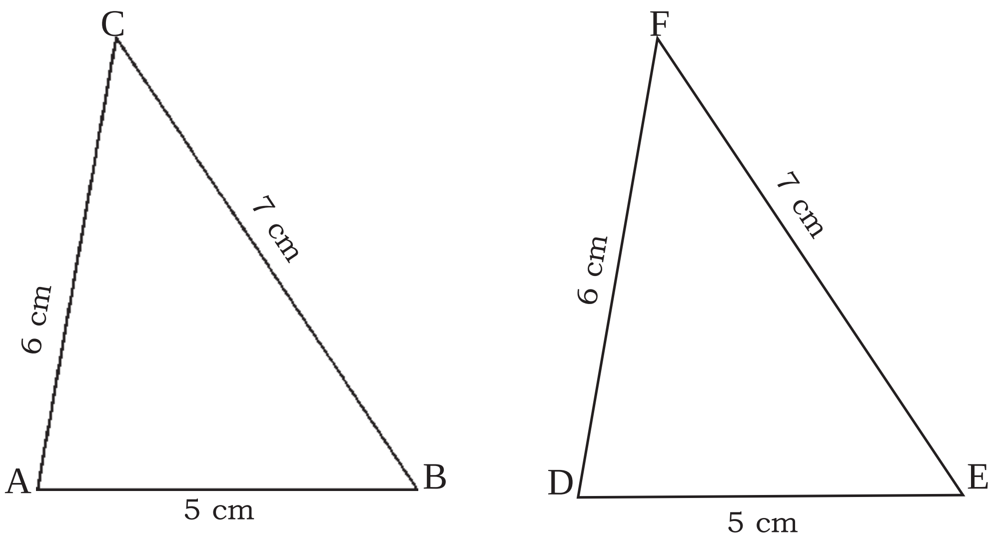


Figure 8.22

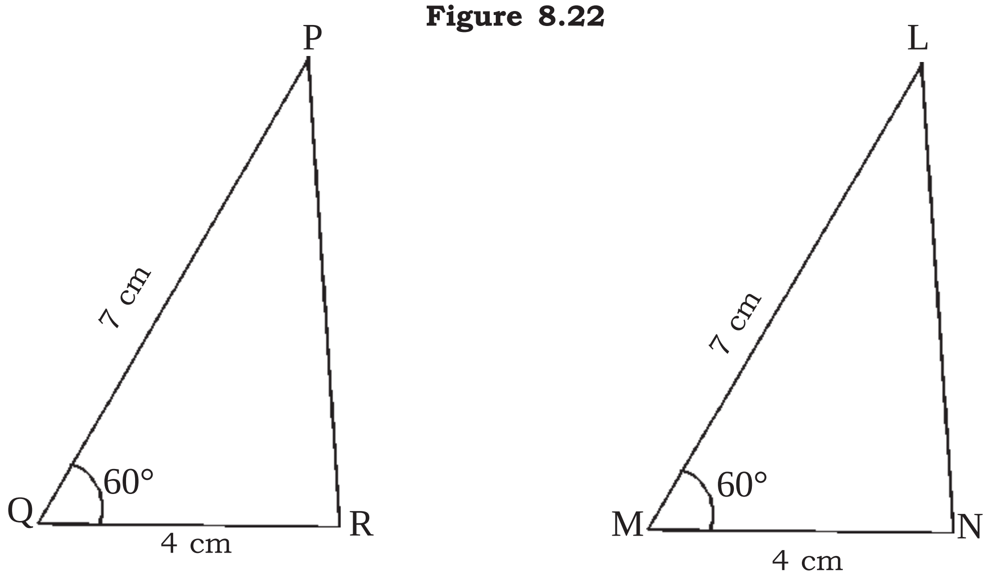


Figure 8.23

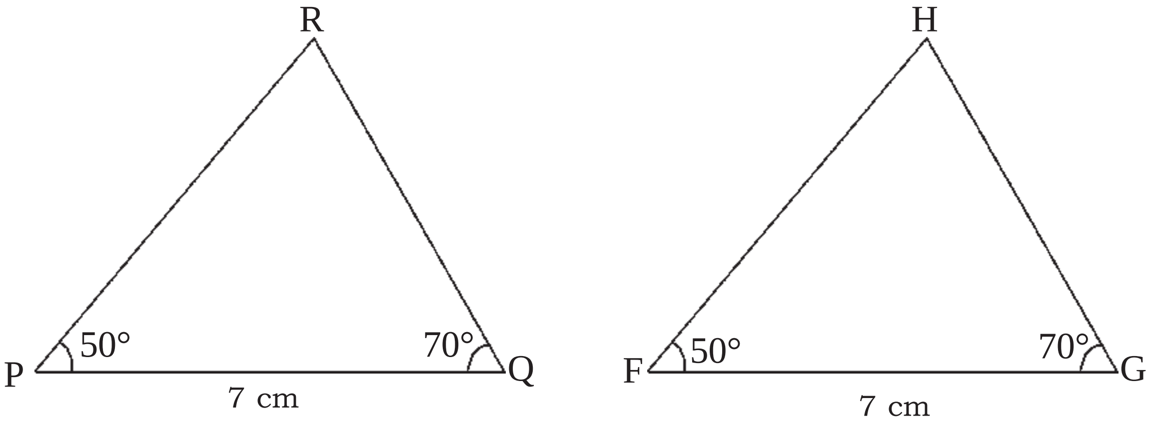


Figure 8.24

In figure 8.22, the corresponding sides of the triangles are equal and you find that the corresponding angles are equal. Hence, the two triangles are congruent. Would whenever the corresponding sides of two triangles are equal, the triangles are congruent?

Similarly, in figure – 8.23 we find that the two corresponding sides and the angle in between is equal and we find that the triangles are congruent. In figure 8.24, two angles and a side of each triangle is equal to the other, and we find the triangles are also congruent.

Can on the basis of these properties, we always say that the two triangles would are congruent? Let us see-

Rule of side-side-side (S.S.S) congruence

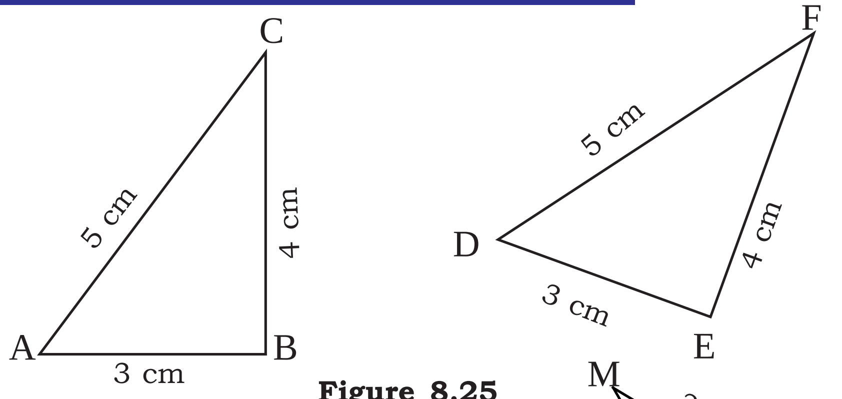


Figure 8.25

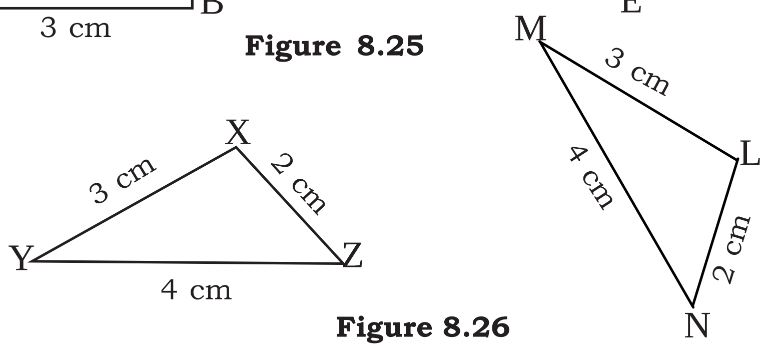


Figure 8.26

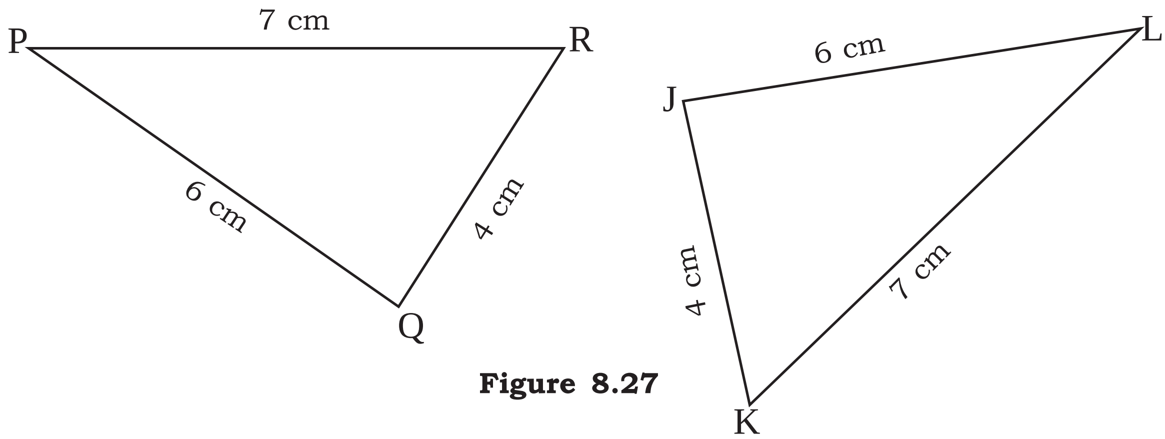


Figure 8.27

The triangles in figure 8.25, 8.26 and 8.27, are congruent. Hence, if the corresponding sides of two triangles are equal, then the triangles are congruent. Such congruency is called **Side-Side-Side congruency** or **S.S.S. congruence**.

Side Angle Side (S.A.S.) Congruence rule

Some pairs of triangles are given below. In each pair two sides and the angle between them for one triangle is equal to the corresponding two sides and the angle between them of the other. Find out whether the triangles are congruent or not? Check the pairs –

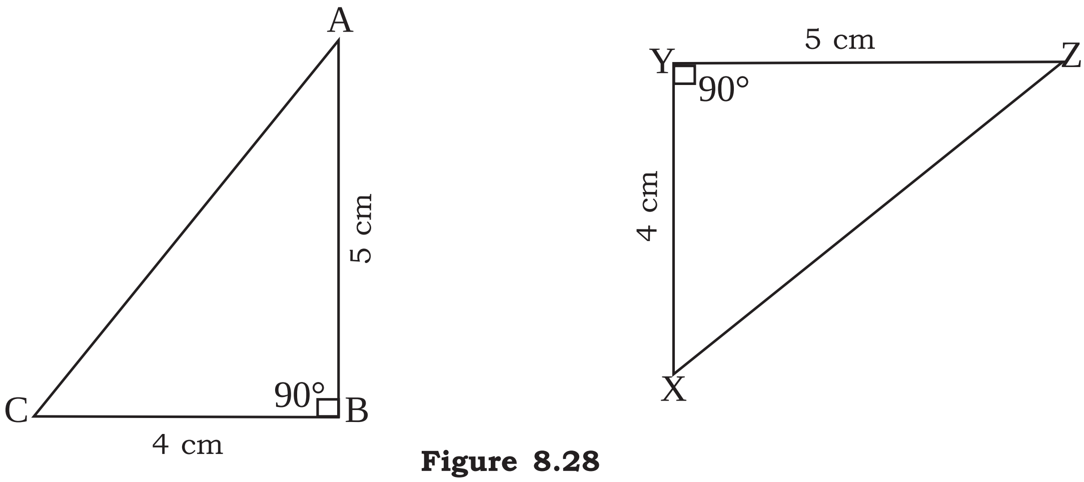


Figure 8.28

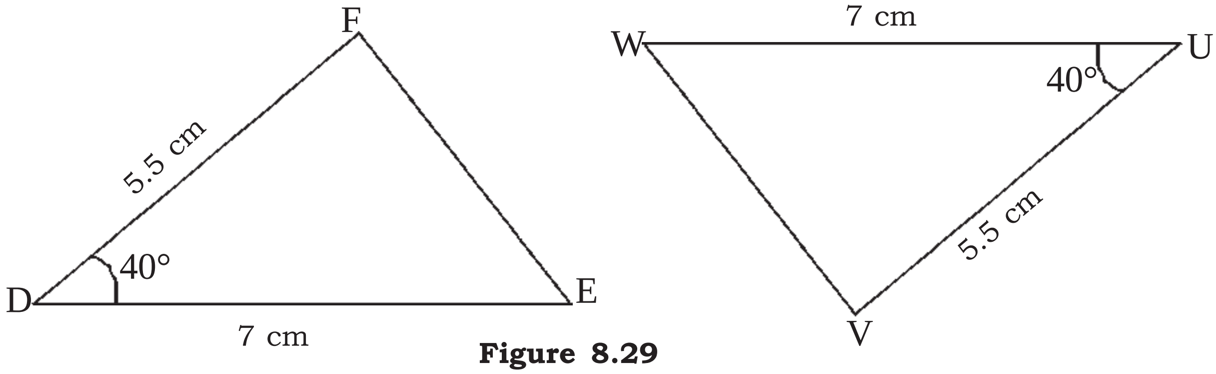


Figure 8.29

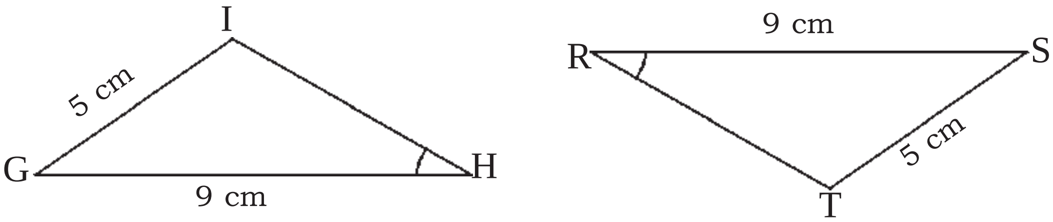


Figure 8.30

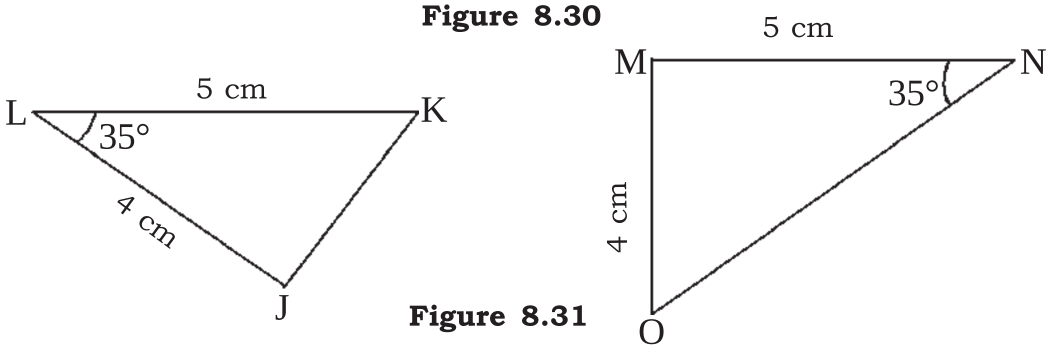


Figure 8.31

In the above figures 8.28, 8.29 and 8.30 the triangles are congruent but the triangles drawn in figure 8.31 are not congruent. Why not? Write the reasons in your note book.

In figure 8.31, the corresponding sides are of the same length but the angles between them are not equal. In the first triangle the angle between the 5 cm and the 4 cm sides is of 35°, but in the second triangle, the angle of 35° is between the 5 cm side and the third side.

Due to this all the corresponding elements in the second triangle are not equal to the elements of the first triangle. Therefore the triangles are not congruent.

If the two adjacent sides of a triangle and the angle between them is equal to the two adjacent sides and the angle between them of another triangle, then the two triangles are congruent. This congruency is known as **Side–Angle–Side congruency** or **SAS congruence**.

ANGLE SIDE ANGLE (A.S.A.) Congruence r

If one side of a triangle is equal to the corresponding side of an other triangle and if two angles of the first triangle are also equal to the corresponding angles of the other triangle, then the triangles are congruent. Such congruency is known as **Angle–Side-Angle congruency** or **ASA congruence**.

Activity 7

Some pairs of triangles are given below. In each of the pairs, a side and two angles of the first triangle is equal to the corresponding side and two angles of the other triangle. Find the measures of the remaining sides and angles of the triangles. Find out whether the two triangles are congruent or not and if not then why not?

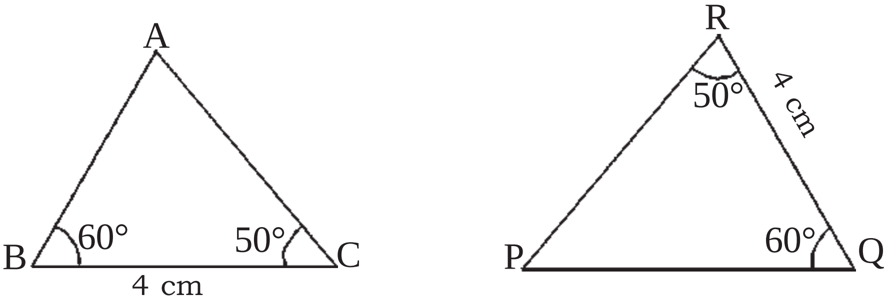


Figure 8.32

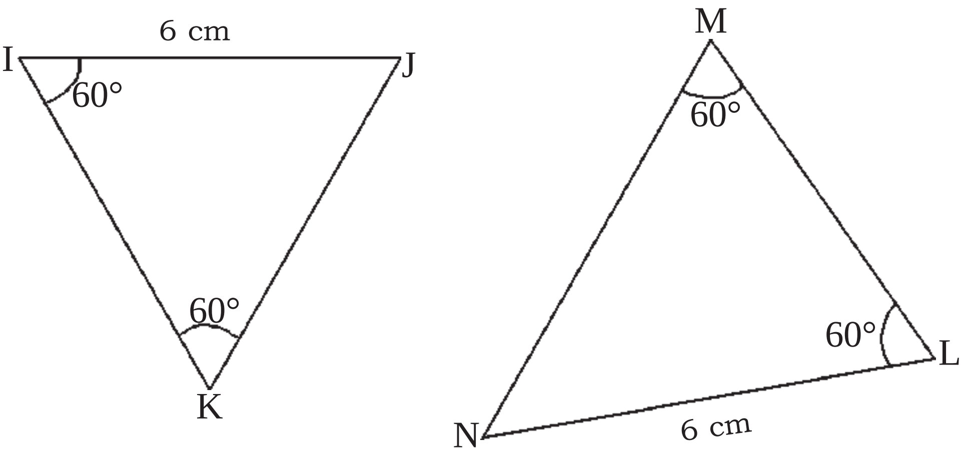


Figure 8.33

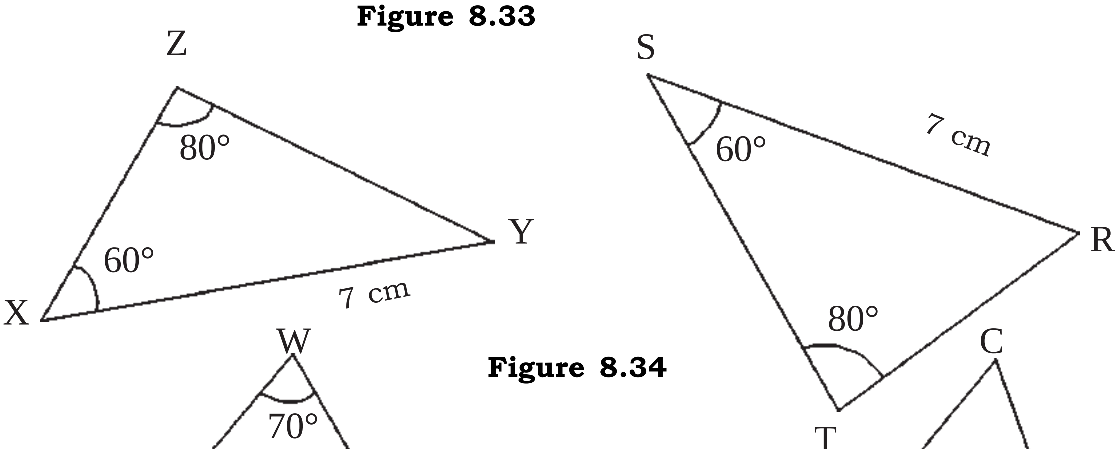


Figure 8.34

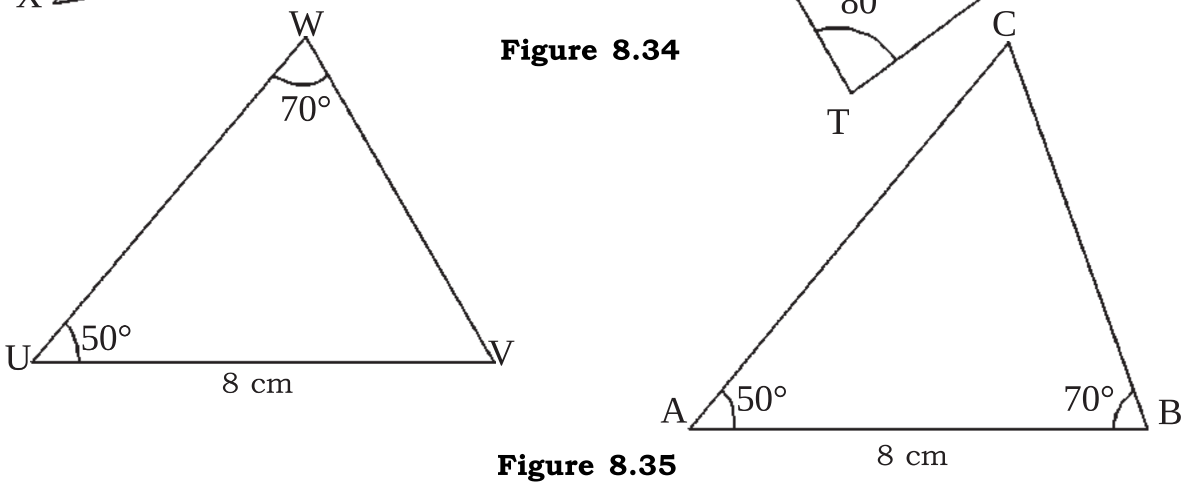
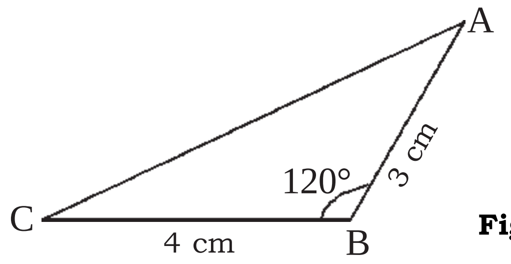


Figure 8.35

Example 2:

Two triangles CAB and RPQ are given below. Are the two triangles congruent or not? Measure the remaining elements of the triangles and write the relation between them.



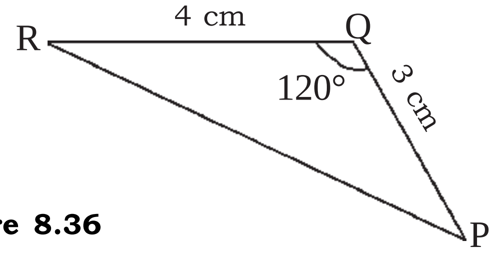


Figure 8.36

Solution: In the given ΔCAB and ΔRPQ

BC= QR = 4 cm

∠B = ∠Q = 120°

And AB = PQ = 3 cm

In Δ CAB the two sides and the angle between them are equal to the corresponding sides and the angle between them in Δ RQP.

Therefore, by Side- Angle-Side congruence rule it in clear that

Δ CAB ≅ Δ RPQ

Again, in the two triangles

AC = PR = 6.1 cm

∠C=∠R=26° And ∠A= ∠P = 34°

Therefore, side AC ↔ side PR, ∠C ↔ ∠R and ∠A ↔ ∠P

Example 3

Two triangles are given below. The equal corresponding elements in the triangles are shown. Is Δ ABD ≅ Δ ACD or not?

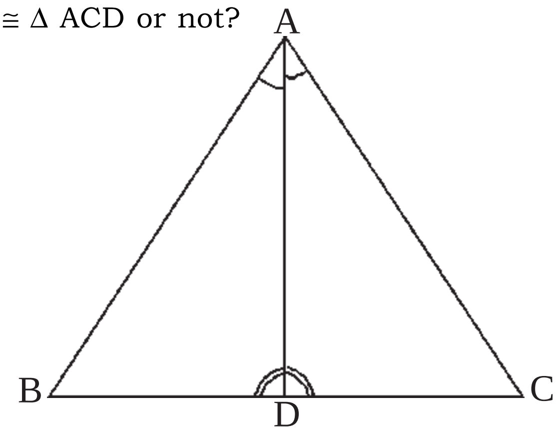


Figure 8.37

Solution:

In the given figure we have Δ ABD and ΔACD

In these ∠BAD = ∠CAD (given in the figure)

AD = AD (common)

And ∠ADB = ∠ADC (given in the figure)

Therefore by Angle- Side-Angle congruence rule, Δ ABD ≅ Δ ACD

Example 4: Consider the measures of the sides of the triangles (given below) and prove their congruency.

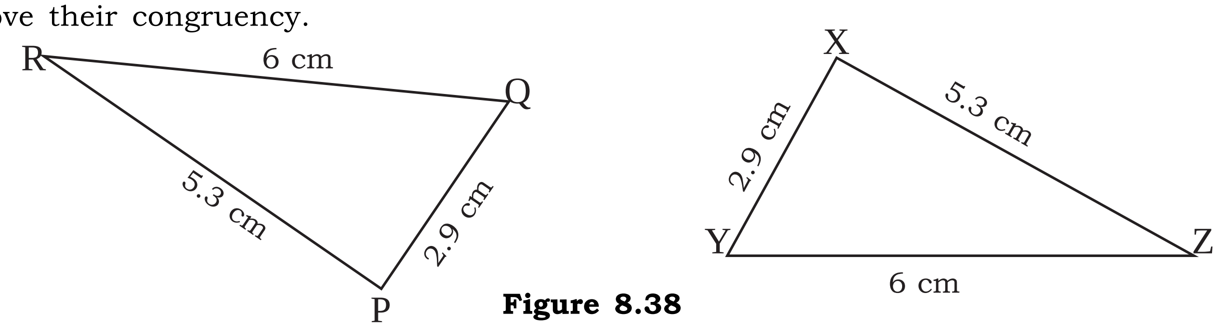


Figure 8.38

Solution: According to the given figure in ΔPQR and ΔXYZ ,we have

PQ = XY = 2.9 cm (given in the figure)

QR = YZ = 6 cm (given in the figure)

RP=ZX=5.3 cm (given in the figure)

Therefore by Side-Side-Side congruence rule, ΔPQR ≅ Δ XYZ

Thus, the two triangles are congruent.

Right Angle– Hypotenuse – Side (RHS) congruence rule

All the three rules are applicable to all triangles But the R.H.S rule is applicable only to right angled triangle.

If the hypotenuse and a side of a right angle triangle is equal to the hypotenuse and a side of the other right angled triangle, then both the triangles are congruent. Such congruence is called **Right-Angle– Hypotenuse- side congruence** or **RHS congruence rule**.

Activity 8

Some pairs of right angle triangles are given below. In each pair the hypotenuse and a side of one triangle is equal to the hypotenuse and a side of the other triangle. Measure the third side and the angles of the two triangles and check whether they are congruent or not. If they are not congruent, then why not?

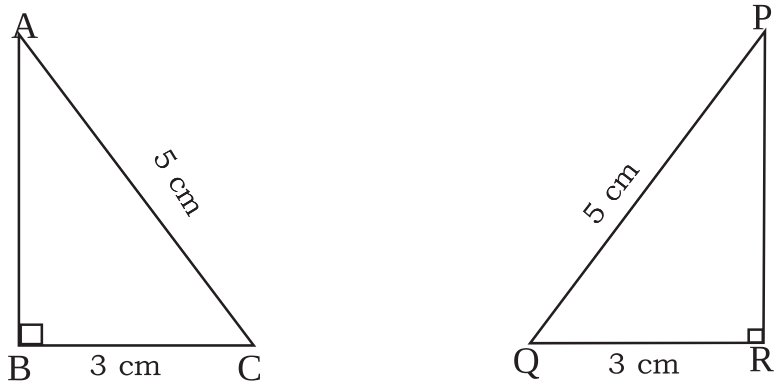


Figure 8.39

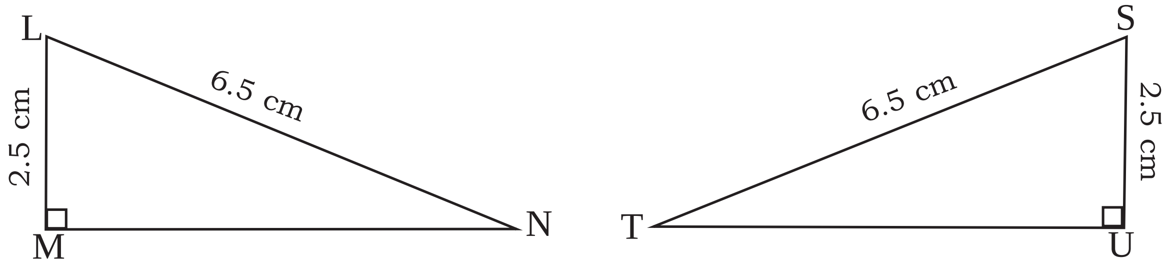


Figure 8.40

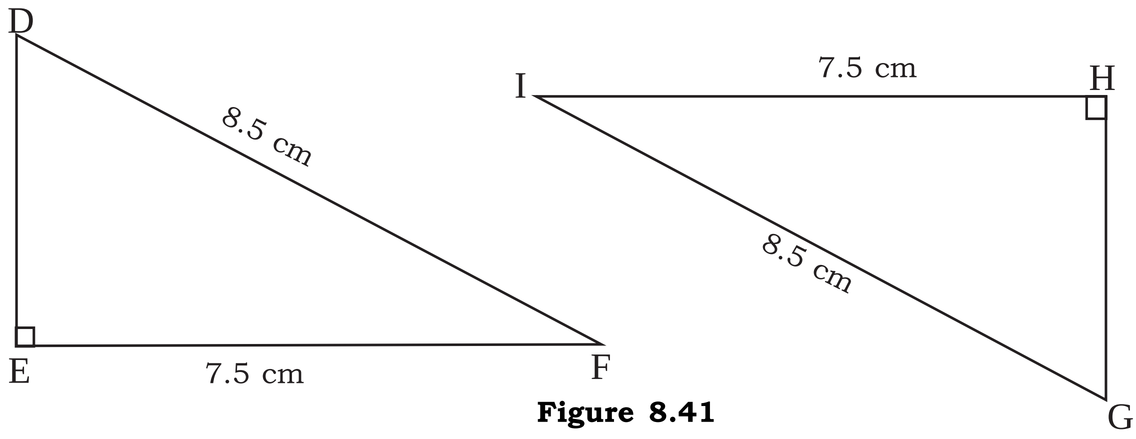


Figure 8.41

Example 5: Consider the triangles below. Is ΔABC ≅ ΔDEF in figure 8.42? Give reasons.

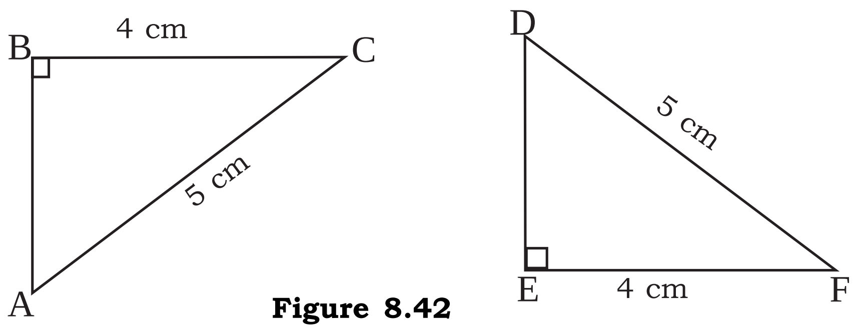


Figure 8.42

Solution: In triangles ABC and ΔDEF

AC = DF = 5cm (Hypotenuse)

BC = EF = 4 cm (Side)

∠B = ∠E = 90° (right angle)

Hence by R.H.S. congruence rule Δ ABC ≅ Δ DEF

Exercise 8.2

Q.1. The measures of some elements of triangles ABC and DEF are given below. On the basis of the measures find out whether the two triangles are congruent or not? If they are congruent write the rule of congruence? One example is given, solve the remaining problems according to that.

S. No.	Measures of Triangles		Congruent or not	Rule of congruence
i.	AB=7 cm, BC=5 cm CA=9 cm	DE=7 cm, EF=5 cm FD=9 cm	Yes	S.S.S
ii.	BC=3.5 cm, CA=6.2 cm ∠C=47°	EF=3.5 cm, FD=6.2 cm ∠F=45°	------------	-----------
iii.	∠B=90°, BA=5 cm, AC=13 cm	∠E=90°, ED=5 cm, DF=13 cm	------------	-----------
iv.	AB=7.1 cm, ∠A=30°, ∠B=43°	DE=7.1 cm, ∠D=30°, ∠E=43°	------------	-----------
v.	∠C=110°, ∠B=30° BC=5.5 cm	∠F=30°, ∠E=110° EF=5.5 cm	------------	-----------
vi.	CB=8 cm, ∠C=90°, AB=10 cm	FE= 8 cm, ∠E=90°, DF=10 cm	------------	-----------
vii.	AB=6 cm, BC=8.2 cm CA=7.8 cm	DF=6 cm, EF=8.2 cm ED=7.8 cm	------------	-----------

Q.2 In the figures given below check for congruence. Also write the rule used.

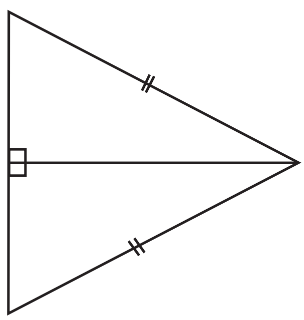


Figure 1

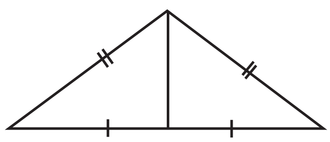


Figure 2

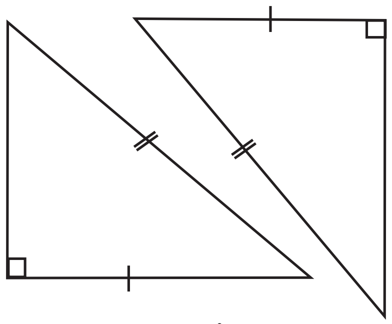


Figure 3

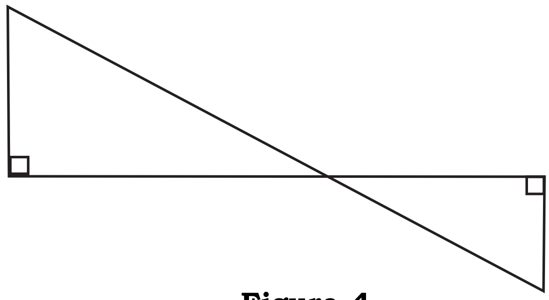


Figure 4

Q3. If AB = AD, ∠BAC= ∠DAC in the following figure, can we write ΔABC ≅ Δ ADC? If yes, why?

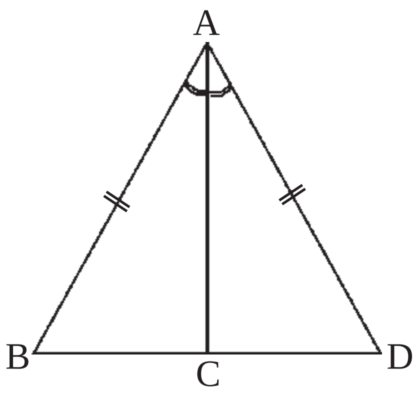


Figure 5

Q4. ΔPOQ is an isosceles triangle in which PQ=PR and the side PO bisects ∠P. Which of the following statements are true or false?

(i) ΔPOQ ≅ Δ POR

(ii) ΔPQR ≅ Δ PQO

(iii) PRQ ≅ Δ PRO

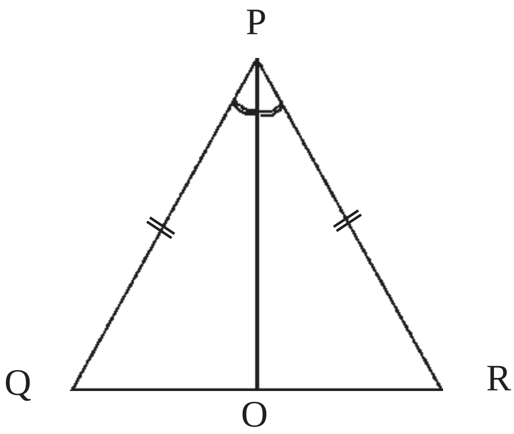


Figure 6

Q5. In the following figure, ∠P=∠S=90° and PQ = SR. Are ΔPQR and ΔSQR congruent? Write reasons.

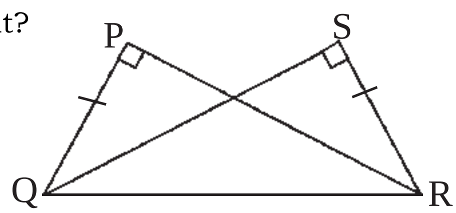


Figure 7

Q6. Some triangle pairs are given. Choose the correct congruent pair?

(i) ΔABC ≅ ΔABD

(ii) ΔABC ≅ ΔBAD

(iii) ΔABC ≅ ΔDBA

(iv) ΔABC ≅ ΔDAB

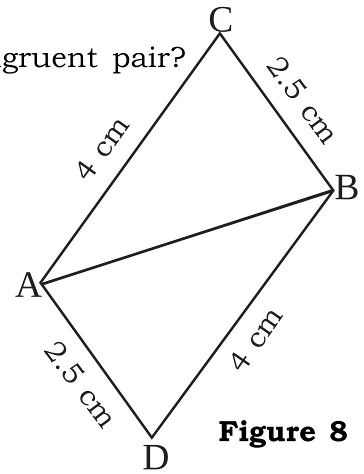


Figure 8

Q7. In two triangles ΔPQR and ΔSRQ, PR=SQ and PQ=SR, then show that the triangles are congruent and indicate the correspondence.

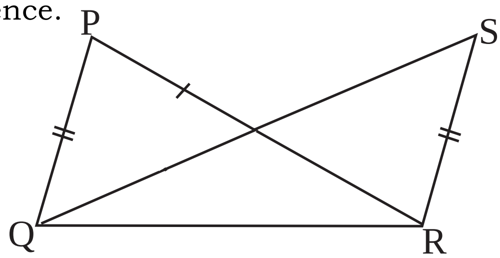


Figure 9

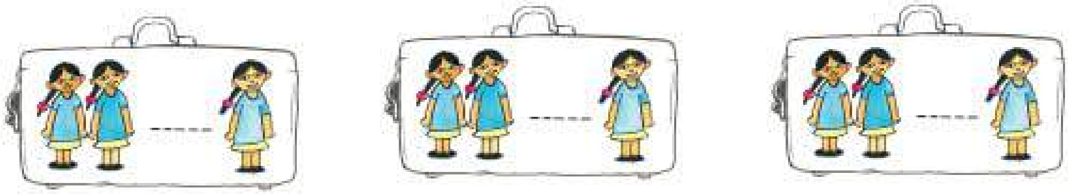
We have learnt

1. Two line segments of the same length cover each other completely, therefore they are congruent.
2. Two figures of the same size and shape are congruent.
3. Two triangles are congruent if all the three sides and the three angles of one triangle are equal to the corresponding three sides and angles of the other triangle.
4. In congruent, triangles the three sides and the three angles of a triangle are equal to the three side and angles of the other triangle.
5. If two sides of a triangle and the angle between them is equal to the corresponding sides and the angle between them for the other triangle, then the triangles are congruent. This congruence is called SAS congruence.
6. If all the sides of a triangle are equal to the corresponding sides of the other triangle then the triangles are congruent. This congruence is known as SSS congruence.
7. If two angles and one of the sides of a triangle are equal to the corresponding two angles and a side of the other triangle, then the triangles are congruent. This congruence is known as ASA congruence.
8. If the hypotenuse and one side of a right angle triangle is equal to the hypotenuse and side of the other right angle triangle then these triangles are congruent. This congruence is known as Right Angle – Hypotenuse-Side congruence (RHS).

Chapter -9

OPERATIONS ON ALGEBRAIC EXPRESSIONS

Ankita has 3 boxes of toys. Each box has the same number of toys. How many toys are there in all?



If each box has 5 toys, total number of toys = 3×5

Similarly if each box has 9 toys then total number of toys = 3×9

If each box has x toys, then total number of toys = $3 \times x$

If the number of toys in one box is x, the number of toys in 3 boxes equals 3 x. In the same way if the number of toys in one box is p, then the number of toys in 3 boxes is 3p.

Similarly if the number of toys in one box is z, what will be the numbers of toys in 11 boxes? [ ]

If the number of toys in one box is S, what will be the numbers of toys in 21 boxes? [ ]

If number of toys in one box is x what will be the number of toys in y boxes? [ ]

Let us see some more use of variables.

Addition and subtraction of Algebraic Expressions

Radha has twice the number of books and thrice the number of copies that Shyam has. What will be the number of books and copies that they have?

If Shyam has x number of books and y number of copies that total number of books and copies Shyam would have is x + y. Radha has twice the number of books that Shyam has. This means Radha has 2x books and since she has thrice the number of copies than Shyam, therefore, she will have 3y copies. The total number of books and copies that Radha has is 2 x + 3y. Number of books and copies they together have = number of books and copies that Shyam has + number of books and copies that Radha has.

$= (x + y) + (2x + 3y) = 3x + 4y$

Here it is clear that while adding two algebraic expressions like terms add together. Similarly while subtracting algebraic expressions, we subtract the like terms.

Examples 1: Add 5x + 6 y and 3x + 2 y

The algebraic expressions that are to be added are written one below the other such that like terms come immediately below each other.

Solution: (5x + 6y) + (3 x + 2y). Here like terms are 5 x, 3 x and 6 y, 2y. Adding numerical coefficients of the like terms, we get

5x + 3x = 8x and 6y + 2 y = 8y

Or 5x + 6y + 3x + 2y =

= (5x+3x) + (6y + 8y) = 8x + 8y

This can also be solved as follows

$5x + 6y$
$3x + 2y$
$8x + 8y$

(The algebraic expressions that are to be added are written one below the other such that like terms come immediately below each other.)

Example 2: Add 5xy + 3z and 8z + 7 xy

Solutions: (5xy + 3z) + (8z + 7 xy)

= 5xy + 7 xy + 3z +8z

= 12 xy + 11z

Second Method

Or $5xy + 3z$
$7xy + 8z$
$12xy + 11z$

Example3: Subtract $5z^2 – 7\ xy$ from $13\ xy – 8z^2$

Solution: $13xy – 8z^2 – (5z^2-7xy)$

$= 13xy – 8z^2 – 5z^2 + 7\ xy$

Notes -(1) While opening the bracket and multiplying by a negative sign, a positive integer becomes a negative integer and a negative integer becomes positive.

(2) '_' sign before a bracket means multiplication by -1

$= 13\ xy + 7\ xy - 8z^2 - 5z^2$

$= 20xy - 13z^2$

Or $\quad 13\ xy - 8z^2$

$\mp 7\ xy \pm 5z^2$ (on changing the sign)

$20\ xy - 13z^2$

Example 4: Subtract 3x + 7-8xy from $3x^2$ y + 8 + 3y

Solution : $= 3x^2\ y + 8 + 3y - (3x + 7 - 8xy)$

$= 3x^2y + 8 + 3y - 3x - 7 + 8xy$

$= 3x^2y + 1 + 3y - 3x + 8xy$

Or $\quad 3x^2y + 3y + 8$

(–) $\pm 7 \pm 3x \quad 8xy$ (on changing the sign)

$3x^2y + 3y + 1 - 3x + 8xy$

When the algebraic expressions do not have like terms, the number of terms increases after the operation of addition or subtraction.

EXERCISE 9.1

Q1. Add the following:-

(a) 2 pq and 7pq

(b) 2 xy – 4xy and 8xy

(c) 3x + 8y and 7x + 6y

(d) 7y + 3z and 3x + 4y

(e) x+y - z, x-y-z and y +z-x

(f) 5x+ 4y-12, 6x+5y and 12z - 7x+9y

(g) 3x -7xy, 6xy-4y and x+2

(h) $x^2\ y^2 + 3x^2 - 7$, $-5x^2y^2 - 5x + 7$

Q2. Subtract

(a) 3x from 8x

(b) -4x from 12x

(c) x from -9x

(d) -8x from -5x

(e) $-3x^2-4x-2$ from x^2-3x+7

(f) 5y-x+3z from x-3y

(g) 3ab+2a-3b from xy-5a-9b

Q.3 Simplify the following:-

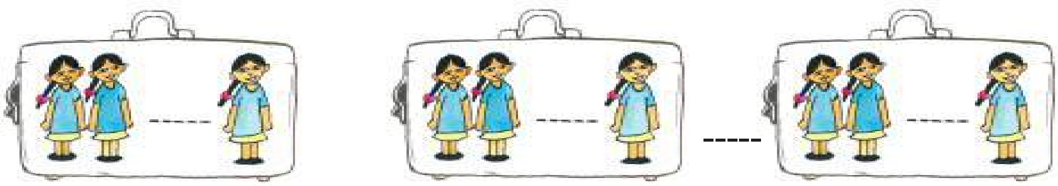
1. $5ab-7b^2c - 6ab + 2bc^2 - 4b^2c-3bc^2$

2. $m^2-2n^2+7mn-5m^2-11mn-3n^2+2n^2$

Q4. From a book fair Shashank bought x books at the rate of Rs. 4 per book, y books at the rate of Rs. 5 per book and 7 books at the rate of Rs. x per book and 8 books at the rate of Rs. y per book. How many rupees did he spend?

Multiplication of Algebraic expressions

Radha has m boxes of toys and each box has n toys then, how many toys does Radha have?



n Toys n Toys n Toys

I^{st} Box II^{nd} Box m^{th} Box

Figure 9.2

Total no. of toys = Toys of 1^{st} box + Toys of 2^{nd} box + ———— + Toys of m^{th} box

= n+ n+ ———+ n (m times)

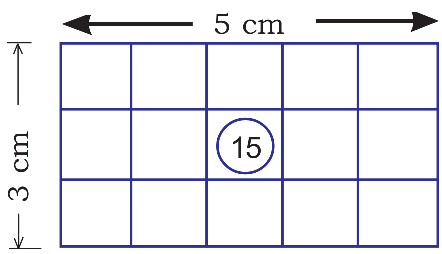
= n × m(Total no. of boxes)

= n×m

= mn

Here, total no. of toys = m × n = mn

Think about a rectangle of length 5 cm and breadth 3cm, what is the area of the rectangle?



Area = l × b = 5 × 3 = 15 cm^2

Now if the length of the rectangle is 8 cm and the breadth is 3 cm then what is the area of rectangle? Similarly, if length of the rectangle is x cm and breadth is 3 cm, what will be the area of the rectangle?

Example : 5

If the length of rectangle is p cm and breadth is q cm, what will be the area of rectangle?

Solution: - Area of Rectangle = length × breadth

= p cm × q cm

= pq cm^2

Every variable that has been used here has some numerical value and so will follow the same rules that the numerals follow. Such rules about which you have already read are the closure property, commutative property, Associative property. Let us see how these rules are followed by the Algebraic Expression.

Activity 1

In the table given below we are given two algebraic expressions and their product and some spaces are blank. The blank spaces are for you to write any two algebraic expression and then their product as done in the examples above:-

S.No.	I^{st} algebraic Expression	II^{nd} algebraic Expression	I^{st} alg. exp. × II^{nd} alg. exp.	II^{nd} alg. exp. × I^{st} alg. exp.	Product
1	-3	a	-3 a	a × (-3)	-3a
2	x	5	x × 5	5 × x	5x
3	2a	5a	2a × 5a	5a × 2a	$10a^2$
4	....	....			
5	....	....			
6	....	....			

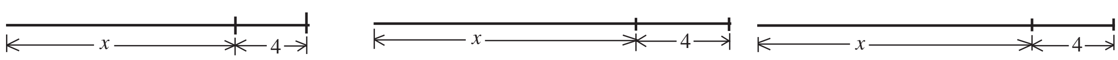
Does the result change by changing the position of terms?

From the table what do you conclude regarding the multiplication of algebraic expressions? **Write.**

When two algebraic expressions are multiplied, the corresponding integers are multiplied and the corresponding variables are multiplied. It is also clear from the table that for multiplication algebraic expressions follow commutative property.

Now let us discuss some problems:-

Question: Rajni has 3 ropes of length x meter each. If she joins each rope with a rope 4 meter in length, what will be the total length of the ropes?



Solution: By adding a rope of length 4 meter to the rope of length x meter, the length of each rope is x+4 meters.

Total length of 3 rope =$(x+4) \times 3$ meters or 3x+12 meters.

This question can also be discussed as follows:-

The sum of length of the given 3 ropes = x+x+x=3x meter.

The total increase in length = $4\times3=12$ meter

Therefore, the total length after the increase in length = 3x +12 meter

In both cases the total length is the same.

That is; $3(x+4) = 3x + 12$

Or $(x+4) 3 = x \times 3 + 4\times3 = x\times3+12 = 3x+12$

Again consider, if there are 5 ropes of length x meter each and ropes y meter is added to each, what will be the total length of the 5 ropes?

Adding the rope of length x to the rope of length y the length of each rope = x +y

Now, the total length of all the ropes = $(x + y) \times 5$ { there are 5 ropes of this type]

$= 5 (x + y)$

$= 5x + 5y$

∴

Again consider, length of 5 ropes of x m length = x ×5

We have to add a rope of length y m to each of these. Therefore, the increase in the total length of all the 5 ropes = y x 5.

After the increase the total length = x5 +y5

In both the situations the total length has to be the same. This means that (x + y) 5

= x 5 + y5

= 5 x +5 y

Example5: Multiply the expression –5a with (6b+3c)

Solution: $-5a\times (6b+3c) = (-5a) \times (6b) + (-5a) \times (3C)$

(Since $a(b+c) = ab+ac$)

$= -30ab-15ac$

Example 6: Multiply 4b by (7b-3c).

Solution: $(7b-3c) (4b) = 7b \times 4b + (-3c) \times 4b$

[Using $(a + b) c = ac + bc$]

$= 28b^2 -12bc$

Example 7: Multiply $(-5x + \quad y)$ with 4a

Solution : $(-5x + \frac{1}{2}y) \times 4a = (-5x) \times (4a) + \frac{1}{2}y \times (4a)$ $(a+b)c = ac + bc$

$= -20x\,a + 2y\,a$

Or $= -20ax + 2ay$

EXERCISE 9.2

1. Fill in the blanks:-

(i) $(2x + 3y) \times 2z = 2x \times 2z + 3y. \ldots.. = 4xz + 6yz$

(ii) $a(12x + xy) = a \times 12x + a \times xy = \ldots\ldots + axy$

(iii) $= \ldots\ldots\ldots + \ldots\ldots\ldots = 3x^2y - \frac{1}{2}xz$

(iv) $\left(\frac{5}{2}m - 6n\right) \times p^2 = \ldots\ldots + (-6n)p^2 = \ldots\ldots + \ldots\ldots$

(v) $(-3x^2y + 2z) \times y^2 = \ldots\ldots + \ldots\ldots = \ldots\ldots + \ldots\ldots$

(vi) $(\ldots\ldots + 7y^2) \times (-z^3) = (5x) \times (-z^3) + (7y^2) \times (-z^3) = -5xz^3 - \ldots\ldots$

2. Solve the following:- $\frac{1}{2}$

(i) $xy(7 + 8x)$ (ii) $(3r^2 - 5s)\,2t^2$ (iii) $\frac{1}{2}m\left(m^3 + \frac{3}{2}n\right)$

(iv) $mst(r^3 - st)$ (v) $\frac{4}{3}a\left(2b^2 + \frac{1}{2}c\right)$

We have learnt

1. We add or subtract groups of like terms in an algebraic expressions
2. In the subtraction of expressions, the terms to be subtracted are added to the rest after changing their respective sign.
3. In the multiplication of algebraic expressions, we multiply the term in the expression with integers first and then the terms that have variables.

Chapter -10

GRAPH

One day Suresh complained to his teacher that lily sat on his place.

The teacher asked Suresh where his place was.

Suresh – The first place of this row.

Teacher- Lily which is your place?

Lily – The second position from the starting, of forth row the teacher told lily and Suresh to sit on their respective place. The teacher asked Mohan- Mohan how you recognized your place.

Mohan – My seat (place) is second from starting of fifth row.

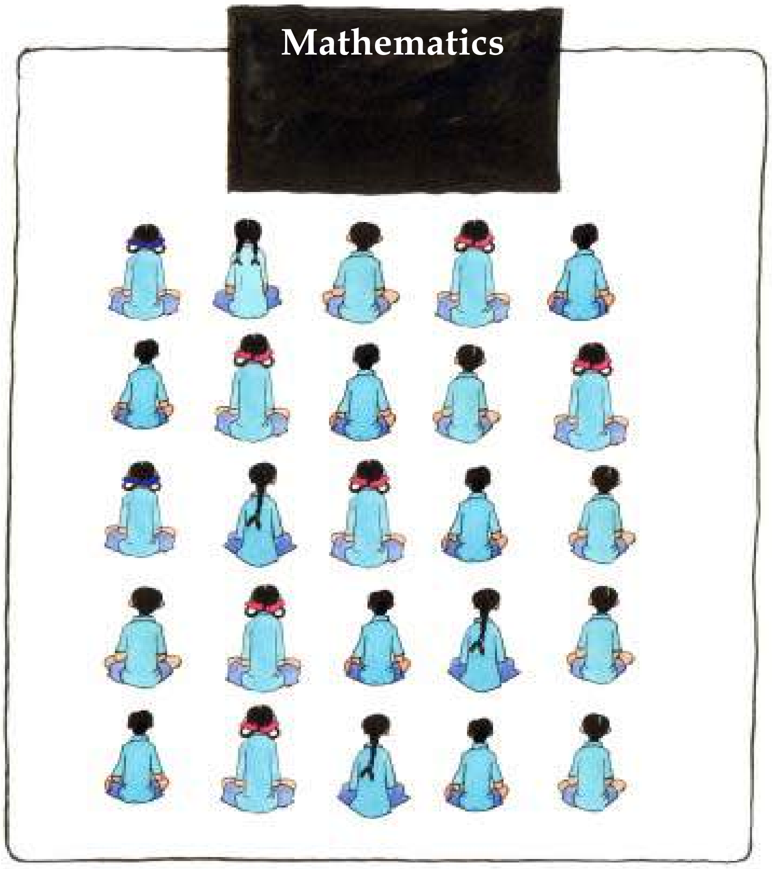


Fig. 10.1

The teacher asked from Hamid, Sandhya and Akbar about their places.

The teacher said – All of your places are fixed wherever you sit. Come, now we will play a game related to position.

Activity-1

There is a diagram of a colony of a town given below you have to reach to houses from chowk (O).

Conditions are:-

1. You have to walk on lines only.
2. You have to select the least distance.
3. Either don't take a turn or turn once.

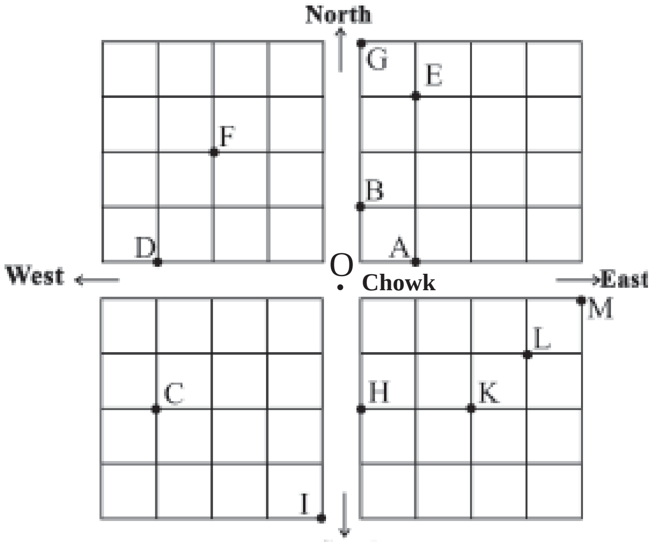


Fig. 10.2

Do reach the places, as directed in the diagram select the paths according to the examples given below.

Table 10.1

S.N.	Symbol of house	The way to reach the house from the chowk	No. of total crossed blocks
1.	A	One block to east direction form 'O'.	1
2.	B	One block to North direction from 'O'.	1
3.	C	3 block to west direction from 'O' then 2 blocks to south direction from 'O' or 2 block to south direction from 'O' then 3 blocks west direction from 'O'.	5
4.	D		
5.	E		
6.	F		
7.	K		

Exhibit the position with pairs:

On occasion of Annual sports competition the school was divided in to 4 divisions. Flags were the students of classes V^{th}, VI^{th}, VII^{th} and $VIII^{th}$ have to reach the flag.

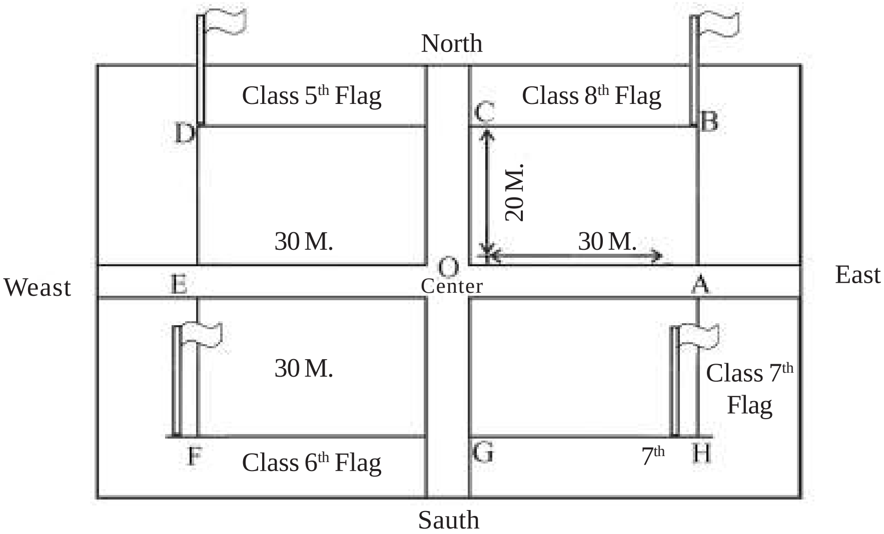


Fig. 10.3

Mohan and lily of class VIII went to the flag from two different ways. Mohan went 30 meter in east then from there 20 meters towards North and he reached flag 'B'.

Lily first with 20 meters in North direction and reached to flag 'C' from there she went 30 meters towards east and reached flag 'B'.

Position of flag form centre is like this:

Direction	Distance
East	30 meter
North	20 meter

Position of B (30 meter. 20 meter) → (Distance towards East distance towards North)

In short (30, 20) → (From East, North)

Write position of other points in same manner.

Determination of position and number lines:

You learnt about the use of number lines in class VIth and VII th. Come and draw number lines in diagram of a field or a colony. One horizontal line for East_ West and one vertical line for North South. Let, the center point 'O' at which both the number.

On the horizontal (East–West) numbers line the right hand side of 'O' is positive (+) and left hand side of 'O' is negative (–).

On the vertical (North-South) number line the upward direction of 'O' is positive (+) and the down- ward direction of 'O' is negative (–) for convenience, first write the numbers on horizontal numbers line and then on vertical line to show the positive of any point.

In this way, determine the positive of any

Class 8 flag B (30,20)

Class 5 flag D(–30, 20)

Class 6 flag F(–30, –20)

Class 7 flag H(30, –20)

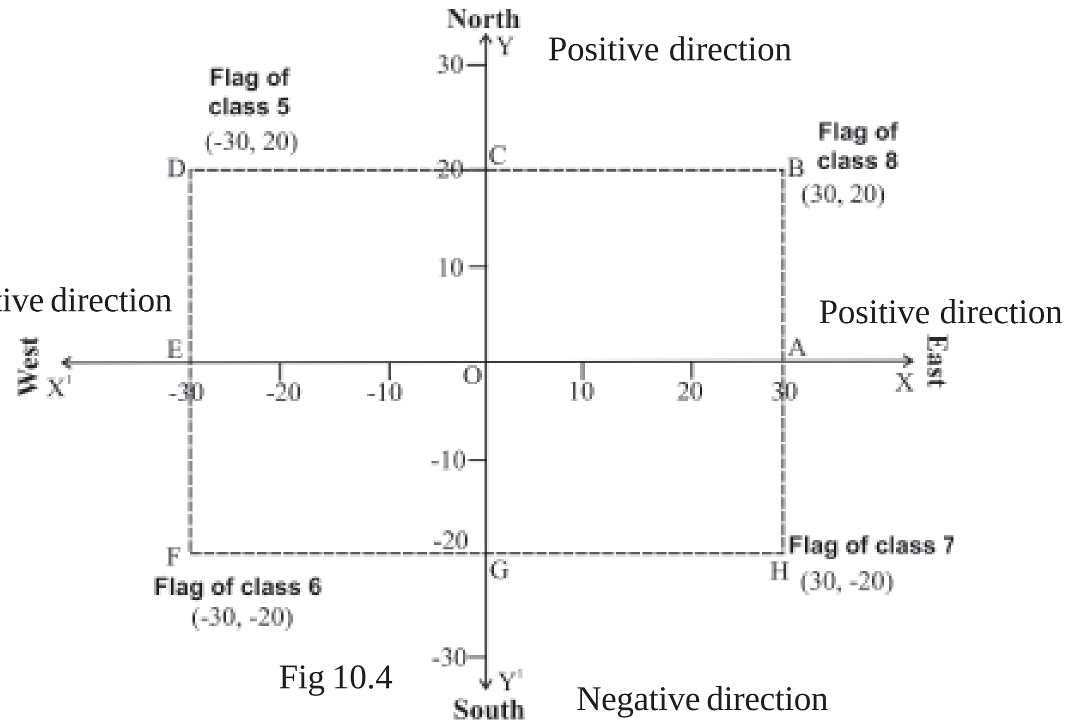


Fig 10.4

Is the position of center (0,0)?

In this way the position of any point is represented by a pair of numbers.

Co-ordinate axis surface:

Both the number lines which are told above are called Co- ordinate axis.

Horizontal Co-ordinate axis :- The number line which is exhibited by transverse line or, horizontal line is known as XX'.

Vertical – Co- ordinate axis – is exhibited by vertical line or straight line is known as YY'.

The surface of the paper is known as Co-ordinate surface. This system. Was first used by mathematician Rene Des castes.

In his memory co-ordinate surface is also known as cartesian surface.

Co-ordinate number:- In the position of class VIII the flag (30,20) the first co-ordinate numbers 30 is known as 'x' Co-ordinate, because it is exhibited in xx' axis. Second Co-ordinate number 20 is known as 'Y' Co-ordinate because it is exhibited in YY' axis.

Position of points on Co-ordinate axis:-

1. Position of point on XX' axis- point A is at a distance of 30 units on XX' axis. How much distance we have to cover in upward direction to reach the point 'A'. clearly this distance is zero. So the position of point A will be (30,0). In this way find out the position of point E. For each point which are at XX' axis their Y Co-ordinate number will be zero.

2. Will the Co-ordinate C(0,20),G(0-20) be pointed on YY' axis?

   We can exhibit the points (0,3), (0,12),(0-4) = as (0,Y) on YY' axis. The Co-ordinate number of principal point is (0,0) is center of Cartesian surface.

To exhibit the position of point on plane. The position of different points in figure 10.5.

1. Starting from point 'O' in the +ve direction of XX' direction , to point P (4,3) and covering 4 units (ON=4) reaching point N. stating from N towards position direction of YY' axis and covering 3 units (NP=3). When we reach these, we see that the point is p (4,3) [co-ordinate number of N= 4,0]

2. For point Q (-2,5) negative direction of XX' axis at a distance of 2 units from M. +ve direction of YY' axis after covering 5 units we reach the point (-2,5) [Co-ordinate of M (-2,5)].

3. For point A (6,o) in the +ve direction of XX' axis at a distance of '6' units and from here we have to move zero unit in the direction of yy' axis, then the point 'A' in on xx' axis.

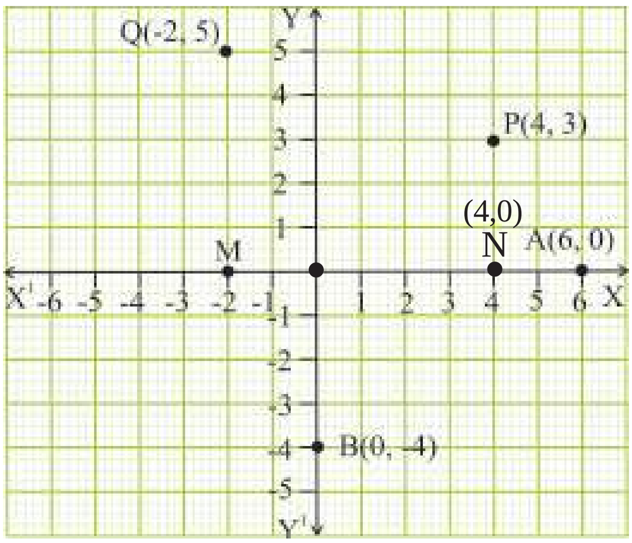


Fig. 10.5

4. For point B (0,-4) we have to move '0' unit in the direction of XX' axis from the original point.

Mark the following points on cartisan plane:-

(i) (3, 2) (ii) (3, - 4) (iii) (- 1, 5) (iv) (4, 4) (v) (0, 0)

(vi) (0, - 2) (vii) (4, 0) (viii) (- 3, - 5)

Find out the co-ordinate number of following marked points in cortisone plane –

(1) To find out the co-ordinate number of 'H' in fig. 10.6 i.e. to reach upto H first we have to find out the distance covered on XX' axis as well as YY' axis.

In this way

Co- ordinate number of H on X = 2.5

Co-ordinate number of H on Y = –3

So Co- ordinate number of H is (2.5,–3)

(2) Point k, is on YY' axis. So distance towards direction of XX' axis = 0 and distance towards direction of YY' axis = 2 thus 'k' is (0,2)

(3) To reach point L, the distance on XX' axis is – 3 and the distance on YY' axis is 4. So Co-ordinate number of L is (–3,4)

In this way find out the co-ordinate number of points M, P, Q, R,S.

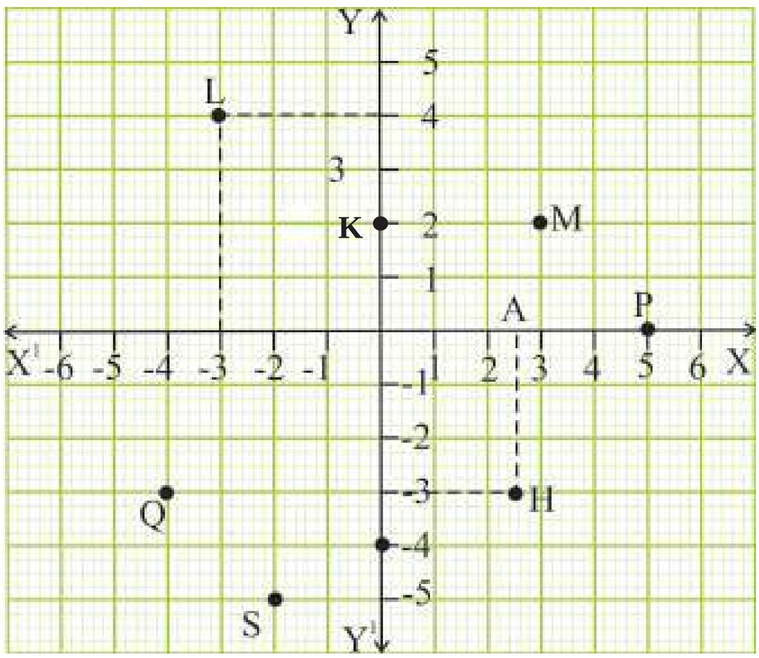


Fig.10.6

Graph between number and their multiples :-

In previous classes you have read about multiples suppose 3 is a number, its multiples will be 6,9,12,15,18........... etc. In this way of the number is x then multiples of x will be 2x, 3x, 4x ………. To exhibit these numbers on graph given number are exhibited on XX' axis and multiples are exhibited on YY' axis.

Example : y = 2x

Because y is equal to two times of ‘x’. We exhibit all the values of ‘x’ on XX' axis and its multiples 2x is exhibited on YY' axis. So co-ordinate numbers for point (x,y) will be following.

Activity -2

After filling the blanks join the points in the graph.

Table 10.2

S.No.	First No. (x)	Second No. (y=2x)	Point (x, y)
1	1	2	(1, 2)
2	2	4	(2, 4)
3	3	------	------
4.	0	------	------
5.	-1	------	------
6.	-4	-8	------
7.	----	------	------

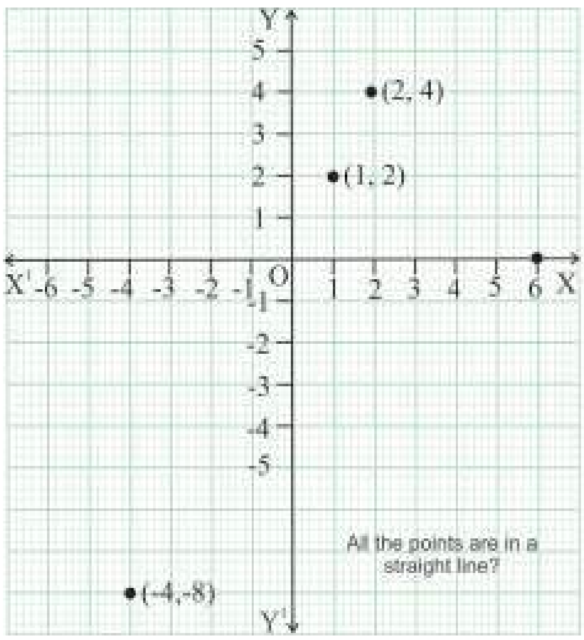


Fig. 10.7

Measurement Scale –

If the obtained numbers in co-ordinate numbers are greater, which is not possible to exhibit in graph paper, while drawing it on graph, you can take appropriate scale for the co-ordinates.

In the same way, if the co-ordinate numbers are very small, then also you can take appropriate scales for these number while drawing these on graph.

Example – The population of a village in different years is given below.

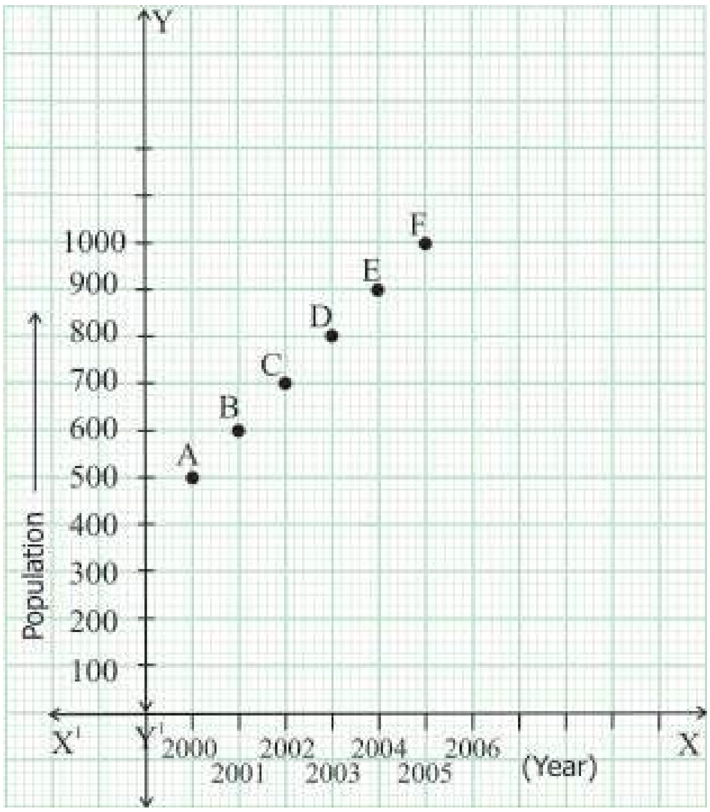
Year x	2000	2001	2002	2003	2004	2005
Population y	500	670	720	860	940	1000

From the above table, the graph has been drawn by taking year on 'X' axis and population on 'Y' axis figures of obtained from the population are 500 and 1000. So it is drawn in graph after taking appropriate states because such large numbers are not possible to exhibit on graph papers.

Scale : On XX' axis, 10 small block (1cm) = 1 year

On YY' axis, 10 small block (1cm) = 100 (population)

So we have to take 50 small blocks or 5 big blocks for the population of 500.



Graph for yearly population of village
Fig. 10.8

The graph between sides and perimeter of square:

In previous classes you have learnt to calculate perimeter (4 x side) of a square. If the side of a square = 5cm, then perimeter = 4 × 5 = 20 cm.

In the same way if the side of a square is 'x' then its perimeter will be 4 x.

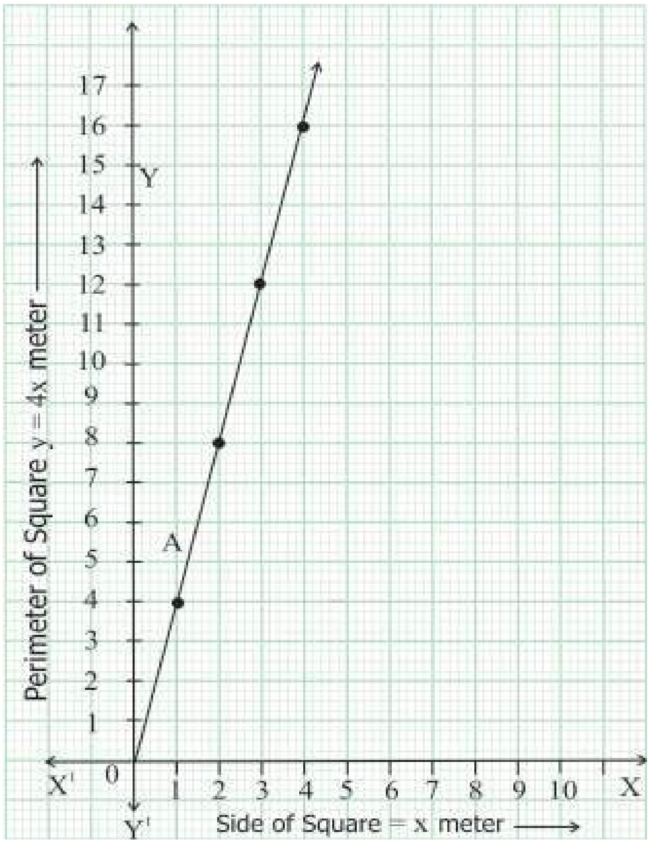
Now, let us do some activity.

Activity 3

Draw graph after completing the table

Table 10.3

S.No.	Length of the side of square x	Perimeter of square y = 4x	Point
1.	1	4	(1,4)
2.	2	......	
3.	3	......	
4.	......	......	(4, 16)
5.	......	......	
6.	......	......	



Graph between side and perimeter of square
Fig. 10.9

Graph between time and simple interest-

Simple interest = principle × rate × time/100

If the principle and rate is fixed then simple interest = K × time

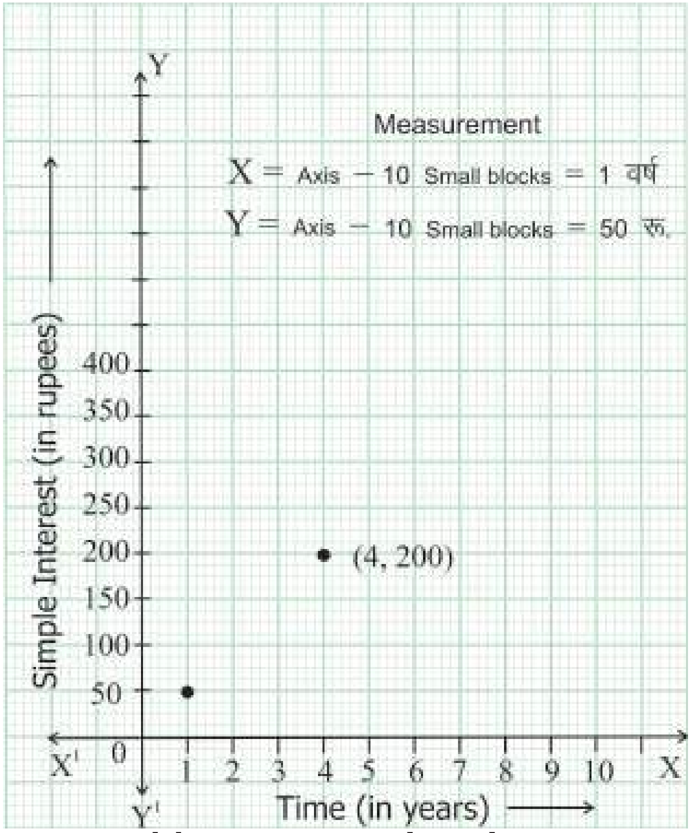
Thus, if principle and rate is fixed then simple interest is directly proportional to time.

Activity 4

Draw graph after completing the table when principle is Rs 1000 rate is 5% annum.

Table 10.4

S.No.	Time (in years)	S.I. = Rs. $\frac{1000 \times 5 \times \text{Time}}{100}$	Point (x, y)
1.	1	....	(1, 50)
2.	2	100	....
3.	3	....	....
4.	....	....	(4,200)
5.	....	....	....



Graph between time and simple interest
Fig. 10.10

Reading of graph

Linear graph :-

Till now, we drawn graph between multiple of numbers, sides and perimeter of square, and between time and simple interest. Now we will learn to read given graph.

Activity -5

Observe the given graph carefully-

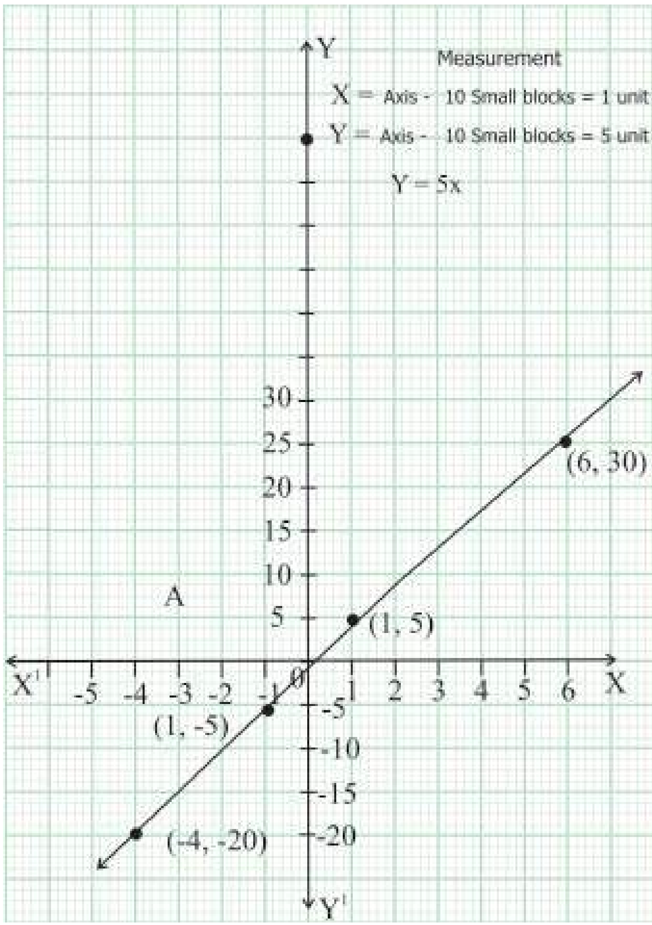


Fig. 10.11

Write the points which are straight line in above graph.

Table 10.5

S.No.	Point	Value of x	Value of y
1.	(–4, –20)	–4	–20
2.	(–1, –5)	....	....
3.	(1, 5)	....	....
4.	....	....	....
5.	....	....	....
6.	....	....	....

In the above table, each value of 'y' is how many times of the appropriate value on x – axis? Tell the relation between y and x ?

It is known from the table that each value of y is 5 times of the corresponding value of x. In this way, the relation between y and x is y = 5 x. In the graph, we see that when we increase the value of 'x' the value of 'y' is also increases and just movers.

When we decrease the value of 'x' the value of 'y' is also decreases. It means 'y' is proportional to 'x'. Thus and two variables, which are proportional to one – another, if we draw a graph between then we will get a straight line.

Graph between time and distance :-

In previous classes you have learnt that speed = distance/time

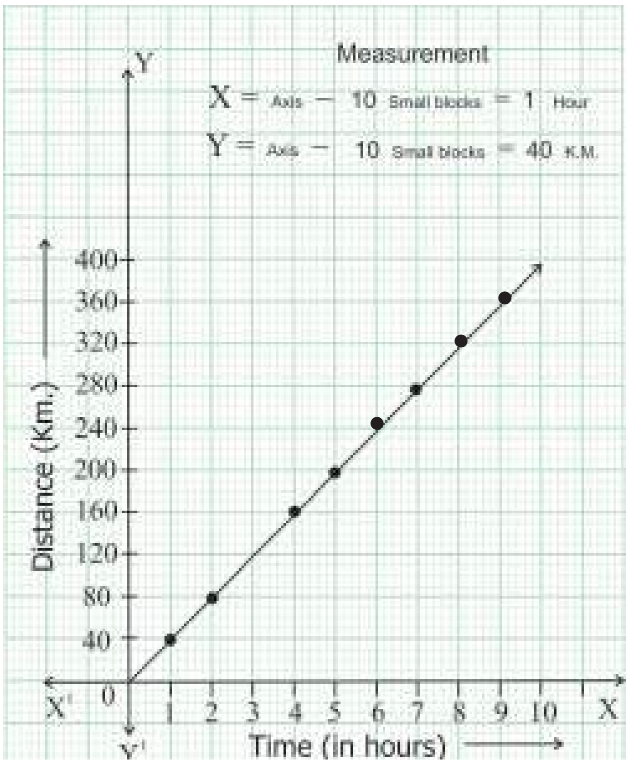
Speed × time = Distance

If the speed remains constant then distance is proportional to time. It means, Distance = Constant × time

Now if the time is taken on X-axis and distance is taken on Y-axis then also a straight line will be formed between them.

Activity 6

One motorcyclist is traveling at a constant speed. The assembled value of distance covered by him in different time is exhibited on the graph. Fill the blanks in the given table from the following graph and find out the speed.



Graph between time & distance

Table 10.6

S.No.	Point	X-axis	Y-axis	Speed = (km/hr)
1.	(2, 80)	2	80	$\frac{80}{2}$ = -----
2.	(4, 160)	....	....	....
3.	....	....	....	....
4.	....	....	....	....
5.	....	....	....	....
6.	....	....	....	....

It is known from the above table that the speed of motorcyclist is __________ km/hr

Excercise 10

Q.1 Exhibit following points on graph-

(2,5) (-3,4) (1,-1) (-3-2) (0,6) (-3,0) (10,-4) (0,0)

$\frac{\text{Distance}}{\text{Time}}$

Q.2 Answer in which quadrant the following points.

(2,-2) (-4,4) (-5,-4) (5,4)

Q.3 Exhibit the following points in the graph, join them, is the obtained figure is a straight line?

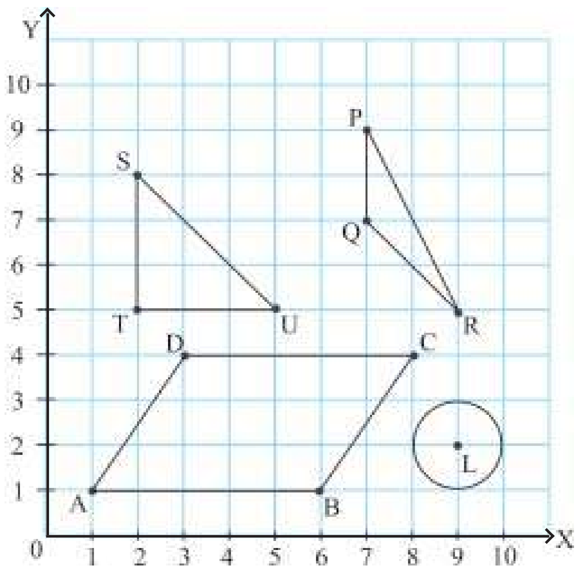
(3,-1), (1,1) (5,-3), (6,-4), (-2,4), (-4,6), (8,-6)

Q.4 Draw the following figure in the graph copy and answer the following questions-

(i) Find out the co-ordinates of the vertices the triangle PQR.

(ii) Find out the length of the sides ST and TU. After finding co-ordinates of the vertices right angled triangle STU.

(iii) Find out the length of sides AB and DC after the co-ordinates of the vertices of a parallelogram ABCD.



(iv) Find out the diameter of a circle after finding the co-ordinates of the center 'L' of circle.

Q.5 Do the graphical representation of the number and its thrice.

Q.6 Draw a graph, after finding the simple interest for different years with the given principle Rs. 800 at 10% rate per annum. according to the following tabel :

Time	1 year	2 year	3 year
Simple interest	Rs 80	Rs. 160	Rs. 240

Q.7 A train is at a constant speed of 60 km/hr. Draw a graph between time and distance.

Q.8 From relation between following co-ordinates .

1. (8,24)   2. (2,12)   3. (16,4)   4. (2,   )

We have Learnt

1. In the cartisan plane we learnt any point is exhibited be pair of number (co-ordinate).
2. Co-ordinate of 'Y' at X axis 'X' y axis in always zero.
3. Co-ordinate of origin is (0,0). This is known as centre of cartisan plane.
4. Cartisan plane divided in to 4 parts (division). Each division is known as quad-rant.
5. If there is proportional relation between two variables then the graph formed between them is straight line.

Chapter -11

DECIMAL REPRESENTATION OF RATIONAL NUMBERS AND OPERATIONS

In the previous chapter we have seen that rational numbers can be represented in the form of $\frac{p}{q}$ where q ≠ o and p, q are integers. means q^{th} part of p or it is the number obtained on dividing p by q.

While studying rational numbers Manohar wondered as to what would happen if we divide the numerator by the denominator.

Division in Rational numbers

Let us think about Manohar's question. Rational number means 5^{th} part of 2. This is obtained when we divide 2 by 5 → $5\overline{)2}$

To divide 2 by 5 we require decimal.

```
   0.4
5 )2
   0
   20
   20
   00
```

$\therefore \frac{2}{5} = 0.4$

We can represent in the from of 0.4.

Let us see what we get if we solve or on dividing 13 by 4.

Thus $\frac{13}{4} = 3.25$

```
     3.25
4   )13
     12
      10
       8
       20
       20
       00
```

In the above the dividends 0.4 and 3.25 are decimal representation of $\frac{2}{5}$ and $\frac{13}{4}$ respectively. What are the decimal representations of the following rational numbers?

(i) $\frac{3}{5}$ (ii) $\frac{17}{4}$ (iii) $\frac{15}{6}$ (iv) $\frac{19}{2}$ (v) $\frac{20}{3}$

Let us think about $\frac{20}{3}$

$$\begin{array}{r|l} & 6.666 \\ 3 & 20 \\ & 18 \\ \hline & 2\,0 \\ & 1\,8 \\ \hline & \;\;2\,0 \\ & \;\;1\,8 \\ \hline & \;\;\;\;2\,0 \\ & \;\;\;\;1\,8 \\ \hline & \;\;\;\;\;\;2 \end{array}$$

$\therefore \frac{20}{3} = 6.666...$

Terminating and non-terminating decimals:-

In the earlier questions we had seen that after some finite steps division process ended. But in representation of $\frac{20}{3}$ we will always obtain 2 as remainder and 6 in the quotient again and again. In this way we do not get 0 as remainder. Therefore, when the division gets completed in some finite steps and we reach remainder '0' then it is known as terminating. When the division does not ever get completed and we keep getting some remainder then it is known as a non-terminating number.

Find out the decimal representation of following numbers and state whether they are terminating or non- terminating.

(i) $\frac{3}{8}$ (ii) $\frac{15}{4}$ (iii) $\frac{1}{6}$ (iv) $\frac{1}{7}$ (v) $\frac{2}{9}$ (vi) $\frac{2}{11}$

On the left side some rational numbers are shown as terminating or non-terminating decimal numbers. On the right side some rational numbers are given. Follow the same method and find terminating or non-terminating numbers.

(i) $\frac{3}{8}$

$$\begin{array}{r} 0.375 \\ 8\overline{)3} \\ 0 \\ \hline 30 \\ 24 \\ \hline 60 \\ 56 \\ \hline 40 \\ 40 \\ \hline \text{xx} \end{array}$$

$\therefore\ \frac{3}{8} = 0.375$ Terminating decimal

(ii) $\frac{15}{4}$

$$\begin{array}{r} 3.75 \\ 4\overline{)15} \\ 12 \\ \hline 30 \\ 28 \\ \hline 20 \\ 20 \\ \hline 00 \end{array}$$

$\frac{1}{6} = 0.1666........,$

$\therefore\ \frac{15}{4} = 0.375$ terminating decimal

(ii) $\frac{1}{6}$

$$\begin{array}{r} 0.1666 \\ 6\overline{)1} \\ 0 \\ \hline 10 \\ 6 \\ \hline 40 \\ 36 \\ \hline 40 \\ 36 \\ \hline 40 \\ 36 \\ \hline 4 \end{array}$$

$\therefore$ non-terminating decimal

Find terminating or non-terminating from the following fractions:-

$\frac{5}{8}$ $\quad 8\overline{)5}$

$\frac{13}{4}$ $\quad 4\overline{)13}$

$\frac{1}{12}$ $\quad 12\overline{)1}$

(iv) $\frac{1}{7}$

$$
\begin{array}{r|l}
 & 0.14285714 \\
7 & 1 \\
 & 0 \\ \hline
 & 10 \\
 & 7 \\ \hline
 & 30 \\
 & 28 \\ \hline
 & 20 \\
 & 14 \\ \hline
 & 60 \\
 & 56 \\ \hline
 & 40 \\
 & 35 \\ \hline
 & 50 \\
 & 49 \\ \hline
 & 10 \\
 & 7 \\ \hline
 & 30 \\
 & 28 \\ \hline
 & 2
\end{array}
$$

$\therefore$ non-terminating decimals

(v) $\frac{2}{9}$

$$
\begin{array}{r|l}
 & 0.222 \\
9 & 2 \\
 & 0 \\ \hline
 & 20 \\
 & 18 \\ \hline
 & 20 \\
 & 18 \\ \hline
 & 20 \\
 & 18 \\ \hline
 & 2
\end{array}
$$

$\therefore \frac{2}{9} = 0.222$ non-terminating decimal

$\frac{1}{14}$ $\quad 14\overline{)1}$

$\frac{4}{9}$ $\quad 9\overline{)4}$

$\frac{1}{7}$

(vi) $\frac{2}{11}$

$$\begin{array}{r|l} 11 & 2 \;(0.1818 \\ & 0 \\ \hline & 20 \\ & 11 \\ \hline & 90 \\ & 88 \\ \hline & 20 \\ & 11 \\ \hline & 90 \\ & 88 \\ \hline & 2 \end{array}$$

$\therefore \frac{2}{11} = 0.1818$ non-terminating decimals

$22\overline{)1}$

Here (i) and (ii) are terminating decimals while rational numbers (iii), (iv), (v) & (vi) are non-terminating decimals.

$\frac{1}{22}$

Think of some rational numbers and ask your friends to find whether they are terminating or non-terminating decimals.

Non-terminating repeating decimals:

Quotient of (iii) is 0.1666.... here 6 is repeated. Quotient of (iv) is 0.14285714. When we see this carefully we find that 1,4,2,8,5,7 is repeated. Similarly, in (v) 2 and in (vi) one eight are repeated and the division is never completed. These are non-terminating digits after decimals. As there is repeated repetition of one or more digits the division would never end and therefore they are called non-terminating repeating decimals.

After the decimal sign if same digits are repeated then those digits that are repeated are denoted with a bar like.

$\frac{1}{6} = 0.1666......... = 0.1\overline{6}$ or $0.1\dot{6}$

If after decimals one or more than one digits are repeated then a - is placed on each of the repeated number and denoted by '———' or dots are put on the first and last digit like:-

$$\frac{1}{7} = 0.14285714...... = 0.\overline{142857} \text{ or } 0.\dot{1}4285\dot{7}$$

$$\frac{2}{9} = 0.222...... = 0.\overline{2} \text{ or } 0.\dot{2}$$

$$\frac{2}{11} = 0.1818...... = 0.\overline{18} \text{ or } 0.\dot{1}\dot{8}$$

Activity 1

Below some rational numbers are given find their decimal representations and say whether they are terminating or non-terminating.

S.No.	Rational number	Decimal representation	Terminating or non-terminating
1.	$\frac{1}{2}$		
2.	$\frac{1}{3}$		
3.	$\frac{1}{4}$		
4.	$\frac{1}{5}$		
5.	$\frac{1}{6}$		
6.	$\frac{1}{7}$		
7.	$\frac{1}{8}$		
8.	$\frac{1}{9}$		

Sort the terminating and the non-terminating decimals? What kind of rational numbers can be written in the form of terminating decimals? What are their characteristics? Write.

You have seen that $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$ and $\frac{1}{8}$ are the numbers whose decimal representations are terminating.

If $\frac{1}{2}$ is terminating then why is the decimal representation of $\frac{1}{6}$ non-terminating. Think and find the answer.

Now you know that we get terminating or non-terminating rational numbers because of the nature of their denominators.

Thus, let us see how on the basis of the prime factors of the denominator we can say whether the number after the decimal would be terminating or non-terminating.

Activity 2

Find the decimal representations of $\frac{5}{2}, \frac{24}{25}, \frac{3}{10}, \frac{21}{8}$ and check whether each is terminating or non- terminating?

Manohar stood up and said that they were all terminating. How did he say that?

Find the Prime Factors of denominators of the rational numbers given above.

You would see that the **prime factors of the these denominators have 2,5 or both. Therefore, the decimal representations of this kind of rational numbers are terminating.** We have seen that the decimal representations of $\frac{1}{6}, \frac{1}{7}, \frac{2}{9}, \frac{2}{11}$ are non-terminating. Find the prime factors of the denominators of these numbers. Do the prime factors of the denominators of these numbers have numbers other than 2 and 5? If the prime factors of denominator have numbers other than 2 and 5, then the decimal representations of such numbers would be non-terminating.

Example 1

Find the terminating and non-terminating numbers from the following numbers

(i) $\frac{4}{125}$ (ii) $\frac{5}{18}$ (iii) $\frac{11}{8}$ (iv) $\frac{13}{100}$

Solution

(i) In $\frac{4}{125}$, the prime factors of 125 are 5×5×5. The denominator has only 5 in its prime factors therefore the number has a terminating decimal representation.

(ii) In $\frac{5}{18}$ prime factorization of 18 is 2 × 3 × 3

Here the denominator 18 has a number other than 2, 5 i.e.3 as a prime factor.

Therefore, it has a non-terminating decimal.

(iii) $\frac{11}{8}$ has a denominator 8, its prime factorization is 2×2×2.

Here the prime factor is only 2 and therefore it is a terminating decimal.

(iv) In $\frac{13}{100}$, the prime factorization of the denominator 100 is = 2×2×5×5. Here the prime factors are only 2 and 5 and hence the number has a terminating decimal representation.

All the examples considered above were positive rational numbers.

How will we represent negative rational numbers in decimal form? $\frac{23}{3}$

Decimal representation of negative numbers:

To find the decimal representation of a negative number, first find the decimal representation of the same number without a negative sign. After that we can put the negative sign.

Example 2

Find the decimal representation of $\frac{-23}{3}$.

Solution: consider in place of $\frac{-23}{3}$. For the decimal representation is:

Or $\frac{23}{3} = 7.666 = 7.\overline{6}$

That is $\frac{-23}{3} = -7.\overline{6}$

Exercise 11.1

1. Without dividing find out the terminating and non-terminating decimal numbers.

$$\frac{4}{5}, \frac{8}{7}, \frac{-15}{49}, \frac{7}{50}, \frac{3}{28}$$

[illegible]

21

20

18

20

18

20

18

2

2. Change given rational numbers to decimal numbers.

3. Write the following rational numbers as decimal numbers.

Converting Decimal Numbers into Rational Numbers:

You have studied how to convert rational numbers into terminating and non-terminating decimal numbers. But can decimal numbers be converted into rational numbers? Let us discuss through some examples.

$$0.25 = \frac{0.25 \text{x} 100}{100} = \frac{25}{100} = \frac{1}{4} \quad \text{(Simplest form)}$$

$$2.6 = \frac{2.6 \text{x} 10}{10} = \frac{26}{10} = \frac{13}{5} \quad \text{(Simplest form)}$$

$$0.317 = \frac{0.317 \text{x} 1000}{1000} = \frac{317}{1000}$$

$$4.625 = \frac{4.625 \text{x} 1000}{1000} = \frac{4625}{1000} = \frac{37}{8}$$

From the above examples we see that it is easy to convert decimal numbers into rational numbers. We will write 1 in the denominator and after 1 put as many zeros as the digits after the decimal point in the number, then remove the decimal point. This will give us rational numbers like:

$$7.21 = \frac{721}{100}$$

$$4.2 = \frac{42}{10} = \frac{21}{5} \quad \text{etc.}$$

Geeta said that by this procedure we can only convert terminating numbers to rational number but how can we convert non-terminating and repeating decimals to rational numbers?

For e.g. 1.666 ... = $1.\overline{6}$

Let us consider how we can convert non-terminating and repeating decimal numbers to rational numbers.

Example 3

Convert $0.\overline{6}$ into rational number.

Let $x = 0.\overline{6}$

x = 0.666...........(i)

Multiply both sides by 10,

10 x = 6.66 – (ii)

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Subtract (i) from (ii)

Or 10 $x - x$ = 6.666 .... % 0.666 ...

Or 9 x = 6

Or $x = \frac{6}{9} = \frac{2}{3}$, hence $0.\overline{6} = \frac{2}{3}$

Example 4

Convert $0.\overline{234}$ into rational number.

Let x = $0.\overline{234}$

x = 0.234 234 ....(i)

Multiply both sides by 1000

1000 x = 234.234234..... (ii)

Subtract (i) from (ii)

999 x = 234 or $x = \frac{234}{999} = \frac{26}{111}$

Hence $0.\overline{234} = \frac{26}{111}$

In the above examples we adopted the following procedure:-

(i) First we considered the given decimal number as x and wrote it as equation (i).

(ii) The number repeating itself after decimal is written 2-3 times.

(iii) The number of digits repeating is counted. We multiply both sides by as many powers of 10 as the number of repeating digits. This is written as equation (2)

(iv) Subtract (ii) from (i) and find the value of x. Notice the non-terminating part gets subtracted.

Manohar asked Geeta if the repeating digits come after some digits in a decimal, e.g. 1.25666 then how do we convert it to a rational number form? Geeta did not know either?

Let us solve some such examples:-

Example5 Write $3.21\overline{6}$ in the form of a rational number.

Solution: Let $x = 3.21\overline{6}$

Or $x = 3.21666....$ (i)

Multiply both sides by 100

We have 100 x = 321.666...... (ii)

Then multiplying both sides of equation (ii) by 10

1000x = 3216.66....... (iii)

Subtract (ii) from (iii)

Or 1000x – 100 x = 3216.66 – 321.66

900x = 2895

$$x = \frac{2895}{900} = \frac{193}{60}$$

Hence $3.21\overline{6} = \frac{193}{60}$

Example 6

Convert $0.15\overline{23}$ into a rational form.

Solution: Let $x = 0.15\overline{23}$

Or x = 0.152323.......(i)

Multiply both sides of (1) by 100

100 x = 15.2323.........(ii)

Then multiply both sides of (ii) by 100

We have 10000x = 1523.2323 .... (iii)

Subtract (ii) from (iii)

10000 x - 100x = 1523.2323 –15.2323

9900 x = 1508

$$x = \frac{1508}{9900} = \frac{377}{2475}$$ Hence $0.15\overline{23} = \frac{377}{2475}$

In both the examples the not repeating digits are counted and as many zeros written after 1. Multiply the given number by this. After this only the repeating digits remain. Then follow the method for numbers with only the repeating digits to find the rational number.

Exercise 11.2

(1) Convert the following numbers into rational number forms.

(a) 0.2 (b) 0.55 (c) 6.25

(d)2.175 (e) 14.53

(2) Write the following in rational form.

(a) $0.\overline{4}$ (b) 7.25 (c) $0.05\overline{6}$

(d) 0.27 (e) $0.5\overline{4}$

Multiplication of Decimal Numbers:-

You have learnt how to write rational numbers as decimal number forms. In earlier classes, we have also studied multiplication of integers. Let us learn multiplication of decimal numbers.

We want to multiply 0.2 × 0.3 then

$$0.2 = \frac{2}{10} \text{ and } 0.3 = \frac{3}{10}$$

Now $0.2 \times 0.3 = \frac{2}{10} \times \frac{3}{10} = \frac{6}{100} = 0.06$

Now, we can see that there is a big difference between the products of positive integers 2 and 3 and of positive numbers 0.2 and 0.3. Clearly 6 is 100 times more than 0.06. We can also see that 6 is larger than either 2 or 3 while .06 is smaller than either of them.

Example 7 Find the value of 0.31 × 0.04

Solution: $0.31 = \frac{31}{100} \text{ and } 0.04 = \frac{4}{100}$

Now $0.31 \times 0.04 = \frac{31}{100} \times \frac{4}{100} = \frac{124}{10000}$

$= 0.0124$ Answer

Example 8 Find the value of :

Solution: , $0.3 = \frac{3}{10}$ and $0.02 = \frac{2}{100}$

Now $0.015 \times 0.3 \times 0.02$

$$= \frac{90}{1000000} = 0.00009$$

Activity3

Fill up the following blanks as per column 1:

S.No.	Numbers	Multiplication process	Product as a fraction	Answer	No. of digits after decimal in answer
1.			$\frac{2}{100000}$	0.00002	5
2.	0.502×0.45			0.22590	5
3.	0.22×0.101	....	....	....	....
4.		....	....	....	....
5.		....	....	....	....
6.		....	....	....	....

To place the decimal point in the product of two numbers, we count the digits after decimal point in both numbers and then put the decimal point in the product after leaving as many digits as the sum of these from right to left. If the number of digits is less than the sum then write 0's on the left of the product till we match the sum and then place the decimal point.

Activity 4

Place the decimal point at appropriate places in the following products:

(1) 4.283 × 3.41 = 1460503

(2) 326.7 × 0.319 = 1042173

(3) 9.07 × 13.4 = 121538

(4) 69.05 × 5.044 × 19.5 = 67916199

Division in Decimal numbers

We divide decimal numbers as we divide integers.

If the divisor is an integer then-

Example 9 Find the value of :

Solution: 25) 25.2025 (1.0081

```
25 ) 25.2025 (1.0081
   - 25
     ××  2
       - 0
         20
       - 00
         202
       - 200
         ××25
       -   25
           ××
```

If the divisor and the dividend are both decimal numbers:-

Example 10 Find the value of :

(I)

$$45.27 \div 1.5 = \frac{4527}{100} \times \frac{10}{15}$$

$$= \frac{4527}{100} \times \frac{10}{15}$$

$$= \frac{4527}{15 \times 10}$$

$\Rightarrow$

```
150) 4527 (30.18
   -  450
      ××27
   -    00
        270
   -    150
        1200
   -    1200
         ×××
```

= 30.18 Answer

(II)

$$\Rightarrow \frac{452.7}{15}$$

```
15 ) 452.7 (30.18
   - 45
     xx 2
      - 0
        27
      - 15
        120
      - 120
        xxx
```

= 30.18 Answer

If the divisor is a decimal number than to convert it into a whole number we multiply dividend and divisor both by 10, 100, ….. This is done so that the divisor becomes a whole number. Then the number obtained in the numerator is divided by the denominator.

Activity 5

S.No.	I Number × II Number	Product	$\frac{\text{Product}}{\text{I Number}}$ = II Number	$\frac{\text{Product}}{\text{II Number}}$ = Distance
1.	0.4×0.6	0.24		$\frac{0.24}{0.6} = 0.4$
2.	0.7×0.02	0.014		$\frac{.....}{0.2}$ =.....
3.	0.12×0.35	0.0420		$\frac{0.0420}{0.35}$ =.....
4.	7.2×0.3	……….		
5.	4.52×0.06	……….		$\frac{0.2712}{0.06}$ =....
6.	0.008×0.0007	……….		$\frac{0.0000056}{.........} = 0.008$

It is clear from the table that if the product of two numbers is divided by the first number, we get the second number. And if the product is divided by the second number, we get the first number. This means;

$$x \times y = p \quad \Rightarrow \quad x = \frac{p}{y} \text{ and } y = \frac{p}{x}$$

Activity 6

Place decimal point at appropriate place in the following divisions–

1. 68.64 ÷ 4.4 = 156
2. 400.14 ÷ 85.5 = 468
3. 0.735 ÷ 0.7 =105
4. 51.1875 ÷ 1.05 = 4875
5. 3.773 ÷0.98 = 385

Example 11: Solve 0.512 × 4.375

Solution $\frac{512}{1000} \times \frac{4375}{1000}$

$= \frac{2240000}{1000000}$

$= \frac{224}{100}$

Answer

Example 12. Solve 3.15 ÷0.02

$= \frac{3.15}{0.02} \times \frac{100}{100}$ $= \frac{0.3942}{0.018}$ **Solution:** 3.15 ÷ 0.02

$= \frac{315}{2}$

```
2 ) 315 (157.5
  - 2
    11
  - 10
     ×15
  -  14
      ×10
  -   10
       ××
```

Answer = 157.5

Example 13: Find the value of 0.3942 ÷ 1.8

Solution: 0.3942 ÷ 1.8

$= \frac{3.942}{18}$

18) 3.942 (0.219
- 0
39
- 36
×34
- 18
162
- 162
×××

Answer = 0.219

Example 14. : Solve .

Solution: $\frac{0.005 \times 0.84 \times 2.25}{0.021 \times 0.05 \times 1.10} \times \frac{10^7}{10^7}$

$\frac{5 \quad 84 \quad 225}{21 \quad 5 \quad 110}$

$= \frac{900}{110}$

$= \frac{90}{11}$ $= 8.\overline{19}$ Answer

$\frac{0.0}{0.0}$

Exercise 11.3

1. Add

(i) 1.0087 + 0.321 (ii) 0.2+0.02+0.0202+0.20204

(iii) 3.81+0.009+10.0023 (iv) 2.45+6.908+0.125 +1.0074

2. Find the value

(i) 7.89-2.324 (ii) 5.01-0.00729

(iii) 1.01-0.1-0.001+10.001 (iv) 7.802-1.4+2.8-0.00107

3. Solve

(i) 243 × 0.15 (ii) 0.85 ×0.022 (iii) 0.1 × 0.1 ×0.1 × 0.1

4. Solve

   (i) 2.25 ÷ 15 (ii) 10.206 ÷ 0.06

   (iii) 0.324 ÷ 1.8 (iv) 46.225 ÷ 2.15

5. Suneeta purchased oil for Rs. 23 and 50 paisa, soap for Rs. 8 and 15 paisa and powder for Rs. 12 and 39 paisa. How much was the bill for?

6. Simran usually pays Rs. 472.5 as the electricity bill of the house. Per unit Rs. 3.50 is charged for electricity. How many units does she use in her house?

7. Rahim pays Rs. 2075 as house rent per month. How much will he pay in two and a half years?

8. Find in cubic meters the air contained in a room of length, breadth and height 5.5m, 4.6m and 3.2 m respectively.

We Have Learnt

1. Each rational number can be expressed as a decimal number form.

2. We can convert decimal numbers into rational form.

3. In converting rational numbers into decimal number forms if the division ends after a finite number of steps, then it is terminating other wise non-terminating

4. In a terminating decimal, the denominator has only 2 and 5 as the prime factors.

5. If in converting a rational number form into a decimal number, we see that one or more digits start being repeated after the decimal point. Then these digits are called repeating numbers and the number is a non- terminating decimal. Mark a line or a bar on the repeating digit or mark a point on the first and the last digit of the repeating digits in the numbers to avoid repeatedly writing them.

6. In multiplication of decimal numbers, we count the digits after decimal in each of the given decimal numbers. We place the decimal in the product after adding the two and counting the required digits from right to left.

Chapter -12

THE ANGLE,PAIR OF STRAIGHT LINES AND TRANSVERSALS

In previous class you studies about angle, types of angle and their measurement. Now, lets us discuss about pair of angles.

Pairs of Angles

You must have noticed that more than one angles are formed at every point. Think about these:

1. Adjacent Angles

When the anlges formed on both sides of a common arm at an apex, are considered they are known as adjacent angles.

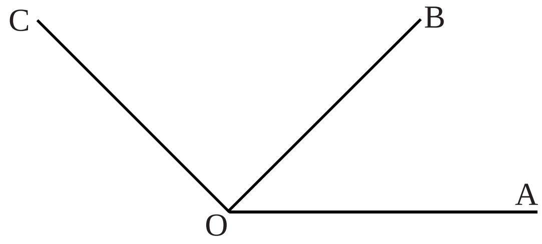


Fig 12.01

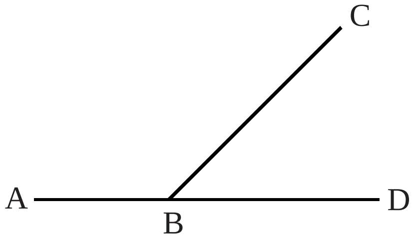


Fig 12.02

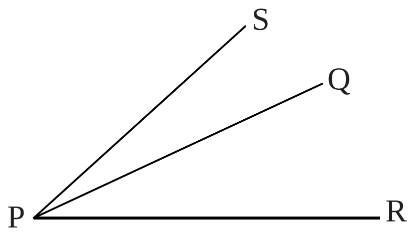


Fig 12.03

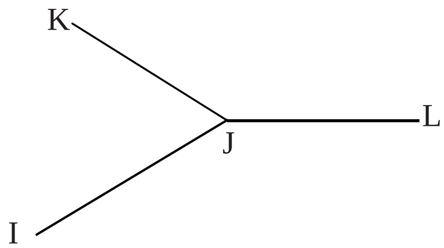


Fig 12.04

In fig 12.1 above ∠COB and ∠BOA are *adjacent angles*. In fig 12.2 ∠ABC and ∠CBD are *adjacent angles*. Similarly, in fig 12.3 ∠SPQ and ∠QPR are *adjacent angles*, while in fig 12.4 ∠IJK and ∠KJL are *adjacent angles*.

Let us find out some more imformation about adjacent angles.

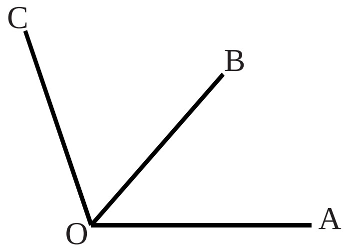


Fig 12.05

In the above fig 12.5, if O is an apex and OA is a common arm, then are ∠AOC and ∠AOB *adjacent angles*? If no, why?

In fig 12.5, you can see that ∠AOB and ∠AOC are angles formed on the same side of the common arm OA. Therefore they are not adjacent angles, but angles ∠AOB and ∠BOC are adjacent angles because they are formed on both sides of the common arm OB and not on the same side.

Linear Pairs

When the arms of adjacent angles are not common and are in a straight line, then the adjacent angles thus formed are called linear pairs. This means at this state, **the sum of the measures of the adjacent angles equal 180°. These are also called *linear adjacent angles* or *linear angles*.**

e.g.

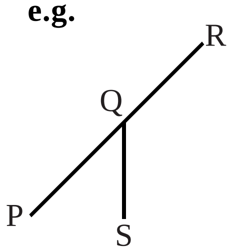


Fig 12.06

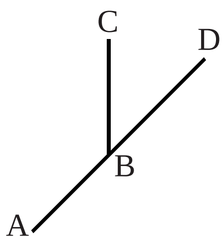


Fig 12.07

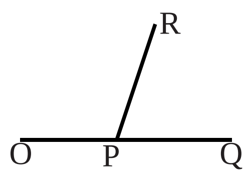


Fig 12.08

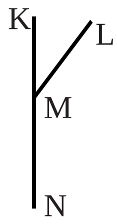


Fig 12.09

Look at the fig 12.6, 12.7, 12.8 and 12.9. The sum of the adjacent angles in these figures are as follows.

fig 12.6	sum of ∠PQS and ∠RQS = 180°
fig 12.7	sum of ∠ABC and ∠CBD = 180°
fig 12.8	sum of ∠OPR and ∠RPQ = ?
fig 12.9	sum of ∠KML and ∠LMN = ?

Activity 1

Recognise the pairs of adjacent angles and linear angles and write them in the table

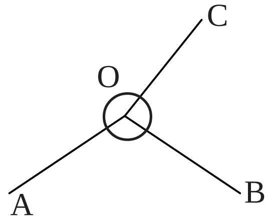


Fig 12.10

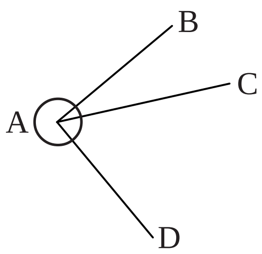


Fig 12.11

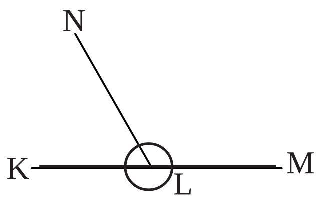


Fig 12.12

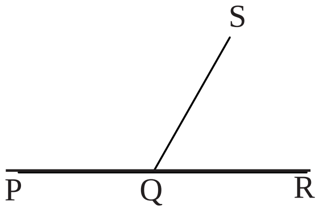


Fig 12.13

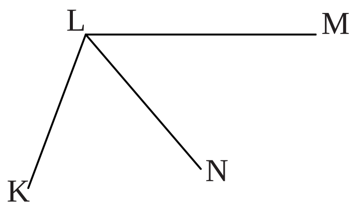


Fig 12.14

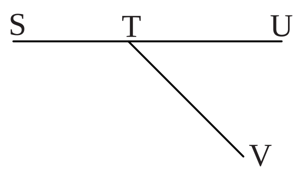


Fig 12.15

Table No. 1

Figure No.	Adjacent angle	Linear angle
12.10	----------------	----------------
12.11	----------------	----------------
12.13	----------------	----------------
12.14	----------------	----------------
12.15	----------------	----------------

Vertically opposite Angles

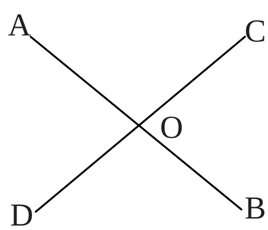


Fig 12.16

In the above picture

The angle opposite to $\angle AOC$ is ___________.

Similarly, the angle opposite to $\angle AOD$ is __________.

Thus you have noted that when two straight lines or a line segment cross each other (intersect) at a point, then 4 angles are formed at the point of intersection, out of which the two angles that are equal and opposite and face each other vertically are known as vertically opposite angles.

ACTIVITY 2

Take two broom sticks and fix them upon each other at the centre. We can rotate the sticks round the pin. Now rotate the sticks at different places, measure the angles opposite to each other and write them down.

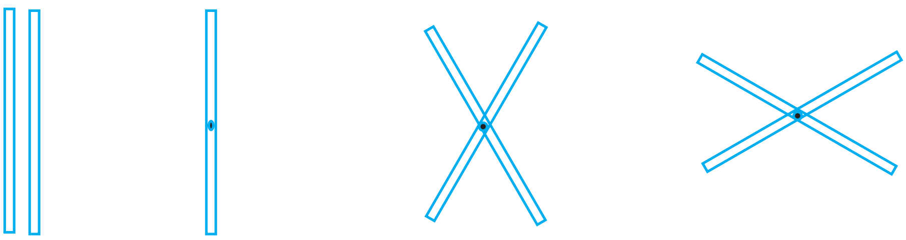


Fig 12.17

Draw two line segments in your notebook in such a way that they intersect each other. Measure the adjacent angle formed by these lines.

Supplementary and Complementory Angles

Given below are two types of angles. Measure them carefully and write them down.

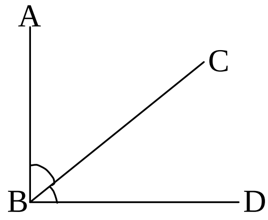


Fig 12.18

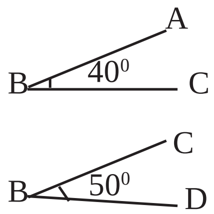


Fig 12.19

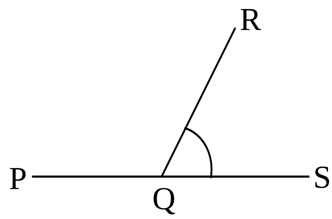


Fig 12.20

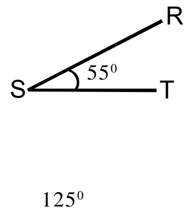


Fig 12.21

In fig 12.18, adjacent angles ∠ABC + ∠CBD = ______________

In fig 12.20, adjacent angles ∠PQR + ∠RQS = ______________

In fig 12.19, the sum of both the angles is 90°.

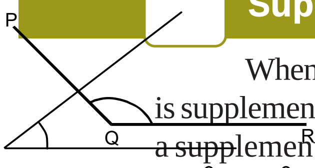
In fig 12.21, the sum of the both angle is 180°.

Complementory Angles

When the sum of two angles is a right angle or 90°, then one angle is complementory to the other e.g. fig 12.19.

∠ABC is complementory to ∠CBD, i.e. if ∠ABC = 40°, then the complementory angle ∠ABC will be $90^{\circ} - 40^{\circ} = 50^{\circ}$.

Supplementary Angles



When the sum of two angles is equal to two right angles or a straight angle (180°), then one angle is supplementary to the other angle. Example, in fig 12.21, ∠PQR + ∠RQS = 180°, therefore, ∠PQR is a supplementary angle to ∠RQS. This means if ∠PQR = 125°, then the supplementary ∠RQS 180°- $125^{\circ} = 55^{\circ}$.

ACTIVITY 3

Below are given pictures, in which measures of anlges are given. Write the measures of the complementary and supplementary angles. Point out if such angles cannot be formed.

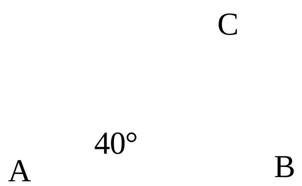


Fig 12.22

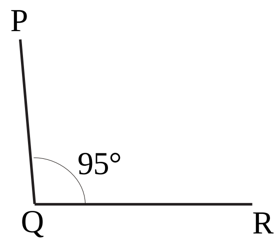


Fig 12.23

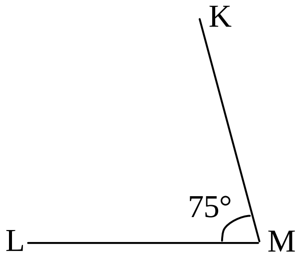


Fig 12.24

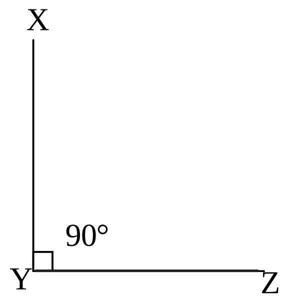


Fig 12.25

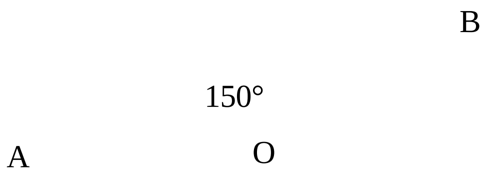


Fig 12.26

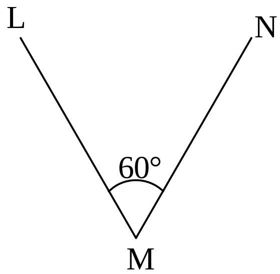


Fig 12.27

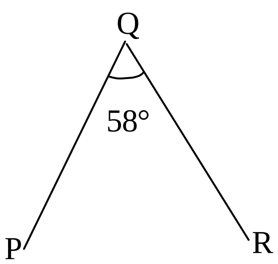


Fig 12.28

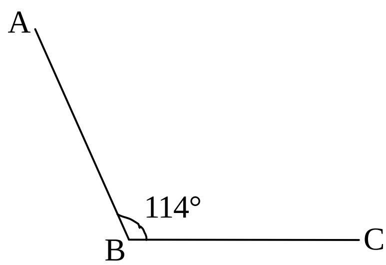


Fig 12.29

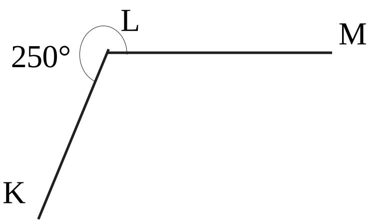


Fig 12.30

Table No. 2

Figure No.	Angles	Measures of complementroy angles	Measures of Supplementary angles	If not possible why ?
12.22	CAB	$90^\circ - 40^\circ = 50^\circ$	$180^\circ - 40^\circ = 140^\circ$	Possible
12.23	PQR			
12.24	∠KML			
12.25	XYZ			
12.26	AOB			
12.27	LMN			
12.28	PQR			
12.29	ABC			
12.30	MLK			

EXERCISE 12.1

1. Write the definitions of the given angles:

   (i) adjacent angles (ii) supplementary angles

∠

   (iii) vertically opposite angles

2. What are the complementory angles for the following:

   (i) 40° (ii) 50° (iii) 60°

   (iv) 75° (v) 0° (vi) 70°

3. What are the supplementary angles for the following:

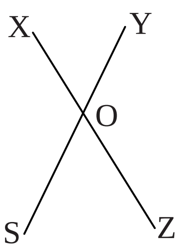
   (i) 110° (ii) 70° (iii) 0°

   (iv) 120° (v) 45° (vi) 50°

4. An angle is twice the value of its complementory angle, find the measures of both the angles ?

5. An angle is half the measure of its sumpplementary angle, find the measures of that angle ?

6. XOZ and SOY are two straight lines. If $\angle XOY = 40^\circ$, find $\angle SOZ$ and $\angle XOS$.

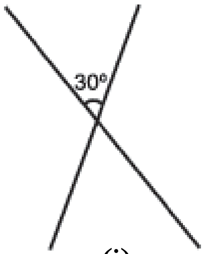


7. If the sum of two adjacent angles is 180°, what type of angles are they?

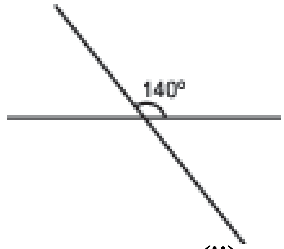
8. One angle of linear pair angle is given. Find out the second angle of that pair.

(i) (ii) (iii) (iv) (v) (vi)

9. In the given figures measure of one angle is given. Find the measurement of its vertically opposite angle.

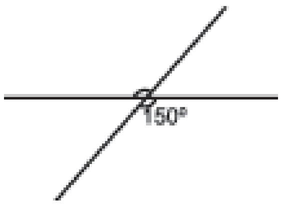


(i)

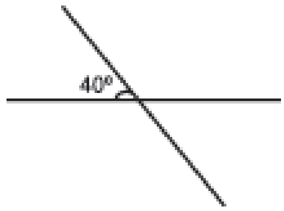


(ii)

10. In the given figure only one the angle is given. Find out the value of reminding three angles.



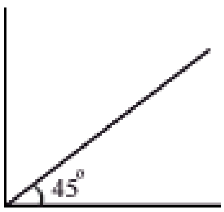
(i)



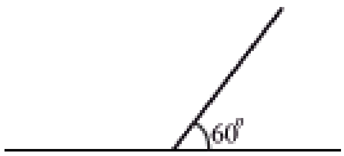
(ii)

11. In the given figure one angle of adjacent angles is given find out the another angle.

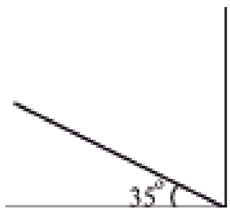
(i)



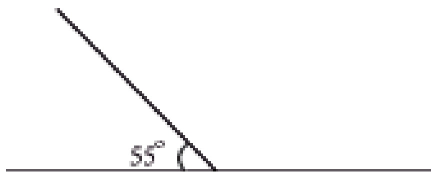
(ii)



(iii)



(iv)



ACTIVITY 4

Sketch two straight lines in your note book. Look at these lines carefully and answer the following questions.

1. Are these lines intersecting each other? If not, will they intersect after extending the lines?
2. If in both situations, your answer is 'no', then what kind of lines are these?

Mary, Raju and Anu draw a few such lines.

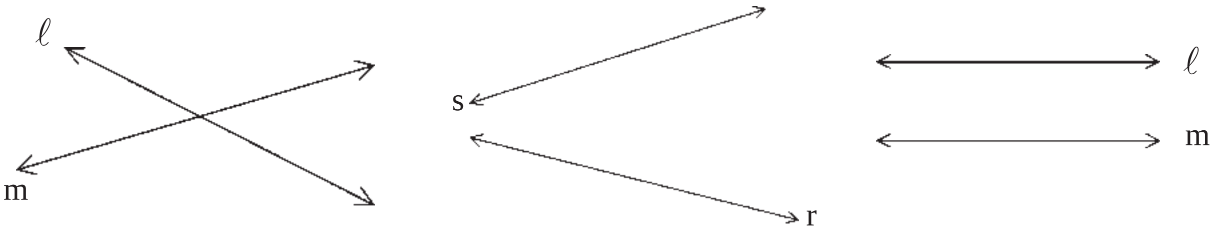


Figure 12.31

Lines drawn by mary Lines drawn by Raju Lines drawn by Anu

Here the lines drawn by Anu never intersect. These are parallel lines. Lines draw by Mary and Raju intersect each other or would intersect when they are extended, these are intersecting lines.

You have seen above how two lines can be drawn in different ways. In the same manner draw three straight lines in your note book. In how many ways can you draw these lines?

Let us look at the possibilities:-

1. All the lines are parallel like –

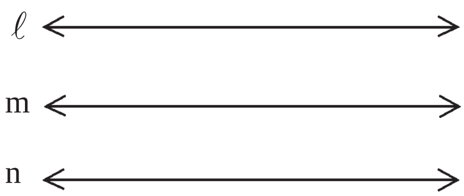


Figure 12.32

2. When three lines intersect at one point like -

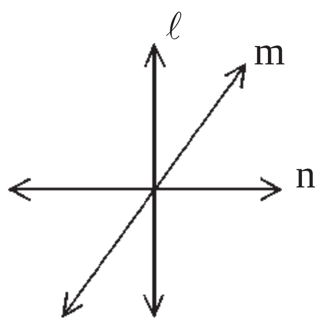


Figure 12.33

In figure 12.33 ℓ, m and n are concurrent lines.

3. When one straight line intersects the other two straight lines at different points p and q. For example

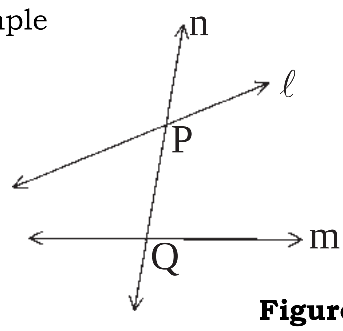


Figure 12.34

Here the line n intersects the lines ℓ and m at points p and q, thus the line n is a transversal line on the lines ℓ and m.

If we extend l and m so that they intersect each other at one point, then would line m be transversal lines for line ℓ and line n?

Would the line be a transversal on the lines m and n? If yes? Then why? Write reasons in support of your answer.

Since the line m intersects line and line n at different points and similarly the line intersect line m and n at different points, therefore, line and line m are transversal lines.

'The line that intersects two or more than two straight lines at different points in a plane, is called a transversal line'.

Concurrent lines

Look at figure 12.35 and 12.36

Are the lines given in figure 12.35 and 12.36 transversal?

In figure 12.35, line n, line and m intersect at one point, therefore these lines are concurrent lines.

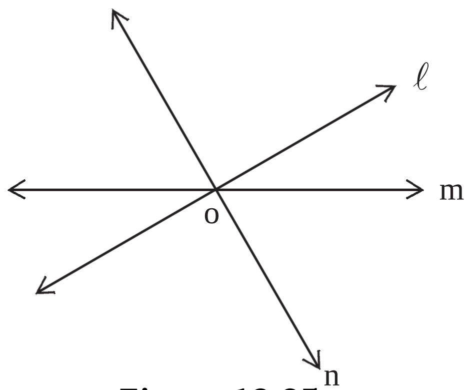


Figure 12.35

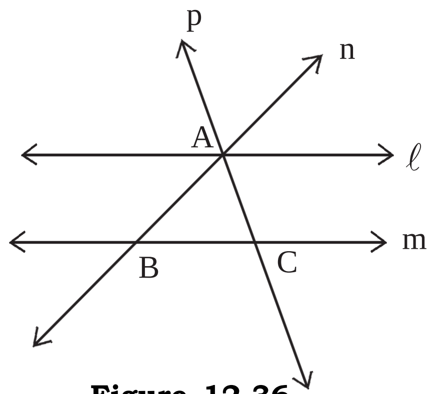


Figure 12.36

In figure 12.36, line p does not intersect the three lines , m and n at three different points, thus the line p is not transversal to , m and n. But line p is a transversal line to and n. Similarly, the line n is a transversal line for and m.

Name the concurrent lines in figure 12.36

Angles made by transversal line on two lines

In figure 12.37, ℓ and m are two lines and n is a transversal line because it intersects lines ℓ and m at two different points A and B respectively.

The line n in fig. 12.37 makes 4 angles at the point A with the line ℓ.

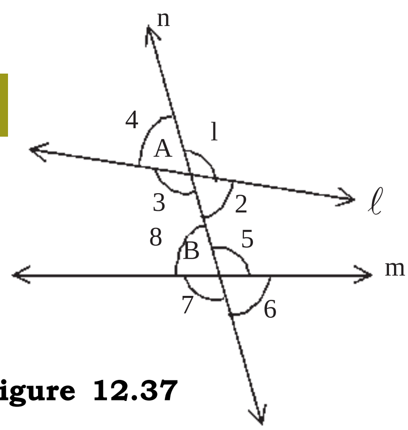


Figure 12.37

The line n also makes 4 angles at the point B with the line m.

Thus a line makes 8 angles on any two lines to which it is transversal, Angles are shown as ∠1, ∠2, ∠3, ∠4, ∠5, ∠6, 7 and ∠8.

Exterior angles and Interior angles

In figure 12.38 angles ∠1, ∠4, ∠6 and ∠7 are made on the outer side of line ℓ and m, therefore these angles are called exterior angles. The exterior angles are not those formed with the line segment AB trapped between the lines ℓ and m.

In figure 12.39 ∠2, ∠3, ∠5 and ∠8 are interior angles as all these angles are made with line segment AB lying between and m.

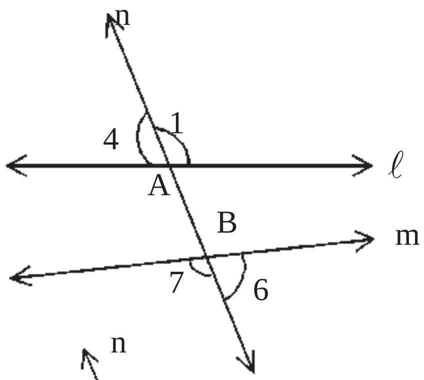


Figure 12.38

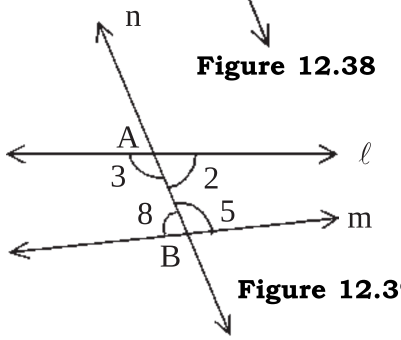


Figure 12.39

Activity 5

Identify the transversal line, interior angels and the exterior angles of the following figures and write in the table.

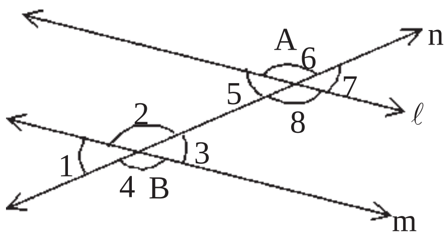


Figure 12.40

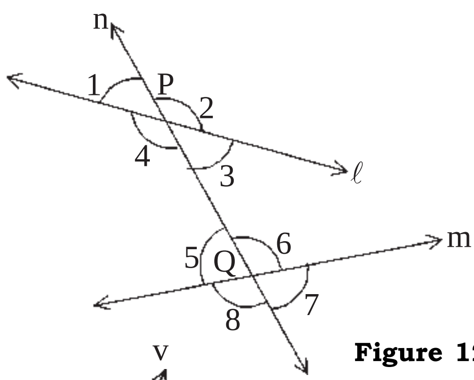


Figure 12.41

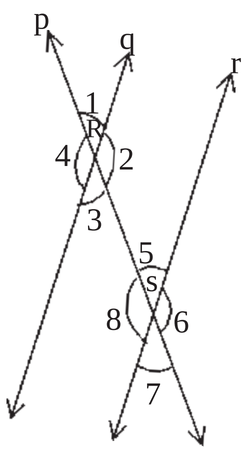


Figure 12.42

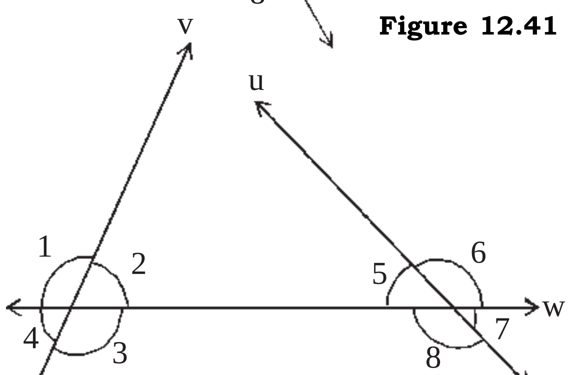


Figure 12.43

Table 3

Figure No.	Transversal line	Exterior angles	Interior angles
12.40	Line n	∠1, ∠4,∠6,∠7	∠2,∠3,∠5,∠8
12.41	....	....	....
12.42	....	....	....
12.43	....	....	....

We observe from the above examples that on each side of the transversal line 2 exterior and 2 interior angles are formed. In the same way on the other side of the transversal line also 2 exterior and 2 interior angles are formed.

Activity 6

Look at the figure given below and find out the answers of following questions.

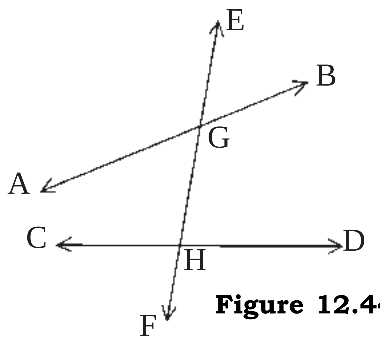


Figure 12.44

Q1. Write the exterior angles on the right of EF in fig. 12.44

(i) ___________________ (ii) _____________

Q2. Write the interior angles to the right of EF.

(i) ___________________ (ii) _____________

Q3. Write the exterior angles to the left of EF

(i) ___________________ (ii) _____________

Q4. Write the interior angles to the left of EF.

(i) ___________________ (ii) _____________

Q5. Write the pairs of the exterior angles on the right and the left of EF such that they are on opposite sides of the transversal line but are not adjacent. For example - ∠EGB and ∠CHF are exterior angles and they are on opposite side of the transversal line and not adjacent. ∠ AGE and ∠ ___________ are similar.

Q6. Make the pair of interior angles on the right and left of EF such that they are not adjacent but are opposite to each other.

(i) ∠___________________ and _____________

(ii) ∠___________________ and ∠_____________

Thus the pair of exterior angle that is on opposite side of the transversal line and not adjacent is called **exterior alternate angles**.

Similarly, the pair of interior angles that is on opposite side to the transversal line but not adjacent is called **interior alternate angle**.

Write the exterior alternate angles and the interior alternate angles in fig. 12.40 to fig. 12.43.

Corresponding Angels

You know that when a transversal line cuts two straight lines it makes 8 angles (4 angles are on one side of the transversal line and 4 angles are on the other side of the transversal line).

Like in figure 12.45, ∠1, ∠2, ∠5 and ∠6 are on right side of transversal line and ∠4, ∠3, ∠8 and ∠7 are on left side of transversal line

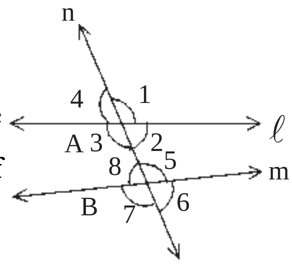


Figure 12.45

Angles formed on one side of the transversal line that are either both of the upper side of the two lines or both on the lower side of the two lines are called **corresponding angles**.

In figure 12.45 the angles ∠1 and 5 are on the right side of transversal line n. These are both on upper side line ℓ and m respectively. These are corresponding angles.

Similarly angles ∠2 and ∠6 formed on the right side of the line n lie below the lines ℓ and m respectively. These are also corresponding angles.

Write the pairs of corresponding angles on the left side of line n.

Activity 7

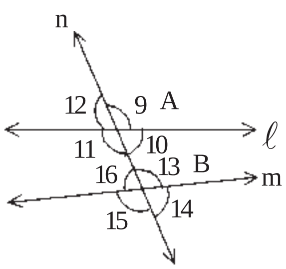


Figure 12.46

Write 4 pairs of the corresponding angles in figure 12.46

(i) ___________ and ____________

(ii) ___________ and ____________

(iii) ___________ and ____________

(iv) ___________ and ____________

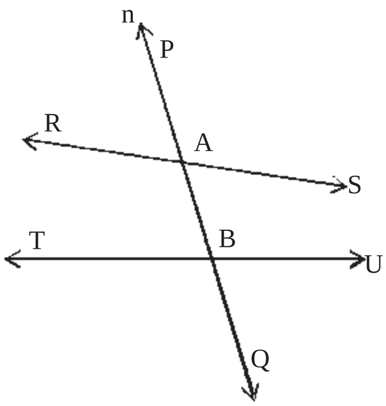


Figure 12.47

Write 4 pairs of corresponding angles in figure 12.47

(i) ∠PAS and ∠ABU

(ii) ∠______ and ∠________

(iii) ∠______ and ∠________

(iv) ∠______ and ∠________

We have seen that **the corresponding angles are formed on one side of the transversal line. From these one is an exterior angle and the other one is an interior angle and these angles are not formed at the same point.**

Activity 8

Write in table pairs of the corresponding angles in figure 12.48 and 12.49.

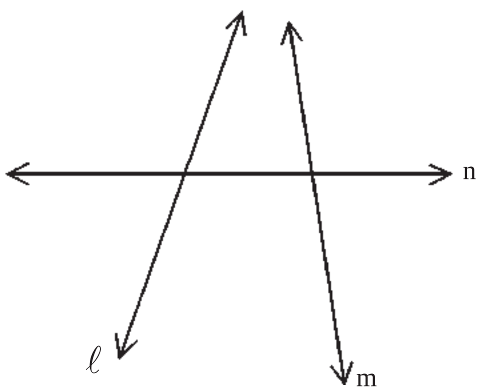


Figure 12.48

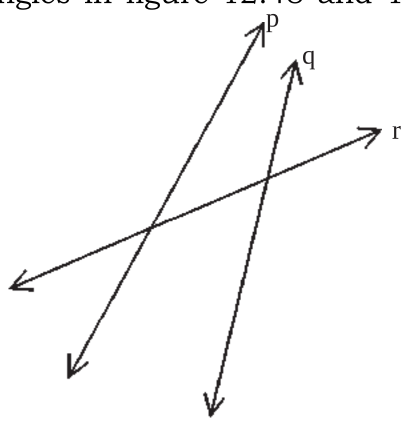


Figure 12.49

Table 4

Figure No.	Pairs of corresponding angles
Figure 12.48	(i)........, (ii).........., (iii).........., (iv)...........
Figure 12.49	(i)........, (ii).........., (iii).........., (iv)...........

Sum of the interior angles

A transversal line makes four interior angles on the two straight lines. Thus we get 2 pairs of interior angles. Lets us see in the following figures.

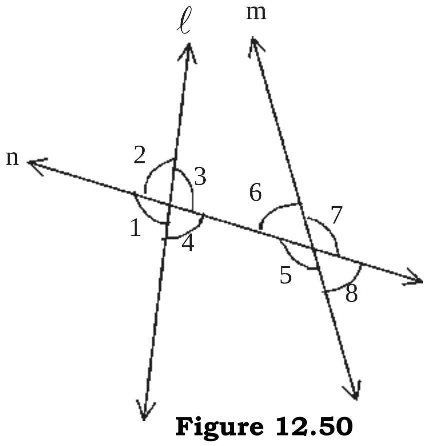


Figure 12.50

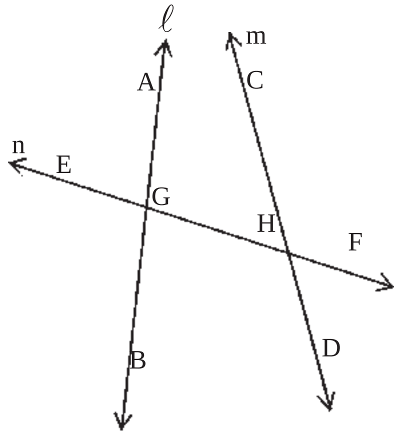


Figure 12.51

In figure 12.50, one of the pair of interior angles is ∠3 and ∠6.

These two angles are situated on one side of the transversal line. Similarly, the other pair of interior angle is ∠4 and ∠5 which are on the other side of the transversal line.

Similarly, find the pairs of interior angles of figure 12.51 and write them in the blanks (i) ____________, ________ (ii) ______, _________

Thus the pairs of the interior angles are formed on the same side of the transversal line but are not made at the same point.

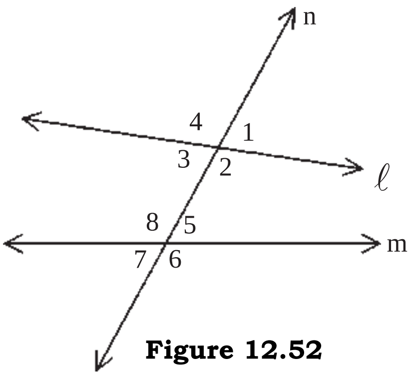


Figure 12.52

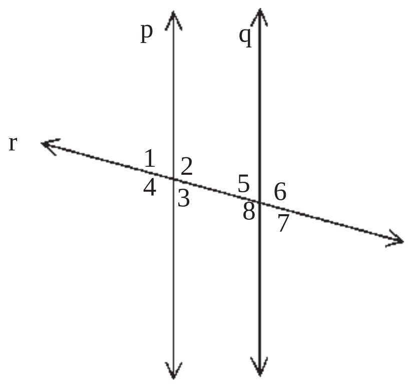


Figure 12.53

In figures 12.52 and 12.53 measure the interior angles on one side of the transversal line with help of protractor 'D' and find their sum too.

Figure 12.52 ∠2__________, ∠5 __________, ∠2+ ∠5 = __________

∠3__________, ∠8 __________, ∠3+ ∠8 = __________

Figure 12.53 ∠2__________, ∠5 __________, ∠2+ ∠5 = __________

∠3__________, ∠8 __________, ∠3+ ∠8 = __________

Is the sum in figure 12.52 equal to the sum in figure 12.53? If yes, why?

Activity 9

Find the pairs of corresponding angles, alternate angles and the interior angles in the given figures and fill up the table.

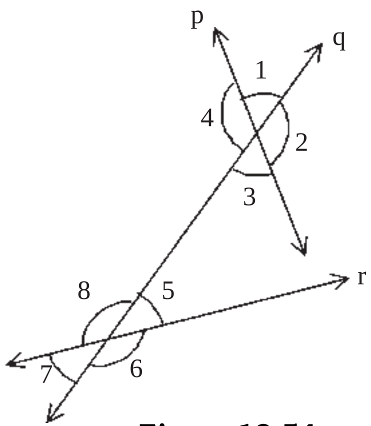


Figure 12.54

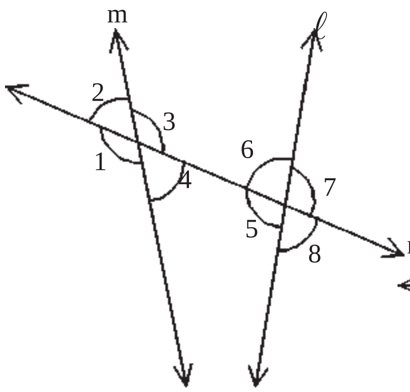


Figure 12.55

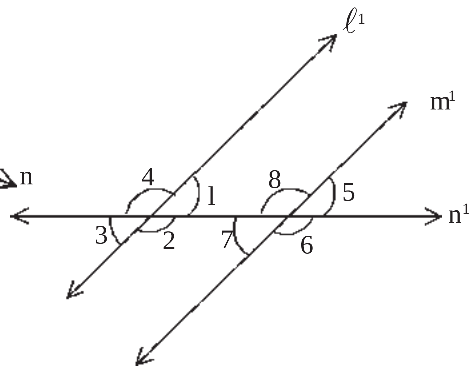


Figure 12.56

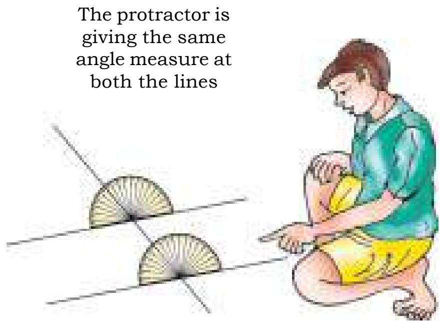
Table 5

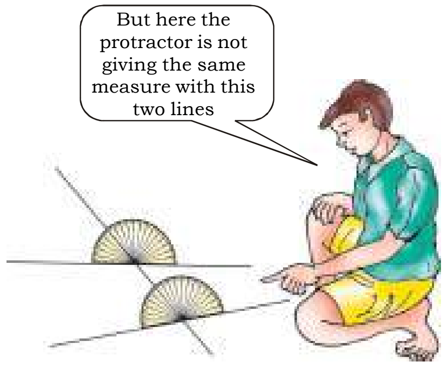
Figure No.	Pair of corresponding angles	Pair of Alternate angles		Pairs of interior angles
		Interior alternate angles	Exterior alternate angles	
12.54	(i) $\angle 1$ and $\angle 8$ (ii)....... (iii) .... (iv).......	(i) .... (ii)	(i) $\angle 1$ & $\angle 6$ (ii)...........	(i) $\angle 3$ & $\angle 5$ (ii).........
12.55	(i) .... (ii)....... (iii) .... (iv).......	(i) $\angle 3$ and $\angle 5$ (ii)	(i)........... (ii)...........	(i)......... (ii).........
12.56	(i) .... (ii)....... (iii) .... (iv).......	(i) .... (ii)	(i)........... (ii)...........	(i)......... (ii).........

Parallel lines and the transversal line

So far we have learnt about the corresponding angles and the pairs of exterior and interior alternate angles formed when a transversal line cuts two straight lines.

When a transversal line cuts two parallel lines we get corresponding angles, alternate angles and interior angles. Let us measure these pairs of angles and learn the special character of these angles.





Activity 10

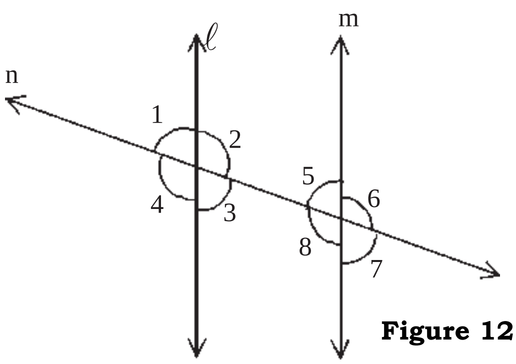


Figure 12.57

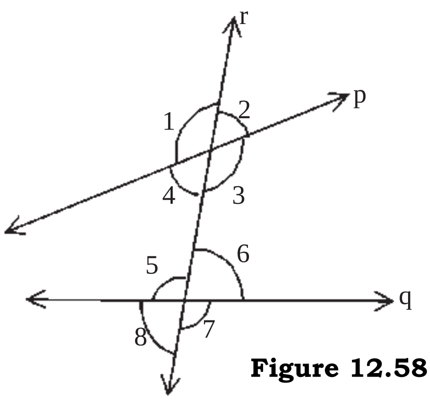


Figure 12.58

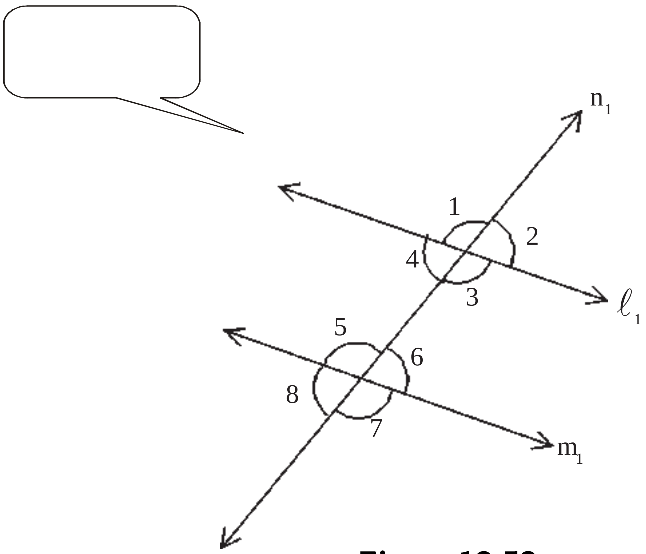


Figure 12.59

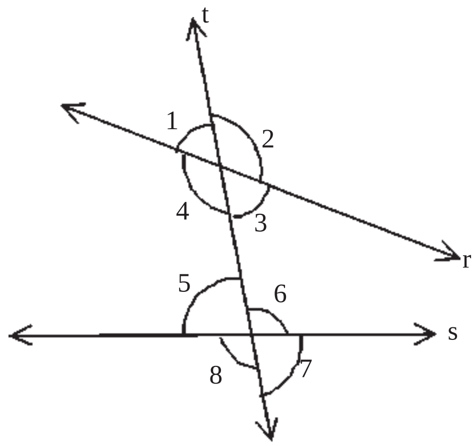


Figure 12.60

Measure each angle of the above figures and fill up table 4

Table 6

Fig.No.	Corresponding angles							
	First pair		Second pair		Third pair		Fourth pair	
12.57	∠1=.....,	∠5=.....	∠2=.....	∠6=.....	∠3=.....	∠7=....	∠4=....	∠8=....
12.58	∠1= .....,	∠5=.....	∠2=.....	∠6=.....	∠3=.....	∠7=....	∠4=....	∠8=....
12.59	∠1=.....,	∠5=.....	∠2=.....	∠6=.....	∠3=.....	∠7=....	∠4=....	∠8=....
12.60	∠1=.....,	∠5=.....	∠2=.....	∠6=.....	∠3=.....	∠7=....	∠4=....	∠8=....

Look at the table 6 carefully and then write the figure numbers in which the corresponding angles are equal _______, _________, ______, _______

Consider the lines of those figures in which corresponding angles are equal. Is there anything special about these lines?

Yes! You are right. In figure 12.57 and in figure 12.59, the transversal line cuts lines that are parallel.

Can we say that the corresponding angles made by the transversal line on parallel lines are equal?

Let us draw some more parallel lines and a transversal line to these and check this.

Activity 11 (i)

If the given lines are parallel than draw a transversal line and check if the corresponding angles are equal.

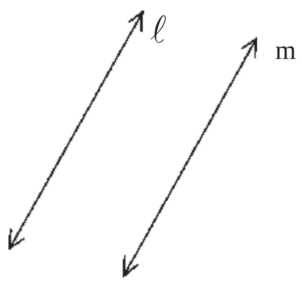


Figure 12.61

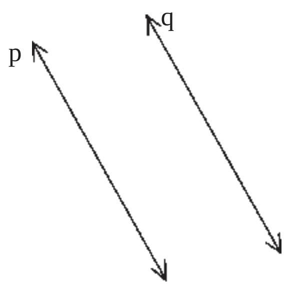


Figure 12.62

Activity 11 (ii)

Which of the lines in figure 12.63 and 12.64 are parallel? Why do you say that they are parallel? Write reasons.

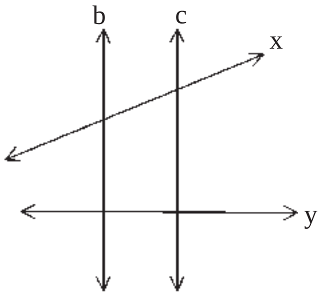


Figure 12.63

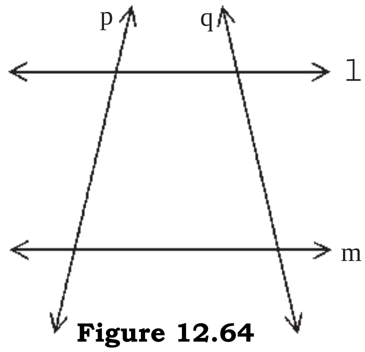


Figure 12.64

Activity 12

Measure the angles of the given figures and fill up the table.

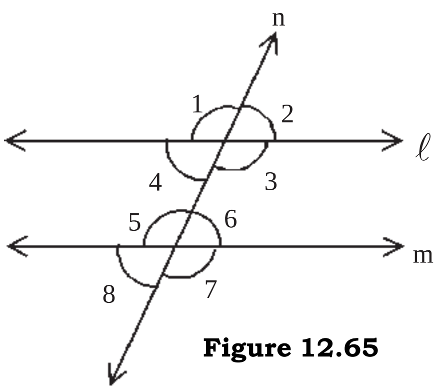


Figure 12.65

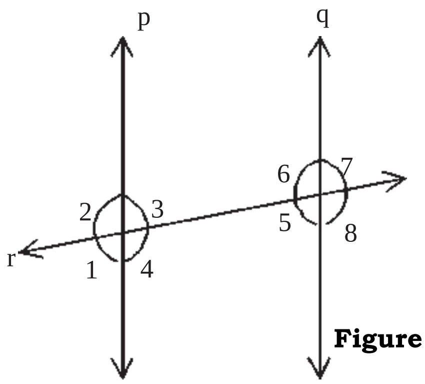


Figure 12.66

Table 7

Figure no.	Pair of alternate angles		Interior angle pair	
	Exterior alternate ∠s	Interior alternate ∠s	Measure	Sum
12.65	∠1=.......,∠7=....... ∠2=.......,∠8=.......	∠3=......., ∠5=....... ∠4=......., ∠6=.......	∠3 +∠6=.......+....... ∠4 +∠5=.......+.......	
12.66	∠1=.......,∠7=....... ∠2=.......,∠8=.......	∠3=......., ∠5=....... ∠4=......., ∠6=.......	∠3 +∠6=.......+....... ∠4 +∠5=.......+.......	

What is the similarity between the value of the sum of the pairs of alternate angles?

Are the pairs of exterior alternate angles equal?

Similarly, are the pairs of interior alternate angles also equal?

Can we conclude that when a transversal line cuts the two parallel lines, the alternate angels are equal?

Draw more such parallel lines and write the pair of alternate angles with their measures.

Can we say that if two alternate angles formed by the same transversal with two straight lines are equal, the given straight lines are parallel?

Is the sum of the pair of interior angles equal in the above table?

The measures of angles are different but the sum of the angles is 180°. We got a similar value in figure 12.53.

Can we conclude that whenever a transversal line cuts two parallel lines, the sum of the interior angles on one side of the transversal line is180°?

Example 1. In figure 12.67, ℓ || m and $\angle 3$ = 65°, find the remaining angles.

Solution: Given $\angle 3 = 65°$

$\angle 5 = 65°$ ($\angle 3=65°$, interior alternate angle)

$\angle 7=65°$ ($\angle 5=\angle 7$, vertical opposite angles)

$\angle 1=65°$ ($\angle 5=\angle 1$, corresponding angles)

Since $\angle 3+\angle 6 = 180°$ (sum of interior angles)

$65°+\angle 6=180°$

$\angle 6=180-65°$

$\angle 6=115°$

$\angle 8=115°$ ($\angle 6=\angle 8$ vertical opposite angles)

$\angle 4=115°$ ($\angle 8=\angle 4$ corresponding angles)

$\angle 2=\angle 115°$ ($\angle 6=\angle 2$ corresponding angles)

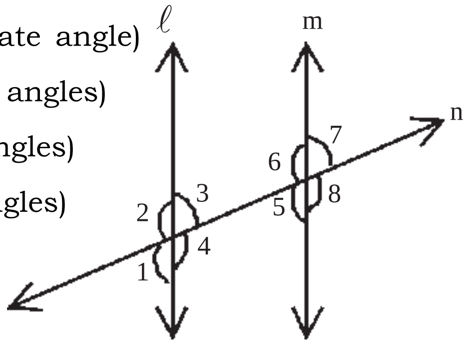


Figure 12.67

Example 2. In figure 12.68, $\angle 1=45°$ and $\angle 2=45°$, prove that segment AD is parallel to segment BC.

Solution: According to the figure, $\angle 1$ and $\angle 2$ are interior alternate angles And $\angle 1 = \angle 2 = 45°$

Thus AD is parallel to BC or AD || BC which is the required result.

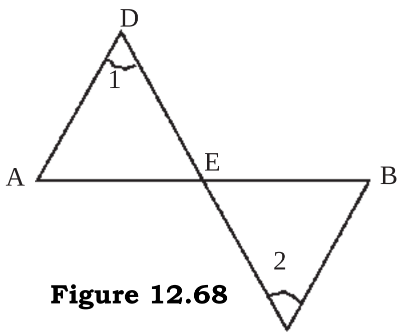


Figure 12.68

Example 3: In figure 12.69, $\angle 3=35°$ and $\angle 4=40°$. Are the lines and m parallel? Give reasons.

Solution: In figure 12.69, $\angle 3$ and $\angle 4$ are corresponding angles. Since $\angle 3 \neq \angle 4$, therefore $\ell$and m are not parallel.

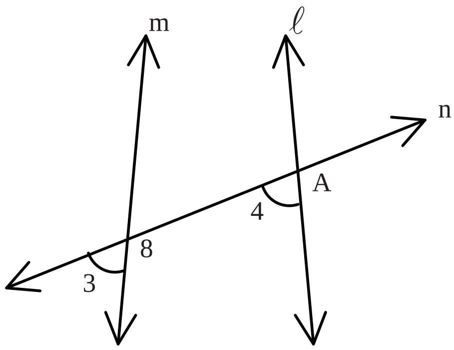


Figure 12.69

Example 4 : In figure 12.70, it is given that $\ell||m$ and $p||q$ find $\angle x$, $\angle y$ and $\angle z$.

Solution: Since $p||q$ and is a transversal line and one angle is given = 70°.

$\therefore$ $\angle x=70°$ (Interior alternate angle)

Since || m and q is a transversal line

$\therefore$ $\angle y=70°$ (corresponding angle)

Since ||m and p is a transversal line

$\therefore \angle z = \angle x = 70°$ (corresponding angle)

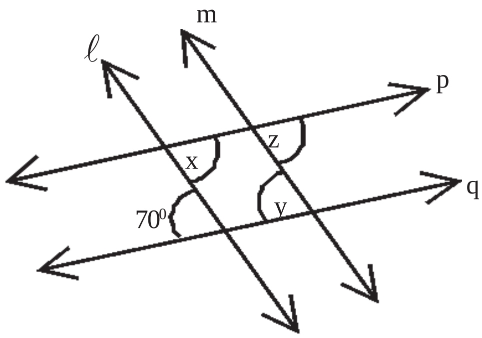


Figure 12.70

Exercise 12.2

1. Fill up the following blanks.

   (i) If the alternate angles are equal, the two straight lines are ______.

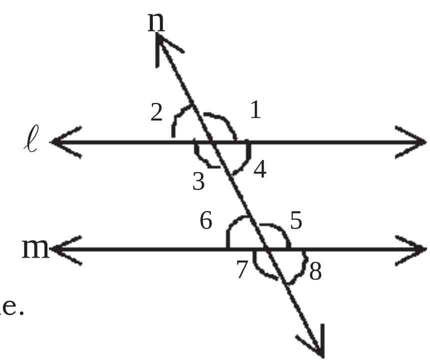
   (ii) If a transversal line cuts two parallel lines, the corresponding angles are ____________.

   (iii) If one of the angles of a pair of alternate angles is 127°, the other angle is ____.

   (iv) If one angle of a pair of interior angles is 87.5°, the second angle is ________.

   (v) If three straight lines intersect at one point, then straight lines are_____.

2. In the given figure ||m and n is the transversal line, find the statements that are true;



   (i) If $\angle 2=60°$, $\angle 4=60°$

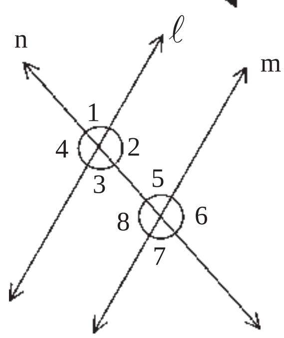
   (ii) If $\angle 2=60°$, $\angle 3=60°$

   (iii) If $\angle 2=60°$, $\angle 6=60°$

   (iv) If $\angle 2=60°$, $\angle 8=60°$

3. In the given figure $\ell||m$ and n is a transversal line.

   Answer the following



   (i) Write all the pairs of alternate angles.

   (ii) Write the exterior angles.

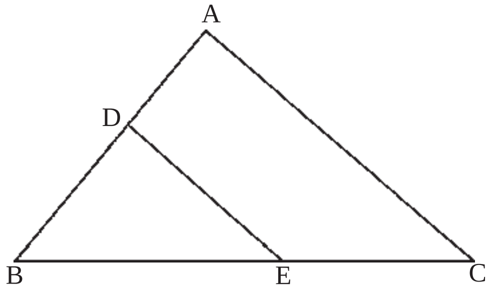
   (iii) Write all the interior angles.

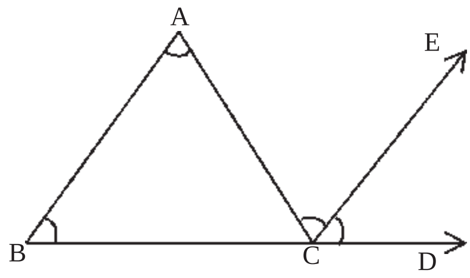
   (iv) Write all the pairs of corresponding angles.

   (v) Write all pairs of interior angles.

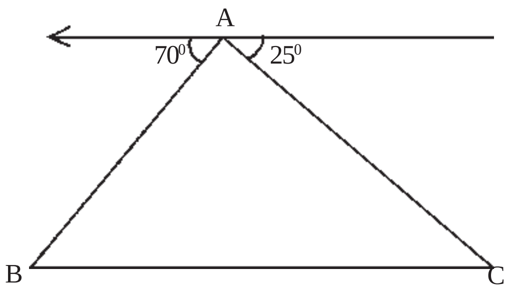
   (vi) If $\angle 5 = 70°$, find the remaining angles.

4. Identify the pairs of parallel lines and the transversal line in the given figure.

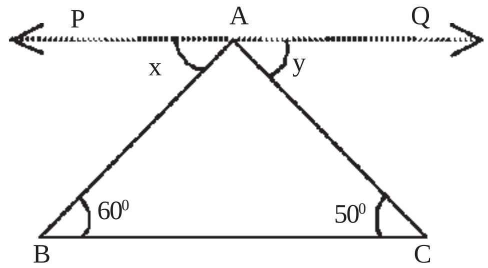




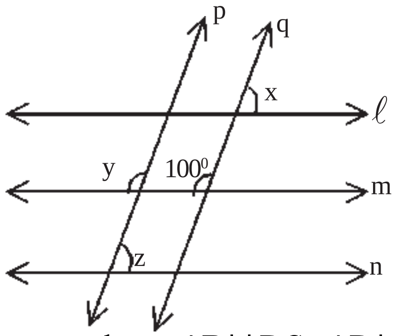
5. Find ∠ABC and ∠ACB in the figure given below if AE||BC.



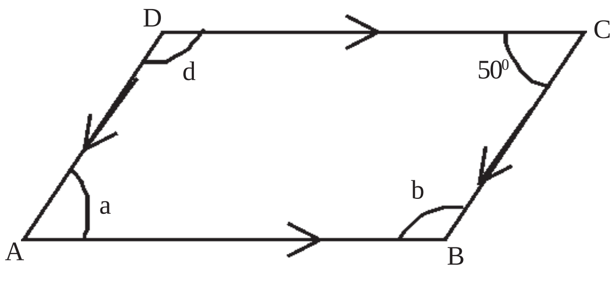
6. In Δ ABC, PQ||BC. Find the value of x and y.



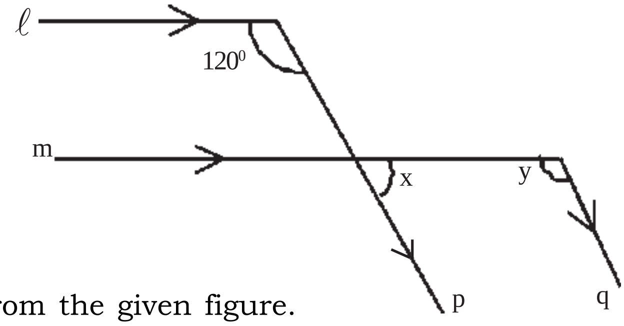
7. In the given figure ||m||n and p||q, find x, y and z.



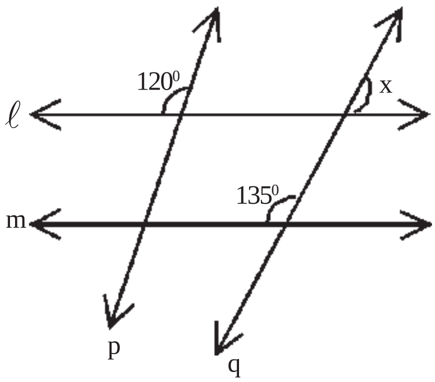
8. In the following figure, where AB||DC, AD|| BC, one angle is given, find a, b and d.



9. Find x and y from the following figure, given ||m, p||q.



10. If ||m, find x from the given figure.



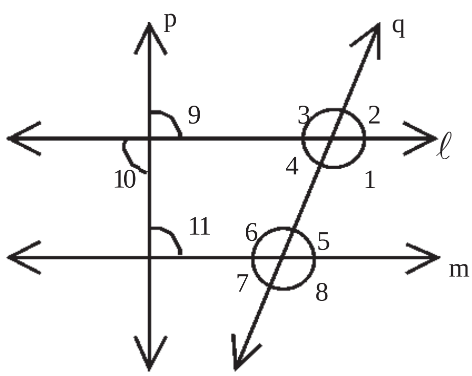
11. ||m and p and q are transversal lines on them.

Find the sum of the following angles from the figure.

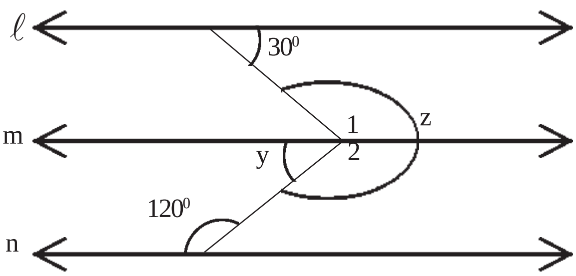
(i) ∠1 + ∠5

(ii) ∠3 + ∠5

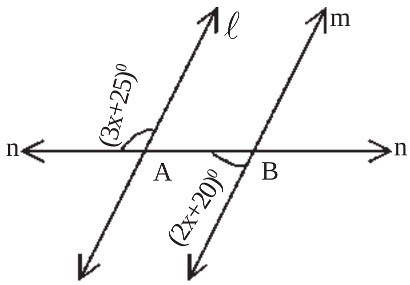
(iii) If ∠11 = 90°, find ∠10 and ∠9



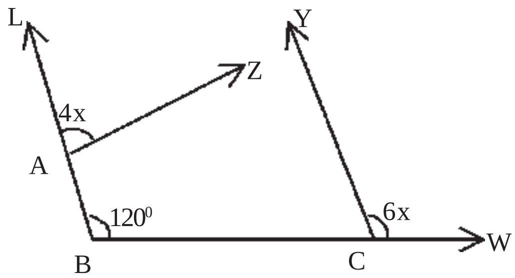
Q12 In the given figure ℓ||m||n, find ∠y and ∠2 while ∠z=∠1+∠2.



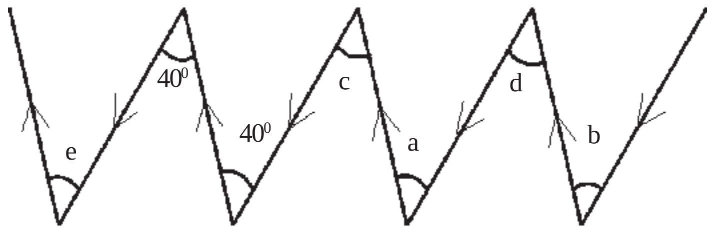
Q13. l || m, find x from the given figure.



Q14. BL || CY, Find A from the given figure.



Q15. Find a, b, c, d and e from the given figure where lines with arrow.. in the same direction are parallel.



We have learnt

1. Adjacent angles - The angle formed opposite to a common arm on a common vertex are called adjacent angles.
2. Linear pairs are specific type of adjacent angles. The arms other than the common arm in such angles are in a straight line. The pairs of angles are formed on the same side of the straight line.
3. Complementory angles -When the sum of two angles is 90°, then one angle is complement to the other.
4. Supplementary angles - If the sum of two angles equals 180°, then each of the two angles is supplement to the other.

5. A line which cuts two or more than two lines at different points is called a transversal line.

6. A transversal line makes 8 angles on two straight lines. Four of the angles are interior and four of the angles are exterior.

7. When a transversal line cuts two lines, we get 4 pairs of corresponding angles, 2 pairs of exterior alternate angles and 2 pairs of interior alternate angles.

8. The lines which would not meet each other even after extension are called parallel lines.

9. The perpendicular distance between two parallel lines is always the same.

10. If a transversal line cuts two parallel lines,

    (i) the corresponding angles are equal

    (ii) alternate angles are equal.

    (iii) Sum of the interior angles formed on one side of the transversal line are supplementary that is there sum is 180°

11. If a transversal line cuts two lines and any one of the following statement is true;

    (i) one pair of corresponding angles are equal

    (ii) one pair of alternate angles are equal

    (iii) sum of the interior angles on one side of the transversal is 180°, then the two lines are parallel.

Chapter -13

QUADRILATERAL

You know about triangles. Everyday you see shapes like a black board, football playground, Kabaddi playground and pages of your copies, books etc. How many sides do each of these have? Where else have you seen shapes like these? Write down more names.

Choose figures like these from among the following: -

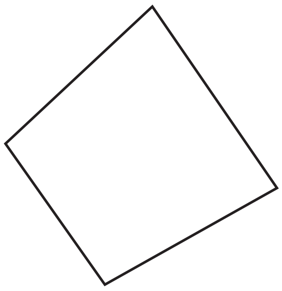


Figure 13.1

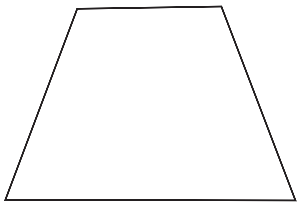


Figure 13.2

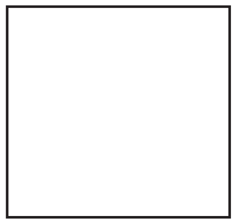


Figure 13.2

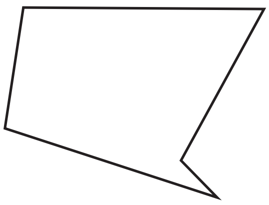


Figure 13.4

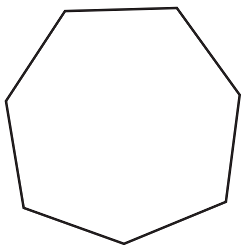


Figure 13.5

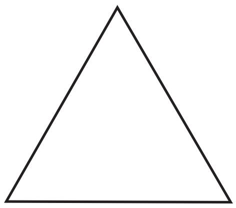


Figure 13.6

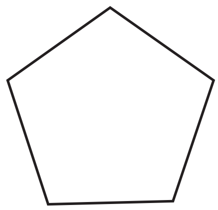


Figure 13.7

You have selected the four cornered shapes from the figures above. These shapes have 4 sides and are therefore called Quadrilaterals.

Some more shapes each of which, is formed by joining four sides are given below. Are all these quadrilaterals? If not, then think of the reason for each of your answers?

Figure 13.8

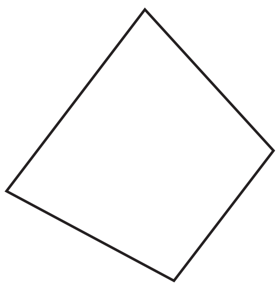


Figure 13.9

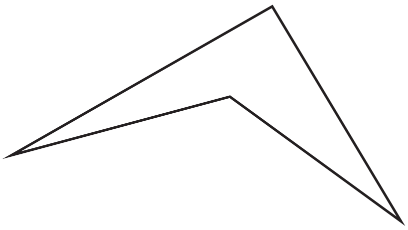


Figure 13.10

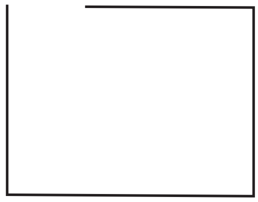


Figure 13.11

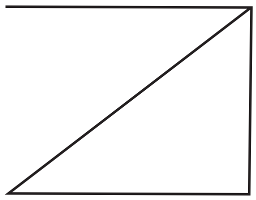


Figure 13.12

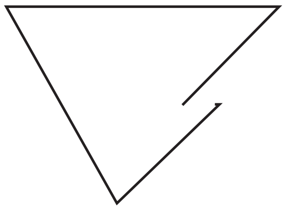


Figure 13.13

You can observe that figures 13.8, 13.9 and 13.10 are closed shapes enclosed by four sides and the enclosed region has four angles. All these are therefore Quadrilaterals.

Figures 13.11, 13.12 and 13.13 are not closed shapes and, therefore are not quadrilaterals. **In this way, we say "Closed shapes having four sides where four angles are formed are called quadrilaterals".**

PARTS OF A QUADRILATERAL

In the quadrilateral ABCD, AB, BC, CD and DA are the four sides and A, B, C, D are the four vertices. Every vertex is formed by the joining of two sides and at every vertex the two sides form one interior angle. In this way four interior angles are formed. These are $\angle BAD$, $\angle ADC$, $\angle DCB$ and $\angle CBA$ respectively.

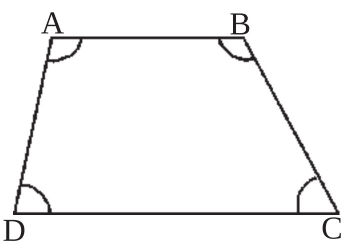
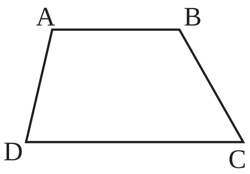
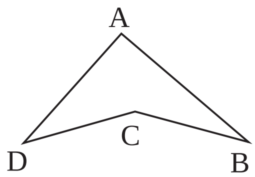
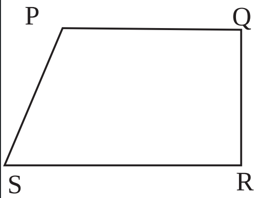


Figure 13.14

Activity 1

In the figures below, identify the sides, angles & vertices and write them at the appropriate places.

Figure No.	Figure	Vertices	Sides	Angles
13.15		(i) A (ii) B (iii) C (iv) D	(i) AB (ii) BC (iii) CD (iv) DA	(i) ADC or CDA (ii) DCB or BCD (iii) CBA or ABC (iv) BAD or DAB
13.16		(i)______ (ii)_____ (iii)_____ (iv)_____	(i)______ (ii)_____ (iii)_____ (iv)_____	(i)______ (ii)_____ (iii)_____ (iv)_____
13.17		(i)______ (ii)_____ (iii)_____ (iv)_____	(i)______ (ii)_____ (iii)_____ (iv)_____	(i)______ (ii)_____ (iii)_____ (iv)_____

Interior Region and Exterior Regions of a Quadrilateral

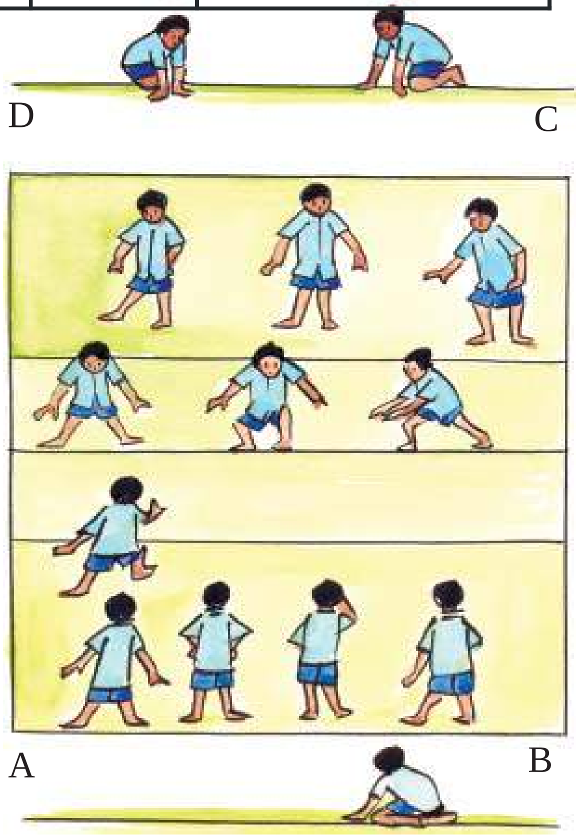
We are familiar with Kabaddi grounds. The adjacent figure shows players playing Kabaddi. Can you tell the number of players in the ground?

We can see in the picture that some players are outside the ground. They are 3 in number.

Is the Kabaddi ground ABCD a quadrilateral?

In the adjacent figure, the region inside the boundary of the quadrilateral is called the interior region of the quadrilateral. In figure 13.18, points P and Q are shown in the interior region of the quadrilateral.

The part of the plane (ground), outside the quadrilateral, is called **exterior region** of the



Kabbadi Ground

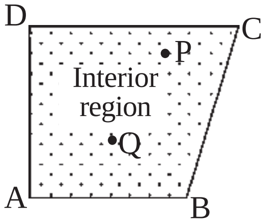


Figure 13.18

quadrilateral. In figure 13.19 points R and S are in the exterior region of the quadrilateral.

Numbers, letters etc written on any page of your book, are located in which region of the page?

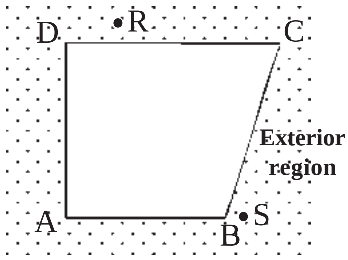


Figure 13.19

ADJACENT SIDES AND OPPOSITE SIDES

In figure 13.20, you can see that the sides SP and QP are meeting at the vertex P. Similarly, the sides PQ and RQ meet at the vertex Q.

The sides of a quadrilateral, that meet each other at a point (vertex), are called **adjacent sides**. Here RS and PS are adjacent sides, which meet at the vertex S. Write the name of the adjacent sides, which meet at the vertices Q and R.

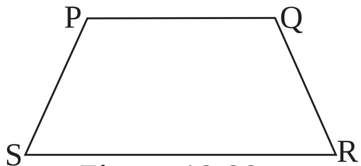


Figure 13.20

In figure 13.20, sides PQ and RS do not meet; therefore these sides are known as opposite sides. In figure 13.20 write the second pair of opposite sides.

Activity 2

Identify the pairs of adjacent sides in the following figures and write them along with their vertices in the following table-

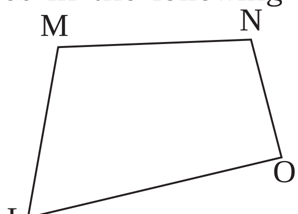


Figure 13.21

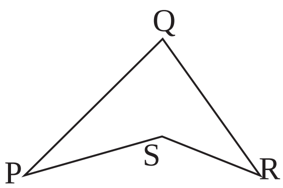


Figure 13.22

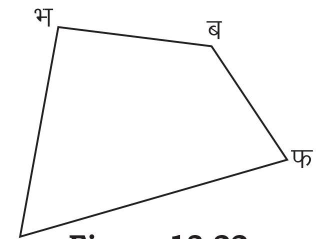


Figure 13.22

Figure No.	Adjacent sides	Vertices	Opposite side
13.21	(i) .... (ii) .... (iii) .... (iv)	(i) .... (ii) .... (iii) .... (iv)	(i) .... (ii) .... (iii) .... (iv) ....
13.22	(i) .... (ii) .... (iii) .... (iv)	(i) .... (ii) .... (iii) .... (iv)	(i) .... (ii) .... (iii) .... (iv) ....
13.23	(i) .... (ii) .... (iii) .... (iv)	(i) .... (ii) .... (iii) .... (iv)	(i) .... (ii) .... (iii) .... (iv) ....

Adjacent angles and Opposite angles

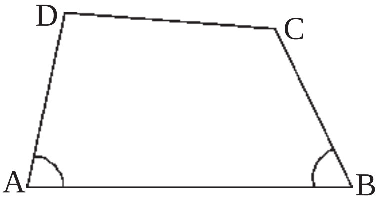


Figure 13.24

We have studied that a quadrilateral has four interior angles. Out of these, two such angles, which are formed by one common side, are called **adjacent angles**.

In figure 13.24, ∠ A is formed by sides DA and AB, and ∠B is formed by sides AB and BC. Here AB is the common side therefore ∠ A and ∠ B are adjacent angles.

Is there any other angle adjacent to ∠ A?

In the same manner, write the adjacent angles of ∠B, ∠C & ∠D.

In the above figure 13.24, ∠B has 2 adjacent angles ∠A & ∠C but ∠ B & ∠ D are not adjacent angles.

Therefore any two angles of a quadrilateral, which are not adjacent are called **opposite angles**.

In figure 13.24, ∠D is the opposite angle of ∠B, and ∠A is the opposite angle of ∠C. Opposite angles face each other.

DIAGONAL OF A QUADRILATERAL AND THE SUM OF THE INTERIOR ANGLES

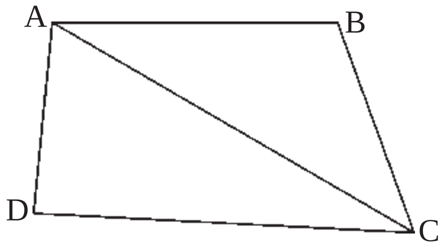


Figure 13.25

ABCD is a quadrilateral. If two opposite vertices of this quadrilateral are joined by a line segment then it gets divided into two triangles.

Line segment AC is the diagonal of the quadrilateral ABCD. This is formed by joining the opposite vertices A and C.

Similarly, the line segment BD will also be a diagonal.

Activity 3

Draw the diagonals in the following quadrilaterals and write their names.

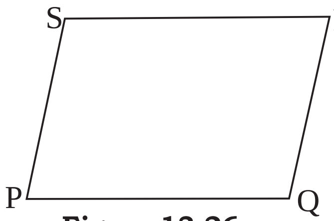


Figure 13.26

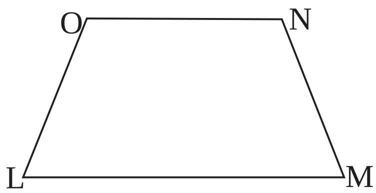


Figure 13.27

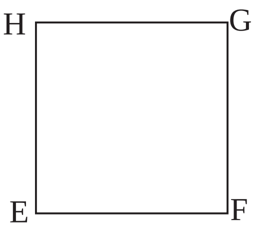


Figure 13.28

(1) __________ (1) __________ (1) __________

(2) __________ (2) __________ (2) __________

You have seen that a diagonal in a quadrilateral divides it into two triangles. In figure 13.27, diagonal AC divides the quadrilateral ABCD into two triangles i.e. ΔABC and ΔADC.

You know that the, sum of the angles of a triangles is 180°.

∴ Sum of the angles of quadrilateral ABCD

= Sum of the angles of ΔABC

+

Sum of the angles of ΔADC

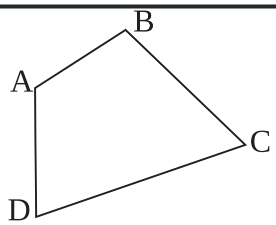
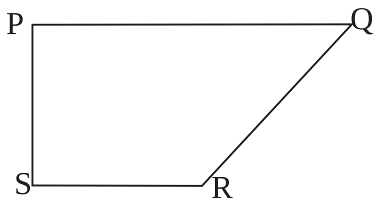
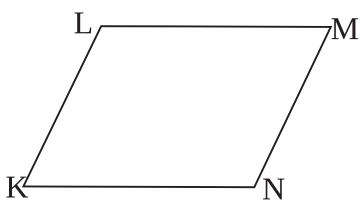
= 180° +180°

= 360°

Thus, the sum of all the four interior angles of a quadrilateral is 360°.

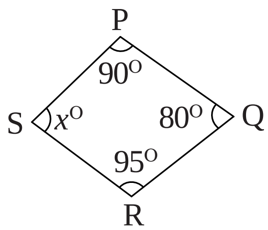
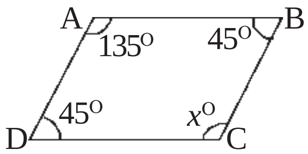
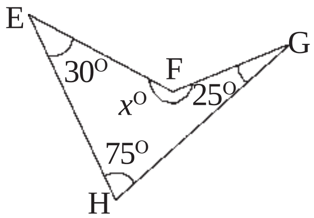
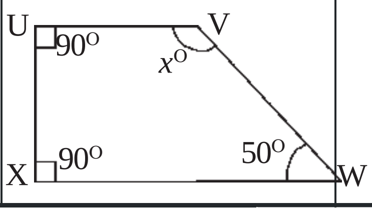
Activity 4

Measure the interior angles of the following figures with the help of a protractor and calculate the sum:-

Fig. Number	Figure	Measure of all the four interior angles	Sum of all the four interior angles
13.29		BAD = ADC = DCB = CBA =	
13.30		QPS = PSR = SRQ = RQP =	
13.31		MLK = LKN = KNM= NML =	

What conclusion can you draw from the above table? Write them in your notebook. Draw more such quadrilaterals and check your conclusion.

In the given quadrilaterals, measure of three interior angles is given, find the fourth angle and fill in the blanks: -

Fig. No.	Quadrilateral	Solution	Obtained angle
13.32		P = 90° Q = 80° R = 95° Thus, S = 360°- (P+Q+R) = 360°-(90°+80°+95) = 360°-265° =95°	95
13.33		A= B= D= Thus, C=360°- (D+B+A)	
13.34		H= G= E=	
13.35		U= X= W=	

EXAMPLE 1: In the quadrilateral ABCD, measure of three angles is equal. If the fourth angle is of the measure 60°, then find the measure of each of the other angles.

SOLUTION: Let ∠ B=∠ C= D=x°

Since ∠ A+∠ B+ C+∠ D=360°

Given that ∠ A =60°

⇒ 60° + x + x + x = 360°

⇒ 60° + 3 x = 360°

⇒ 3x = 360°-60° (Taking 60° to the other side)

$\Rightarrow 3x = 300°$

$\Rightarrow \frac{3x}{3} = \frac{300°}{3} = 100°$ (Dividing both sides by 3)

$x = 100°$

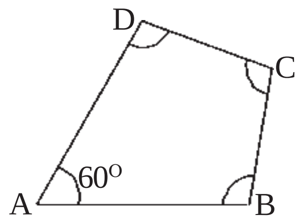


Figure 13.35

Thus, ∠ B = ∠ C = ∠ D = 100°

Example 2: Sum of two angles of a quadrilateral is 150°. If the measure of one of the remaining angles is 130°, find the measure of the fourth angle.

Solution: Given that the sum of two angles = 150°

Sum of remaining two angles = 360°-150° (sum of the angles of a quadrilateral is 360°).

= 210°

Thus, fourth angle = 210°-130°

= 80°

Example 3: Angles of a quadrilateral are in the ratio of 1:2:3:4, determine the measure of all the angles.

Solution: Let the four angles of the quadrilateral be x, 2x, 3x and 4 x respectively.

Sum of the 4 angles of a quadrilateral = 360°

$\Rightarrow x + 2x + 3x + 4x = 360°$

$\Rightarrow 10x = 360°$

$\Rightarrow x = \frac{360°}{10}$

$\Rightarrow$ $x = 36°$

1st angle of the quadrilateral = x = 36°

2nd angle = 2 x = 2 × 36° = 72°

3rd angle = 3x = 3× 36° = 108°

4th angle = 4x = 4× 36° = 144°

Exercise 13.1

Q1. Fill in the blanks-

(i) A quadrilateral has _____ diagonals.

(ii) The diagonal of a quadrilateral divides the quadrilateral into two _____.

(iii)The sum of all the interior angles of a quadrilateral is _______ degrees.

(iv) _______ pair(s) of opposite angle(s) are/is formed in a quadrilateral.

(v) Every quadrilateral has __________ vertices, among which more than __________ vertices cannot lie on a straight line.

Q2. From the given groups of angles which are possible groups for the interior angles of a quadrilateral:

(i) 60°, 70°, 80° and 145° (iii) 75°, 75°, 75°, and 135°

(ii) 102°, 150°, 40° and 68° (iv) 90°, 90°, 90° and 90°.

Q3. Two angles of a quadrilateral are supplementary. If one of the remaining angles is 65°, determine the fourth angle.

Q4. Two angles of a quadrilateral are of 70° each, and the remaining two angles are equal. Determine the measure of the equal angles.

Q5. If all the angles of a quadrilateral are equal, determine the measure of each of them.

Q6. Two angles of a quadrilateral measure 65° and 105° respectively. The remaining two angles are equal. Determine the measure of the equal angles.

Q7. Ratio of the angles of a quadrilateral is 3:5:7:9. Determine the measure of each angle.

Q8. Classify the following as true or false. And correct the false statements.

(i) Sum of the four interior angles of a quadrilateral is four times the measure of a right angle.

(ii) One diagonal of a quadrilateral divides it into four triangles.

(iii) There are four pairs of adjacent angles in a quadrilateral.

(iv) There are four pairs of opposite angles in a quadrilateral.

(v) It is not possible that each angle of a quadrilateral be 90°.

Q9. Three angles of a quadrilateral are of 80° each. Determine the fourth angle.

TYPES OF QUADRILATERALS

Use a scale to take broom sticks of lengths mentioned below. Join them head to head and form quadrilaterals of different shapes.

(i) 8cm, 4cm, 8cm and 4cm.

Following are some of the quadrilaterals so formed by these: -

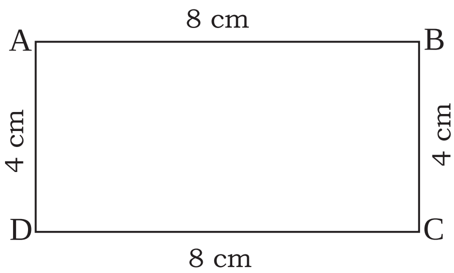


Figure 13.36

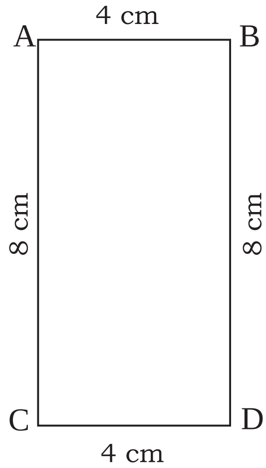


Figure 13.37

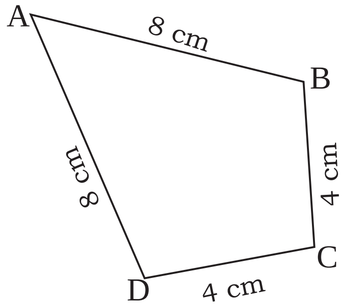


Figure 13.38

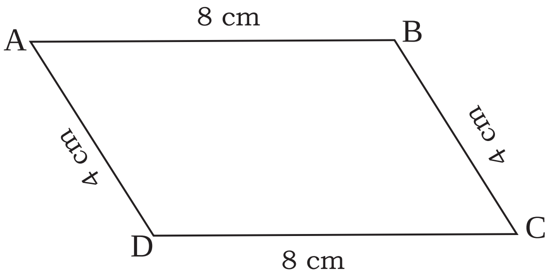


Figure 13.39

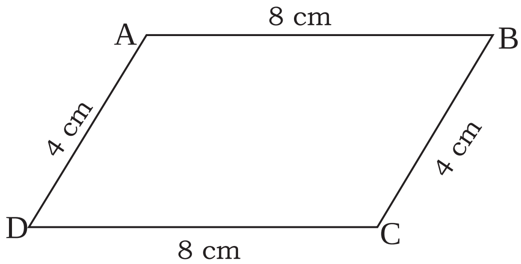


Figure 13.40

Among these figures, 13.36, 13.37, 13.39 and 13.40 have both pairs of opposite sides parallel to each other and equal in length. These are known as parallelograms.

Therefore, those quadrilaterals in which opposite sides are parallel and equal to each other are called parallelograms.

Figures 13.36, 13.37 are parallelograms with all angles of 90° each. These are called rectangles. **Thus, those parallelograms, which have all angles as right angles, are called rectangles.**

In figure 13.38, the opposite sides are neither parallel nor equal. Therefore it is not a parallelogram.

(ii) Take 4 sticks of 4 cm length each and make quadrilaterals: -

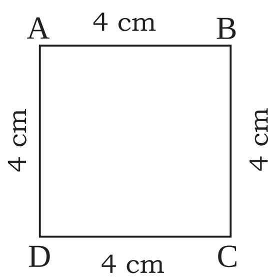


Figure 13.41

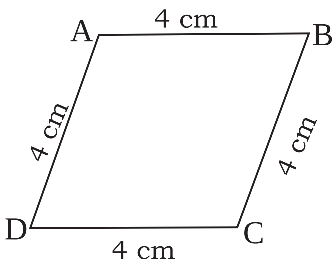


Figure 13.42

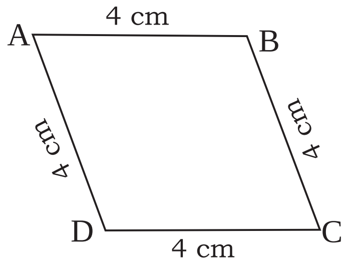


Figure 13.43

Some of the quadrilaterals made by you would be similar to the figures drawn above. Are these quadrilaterals parallelograms?

You will observe that, all the pairs of opposite sides in these figures are parallel and equal. Thus, all these are parallelograms. Since all sides of these quadrilaterals are equal, so they are a special type of parallelogram.

Those parallelograms, which have all sides equal are known as Rhombus.

Figure 13.41 is also a Rhombus. Apart from having all equal sides, this parallelogram has another specialty too. Each angle of this quadrilateral is of 90°.

Such a quadrilateral, which has all equal sides and all angles as right angles, is known as a square. Thus, square is a special type of Rhombus.

(iii) Now take sticks of lengths 3 cm, 4cm, 5 cm and 6 cm respectively, join them head to head and form many - different quadrilaterals. Some of the quadrilaterals formed by you may be of the following types: -

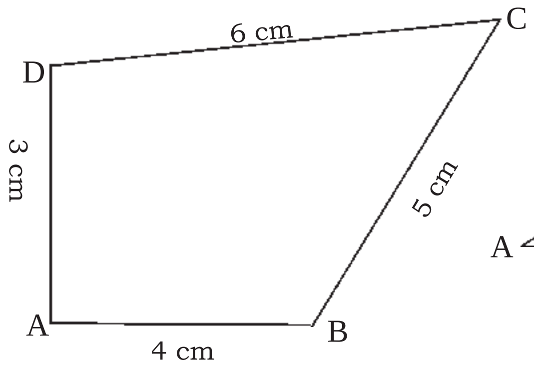


Figure 13.44

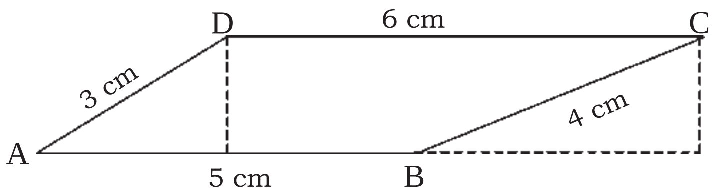


Figure 13.45

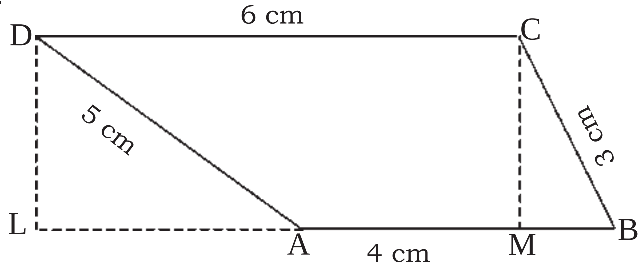


Figure 13.46

With the help of the sticks of specified lengths try and form some more quadrilaterals.

Each side of figure 13.44 is of a different length and opposite sides are not parallel. This is a quadrilateral having all sides of different lengths.

Quadrilaterals shown in figures 13.45 and 13.46 have two of their opposite sides (AB and DC) parallel but of different lengths. These are called Trapeziums. In a trapezium, perpendiculars drawn from the vertex on the opposite parallel side are of equal lengths.

Thus those quadrilaterals, in which only one pair of opposite sides is parallel, are called Trapeziums.

Activity 5

Classify the following figures as rectangles, squares, rhombuses, trapezium and quadrilaterals with all sides of different lengths and fill the table given below:

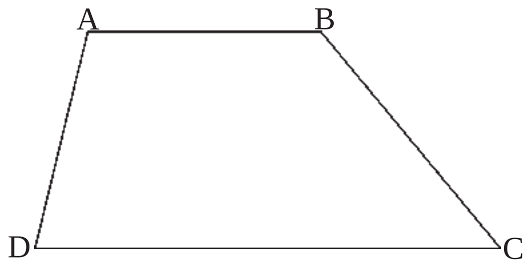


Figure 13.47

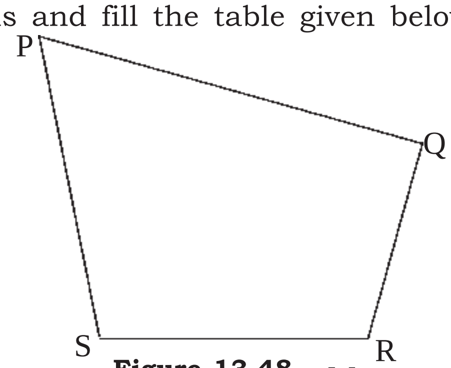


Figure 13.48

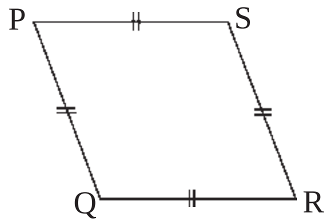


Figure 13.49

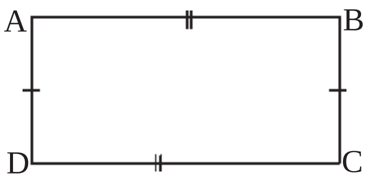


Figure 13.50

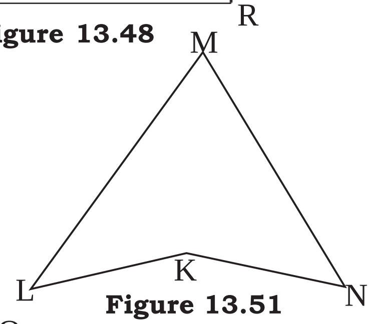


Figure 13.51

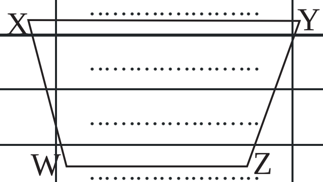


Figure 13.52

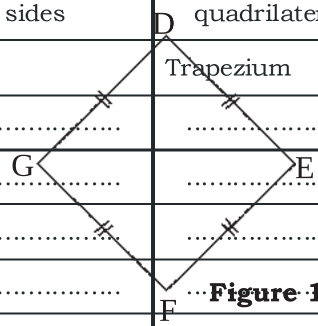


Figure 13.53

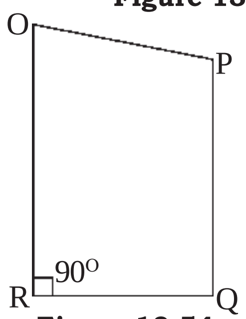


Figure 13.54

Fig. No.	Names of Parallel sides	Names of Equal sides	Type of the quadrilateral
13.47	AB | | DC	None	Trapezium
13.48	....	....	....
13.49	....	....	....
13.50	....	....	....
13.51	....	....	....
13.52	....	....	....
13.53	....	....	....
13.54	....	....	....

Exercise 13.2

Q1. (i) If in a quadrilateral, only one pair of opposite sides is parallel, then such a quadrilateral is called ____________________

(ii) Each angle of a rectangle is of __________degrees

(iii) In a rhombus, opposite sides are _____________ and all four sides are ________________to each other.

(iv) A parallelogram in which each angle is of 90°and all sides are of equal lengths is called ___________________.

(v) A quadrilateral whose all sides are equal is called ________________.

Q2. State true or false and correct the false statements: -

(i) A Rectangle is a parallelogram.

(ii) Every parallelogram is a rectangle.

(iii) Every Rhombus is a square

(iv) Opposite sides of a trapezium are parallel.

Q3. Draw the following quadrilaterals and name them: -

(i) Trapezium (ii) Rectangle

(iii) Square (iv) Parallelogram

We have learnt

1. A closed shape formed by four sides having four interior angles is known as a quadrilateral.
2. There are four vertices, four sides and four angles in a quadrilateral.
3. The line, joining opposite vertices of a quadrilateral is called a diagonal. There are two diagonals in a quadrilateral.
4. Sides of a quadrilateral having one common vertex are called adjacent sides.
5. Sides of a quadrilateral, which do not have any common vertex, are called opposite sides.
6. Interior of the quadrilateral ABCD together with the boundary of the quadrilateral forms the region of quadrilateral ABCD.
7. Sum of all the angles of a quadrilateral is 360°.
8. Opposite sides of a parallelogram are equal and parallel to each other.
9. A parallelogram each of whose angles is of 90° is called a rectangle.
10. A parallelogram in which all the sides are equal is called a rhombus.
11. A quadrilateral in which only one pair of opposite sides is parallel is called a trapezium.
12. A parallelogram each of whose angle is of 90° and all sides are equal, is called a square.

Chapter -14

PROPORTION

Introduction

Shelly had plums and Mishri had grapes. Both decided to divide the plums & grapes between themselves. Shelly gave Mishri 12 plums out of 24 plums she had and Mishri gave her 75 grapes out of 150 grapes she had. Later they had a quarrel because of the distribution because Mishri said that Shelly had given her less plums. Shelly didn't agree with her and went to Nisha auntie to decide the case. Nisha asked Shelly how many plums did she have with her. She answered she had 24. Then Nisha auntie said, "Mishri got 12 out of 24 plums, that means the ratio is 12:12 or 1:1. Similarly Mishri has given 75 grapes out of 150. Shelly and Mishri have both got 75 grapes in the ratio 75:75 or 1:1. Both the ratio are similar which means they are proportional.

In such conditions, we often need to compare ratios also. Let us take some examples:

Example 1.

Ten packets milk cost 150 Rs. & 25 packets cost Rs. 375.

Here the ratio of packets of milk = 10 : 25 = 2 : 5

The ratio of cost of milk packets = 150 : 375 = 2 : 5

Both the ratio are equal.

Example 2.

5 bags of cement cost 550 Rs. and 20 bags of cement cost Rs. 2200.

The ratio of the number of cement bags = 5 : 20 = 1 : 4.

Ratio of the cost of cement bags = 550 : 2200 = 1 : 4

ratio of number of bags = ratio of cost of bags

1 : 4 = 1 : 4

Now two ratios are also equal. So, these ratios are known as equivalent ratio or proportionate ratios.

Is 4 : 5 proportionate to 20 : 25?

Now you also find examlpes where the ratio of two quantities is in proportion to the ratio of two other quantities. Consider some such situations & compare the ratios. When the ratio between two quantities 'a' and 'b' is equal to the ratio between other two quantities 'c' and 'd'. Then a : b = c : d is written as a : b : : c : d.

In this, **: :** is the symbol of proportionateness.

In a : b : : c : d, a and b are known as outer terms and b and c are known as the middle terms, a, b, c and d are first, second, third and fourth terms respectively. Verify whether the ratio of the first & second is equal to the ratio of the third & fourth term.

(i) 1 : 5 and 6 : 30 (ii) 4 : 12 and 18 : 54 (iii) 20 : 10 and 30 : 15

All are the examples above proportionate. In the table below, fill in the blanks on the basis of the law of proportion.

S. No.	Proportionate terms	Product of the outer terms	Product of the middle terms
1.	1 : 2 : : 4 : 8	1 × 8 = 8	2 × 4 = 8
2.	5 : 6 : : 75 : 90	________	________
3.	3 : 4 : : 24 : 32	________	96
4.	2.5 : 2.4 : : 7.5 : 7.2	2.5 × 7.2 = ____	______ = 18
5.	2 : 5 : : 4 : __	________	20

In the above examples, you can see that the product of the outer terms is equal to the product of the middle terms.

In a proportion :

product of middle terms = product of outer terms

Activity 1

Two friends Hamida and Anu went to market to buy kites. They bought 45 kites for Rs. 15. Anu gave Rs. 9 and Hamida gave Rs. 6. When they returned home, Anu divided the kites like this: She said, "Two kites for you and 3 kites for me." Now say :

1. In what ratio is Anu dividing the kites?
2. Hamida thought Anu is not dividing the kites rightly. We must get equal number of kites but Anu said that we have paid for the kite in the ratio 6 : 9 or 2 : 3, so out of 5 kites 2 kites will be yours & 3 will be mine.
   The total number of kites are 45. So, 2 × 9 = 18 kites will be yours and 3 × 9 = 27 kites will be mine.
   Is Anu justified? Give reasons for your answer.

Example 3.

Are 40, 30, 60, 45 proportionate?

We know that

$$40 : 30 = \frac{40}{30} = \frac{4}{3} = 4 : 3$$

$$60 : 45 = \frac{60}{45} = \frac{4}{3} = 4 : 3$$

Therefore, 40 : 30 : : 60 : 45

Hence, 40, 30, 60 & 45 are proportionate or equivalent ratios.

Example 4.

Find out the unknown term in problems of proportion.

8 : ____ : : 7 : 14

In this example, we don't know the second term.

If we write x in its place, the proportion would be :

$8 : x : : 7 : 14$

Since, product of outer terms = product of middle terms

$8 \times 14 = x \times 7$

or $7x = 112$

or $x = \quad = 16$

$\therefore x = 16$

In summer we usually make sweet drinks. We mix sugar in these drinks. If 12 spoons of sugar have been used to make 6 glasses of sweet drinks, it means 2 spoons of sugar have been used for each glass. If we add 3 spoons of sugar to each glass to make it sweeter. Then the ratio of sugar in both types of drinks would be 2 : 3.

Such examples from our everyday life talk about proportions. Think of more such examples. Remember ratios are always exihibited between two equal quantities and equal units only.

Example5

If 100 × 75 = 150 × 50. Verify whether 100, 150 & 50, 75 are proportionate.

Solution :

= 2 : 3

$\frac{50}{75} = \frac{2}{3} = 2 : 3$

$\frac{100}{150} = \frac{2}{3}$ (28/4)

So, it is clear 100 : 150 : : 50 : 75.

Hence, the numbers 100, 150, 50 & 75 are proportionate.

Example 6

The length & breadth of playground in a school is in the ratio 4 : 3. If length is 28 meter, what would be its width?

Solution:

Suppose, the breadth of the playground is x.

4 : 3 and 28 : x are similar ratios.

$\therefore 4 : 3 : : 28 : x$

product of outer terms = product of middle terms

$4 \times x = 3 \times 28$

$x = 3 \times$

x = 21 meters.

Example 7

The ratio of 2 kg of tomatoes is Rs. 16 Find out how much tomatoes will be available for

Rs. 40 ?

Solution :

Suppose, we get x kg of tomatoes for Rs. 40

2 : x and 16 : 40 (2 kg & x kg and Rs. 16 & Rs. 40 are similar quantities)

$\therefore 2 : x : : 16 : 40$

$x \times 16 = 2 \times 40$

x = = 5

Therefore, we shall get 5 kgs of tomatoes for Rs. 40

Example 8

1 : 4 = 8 : 32. How many proportions can be formed by these quantities.

Solution :

The following proportions would be possible:

1) 1 : 4 : : 8 : 32, 1 × 32 = 4 × 8
2) 1 : 8 : : 4 : 32, 1 × 32 = 8 × 4
3) 32 : 8 : : 4 : 1, 32 × 1 = 8 × 4
4) 32 : 4 : : 8 : 1, 32 × 1 = 4 × 8

Can you think of some more equivalent ratios with the above numbers?

Exercise 14.1

1. Apply the rule of proportion and say which of the given statements are true & why?

(i) 10 : 20 : : 300 : 600 _______________

(ii) 38 : 76 : : 250 : 500 _______________

(iii) 22 : 66 : : 66 : 22 _______________

(iv) 24 : 96 : : 16 : 54 _______________

(v) 25 : 65 : : 1 : 3 _______________

(vi) 15 : 30 : : 200 : 400 _______________

(vii) 34 : 136 : : 45 : 180 _______________

(viii) 70 : 350 : : 1 : 4 _______________

(ix) 5 : 25 : : 30 : 150 _______________

(x) 33 : 11 : : 133 : 111 _______________

(xi) 18 : 24 : : 15 : 20 _______________

(xii) 75 : 150 : : 3 : 18 _______________

2. Which of the following groups are proportionate? If they are not proportionate, is it possible to change their order and make them proportionate by rearranging them? For which groups even this can't be done?

(i) 4, 8, 16, 32 (ii) 12, 16, 48, 64 (iii) 4, 6, 18, 12

(iv) 200, 300, 400, 600 (v) 11, 22, 88, 44 (vi) 4, 1, 2, 8

(vii) 25, 15, 3, 5 (viii) 224, 34, 68, 112 (ix) 67, 134, 45, 90

(x) 1, 2, 3, 6 (xi) 5, 7, 9, 13

3. Fill in the blanks with numbers that make the ratios on both side equivalent :

   (i) 32 : _____ = 6 : 12

   (ii) 22 kg : 26 kg = _____ : 260 meters

   (iii) 45 km : 60 km = _____ : 12 hours

4. The cost of 8 kilogram of sugar is 72 rupees. Find out the cost of 15 kilogram of sugar

5. The ratio of the length & width of a playground is 5 :2. Find the length of the playground in meter, if its width is 40 meter ?

6. A man bought 3 copies of a book for Rs. 75. Find out how many copies of books can the person buy for Rs. 300 ?

UNITARY METHOD

The unitary method is a technique which is used for solving a problem by finding the value of a single unit. You have studied about this in previous classes. We can solve problems of equivalent fraction by unitary method.

Example 9 You go to the market and buy two copies for Rs. 20. Now, if you need 5 more copies, how much money should you have?

Two copies cost Rs. 20 i.e. the ratio of copies & their cost is 2 : 20

$\frac{100}{2}$ / $\frac{20}{2}$ = Rs.10

So, for five copies- also the ratio will be the same.If we combine the cost of 5 copies as x. Then

copies : cost = copies : cost

2 : 20 = 5 : x

Since, the product of middle terms = product of outer terms

$5 \times 20 = x \times 2$

or $100 = 2x$.

or $2x = 100$

or $x =$

or $x. = 50$

That means the cost of 5 copies would be Rs.50

Solving by unitary method

If two copies costs Rs. 20

Then 1 copy costs

Now if 1 copy costs Rs. 10

5 copy would cost = 10 × 5 = Rs. 50

Now are these methods to solve the other two situations also.

Some questions of this kind are given in exercise, Solve them.

> In such conditions where the cost of many objects is known and the cost of one object is found in order to get the cost of the number of objects asked for, unitary method or unitary law is used.

Exercise 14.2

1. Three copies cost Rs.16.5 What will be the cost of 7 copies?
2. A car moves 165 kms in 3 hours. Then find out
   (i) How much time would it need to move 440 kms?
   (ii) How much distance will be covered by the car in 6½ hours?
3. 72 books weight 9 kilograms ?
   (i) Find the weigh of 80 books.
   (ii) How many books would weight 6 kgs?
4. A worker earns Rs1500 in 25 days. Find out his income in 30 days ?
5. If 22 metres of cloth cost Rs704 , what would be the cost of 20 metres of cloth?
6. Complete the given table :

Number of books	Price (in Rupees
50	2500
75	____
____	100
____	3000

What Have We Learnt

(1) Equality in two ratios is known as proportion. If a : b and c : d are equal or similar, then they make proportionate ratios.

(2) In a proportion, the first term and the fourth term are called outer terms and the second and third term are called middle terms.

(3) When four numbers are in proportion, then product of the outer terms = product of the middle terms.

(4) If we know a : b and c : d are equal, the following proportions can be formed :
   (a) a : b : : c : d   (b) b : a : : d : c
   (c) c : a : : d : b   (d) b : d : : a : c

(5) When the price or value of a unit quantity is found out from the given number of the quantity and then the value/ price for the asked number of quantity is determined, the method is known as the unitary method.

Chapter -15

Area

Introduction:

Every close figure has some space within itself. Some points are outside it and we cannot reach the point inside without crossing the outline figure. The space inside a closed figure is its area. Some figures have more space inside. Those which have more space are bigger.

In the pairs of figures given below identify which figure covers more space, which of the rectangles is bigger. Identify the figures covering bigger areas.

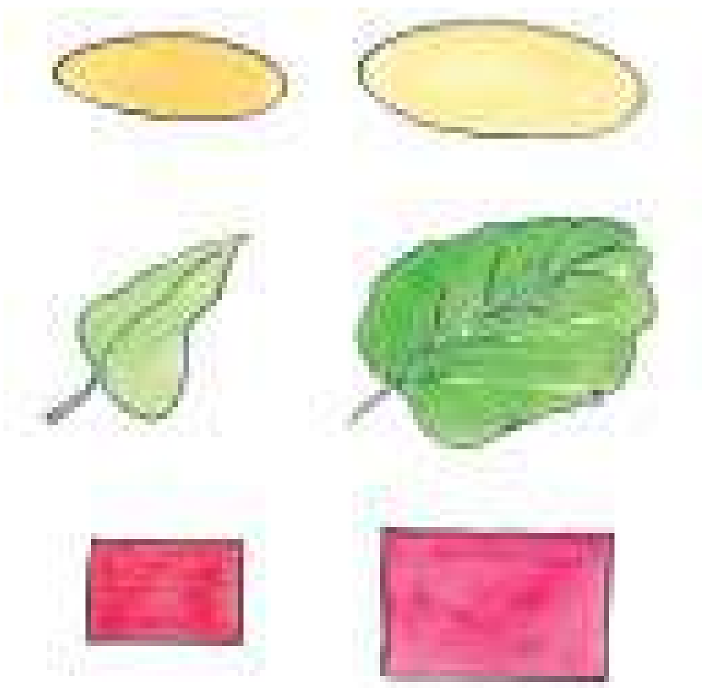


Fig 15.1

In the above figures you have seen that a big or small figure means the space that the figure circumscribes or covers on the same surface.

The place that an object or figure covers on a plane surface then needs to be measured. How do we do that?

One of the methods is as follows.

We can try to imagine and find out that how many small or certain unit measures of that particular object would make the bigger object or figure.

If you draw the outlines of objects like leaves or petals on a graph paper, you can see how many such unit shapes is covered by that space.

Thus, the space that an object or figure covers on any surface is known as its **area.**

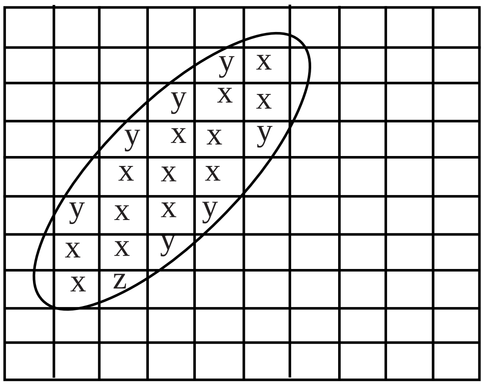


Fig 15.2

To do this :

Trace the object or figure on a piece of graph paper and calculate/measure the area as follows:

Count the number of complete squares inside the closed figure.

Count the number of squares that are bigger than half in size in the closed figure.

Count the number of half sized squares within the closed figure.

Leave the squares within the closed figure that are smaller than half. $\frac{z}{2}$

No. of squares for calculation = (No. of complete squares + No. of squares more than half in size + $\frac{\text{No. of half sized squares}}{2}$)

Therefore,

The area of the figure = (The no. of squares counted above)

The no. of squares that are more than half in size have been counted as complete squares, therefore the squares that are smaller than half in size have been left out and the exactly half sized squares has been counted as half.

The unit of measuring a square is 1cm × 1cm, in which each side of the square is 1cm in length. Therefore Area would be shown as 1 square cm. or 1cm square ($1cm^2$).

The area of the figure shown above is done by this method.

Complete squares (if considered x) = 13

More half sized squares (if taken as y) = 7

Just half sized squares (if taken as z) = 1

∴ Area of the figure = $x + y +$

$$= 13 + 7 + \frac{1}{2}$$

$$= 20.5 \text{ square cm.}$$

ACTIVITY 1

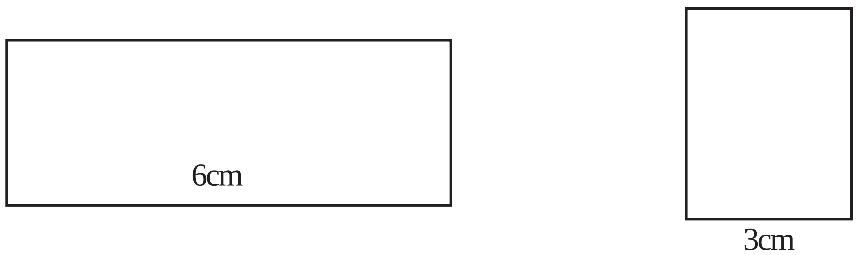
Similarly, you can trace your palm on the graph paper with your pencil and calculate its area.

Area of a Rectangle

You have learnt about rectangles in Class V. It is a quadrilateral whose opposite sides are equal and every angle is a right angle.

ACTIVITY 2

1. A rectangle has a length of 6cm and width of 3cm. Draw vertical and horizontal lines on both sides at a distance of 1cm each.



horizontal state of the rectangle   vertical state of the rectangle

Fig 15.3

2. Divide the rectangle into 1cm × 1cm parts, like this

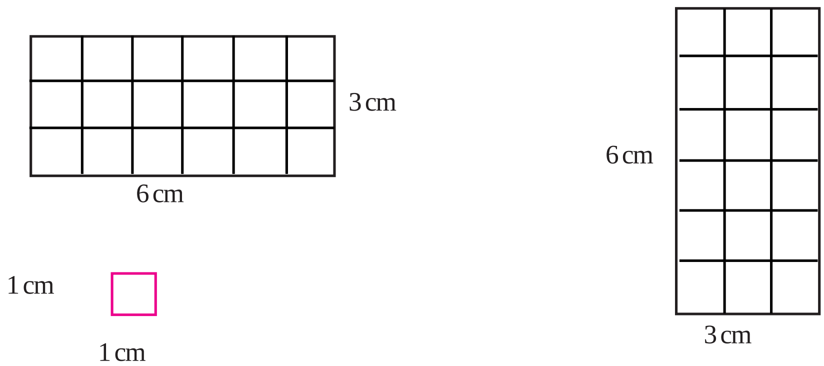


Fig 15.4

In the figure above each small square is 1cm × 1cm. Count there 1cm × 1cm small squares.

No. of squares	= 18
Area of 1 square	= 1 square cm.
18 square area	= 18 square cm.

Conclusion:

The larger the rectangle, the more will be the number of 1square cm squares.

Area = 18 square cm

= 6cm × 3cm

or = 3cm × 6cm

Area of a Rectangle = Length × breadth

Since the operation of multiplication follows the commutative law. Therefore we could also write Area of the Rectangle = Breadth × Length.

ACTIVITY 3

1. Construct the following rectangles on a graph paper and find out how many 1cm×1cm squares can they be divided into?

(i) 7 cm long and 3 cm wide

(ii) 10cm long and 1cm wide

(iii) 5cm long and 5cm wide

Area of a Square

A square is a special kind of rectangle whose sides are equal, that is the length and breadth of a square are equal.

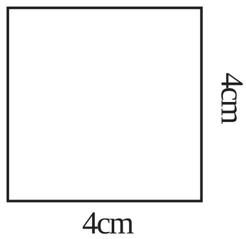


Fig 15.5

If we divide a square of 4cm × 4cm into unit squares of 1cm × 1cm.

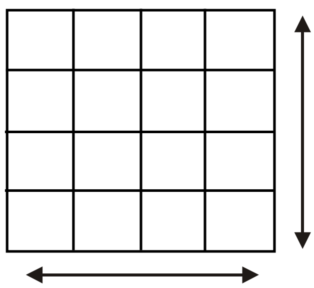


Fig 15.6

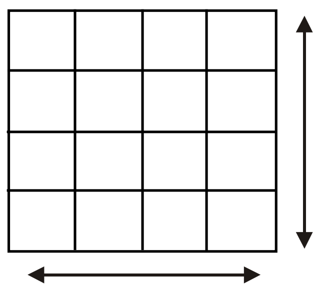


Fig 15.7

1 square cm = 1cm × 1cm

Area of the square = the number of unit squares

= 16

Area of one unit square = 1 square cm

Area of 16 unit square = 16 square cm

Area of a square = 16 square cm

The area represented = 4cm × 4cm

Thus, Area of a square = Side × side

or **Area of a square = $(\text{side})^2$**

Example 1.

If the side of a square is 5cm. What would be its area?

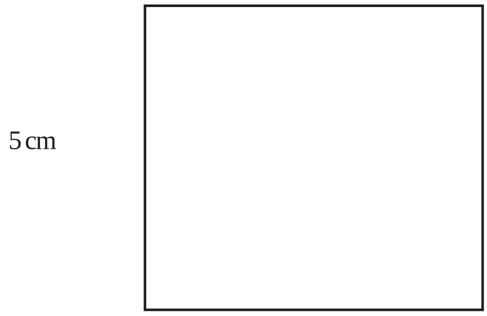


Fig 15.8

In the figure, a square of 5cm has been shown. On each arm mark point at gaps of 1cm.

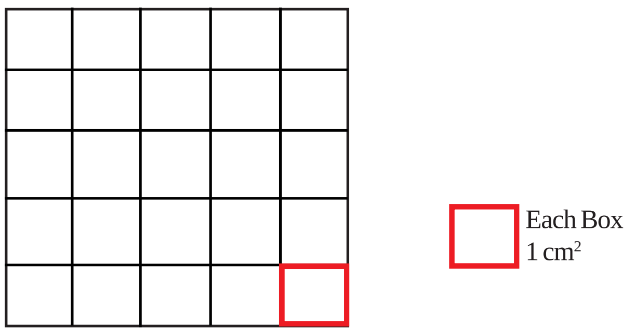


Fig 15.9

Now join the two marks with the help of horizontal and vertical lines. This will divide the bigger square into smaller squares. Now count the 1cm long and 1cm wide boxes inside the square.

Area of the square = No. of 1cm long and 1cm wide boxes inside the square.

= 25

= 25 × Area of 1 box

= 25 × 1square cm

= 25 square cm

∴ The area of the square = side × side

= square of the side

Example 2.

A rectangle is 9cm long and 4cm wide. Find its area.

Solution:

Here length of the rectangle = 9cm

breadth of the rectangle = 4cm

Area of the rectangle = length × breadth

= 9cm × 4cm

= $36cm^2$

or 36 square cm.

Example 3.

Find the area of a square whose side is 6cm. long.

Solution:

Area of a square = side × side

= 6cm × 6cm

= 36 cm^2

or 36 square cm.

Example 4.

A cloth is 2m in length and 100cm in width. Find its area.

Solution:

The cloth is rectangular.

∴ Length of the rectangular piece of cloth = 2 metre

Breadth of the rectangular piece of cloth = 100cm

Here the units of length and width are different, so we need to convert them into same units.

The length of the cloth = 2 metre

(since 1m = 100cm)

= 2 × 100cm

= 200cm.

(A) Now the area of the rectangular piece of cloth = length × width

= 200cm × 100cm

= 20,000 square cm

or 20,000 cm^2

(B) If the sides of the cloth are expressed in metre, then

The length of the rectangular piece of cloth = 2 metre

The width of the cloth = 100cm (since 100cm = 1m)

= 1m.

Therefore,

Area of the rectangular piece of cloth = length × width

= 2m × 1m

= 2 square m.

or 2 metre^2

If we compare (A) and (B), we find

20,000 cm^2 = 2 metre^2

or 10,000cm^2 = 1 metre^2

which means 1 metre^2 = 10,000 cm^2

Example 5.

A rectangular piece of paper is 25cm long and 15cm wide. If we fold a part of the paper to form a square, what will be the area of the biggest possible square that can be obtained from it?

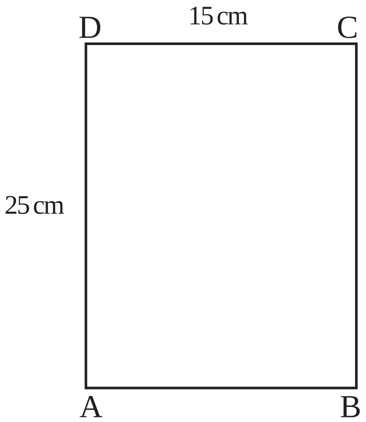
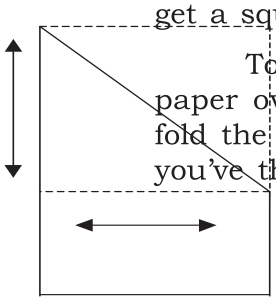


Fig 15.10 (A)

Solution:

As we noticed in the previous activity, we can get a square by folding a rectangular piece of paper.

To get a square fold the longer side of the rectangular paper over the smaller side as shown in the figure. Now fold the paper along CB and open it as in the figure. Now you've the square ABCD.



D C

15 cm

A_1 B_1

15 cm

A B

Fig 15.10 (B)

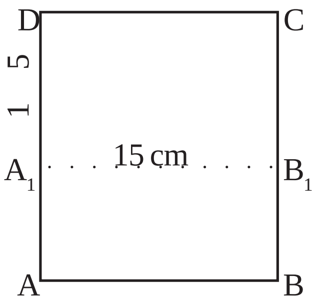


Fig 15.10 (C)

Now area of the square of maximum size = Side × side

= $A_1B_1 \times A_1D$

= 15cm × 15cm

= 225cm^2

Example 6.

The length of the rectangular floor of a room is 12 feet and its breadth is 6 feet. Find the cost of 2 feet × 1 feet tiles to be fitted on this floor if a piece of tile costs 10 rupees.

Solution: Here,

The length of the rectangular floor	=	12 feet
Breadth of the floor	=	5 feet
Area of the floor	=	length × width
	=	12 feet × 5 feet
	=	60 feet^2 or
		60 square feet
Since the area of 1 tile	=	2 feet × 1 feet
	=	2 square feet.
It means on 2 sq. feet floor, tiles required	=	1
∴ On 1sq.feet floor, tiles required	=	
Then on 60 sq.feet floor, tiles required	=	$\frac{1}{2} \times 60$
	=	30 tiles $\frac{1}{2}$

Therefore, 30 tiles will be required to cover the floor of the 12 feet long and 6 feet wide room.

Now, the cost of 1 tile	=	Rs. 10
∴ cost of 30 tiles	=	10 × 30
	=	Rs. 300

Thus, the cost of fitting 2 feet × 1 feet tiles on the floor of that room = 300 rupees.

Exercise 15.1

1. Find out the area of the rectangles whose length and breadth are as follows-
   (i) length = 5 cm; width = 3 cm
   (ii) length = 3.5 cm; width = 2 cm

2. Find the areas of the squares whose sides are :
   (i) 5 cm (ii) 7 cm
3. The area of the square is 1 square metre. Find the area in centimetre.
4. The area of square is 10,000 square cm, find the area in metres.

5. How many 1 square cm squares can you cut out of a 10 square cm piece of paper?
6. How many 2 square cm squares can you cut out of a rectangular piece of paper of 10cm × 2cm size? Do the experiment and find out.
7. The length of the rectangular floor of a room is 6 metre and its breadth is 2 metre. On the floor tiles of 10cm × 5cm are to be fitted. Find out the cost of fitting tiles if Each tile costs 5 rupees.
8. If the side of a square is 10m, find the change in its area if :

   (i) the length of the side is increased 2 times

   (ii) the length of the side is increased 3 times
9. The upper rectangular surface of a table is 200cm long and 50cm wide. Find the cost of covering the surface completely with sunmica (in rupees), if sunmica costs 25 paise per square cm.
10. The walls of a room are rectangular. Each wall is 3m long and 2m wide. Find the cost of white washing the walls when the rate of white washing is 10 paise per square cm.

Area of a Circle

You have drawn circles in Class V and in the previous lessons.

Draw a circle with the help of your compass on a graph paper. Count the no. of squares within the circular area.

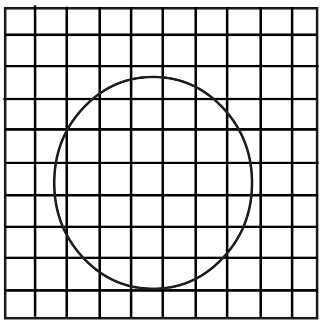


Fig 15.11

$\frac{\text{The number of just half size squares within circle}}{2}$

Count the number of squares as you did in case of finding the area of leaf on the graph paper and find the following:

A = The number complete squares within circle = ________

B = The number of more than half size squares within circle = ________

C = = ________

Total number of squares = A + B + C = ________

The area of a square box = 1cm × 1cm = 1 square cm.

Area of a circle = (A + B + C) square cm.

Finding the Area of a Circle with the help of Formula

In figure 12, O is the centre of the circle. OA and OB are the radii of the circle. We indicate its radius by *r*.

AB = OA + OB

$= r + r$

AB = $2r$

AB = 2 × radius

AB is known as the diameter of the circle

diameter = 2 × radius

. This is the ratio of the circumference and diameter of a circle and its value is approximately $\frac{22}{7}$ or 3.14.

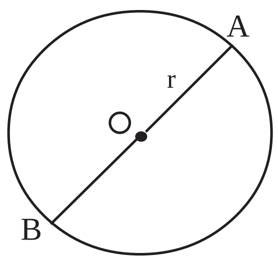


Fig. 15.12

Here, the area of circle = $\frac{22}{7} \times r^2$

A = $\pi \times (\text{radius})^2$

A =

On a graph paper draw a circle of a particular radius and find its area by counting the squares within the circle. Then verify the area with the help of formula using the same radius.

Exercise 15.2

1. Find the area of circle with the given radii.

   (i) 3cm (ii) 7cm (iii) 14cm

2. Find the area of the circle with the given diameter.

   (i) 8cm (ii) 20cm (iii) 14cm

What Have We Learnt

1. The area of an object is the space covered by it on a plane surface.
2. The area of a rectangle = length × width
3. The area of a square = side × side = $(\text{side})^2$
4. diameter of a circle = 2 × radius
5. The area of a circle = πr^2; where r is the radius of the circle.
6. The unit of area is square unit.

Chapter -16

PERCENTAGE

The examination result of a school for three years is as follows:-

Table -1

Year	Student Appeared	Student Passed
2002	200	160
2003	400	360
2004	300	282

Which result is better?

In the above table is the result for 2003 the best?

Can we just by looking at the number of students who passed, decide which result is better? Remember the number of students appearing each year is not the same.

To determine which result was the best we need to find the number of students who passed after bringing the number of students appearing to the same value. To equalize the number of students appearing we can use any number as the base.

Let us assume that we have to calculate the number of students passing with 400 as the base number of students appearing. In this case:

In year 2002, number of students who passed out of 200 = 160

In year 2002, student who passed out of 1 = $\frac{160}{200}$

In year 2002, number of students who would have passed out of 400 $= \frac{160}{200} \times 400$

= 320

In year 2003, number of students who would have passed out of 400 is 360, already given.

Again, in year 2004, number of students who passed out of 300 = 282

In year 2004, number of students who passed out of 1 = $\frac{282}{300}$

In year 2004, students who would have passed out of 400 = $\frac{282}{300} \times 400$

= 376

Therefore, in the years 2002, 2003 and 2004 number of students who would have passed out from the common base 400 is 320, 360 and 376 respectively. The result of year 2004 was the best.

Here we chose 400 as base for comparison. We can also choose 100, 1000, 10000 or any other easier number as base for comparison.

Let us discuss this question again-

Number of students passed in year 2002 out of 100 = $\frac{160}{200} \times 100 = 80$

Number of students passed in year 2003 out of 100 = $\frac{360}{400} \times 100 = 90$

Number of students passed in year 2004 out of 100 = $\frac{282}{300} \times 100 = 94$

Therefore, the number of students who passed in the years 2002, 2003 and 2004 on the common base of 100 is 80, 90 and 94 respectively.

In this way if we take 100 as common base for comparison then we call it percent or per hundred. But if the numbers to be compared are large, then we take 1000, 10000, or 1 lakh as base and we call it per thousand, per ten thousand, or per lakh respectively.

Expressing Percentage into Different Forms

By now you have understood that 25% means 25 out of 100. Similarly 50% means 50 out of 100. Can we write 25 or 50 out of 100 in some other form? You have already learnt one way of writing it. Since percentage is a ratio and fractions can also be written in the form of decimals. Therefore percentage can also be written in the form of ratio as well as decimals.

Let us learn the process one by one.

A Conversion of Percentage into Fraction

You know 50% means 50 out of 100. This can be written as $\frac{50}{100}$. Therefore the fractional form of 50% in terms of 1 is $\frac{1}{2}$.

Activity 1

(i) Change 25% into fraction.

(ii) Change 75% into fraction.

Think of more questions and ask your friends to solve them.

B Conversion of Percentage into Ratio

You have seen how percentage can be converted into fraction. Similarly we can change percentage into ratios also.

The fractional form of 50% = $\frac{50}{100}$

This will be depicted as 50 : 100

or 1 : 2

Activity 2

(i) Change 25% into a ratio.

(ii) Change 75% into a ratio.

Think of problems of this kind also and ask your friends to solve them.

C Converting Percentage into Decimal

You have converted percentages into fraction and ratio. Now let us see how percentage can be converted into decimal.

Fractional form of 50% = $\frac{50}{100}$

Ratio form of 50% = 50 : 100

Now $= \frac{50}{100}$ means 50 is divided by 100

= 0.50.

Therefore the decimal form of 50% is 0.50.

Activity 3

Now try to solve the following -

(i) Change 25% into decimals.

(ii) Change 75% into decimals.

Think of more such questions and ask your friends to solve them.

Conversion of Fractions, Ratios and Decimals into Percentage

You have learnt how to change percentage into fraction, ratio and decimals. You have noticed that each time their values remain the same, only the form changes. Now on the contrary let us convert fractions into ratios and decimals into percentage.

A Converting Ratio into Percentage

If the given ratio is 1:2, it means $\frac{1}{2}$

Therefore $\frac{1}{2} \times \frac{100}{100} = \frac{100}{2} \times \frac{1}{100} = 50 \times \frac{1}{100} = 50\%$

Activity 4

Change the following into percentages

(i) 1:5 into percentage.

(ii) 3:4 into percentage.

Make such questions and solve them.

B Converting Decimal into Percentage

Example 1. 0.75 expressed in percentage

Solution: $\frac{0.75 \times 100}{100} = \quad = 75\%$

C Comparing Percentage and Decimal forms

The figures below show different quantities in percentage and decimal forms.

Example 2.

0.2 = 20%

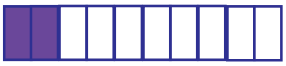


Fig 16.1

0.3 = 30%

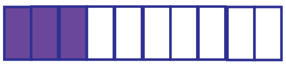


Fig 16.2

0.4 = 40%

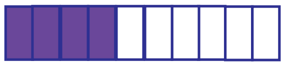


Fig 16.3

$0.2 \times \frac{100}{100} = 20\ \%$ $\quad 0.3 \times \frac{100}{100} = 30\ \%$ $\quad 0.4 \times \frac{100}{100} = 40\ \%$

This means to convert a decimal into percentage, the decimal digit is multiplied by 100 and added with the % symbol.

Activity 5

Change the following decimals in terms of

(i) Change 2.25 into percentage.

(ii) Change 0.60 into percentage.

Make such questions and solve them with your friends.

The mutual relationship between fraction, decimal, ratio and percentage

If a class has 20 students enrolled, out of which 04 students are absent. The fractional form will be $\frac{4}{20}$. Compared to the decimal form will be 0.2. So by changing into percentage we come to know that 20% students are absent.

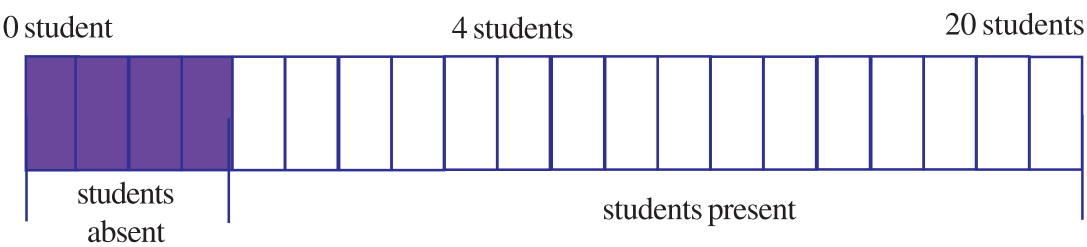


Fig 16.4

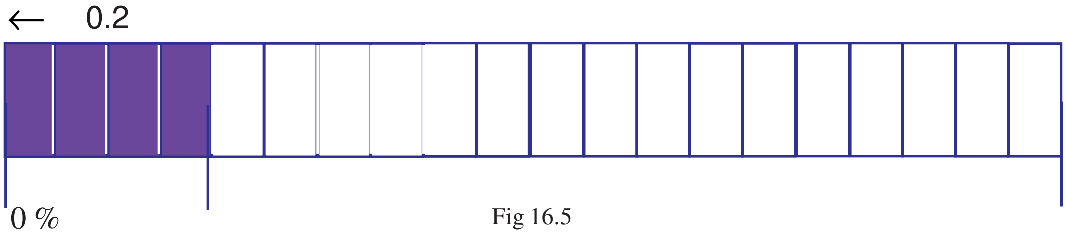


Fig 16.5

Conclusion : , 0.2 and 20% are equivalent quantities. It is clear that it is possible to change a decimal into fraction, decimal into percentage & percentage into decimal or fraction.

Activity 6

Fill in the blanks :-

1) = __________% = __________ (decimal form)

2) 0.45 = __________% = __________ (fraction form)

= __________ (ratio form)

Make more questions of such type.

PERCENTAGE IN FIGURES

Find out the percentage of the shaded & unshaded parts in the figures.

$\frac{4}{20}$

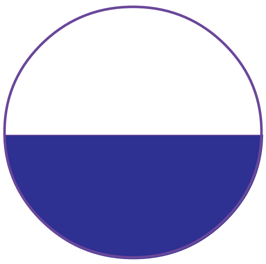


Fig 16.6

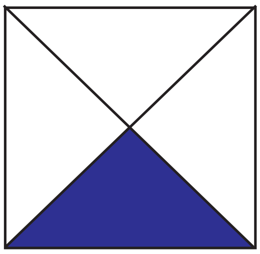


Fig 16.7

Shaded portion $= \frac{1}{2}$

$= \frac{50}{100}$ (how?)

$= 50\%$

Unshaded portion $= \frac{1}{2}$

$= \frac{50}{100}$ (how?)

$= 50\%$

Shaded part $= \frac{1}{4}$

$= \frac{25}{100}$

$= 25\%$

Unshaded part $= \frac{3}{4}$

$= \frac{75}{100}$

$= 75\%$

Chapter -17

STATISTICS

INTRODUCTION

A programme to decorate the classroom was to be organized in the school. The students of class 7 could not decide the colour to be used to paint the walls of the class room. Only 4 colours viz light yellow, pink, light green and sky blue were available in their school. The class monitor asked all the students to write their names and their favorite colour on a paper. This is represented in the following table.

Table 1

No.	Name of the student	Colour
1	Rajesh	Light yellow
2	Ruchi	Pink
3	Meena	Light Yellow
4	Raheem	Sky blue
5	Hameeda	Light yellow
6	Julie	Light green
7	Anita	Light green
8	Francis	Sky blue

No.	Name of the student	Colour
9	Keshav	Light Yellow
10	Basant	Sky blue
11	Shekhar	Light green
12	Reeta	Pink
13	Sunil	Light yellow
14	Anamika	Light yellow
15	Balwant	Pink
16	Raghu	Light yellow

On the basis of this data, can you decide the colour to use on the walls of the classroom? Rita got an idea, she wrote all the colours on the board and asked each student to write his or her name in front of his or her favorite colour.

Now, the following list was formed:-

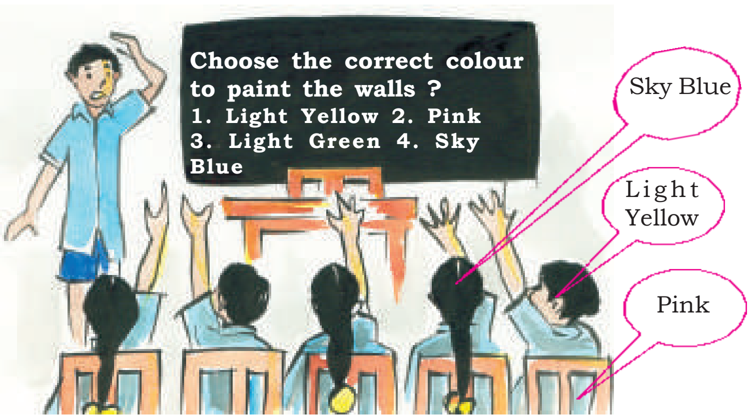


Figure 17.1

Table 2

Colour	Students Name
Pink	Ruchi, Reeta, Balwant
Light Yellow	Anamika, Rajesh, Meena, Hameda, Keshav, Sunil, Raghu
Light Green	Julie, Anita, Shekhar
Sky blue	Raheem, Basant, Francis

Since light yellow was the favorite colour for more students, it was decided to paint the walls with this colour.

Have you ever adopted this method to take a decision in your daily life?

Now, you construct a list classifying students scoring above and below 34% marks in each subject. On the basis of this data, can you say in which subject is the result the best and in which subject the result is the worst?

DATA

We always require some information to take a decision. This necessary quantitative information is called data.

Suppose you have to buy a newspaper for the students of your class. Which newspaper will you buy so that the largest number of students read it? How will you take this decision?

All the students of the class made a table in which they wrote their name in front of their favorite newspaper. The newspaper that the largest number of students liked was selected.

While looking at the tables again and again Julie kept thinking that there was no point writing their names in the table. They only needed the number of students in favour of a particular newspaper. So instead of writing names in the table a symbol could be used to indicate the choice.

Do you agree with Julie? Can you think of a way to count the data using only a symbol instead of having to use names in the table?

Basant suggested that in place of each name we can use a small vertical line to represent the student and then these lines could be counted. Everyone agreed with Basant's suggestion.

Anita said, "Let us find out the order of popularity of some games". She wrote the names of 4 games on the board and asked each student to draw a small vertical line in front of their favourite game. The following table was generated:-

Table 3

Name of the game	Tally sign (Vertical line)	No. of students
Football	| | |	3
Cricket	| | | | | | |	7
Vollyball	|	1
Kabaddi	| | | | |	5

But in such tables, it is inconvenient to count a very large number of vertical lines. So, as in earlier classes while learning counting, we made bundles of 10 units, in the same way if we make bundles of 5 vertical lines here then it will become much easier for us to count the lines. We draw 4 vertical lines and represent the 5th line by a slanted line which cuts these 4 lines (as shown below). E.g. for 5:-

For 5 : ~~||||~~

For 19 : ~~||||~~ ~~||||~~ ~~||||~~ | | | |

This makes counting easier.

According to the data in the table above, the number of students liking Cricket is ~~||||~~ | | i.e. 7 This is called **frequency**. The procedure of representing each data by a vertical line is called **marking a Tally** and the method is known as collection of data using **Tally method**. The table constructed by this, is called the **Frequency Table**.

Use this method to collect data for quantities around you.

Example 1: The number of children in 20 houses of a village are represented by the following table:-

Table 4

House No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
No. of children	2	3	2	1	3	2	0	1	3	4	2	2	1	1
House No.	15	16	17	18	19	20								
No. of children	2	4	3	2	0	3								

Construct an appropriate frequency table for the above data using Tally method.

Solution: Let us make columns for the number of children in the house, for tally marks and for the frequency. Mark the tally sign for each house. For convenience we represent the 5th sign by a slanted line cutting the 4 previously drawn vertical lines.

Table 5

No. of Children	Tally sign	Frequency
0	II	2
1	IIII	4
2	~~IIII~~ II	7
3	~~IIII~~	5
4	II	2

In the above table why have we chosen the no. of children to be between 0 to 4 only?

What would happen if we start with 1 ?

What happens, if we were to write the number of children in the table to be from 0,1,2,3 —— upto 7?

Exercise 17.1

Q1. In a class 20 students obtained the following marks out of 5, in their mathematics test:-

3 2 5 4 0 1 2 3 5 2 2 3 5

4 1 0 3 2 3 4

Construct a table for the above using the Tally method.

Q2. The maximum daily temperature of a city in degree Celsius between 1st April 2005 to 15th April 2005 was recorded as follows:

37.8, 37.8, 37.9, 38.0, 37.9, 37.9, 38.0, 38.1, 38.1,
38.2, 38.3, 38.3, 38.2, 38.1, 38.2

Construct a table for the daily temperature from the above data using the Tally Method.

Q3. The following table represents the results of students of class VI according to divisions obtained. Observe the table and answer the following questions:

Division	**No. of students**
Ist Division	12
IInd Division	14
IIIrd Division	10
Failed	04

(a) In which division do the maximum number of students fall?

(b) How many students appeared for the exams?

(c) How many students passed the examination?

Pictograph

Rajesh was reading the newspaper. The newspaper said that -"Girls score over boys"

In the class 8th board examinations of this year, girls are ahead of boys in all areas. While looking at the figures, Rajesh thought that– "this is a good method of data display. By looking at these figures, it is very easy to see that the girls have scored over boys in all aspects of the result". A similar picture can be seen when we stand in queues during prayer, the number of students in a class can be compared by the length of the queues. Rajesh asked his

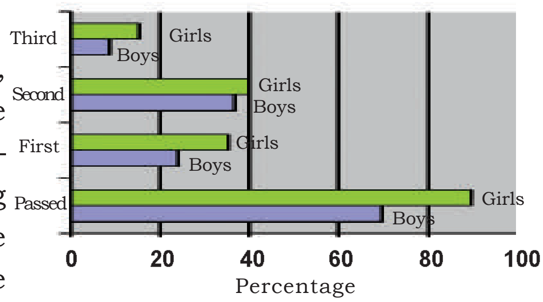


Figure 17.2

friends, “Why do we not represent the popularity of games in the same form using the data collected in table 3?”

The total No. of students in table 3 was 16. In this, 3 students liked Football, 7 liked Cricket, 1 liked Volleyball and 5 liked Kabaddi. How can this be represented in the form of a figure?

Julie said, “If we make a picture for each student, then 3 pictures in front of football, 7 in front of cricket, 1 in front of volleyball and 5 in front of Kabaddi will have to be made.

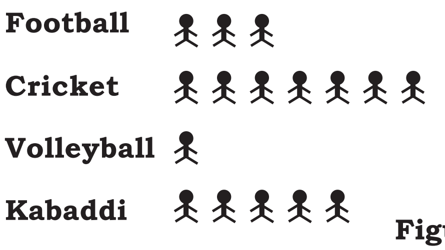


Figure 17.3

The representation of data with the help of pictures in this form is called a Pictograph. Pictograph is easy to understand and conclusions can be drawn by looking at the pictures.

Bar Graph

This method of pictograph requires a lot of pictures to be drawn which sometime becomes impractical. If we take a bar of length 1 cm for each student, then the representation of data becomes even more easy. These bars can be drawn in both horizontal and vertical form.

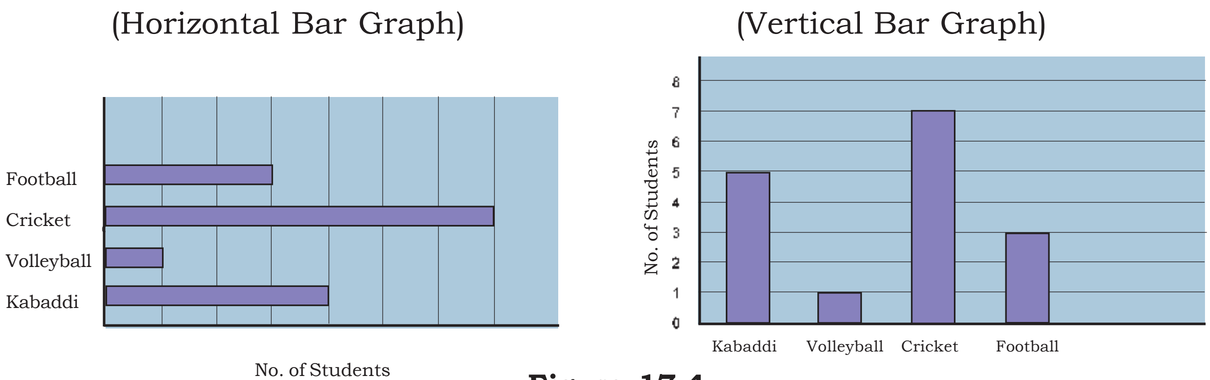


Figure 17.4

Note that width of the bars is kept equal in the above graphs. It is easy to estimate the extent of the popularity of these games by looking at these bar graphs. Since the number of students in the above example is small, the data can be easily represented on a notebook using a bar of 1 cm. length for each student. But in case the number of students is large, how can we depict it on the notebook? In such a situation, the main problem is to choose the height of the bars. Let us think this over-

There are 750 men, 660 women and 140 children in the locality where Rajesh resides. We are required to represent this data in a graph.

What should be the height of the bars in order to represent the above data? If we take 1cm for each person, then we need to draw 750 cm high bar for men, 660 cm bar for women and 140 cm bar for children. But it is impossible to draw such bars in our notebooks.

If we take 1cm bar for every 10 people, then we need to draw bars of 75 cm, 66 cm and 14 cm for men, women and children respectively. Even these heights cannot be represented in our notebooks. But, if we take 1cm bar for every 100 people, then we need to draw bars 7.5 cm, 6.6 cm and 1.4 cm long for men, women and children respectively. These bars can be easily represented on our notebooks. So, let us see how we will represent this data using a Bar graph.

Vertical Bar Graph

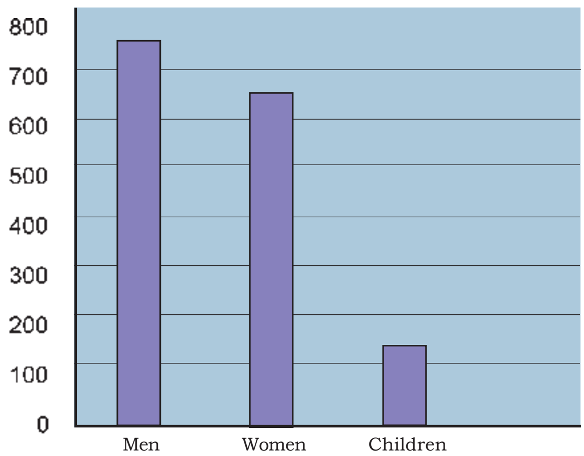


Figure 17.5

This data is represented by vertical bars. This is called a **vertical bar graph**. Bars can also be drawn horizontally.

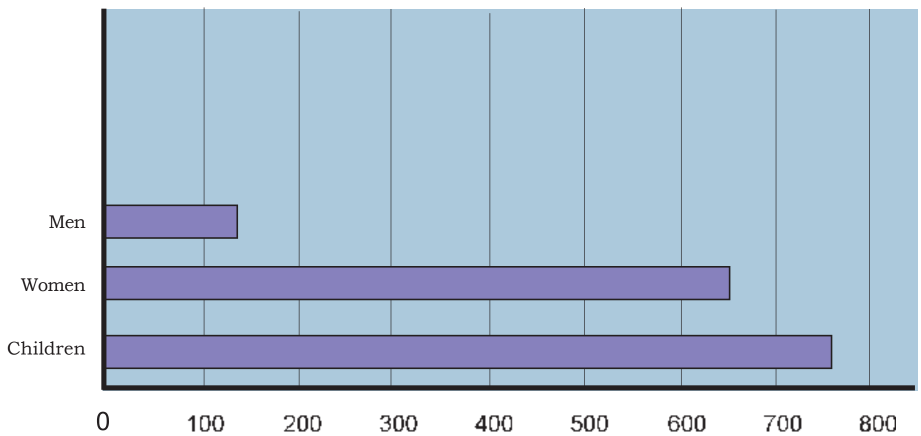


Figure 17.6

If the bars are drawn horizontally, then the graph obtained is called a **horizontal bar graph** (Fig. 17.6). Anita was wondering about the use of these bar graphs. She thought we get the same information from the graphs as we get from the frequency tables.

Let us find a solution to Anita's question.

Following table gives the production of wheat from year the 1991 to 2000:-

Table 6

Year	Wheat Production (in lakh tones)
1991	72
1992	90
1993	82
1994	103
1995	110
1996	94
1997	99
1998	88
1999	90
2000	78

The above data can be represented in the form of a bar graph in the following way:-

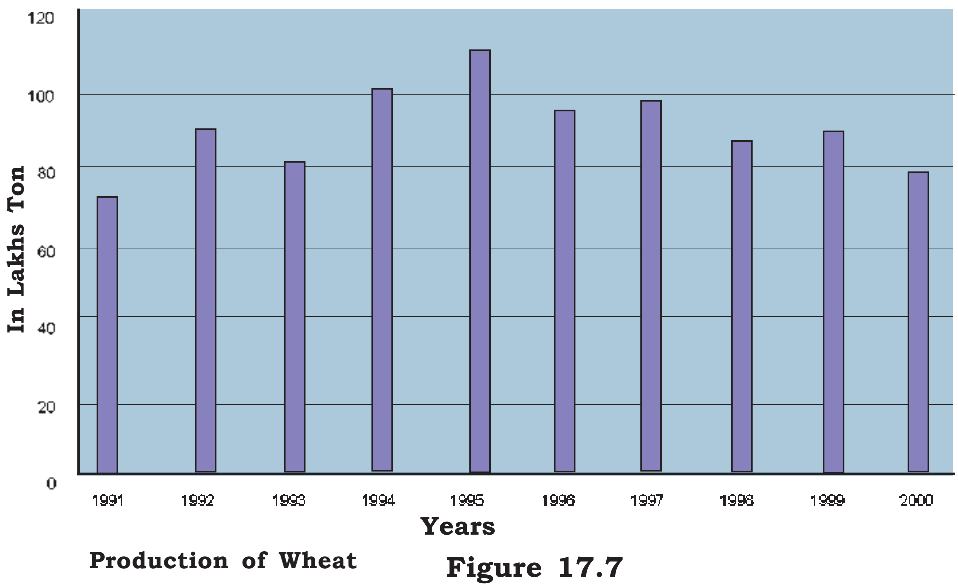


Figure 17.7

By looking at this bar graph, can you tell which year had the minimum wheat production and which year had the maximum production? What other information can you obtain from this graph? Write down.

You will observe that the maximum wheat production was in the year 1995 and the minimum was in the year 1991. We can also observe that the years 1992 and 1999 had equal wheat production; can you make the same observations using a frequency table?

Clearly, it is difficult to draw conclusions just by looking at the data in the table. For this, one needs to examine the data minutely, whereas with just a glance at the bar graphs we can see which year had the maximum and which year the minimum production. Thus, the major advantage of a bar graph is that it can be easily understood just by looking at it and it can be easily compared with other data.

Example 2: The following graph represents the runs scored by Arun from the year 2000 to 2004. Observe the graph and answer the following questions:-

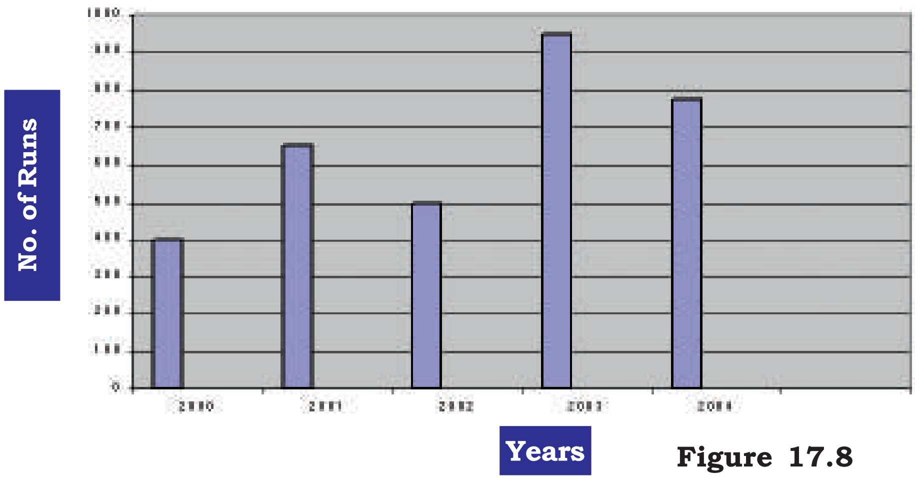


Figure 17.8

(a) What information do we obtain from this bar graph?

(b) In which year did Arun score the minimum number of runs?

(c) In which year did Arun scored the maximum runs?

(d) Is Arun's performance regularly improving from one year to the next?

Solution :

(a) The given bar graph represents the runs scored by Arun from the year 2000 to 2004.

(b) Since the bar representing the year 2000 is the smallest, hence the number of runs scored by Arun in 2000 is the minimum.

(c) Since the bar for the year 2003 is of the largest height, Arun scored the maximum runs in this year.

(d) No, because in comparison to 2001 Arun scored lesser runs in the year 2002. He also scored lesser runs in 2004 as compared to 2003.

Example 3: Following bar graph represents the pass percentage of students in schools A,B,C,D,E. Look at the graph and answer the following questions:-

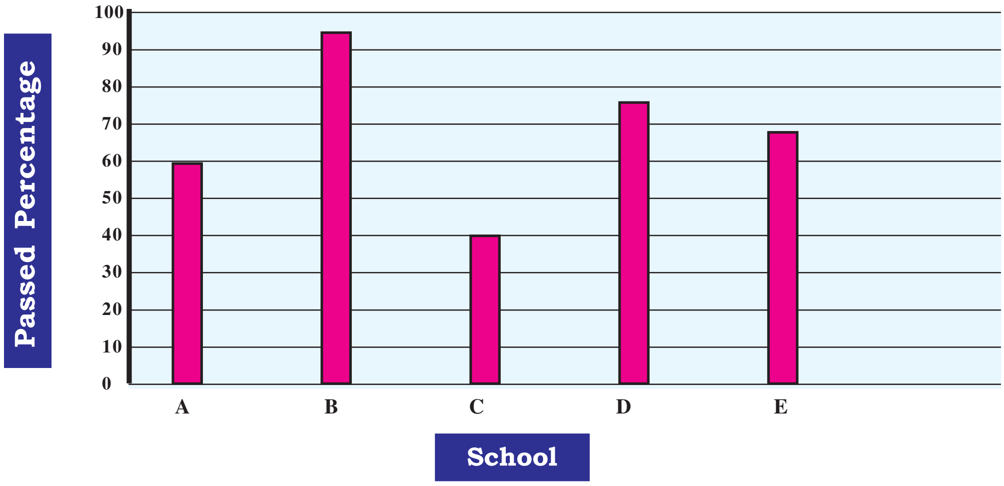


Figure 17.9

(a) Which school has the minimum pass percentage of students?

(b) Which school has the pass percentage of students above 90%?

(c) Which school has the minimum percentage of students who have failed?

(d) In how many schools the pass percentage was 60% or above?

Solution:

(a) School C has the minimum percentage (40%) of passing students.

(b) In school B, the percentage of passing students is 95%, which is more than 90%

(c) School B has the minimum percentage of failing students as the percentage of passing students is the highest.

(d) 4 Schools viz A,B,D and E have pass percentages of 60% and above.

EXERCISE 17.2

Q1. The following table represents the yearly income of a company for 5 years. Represent the data by a bar graph.

Year	**1996**	**1997**	**1998**	**1999**	**2000**
Yearly income (in Lakhs)	10	20	15	12	22

Q2. The following table represents the percentage of people buying different TV sets. Represent the data in a bar graph.

Brand	**% purchased**
p	25
q	30
r	15
S	10
T	10
Others	10

Q3. The following table represents the percentage of average marks obtained by the students of a school, in their annual examinations. Represent the data in a bar graph.

Subject	**Average Marks obtained by students (%)**
English	55
Maths	60
Science	65
Social Science	90
Hindi	70

Q4. The following data collected by Shubham represents the temperature at 11 A.M. for 1 week (Collected at 11'o clock in the morning)

Day	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**
Temp (°C)	50	45	40	45	35	40	48

Represent the above data in a bar graph.

5. See the data in the following bar diagram. The no. of the sale of daily newspapers printed in six languages in a city has been shown. The data are given in thousands study the graph and answer the following questions.

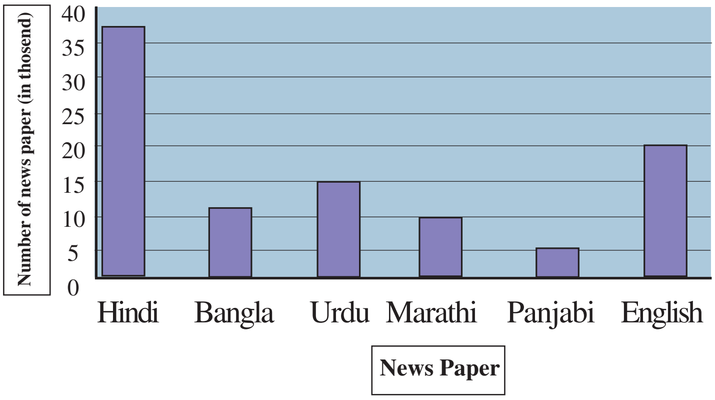


Figure 17.10

1. Write the number of Newspapers read in Hindi, Bangla, Urdu, Marathi, Punjabi and English languages.
2. How many more Newspapers are read in Marathi rather than Punjabi?
3. Which language Newspaper has minimum reader?
4. Write in increasing order, the number of Newspaper read in different languages.

Mean

Radha enjoys feeding the cattle and giving them water to drink. She fills water for the cattle and also keeps record of the fact that from 8 to 11 in the morning, how many cows drink water. The record she has for the last week is as follows :

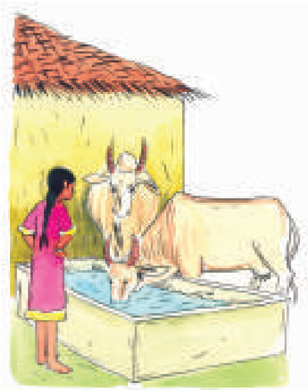


Fig. 17.11

Monday -12, Tuesday - 15, Wednesday - 13, Thusday - 11, Friday -13,Saturday - 13, Sunday - 14.

Can you tell how many cows drink water on average everyday ?

Cricket player A in 10 innings had scored 60, 70, 15, 90, 72, 45, 11, 77, 125, 200 runs respectively. Similarly player B made 220, 110, 70, 37, 15 and 07 runs in 06 innings. Can you say which player had a better performance ?

We can easily compare such comparison with the help of average ? We use average in several centexts in our everday life. For example :

1. The average age of students studying in your class is 14 years.
2. The average duration of sleep at night for you is 8 hours.
3. The average rate of our daily newspaper is Rs. 2.50.
4. The average attendance of students in the class is 45.
5. This year Raipur received rains below average.

The above examples show that the average age of sudents in the class is 14 years. On the average duration of sleep at night is 8 hours. It is neither the maximum limit nor the minimum limit.

In fact average is attained by dividing the sum of the scores in a given data by the number of scores. This is also known as the mean. This is indicated by M.

$$Therefore\ Average\ or\ mean\ (M) = \frac{sum\ of\ Scores}{No.\ of\ Scores}$$

Now we can easily find the number of cattle that Radha served water to drink.

$$Average = \frac{12+15+13+11+13+12+15}{7} = \frac{91}{7} = 13$$

Therefore on an average 13 cattle drink water that Radha served everyday.

Now you can yourself find out which cricket player had a better performance.

Activity 1

Find out the average age of the members of your family.

Activity 2

Find out the average of the scores obtained by you in all the subjects in your halfyearly exams.

Example 4 : In a Fruit shop apples have been kept in five baskets. Containing 46 kg, 21kg, 18kg, 25kg, and 35kg apples. Find out the means.

$$(M) = \frac{46+21+18+25+35}{5}$$

$$= \frac{145}{5} = 29kg$$

Example 5 : Find out the mean of the first 10 natural numbers

Solution :

The first ten natural numbers are :-

1, 2, 3, 4, 5, 6, 7, 8, 9, 10

$$Mean(M) = \frac{Sum\ of\ the\ scores}{No.\ of\ scores}$$

$$M = \frac{1+2+3+4+5+6+7+8+9+10}{10}$$

$$= \frac{55}{10} = 5.5$$

MODE

The school decided to take 30 students of class VIII for an excursion during the Deepawali vacations. The headmaster instructed the students to select a place. out of Sirpur, Ratanpur, Jagdalpur and Ambikapur. Some students wanted to go to Sirpur while others thought about Jagdalpur. Since the place could not be decided, the class teacher wrote the names of all the four places on the blackboard and asked the students to raise their hands for each option. He put tally marks in front of each name which was as follows :

Table 7

Places to be Visited	Tally Marks	No. of Students
Sirpur	~~IIII~~ II	07
Jagdalpur	~~IIII~~ ~~IIII~~ III	13
Ratanpur	~~IIII~~	05
Amtikapur	~~IIII~~	05

After making the table, the class teacher said that the maximum number of student i.e. 13 of the the students want to go to Jagdalpur, so we should proceed towards Jagdapur.

In our daily life, we have many such situations, where selection is made in this manner for example, mostly the size of shirts that people wear are 38 or 40. Therefore we easily get the sizes 38 or 40 in the readymade stores. Generallly shops seldom keep the smaller or bigger sizes beause they are less in demand and the manufacture depends on the maximum demand in the market.

This basis of selection is called the mode.

So, mode is that value in a given data which has been repeated maximum number of times. This is indicated by M_O.

Example 6 : In a football Team the sizes of the shoes put on the eleven players are as follows:

6, 4, 5, 6, 7, 7, 6, 5, 6, 7, 8. Find the mode.

Solution : On writing the given scores in ascending order we get :

4, 5, 5, 6, 6, 6, 6, 7, 7, 7, 8

Clearly, the score 6. appears the maximum number of times (4 times). Therefore the mode of this score would be 6.

$M_O = 6$

Median

Example 7 : In a class of 15 students received the following marks in Mathematics out of 100 :-

15, 35, 16, 25, 45, 76, 90, 99, 50, 16, 57, 60, 86, 17, 95. How many students have scored more than 50% marks ? This is not very clear by the marks. Let us arrange the marks in ascending order :

15, 16, 16, 17, 25, 35, 45, 50, 57, 60, 76, 86, 90, 95, 99.

Now, we can see that 07 students have got more than 50% marks. We can also see that 7 students have received less than 50 marks.

Example 8 : The number of chapatis that 11 people eat in a day are as follows :

3, 7, 9, 8, 6, 5, 4, 2, 12, 10, 11

Find the mean of these numbers Mary quickly wrote it in her notebook and said that the mean is 7. Can you find out how many people ate more than 7 ? Radha calculated it and said that 5 people ate less than 7 chapatis while Aslam reported that 5 people ate more than 7 chapatis.

In the example (4) 7 students have scored above 50% marks while 7 students have got less than 50% while in example (5) also the number of people who ate less than 7 and more than 7 chapaties are equal i.e. 5.

Therefore, we can say that putting in order, the number 50 (in Example 4) and 5 (in Example 5) are the number that occure in the middle. This number is know as the median.

This means, when the scores are arranged in descending or ascending order, the value that occures in the middle is known as the Median. It is denoted by M_d.

A. Finding the Median when the number of scores N is odd.

When the number of scores in a given data is odd., then first we write them in an ascending order and find the value of the $M_d = \left(\frac{N+1}{2}\right)$ to score. This number obtained is the median.

Therefore th item

Example 9 : Find the median of the given data :
3, 5, 10, 9, 8, 14, 6, 12, 13, 11, 7

Solution : On writing the scores in an ascending order :

3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 (here the total numbers of item is 11 i.e. odd)

th item value

th item = 6th item's value

∴

B. Finding the Median when the number of scores N is even.

When the number of scores in a given data is even, then arranging in ascending or descending order shows two items in the middle position.

In such a state, the mean of these two items are taken to find the median. This means,

$$M_d = \frac{\left[\left(\frac{N}{2}\right)^{th} item + \left(\frac{N}{2}+1\right)^{th} item\right]}{2}$$

Example 10 : Find the median of the given descending numbers :
5, 9, 4, 6, 12, 8

Solution : On arranging the scores in ascending order, we get :

4, 5, 6, 8, 9, 12

Here N = 6 (even number)

$$\text{Median } M_d = \frac{\left[\left(\frac{N}{2}\right)^{th} item + \left(\frac{N}{2}+1\right)^{th} item\right]}{2}$$

$$M_d = \frac{\left(\frac{6}{2}\right)^{th} item + \left(\frac{6}{2}+1\right)^{th} item}{2}$$

$$= \frac{(Value\ of\ the\ 3rd\ item + Value\ of\ the\ 4th\ item)}{2}$$

$$= \frac{6+8}{2} = \frac{14}{2}$$

$$M_d = 7$$

Exercise 17.3

Q.1 Find the mean : 81, 74, 69, 73, 91, 55, 61.

Q. 2. Find the mean of the even number between 50 and 70.

Q. 3 Find the median : 4, 5, 10, 6, 7, 14, 9, 15.

Q. 4 The weight of 11 students (in kgs) in a class ar e as follows : 25, 27, 29, 32, 30, 28, 26, 31, 35, 41, 34. Find its median.

Q. 5 In a Science quiz competition, one student of class VIII received the following marks : 83, 61, 48, 73, 76, 52, 67, 61, 79.

Find the median of the above marks.

Q. 6 Find the mode of the given data:

7, 5, 99, 3, 1, 9, 7, 5, 3

1, 1, 9, 7, 7, 5, 5, 5, 3, 1

5, 3, 5, 1, 5, 7, 7, 9, 9, 1

Q. 7 Find the mode of the given distribution :

5, 3, 2, 2, 4, 5, 3, 3, 4, 3, 5, 3.

Q. 8 Find the mean of the first five odd natural numbers.

Q. 9 The mean of the number 8, 5, x, 6, 10, 5 is 7. Find the value of x.

Variability

Change is the important component of nature. there are so many changes are going in nature. Some of them are in constant direction like – Childhood, Adult and old age, change of height and weight in childhood, growing of plants etc. A different type of change is also here in which continuity, certainty and seriality occurs. Like– Sunrise, Sunset, rotation of Earth, Day and night, Change in climate etc.

We can estimate these changes because these changes occur in seriality, Night will occur after day, not day will occur after day. Rainy season will occur after summer season not any other season.

Some changes of nature are uncertain we can't say about its definite results. like – After clouds rain will occur.

There are some more changes occurs around us about which we can't predict any result we can only assume about its results, like on tossing a coin occurrence of head or tail, getting a particular number on throwing a dice. Particular card drawn from a pack of cards, drawn of a particular ball from a bag.

We can tell probably about this event's result. We can't say anything about its fixed result. If head comes an tossing of a coin in first chance then in next turn there will be again probability of occurrence of head or tail.

The result of these types of experiment depends upon nature of events.

1. On tossing a coin two types of result can occur.
   - Head
   - tail
2. On rolling a dice the number appear on its upper phase belongs to these 6 results.
   - 1 point
   - 2 point
   - 3 point
   - 4 point
   - 5 point
   - 6 point
3. One ball drawn from a bag containing 1 red, 1 green, 1 white or 1 black ball. The ball will be
   - Red
   - Green
   - Black
   - White

Now it is clear that number of outcome depends on event occur.

Activity 3

Tell whether the given statement is certain or uncertain -

1. Occurrence of night after day.
2. Occurrence of head on tossing a coin.
3. Occurrence of rainy season after summer season.
4. Rain after appearance of ....
5. On relining a dice accordance of six point on its upper phase.
6. Change in height as per age in childhood.
7. Sickness of a person.
8. Aging of a person.

WE HAVE LEARNT

1. Depiction of quantitative data in the form of pictures is called a pictograph.
2. A bar graph, is a representation of quantitative data using bars of equal width taken at equal distances either horizontally or vertically.
3. It is easy to infer many things by observing a bar graph.
4. Average (Mean) is one unique number, that represents a group of scores or data.
5. Ave

M_d

6. While finding out the median the scores are arranged in ascending order.
7. The medan is the number in the middle of the scores arranged in ascending order.
8. (a) $M_d = \left(\frac{N+1}{2}\right)^{th}$ item (when N is an odd No.)

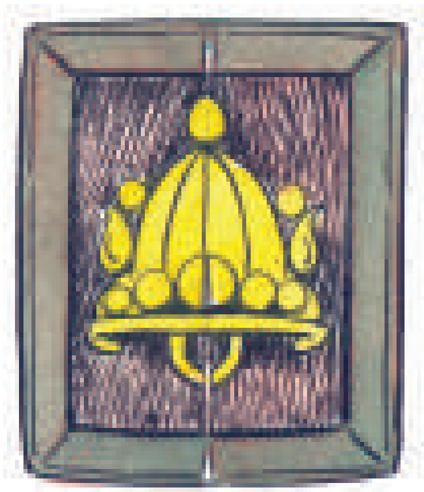
   (b) (when N is an even No.)
9. The mode is the number that has the highest frequency in the scores.
10. Variability is an important component of nature.
11. Some changes in nature are continuous, constant and sequential about which we can predict. But some changes are unpredictable.
12. Some events around us are such that the consequence of its occurrence cannot be estimated only the possibility can be expressed.
13. The number of possible outcomes of an event depends on its nature.

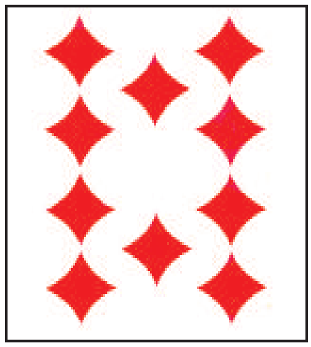
Chapter -18

SYMMETRY

Many types of shapes exist around us. We look at the flowers, beautiful paintings, buildings and other things. In all these, we see symmetry and some kind of harmony.

Many of these shapes are in balanced proportions. Some of these look the same at different positions. Some of these look as if they are made up of two similar figures.





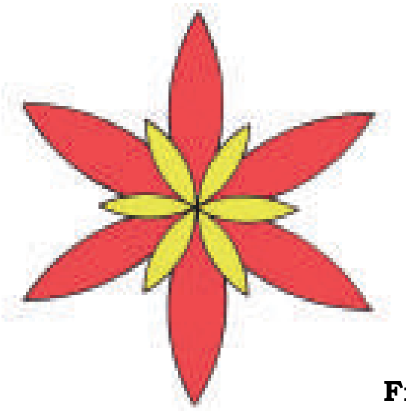


Figure 18 .1

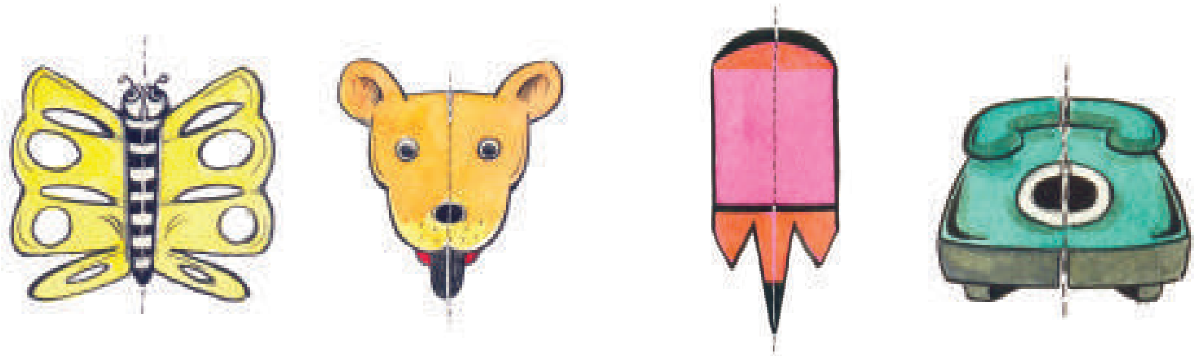
All these are symmetric shapes. When we see such shapes around us everyday then we say these are symmetric shapes.

Activity 1

Axis of symmetry

Observe the given figures. If we can fold any one of these figures in such a way that it's left half portion coincides with the other right half portion OR the upper half part coincides completely with the lower half then we say that these figures have a axis of symmetry. In such a case, both the halves are mirror images of each other.

See figures A. If it is folded along the broken line, then one part will completely hide the other part. Observe similar lines in the remaining figures.



(a) (b) (c) (d)

Figure 18.2

If we place a plane mirror at the line of fold then in symmetrical figures, the mirror image of one part of the figure will completely cover the other part. In these figures make a fold (using a real or imaginary line) and after placing a plane mirror on the broken line, observe the figures.

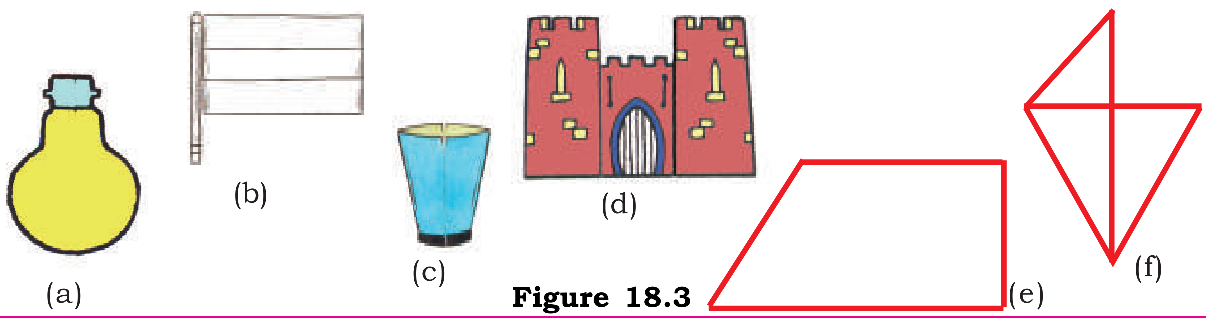
Is the image seen in the mirror same as the remaining part? This mirror line, is called symmetrical line or the axis of symmetry of the figure. According to Rohan all the figures drawn above are symmetric. Do you agree with him? Why?

Try to draw 5 symmetrical figures and draw their axis of symmetry.

Activity 2

Recognize the symmetrical figures:-

Which one of the figures given below are symmetric?



(a) (b) (c) (d) (e) (f)

Figure 18.3

How did you recognize the symmetrical figures?

Now, draw their axis of symmetry. Can you change non-symmetrical figures into symmetric figures by adding something? Choose one figure and think this over.

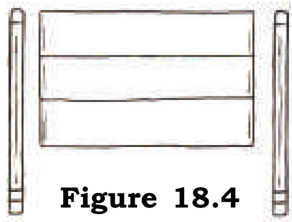


Figure 18.4

In symmetric figures, one half of the figure completely covers the other half on the axis of symmetry.

Figure (B) is not symmetric but if we add one more pole to it then the new structure will be symmetrical. In this figure, where is the axis of symmetry. Do the same with the remaining non-symmetric figures to make them symmetric.

Activity 3

Which of these letters are symmetrical?

Cut out the shapes of letters A,B,..., Y,Z from a thick paper. Take two boxes and paste the slip marked "Symmetrical" on one box and paste the other slip marked "Non-Symmetrical" on the second box.

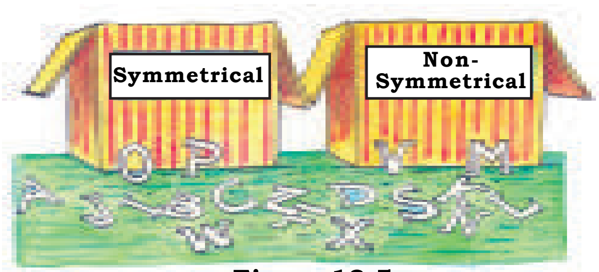


Figure 18.5

Now observe A,B,C,D....... one by one and check whether half part of the letter coincides completely with the remaining half part on the axis of symmetry
In which box, will you put the letter whose parts coincide with each other?
Which letters did you put in the symmetrical box? Which box has more letters?
Do the same practice for the letters क ख ग ....ह
Which of these letters are symmetric?

Activity 4

Another type of Symmetry

Take a paper and fold it into two equal parts. Put some ink or colour drops on one half. Fold the second part over the first and press it. What do you see?

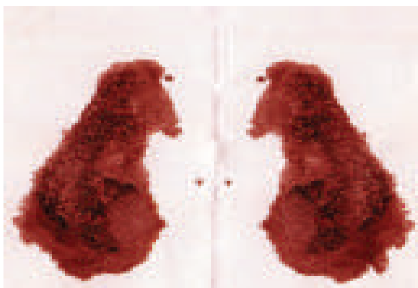


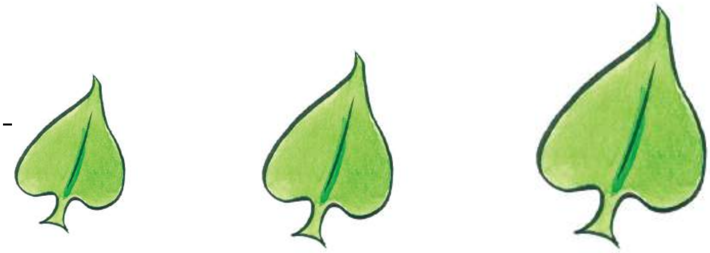
Figure 18.6

Is the obtained figure symmetric? If yes, then where is the line of it's symmetry? Is there any other line along which if the paper is folded, two similar parts would be obtained? Try to make some more symmetrical patterns of this type.

Observe the different objects available in your classroom. List the objects that have symmetrical shapes example the black board, top surface of a table, your notebook etc. Is the shape of the wing of the fan also symmetrical? After discussion, show your list to your teacher. After drawing all the symmetrical figures also draw a line of symmetry for each.

Activity 5

Now, look at the following figures:-



(a) (b) (c)

Figure 18.7

Are they symmetric?

You have seen the figures drawn on the walls of your home or on the walls of other houses located in your village. Draw similar figures in your notebook. Are these figures symmetric? Draw the axis of symmetry for each.

Activity 6

Recognize the symmetry:

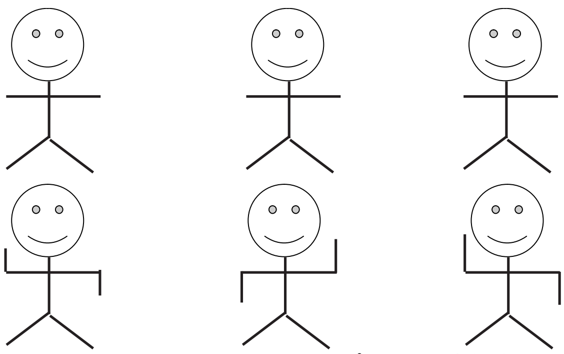
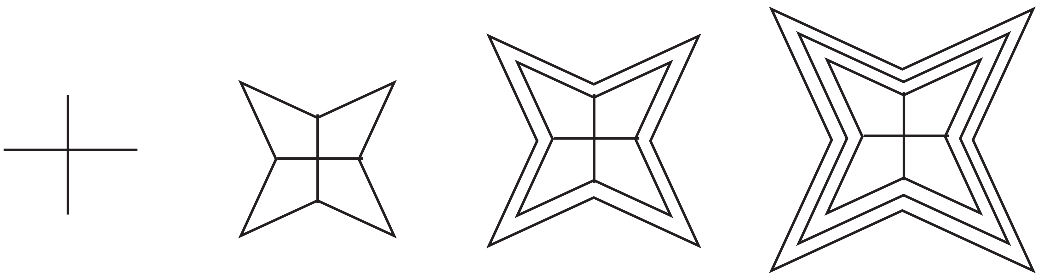


Figure 18.8

Which of the above figures are symmetrical? Convert the non-symmetrical figures into symmetrical ones.

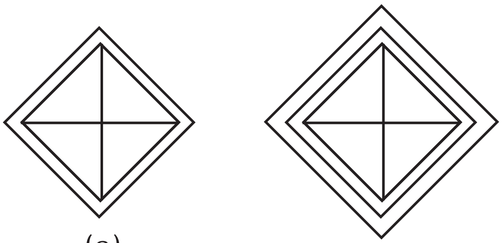
Have you ever made a 'Rangoli'? It involves kind of figures shown below.



(a) (b) (c) (d)

Figure 18.9

In different parts of these figures, different-different colours can be filled.



(a) (b)

Figure 18.10

These figures look beautiful after being coloured. Do they have a line of symmetry? Observe every shape. Does any of the shapes have more than one line of symmetry?

Activity 7

There are two set squares in your geometry box, one of them has angles 90°, 60° and 30°. Take two such set squares.

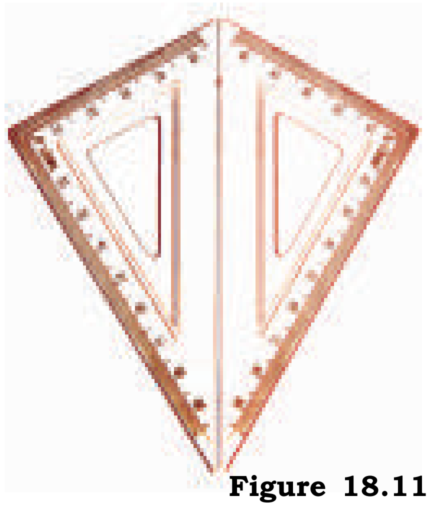


Figure 18.11

Join these two to make the shape of a kite as shown in the figure. How many lines of symmetry does this figure have?

In the same way take two set-squares of the other kind (involving 90°, 45° & 45°) and keep them along side each other as earlier.

What shape do you obtain?

How many lines of symmetry in this?

Think of more shapes having more than one line of symmetry.

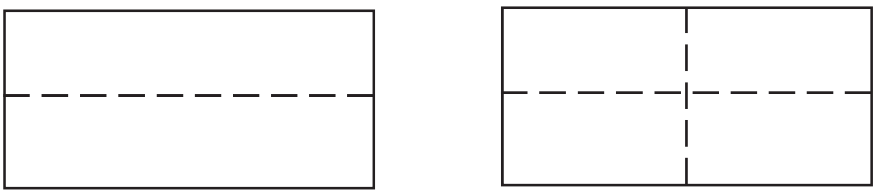
Activity 8

One Rectangle.

Take a postcard. Fold it along its length (Figure 18.12a) so that one part completely covers the other part. Is this fold line, a line of symmetry?

Give reasons for your answer.

Open this postcard and fold it again in the same way along its breadth (Figure 18.12b)



(a) (b)

Figure 18.12

Is the line of the second fold also, a line of symmetry?

Do you think there are only two lines of symmetry in this figure?

Now again think of the square formed above by using set-squares.

How many lines of symmetry does it have?

Activity 9

Mirror and symmetry:

A picture of an umbrella (Figure a) is shown below. In figure B, one half of the umbrella is shown placed in front of a plane mirror. Carefully observe the front half of the umbrella and its reflection in the mirror. Does, the figure of the umbrella look complete?

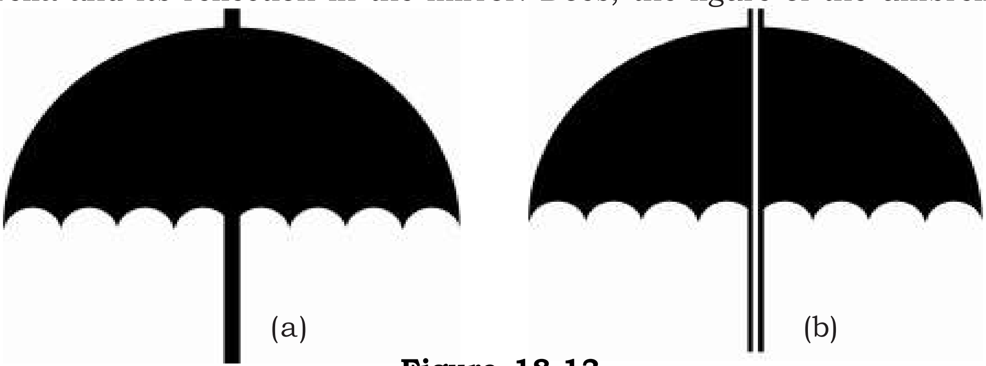


Figure 18.13

Activity 10

One half of a face is shown in the figure. Would you see the full face if you place a mirror along the line AB?

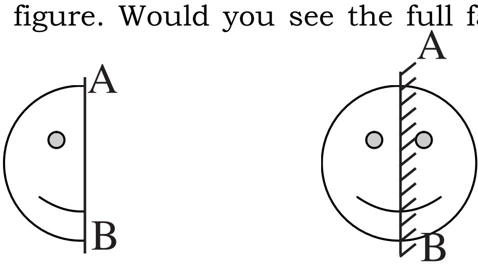


Figure 18.14

AB is the line of symmetry of the complete figure.

Activity 11

In which figures are both parts reflected when a plane mirror is placed on its line of symmetry.

Look at these figures and determine such position for the plane mirror from where the image and the object look the same.

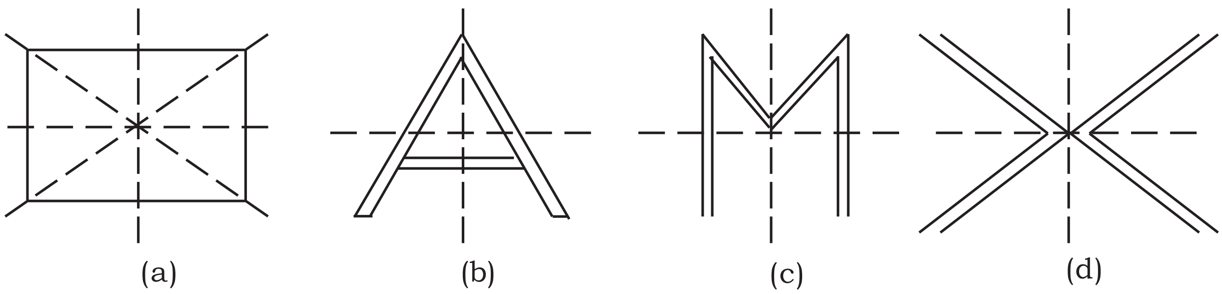


Figure 18.15

EXERCISE 18.1

Identify, which of the following figures are symmetric?

Find the lines of symmetry for these. For the symmetric figures draw the lines of symmetry and for each figure write the number of the lines of symmetry.

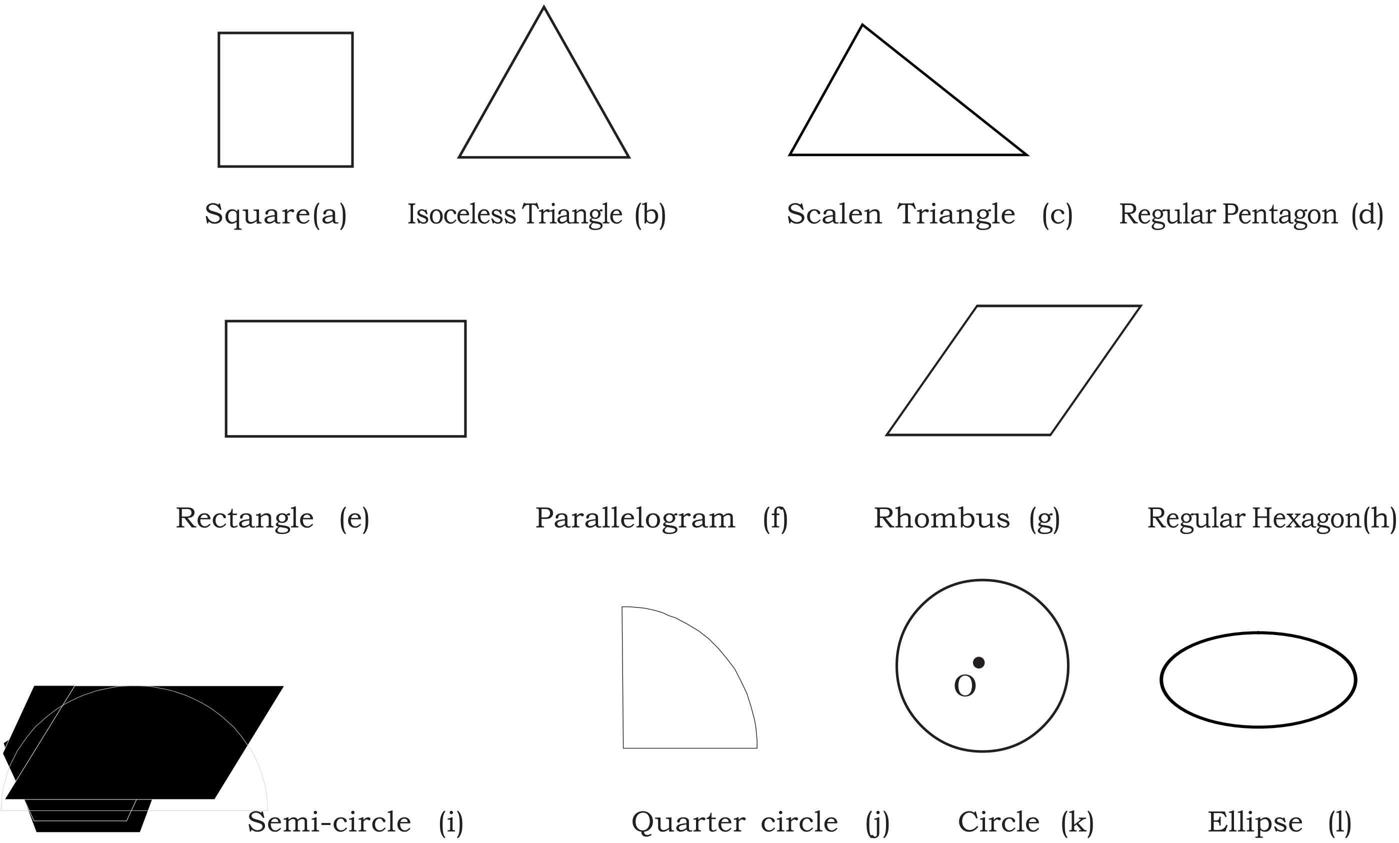


Figure 18.16

How many lines of symmetry are there in a circle?

A circle is symmetrical about each of its diameters. This means, on cutting the circle along any diameter, two equal parts are obtained.

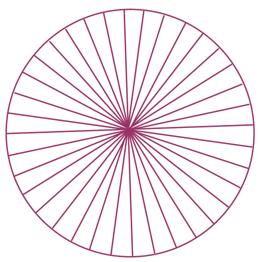


Figure 18.17

Activity 12

Take a square shaped paper. Fold it first from top to bottom and then from left to right. Now make a design on it, according to the given figure. Cut along the border of the figure made, open the paper out.

How many lines of symmetry are there?

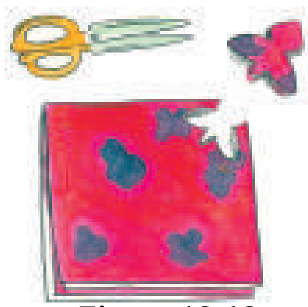


Figure 18.18

Activity 13

Many lines of symmetry.

Take 3 boxes. Now paste a paper strip on each and write '1 line of symmetry on first box '2 lines of symmetry on the second box, and '3 or more lines of symmetry' on the third box.

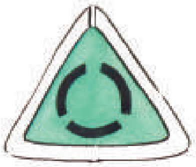
Observe the cut out shapes of letters A, B, C, ... Y, Z and determine the number of lines of symmetry for each. Put the letters having one line of symmetry in 1st box, 2 lines of symmetry in 2nd box and those with 3 or more lines of symmetry in the 3rd box. Discuss what you do with your classmates.

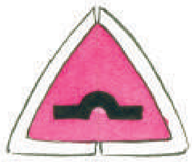
Can you say which English letter has the maximum lines of symmetry? Classify more figures and shapes on the basis of the number of lines of symmetry.

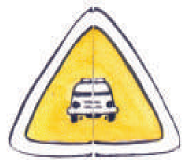
Figure 18.19

Symmetry all around:

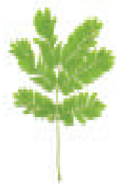
1. We observe different road signs and signals while traveling in a bus. Identify those shapes that have lines of symmetry and draw them in your notebook.


(i)


(ii)

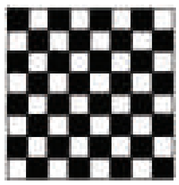

(iii)

2. Look at plants/leaves/petiole of leaves, do they have lines of symmetry?

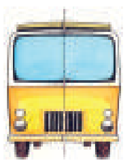


3. Are there any lines of symmetry in the given figure?

4. Are there lines of symmetry in the playing Cards? Classify them into groups of 'no line of symmetry', '1 line of symmetry', 2 lines of symmetry, 3 lines of symmetry and more lines of symmetry.



5. There are lines of symmetry in playgrounds and in play boards. List such play grounds and play boards and discuss them with your teacher.



6. There is symmetry in every kind of a vehicle. Like Buses, trucks etc.

MAKING SHAPES WITH PAPER

Take a rectangular coloured paper. Fold it many times and cut it as shown in the figure below. Now unfold the paper.

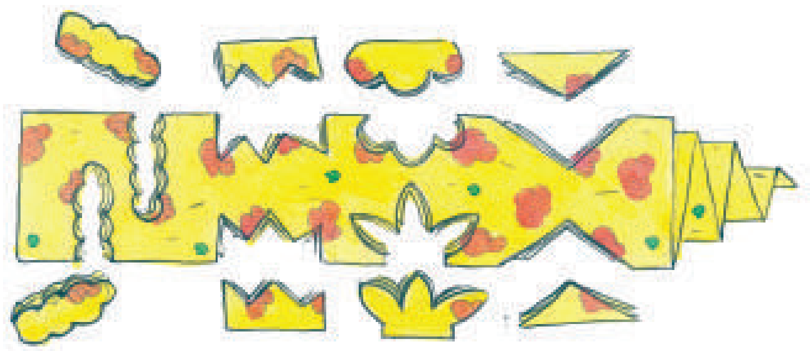


Figure 18.20

Place the shape on your notebook and fill different colours in it. Do you observe any symmetry?

RANGOLI

Have you ever made 'Rangoli' during the festivals? Have you noticed the symmetry in making these figures? Copy these rangoli patterns on paper and make an album of different patterns.

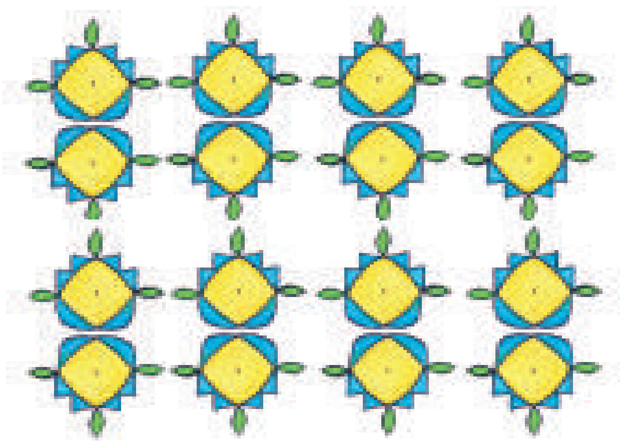


Figure 18.21

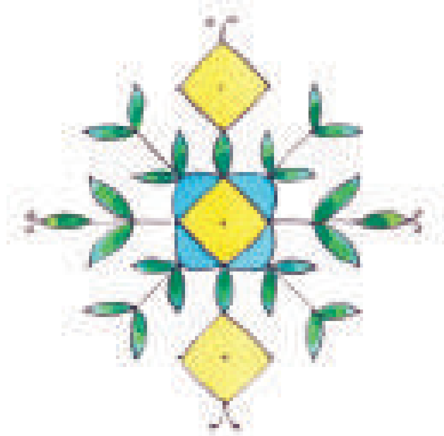


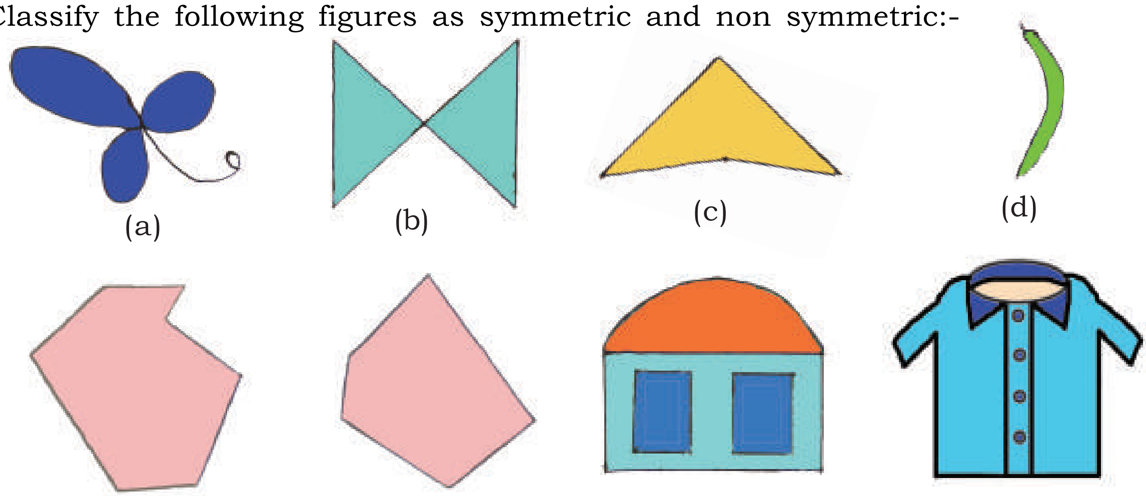
Figure 18.22

MEHANDI

You would have seen ladies applying 'mehandi' on their palms. Is there any symmetry in mehandi designs? Discuss it with the girls of your class.

EXERCISE 18.2

Q1. Classify the following figures as symmetric and non symmetric:-



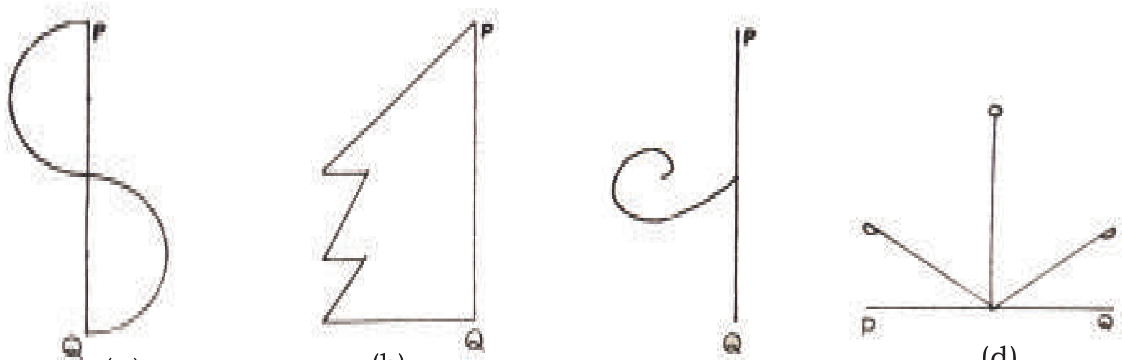
(a) (b) (c) (d) (e) (f) (g) (h)

Figure 18.23

Q2. List any 5 non symmetric figures around you, which have not appeared in this book.

Q3. Draw a line segment of 6 cm and mark a line of symmetry on it.

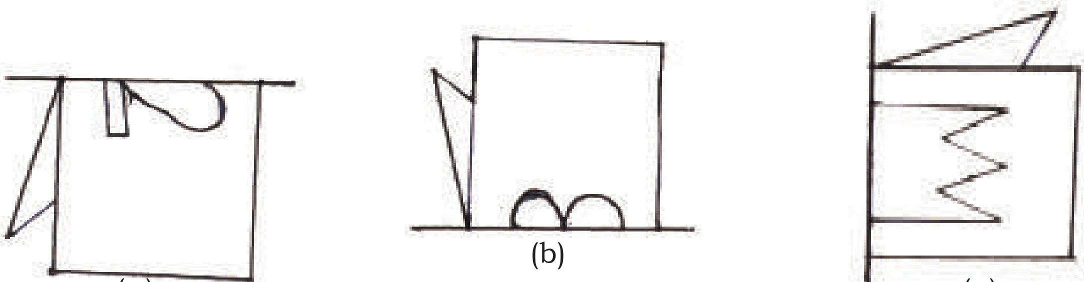
Q4. Complete the following figures. They have PQ as the line of symmetry.



(a) (b) (c) (d)

Figure 18.24

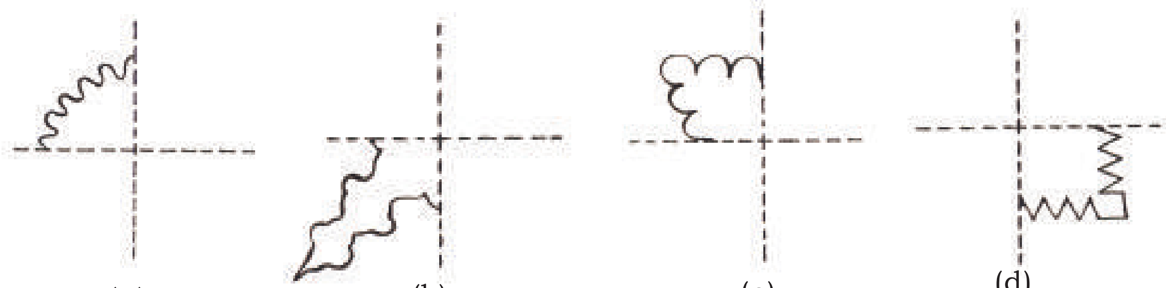
Q5. The shapes of some folded papers are drawn below. On their folds shapes are drawn. In each draw the complete shape that would be obtained when we cut along the design for each of the figures:-



(a) (b) (c)

Figure 18.25

Q6. If the following figure were drawn on one section of a 4 folded paper, then how would the full figure be? Think of the shapes and draw them in your notebook. If you cannot find the shapes by thinking then find them by paper cutting.



(a) (b) (c) (d)

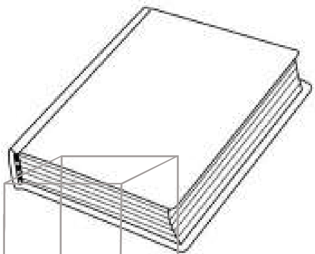
Figure 18.26

Q8. Write down in your notebook the numbers upto 100 which are symmetrical?

Three Dimensional Shapes

In our daily life we see some solid object which are not plane.

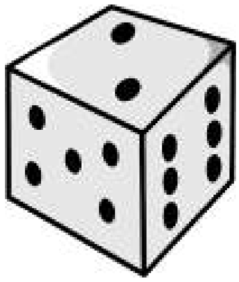
Cane



Book : Shape of cuboid

Icecream – Shape of cone

Ball : Sphere



Dice : Shape of cube

Figure 18.27

faces, Edges and vertices

We can easily identity the faces ,edges and vertices in three dimension shapes.

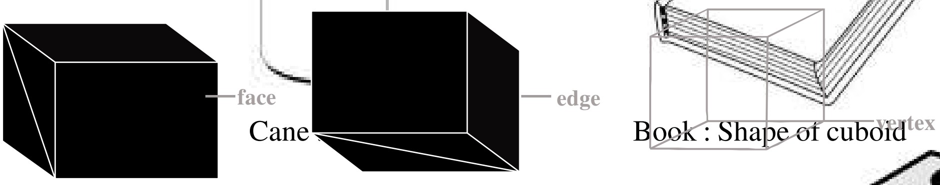


Figure 18.28

For example, take a cuboid. The each upper face of cuboid is a rectangle itself. The two faces of the cuboid meet at a line segment which is known as edge of cuboid. The three adjacent edges of cuboid meets at a point, which is called as vertex.

In this manner a cuboid have 6 rectangular faces, 12 edges and 8 verttices.

Activity 13

1. Match the following –

(i)	Cone	(i)	
(ii)	Sphere	(ii)	
(iii)	Cylinder	(iii)	
(iv)	Cube	(iv)	
(v)	Cuboid	(v)	

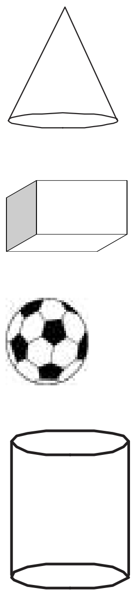
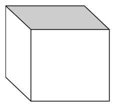
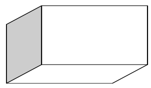
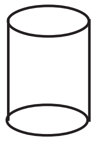
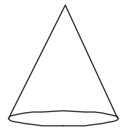


Fig. 18.29

2. Identify the shape of object-
   - (i) Chalk box
   - (ii) Tennis Ball
   - (iii) Pipe
   - (iv) Cap of joker
   - (v) Dice
3. Name four objects whose shape is similar to cuboid.
4. Name three objects whose shape is similar to cylinder.

5. In the table given below, write down the number of faces, edges and vertices.

Table

Shape						
faces	Plane					
	Curve					
Edge	Plane					
	Curve					
vertices						

We have learnt

1. We see flowers, beautiful paintings, buildings and other objects, many of things are symmetric.
2. Objects look beautiful if they have symmetry.
3. So many three dimensional figures are there around us. Some of them are cube, cuboid, sphere, cylinder and cone.

Vedic Ganit

Some new methods of multiplications

In the last classes you have practiced some method of Vedic mathematics. With the help of these you learn to multiply and know how these works.

FORMULA - 'EKADHIKAIN PURVAN AND ANTYAYORDASHAKAPI' formula for multiplication.

This method is use when the sum of ones place number of multiplicand and multiplier is 10 and tens place digits are same.

For example - 15×15 16×14 27×23 36×34

Solve One Example - 24×26

Multipliction of Ones - $4 \times 6 = 24$

Multipliction of Tens - $2 \times (2 + 1)$ [Tens $\times$ One more than Tens]

$= 2 \times 3 = 6$

Total Muliplication - $= 624$

See One More Example - 52×58

	Th H	T O
Multipliction =	(5×6)	(2×8) = 3016
	(Tens x One more than Tens)	(Multiplication of units)

Understand how this happens -

Take two numbers of two digits in which tens place number is x and ones are y and z respectively. These two numbers are xy and xz. Here $y + z = 10$

TENS	ONES	
x	y	The values of these numbers are $10x + y$
x	z	and $10x + z$ respectively.

After multiplication -

$$(10x + y)(10x + z) = 100x^2 + 10xz + 10xy + yz$$
$$= 100x^2 + 10x(y + z) + yz$$
$$= 100x^2 + 10x \times 10 + yz \qquad (y+z = 10)$$
$$= 100x^2 + 100x + yz$$
$$= 100x(x + 1) + yz$$
$$= x(x + 1) \times 100 + yz$$

The term of left side has 100 as multiple. Therefore x(x+1) has hundreds place number (or if necessary on thousands place also) we place it. The multiple of y and z is put on ones and tens position. If the value of yz has 1 and 9 then its multiplication will write as 09.

Is this method applicable in the multiplication of two numbers having three digits? Come and think about 317 x 313. Here the sum of ones digits is 10. (7 + 3 = 10). Both numbers have 31 tens and it means tens and hundreds respectively.

		Ten Th.	Th.	Hund.	Tens	Ones
Multiplication 317 × 313	=	(31	×	32)	(7 ×	3)
	=		9 9	2	2	1
	=		99221			

See one more example -

124 × 126	=	Ten Th.	Th.	Hund.	Tens	Ones
	=	(12	×	13)	(4 × 6)	
	=	1	5	6	2	4
	=	15624				

(Since multiplication is more than 100 × 100 that's why the number of result is more than ten thousand)

URDHVTIRYGBHYAM METHOD FOR MULTIPLICATION -

If we remember how many ones, tens, hundred etc. get for multiplying two numbers and we place them in proper place, than multiplication is easy for us.

Understand with one example -

32×14 = Multiply 4 ones by 32 the result is 8 ones and 12 tens.

Again, multiply 1 tens by 32 the result is 2 tens and 3 hundreds.

So, multiplication = 3 hundreds + 2 tens + 12 tens + 8 ones

= 3 hundreds + 14 tens + 8 ones

= 3 hundreds + 1 hundreds + 4 tens + 8 ones

= 4 hundreds + 4 tens + 8 ones

= 448

See in the pictorial from

3 2

×

1 4

Hundreds	Tens	Ones
3×1	$(2 \times 1) + (3 \times 4)$	2×4
3	2 + 12	8
3 ←	① 4	8
4	4	8

How we multiply two numbers having three digits in each number?

Here the result of multiplication is about ten thousand.

	1	4	7
×	2	6	5

	Ten Th.	Thousand	Hundred	Tens	Ones
I	1 2	1 4 2 6	1 4 7 2 6 5	4 7 6 5	7 5
II	2 × 1	1 × 6 + 2 × 4	1 × 5 + 2 × 7 + 4 × 6	4 × 5 + 6 × 7	7 × 5
III	2	6 +8	5 +14 +24	20 +42	35
IV	2 ←	(1) 4 ←	(4) 3 ←	(6) 2 ←	(3)5
V	3	8	9	5	3

While seeing this method it looks long but after some exercise we write the answer directly.

Solve one more example -

143 × 25

Here multiplier 25, we can write as 025 -

143
× 025

143
× 25
3575

Ten Th.	Thousand	Hundred	Tens	Ones
1 0	1 4 0 2	1 4 3 0 2 5	4 3 2 5	3 5
0	2 3	13 5	26 7	1 5

USE OF 'EK NUENAN PURVEN' FORMULA FOR MULTIPLICATION

You have learned the use of 'EK NUENAN PURVEN' formula for multiplication in class 6th. You remembered that this method is use only when one of the numbers is 9. Once again see an example.

Example - (1) Solve 17 x 99

	Th.	H.	T.	O.
17 × 99 =	(17 - 1)		9	9
		-	1	6
	1	6	8	3

17 × 99 = 1683

17 × 99	=	17 × (100 - 1)
	=	1700 - 17
	=	1600 + (100 - 17)
	=	1600 + (99 - 16)
	=	1600 + 83
	=	1683

Example - (2) Solve 275 × 999

	L.	T.Th.	Th.	H.	T.	O.
275 × 999 =	(275 - 1)			9	9	9
			-	2	7	4
	2	7	4	7	2	5

275 × 999 = 274725

Example - (3) 110 × 999

= (110 - 1) 999

- 109

= 109 890

= 109890

If 9 is less in number in multiplier then - (as 318 x 99, 231 x 99 etc.)

Let solve and see -

(i) 318 × 99 =

	T. Th.	Th.	H.	T.	O.
	(318 – 1)			9	9
	–		3	1	7
=	3	1	7	9	9
			– 3	1	7
=	3	1	4	8	2

(ii) 213 × 99 =

T. Th.	Th.	H.	T.	O.
(213 – 1)			9	9
–		2	1	2
2	1	0	8	7

If 9 is more in number in multiplier then - (as 5 × 99, 87 × 999 etc.)

Let solve and see -

(i) 5 × 99 = 05 × 99 =

H.	T.	O.
(5 – 1)	9	9
	0	4
4	9	5

(ii) 87 × 999 =

T. Th.	Th.	H.	T.	O.
(87 - 1)		9	9	9
		0	8	6
8	6	9	1	3

USE OF BIJANK TO CHECK THE ANSWER -

In last class you have learn the use of BIJANK to check the multiplication. In the case of multiplication we can say that.

Bijank of Multiplicand × Bijank of Multiplier = Bijank of Product

Example - (1)

$24 \times 26 = 624$

Bijank of multiplicand	$24 = 2 + 4 = 6$
Bijank of multiplier	$26 = 2 + 8 = 8$
Product of bijank	$6 \times 8 = 48$
Bijank of	$48 = 4 + 8 = 12,\ 1 + 2 = 3$
Bijank of product	$624 = 6 + 2 + 4 = 12, \quad 1 + 2 = 3$

Since both Bijank are equal.

24 x 26 = 624 is the correct answer.

Example - (2)

$317 \times 313 = 99221$

Bijank of multiplicand	$317 = 3 + 1 + 7 \implies 1 + 1 = 2$
Bijank of multiplier	$313 = 3 + 1 + 3 = 7$
	$2 \times 7 = 14,\ 1 + 4 = 5$
Bijank of product	$99221 = 9 + 9 + 2 + 2 + 1 = 23,\ 2 + 3 = 5$

Since both Bijank are equal.

Therefore $317 \times 313 = 99221$ is the correct answer.

Exercise

Solve by using the above suitable method and check the answer -

(i)	25×29	(ii)	17×9	(iii)	387×999
(iv)	211×99	(v)	84×999	(vi)	203×99
(vii)	98×92	(viii)	143×147	(ix)	74×76
(x)	432×438	(xi)	36×45	(xii)	107×234
(xiii)	201×104	(xiv)	123×45	(xv)	28×317

Answers

Exercise 1

1	(i) >,	(ii) >	(iii) =	(iv) >	(v) <
2	(i) –160	(ii) 0	(iii) –756	(iv) 2625	(v) 6000 (vi) –2880
3	(i) –5	(ii) 3	(iii) –50	(iv) 10	(v) –16
	(vi) 170	(vii) 0	(viii) –321	(ix) –1	(x) –20
4	(i) =	(ii) >	(iii) =	(iv) >	(v) =
	(vi) >	(vii) =	(viii) >	(ix) =	(x) <
5	(i) $\frac{3}{7}$	(ii) $\frac{3}{4}$	(iii) 1	(iv) $\frac{5}{12}$	(v) $\frac{6}{5}$
	(vi) $\frac{2}{3}$				

6 $\frac{3}{4}$

7 18

8. Rs. 275

9. Rs.30000 , Rs.15000, Rs.15000

Exercise 2.1

1. $\frac{4}{1}, \frac{-3}{7}, -27, \frac{-3}{-5}$

2. $\frac{-38}{1}, \frac{17}{1}, \frac{0}{1}, \frac{-100}{1}, \frac{79}{1}$

3. $\frac{1}{5} = \frac{2}{10} = \frac{3}{15} = \frac{4}{20}$

$\frac{-3}{4} = \frac{-6}{8} = \frac{-9}{12} = \frac{-12}{16}$

$\frac{-5}{8} = \frac{-10}{16} = \frac{-15}{24} = \frac{-20}{32}$

$\frac{6}{11} = \frac{12}{22} = \frac{18}{33} = \frac{24}{44}$

$\frac{4}{3} = \frac{8}{6} = \frac{12}{9} = \frac{16}{12}$

4. $\frac{5}{8}, \frac{-4}{9}, \frac{1}{3}, \frac{-1}{2}, \frac{-7}{10}$

5. (i) $\frac{4}{12}, \frac{8}{24}, \frac{1}{3}, \frac{25}{75}$ (ii) $\frac{-3}{5}, \frac{-6}{10}, \frac{-15}{25}, \frac{-27}{45}$

6. No

7. (i) $-\frac{6}{16}$ (ii) $\frac{12}{-32}$ (iii) $\frac{9}{-24}$ (iv) $\frac{12}{-32}$

8. (i) -15 (ii) 12 (iii) 15 (iv) 3 (v) 8

Exercise 2.2

3. (i) > (ii) < (iii) = (iv) > (v) >

4. (i) $\frac{7}{13}$ (ii) $\frac{2}{5}$ (iii) $\frac{-21}{20}$ (iv) $\frac{7}{9}$ (v) 0

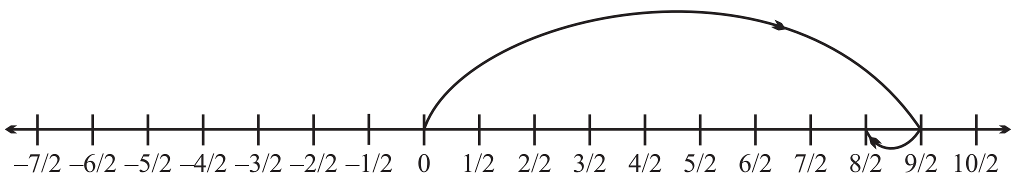
5. (i) $\frac{13}{3}$ (ii) $\frac{-7}{3}$ (iii) $\frac{-17}{11}$ (iv) $\frac{-17}{11}$

6. $\frac{-4}{12} < \frac{-5}{18} < \frac{-9}{-27} = \frac{2}{6}$ 7. $\frac{2}{21} > \frac{1}{28} > \frac{-5}{14} > \frac{-8}{7}$

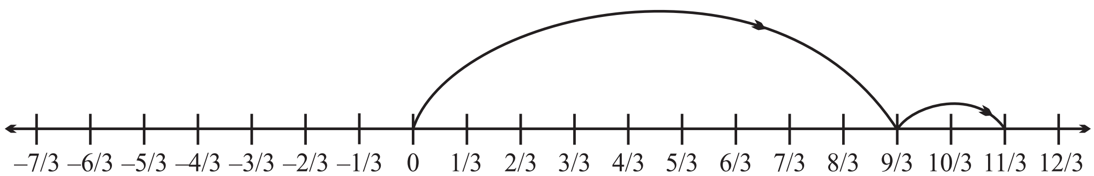
8. (i) False (ii) True (iii) False (iv) True

Exercise 2.3

1. Equal time 2. Yes

3.


He is $\frac{8}{2} = 4$ kilometers away from school

4.


He is $\frac{11}{3}$ kilometers away from school

Exercise 3.1

1. (i) Equal (ii) Isosceles (iii) 60 (iv) 35° (v) Big (vi) Short
2. (i) $\angle C=50° \ \angle C = 80°$ (ii) $\angle R=45° \ \angle P = 90°$ (iii) $\angle E = \angle F = 48°$ (iv) $\angle L = \angle M = \angle N = 60°$
3. XY and YZ, $\angle Y = 100°$
4. (i) No, Because thier opposit angles are not equal.
   (ii) AC > AB.
   (iii) Opposit of biggest angle.
5. $\angle Q = 28°, \angle P = 124°$
6. 30° and 120°
7. 55°
8. $x = 45°$
9. $\angle A = 36°, \angle B = \angle C = 72°$
10. PQ and RQ, Biggest side = PR
11. 40°, 60°, 80°

Exercise 3.2

1. (i) Mid point (ii) Altitude (iii) Concurrent (iv) Centroid (v) Orthocentre (vi) 2:1

Exercise 4.1

1. (i) 2 (ii) 9 (iii) 2 (iv) 5
2. (i) 4 (ii) $\frac{-17}{4}$ (iii) -3 (iv) $\frac{18}{5}$ (v) 2 (vi) 34 (vii) $\frac{1}{4}$ (viii) 5 (ix) 2

Exercise 4.2

1. (i) $\frac{2}{3}x = 24$ (ii) $2x + x = 51$ (iii) $\frac{x}{10} = 2500$ (iv) $x+(x+1) = 15$ (v) $\frac{x}{x+5} = \frac{19}{24}$

(2) 4 (3) Rs. 100, Rs. 200 (4) 14 (5) =9 cm, b=6cm (6) = 18 cm, b= 27 cm (7) Boys = 25 and girls = 10 (8) 32 (9) 17 and 18 (10)12 and 36, (11) 275m;100m (12) (i) $\angle A = 69°, \angle B=75°, \angle C=45°$ (ii) 120°, (iii) 30°

Exercise 5.1

1- (i) (10 -2) ÷ 30 (ii) (12 -5) × 27 (iii) (4.5+ 2.3) ÷ 3.8
   (iv) $\frac{8}{27} \div \left(\frac{2}{3} + \frac{7}{15}\right)$

2- (i) (15+27) × (8-6) (ii) (37×28) + (11 ÷29)
   (iii) (8.45 - 6.75)× (3.2+2.4) (iv) 2(5+11) - (8-3)
   (v) $\left(\frac{4}{27} + \frac{5}{9}\right) \div \frac{7}{8}$ (vi) (5 + 10) + (7-3) + (8 × 25)

Exercise 5.2

1. $6a + 20b$ 2. $3a - 6b$ 3. $2x$ 4. $2x+10$

5. $30 - 60x + 30y$ 6. 34.5 7. 8.96 8. $23a + 18b - 9$ 9. $1\frac{1}{4}$

Exercise 5.3

1. (i) 2 (ii) −11 (iii) 5 (iv) 15 (v) 16 (vi) 38 (vii) 21

2. (i) $-4x$ (ii) 0 (iii) 18 (iv) 507 (v) $6a^2 - 2a$ (vi) $3\frac{3}{4}$ (vii) $3a - b$

3. (i) False (ii) True (iii) True (iv) False

4. (i) 35047 (ii) 42042 (iii) 8.7 (iv) 43.2 (5) Rs. 35 (6) Rs. 30 (7) Rs. 21

Exercise 6.1

(1) (a) 3^5 (b) 5^8 (c) a^7 (d) b^4

(2) (a) $3^4 \times 5^4$ (b) $2^5 \times 3^5 \times 7^5$ (c) $3^3 \times 17^3$

(d) $3^m \times 7^m$ (e) $5^6 \times 13^6$

(4) (a) 42^8 (b) $(ab)^3$ (c) $(pqr)^9$ (d) $(abcd)^n$

(5) (a) True (b) True (c) False (d) True (e) True

Exercise 6.2

(1) (a) 6^2 (b) 27^6 (c) 13^{m-n} (d) $(mn)^5$ (e) x^7

(3) (a) $\frac{1}{12^3}$ (b) $\frac{1}{19^5}$ (c) $\frac{1}{3^4}$ (d) $\frac{1}{5^3}$

(4) (a) 35^{-4} (b) x^{-5}

(5) (a) $\frac{1}{3^9}$ (b) $\frac{1}{a^{-m}b^m}$ (c) $\frac{1}{p^9}$

(6) (a) 1 (b) 1 (c) 1

(7) (i) $x = 2$ (ii) $x = 3$ (iii) $x = 0$ (iv) $x = -7$ (v) $x = -3$ (vi) $x = 1$. (8) 1.25×10^{-2}

Exercise 8.1

1. (i) $\angle X$ (ii) $\angle B$ (iii) $\angle C$

(iv) XY v) BC (vi) AB

2. XY=4.5 cm OX=3.7 cm, OY=2.9 cm, $\angle XOY = \angle MON = 70°$, $\angle Y = 60°$, $\angle X = 50°$

3. (i) True (ii) False (iii) True

(iv) False (v) True (vi) True (vii) False

Exercise 8.2

1. (ii) Yes, S-A-S (iii) Yes, Right angle hypotenuse side

(iv) Yes, A-S-A (v) Yes, A-S-A (vi) Right angle hypotenuse side

(vii) S-S-S

2. (i) Right angle hypotenuse side (ii) S-S-S (iii) Right angle hypotenuse side

(iv) No

3. S-A-S

4. (i) True (ii) False (iii) False

5. ΔPQR≅ΔSQR right angle hypotenuse side 6. (ii) 7. S-S-S

Exercise 9.1

1. (a) 9pq (b) 6xy (c) 10x + 10y

(d) 3xy + 11 y + 3z (e) x + y – z (f) 4x + 18y + 12z – 12

(g) 4x – xy – 4y + 2 (h) $-4x^2y^2 + 3x^2 - 5x$

2. (a) 5x (b) 16x (c) –10x

(d) 3x (e) $4x^2 + x + 9$ (f) 2x – 8xy + 3z

(g) xy – 7a – 6b – 3ab

3. (i) $-ab - 11b^2c - bc^2$ (ii) $-4m^2 - 4nm - 3n^2$

4. 11x + 13y

5. $5x^2 + 2x + 5$.

[illegible] + xz

$\frac{5}{2}m^2 + \frac{[illegible]}{2}mn$

Exercise 9.2

1. (i) 2 z (ii) 12ax (iii)

(iv) (v) $(-3x^2y).y^2$, $(2z).y^2$ - $3x^2y^3$, zy^2z

(vi) 5x, $7y^2z^3$

2. (i) $7xy + 8x^2y$ (ii) $6r^2t^2 - 10st^2$ (iii)

(iv) $-8ab + 12ac$ (v) $\frac{8}{3}ab^2 + \frac{2}{3}ac$

Exercise 10

Q. 2. 4^{th}, 2^{nd}, 3^{rd}, 1^{st}, 1^{st}, 4^{th}, 1^{st} Q. 3. Yes

Q. 4. (i) (7, 9), (7, 7), (9, 5) (ii) (2, 8), (2, 5), (5, 5)

ST = 3 cm, TU = 3 cm, SU = cm

(iii) (1, 1), (6, 1), (8, 4), (3, 4)

AB = 5 cm, DC = 5 cm

(iv) (9, 2), Diameter = 2 cm

Exercise 11.1

(1) $\frac{4}{5}, \frac{7}{50}$ Terminating, $\frac{8}{7}, \frac{-15}{49}, \frac{3}{28}$ Non-terminating

(2) $\frac{3}{5} = 0.6$, $\frac{4}{25} = 0.16$, $\frac{7}{10} = 0.7$, $\frac{-13}{125} = -0.104$ $\frac{9}{40} = 0.225$

(3) $\frac{2}{3} = 0.\overline{6}$, $\frac{-5}{6} = -0.8\overline{3}$, $\frac{8}{15} = 0.5\overline{3}$ $\frac{3}{11} = 0.\overline{27}$, $\frac{19}{45} = 0.4\overline{2}$

Exercise 11.2

(1) (a) $\frac{1}{5}$ (b) $\frac{11}{20}$ (c) $\frac{25}{4}$ (d) $\frac{87}{40}$ (e) $\frac{1453}{100}$

(2) (a) $\frac{4}{9}$ (b) $\frac{718}{99}$ (c) $\frac{17}{300}$ (d) $\frac{5}{18}$ (e) $\frac{6}{11}$

Exercise 11.3

(1) (i) 1.3297 (ii) 0.44224 (iii) 13.8213 (iv) 10.4904

(2) (i) 5.566 (ii) 5.00271 (iii) 10.910 (iv) 9.20093

(3) (i) 36.45 (ii) 0.018459 (iii) 0.0001 (iv) 29.173632

(4) (i) 1.5 (ii) 170.06 (iii) 0.219 (iv) 30.15

(5) Rs. 44.04 (6) 365.58 (7) Rs. 493800

(8) 80.96 m^3

Exercise 12.1

2. (i) 50° (ii) 40° (iii) 30° (iv) 15°
   (v) 90° (vi) 20^0

3. (i) 70° (ii) 110° (iii) 180° (iv) 60°
   (v) 135° (vi) 130^0

4. 60°, 30°

5. 60°, 120°

6. $\angle SOZ = 40°$, $\angle XOS = 140°$

7. Angles of Linear pairs

8. (i) 145º (ii) 75º (iii) 108º (iv) 40º (v) 55º (vi) 126º

9. (i) 30º (ii) 140º

10. (i) 30º, 150º, 30º (ii) 140º,40º,140º

11. (i) 45º (ii) 120º (iii) 55º (iv) 125º

Exercise 12.2

1. (i) Parallel (ii) Equal (iii) 127^0 (iv) 92.5^0 (v) Concurrent

2. (i) True (ii) False (iii) True (iv) True

3. (i) $\angle 1$ and $\angle 7$, $\angle 4 = \angle 6$, $\angle 2$ and $\angle 8$, $\angle 3 = \angle 5$ (ii) $\angle 1$, $\angle 4$, $\angle 6$ and $\angle 7$

   (iii) $\angle 2$, $\angle 3$, $\angle 5$, $\angle 8$ (iv) $\angle 1$ and $\angle 5$, $\angle 4$ and $\angle 8$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$

   (v) $\angle 2$,oa $\angle 5$, $\angle 3$,oa $\angle 8$ (vi) $\angle 1 = 70°$, $\angle 7 = 70°$, $\angle 3 = 70°$, $\angle 2 = 110°$, $\angle 4 = 110°$, $\angle 6 = 110°$, $\angle 8 = 110°$

4. (a) DE || AC and transversal lines AB and BC

   (b) AB || EC and transversal lines AC and BD

5. $\angle ABC=70°$ $\angle ACB=25^0$

6. $x = 60°$, $y = 50°$ 7. $X=70°$, $y= 110^0$, $z=70°$

8. b=130°, a=50°, d=130°   9. $x=60°$, $y=120°$

10. $x= 45°$ since q is transverval line   11. $\angle 1+\angle 5= 180°$ (Pair of interior angle $\angle 3+\angle 5= 180°$ ($\angle 1= \angle 3$,$\angle 9 = 90°$ ($\angle 9= \angle 11$, Corresponding angles)

    $\angle 10 = 90°$ ($\angle 9= \angle 10$, Opposite angle)

12. $y= 60°$, $\angle 2 = 120°$, $\angle 1 = 150°$, $z= 270°$

13. $x= 27°$ 14. $y= 80°$ 15. a= 40°, b=40°, c=40°, d=40°, e=40°

Exercise 13.1

(1) (i) Two (ii) Triangles (iii) 360° (iv) Two (v) Four, Three

(2) (i) No (ii) Yes (iii) Yes (iv) Yes

(3) 115° (4) 110° (5) 90° (6) 95°

(7) 45°, 75°, 105° and 135°

(8) (i) True (ii) False (iii) True (iv) False (v) False (9) 120°

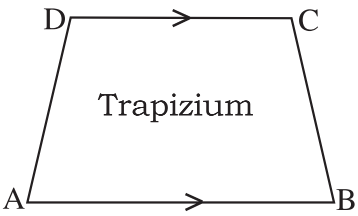
Exercise 13.2

(1) (i) Trapizium (ii) 90° (iii) Parallel, eqaul (iv) Square

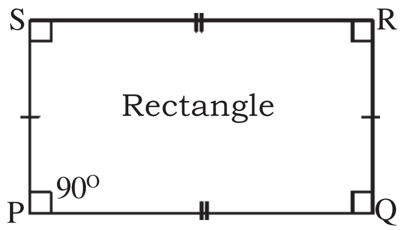
(v) Rhombus

(2) (i) True (ii) False (iii) False (iv) True (v) False

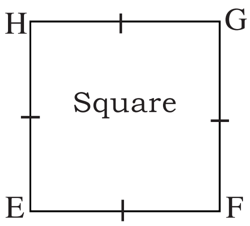
(3) (i)



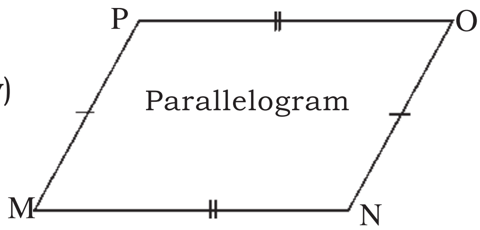
(ii)



(iii)



(iv)



Exercise 14.1

1- (i), (ii), (vi), (vii), (ix), (xi)

2. Proportional (i), (ii), (iv), (ix), (x)

(iii) 4 : 6 :: 12 : 18 (v) 11 : 22 :: 44 : 88

(vi) 1 : 4 :: 2 : 8 (vii) 15 : 25 :: 3 : 5 (viii) 34 : 68 :: 112 : 224

(xi) Cannot be arranged to have a relationship of proportionality.

3. (i) 64 (ii) 220 metre (iii) 9 hours

4. Rs. 135 5- 100 metre 6. 12 copies

Exercise 14.2

1. Rs. 38.50 2. (i) In 8 hours (ii) 357.50 Kilometre

3. (i) 10 Kilograms (ii) 48 Kilograms 4. Rs.1800 5. Rs. 640

6.

No. of Books	Price (in rupees)
50	2500
75	3750
2	100
60	3000

Exercise 15.1

(1) (i) 15 sq.cm (ii) 7 sq.cm

(2) (i) 25 sq.cm (ii) 49 sq.cm

(3) 10,000 sq.cm

(4) 1 sq.m

(5) 10 metre

(6) 10 pieces

(7) Rs. 12,000

(8) (a) Four times (b) 9 times

(9) Rs. 25,00 (10) Rs. 24,000

Exercise 15.2

1. (i) Approximate 28.29 sq.cm (ii) 154 sq.cm (iii) 616 sq.cm

2- (i) Approximate 50.29 sq.cm (ii) 314.29 sq.cm (iii) 154 sq.cm

Exercise 16.1

1- (a) , 0.25 and 25% (b) , 0.5 and 50% (c) , 0.30 and 30%

(d) , 0.125 and 12.5% (e) , 0.6 and 60% (f) , 0.50 and 50%

(g) and 100%

2. (a) , 25% (b) 0.25, 25% (c) Per hundred (d) , 0.75 (e)

3. 4. 5. 6. , 0.25

Exercise 16.2

Q. 1 (iii) loss = Rs. 21 loss % = 3% (iv) profit = Rs. 24 profit % = 8%

(v) loss = Rs. 22 loss % = 20%

2 (ii) S. P. = Rs. 340, profit % = $13\frac{1}{3}$ % (iii) S. P. = Rs. 525, profit % = 5%

(iv) S. P. = Rs. 1120, $6\frac{2}{3}$% loss (v) Rs. 360, 10% loss

3. (ii) Cost Price = Rs.180, loss% = (iii) Cost Price = Rs. 1120,

profit= 25% (iv) Cost Price = Rs. 1000, loss = 5%

(v) Cost Price= Rs.350, profit= $7\frac{1}{7}$%

4. 25% 5. 10% 6. $3\frac{19}{27}$% 7. Rs. 600

8. Rs. 16.32 per dozen 9. Rs. 165 10. Rs. 5775 11. Rs. 1800

12. Rs. 2079 13. 25% 14. loss $7\frac{7}{19}$% 15. Rs. 1035

Exercise 16.3

16

1(i) Rs. 150 (ii) Rs. 750 (iii) Rs. 1000 (iv) Rs. 1450.75 (v) Rs. 1145.25

2(i) Rs. 720 (ii) Rs. 148.50 (iii) Rs. 93 (iv) Rs. 42.02 (v) Rs. 104

3(i) Rs. 3078 (ii) Rs. 4400 (iii) 1539 (iv) Rs. 1048.33

Exercise 16.4

1. (i) 3½ year (ii) Rate 7% (iii) Principal = Rs. 150 (iv) Principal = Rs. 576

   (v) Time 1 year 8 months
2. Rs. 500
3. Rate 7.2%
4. 8%
5. Rs. 900
6. 6 years
7. 7%

8. 1¼ years or 1 year 3 months

9. 13% 10. 10 years

Exercise 17.1

Q. 1

No.	Tally sign	Frequency
0	II	2
1	II	2
2	卌	5
3	卌	5
4	III	3
5	III	3

Q. 2

Temperature	Tally sign	Frequency
37.8	II	2
37.9	III	3
38.0	II	2
38.1	III	3
38.2	III	3
38.3	II	2

Q. 3. (a) Second class (b) 40 (c) 36

Exercise 17.2

5. (1) Hindi—37000, Bangla—12000, Urdu—15000, Marathi—10000, Panjabi—5000, English—20000

(2) 5000

(3) Panjabi

(4) Panjabi—5000, Marathi—10000, Bangla—12000, Urdu—15000, English—20000, Hindi—37000

Exercise 17.3

1. 72 2. 60 3. 8

4. 30 kg. 5. 67 6. 5 7. 3

8. 5 9. $x = 8$